MATTER BISPECTRUM BEYOND HORNDESKI

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(進一　平野)

based on

SH, T. Kobayashi, S. Yokoyama (Nagoya U.), T. Hiroyuki (Nagoya U.) **1801. 07885**

SH, T. Kobayashi, D. Yamauchi (Kanagawa U.), S. Yokoyama, in preparation

MOGRA2018, August 8-10 ,2018

Introduction

- **Modified gravity: alternative to cosmological constant**
  - Cosmological scale: late-time acceleration
  - Small scale: recovering the result of gravitational test

    ⇒ screening mechanism

- **Horndeski theory** Horndeski (1972), Kobayashi+ (2011), Deffaiyet+ (2011)
  - Most general scalar-tensor theory with 2nd-order EoMs
  - Vainshtein screening thanks to 2nd-order derivative non-linear ints.
  - GW170817, GRB170817A: $|c_T - 1| < 10^{-15}$ Abbott+ (2017)

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{f(\phi)}{2} R + G_2(\phi, X) - G_3(\phi, X)\Box\phi$$

Recent progress & our work

- **Beyond Horndeski** (higher-order EoMs, no Ostrogradski ghost)

    GLPV theory   Gleyzes+ (2014), Gleyzes+ (2015)

    DHOST theory   Langlois & Noui (2015), Achour+ (2016), Achour+ (2016)

✓ Models with $(\partial\partial\phi)^2$ and $c_T = 1$

✓ Partial breaking of Vainshtein screening inside matter ($\delta \gg 1$)

non-linear int.   Kobayashi+ (2015), Langlois+ (2017), …

Our aim

How much is the effect of non-linear ints. at "*cosmological scale*" ?

$\delta \ll 1$

Matter bispectrum beyond Horndeski

Plan of talk

- Our setup
- Cosmological perturbations
- Matter bispectrum beyond Horndeski
- Summary

OUR SETUP

quadratic DHOST

Langlois, Noui (2015,2016), Koyama+ (2016), de Rham, Matas (2016)

$$\frac{\mathcal{L}_{\rm qD}}{\sqrt{-g}} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma}$$

$$C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma} = a_1(\phi, X)\phi_{\mu\nu}^2 + a_2(\phi, X)(\Box\phi)^2 + a_3(\phi, X)\Box\phi(\phi^\mu\phi_{\mu\nu}\phi^\nu)$$
$$+ a_4(\phi, X)\phi^\mu\phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho + a_5(\phi, X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2$$

($X = -\frac{1}{2}(\nabla\phi)^2, \phi_\mu = \nabla_\mu\phi, \Box\phi = \nabla^2\phi, \phi_{\mu\nu} = \nabla_\nu\nabla_\mu\phi$)

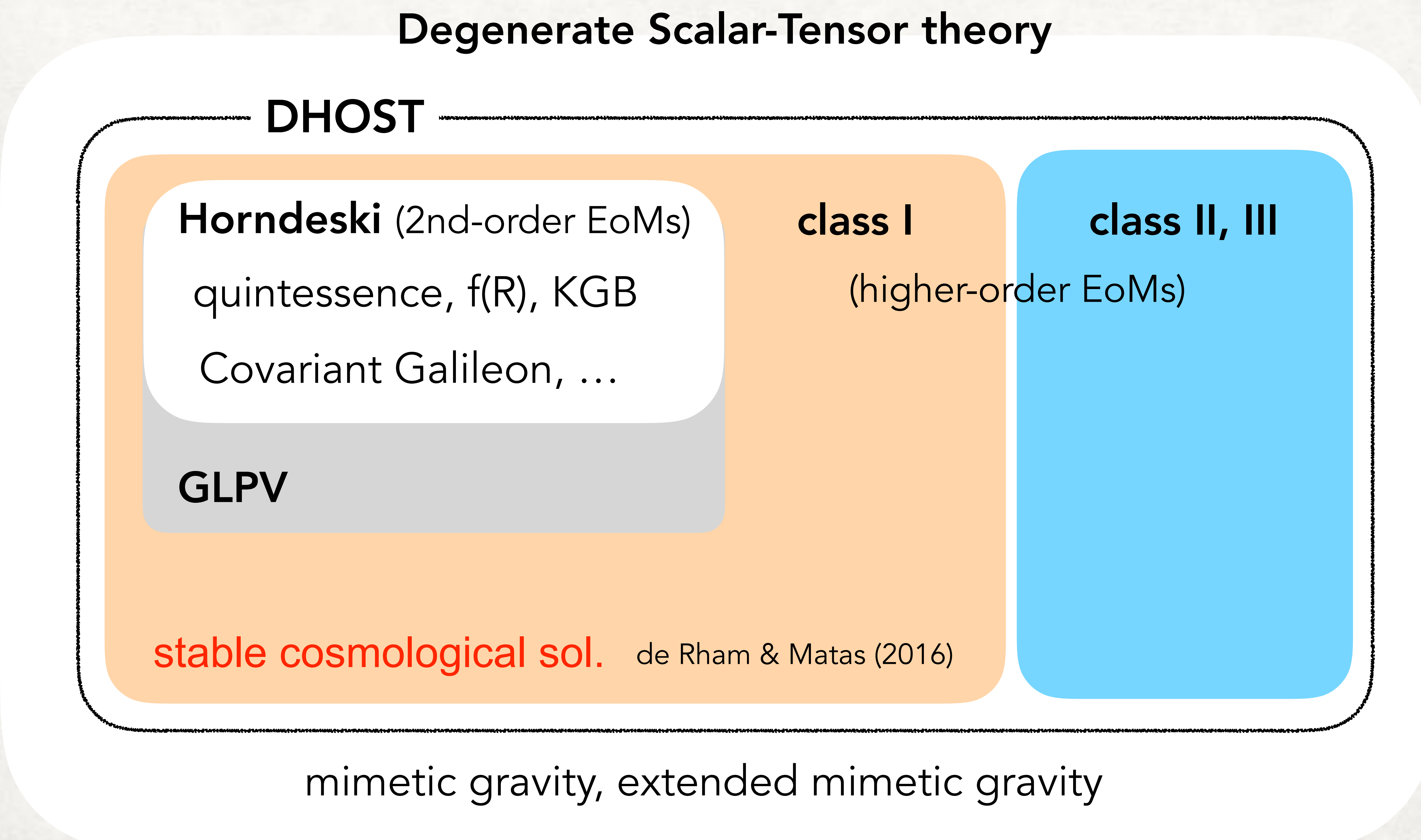
non-linear ints.

- includes Horndeski and GLPV at Lagrangian level

  Horndeski: $a_1 = -a_2 = -G_{4X},\ a_3 = a_4 = a_5 = 0$, GLPV: …

- The non-trivial relation between arbitrary func. $(G_4, C_{(2)})$ in order to evade Ostragradski ghost ← "degeneracy conditions"

