BOTTOM-UP APPROACH TO THE MASSIVE SPIN-TWO THEORY IN ARBITRARY SPACETIME

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INTRODUCTION

In the flat spacetime, the ghost-free model describing the massive spin-two field is well known. It is called the Fierz-Pauli model. On the other hand, in the ``fixed" curved background with the ``arbitrary" metric, it is well known that the infinite series of the nonminimal coupling terms are necessary for the ghost-freeness. Buchbinder *et. al.* denote the nonminimal coupling terms as the powers of the curvature and consider the conditions for the ghost-freeness. They solve the condition in the leading order approximation with respect to R/m^2. The action of the solution is given by,

$$S_{\text{general}} = \int d^D x \sqrt{-g} \left[\frac{1}{2} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \nabla_{\mu_1} h_{\mu_2\nu_2} \nabla_{\nu_3} h_{\mu_3\nu_3} + \frac{1}{2} \left\{ m^2 g^{\mu_1\nu_1\mu_2\nu_2} + \gamma_1 R g^{\mu_1\nu_1\mu_2\nu_2} \right. \right.$$
$$\left. \left. + \frac{\gamma_2}{2} \left(R^{\mu_1[\nu_1} g^{\nu_2]\mu_2} - R^{\mu_2[\nu_1} g^{\nu_2]\mu_1} \right) + \gamma_3 R^{\mu_1\mu_2\nu_1\nu_2} \right\} h_{\mu_1\nu_1} h_{\mu_2\nu_2} + \mathcal{O}\left(R^2/m^2 \right) \right]$$

There are three free parameters which are not restricted by the leading order condition. However, Buchbinder *et.al.* have not removed the possibility that these free parameters are restricted in the higher order calculation.

INTRODUCTION

Recently, some class of full completion of the nonminimal coupling terms have been obtained by linearizing the dRGT Model. The leading teams of the linearized dRGT model are included in the three free parameters of the Buchbinder's model, but it depends only on one free parameter s.

$$\gamma_1 = -\frac{1}{2}\left(\frac{1}{D-1} - s\right), \quad \gamma_2 = -4s, \quad \gamma_3 = 1$$

In contrast to the Buchbinder's model, it is guaranteed that there is the full completion corresponding to the above free parameter region [L. Bernard *et. al.* (2015)].

In this talk, we solve the condition proposed by Buchbinder *et. al.* up to fourth order, and investigate whether larger class than the dRGT class can be allowed or not. As a result, we obtain a restriction between the three free parameters at the fourth order condition with respect to curvatures.

NOTATION

$\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}$: anti-symmetrization of $\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\cdots\eta^{\mu_n\nu_n}$ with respect to $\nu_1\nu_2\cdots\nu_n$

Ex.

$$\eta^{\mu_1\nu_1\mu_2\nu_2} \equiv \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2} - \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}$$

$$\begin{aligned}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \equiv& \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3} + \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_1} + \eta^{\mu_1\nu_3}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_2} \\ &- \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_3} - \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_2} - \eta^{\mu_1\nu_3}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_1}\end{aligned}$$

Using this notation, the Fierz-Puali theory can be expressed as:

$$\begin{aligned}\mathcal{L}_{\mathrm{FP}} =& -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) \\ =& \frac{1}{2}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\partial_{\mu_1}h_{\mu_2\nu_2}\partial_{\nu_1}h_{\mu_3\nu_3} + \frac{m^2}{2}\eta^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}\end{aligned}$$

As the same way, we would like to define $g^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}$ $\delta^{i_1j_1i_2j_2\cdots i_nj_n}$.

OUTLINE

1. Lagrangian analysis
2. Irreducible decomposition of nonminimal coupling terms
3. Condition for ghost-freeness
4. Perturbative solution
5. Summary

LAGRANGIAN ANALYSIS

In order to count the DoF, we use the ``lagrangian analysis". Here, we introduce the ``lagrangian analysis" by counting the DoF of Fierz-Pauli theory in the flat space.

