VALIDITY OF CHAMELEON MECHANISM IN INHOMOGENEOUS DENSITY PROFILE

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with

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Introduction

- One of the important motivation for modified gravity is to explain accelerated expansion of the universe.

Consider the models GR + scalar fields

Solar system scale → cosmological scale

For scalar field

fifth force experimental constraint ← **Scale-dependent behavior** → accelerated expansion

Screening mechanism

chameleon, symmetron, Vainstein mechanism etc.

Chameleon field

Action

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right]$$

$$+ \int d^4x \mathcal{L}_m \left(\psi_m, \Omega^2(\phi) g_{\mu\nu}\right) \qquad \Omega(\phi) = e^{\frac{\beta}{2M_{pl}}\phi}$$

Typical potential $V(\phi) = \frac{M^{4+n}}{\phi^n}$

matter : non-relativistic perfect fluid with density ρ

EoM for the scalar field

$$\nabla_\mu\nabla^\mu\phi = \frac{\beta}{M_{pl}}\rho e^{\frac{\beta}{M_{pl}}\phi} - n\frac{M^{4+n}}{\phi^{n+1}}$$

$$= \frac{dV_{eff}}{d\phi}(\phi)$$

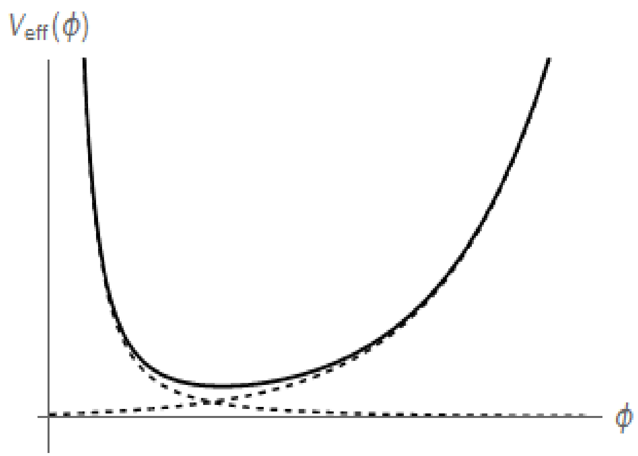
Chameleon mechanism

Mass of the field change depending on the environmental matter density.

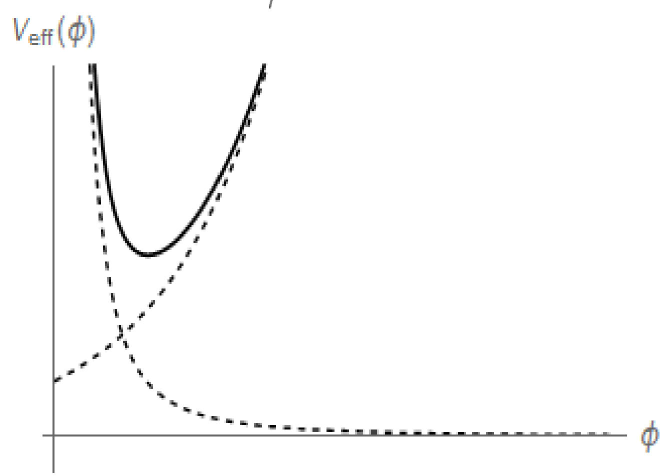
EoM for the scalar field

$$\nabla_\mu \nabla^\mu \phi = \frac{\beta}{M_{pl}} \rho e^{\frac{\beta}{M_{pl}}\phi} - n\frac{M^{4+n}}{\phi^{n+1}}$$

$$= \frac{dV_{eff}}{d\phi}(\phi)$$



Small ρ



Large ρ

ϕ : Small mass Large mass

Screening mechanism

Considering **a star** which has **constant density ρ_c**

For large ρ_c,

R_{roll} is sufficiently close to R_c

$$\left|\frac{F_\phi}{F_{\rm Newton}}\right| \sim \beta^2 \frac{R_c - R_{\rm roll}}{R_c}$$

For small ρ_c,

$R_{roll} \sim 0$

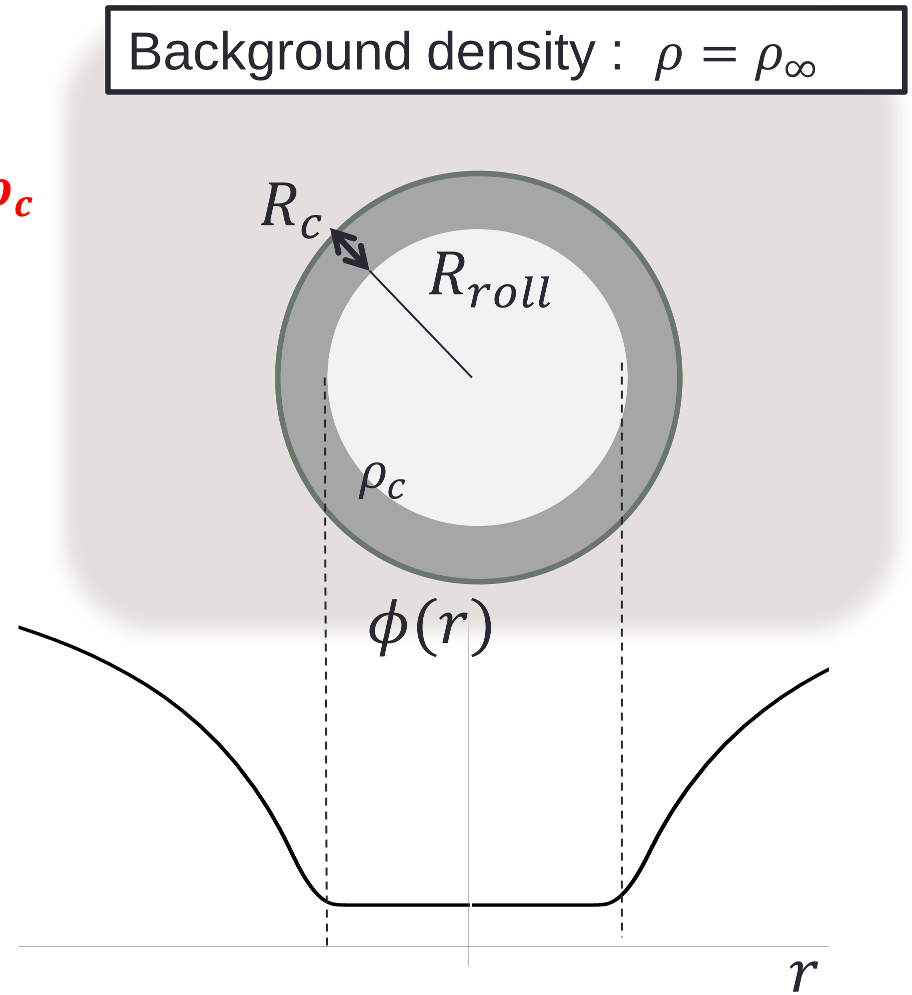
$$\left|\frac{F_\phi}{F_{\rm Newton}}\right| \sim \beta^2$$