Viable DHOST after GW170817



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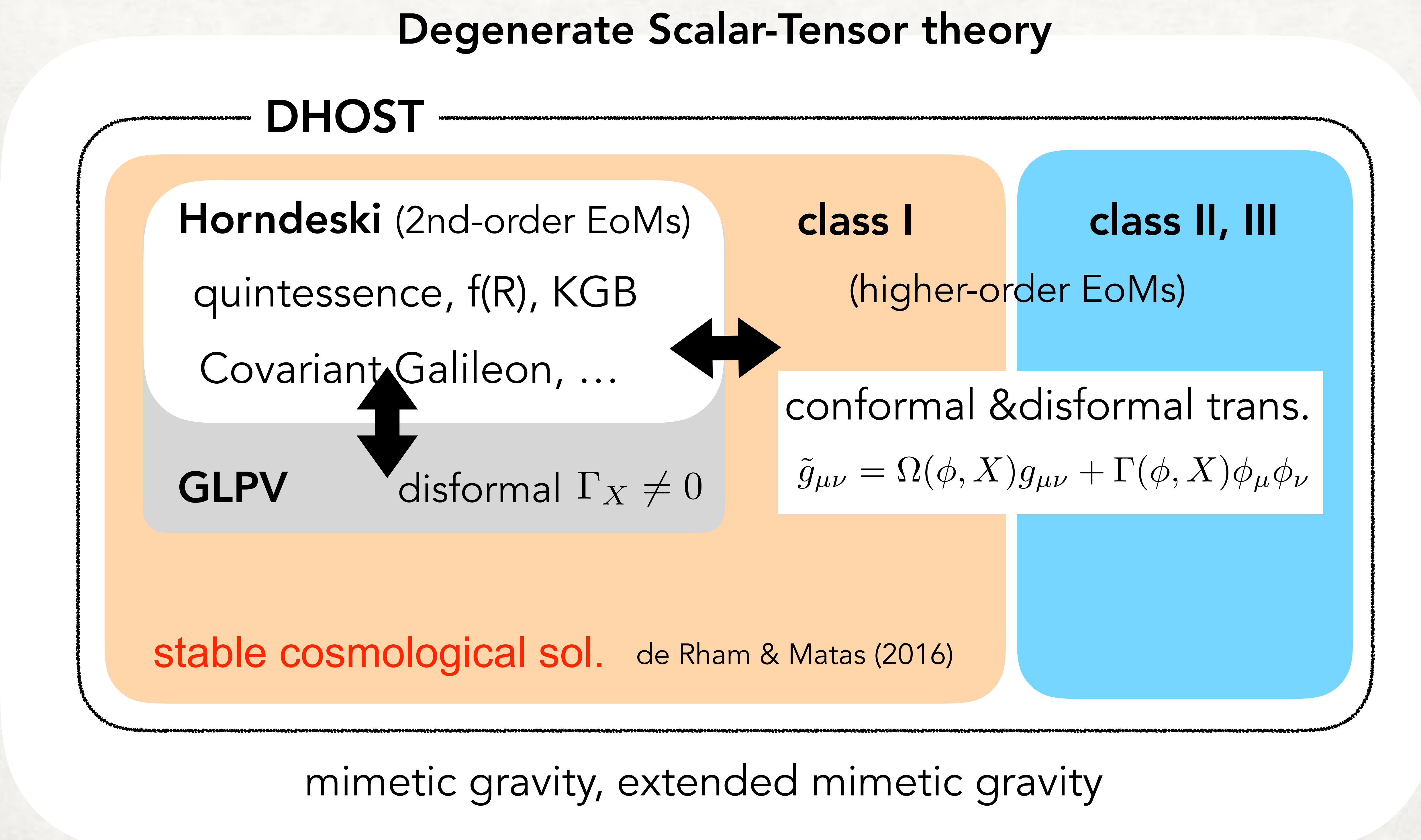

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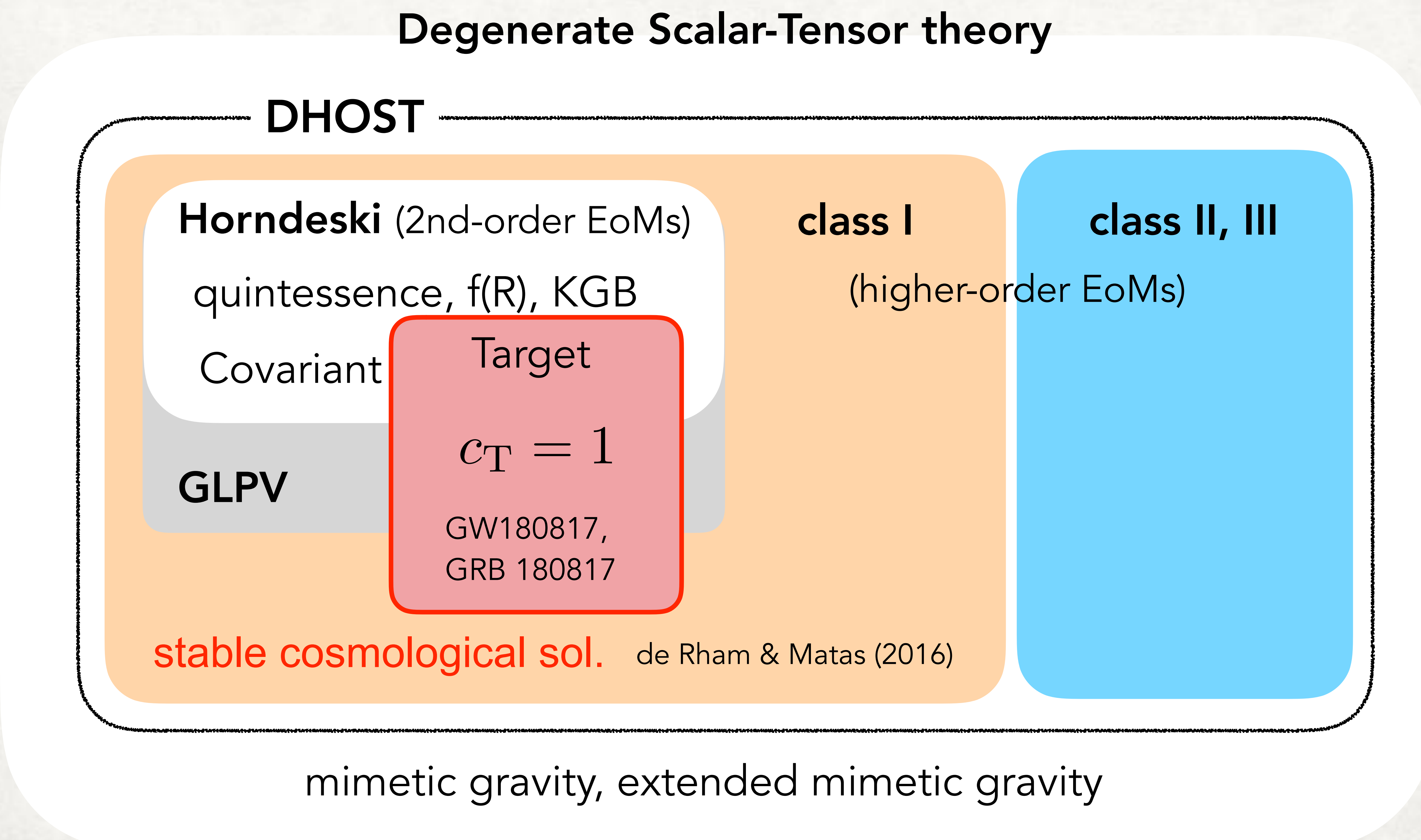


Parametrization

Bellini & Sawicki (2014), Gleyzes et al. (2015)
Langlois+ (2017), Dima & Vernizzi (2017)

$$S^{\rm eff} = \int d^4x\,\sqrt{\gamma}\,\frac{M^2}{2}\Big[-\mathcal{K}_2 + c_T^2 R^{(3)} + H^2\alpha_K \delta N^2 + 4H\alpha_B \delta K \delta N$$

$$+ (1+\alpha_H)R^{(3)}\delta N + (1-\alpha_H)\delta N \delta\mathcal{K}_2 + 4\beta_1 \delta K \tilde{V} + \beta_2 \tilde{V}^2 + \beta_3 a_i^2\Big].$$

$$\mathcal{K}_2 := K_{ij}^2 - K^2,\ \tilde{V} := \frac{1}{N}(\dot{N} - N^i\partial_i N),\ a_i := \partial_i N/N$$

depend on β_1 through degeneracy cond.

- **alpha-parameters**

α_K: kineticity … non-standard kinetic terms

α_B: braiding … kinetic mixing between scalar and metric

α_M : time evolution of M

α_H : disformal coupling to matter → **GLPV**

β_1 : conformal & disformal coupling to matter → **DHOST**

Viable conditions

Kimura+ (2011)
Kobayashi+ (2015)

- **Early time**

$M_{\rm pl}^2/2$ (GR)

$$M^2 \approx 2G_4 := \mathcal{O}(M_{\rm pl}^2),\ (\alpha_i, \beta_1) \ll 1$$

$$\Rightarrow\ 3M^2H^2 \approx \rho_{\rm m},\ \rho_{\rm m} \gg \rho_\phi$$

※ we do not consider quintessential inflation and early dark energy scenarios.

