10. Aug. 2018 in MOGRA @ Nagoya

Stochastic Inflation in a General Field Space

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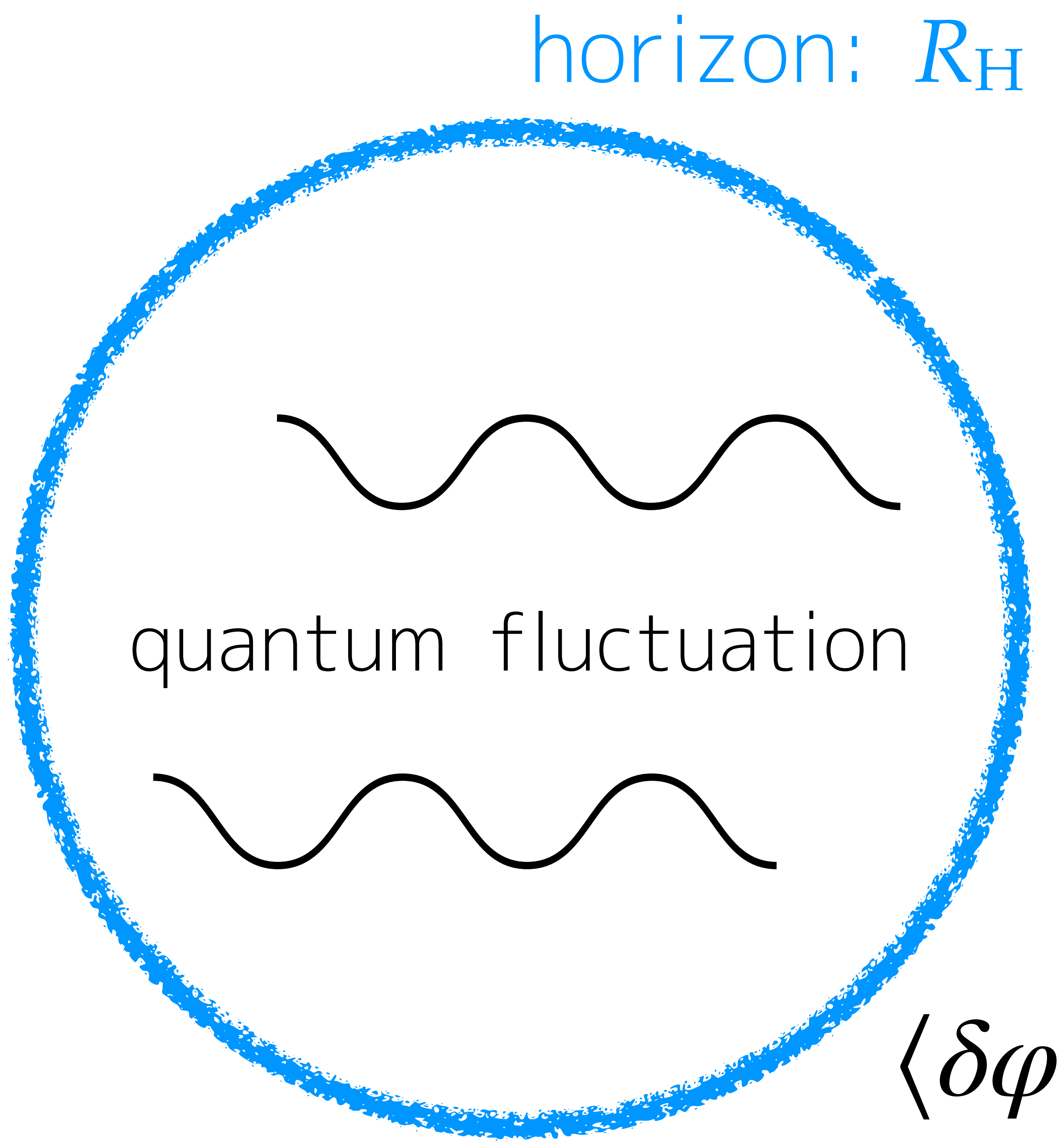
T. Fujita, J. Tokuda (Kyoto)

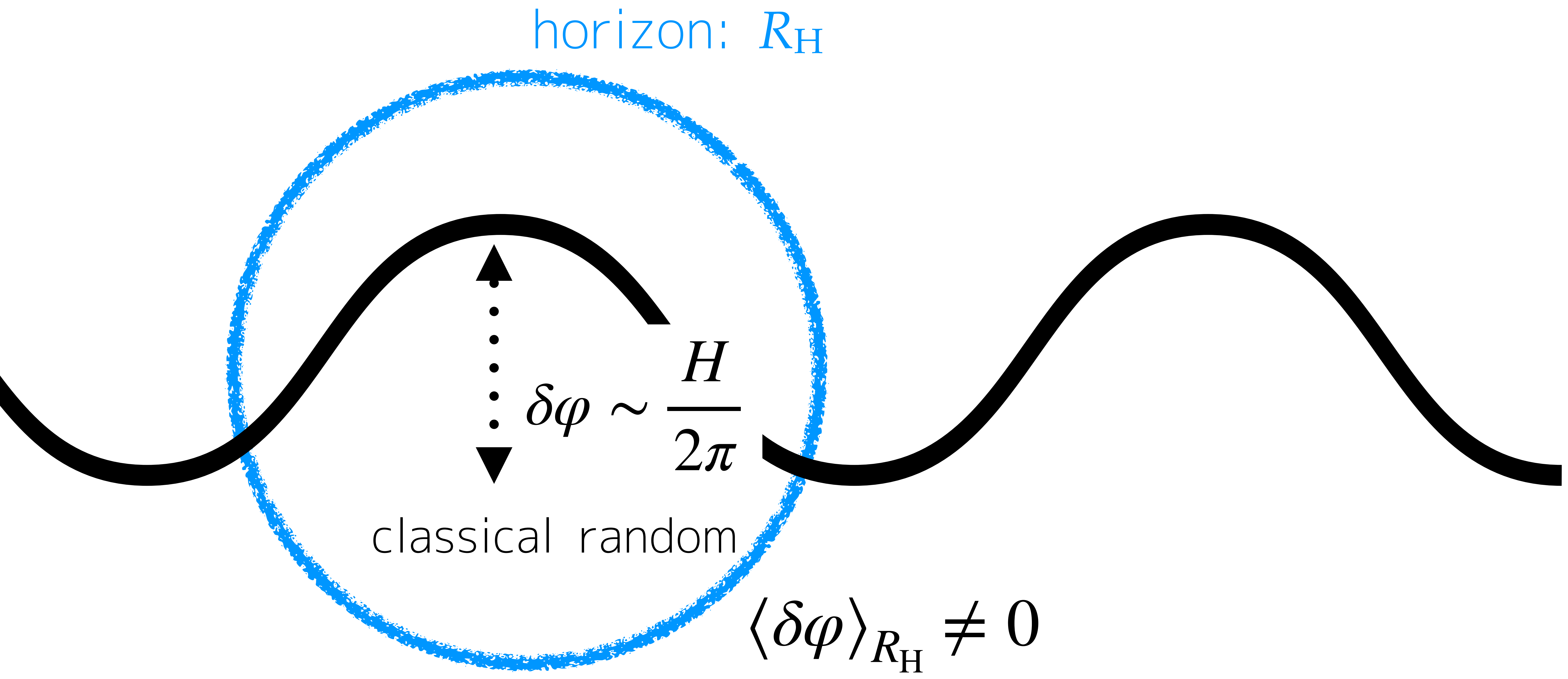
arXiv: 1806.10126 & in prep.