Fierz-Pauli theory

$$E^{\mu\nu} \equiv -\eta^{\mu\nu\mu_1\nu_1\mu_2\nu_2}\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2} + m^2\eta^{\mu\nu\mu_1\nu_1}h_{\mu_1\nu_1} = 0$$

Each component

$$E^{ij} = \delta^{iji_1j_1}\ddot{h}_{i_1j_1} + (\text{terms without } \ddot{h})^{ij} = 0 \qquad \text{(a)}$$

$$\phi^{(1)\nu} \equiv E^{0\nu} = -\eta^{0\nu i_1\nu_1 i_2\nu_2}\partial_{i_1}\partial_{\nu_1}h_{i_2\nu_2} + m^2\eta^{0\nu i_1\nu_1}h_{i_1\nu_1} = 0 \qquad \text{(b)}$$

From Eqs.(a), $\ddot{h}_{ij}$ can be decided. On the other hand, Eqs.(b) do not contain $\ddot{h}_{0\mu}$. We would like to decide the acceleration of the (0μ) component by taking the time derivative of Eqs.(b).

LAGRANGIAN ANALYSIS

$$\phi^{(1)\nu} \equiv E^{0\nu} = -\eta^{0\nu i_1 \nu_1 i_2 \nu_2} \partial_{i_1} \partial_{\nu_1} h_{i_2 \nu_2} + m^2 \eta^{0\nu i_1 \nu_1} h_{i_1 \nu_1} = 0 \qquad \text{(b)}$$

We consider the time derivative of the Eqs.(b),

$$\dot{\phi}^{(1)\nu} = 0 \, . \qquad \text{(c)}$$

Eqs.(c) guarantee the conservation of the original Eqs (b). Then if the Eqs.(b) are only satisfied in the initial time, the Eqs.(b) are automatically valid in any time. Therefore, the Eqs.(b) can be regarded as the ``constraints" restricting the initial values. We would like to express the equivalence on the initial time by using $\approx$.

$$\phi^{(1)\nu} \approx 0 \, .$$

We continue this procedure until the acceleration $\ddot{h}_{0\mu}$ appears in the equations. By counting the numbers of all the constraint obtained through this procedure, the DoF can be decided.

Next, we would like to deform the consistency condition (c).

LAGRANGIAN ANALYSIS

The consistency condition Eqs.(c) can be deformed by adding the EoM as follows,

$$0 = \dot{\phi}^{(1)\nu} \approx \partial_\mu E^{\mu\nu} = m^2 \eta^{\mu\nu\mu_1\nu_1} \partial_\mu h_{\mu_1\nu_1} \equiv \phi^{(2)\nu} . \quad \text{(c')}$$

We find that $\dot{\phi}^{(1)\nu}$ does not also contain any second time derivative terms up to EoM and constraints. Then, we regard again Eqs.(c') as constraint and require the consistency condition of (c') as follows,

$$\dot{\phi}^{(2)i} = m^2 \ddot{h}_{0i} + (\text{terms without } \ddot{h}) = 0 , \quad \text{(d)}$$

$$0 = \dot{\phi}^{(2)0} \approx \frac{D-1}{D-2} m^4 h \equiv \phi^{(3)} . \quad \text{(e)}$$

From the Eqs.(d), the acceleration $\ddot{h}_{0i}$ is decided. On the other hand, Eq.(e) is the constraint. We obtain more one constraint from the consistency condition of (e), $\phi^{(4)} = \dot{\phi}^{(3)} \approx 0$. Because the consistency condition of $\phi^{(4)} \approx 0$ contains the acceleration $\ddot{h}_{00}$, there are no more constraints.

IRREDUCIBLE DECOMPOSITION

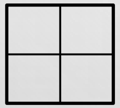
We consider the action with the non-derivative nonminimal coupling terms.

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \nabla_{\mu_1} h_{\mu_2\nu_2} \nabla_{\nu_1} h_{\mu_3\nu_3} + \frac{1}{2} \left\{ m^2 g^{\mu_1\nu_1\mu_2\nu_2} + \Delta^{\mu_1\nu_1\mu_2\nu_2} \right\} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \right]$$

$\Delta^{\mu_1\nu_1\mu_2\nu_2}$: arbitrary ``algebraic" tensor constructed by the curvature and the metric

For any $\Delta^{\mu_1\nu_1\mu_2\nu_2}$, we can prove that there are the tensor $T^{\mu_1\nu_1\mu_2\nu_2}$ $N^{\mu_1\nu_1\mu_2\nu_2}$ satisfying the relation:

$$\Delta^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} = T^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} + N^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} .$$