from detailed calculation

$$\frac{R_c - R_{roll}}{R_c} \sim \frac{1}{\beta M_{\rm pl}} \frac{\phi_\infty}{\Phi_N} \equiv \epsilon$$

Screening condition

$$\beta^2 \epsilon \sim \frac{\beta}{M_{pl}} \frac{\phi_\infty}{\Phi_N} \ll 1$$

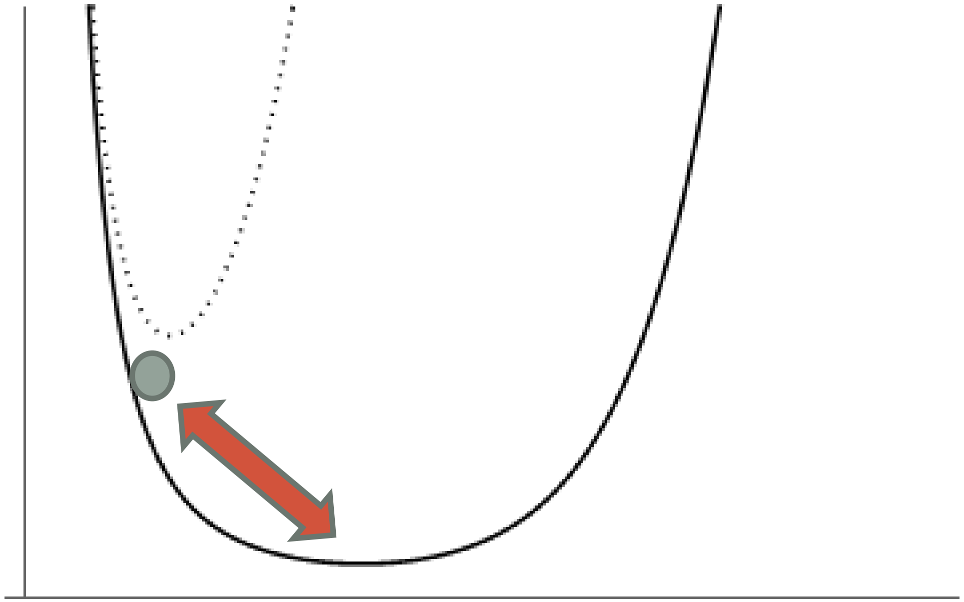


Φ_N : Newton potential at the surface of the star
ϕ_∞ :the potential minimum of effective potential with ρ_c.

In inhomogeneous density profile

The key point in previous calculation is that the scalar field stays at the potential minimum in the object.

If there is a sufficiently large inhomogeneity in the density profile, the potential form will change quickly..



The scalar field cannot stay at the minimum.

Can screening really occur in that situation?

In inhomogeneous density profile

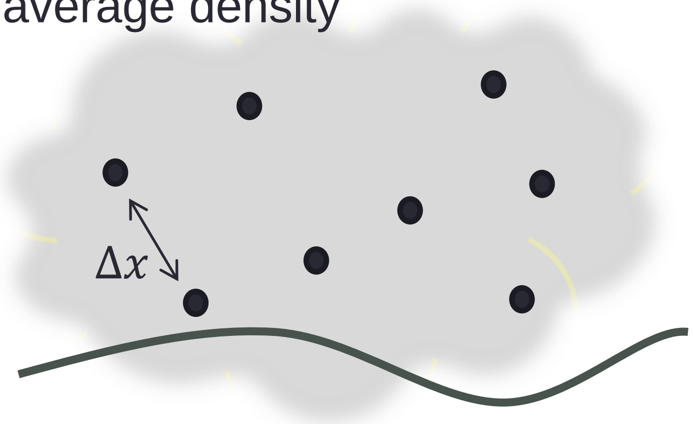
We use an **averaged density** to show the screening .
Furthermore, we ignore the effect of scalar field in inside the object because it has high density.
Is it really valid for the system where its components are separated?