Scalar Perturbation: main Obs. of Inflation



$$\langle \delta\varphi \rangle_{R_{\mathrm{H}}} = 0$$

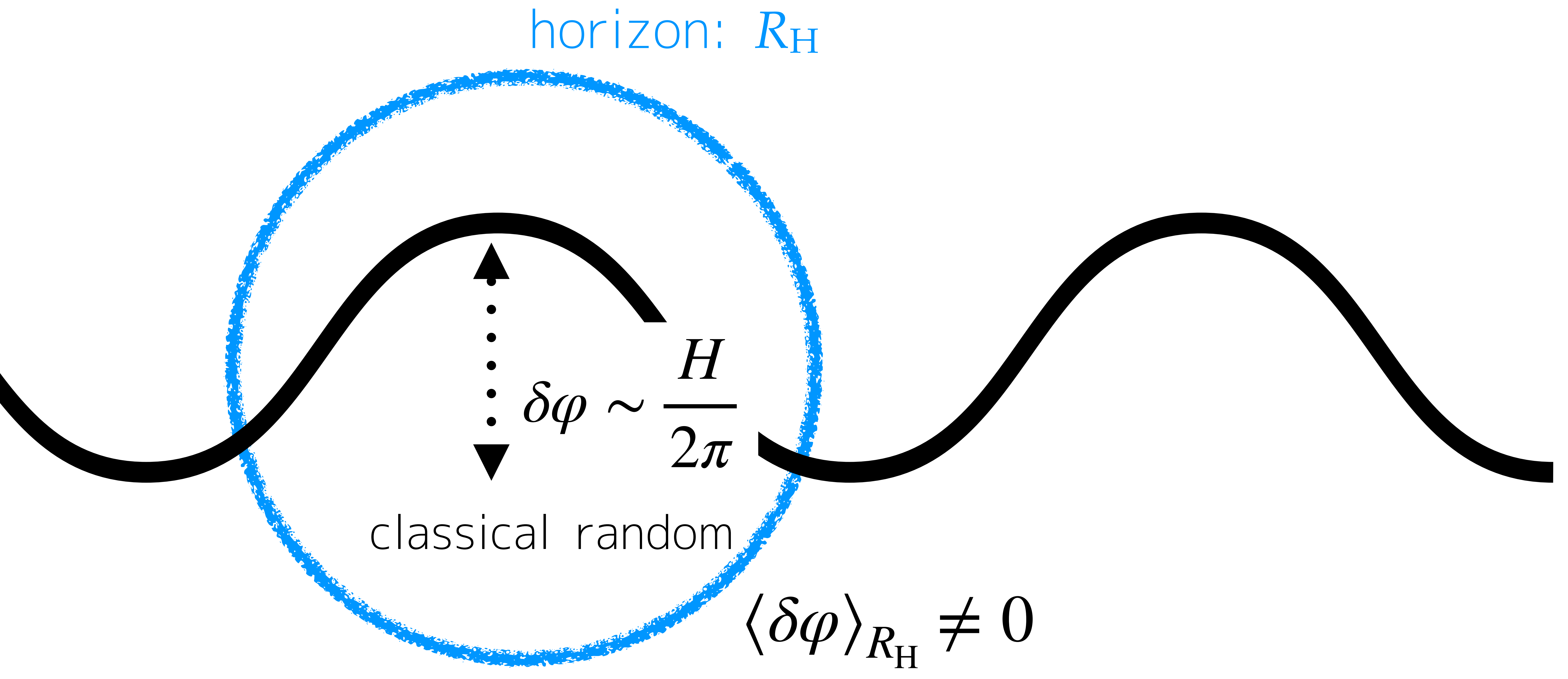
Scalar Perturbation: main Obs. of Inflation



