Gravitational Waves modes in Extended Teleparallel Gravity

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based on **H. Abedi & S. Capozziello EPJC78(2018)474**

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Plan of the talk

- Gravitational waves in General Relativity
- Extended Theories of Gravity
- Gravitational waves modes in $f(T, \varphi, B)$ gravity
- Gravitational waves modes in $f(T, B)$ gravity
- The case of kinetic coupling with a scalar field
- Polarization modes of GWs
- Detector response
- The stochastic background of GWs
- Conclusions and Outlooks

Gravitational Waves in GR

- In vacuum $\longrightarrow$ $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$

Considering a perturbation of the Minkowski flat metric: $g_{\mu\nu} \sim \eta_{\mu\nu} + h_{\mu\nu}$

At first order the trace of field equations is:

$$R = 0 \longrightarrow \Box h = 0$$

Two polarization in T-T Gauge: $h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{+} & h_{x} \\ 0 & 0 & h_{x} & -h_{+} \end{pmatrix}$

Extending General Relativity

$S = \int \sqrt{-g}\, R\, d^4x$ ⟶ $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$

can be extended:

$$S = \int \sqrt{-g}\, [f(R,\varphi) + \frac{\omega}{2} g^{\mu\nu} \varphi_{,\mu}\varphi_{,\nu} + V(\varphi)] d^4x$$

$f(R,\varphi) = F(\varphi)$

$\omega = 0 ; V(\varphi) = 0 ; f(R,\varphi) = f(R)$

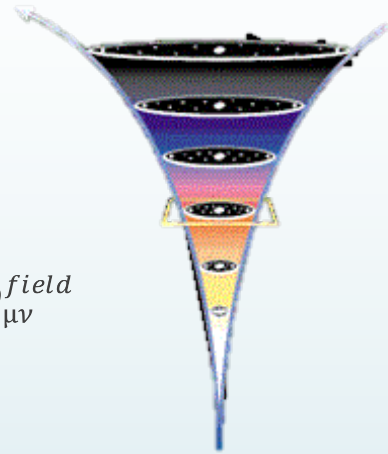
$$G_{\mu\nu} = \frac{1}{f_R(R)} \left\{ \frac{1}{2} g_{\mu\nu} [f(R) - R f_R(R)] + f_R(R)_{;\mu;\nu} - g_{\mu\nu} \Box f_R(R) \right\}$$

- $G_{\mu\nu} = -\frac{1}{2\,F(\varphi)} \left\{ -\frac{\omega}{2} g_{\mu\nu}\, \varphi^{,\alpha} \varphi_{,\alpha} + \omega\, \varphi_{,\mu}\varphi_{,\nu} + g_{\mu\nu} V(\varphi) + 2\, g_{\mu\nu} F(\varphi) - 2F(\varphi)_{,\mu,\nu} \right\}$

Extending General Relativity

$$S = \int \sqrt{-g}\, f(R)\, d^4x \longrightarrow G_{\mu\nu} = \Theta^{curv}_{\mu\nu}$$

$$S = \int \sqrt{-g}\, [F(\varphi) + \frac{\omega}{2}\, g^{\mu\nu}\, \varphi_{,\mu}\varphi_{,\nu} + V(\varphi)] d^4x \longrightarrow G_{\mu\nu} = \Theta^{field}_{\mu\nu}$$



Inflation, Dark Matter, Dark Energy can be addressed under this standard

See for example **S. Nojiri and S.D. Odintsov, Phys. Rept. 505 (2011) 59**
S. Capozziello, M. De Laurentis, Phys. Rept 509 (2011) 167

Gravitational Waves in ETGs

$ETGs$ theory → extra polarizations:

- × → Cross
- + → Plus
- b → Breathing
- y → Vector-y
- x → Vector-x

→ *These modes can be described by independent Newman-Penrose quantities:*

$$\{\Psi_2, \Psi_3, \Psi_4, \Phi_{22}\}$$

Extending Teleparallel Gravity

$S = \int \sqrt{-g}\, R\, d^4x$ ⟶ $S = \int \sqrt{-g}\, f(R)\, d^4x$ metric description by curvature R

$S = \int h\, T\, d^4x$ $S = \int e\, f(T)\, d^4x$ tetrad description by torsion T

Or, considering: ⟶ R=-T+B B is a boundary term

$$\mathcal{L}_{H-E} = -\mathcal{L}_{TEGR} + B \qquad B = 2\nabla_\mu T_\nu^{\nu\mu}$$

- We can, in general, extend *f(T)* gravity: ⟶

$$T^\alpha{}_{\mu\nu} = e_A{}^\alpha\left(\partial_\mu e^A{}_\nu - \partial_\nu e^A{}_\mu\right)$$

$$K^{\mu\nu} := -\frac{1}{2}\left(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}\right),$$

$$S_\rho{}^{\mu\nu} := \frac{1}{2}\left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho T^{\alpha\nu}{}_\alpha - \delta^\nu_\rho T^{\alpha\mu}{}_\alpha\right)$$

$$S = \frac{1}{2}\int e\, d^4x\left[f(T,B,\varphi) - \partial_\mu\varphi\partial^\mu\varphi - 2V(\varphi) + 2\mathcal{L}_m\right] \qquad T := S_\rho{}^{\mu\nu}T^\rho{}_{\mu\nu}$$