$T^{\mu_1\nu_1\mu_2\nu_2} \sim$ 

$$T^{\mu_1\nu_1\mu_2\nu_2} = -T^{\mu_2\nu_1\mu_1\nu_2} = -T^{\mu_1\nu_2\mu_2\nu_1} = T^{\mu_2\nu_2\mu_1\nu_1}$$

$T^{[\mu_1\nu_1\mu_2]\nu_2} = 0$: same symmetry as $R^{\mu_1\mu_2\nu_1\nu_2}$

$N^{\mu_1\nu_1\mu_2\nu_2} \sim$ 

$N^{\mu_1\nu_1\mu_2\nu_2} = N^{(\mu_1\nu_1\mu_2\nu_2)}$: perfect symmetric tensor

We would like to proceed our calculation with using the symmetries of these tensors, without giving the specific forms, until the specific forms become necessary.

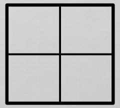
CONDITION FOR GHOST-FREENESS

Let us start the lagrangian analysis in the arbitrary curved background. The EoM obtained by variating the action S is given by,

$$E^{\mu\nu} \equiv \left\{ -g^{(\mu\nu)\mu_1\nu_1\mu_2\nu_2} \nabla_{\mu_2} \nabla_{\nu_2} + m^2 g^{\mu\nu\mu_1\nu_1} + T^{\mu\nu\mu_1\nu_1} + N^{\mu\nu\mu_1\nu_1} \right\} h_{\mu_1\nu_1} = 0$$

Same as the case of flat space, Eqs. $E^{ij} = 0$ decide the acceleration $\ddot{h}_{ij}$, and Eqs $\phi^{(1)\nu} \equiv E^{0\nu} \approx 0$ can be regarded as constraints. The consistency conditions of the constraints are given by,

$$\begin{aligned} \dot{\phi}^{(1)\nu} \approx \nabla_\mu E^{\mu\nu} = &\left\{ m^2 g^{\mu\nu\mu_1\nu_1} + S^{\mu\nu\mu_1\nu_1} + N^{\mu\nu\mu_1\nu_1} + P^{\mu\nu\mu_1\nu_1} \right\} \nabla_\mu h_{\mu_1\nu_1} \\ &+ (\text{terms without any derivatives of } h_{\mu\nu}) \end{aligned}$$

$$S^{\mu\nu\mu_1\nu_1} \equiv T^{\mu\nu\mu_1\nu_1} - R^{\mu\mu_1\nu\nu_1} + \left(R^{\mu[\nu} g^{\nu_1]\mu_1} - R^{\mu_1[\nu} g^{\nu_1]\mu} \right) \sim$$ 

$$P^{\mu\nu\mu_1\nu_1} \equiv \frac{1}{2} \left(R^{\mu\nu} g^{\mu_1\nu_1} - R^{\mu_1\nu_1} g^{\mu\nu} \right) = -P^{\mu_1\nu_1\mu\nu}$$: Pure nonintegrable terms

CONDITION FOR GHOST-FREENESS

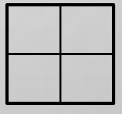
$$\dot{\phi}^{(1)\nu} \approx \nabla_\mu E^{\mu\nu} = \left\{ m^2 g^{\mu\nu\mu_1\nu_1} + S^{\mu\nu\mu_1\nu_1} + N^{\mu\nu\mu_1\nu_1} + P^{\mu\nu\mu_1\nu_1} \right\} \nabla_\mu h_{\mu_1\nu_1}$$
$$+ (\text{terms without any derivatives of } h_{\mu\nu})$$

This conditions can be regarded as the constraints $\phi^{(2)\nu} \equiv \dot{\phi}^{(1)\nu} \approx 0$. However, for the existence of additional constraints, we have to restrict the integrable terms $S^{\mu\nu\mu_1\nu_1},\ N^{\mu\nu\mu_1\nu_1}$. The condition for the existence of additional constraint is given by,