Cf. Compton wavelength in the Galaxy ∼1pc ∼mean stellar distance

- Course-graining and Compton wavelength

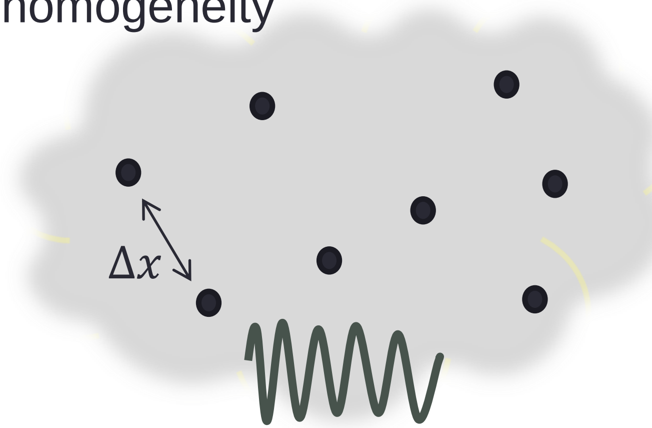
$1/m_\rho > \Delta x$

We can deal the system with average density



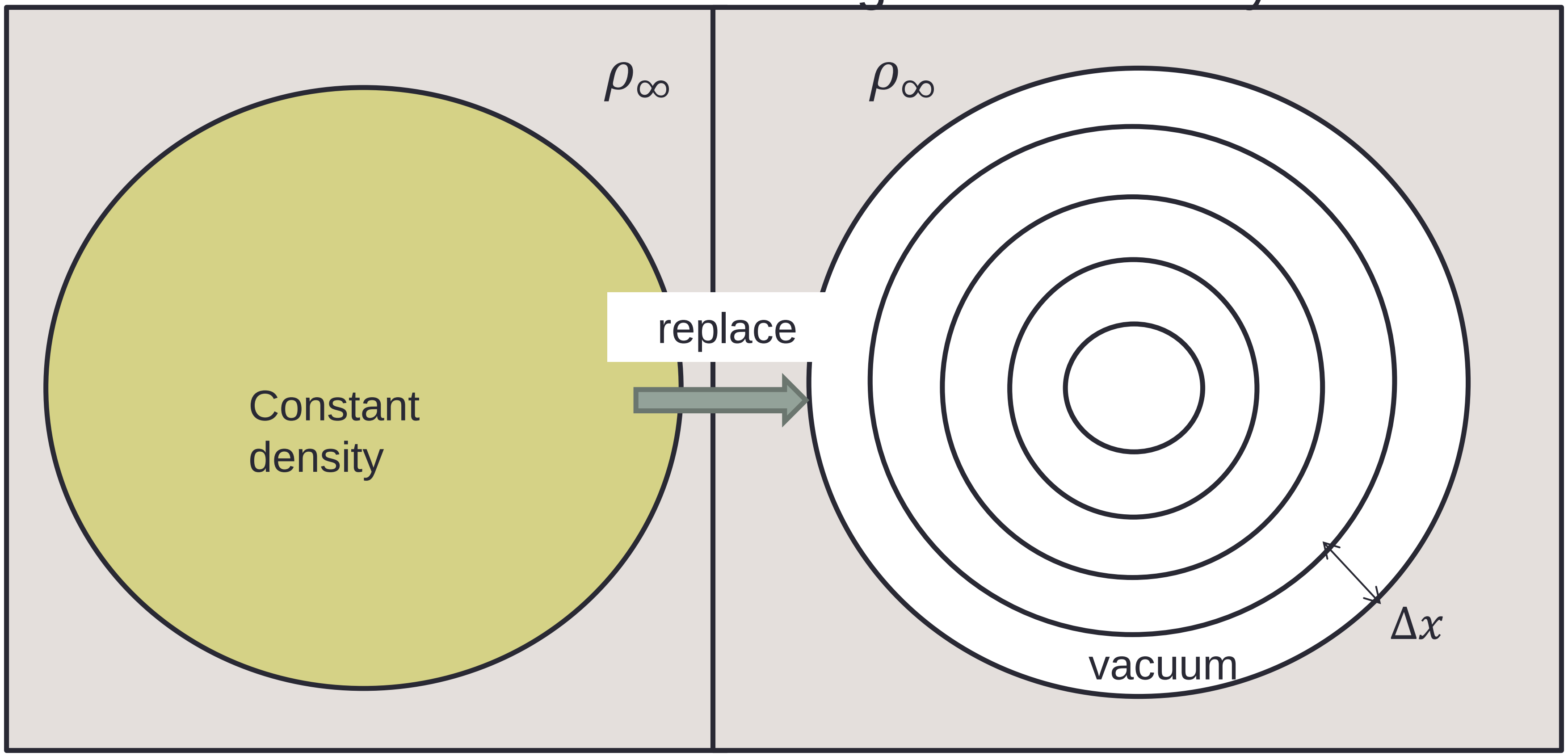
$1/m_\rho < \Delta x$

We have to consider the inhomogeneity



Spherical shell system

Considering a static and spherically symmetric object, we construct a model which has inhomogeneous density as below.



Each shell is infinitely thin, has same density and separated by vacuum regions with equal intervals.