Gravitational waves in $f(T,\varphi,B)$ gravity

$$S = \frac{1}{2}\int e\, d^4x\left[f(T,B,\varphi) - \partial_\mu\varphi\partial^\mu\varphi - 2V(\varphi) + 2\mathcal{L}_m\right]$$

$$-f_T G_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu) f_B + \frac{1}{2}(f_B B + f_T T - f) g_{\mu\nu}$$
$$+ 2S_\nu{}^\alpha{}_\mu\, \partial_\alpha(f_T + f_B) - g_{\mu\nu}\left[\frac{1}{2}\partial_\alpha\phi\, \partial^\alpha\phi + V(\phi)\right]$$
$$+ \partial_\mu\phi\, \partial_\nu\phi = \Theta_{\mu\nu},$$

$$G^\nu_\sigma = -2\left(e^{-1}\partial_\mu(eS_A{}^{\mu\nu}) - T^\rho{}_{\mu A}S_\rho{}^{\nu\mu} - \frac{1}{4}e_A{}^\nu T\right) e^A{}_\sigma$$

$$\Box\phi + \frac{1}{2}f' - V' = 0$$

A perturbation of the tetrads, up to first order gives:

$$e^A{}_\mu = \delta^A{}_\mu + h^A{}_\mu,$$

$$R^{(1)} = -T^{(1)}$$

B is second order in perturbations

Gravitational waves in $f(T, \varphi, B)$ gravity

$$f(T,B,\phi^i) = [-1+\xi F(\phi)]T + \chi E(\phi) B$$

$$(-1+\xi F)G_{\mu\nu} + \chi (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) E + 2S_\nu{}^\alpha{}_\mu \partial_\alpha (\xi F + \chi E) - g_{\mu\nu}\left(\frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi + V\right) + \partial_\mu \phi \partial_\nu \phi = \Theta_{\mu\nu}.$$

At the first order, the trace is:

$$-(-1+\xi F_0)R^{(1)} + 3\xi E_0' \Box \delta\phi - hV_0 - 4V_0' \delta\phi = \Theta^{(1)} \longrightarrow \Box \bar{h}_{\mu\nu} = 0$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h + \frac{\chi E_0'}{-1+\xi F_0}\eta_{\mu\nu}\delta\phi,$$
$$\bar{h} = -h + \frac{4\chi E_0'}{-1+\xi F_0}\delta\phi,$$
$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h} + \frac{\chi E_0'}{-1+\xi F_0}\eta_{\mu\nu}\delta\phi$$

In Fourier space:

$$\bar{h}_{\mu\nu}(\mathbf{k}) = A_{\mu\nu}(\mathbf{k}) \exp(ik^\alpha x_\alpha) + \text{c.c.}$$

Gravitational waves in $f(T,\varphi,B)$ gravity

Furthermore….

Assuming $\overline{\Phi}$ as the minimum of the potential: ⟶ $V \simeq V_0 + \frac{1}{2}\gamma\,(\delta\phi)^2$

Field equation $\Box\phi + \frac{1}{2}f' - V' = 0$ at first order becomes $(\Box - m^2)\,\delta\phi = 0$

$V_0' = 0$; $m^2 = \dfrac{2V_0''(-1+\xi F_0)}{2(-1+\xi F_0) - 3\xi\chi F_0' E_0'}$

↓

Effective mass

SOLUTION: $\delta\phi(\mathrm{q}) = a(\mathrm{q})\,\exp\left(iq^\alpha x_\alpha\right) + \mathrm{c.c}$

Gravitational waves in $f(T,\varphi,B)$ gravity

Assuming:

- $\bar{h}_{ij} = 0$
- $\mathbf{q} = \left(\Omega, 0, 0, \sqrt{\Omega^2 - m^2}\right)$
- $\ddot{x}_i = -R_{itjt}x_j = \dfrac{\chi E_0'}{2(-1+\xi F_0)}\left(\eta_{ij}\,\ddot{\delta\phi} + (\delta\phi)_{,ij}\right)x_j$ ⟶ Geodesic deviation
- $$\mathbf{k} = \frac{1}{\sqrt{2}}(\mathbf{e}_t + \mathbf{e}_z), \qquad \mathbf{l} = \frac{1}{\sqrt{2}}(\mathbf{e}_t - \mathbf{e}_z),$$
  $$\mathbf{m} = \frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y), \qquad \bar{\mathbf{m}} = \frac{1}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y)$$