<u>Condition for ghost-freeness</u>

$\mathrm{Det}(V^0{}_\nu{}^{\mu 0}) = 0$: the determinant is defined on ${}_\nu{}^\mu$

$$V^{\mu\nu\mu_1\nu_1} \equiv m^2 g^{(\mu\nu)(\mu_1\nu_1)} + \bar{S}^{\mu\nu\mu_1\nu_1} + N^{\mu\nu\mu_1\nu_1} + P^{\mu\nu\mu_1\nu_1}$$

$\bar{S}^{\mu\nu\mu_1\nu_1} \equiv S^{(\mu\nu)(\mu_1\nu_1)}$: the symmetric basis of the mixed tableau 

PERTURBATIVE SOLUTION

We would like to find the covariant tensor $S^{\mu\nu\mu_1\nu_1}$, $N^{\mu\nu\mu_1\nu_1}$satisfying the condition $\mathrm{Det}(V^0{}_\nu{}^{\mu 0})=0$ by taking the perturbation with respect to R/m^2 . Let us expand the tensor $S^{\mu\nu\mu_1\nu_1}$, $N^{\mu\nu\mu_1\nu_1}$ with respect to the powers of the curvatures,

$$\bar{S}^{\mu_1\nu_1\mu_2\nu_2}=\sum_{n=1}^{\infty}\frac{1}{m^{2(n-1)}}\bar{S}^{(n)\mu_1\nu_1\mu_2\nu_2} \qquad N^{\mu_1\nu_1\mu_2\nu_2}=\sum_{n=1}^{\infty}\frac{1}{m^{2(n-1)}}N^{(n)\mu_1\nu_1\mu_2\nu_2}$$

Here, the superscript (n) means the n th-order terms with respect to curvature.

<u>(1) Leading order</u>

$$N^{(1)0000}=0 \longrightarrow N^{(1)\mu_1\nu_1\mu_2\nu_2}=0$$

There are no restrictions for the mixed symmetric tensor$S^{(1)\mu_1\nu_1\mu_2\nu_2}$. This result is consistent with the Buchbinder's result:

$$S_{\text{general}}=\int d^Dx\sqrt{-g}\left[\frac{1}{2}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\nabla_{\mu_1}h_{\mu_2\nu_2}\nabla_{\nu_3}h_{\mu_3\nu_3}+\frac{1}{2}\left\{m^2g^{\mu_1\nu_1\mu_2\nu_2}+\gamma_1Rg^{\mu_1\nu_1\mu_2\nu_2}\right.\right.$$
$$\left.\left.+\frac{\gamma_2}{2}\left(R^{\mu_1[\nu_1}g^{\nu_2]\mu_2}-R^{\mu_2[\nu_1}g^{\nu_2]\mu_1}\right)+\gamma_3R^{\mu_1\mu_2\nu_1\nu_2}\right\}h_{\mu_1\nu_1}h_{\mu_2\nu_2}+\mathcal{O}\left(R^2/m^2\right)\right]$$

PERTURBATIVE SOLUTION

(2) Second order

$$N^{(2)0000} - \frac{1}{2}g^{00\alpha\beta}R^0_\alpha R^0_\beta = 0 \longrightarrow N^{(2)\mu_1\nu_1\mu_2\nu_2} = \frac{1}{2}g^{\alpha\beta(\mu_1\nu_1}R_\alpha^{\mu_2}R_\beta^{\nu_2)}$$

Because $N^{(n)\mu_1\nu_1\mu_2\nu_2}$ is the perfect symmetric tensor, it is not necessary to note the order of the superscript $(\mu_1\nu_1\mu_2\nu_2)$. We find that the nonminimal coupling terms cannot truncate at the leading order, because $N^{(2)\mu_1\nu_1\mu_2\nu_2}$ must be not equal to zero no matter how we choose the $S^{(1)\mu_1\nu_1\mu_2\nu_2}$.

(3) Third order

$$N^{(3)0000} - \bar{S}^{(1)0\alpha\beta0}R^0_\alpha R^0_\beta = 0 \longrightarrow N^{(3)\mu_1\nu_1\mu_2\nu_2} = -\frac{1}{2}S^{(1)\alpha\beta(\mu_1\nu_1}R_\alpha^{\mu_2}R_\beta^{\nu_2)}$$

In contrast to second order, $N^{(3)\mu_1\nu_1\mu_2\nu_2}$ depends purely on $S^{(1)\mu_1\nu_1\mu_2\nu_2}$. Then, we find that $N^{\mu_1\nu_1\mu_2\nu_2}$ cannot be chosen independently of $S^{\mu_1\nu_1\mu_2\nu_2}$.