- **Late time** (after MD)

$$\phi \sim M_{\rm pl},\ \dot{\phi} \sim M_{\rm pl}H_0,\ \ddot{\phi} \sim M_{\rm pl}H_0^2,$$
$$G_2 \sim M_{\rm pl}^2 H_0^0,\ G_3 \sim M_{\rm pl},\ G_4 \sim M_{\rm pl}^2,\ \cdots$$

$$\Rightarrow\ 3M^2H^2 \approx \rho_\phi,\ \rho_\phi \gg \rho_{\rm m},\ \alpha_i = \mathcal{O}(1),\ \beta_1 = \mathcal{O}(1)$$

Vainshtein screening around matter, **its breaking inside matter**

COSMOLOGICAL PERTURBATIONS

Cosmological perturbations

Sub-horizon ($aH \ll k$), late time (after MD)

$Q = H\pi/\dot{\phi}$

- **perturbations**

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Psi)d\boldsymbol{x}^2.$$

$$\phi(t,\boldsymbol{x}) = \phi(t) + \pi(t,\boldsymbol{x}),\;\; \rho(t,\boldsymbol{x}) = \rho(t)[1+\delta(t,\boldsymbol{x})].$$

- **Quasi-static approximation (QSA)**

$$k_{sh} := \frac{aH}{c_s} \ll k \quad \Rightarrow \quad |\dot{\epsilon}| \approx |H\epsilon|,\;\; \epsilon = \Psi, \Phi, Q$$

$$|\dot{\Psi}|^2, |\dot{\Phi}|^2, |\dot{Q}|^2 \ll k^2\Psi^2, k^2\Phi^2, k^2Q^2$$

Note: $0 \neq \alpha_i \ll 1 \quad \Rightarrow \; H^2\epsilon^2 \sim \alpha_i k^2 \epsilon^2$

cf) f(R) $\quad G_{\rm eff} = G_{\rm eff}(k,t)$

In this work

$\alpha_i \sim \alpha_j = \mathcal{O}(1)$

Evolution of density fluctuations

1. Perturbative expansion: $\epsilon = \epsilon_1 + \epsilon_2, \ \epsilon = \Psi, \Phi, Q, \delta$

2. **EoMs**: $\delta\Psi, \delta\Phi, \delta Q \quad \Rightarrow \quad \Phi_1, \Phi_2$

↑ **include the effect of modified gravity**

3. Fluid equations:

continuity/ Eular

(usual forms)

$$\frac{\partial \delta(t, \boldsymbol{x})}{\partial t} + \frac{1}{a}\partial_i[(1+\delta)u^i(t, \boldsymbol{x})] = 0,$$

$$\frac{\partial u^i}{\partial t} + Hu^i + \frac{1}{a}u^j\partial_j u^i = -\frac{1}{a}\partial^i \Phi(t, \boldsymbol{x})$$

⇒ One obtain the evolution equation of density contrast

EoMs of gravitational fields

Green: GLPV, **Red**: DHOST $\quad A, B \supset \alpha_i, \beta_1$

$(\delta\Phi)$

$$\mathcal{G}_T\partial^2\Psi + \tilde{A}_2\partial^2 Q - A_6\partial^2\Phi + A_8\frac{\partial^2\dot{Q}}{H} - \frac{a^2}{2}\rho_{\rm m}\delta = -\frac{B_2}{2a^2H^2}\mathcal{Q}_2 + \frac{B_5}{a^2H^2}\left[(\partial_i\partial_j Q)^2 + \partial_i Q\partial^i\partial^2 Q\right]$$

$(\delta\Psi)$

$$\mathcal{F}_T\partial^2\Psi - \mathcal{G}_T\partial^2\Phi - \tilde{A}_1\partial^2 Q + A_4\frac{\partial^2\dot{Q}}{H} = \frac{B_1}{2a^2H^2}\mathcal{Q}_2 + \frac{B_4}{a^2H^2}\left[(\partial_i\partial_j Q)^2 + \partial_i Q\partial^i\partial^2 Q\right]$$

(δQ)

$$A_0\partial^2 Q - A_1\partial^2\Psi - A_2\partial^2\Phi - A_4\frac{\partial^2\dot{\Psi}}{H} + A_8\frac{\partial^2\dot{\Phi}}{H} - \tilde{A}_9\frac{\partial^2\dot{Q}}{H} - A_9\frac{\partial^2\ddot{Q}}{H^2}$$
$$= -\frac{B_0}{a^2H^2}\mathcal{Q}_2 + \frac{B_2}{a^2H^2}(\partial^2\Phi\partial^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q)$$
$$-\frac{B_4}{a^2H^2}\left(\partial^2\Psi\partial^2 Q + \partial_i Q\partial^i\partial^2\Psi\right) + \frac{B_5}{a^2H^2}(\partial^2\Phi\partial^2 Q + \partial_i Q\partial^i\partial^2\Phi)$$
$$-\frac{\tilde{B}_6}{a^2H^2}\left[(\partial_i\partial_j Q)^2 + \partial_i Q\partial^i\partial^2 Q\right]$$
$$-\frac{B_6}{a^2H^2}\frac{1}{H}\left(\partial^2 Q\partial^2\dot{Q} + 2\partial_i Q\partial^i\partial^2\dot{Q} + 2\partial_i\partial_j Q\partial^i\partial^j\dot{Q} + \partial_i\partial^2 Q\partial^i\dot{Q}\right)$$

EoMs of gravitational fields

- **linear level, GR** $\mathcal{G}_T = \mathcal{F}_T = M_{\rm pl}^2$

$(\delta\Phi)$ Poisson equation

$$\mathcal{G}_T \partial^2 \Psi - \frac{a^2}{2}\rho_{\rm m}\delta = 0$$

$(\delta\Psi)$ trace component of Einstein tensor

$$\mathcal{F}_T \partial^2 \Psi - \mathcal{G}_T \partial^2 \Phi = 0$$

(δQ) $\delta Q = 0$

EoMs of gravitational fields

- **linear level, Horndeski**

contribution of scalar field

$(\delta\Phi)$

$$\mathcal{G}_T\partial^2\Psi + \tilde{A}_2\partial^2 Q - \frac{a^2}{2}\rho_{\rm m}\delta = 0$$

as anisotropic stress

$(\delta\Psi)$

$$\mathcal{F}_T\partial^2\Psi - \mathcal{G}_T\partial^2\Phi - \tilde{A}_1\partial^2 Q = 0$$

(δQ) $$A_0\partial^2 Q - A_1\partial^2\Psi - A_2\partial^2\Phi = 0$$

$\Rightarrow$ **increase of gravitational constant**

EoMs of gravitational fields

- **linear level, beyond Horndeski**

Green: GLPV, Red: DHOST

$\mathcal{O}(\dot{\delta})$

$(\delta\Phi)$

$$\mathcal{G}_T\partial^2\Psi+\tilde{A}_2\partial^2 Q-A_6\partial^2\Phi+A_8\frac{\partial^2\dot{Q}}{H}-\frac{a^2}{2}\rho_{\rm m}\delta=0$$

$(\delta\Psi)$

$\mathcal{O}(\ddot{\delta})$

$$\mathcal{F}_T\partial^2\Psi-\mathcal{G}_T\partial^2\Phi-\tilde{A}_1\partial^2 Q+A_4\frac{\partial^2\dot{Q}}{H}=0$$

(δQ)

$$A_0\partial^2 Q-A_1\partial^2\Psi-A_2\partial^2\Phi-A_4\frac{\partial^2\dot{\Psi}}{H}+A_8\frac{\partial^2\dot{\Phi}}{H}-\tilde{A}_9\frac{\partial^2\dot{Q}}{H}-A_9\frac{\partial^2\ddot{Q}}{H^2}=0$$

$\Rightarrow$ **increase of gravitational constant,**

additional friction term

1st-order solution

Kobayashi+ (2015), D'Amico+ (2017), Chrisostomi & Koyama (2017)

$$\ddot{\delta}_1 + (2+\varsigma)H\dot{\delta}_1 - 4\pi G_{\rm eff}\rho_{\rm m}\delta_1 = 0$$

- $G_{\rm eff}(t)$: G (GR), $G \rightarrow G_{\rm eff}$ (Horndeski, beyond Horndeski)

  $\varsigma(t) \propto \alpha_H, \beta_1$: 0 (GR, Horndeski), $0 \rightarrow \varsigma_0$ (beyond Horndeski)

- growing mode: $\delta_1(\mathbf{p},t) = D_+(t)\delta_L(\mathbf{p})$

  $D_+(t)$: growth factor, $\delta_L(\mathbf{p})$: initial density fluc.