NP quantities $\{\Psi_2, \Psi_3, \Psi_4, \Phi_{22}\}$ take the form: ⟶

$$\begin{cases} \Psi_4 = -R_{\mathbf{l}\bar{\mathbf{m}}\mathbf{l}\bar{\mathbf{m}}} \sim + \text{ and } \times \text{ modes} \\ \Psi_3 = -\dfrac{1}{2}R_{\mathbf{l}\bar{\mathbf{m}}} \sim x \text{ and } y \text{ modes,} \\ \Psi_2 = \dfrac{1}{6}R_{\mathbf{lk}} \sim l \text{ mode,} \\ \Phi_{22} = -\dfrac{1}{2}R_{\mathbf{ll}} \sim b \text{ mode.} \end{cases}$$

Gravitational waves in $f(T, \varphi, B)$ gravity

SOLUTIONS:

$$\Psi_4 = -R_{\mathbf{l\bar{m}l\bar{m}}} \sim + \text{ and } \times \text{ modes}$$
$$\Psi_3 = -\frac{1}{2}R_{\mathbf{l\bar{m}}} \sim x \text{ and } y \text{ modes},$$
$$\Psi_2 = \frac{1}{6}R_{\mathbf{lk}} \sim l \text{ mode},$$
$$\Phi_{22} = -\frac{1}{2}R_{\mathbf{ll}} \sim \ b \text{ mode}.$$

$\rightarrow$

$$\Psi_3 = 0,$$
$$\Psi_2 = \frac{\chi E_0' m^2 a \exp(i q_\alpha x^\alpha)}{12}\left[\frac{1}{-1+\xi F_0} - \frac{3}{2}\right]$$
$$\Phi_{22} = -\frac{\chi E_0'}{2(-1+\xi F_0)}\exp(i q_\alpha x^\alpha)(q_t - q_z)^2$$

For massless waves, we have $V_0'' = 0$ and there are just three modes:

- $\Psi_2 = 0$
- $\Psi_3 = 0$
- $\Psi_4 \neq 0$
- $\Phi_{22} \neq 0$

} modes: $\times$, $+$ and b

Gravitational waves in $f(T,B)$ gravity

The action without scalar field is equivalent to f(R)

x and +, like in GR, plus a scalar mode

$$S = \frac{1}{2}\int \mathrm{d}^4x\, e\, f(T,B)$$

First order field equations:

$$f_{T_0}\left(R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)}\right) + f_{T_0B_0}\left(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu\right)R^{(1)} = 0$$

Trace equation:

$$\Box R^{(1)} + m^2 R^{(1)} = 0$$

$$m^2 = -\frac{f_{T_0}}{3f_{T_0B_0}}$$

Solution:

$$R^{(1)} = \hat{R}\left(q^\rho\right)\exp\left(iq_\rho x^\rho\right)$$

Kinetic coupling

Kinetic coupling:

$$S \supset \int d^4x \sqrt{-g} R X \longrightarrow X = \frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi$$

In the Arnowitt-Deser-Misner formalism:

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

$$X = -\frac{1}{2N^2}\dot{\phi}^2 + \frac{N^i}{N^2}\dot{\phi}\,\partial_i\phi + \frac{1}{2}\left(h^{ij} - \frac{N^i N^j}{N^2}\right)\partial_i\phi\,\partial_j\phi \longrightarrow S \supset \int d^4x \frac{\dot{\phi}^2\dot{\Sigma}}{N^2}$$

$$\bar{\Sigma}_{ij} = \frac{1}{N}\left(\dot{h}_{ij} - \bar{D}_i N_j - \bar{D}_j N_i\right) \qquad R = {}^{(3)}R + \bar{\Sigma}^{ij}\bar{\Sigma}_{ij} - \bar{\Sigma}^2 + \mathcal{D}_R$$

$$\mathcal{D}_R = \frac{2}{N\sqrt{\gamma}}\partial_t\left(\sqrt{\gamma}\bar{\Sigma}\right) - \frac{2}{N}\bar{D}_i\left(\bar{\Sigma}N^i + \gamma^{ij}\partial_j N\right)$$

$\bar{D}_i \longrightarrow$ 3D Levi-Civita covariant derivative

Result: the coupling of $\mathcal{D}_R$ or B to the kinetic term will result in instability..

Kinetic coupling

But, the action

$$S = \int \mathrm{d}^4x\, e \left[-\frac{T}{2}(1+\xi\partial_\mu\phi\,\partial^\mu\phi) + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) + \mathcal{L}_{\mathrm{m}} \right].$$

for ξ=0, it is

equivalent to GR

minimally coupled with a scalar field

Leads to:

$$-2(1+\xi\partial_\mu\phi\,\partial^\mu\phi)\left[e^{-1}e^A_\mu\partial_\alpha(eS_A^{\ \alpha\nu})+T\right]+2T$$
$$-2\xi S_\nu^{\ \alpha\nu}\,\partial_\alpha(\partial_\gamma\phi\,\partial^\gamma\phi)-\partial_\gamma\phi\,\partial^\gamma\phi+4V=\Theta.$$

trace of field equations

with respect to

the tetrads field

$$\Box\phi + V_{,\phi} = \xi\,\partial^\mu\phi\,\partial_\mu T$$

field equation

with respect to

the scalar field

Kinetic coupling

- First order field equations

$$\left[G^{\nu}_{\sigma} - \delta^{\nu}_{\sigma}\left(\frac{1}{2}\partial_{\gamma}\phi\,\partial^{\gamma}\phi - V(\phi)\right) + \partial^{\nu}\phi\,\partial_{\sigma}\phi\right]^{(1)} = (\Theta^{\nu}_{\sigma})^{(1)}$$