slow-roll EoM

$$\frac{\mathrm{d}\varphi_0}{\mathrm{d}N} = -\frac{V'}{3H^2}$$

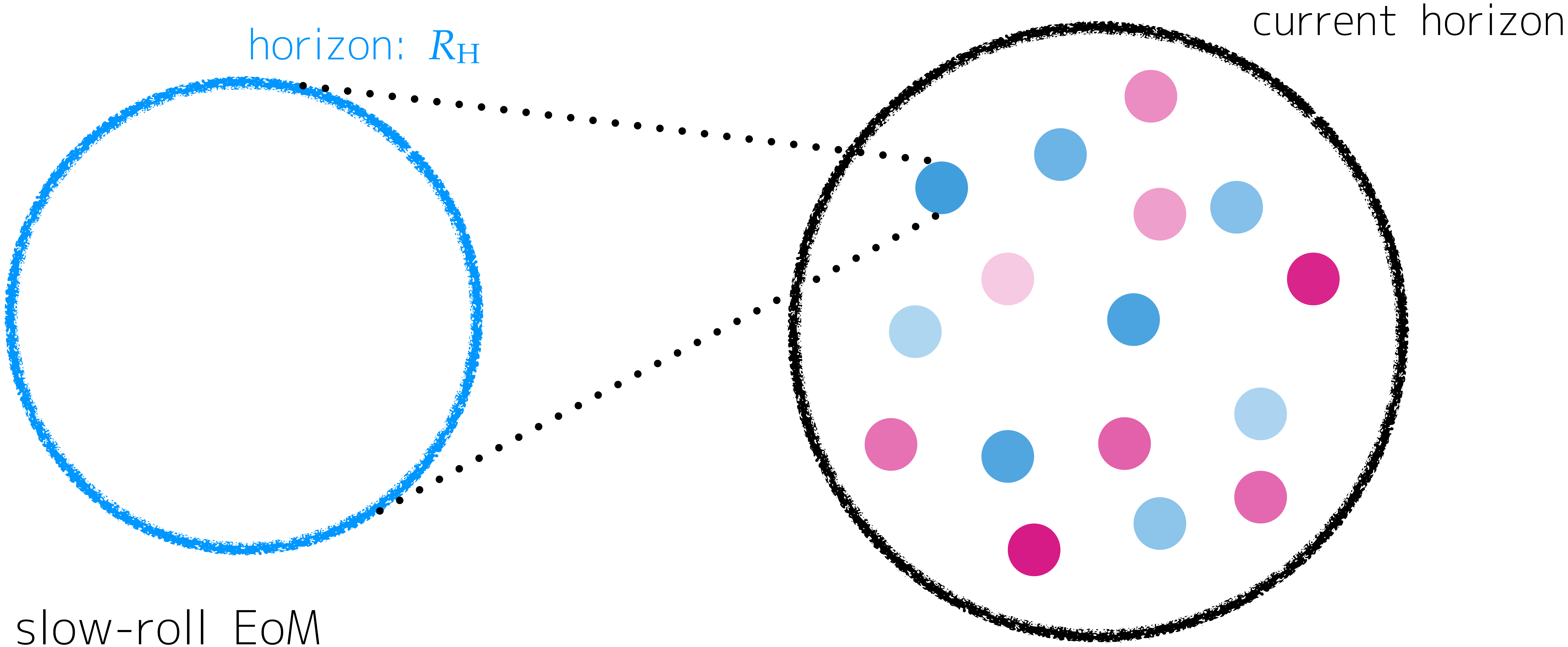
Scalar Perturbation: main Obs. of Inflation



slow-roll EoM

$$\frac{\mathrm{d}\varphi_{\mathrm{IR}}}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi, \qquad \langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

Scalar Perturbation: main Obs. of Inflation

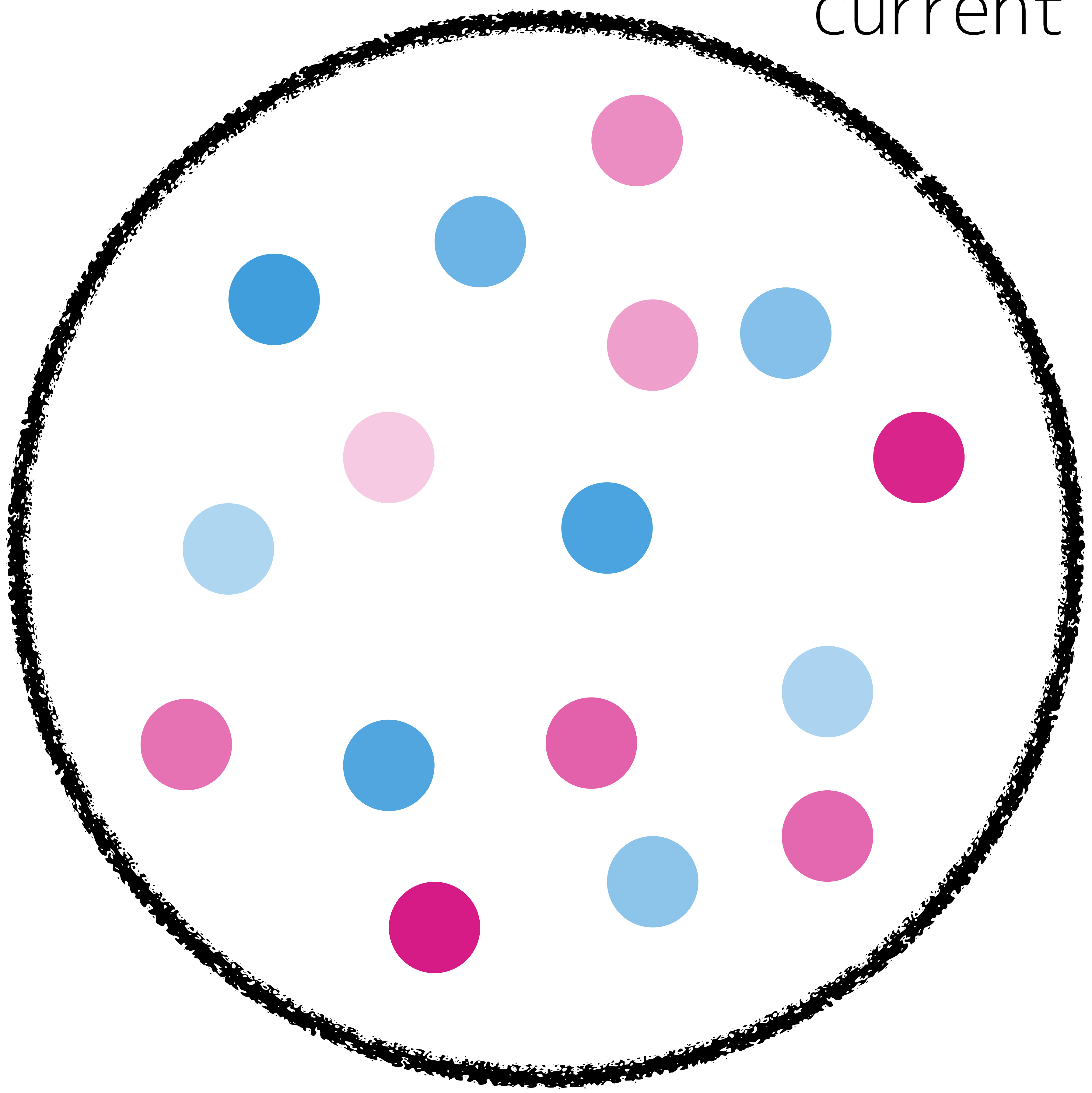


slow-roll EoM

$$\frac{\mathrm{d}\varphi_\mathrm{IR}}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi, \qquad \langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