- change in the growth of density fluctuation due to ς

  cf.) improvement of $f\sigma_8$ Tsujikawa (2015), D'Amico+ (2017)

EoMs of gravitational fields

- **non-linear level, GR** $\mathcal{G}_T = \mathcal{F}_T = M_{\rm pl}^2$

$(\delta\Phi)$ Poisson equation

$$\mathcal{G}_T \partial^2 \Psi - \frac{a^2}{2}\rho_{\rm m}\delta = 0$$

remains usual form

$(\delta\Psi)$ trace component of Einstein tensor

$$\mathcal{F}_T \partial^2 \Psi - \mathcal{G}_T \partial^2 \Phi = 0$$

remains usual form

(δQ) $\delta Q = 0$

⇒ Non-linearity only derives from fluid equations

EoMs of gravitational fields

- **non-linear, Horndeski**

$$\mathcal{Q}_2 = (\partial^2 Q)^2 - (\partial_i \partial_j Q)^2$$

$(\delta\Phi)$

$$\mathcal{G}_T \partial^2 \Psi + \tilde{A}_2 \partial^2 Q - \frac{a^2}{2}\rho_{\rm m}\delta = -\frac{B_2}{2a^2H^2}\mathcal{Q}_2$$

$(\delta\Psi)$

$$\mathcal{F}_T \partial^2 \Psi - \mathcal{G}_T \partial^2 \Phi - \tilde{A}_1 \partial^2 Q = \frac{B_1}{2a^2H^2}\mathcal{Q}_2$$

(δQ)

$$A_0 \partial^2 Q - A_1 \partial^2 \Psi - A_2 \partial^2 \Phi = -\frac{B_0}{a^2H^2}\mathcal{Q}_2 + \frac{B_2}{a^2H^2}(\partial^2\Phi\partial^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q)$$

additional non-linearity from scalar field

⇒ novel probe of modified gravity !

(at a quasi non-linear regime)

2nd-order solution

Horndeski: Takushima+ (2013)
GLPV: Hirano+ (2018)

DHOST: Hirano+ in prep.

$$\ddot{\delta}_2 + (2+\varsigma)H\dot{\delta}_2 - 4\pi G_{\rm eff}\rho_{\rm m}\delta_2 = S_\delta \quad \leftarrow \delta_1^2$$

Primordial fluc. : **Gaussian** ⇒ inhomogeneous sol.

$$\Rightarrow \quad \delta_2(\mathbf{p},t) = D_+^2(t)[\kappa(t)\mathcal{W}_\alpha(\mathbf{p}) + \lambda(t)\mathcal{W}_\gamma(\mathbf{p})]$$

$$\mathcal{W}_i(\boldsymbol{p}) = \frac{1}{(2\pi)^3}\int d^3k_1 d^3k_2\, \delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{p})\mathcal{E}(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\delta_L(\boldsymbol{k}_1)\delta_L(\boldsymbol{k}_2)$$

$$\mathcal{E} = \alpha,\gamma\ ,\ \alpha(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)(k_1^2+k_2^2)}{2k_1^2k_2^2}\ ,\ \gamma(\mathbf{k}_1,\mathbf{k}_2) = 1 - \frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}$$

$\lambda(t)$: 1 (GR) , $1 \to \lambda_0 \neq 1$ (Horndeski, beyond Horndeski)

New! $\kappa(t) \supset \alpha_H, \beta_1$: 1 (GR, Horndeski), $1 \to \kappa_0 \neq 1$ (beyond Horndeski)

MATTER BISPECTRUM BEYOND HORNDESKI

Hirano+ (2018), Hirano+ in prep.

Matter bispectrum

cf) Scoccimarro+ (1998)
Barnardeau+ (2000)

- **Correlation function**

$$\langle \delta(t,\boldsymbol{k}_1)\delta(t,\boldsymbol{k}_2)\delta(t,\boldsymbol{k}_3)\rangle := (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$

- **Leading order (tree-level)**

$$B(t,k_1,k_2,k_3) = 2D_+^4 F_2(t,\boldsymbol{k}_1,\boldsymbol{k}_2)P_{11}(k_1)P_{11}(k_2) + 2 \text{ cyclic terms}$$

Kernel $F_2(t,\mathbf{k}_1,\mathbf{k}_2) = \kappa(t)\alpha(\mathbf{k}_1,\mathbf{k}_2) - \frac{2}{7}\lambda(t)\gamma(\mathbf{k}_1,\mathbf{k}_2)$

$P_{11}(k)$: initial power spectrum

$W_\alpha,\ W_\gamma$

Matter bispectrum

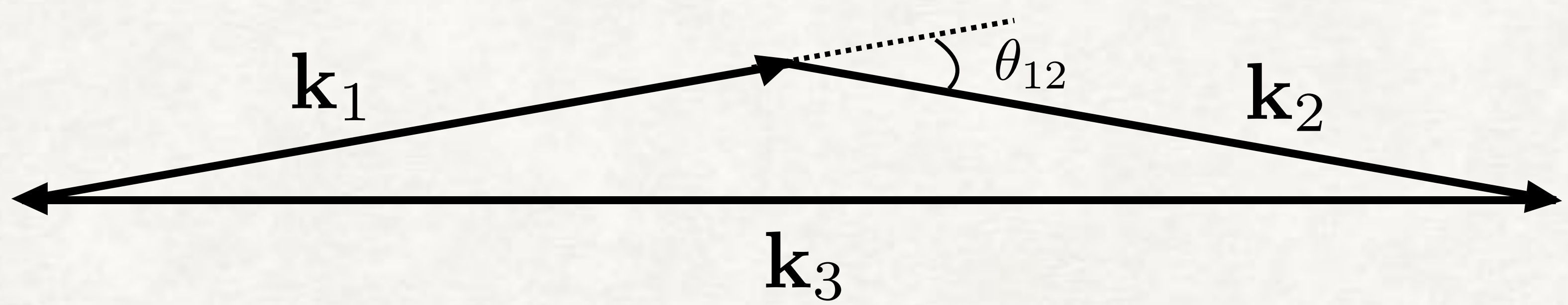
cf) Scoccimarro+ (1998)
Barnardeau+ (2000)

- **Reduced bispectrum**

$$Q_{123}(t,k_1,k_2,k_3) = \frac{B(t,k_1,k_2,k_3)}{D_+^4(t)[P_{11}(k_1)P_{11}(k_2) + 2 \text{ cyclic terms}]}$$

is sensitive to time evolution of κ and λ

✓ $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = \mathbf{0} \Rightarrow \boldsymbol{k}_1 = (0,0,k_1), \ \boldsymbol{k}_2 = (0, k_2 \sin\theta_{12}, k_2\cos\theta_{12}),$
$\boldsymbol{k}_3 = (0, -k_2\sin\theta_{12}, -k_1 - k_2\cos\theta_{12})$