- First order Klein Gordon equation

$$(\Box\phi + V_{,\phi})^{(1)} = 0$$

Same equations as Ricci scalar minimally coupled to scalar field. Same number of GW polarizations as in GR

f(T) gravity

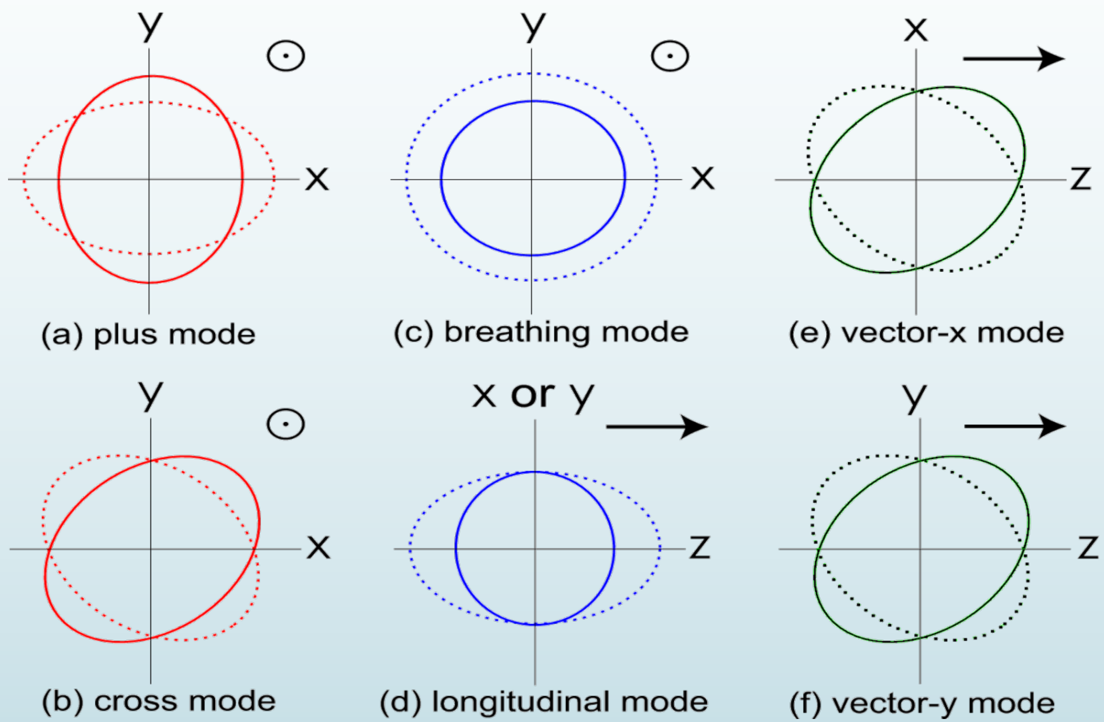
f(T) like GR, only + and x, no further mode

Bamba, Capozziello, De Laurentis, Nojiri, Saez-Gomez PLB 727 (2013) 194

Considering all polarizations

- $V_0'' = 0$
- $\chi = 0$

$\rightarrow$ $\times$ and $+$ polarizations $\longrightarrow$ GR limit



Polarization modes of GWs

The above conditions depend on the value of k^2

For $k^2=0$ modes ➡ massless spin-2 field with two independent polarizations (GR)

For $k^2 \neq 0$ modes ➡ massive spin-2 (ghost) modes
+
scalar modes

$M= (2s+1)$ with $s=0,2$ and $M=6$
6 modes instead of 2!

Polarization modes of GWs

Riemann's Theorem:

A n-dimensional metric has
f=n(n−1)/2 degrees of freedom
n=4, f=6.

In GR we have only 2 polarizations.
The full budget is 6. We MUST search for further polarizations.
This issue comes from first principles!

First case: massless spin-2 modes

In the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different from zero can be chosen. The condition $h = 0$ gives $A_{11} = -A_{22}$.