Scalar Perturbation: main Obs. of Inflation

current horizon



Stochastic Formalism

behaves like Brownian motion

prim. pert. as source of
stars, galaxies, clusters, $\cdots$

slow-roll EoM

$$\frac{\mathrm{d}\varphi_{\mathrm{IR}}}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi, \qquad \langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

How stochastic formalism generalized?

$$\frac{\mathrm{d}\varphi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi \longrightarrow \frac{\mathrm{d}\varphi^I}{\mathrm{d}N} = -\frac{G^{IJ}\partial_J V}{3H^2} + \frac{H}{2\pi}e^I_a\xi_a$$

$$\begin{cases} \langle \xi_a(N)\xi_b(N')\rangle = \delta_{ab}\delta(N-N'), \\ e^I_a e^J_a = G^{IJ}, \\ \mathscr{L}_{\mathrm{kin}} = -\frac{1}{2}G_{IJ}\partial_\mu\varphi^I\partial^\mu\varphi^J \end{cases}$$

depending on noise def. …

- NOT inv. under "Field Coord." trs.

$$\varphi^I \to \tilde{\varphi}^A = \tilde{\varphi}^A(\varphi)$$

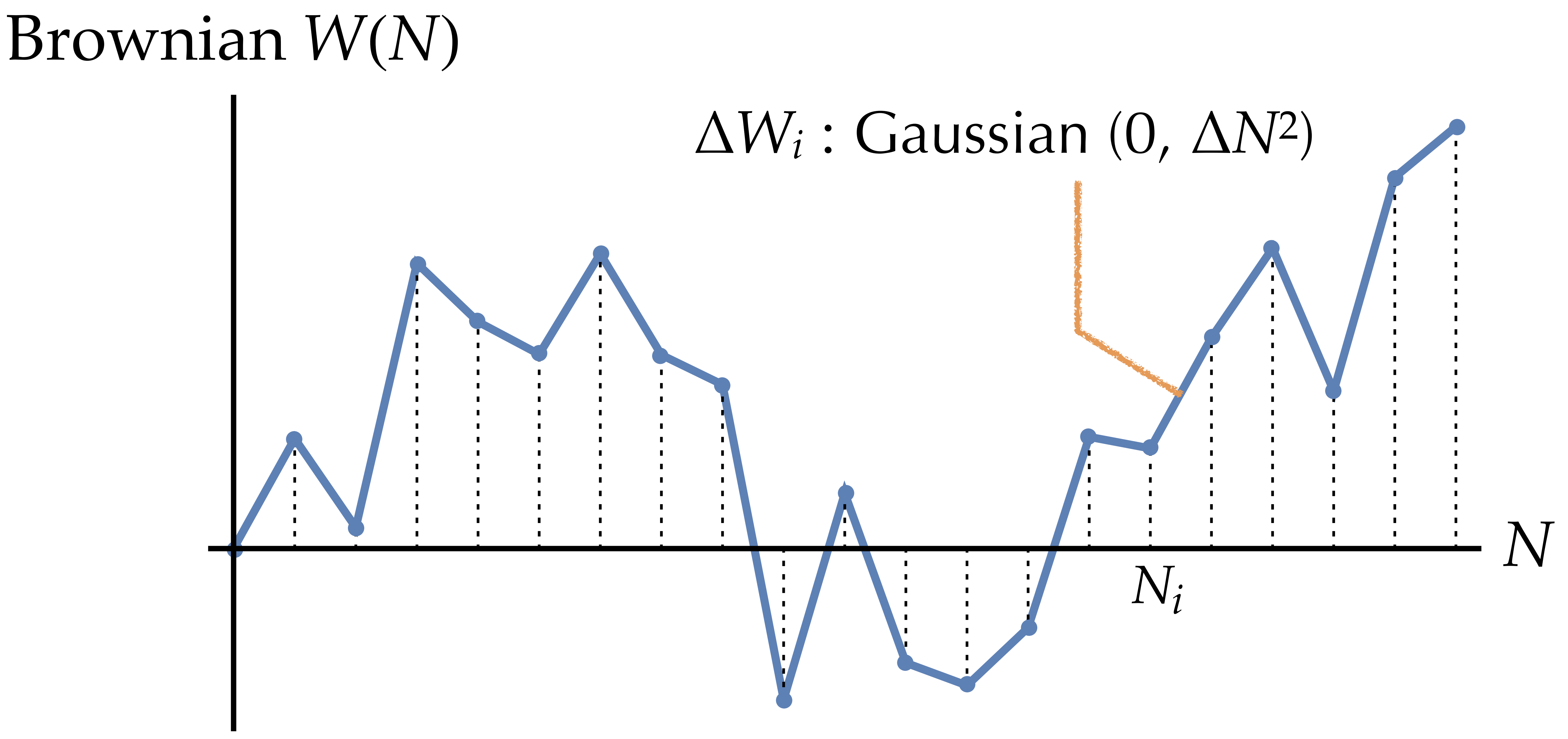
- NOT inv. under "Noise Frame" rot.

$$e^I_a \to \tilde{e}^I_i = e^I_a R_{ai}$$

Stochastic integral

- Langevin eq.

