

# Higgs-like scalars from non-perturbative Yang-Mills dynamics

V.I. Zakharov ( MPI, Munich & ITEP, Moscow)  
based on collaboration with H. Verschelde (Gent)

Shoichi Sakata Centennial Symposium  
Nagoya University, 27-28 Oct 2011

28 октября 2011 г.

# Motivation

Conference is focused on Higgs fields, for obvious reasons

Also very appropriate in view of the role of Sakata in solving mysteries of the first-ever (pseudo)scalar and his search for further scalars

Dynamical Higgs, associated with new strong interactions is an alternative to the SM

Usually QCD lessons on  $\bar{q}q$  states are used in this connection

We will consider confinement-related scalars, in pure YM. Much less studied. Might be closer to the Higgs dynamics and interesting also on their own

## Reservations on Higgs-implications

- No immediate suggestions for an SM Higgs  
Consider only dynamics in a broad context
- the Yang-Mills dynamics in the scalar sector is highly non-trivial, might be potentially important  
But rather exotic to apply straightforwardly
- Roughly speaking, the Higgs condensates are there but massless phases are not

# Why Yang-Mills scalars at all?

Dual-superconductor model of confinement

For monopoles placed into a superconductor

$$\lim_{r \rightarrow \infty} V_{M\bar{M}}(r) \sim \langle \phi_{G-L} \rangle^2 r$$

There is an explicit solution, Abrikosov string

By analogy,

$$\lim_{r \rightarrow \infty} V_{Q\bar{Q}}(r) = \sigma \cdot r$$

$$\sigma \sim \langle \Phi_M \rangle^2$$

where  $\Phi_M$  is **magnetically** charged 4d scalar

Dual-superconductor idea is most popular way of thinking about confinement for about 30 years

Explicit  $U(1)$  examples known since long and boosted greatly by the Seiberg-Witten papers

However, it is not clear at all, what are the magnetic degrees of freedom to be condensed in the non-Abelian case we are interested in.

Very rich lattice phenomenology but mostly in specific lattice language

We will use the data interpreted in continuum terms, without explaining

# Low-viscosity Quark-gluon plasma

Probably, at  $T > T_c$  confinement problem becomes "low-viscosity-plasma" problem.

$\Delta E \cdot \Delta t \sim 1$  +holography  $\rightarrow$  Kovtun-Son-Starinets bound:

$$\frac{\eta}{\mathbf{s}} \geq \frac{1}{4\pi}$$

where  $\eta$  is viscosity,  $\mathbf{s}$  is entropy density

The actual plasma is only factor-of-order-unit away:

$$\left(\frac{\eta}{\mathbf{s}}\right)_{QGP} \approx \frac{1}{4\pi}$$

Superfluid-component scenario is therefore **encouraged**  
(not granted of course)

# Relativistic superfluidity and scalars

Non-relativistically, there is identical-particles wave function:

$$\Psi_{\text{superfluid}} \sim \sqrt{N} \exp i\phi(\mathbf{x}, t)$$

$$\partial_t \phi(\mathbf{x}, t) = \mu, \quad (\partial_i)^2 \phi(\mathbf{x}, t) = 0$$

( $\mu$  is the chemical potential)

Relativistically, no wave function. Instead, postulate, an extra 4d conserved current and

$$\langle \phi_{3d} \rangle \neq 0, \quad \phi_{3d}^* \neq \phi_{3d}$$

Spontaneous breaking implies **3d** Nambu-Goldstone particle needed for superfluidity

# Thermal scalar (1986)

Scalar particles do arise within stringy models of QCD.  
First example discovered is the thermal scalar

For strings, number of states grows fast with mass and at temperature  $T_H \sim \Lambda_{QCD}$  the sum over states diverges, Hagedorn phase transition.