In this frame we may take the bases of polarizations defined as

$$e^{(+)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e^{(\times)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e^{(s)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

...the characteristic amplitude

$$h_{\mu\nu}(t,z) = A^{+}(t-z)e^{(+)}_{\mu\nu} + A^{\times}(t-z)e^{(\times)}_{\mu\nu} + h_s(t - v_G z)e^{s}_{\mu\nu}$$

two standard polarizations of GW arise from GR

the massive field arising from the generic high-order theory

Second case: massive spin-2 modes

We get 6 polarizations defined as the number of independent components of metric

$$e^{(+)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad e^{(\times)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e^{(B)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad e^{(C)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$e^{(D)}_{\mu\nu} = \frac{\sqrt{2}}{3}\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad e^{(s)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

.... the amplitude in terms of the 6 polarization states is

$$h_{\mu\nu}(t,z) = A^{+}(t - v_{G_{s2}}z)e^{(+)}_{\mu\nu} + A^{\times}(t - v_{G_{s2}}z)e^{(\times)}_{\mu\nu} + B^{B}(t - v_{G_{s2}}z)e^{(B)}_{\mu\nu} + C^{C}(t - v_{G_{s2}}z)e^{(C)}_{\mu\nu} + D^{D}(t - v_{G_{s2}}z)e^{(D)}_{\mu\nu} + h_s(t - v_G z)e^{s}_{\mu\nu}.$$

the group velocity of the massive spin-2 field is given by →

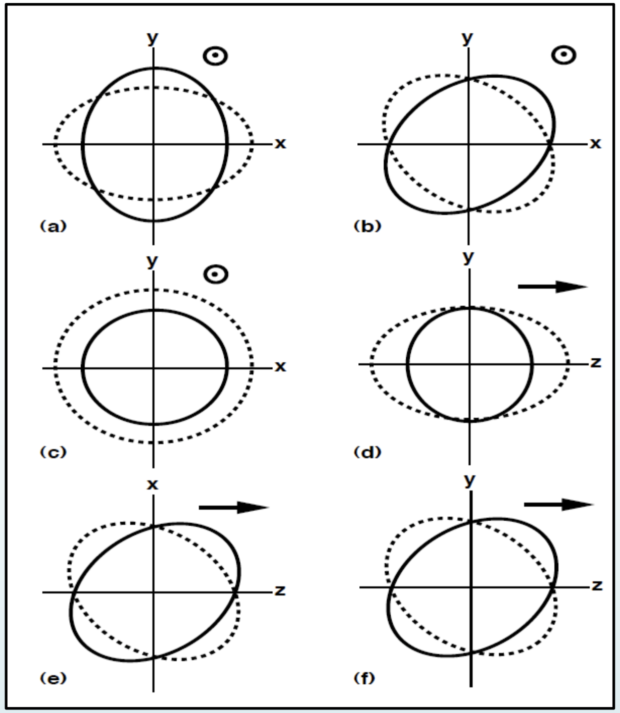
$$v_{G_{s2}} = \frac{\sqrt{\omega^2 - m_{s2}^2}}{\omega}$$

Polarization modes of GWs

Displacement induced by each mode on a sphere of test particles.

The wave propagates out of the plane in (a), (b), (c),
and it propagates in the plane in (d), (e) and (f).

In (a) and (b) we have respectively the plus mode and cross mode, in (c) the scalar mode, in (d), (e) and (f) the D, B and C mode.



C. Bogdanos, S.Capozziello, M. De Laurentis, S. Nesseris, *Astrop.Phys.* 34 (2010) 236
S. Capozziello and A. Stabile, *Astr. Sp. Sc.* 358 (2015) 27

Detector response

Now one can study the detector response to each GW polarization without specifying, a priori, the theoretical model

...the angular pattern function of a detector to GWs is

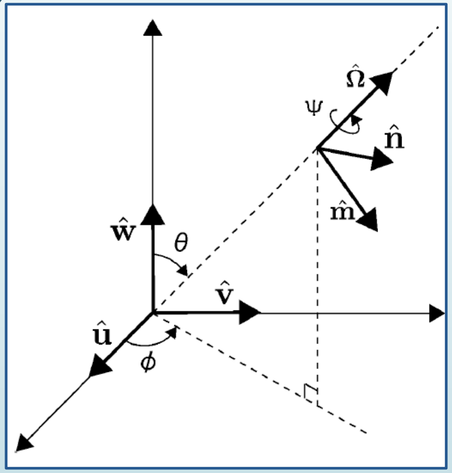
$$F_A(\hat{\Omega}) = \mathbf{D} : \mathbf{e}_A(\hat{\Omega}) \,,$$
$$\mathbf{D} = \frac{1}{2}\left[\hat{\mathbf{u}} \otimes \hat{\mathbf{u}} - \hat{\mathbf{v}} \otimes \hat{\mathbf{v}}\right]$$

holds only when the arm length of the detector is smaller and smaller than the GW wavelength that we are taking into account

$A = +, \times, B, C, D, s$

detector tensor = response of a laser-interferometric detector

maps the metric perturbation in a signal on the detector



This is relevant for dealing with ground-based laser interferometers but this condition could not be valid when dealing with space interferometers like LISA

Detector response

the coordinate system for the GW, rotated by angles (θ,φ), is

$$\begin{cases} \hat{\mathbf{u}}' = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) \\ \hat{\mathbf{v}}' = (-\sin\phi, \cos\phi, 0) \\ \hat{\mathbf{w}}' = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \end{cases}$$