$$\frac{\mathrm{d}X(N)}{\mathrm{d}N} = A\left(X(N)\right)\xi(N), \qquad \langle \xi(N)\xi(N')\rangle = \delta(N-N')$$



$$\Delta X_i = A\left(X(N_i^*)\right)\Delta W_i$$

$$N_i^* \in [N_i, N_{i+1}]$$

X in continuous time

$\lim \Delta N \to 0$

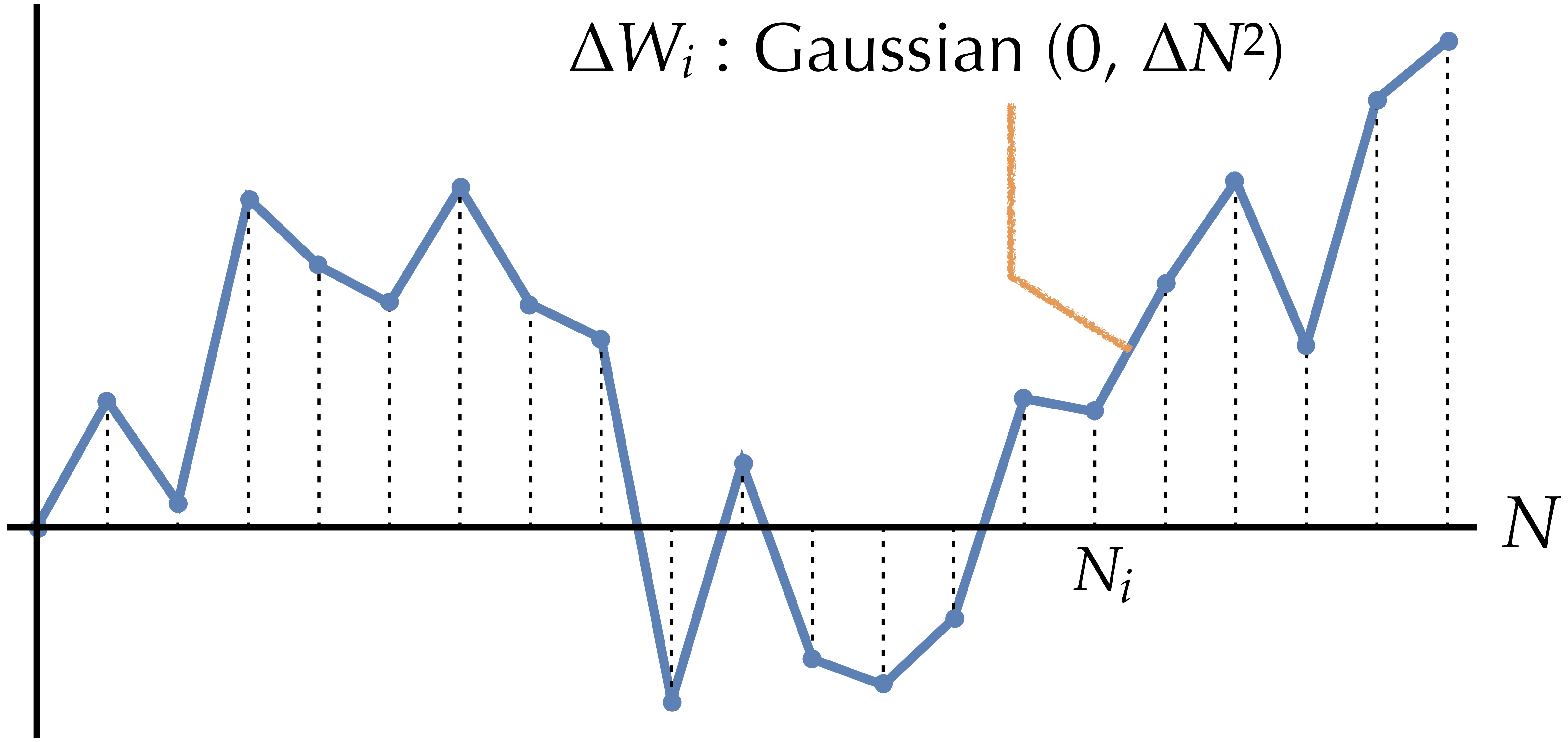
Stochastic integral

- Langevin eq.

$$\frac{\mathrm{d}X(N)}{\mathrm{d}N} = A\left(X(N)\right)\xi(N), \qquad \langle\xi(N)$$

X's property (PDF) depends on the choice of N_i^*!

Brownian $W(N)$



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$$N_i^* \in [N_i, N_{i+1}]$$

X in continuous time

$\lim \Delta N \to 0$

Ito vs. Stratonovich

✱ Ito integral: $N_i^* = N_i$

$$\mathrm{d}X = A(X)\mathrm{d}W$$

✓ causal

✓ Martingale: $\langle X \rangle = 0$

Vilenkin 1999,
Fujita, Kawasaki, YT 2014,
Tokuda & Tanaka 2017, …

✗ Ito's lemma

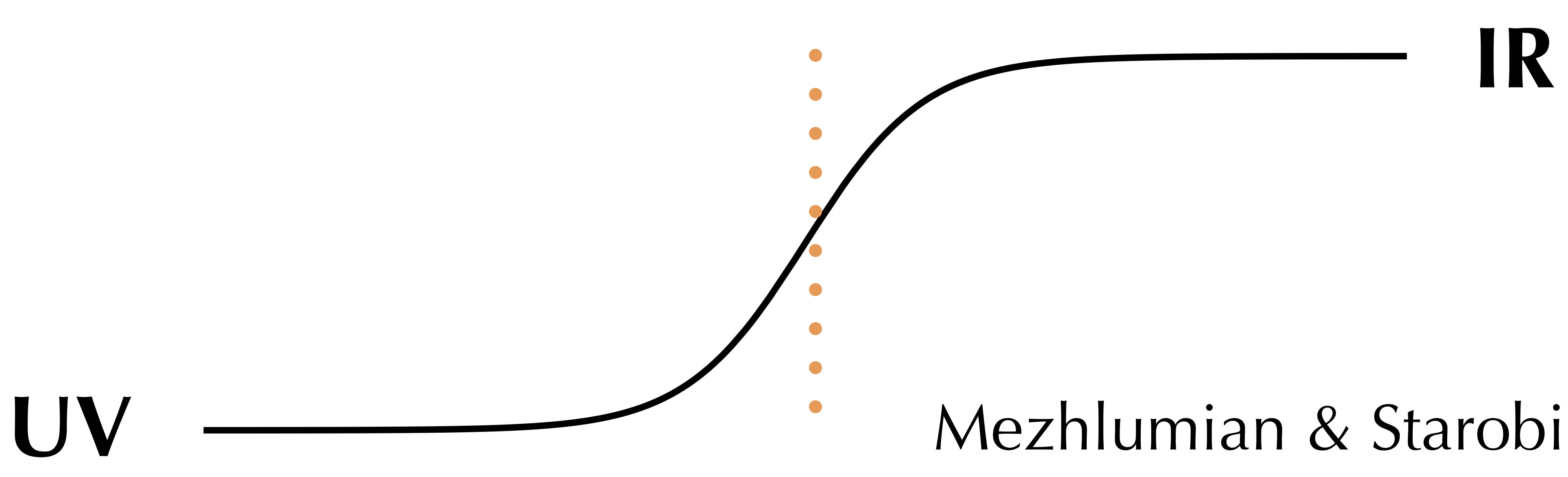
$$\mathrm{d}f(X) = f_X \mathrm{d}X + \underbrace{\frac{1}{2} f_{XX} A^2 \mathrm{d}N}_{\text{accumulation of noise}}$$