If  $T \rightarrow T_H$ ,  $T < T_H$  the whole partition function reduces to that of a static scalar with mass:

$$m_\beta^2 = \frac{(\beta - \beta_H)\beta_H}{(2\pi^2)(\alpha')^2}$$

$\beta = 1/T$ ,  $\beta_H = 1/T_H$ . Hits the Higgs region at  $T = T_H$ .  
This Higgs-like instability is equivalent to Hagedorn instability due to heavy stringy states



# Another language for the massless scalar

In Euclidean picture, time  $\tau$  is periodic,

$$\tau \sim \tau + 1/T$$

The thermal scalar then is once-wrapped mode. Time dependence is fixed by periodicity – lives in spatial dims

Most important: wrapping number is a new, **topological stringy quantum number**. Condensation of the thermal scalar at  $T > T_c$  would spontaneously break a 3d U(1) symmetry, as needed for superfluidity.

Possible condensation of the thermal scalar at  $T > T_c$  has been discussed in many papers, with varied conclusions

## Conclusions to introductory remarks

- Spontaneously broken, non-pert  $U(1)$  symmetries, both 4d and 3d are welcome, to explain confinement and low viscosity (\*), resp.
- The only example relevant seems to be thermal scalar, or stringy topological quantum numbers  
Will try to exploit stringy models through the talk

(\*) *Not a common viewpoint*

# Confinement-related geometry (W-S-S model)

Deforming the  $N=4$  SUSY YM one gets theory in the same universality class [in infrared](#) as ordinary, large  $N_c$ , YM

Geometry: begin with ordinary 4d + 5th dimension with (common in holography) properties:

- 1 conjugated to inverse momentum.
- 2 The 4d world is the UV limit, ( $u \rightarrow \infty$ )
- 3 there is horizon,  $u > u_h$ , confinement-related

# Extra compact dimension

Central point: there is **extra compact,  $x_4$  dim**

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left( f(u) dx_4^2 - dt^2 + \delta_{ij} dx^i dx^j \right) + \left(\frac{u}{R}\right)^{-3/2} \left( \frac{du^2}{f(u)} + u^2 d\Omega_4^2 \right)$$

$$f(u) = (1 - u_h^3/u^3) \quad , \quad x_4 \sim x_4 + \beta_4, \quad \beta_4 = 4\pi/3(R^3/u_h^3)$$

$x_4$  **coordinate survives in the ultraviolet**, does not fit reality, limits the applicability of the model to distances  $d$

$$d \gg \beta_4$$

Finally, the 4-sphere,  $d\Omega_5^2$ , is relevant to baryons and does not concern us here

# Cigar-shape geometries, phase transition

At  $T < T_c$  cigar-shape in the  $(u, x_4)$  coordinates:

$$R_4(u = u_h) = 0$$

For compact ( $T \neq 0$ ) Euclidean time,  $\tau$  cylinder-like geometry

$$R_\tau(u) = \text{const}$$

At phase transition the geometries are interchanged:

$$R_\tau(u = u_h) = 0 \quad R_4(u) = \text{const}$$

Cigar-shape geometry in  $(u, \tau)$  coordinates

# Defects

In YM field theory, in practice

*(nonperturbative physics)  $\approx$  (physics of instantons)*

In the stringy formulation, there are various D-branes,  
stable topologically if wrapped on compact dimensions

*nonperturbative physics is physics of lower dim. defects*

much richer than in field theoretic formulation

In particular, instantons are D0 branes wrapped on  $x_4$   
circle. That, is

wrapping around  $x_4$  responsible for the  $\theta$ -dependence

# Geometry and vanishing actions

Signs of cigar-shape is similar to thermal scalar:

vanishing  $R_4(R_\tau) \rightarrow$  defect's mass/tension vanishes

$T < T_c$ :

*all  $x_4$  – wrapped branes are tensionless in this approx.*

$T > T_c$ :

*all  $\tau$  – wrapped branes are tensionless in this approx.*

Similar to the thermal scalar (static)

Some defects percolate to infinity, or condense

D2 branes, wrapped on  $x_4$  are identified as magnetic strings which can be open on the 't Hooft line

D2 condensation implies no confinement for the external monopoles

Life at distances  $d \gg \Lambda_{QCD}^{-1}$ 

We have restriction,  $d \gg \beta_4 \sim \Lambda_{QCD}^{-1}$

No hadron-spectrum physics survives at such distances, but confinement and plasma are there. Also massless particles, if exist.