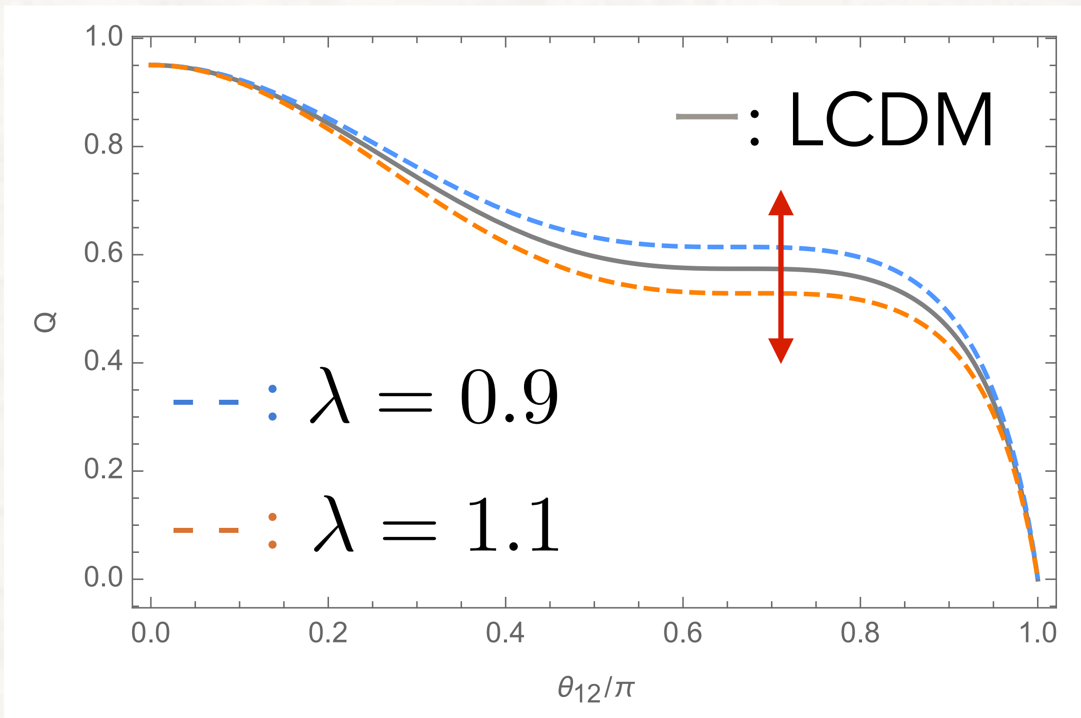
✓ We estimate matter bispectrum on the given $\kappa, \ \lambda$ (z=0) at $k_1 = k_2 = 0.01 h/\mathrm{Mpc}$ and $k_1 = 5k_2 = 0.05h/\mathrm{Mpc}$.
(cosmological parameters: Planck 2015)

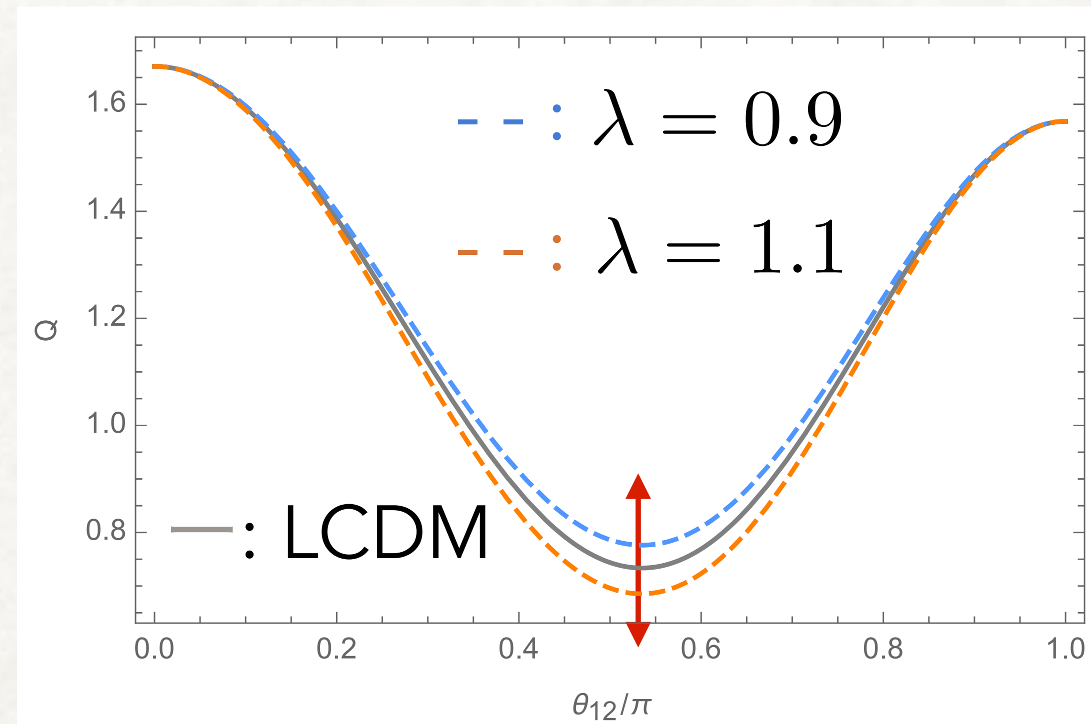
k-dependence

Horndeski: Takushima+ (2013)
GLPV: Hirano+ (2018)
DHOST: Hirano+ in prep.

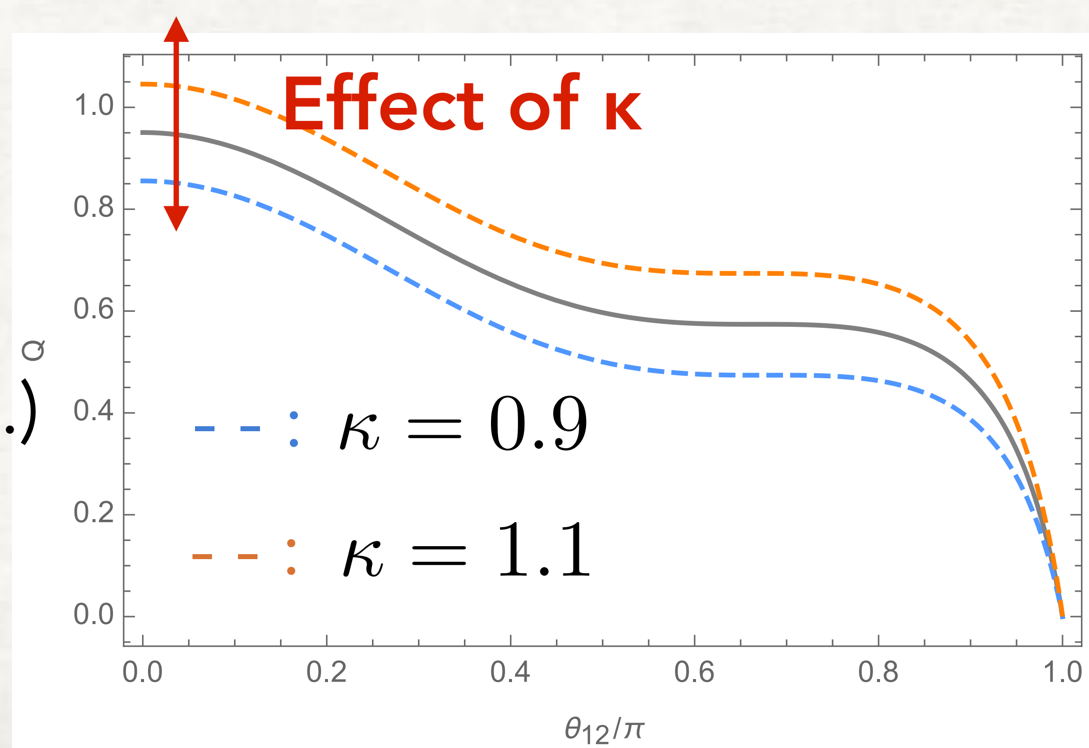
$k_1 = k_2 = 0.01h/Mpc$ | $k_1 = 5k_2 = 0.05h/Mpc$