✱ Stratonovich integral: $N_i^* = \frac{1}{2}(N_i + N_{i+1})$

$$\mathrm{d}X = A(X) \circ \mathrm{d}W$$

✗ "uncausal"

✗ non-Martingale: $\langle X \rangle \neq 0$



Mezhlumian & Starobinsky 1994

✓ std. chain rule

$$\mathrm{d}f(X) = f_X \mathrm{d}X = f_X A \circ \mathrm{d}W$$

Fokker-Planck eq.

$$\frac{\mathrm{d}\varphi^I}{\mathrm{d}N} = -\frac{V^I}{3H^2} + \frac{H}{2\pi} e^I_a \xi_a$$

independent of $e^I{}_a$
but not field-space covariant

$$\cdots\blacktriangleright \begin{cases} \partial_N P = \partial_I\left(\dfrac{V^I}{3H^2}P\right) + \dfrac{1}{2}\partial_I\partial_J\left[\left(\dfrac{H}{2\pi}\right)^2 G^{IJ}P\right] & \text{: Ito} \\ \partial_N P_s = \nabla_I\left(\dfrac{V^I}{3H^2}P_s\right) + \dfrac{1}{2}\nabla_I\left[\dfrac{H}{2\pi}e^I_a\nabla_J\left(\dfrac{H}{2\pi}e^J_a P_s\right)\right] & \text{: Stratonovich} \end{cases}$$

$P_s = P/\sqrt{G}$: scalar

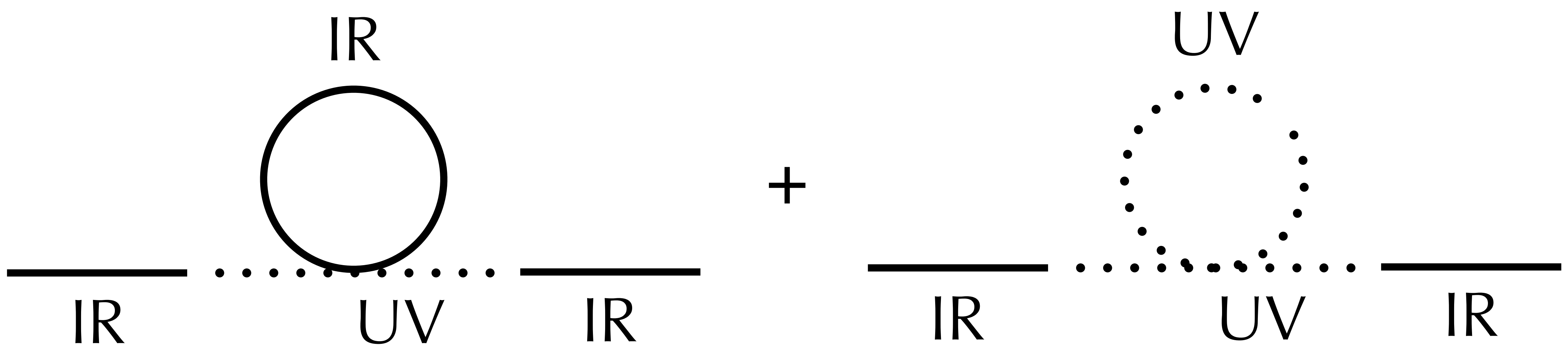
field-space covariant
but depends on $e^I{}_a$

Comparison w/ QFT

Let's compare leading corrections to $\langle\phi^2\rangle = (H/2\pi)^2 N$ in stochastic form. and QFT

- stochastic formalism (IR theory)

  UV integrate out w/ 1-loop



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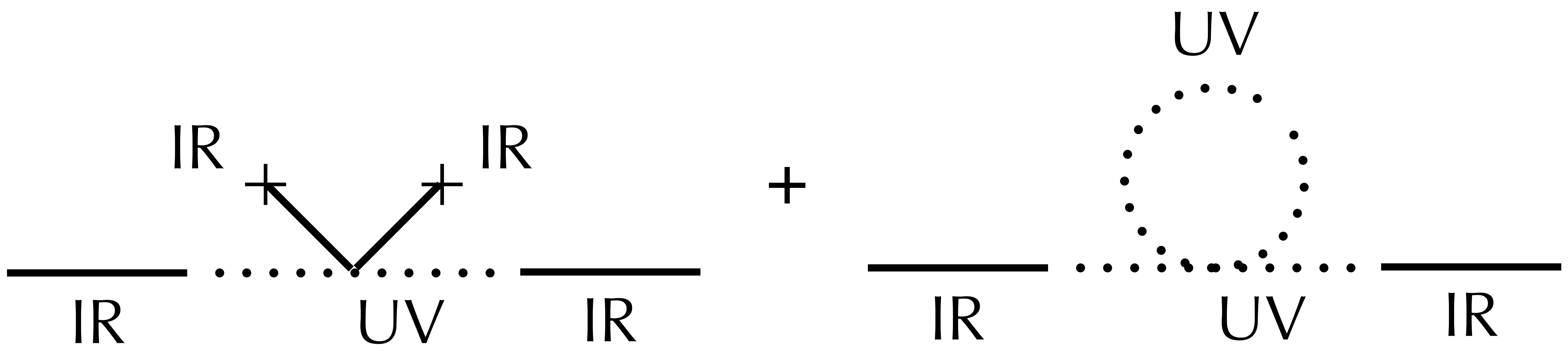