Also, the defects survive. Worth emphasizing that usually defects action exponentially suppressed at large  $N_c$ . We assume action of order  $N_c \cdot 0$  to still vanish. The geometry is borrowed from large  $N_c$ .



## From Euclidean 4d to 3d

According to the theory considered,  
Non-perturbative physics becomes **3d** at  $T = T_c$   
since defects are becoming wrapped on  $\tau$

The phenomenon is well known in the limit of high temperature,  $T \rightarrow \infty$  as dimensional reduction. Here, the dimensional reduction is predicted to happen sharply at  $T = T_c$  (Gorsky et al) For not large  $N_c$  the transition is smoother.

In particular, magnetic strings are expected to look in  $\tau$ -direction and geometrically reduce to their 3d projection, 1d defects, or particles

## Conclusions to holographic models

- suggest condensation of topologically charged magnetic strings below  $T_c$  as confinement mechanism
- thus, physics of confinement is rather physics of strings, not field theory
- Euclidean-time oriented defects above  $T_c$
- suggest condensation of scalars above  $T_c$ . (probably) back to field theory

# Condensation of strings. Hints from the lattice

In the absence of theory, the only source is lattice.

Remarkable results (pure numerical, though):

Condensation of strings has been observed on the lattice

Turns equivalent to a **single scalar living on a string**:

$$(\textit{condensation of strings}) \equiv \langle |\phi_M|^2 \rangle \sim \Lambda_{\text{QCD}}^2$$

$$\langle \phi_M \rangle^2 \sim \Lambda_{\text{QCD}}^3 \cdot \mathbf{a}, \quad \langle \phi_M \rangle_{\text{continuum}} = 0$$

and vanishes, although at any finite lattice spacing  $\mathbf{a}$  suffices to produce confinement (!)

# Condensation of magnetic strings, cn'd

In field theory one would have instead

$$\langle \phi^2 \rangle \sim a^{-2} \quad \langle \phi \rangle^2 \sim \Lambda_{QCD}^2$$

That is, strings are 'milder' by two dimensions

Vanishing of  $\langle \phi_M \rangle$ ,  $\langle \phi_M \rangle \sim (a\Lambda_{QCD})^{1/2}$   
opens the possibility

$$\frac{\langle \phi_M \rangle}{\Lambda_{QCD}} \ll 1$$

upon inclusion of (say, e-m) corrections.

At  $T > T_c$  strings become time-oriented and condensation of strings becomes condensation of a 3d scalar (observed)

Effective 3d scalars at  $T > T_c$ 

At  $T > T_c$  we expect to deal with a field theory, which is easier for theory than strings

Begin with Euclidean time and assume condensation of thermal scalar

Then static correlator of components of energy-momentum tensor has a singularity:

$$\langle T_{0i}, T_{0k} \rangle_{\mathbf{q} \rightarrow 0, \omega \equiv 0} \sim \frac{q_i q_k}{q^2} \langle \phi_{3d} \rangle^2$$

Such a behaviour is the standard symptom of superfluidity (continuation to Minkowski is trivial for a static correlator)

# Action of 3d defects

Another input is the action for 3d defects in Euclidean:

$$S_E = -T \int d\tau d^3x \sqrt{1 + (\partial_i \phi)^2}$$

where the minus sign is due to condensation (similar to supercritical phase in percolation theory). The meaning of  $\phi$  is the extra fluctuating coordinate ( $\tau$ ).