The rotation with respect to the angle ψ, around the GW propagating axis, gives the most general choice for the coordinate system

$$\begin{cases} \hat{\mathbf{m}} = \hat{\mathbf{u}}'\cos\psi + \hat{\mathbf{v}}'\sin\psi \\ \hat{\mathbf{n}} = -\hat{\mathbf{v}}'\sin\psi + \hat{\mathbf{u}}'\cos\psi \\ \hat{\mathbf{\Omega}} = \hat{\mathbf{w}}' \end{cases}$$

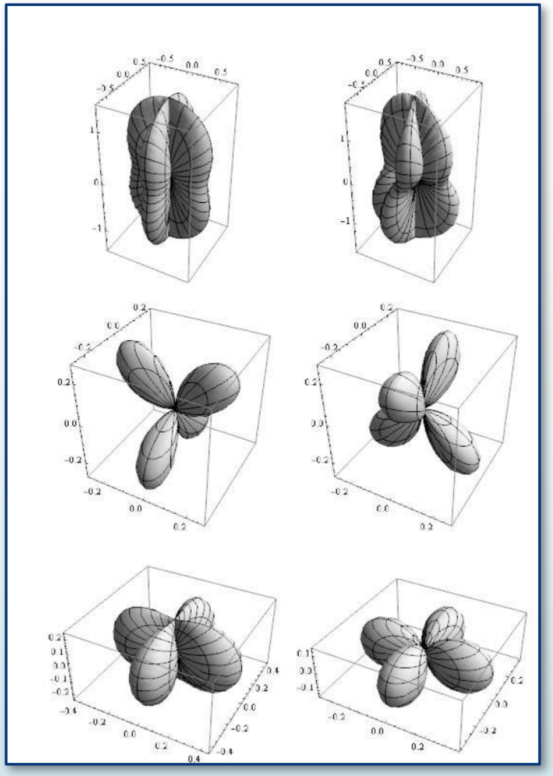
the polarization tensors are

$$\begin{aligned}
\mathbf{e}_+ &= \frac{1}{\sqrt{2}}(\hat{\mathbf{m}}\otimes\hat{\mathbf{m}} - \hat{\mathbf{n}}\otimes\hat{\mathbf{n}}) \ , \\
\mathbf{e}_\times &= \frac{1}{\sqrt{2}}(\hat{\mathbf{m}}\otimes\hat{\mathbf{n}} + \hat{\mathbf{n}}\otimes\hat{\mathbf{m}}) \ , \\
\mathbf{e}_B &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{m}}\otimes\hat{\mathbf{\Omega}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{m}}\right) \ , \\
\mathbf{e}_C &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{n}}\otimes\hat{\mathbf{\Omega}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{n}}\right) \ . \\
\mathbf{e}_D &= \frac{\sqrt{3}}{2}\left(\frac{\hat{\mathbf{m}}}{2}\otimes\frac{\hat{\mathbf{m}}}{2} + \frac{\hat{\mathbf{n}}}{2}\otimes\frac{\hat{\mathbf{n}}}{2} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{\Omega}}\right) \\
\mathbf{e}_s &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{\Omega}}\otimes\hat{\mathbf{\Omega}}\right) \ ,
\end{aligned}$$

Detector response

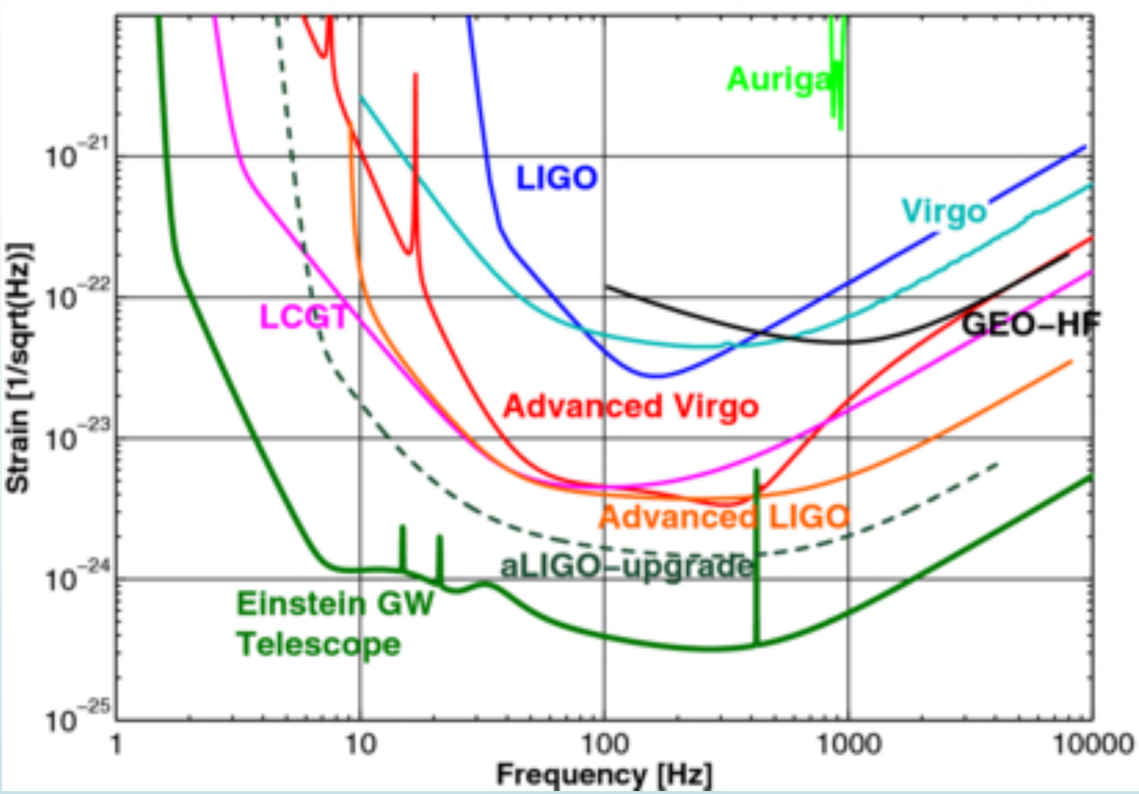
the angular patterns for each polarization