Calculation

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + n\frac{M^{4+n}}{\phi^{n+1}} = 0$$ (inside the object)

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \rho_\infty\frac{\beta}{M_{pl}} + n\frac{M^{4+n}}{\phi^{n+1}} = 0$$ (outside the object)

Junction condition at shells

$$[\phi]^+_- = 0 \qquad \left[\frac{d\phi}{dr}\right]^+_- = \beta\frac{\sigma}{M_{pl}}$$

σ : surface density of the shell

Boundary condition

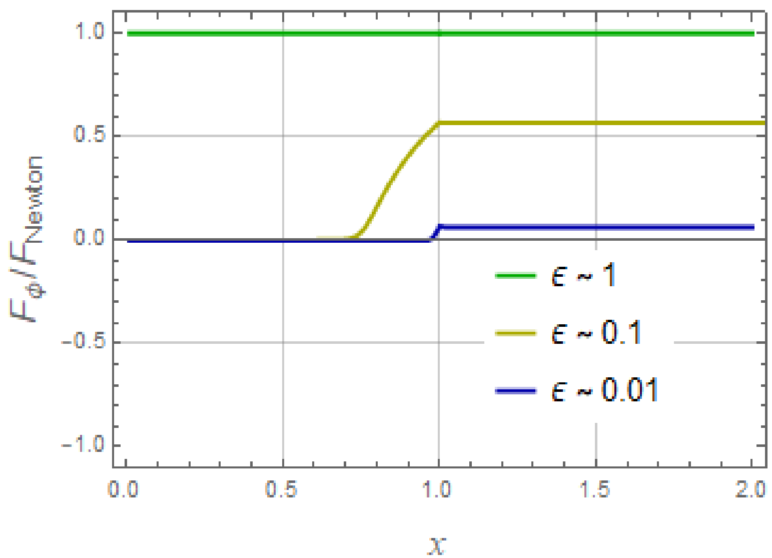
$$\lim_{r\to 0}\frac{d\phi}{dr} = 0$$

$$\lim_{r\to\infty}\phi = \phi_\infty$$

Result

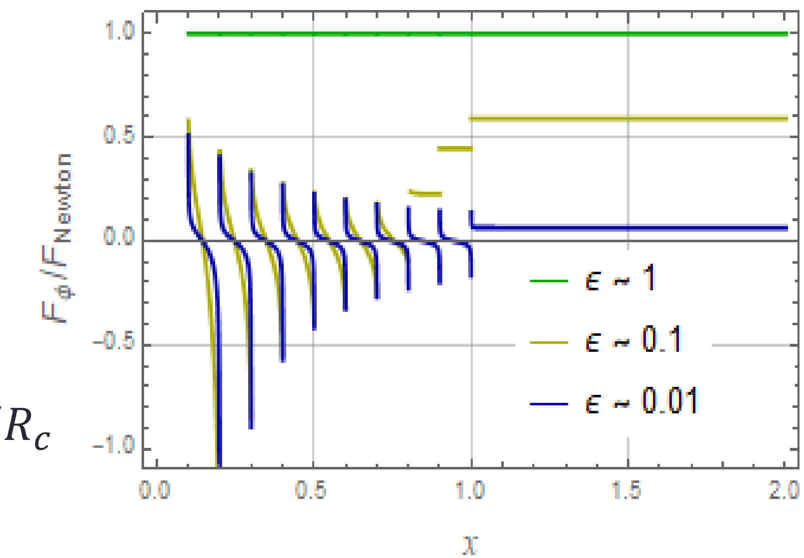
The ratio between **fifth force** and corresponding **Newtonian gravity**

For constant density



For 10 shells $(\Delta x = 0.1)$

$x \equiv r/R_c$

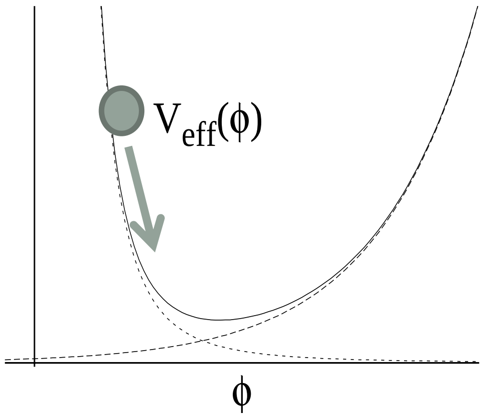


Here, we take β such that $\dfrac{F_\phi}{F_{\rm Newton}} = 6\epsilon$ for constant density case.

outside the object : almost same to the result for the constant case

Inside the object : obvious deviation from the constant case

Analytical calculation

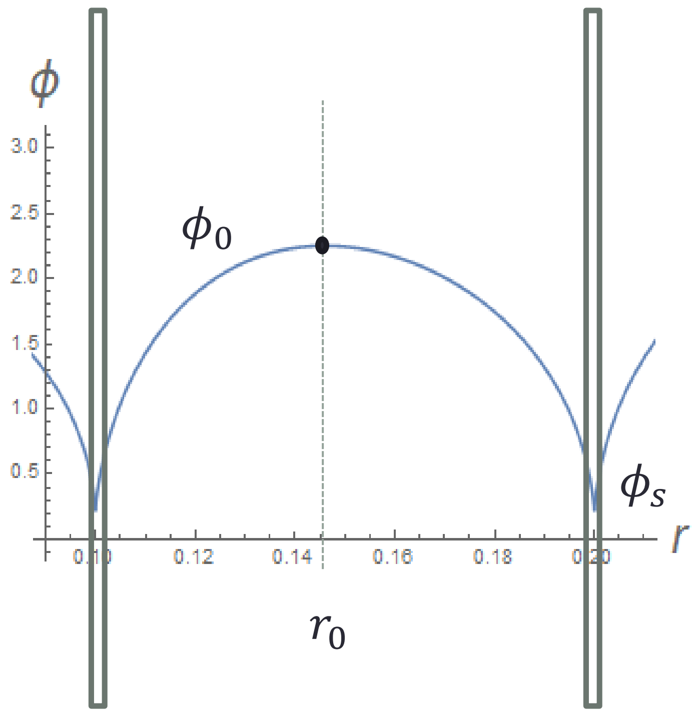


The field is far away from the potential minimum

$$\frac{d^2\phi}{dr^2} + \cancel{\frac{2}{r}\frac{d\phi}{dr}} + n\frac{M^{4+n}}{\phi^{n+1}} = 0$$

For n=2

$$\phi = \sqrt{\phi_0^2 - \frac{M^6}{\phi_0^2}(r - r_0)^2}$$



Using this form and junction condition at shell

$$\frac{F_\phi}{F_{\rm Newton}} \sim \frac{\phi_\infty - \phi_s}{\phi_\infty - \phi_c}\epsilon \qquad \text{(outside the object)}$$

$$\left|\frac{F_\phi}{F_{\rm Newton}}\right| \lesssim \frac{\Delta x}{r} \qquad \text{(inside the object)}$$

Is it an artificial effect from the delta function?