Now we are set to continue to the Minkowski space (where plasma lives)

## An exotic liquid

Luckily enough, liquid holographic dual to the Rindler space was found recently (Compere et al 1103.3022). It was shown that in the dissipation-less approx. the liquid properties are reproduced by the action

$$S_M = (\text{const}) \int d^4x \sqrt{-\gamma} \sqrt{-(\partial_\mu \phi)^2}$$

where metric  $\gamma_{ab} dx^a dx^b \equiv -r_c dt^2 + dx_i^2$  ( $r_c$  is a parameter) with the ground-state solution

$$\phi_{\text{equilibrium}} = t$$

One can readily check that this action can be considered as the Minkowskian counterpart to our Euclidean construction

# Exotic liquid (cn'd)

Equilibrium energy-momentum tensor looks as

$$(T_{ab})_{equilibrium} = (0, \rho, \rho, \rho)$$

( $\rho = 1/\sqrt{r_c}$ ) and we note that it is the same as for the liquid living on the stretched horizon ( $r_c$  is then small)

Moreover, the "Einstein-Navier duality" allows also to find

$$\left(\frac{\eta}{s}\right) = \frac{1}{4\pi}$$

Thus, our non-perturbative component of plasma appears to be close in properties to black-hole liquid



# Partial check on the lattice

The only relevant measurement on the lattice

$$(\epsilon - 3\rho)_{non-pert} < 0$$

and large,

$$|(\epsilon - 3\rho)_{non-pert}| \approx 4 \cdot (\epsilon - 3\rho)_{total}$$

although occupies a tiny fraction of the whole volume

That is, rather in agreement with theory

## Conclusions on condensates properties

- Both 4d and 3d condensates seem to exist and relevant to the confinement and low viscosity of YM plasma
- theoretical guesses tend to agree with the lattice measurements. Especially, in the much better studied case of  $T = 0$

# Absence of massless excitations

A generic manifestation of extra dimensions is massless excitations living on the brane (the Luscher term is a well known example)

More specifically, found massless 3d scalar at  $T > T_c$  manifested in the correlator of the momentum components and present in the action derived from duality

Similarly, at  $T = 0$  could expect massless 4d scalar coupled to topological  $K_\mu$  current

None is there

# Nonperturbative component and unitarity

At closer look, the 3d scalar turns superluminal with respect to the perturbative vacuum and is not carried on to the full theory

The 4d scalar would be ghost-like  
(Since non-pert effects lower the energy of the vacuum)

In other words, the non-pert component is non-unitary by itself and is incomplete in this sense even in the infrared.

# Wrong-sign correlators

Some correlators have wrong sign non-perturbatively

Still, the wrong-sign contribution survives as a local term

In full theory

$$\langle G\tilde{G}(\mathbf{x}), G\tilde{G}(\mathbf{0}) \rangle_{full\ theory} < 0, \quad \mathbf{x}_{Euclidean} \neq \mathbf{0}$$

$$\langle G\tilde{G}(\mathbf{x}), G\tilde{G}(\mathbf{0}) \rangle_{non-pert} < 0, \quad \mathbf{x}_{Euclidean}$$

As a result, the non-pert term squeezed to  $\delta^4(\mathbf{x})$  but still controls the integral over  $d^4\mathbf{x}$  ( topological susceptibility)

The story seems to be repeated in case of

$$\langle T_{0i}(\mathbf{x}), T_{0k}(\mathbf{0}) \rangle \text{ correlator}$$

## Overall conclusions

- Stringy holography fits well lattice phenomenology
- Condensates associated with nonperturbative scalars, 4d and 3d, seem to exist and exhibit exciting properties
- Associated with the condensation massless excitations are not observed. Non-perturbative physics cannot be treated as an independent component in this respect
- Wrong-sign nonperturbative contributions to correlator seem to squeeze to local terms and continue to control integrals