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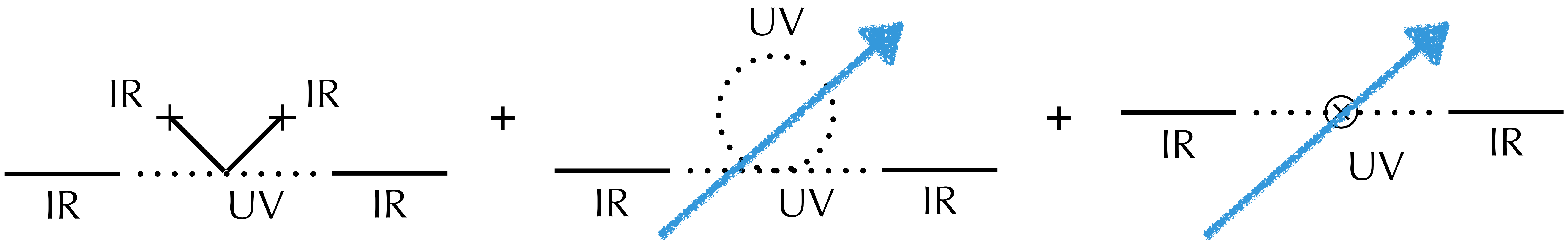

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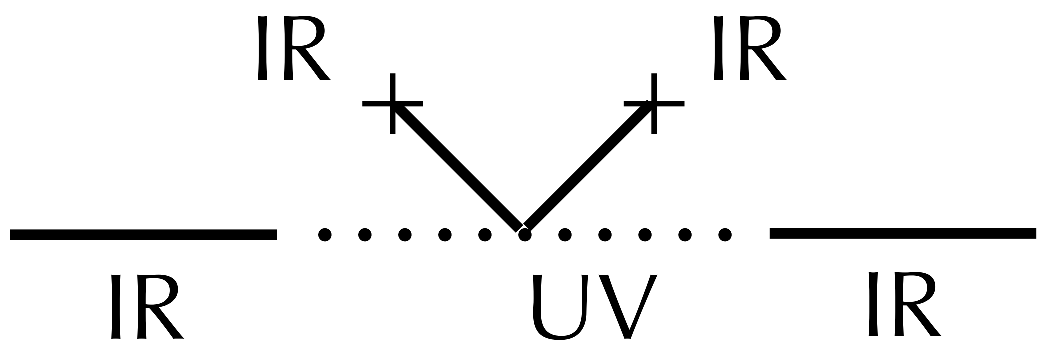
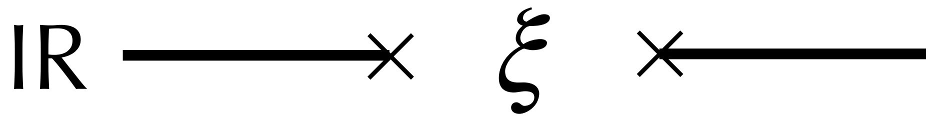
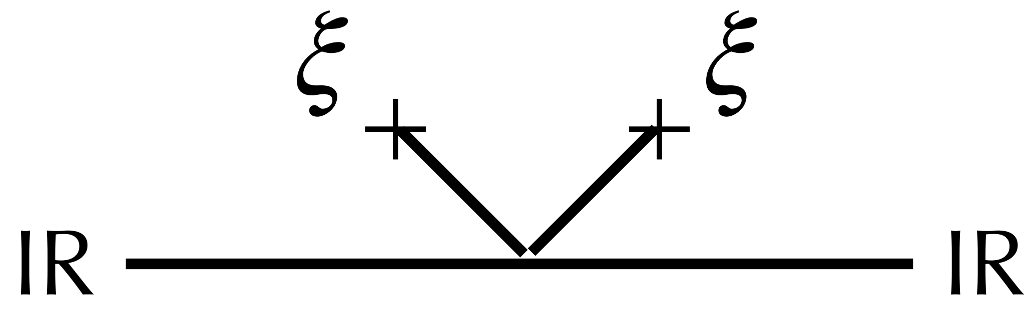
IR EFT (Fokker-Planck)

solve perturbatively





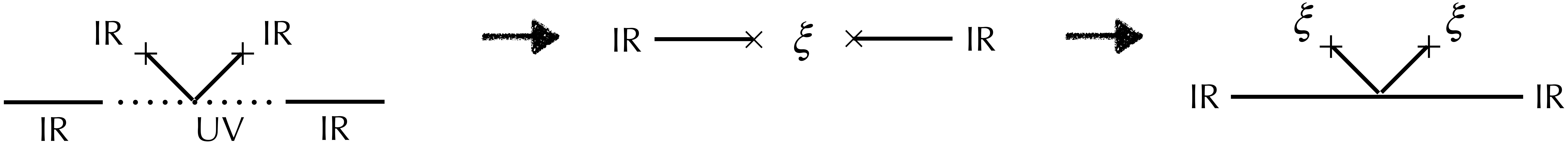


Comparison w/ QFT

Let's compare leading corrections to $\langle \phi^2 \rangle = (H/2\pi)^2 N$ in stochastic form. and QFT

- stochastic formalism (IR theory)