$$\begin{aligned}
F_+(\theta,\phi,\psi) &= \frac{1}{\sqrt{2}}(1+\cos^2\theta)\cos 2\phi\cos 2\psi \\
&\quad -\cos\theta\sin 2\phi\sin 2\psi\ , \\
F_\times(\theta,\phi,\psi) &= -\frac{1}{\sqrt{2}}(1+\cos^2\theta)\cos 2\phi\sin 2\psi \\
&\quad -\cos\theta\sin 2\phi\cos 2\psi\ , \\
F_B(\theta,\phi,\psi) &= \sin\theta\,(\cos\theta\cos 2\phi\cos\psi - \sin 2\phi\sin\psi)\ , \\
F_C(\theta,\phi,\psi) &= \sin\theta\,(\cos\theta\cos 2\phi\sin\psi + \sin 2\phi\cos\psi)\ , \\
F_D(\theta,\phi) &= \frac{\sqrt{3}}{32}\cos 2\phi\left(6\sin^2\theta + (\cos 2\theta + 3)\cos 2\psi\right) \\
F_s(\theta,\phi) &= \frac{1}{\sqrt{2}}\sin^2\theta\cos 2\phi\ .
\end{aligned}$$



Detector response

Combining detectors is the right strategy to reveal further modes



The stochastic background of GWs

The contributions to the gravitational radiation coming from Extended Gravity could be efficiently selected by investigating gravitational sources in extremely strong field regimes.

The further polarizations coming from the higher order contributions could be, in principle, investigated by the response of combined GW detectors
K. Hayama and A. Nishizawa, PRD 87 (2013) 062003

Another approach is to investigate these further contributions from the cosmological background

The stochastic background of GWs

The stochastic background of GWs

GW background can be roughly divided into two main classes of phenomena:

- the background generated by the incoherent superposition of gravitational radiation emitted by large populations of astrophysical sources;
- the primordial GW background generated by processes in the early cosmological eras (see also BICEP2 and PLANCK results)

it can be described and characterized by a dimensionless spectrum

$$\Omega_{sgw}(f) = \frac{1}{\rho_c}\frac{d\rho_{sgw}}{d\ln f},$$

→ energy density of part of the gravitational radiation contained in the frequency range f to f + df.

$$\rho_c \equiv \frac{3H_0^2}{8\pi G}$$

→ H_0: today observed Hubble expansion rate

→ ρ_c: today critical energy density of the Universe

The stochastic background of GWs

The GW stochastic background energy density of all modes can be written as

$$\Omega^A_{\rm gw} \equiv \Omega^+_{\rm gw} + \Omega^\times_{\rm gw} + \Omega^B_{\rm gw} + \Omega^C_{\rm gw} + \Omega^D_{\rm gw} + \Omega^s_{\rm gw}$$

GR

$$\Omega^{GR}_{\rm gw} = \Omega^+_{\rm gw} + \Omega^\times_{\rm gw}$$

$$\Omega^+_{\rm gw} = \Omega^\times_{\rm gw}$$

Higher-order

$$\Omega^{HOG}_{\rm gw} = \Omega^B_{\rm gw} + \Omega^C_{\rm gw} + \Omega^D_{\rm gw}$$

$$\Omega^B_{\rm gw} = \Omega^C_{\rm gw} = \Omega^D_{\rm gw}$$

Scalar term

S. Capozziello, M. De Laurentis, S. Nojiri, S.D. Odintsov PRD95 (2017) 083524

The stochastic background of GWs

The equation for the characteristic amplitude adapted to one of the components of the GWs can be used

$$h_A(f) \simeq 8.93 \times 10^{-19} \left(\frac{1Hz}{f}\right) \sqrt{h_{100}^2 \Omega_{gw}(f)}$$