PERTURBATIVE SOLUTION

(4) Fourth order

$$N^{(4)0000} - \left(\bar{S}^{(2)0\alpha\beta 0} + N^{(2)0\alpha\beta 0}\right) R^0_\alpha R^0_\beta$$
$$+ \frac{1}{8} g^{00\alpha\beta} \left(R^2\right)^0_\alpha \left(R^2\right)^0_\beta - \frac{2}{g^{00}} R^{0\nu} \bar{S}^{(1)0}{}_\nu{}^{\rho 0} \bar{S}^{(1)0}{}_\rho{}^{\sigma 0} R^0_\sigma = 0$$

In contrast to the case of lower order conditions, the fourth order condition includes the noncovariant term, that is the fourth term. This term cannot be canceled by the first term. This fact means that $S^{(1)\mu_1\nu_1\mu_2\nu_2}$ is restricted so that it satisfies the condition,

$$R^{0\nu} \bar{S}^{(1)0}{}_\nu{}^{\rho 0} \bar{S}^{(1)0}{}_\rho{}^{\sigma 0} R^0_\sigma = g^{00} M^{0000}$$

for some covariant tensor M^{0000}. This condition reduce the three free parameters of $S^{(1)\mu_1\nu_1\mu_2\nu_2}$ to the following two free parameters.

$$S^{(1)\mu_1\nu_1\mu_2\nu_2} = \gamma_1^{(1)} R g^{\mu_1\nu_1\mu_2\nu_2} + \frac{\gamma_2^{(1)}}{2} \left(R^{\mu_1[\nu_1} g^{\nu_2]\mu_2} - R^{\mu_2[\nu_1} g^{\nu_2]\mu_1}\right)$$

SUMMARY

In this talk, we investigate the restriction of the massive spin-two theory in arbitrary background. We decompose the general nonminimal coupling terms into the perfect symmetric tensor and the mixed symmetric tensor, and solve perturbatively the condition for the ghost-freeness up to fourth order.

In all the order, all the free parameters belong to the mixed symmetric tensor. At the time of solving the leading order condition, the three free parameters are allowed as the leading nonminimal coupling terms with respect to curvature. However, by the fourth order condition, these free parameters are reduced to the two free parameters.

On the other hand, perfect symmetric parts are uniquely defined as the function of the mixed symmetric tensor and the Richi curvatures.

Although the mixed symmetric tensors over the second order are not restricted by up to the forth order conditions, we can easily predict that all the mixed symmetric tensors will be restricted by higher order conditions. This is the future works for this direction.

THANK YOU FOR YOUR ATTENTION

CONDITION FOR GHOST-FREENESS

$$\mathrm{Det}(V^{0}{}_{\nu}{}^{\mu 0}) = 0 \qquad V^{0\mu\nu 0} \equiv m^2 g^{(0\mu)(\nu 0)} + \bar{S}^{0\mu\nu 0} + N^{0\mu\nu 0} + P^{0\mu\nu 0}$$

$\Updownarrow$

<u>Properties of decomposition in time+space direction</u>

$$\theta^{\mu}_{\nu} \equiv \delta^{\mu}_{\nu} - \frac{g^{\mu 0} g^{0}_{\nu}}{g^{00}} \qquad g^{(0\mu)(\nu 0)} = -\frac{1}{2} g^{00} \theta^{\mu\nu}$$

$$\bar{S}^{0\mu\nu 0} = \theta^{\mu}_{\alpha} \bar{S}^{0\alpha\beta 0} \theta^{\nu}_{\beta} \qquad P^{0\mu\nu 0} = R^{0[\mu} g^{\nu]0} = -g^{0[\mu} \theta^{\nu]}_{\rho} R^{0\rho}$$

$$0 = \hat{N}^{00} + \sum_{k=1}^{\infty} \left(\frac{2}{m^2 g^{00}} \right)^k \left[\left[\hat{N}\theta \left\{ (\hat{S} + \hat{N})\theta \right\}^{k-1} \hat{N} \right]^{00} - \left(\frac{g^{00}}{2} \right)^2 \left[R\theta \left\{ (\hat{S} + \hat{N})\theta \right\}^{k-1} R \right]^{00} \right]$$

$$\hat{N}^{\mu\nu} \equiv N^{0\mu\nu 0} \qquad \hat{S}^{\mu\nu} \equiv \bar{S}^{0\mu\nu 0} \qquad [XY]^{\mu\nu} \equiv X^{\mu}{}_{\rho} Y^{\rho\nu}$$