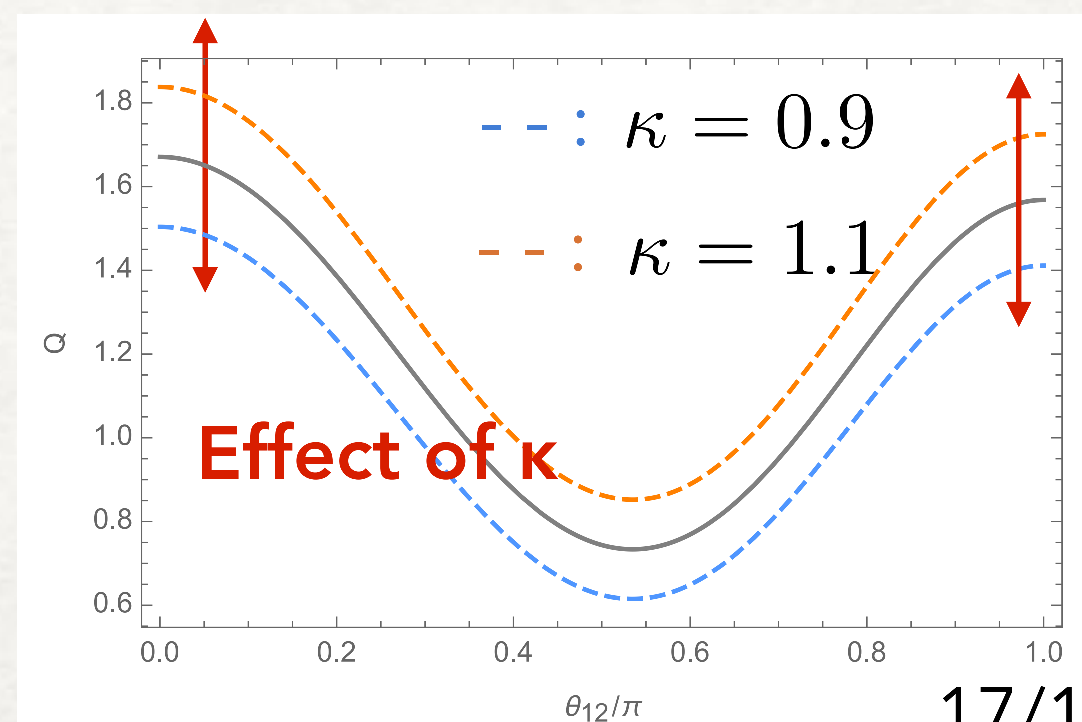
$\kappa = 1$
(Horndeski)





$\lambda = 1$
(beyond H.)





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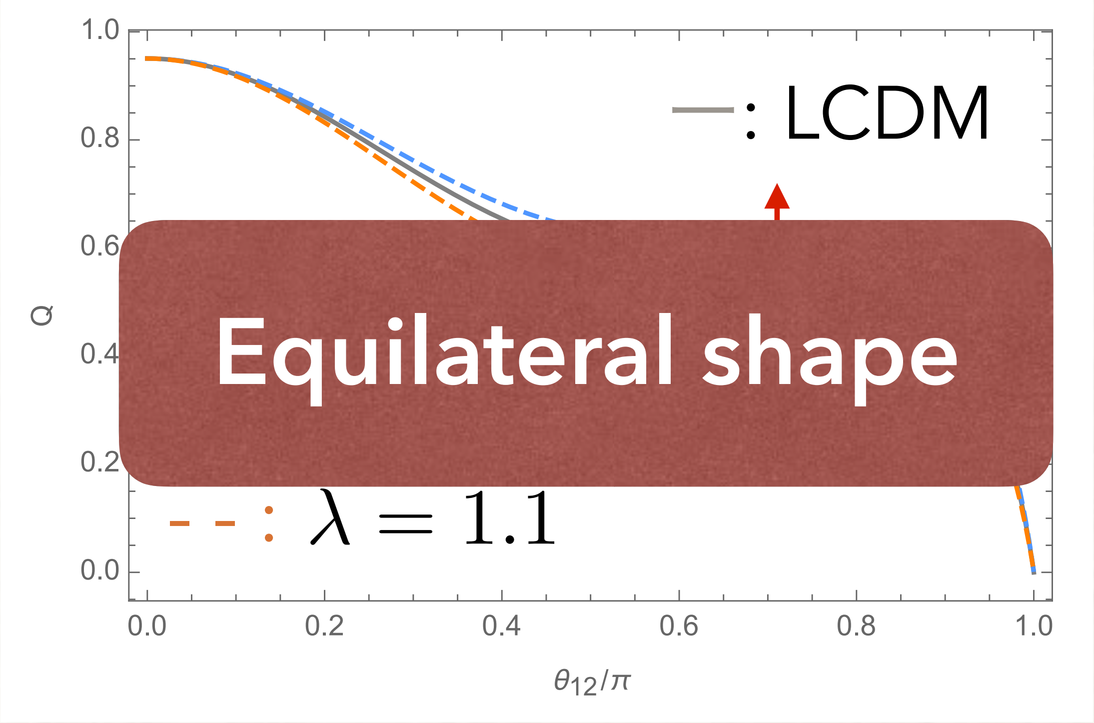
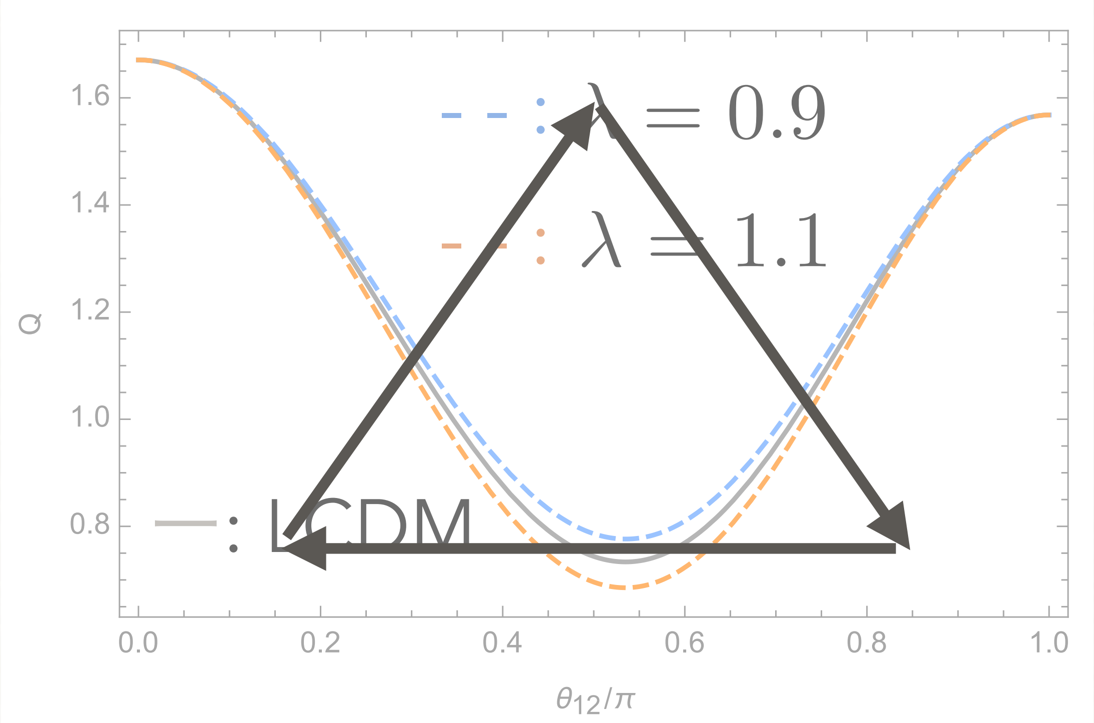
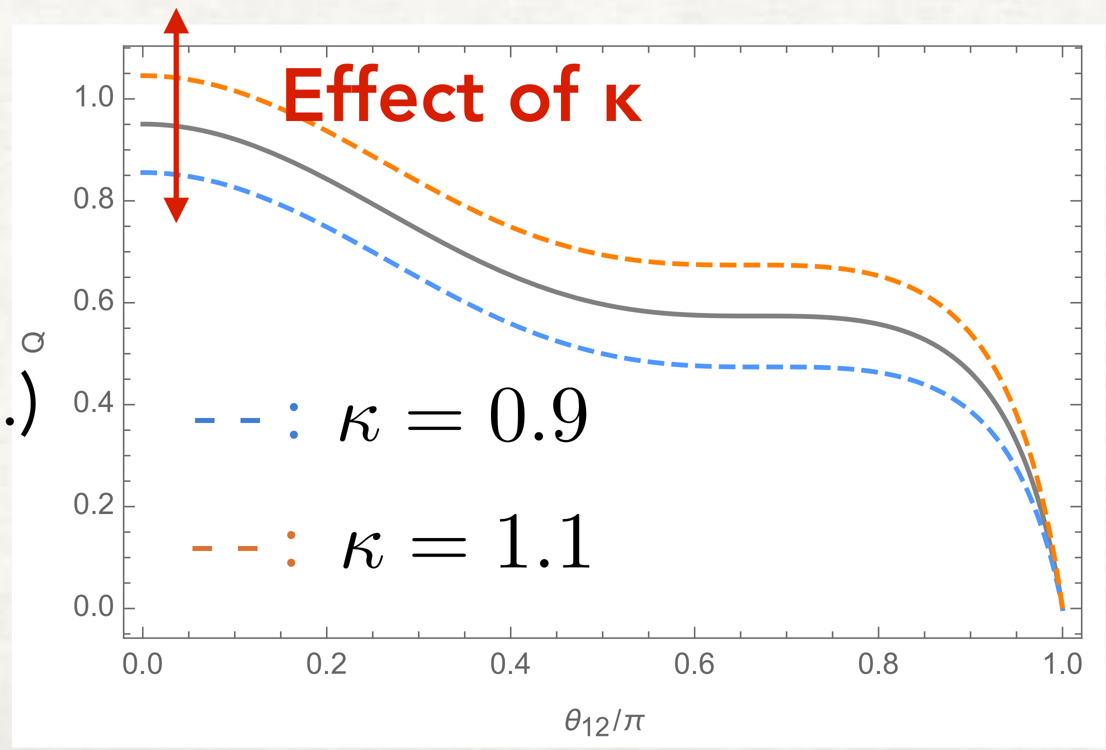
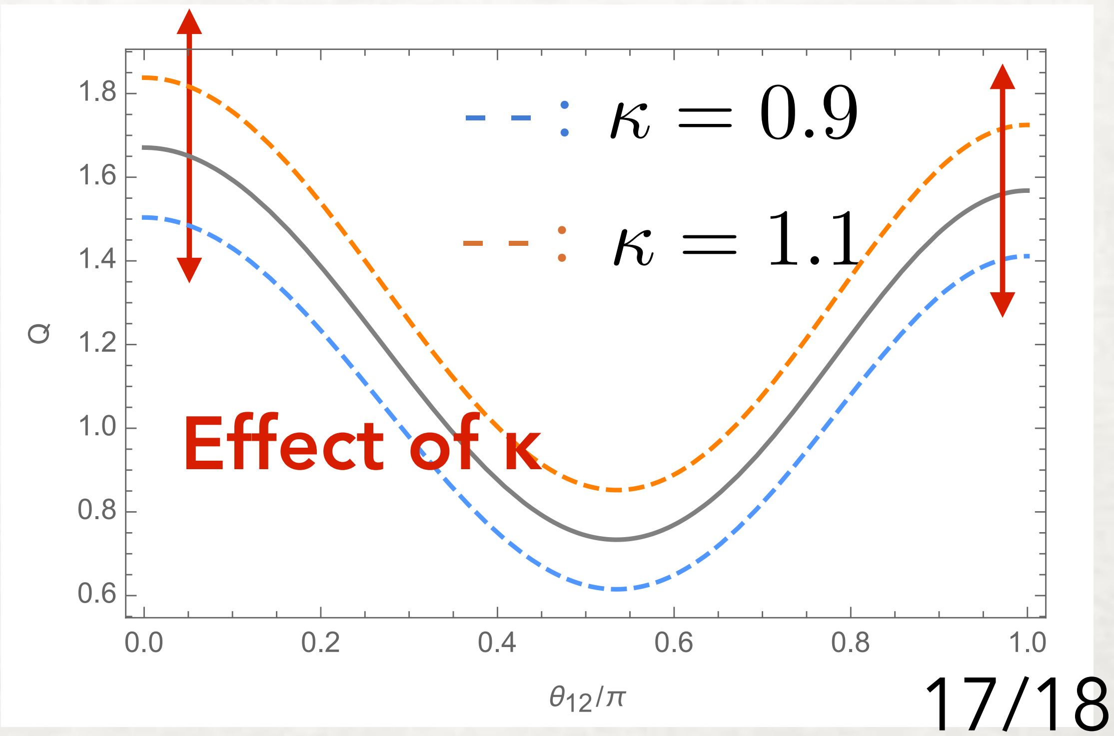
$\kappa = 1$ (Horndeski)