Junction condition for infinitely thin shells

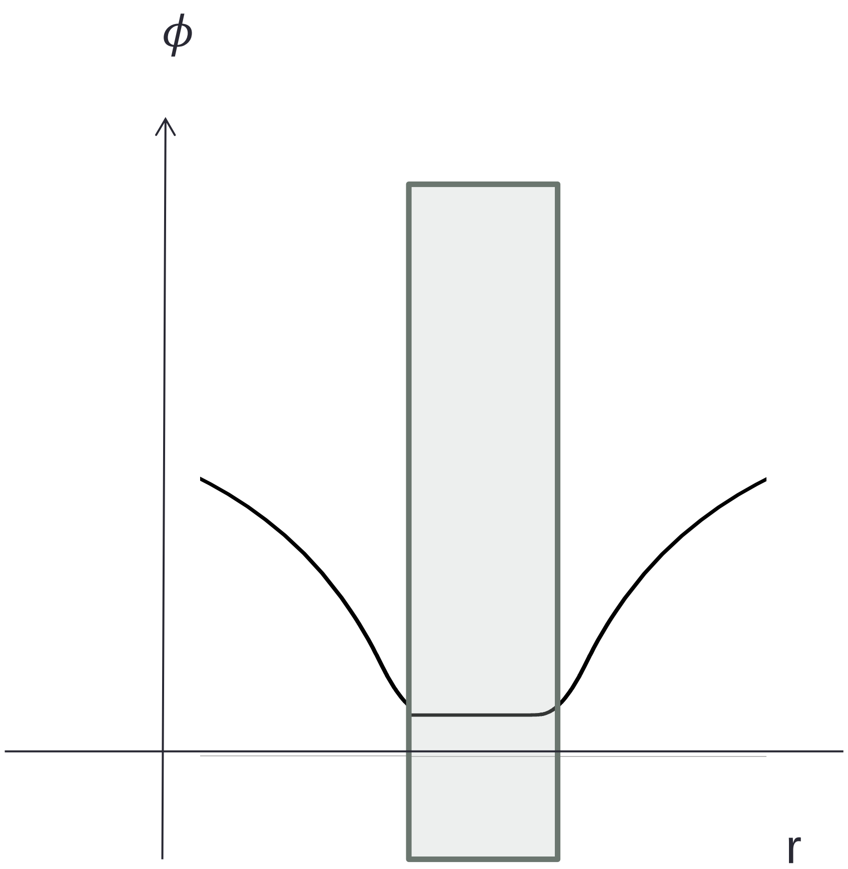
$$\left[\frac{d\phi}{dr}\right]_-^+ = \beta\frac{\sigma}{M_{pl}}$$

For the shells with finite width

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = \frac{\beta}{M_{pl}}\rho - n\frac{M^{4+n}}{\phi^{n+1}}$$

$$\int_V drr^2\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = \int_V drr^2\left(\frac{\beta}{M_{pl}}\rho - n\frac{M^{4+n}}{\phi^{n+1}}\right)$$

$$\left[r^2\frac{d\phi}{dr}\right]_-^+ = \int_V drr^2\left(\frac{\beta}{M_{pl}}\rho - n\frac{M^{4+n}}{\phi^{n+1}}\right)$$



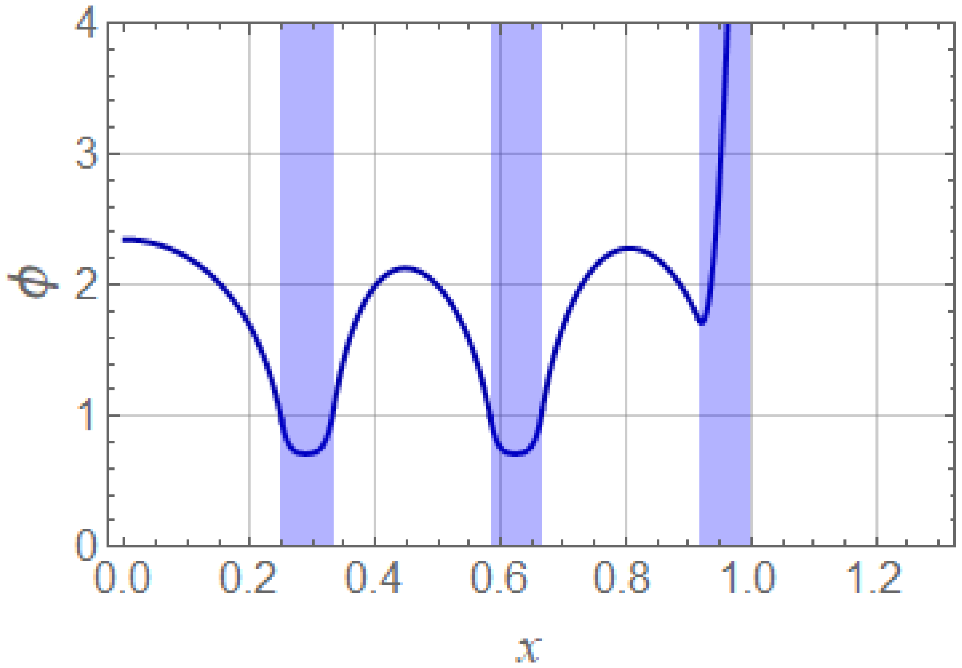
$\phi \to \phi_\rho$ at sufficiently inside the shell

In that region, the right hand side becomes zero.