IRREDUCIBLE DECOMPOSITION

We consider the model with the nonderivative nonminimal coupling terms expressed by general tensor $\Delta^{\mu_1\nu_1\mu_2\nu_2}$ constructed by curvature and metric,

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \nabla_{\mu_1} h_{\mu_2\nu_2} \nabla_{\nu_1} h_{\mu_3\nu_3} + \frac{1}{2} \left\{ m^2 g^{\mu_1\nu_1\mu_2\nu_2} + \Delta^{\mu_1\nu_1\mu_2\nu_2} \right\} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \right]$$

The tensor $\Delta^{\mu_1\nu_1\mu_2\nu_2}$ can be decomposed as follows,

$$\Delta^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} = T^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} + N^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2},$$

$$T^{\mu_1\nu_1\mu_2\nu_2} = \frac{1}{2} \left(\Delta^{[\mu_1|\nu_1|\mu_2]\nu_2} - \Delta^{[\mu_1|\nu_2|\mu_2]\nu_1} \right) + \frac{1}{6} \left(\Delta^{\mu_1[\nu_1\nu_2]\mu_2} - \Delta^{\mu_2[\nu_1\nu_2]\mu_1} \right) + \frac{1}{3} \Delta^{[\nu_2\nu_1][\mu_1\mu_2]},$$

$$N^{\mu_1\nu_1\mu_2\nu_2} = \Delta^{(\mu_1\nu_1\mu_2\nu_2)}$$

$$\Delta^{[\mu_1|\nu_1|\mu_2]\nu_2} \equiv \frac{1}{2} \left(\Delta^{\mu_1\nu_1\mu_2\nu_2} - \Delta^{\mu_2\nu_1\mu_1\nu_2} \right)$$

$\Delta^{(\mu_1\nu_1\mu_2\nu_2)}$: perfect symmetrization of $\Delta^{\mu_1\nu_1\mu_2\nu_2}$

PERTURBATIVE SOLUTION

Restriction up to fourth order caluculation

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \nabla_{\mu_1} h_{\mu_2\nu_2} \nabla_{\nu_1} h_{\mu_3\nu_3} + \frac{1}{2} \left\{ m^2 g^{\mu_1\nu_1\mu_2\nu_2} + T^{\mu_1\nu_1\mu_2\nu_2} + N^{\mu_1\nu_1\mu_2\nu_2} \right\} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \right]$$

$$\begin{aligned} T^{(1)\mu_1\nu_1\mu_2\nu_2} &= S^{(1)\mu_1\nu_1\mu_2\nu_2} + R^{\mu_1\mu_2\nu_1\nu_2} - \left(R^{\mu_1[\nu_1} g^{\nu_2]\mu_2} - R^{\mu_2[\nu_1} g^{\nu_2]\mu_1} \right) \\ &= \gamma_1^{(1)} R g^{\mu_1\nu_1\mu_2\nu_2} + \frac{\gamma_2^{(1)} - 2}{2} \left(R^{\mu_1[\nu_1} g^{\nu_2]\mu_2} - R^{\mu_2[\nu_1} g^{\nu_2]\mu_1} \right) + R^{\mu_1\mu_2\nu_1\nu_2} \end{aligned}$$

$$N^{(1)\mu_1\nu_1\mu_2\nu_2} = 0$$

$$N^{(2)\mu_1\nu_1\mu_2\nu_2} = \frac{1}{2} g^{\alpha\beta(\mu_1\nu_1} R_\alpha^{\ \mu_2} R_\beta^{\ \nu_2)}$$

$$N^{(3)\mu_1\nu_1\mu_2\nu_2} = -\frac{1}{2} S^{(1)\alpha\beta(\mu_1\nu_1} R_\alpha^{\mu_2} R_\beta^{\nu_2)}$$