$\lambda = 1$ (beyond H.)





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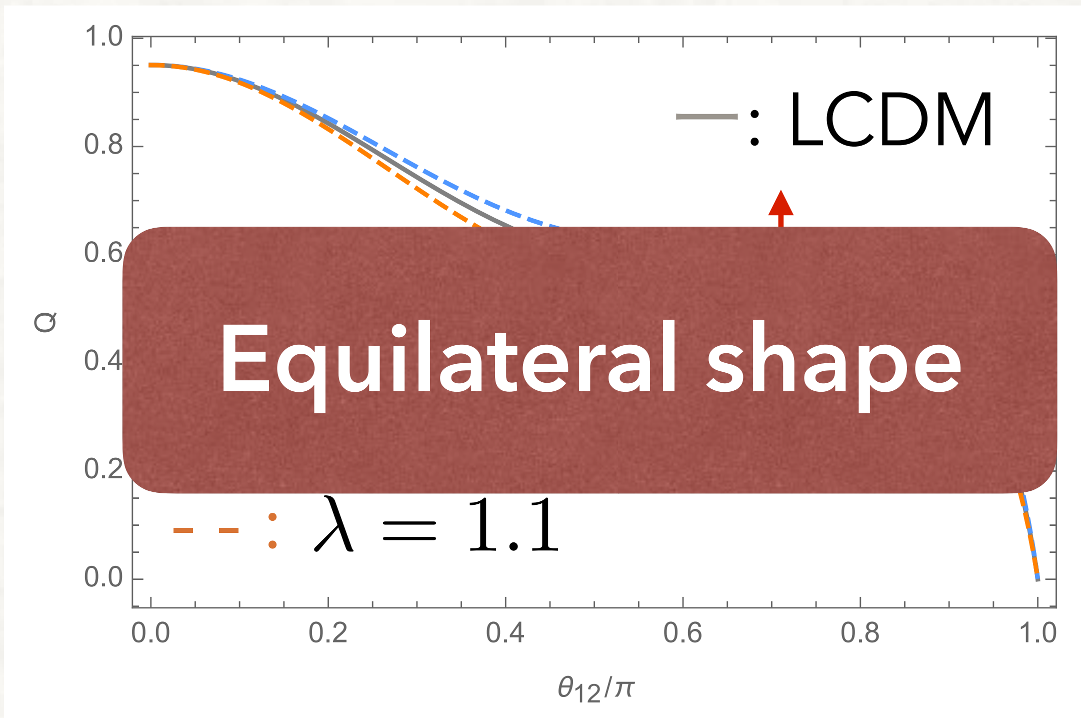