  UV integrate out w/ 1-loop   IR EFT (Fokker-Planck)   solve perturbatively

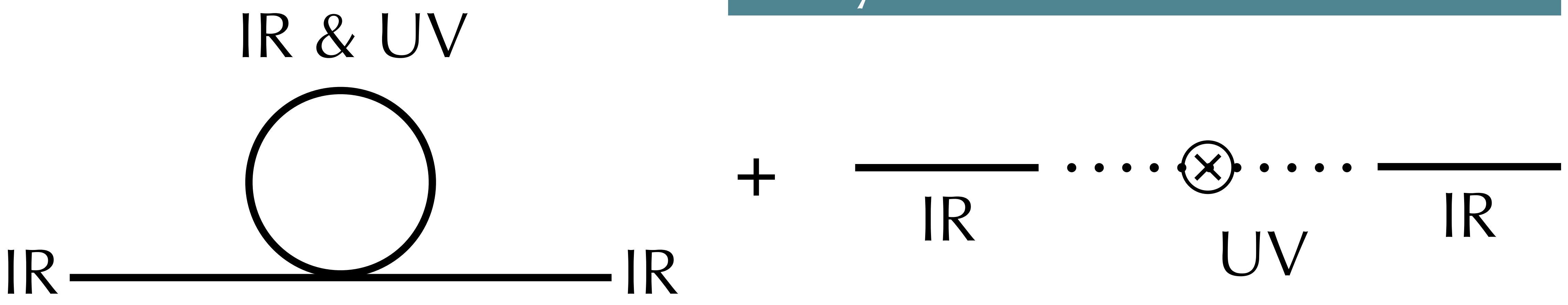


- QFT

  directly calc. 1-loop corrections

  w/ both of IR & UV loop momenta

They should be consistent



Potential-type

Tokuda & Tanaka 2017

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4!}\varphi^4$$

- QFT

param. related
w/ coarse-graining scale

$$\langle\varphi^2\rangle = \left(\frac{H}{2\pi}\right)^2\left[N + \lambda\left(-\frac{1}{36\pi^2}N^3 + \frac{1+9l}{72\pi^2}N^2 + \mathcal{O}(N)\right)\right]$$

ITO works for
potential-type interaction

- stochastic form.

$$\langle\varphi^2\rangle = \left(\frac{H}{2\pi}\right)^2\left[N + \lambda\left(-\frac{1}{36\pi^2}N^3 + \frac{1-2\alpha+9(1+2\alpha)l}{72\pi^2}N^2 + \mathcal{O}(N)\right)\right]$$

$$\alpha = \begin{cases} 0 & \text{:Ito} \\ 1/2 & \text{:Strato} \end{cases}$$

Metric-type

Fujita, Renaux-Petel, Tada, Tokuda in prep.

$$\mathscr{L} = -\frac{1}{2}\left[\partial_\mu\varphi\partial^\mu\varphi - \frac{2\varphi}{M}\partial_\mu\varphi\partial^\mu\chi + \left(1+\frac{\varphi^2}{M^2}\right)\partial_\mu\chi\partial^\mu\chi\right]\left(= -\frac{1}{2}\left[\mathrm{e}^{2\chi/M}\partial_\mu\psi\partial^\mu\psi + \partial_\mu\chi\partial^\mu\chi\right]\right)$$

$$e_a^I = \begin{pmatrix} \varphi/M & 1 \\ 1 & 0 \end{pmatrix}$$

- QFT

$$\langle\varphi^2\rangle = \left(\frac{H}{2\pi}\right)^2\left[N + \frac{H^2}{4\pi^2M^2}N^2\right]$$

- stochastic form.

$$\langle\varphi^2\rangle = \left(\frac{H}{2\pi}\right)^2\left[N + \frac{(1+2\alpha)H^2}{8\pi^2M^2}N^2\right]$$

STRATONOVICH works for metric-type interaction

$$\alpha = \begin{cases} 0 & \text{:Ito} \\ 1/2 & \text{:Strato} \end{cases}$$

diff. btw. Ito and Stratonovich: $\left(\frac{H}{2\pi}e_a^J\right)\partial_J\left(\frac{H}{2\pi}e_a^I\right)$

- variation of noise amp. by noise kick itself → noise non-Gaussianity

- it should be smaller than noise amp. itself for Gaussian anzats

$$\boxed{\left|\left(\frac{H}{2\pi}e_a^J\right)\partial_J\left(\frac{H}{2\pi}e_a^I\right)\right| \ll \left|\frac{H}{2\pi}e_a^I\right|}$$

- small NG
- not stupid vielbein

- rather diff. btw. Ito and Stratonovich can be a Criterion of validity of Stoc. Form.!

stupid rotation:

$$e_a^I = \begin{pmatrix} \varphi/M & 1 \\ 1 & 0 \end{pmatrix} \quad \to \quad \tilde{e}_i^I = \begin{pmatrix} \varphi/M & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\chi/R & -\sin\chi/R \\ \sin\chi/R & \cos\chi/R \end{pmatrix}$$

$$\boxed{\langle\varphi^2\rangle_{\mathrm{QFT}} = \left(\frac{H}{2\pi}\right)^2 \left[N + \frac{H^2}{4\pi^2M^2}N^2\right], \qquad \langle\varphi^2\rangle_{\mathrm{stoc}} = \left(\frac{H}{2\pi}\right)^2 \left[N + \left(\frac{(1+2\alpha)H^2}{8\pi^2M^2} + \alpha^2\frac{H^2}{4\pi^2R^2}\right)N^2\right]}$$

- Ito ($\alpha = 0$) is NOT affected by noise rot.
- $$\left| \frac{\left(\frac{H}{2\pi}e_a^I\right)\partial_I\left(\frac{H}{2\pi}e_a^\varphi\right)}{\sqrt{\left(\frac{H}{2\pi}\right)^2 e_b^\varphi e_b^\varphi}} \right| \ll 1 \quad \to \quad \frac{H}{R}, \frac{H}{M} \ll 1$$

Conclusions

- Ito-Strato ambiguity

  Ito: field-coord. dependent

  Strato: noise-frame dependent

- Ito for potential, Strato for metric

  → no choice for general mixed case

- small Ito-Strato diff. ≃ noise Gaussian condition

$$\left|\left(\frac{H}{2\pi}e_a^J\right)\partial_J\left(\frac{H}{2\pi}e_a^I\right)\right| \ll \left|\frac{H}{2\pi}e_a^I\right|$$