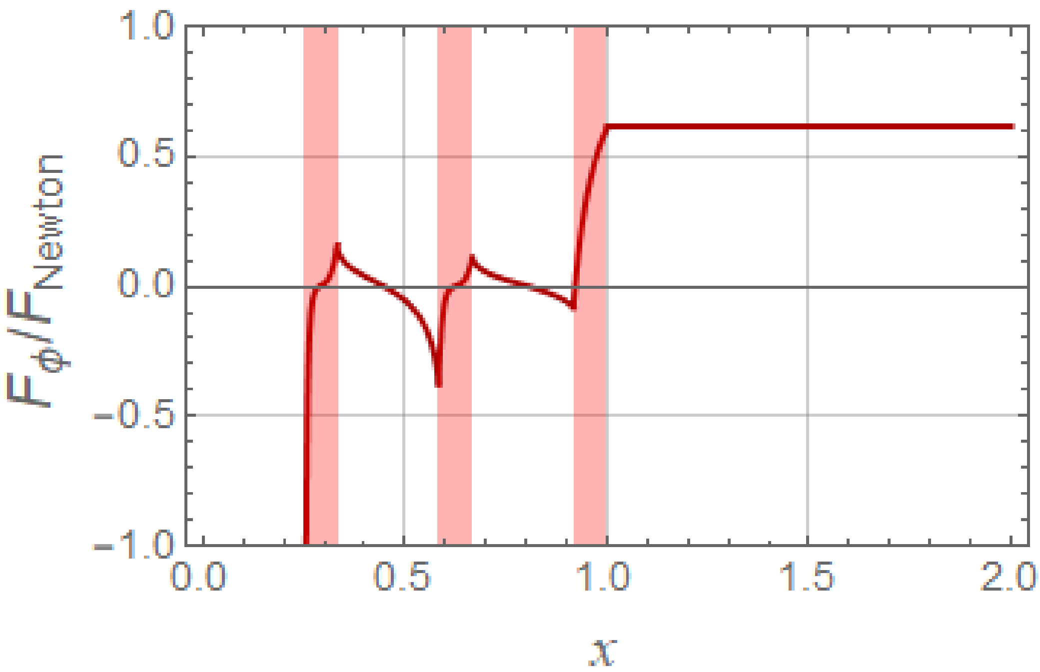
Calculation for the shells with a finite width

we consider the shells have finite width.
Each shell has same width, density and they are separated by equal distance.

Field profile

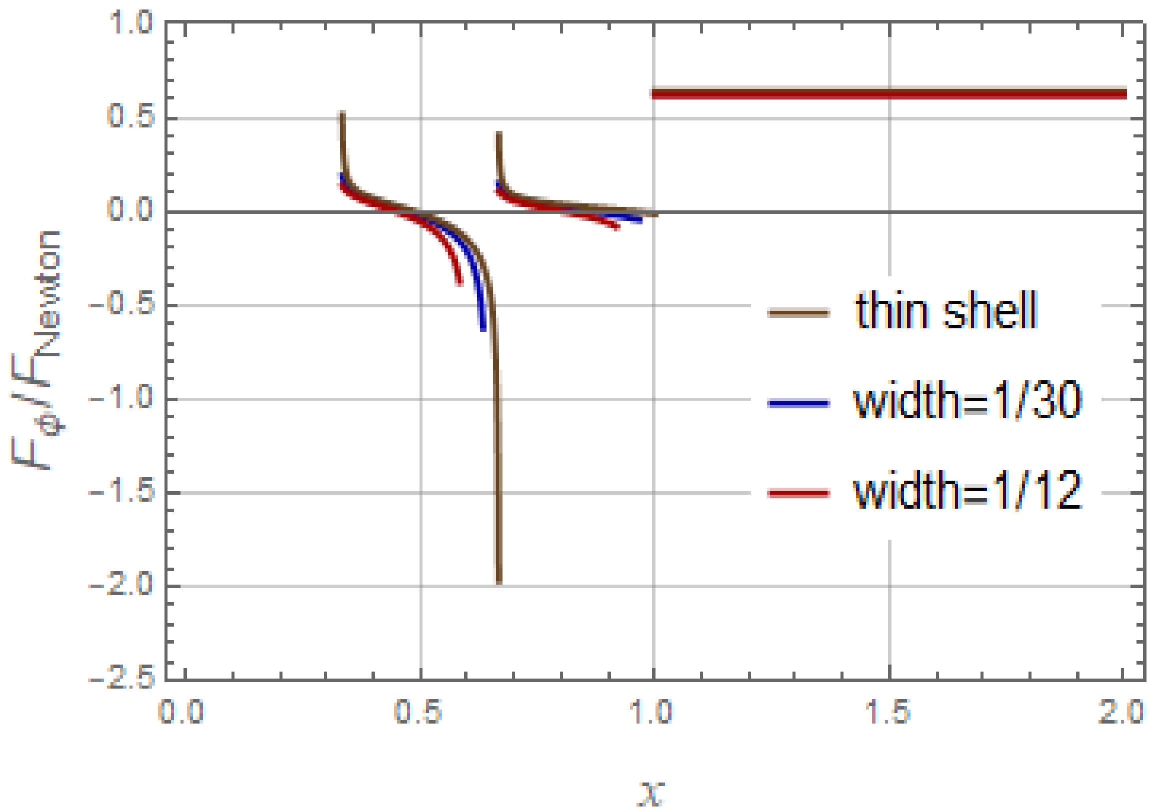


Fifth force strength



$x \equiv r/R_c$

Calculation for the shells with a finite width



The outside is not so much different from previous case.