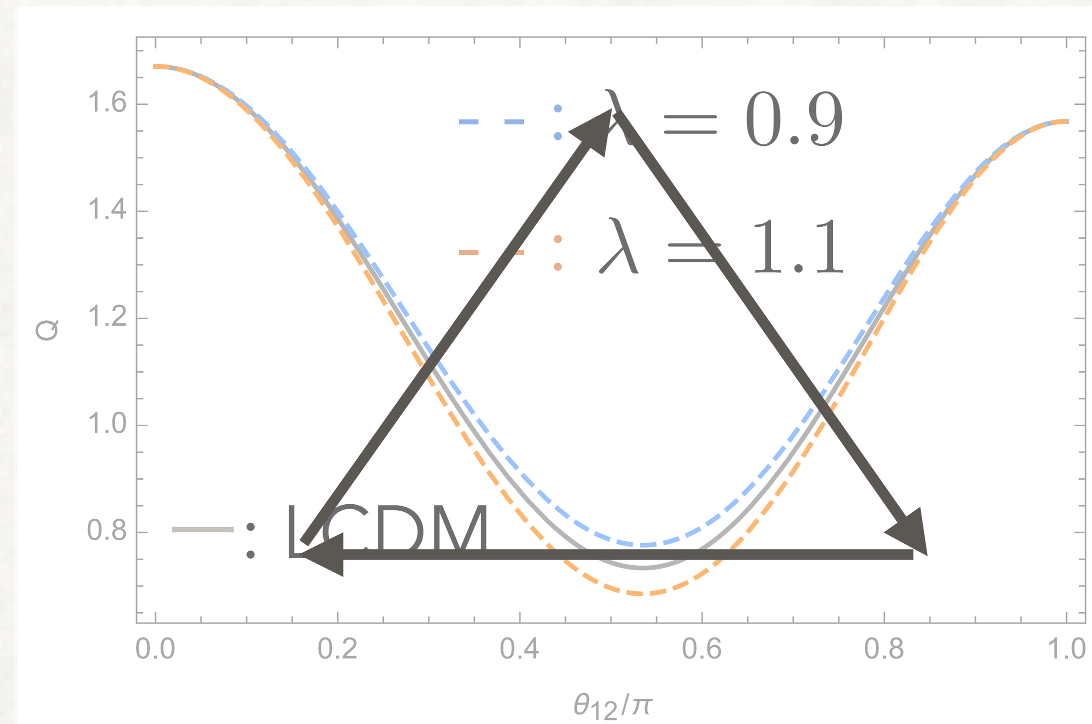

$\lambda = 1$
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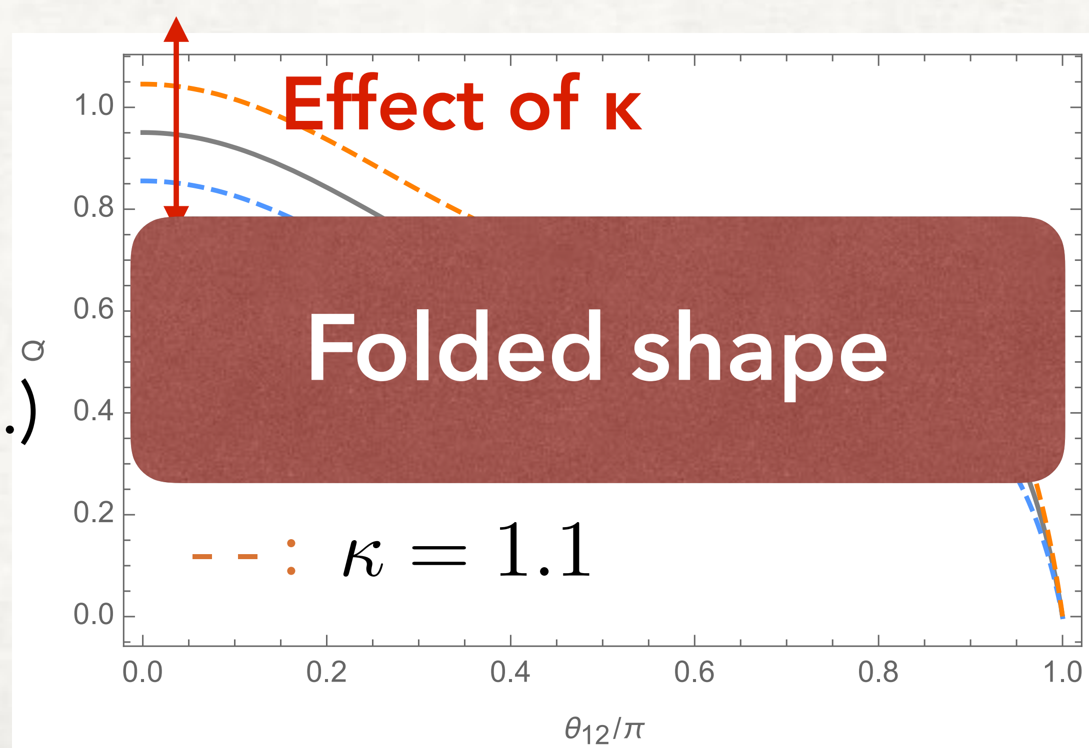

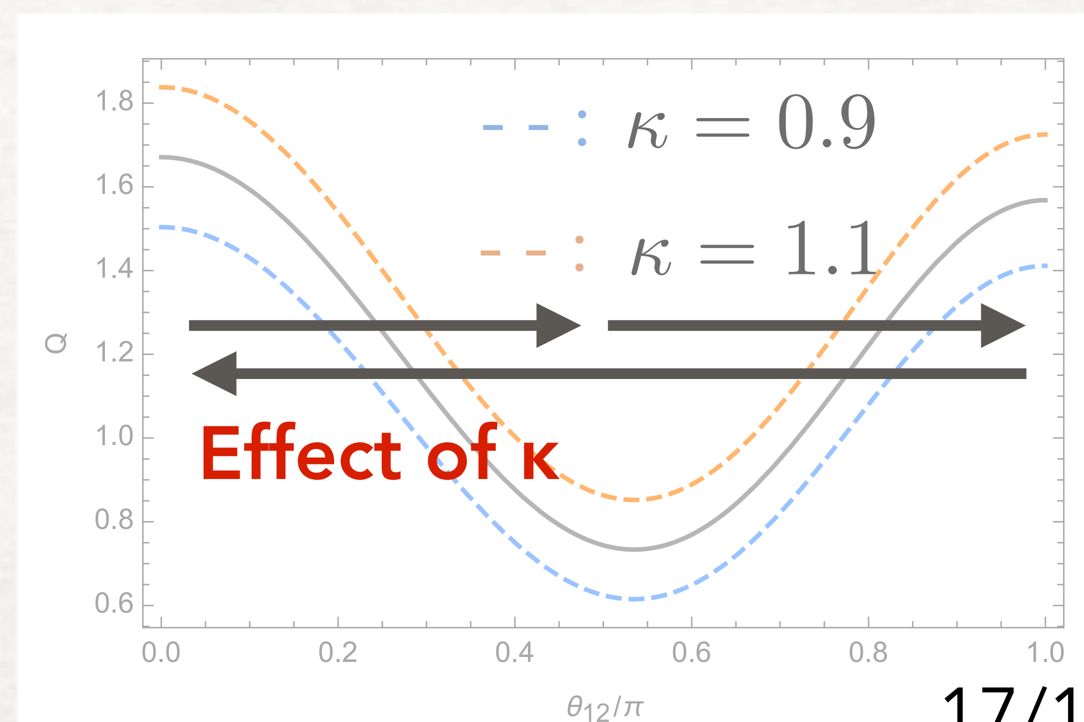

SUMMARY

Summary

- We discuss **beyond Horndeski** on density fluctuations
  at cosmological scale under some assumptions (QSA, $\alpha_i \sim \beta_1 = \mathcal{O}(1)$)

- Non-linear int. … (small scale, early universe) Vainshtein screening
  (cosmological scale) **Matter bispectrum**, …

- Cosmological perturbations
  linear level: friction term $\varsigma(t)$
  non-linear level: new time-evolution $\kappa(t)$ on matter bispectrum
  k-dependence … **folded shape** (beyond Horndeski)

<Future direction>

- Typical value of κ and λ beyond Horndeski? Hirano+ in prep.
- The effect of partial breaking on the density perturbations?