Fifth force is smaller than infinitely thin shell case at the inside.

As the width becomes smaller, the fifth force value approaches to the previous case.

Summary

- We study the validity of chameleon mechanism in inhomogeneous density profile.
- Screening does work for the outside of the object independent on the inner structure.
- However, for marginal screening case, it may have a significant effect.

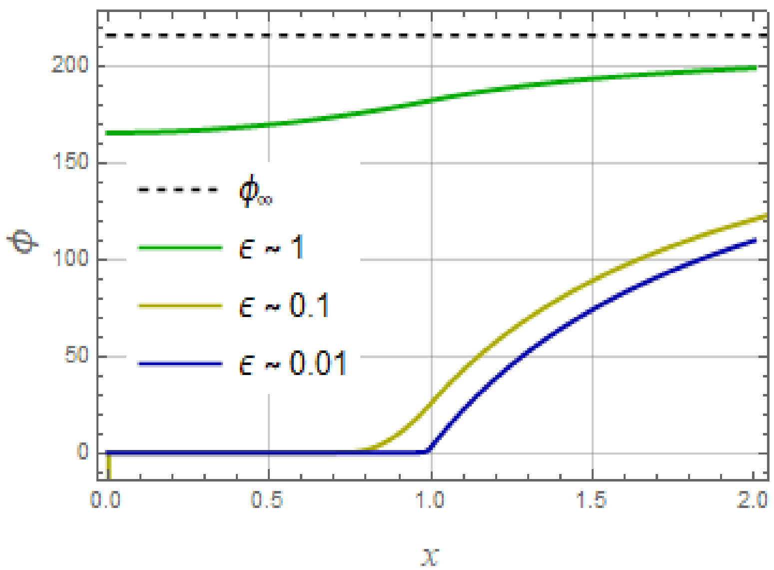
- For the shell system, there is a large fifth force in inside (almost) vacuum region.

- This may indicate the undesirable instability of objects.

Result

Field profile

For constant density



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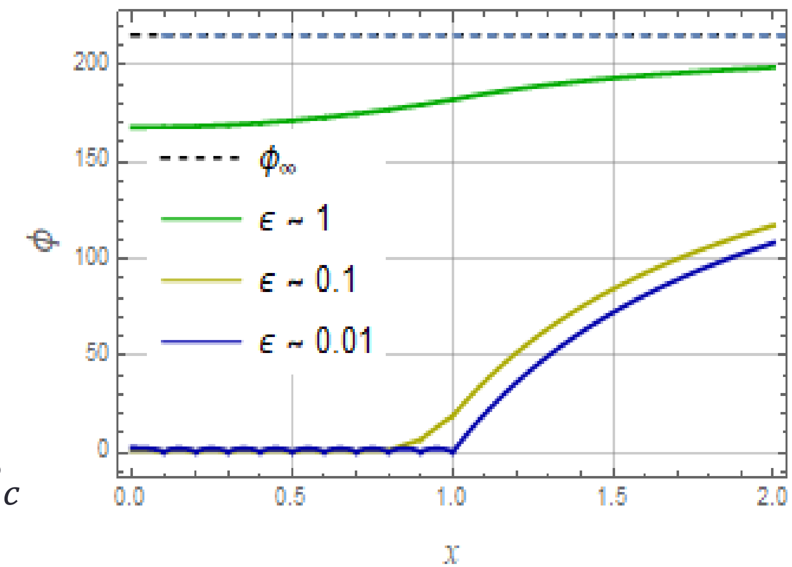


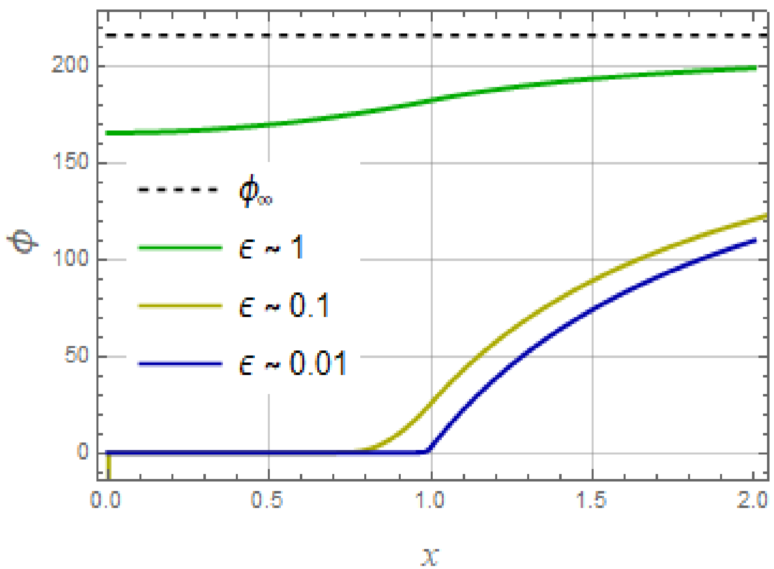
outside the object : almost same to the result for the constant case

Inside the object : obvious deviation from the constant case

Result

Field profile

For constant density



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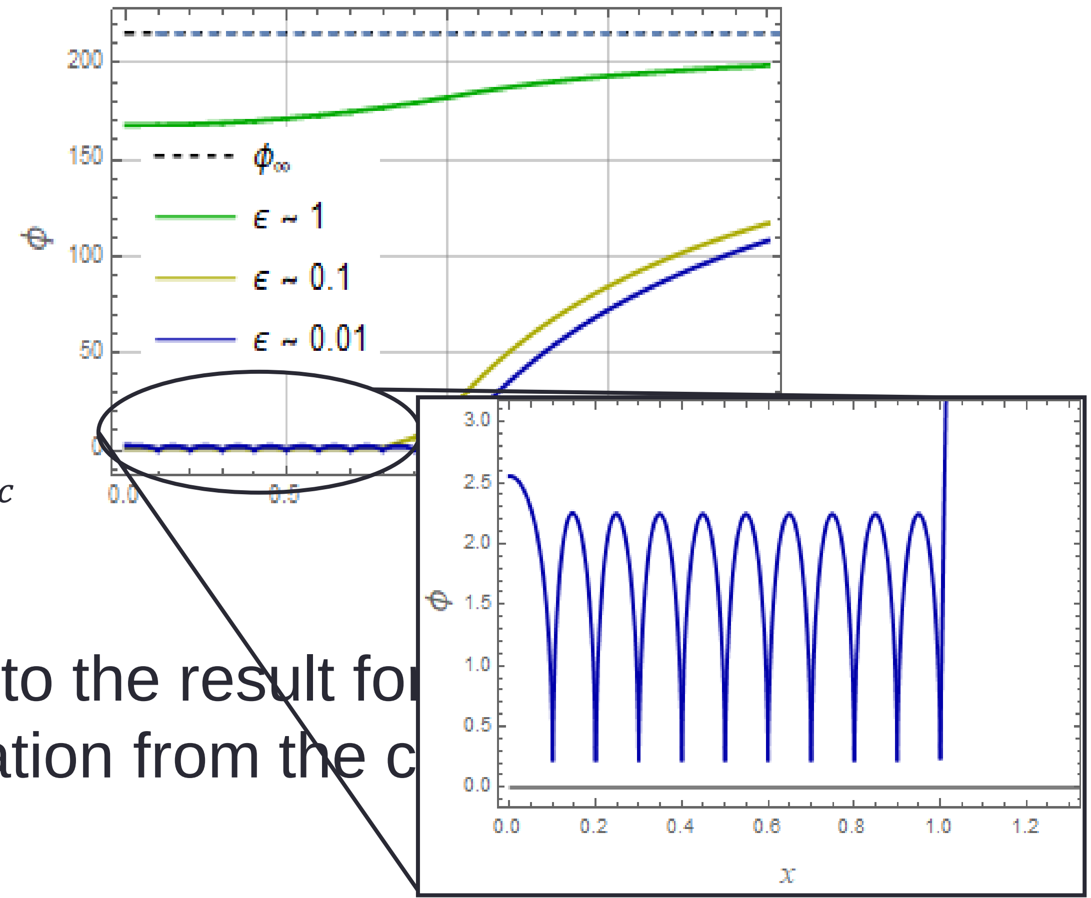

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Instability by the fifth force

Consider the instability of a spherical object

Linear stability of density perturbation for some star models has been already calculated.[Sakstein (2013)]

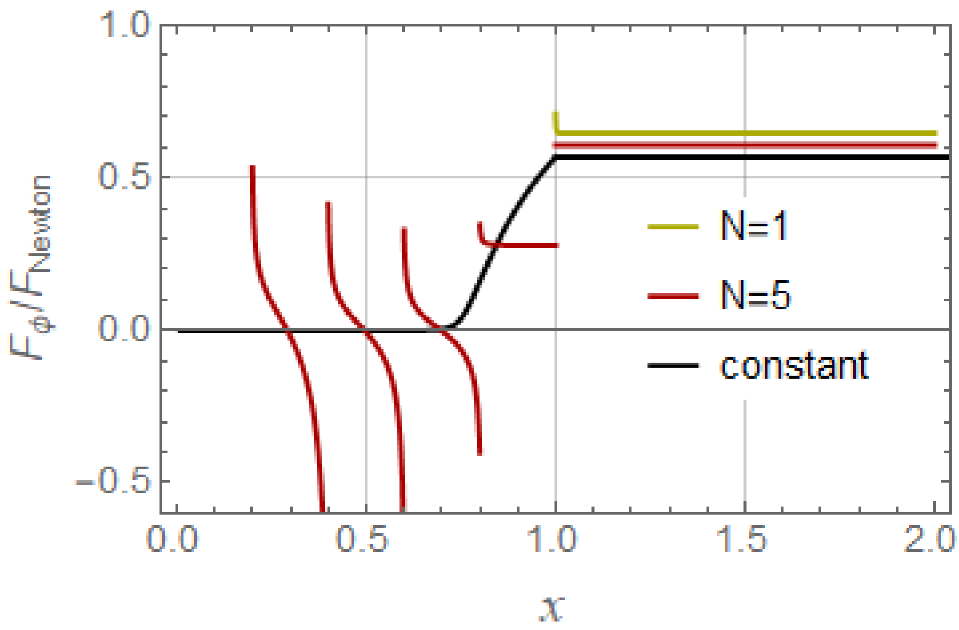
Linear stability of scalar field perturbation has also been calculated.[Silvestri(2011)]

Our result may suggest a non-linear level instability which does not occur in GR case.

Marginal screening case

As indicated from analytical form, if the thin shell parameter ϵ is not so small, there is a certain level of difference in fifth force strength

$\epsilon \sim 10^{-3}$



Inhomogeneity has significant effect for marginal screening case.