and then we obtain for the GR modes

$$h_{GR}(100Hz) < 1.3 \times 10^{-23}$$

for the higher-order modes

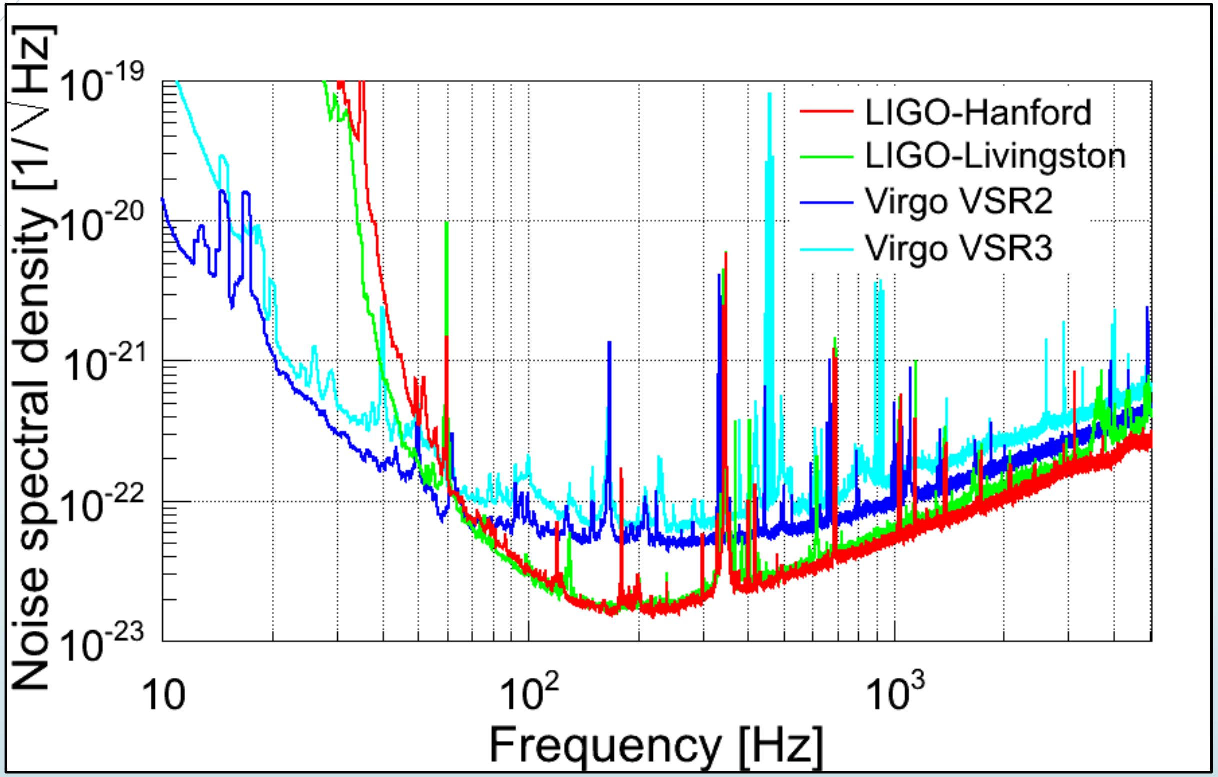
$$h_{HOG}(100Hz) < 7.3 \times 10^{-25}$$

and for scalar modes

$$h_s(100Hz) < 2 \times 1.410^{-26}$$

The stochastic background of GWs

Then, since we expect a sensitivity of the order of 10^{-22} for the above interferometers at ≈ 100Hz, we need to gain at least three orders of magnitude.



The stochastic background of GWs

Let us analyze the situation also at smaller frequencies.

The sensitivity of the VIRGO interferometer is of the order of 10^{-21} at ≈ 10Hz and in that case it is for the GR modes

$$h_{GR}(100Hz) < 1.3 \times 10^{-22}$$

for the higher-order modes

$$h_{HOG}(100Hz) < 7.3 \times 10^{-24}$$

and for scalar modes

$$h_s(100Hz) < 1.4 \times 10^{-25}$$

Still, these effects are below the sensitivity threshold to be observed today but new generation interferometers could be suitable (e.g. Advanced VIRGO-LIGO)

The stochastic background of GWs

The sensitivity of the LISA interferometer should be of the order of 10^{-22} at $\approx 10^{-3}$Hz and in that case it is

$$h_{GR}(100Hz) < 1.3 \times 10^{-18}$$

while for the higher-order modes

$$h_{HOG}(100Hz) < 7.3 \times 10^{-20}$$

and for scalar modes

$$h_s(100Hz) < 1.4 \times 10^{-21}$$

This means that a stochastic background of relic GWs could be, in principle, detected by the LISA interferometer, including the additional modes.

Conclusions and outlooks

- The number of GW polarizations depends on the considered theory of gravity
- There is no extra polarization in TEGR and in *f(T)* theory with respect to GR
  see **Bamba, Capozziello, De Laurentis, Nojiri, Saez-Gomez PLB 727 (2013) 194**
- *f(T,B)* is equivalent to *f(R)* with +, x and scalar mode
- Scalar field, non-minimally coupled to torsion, has only the two polarization of GR plus the scalar mode related to the scalar field itself
- New polarizations appear when the scalar field is coupled to the boundary term B
- Extra massless and massive modes arise **only** when the scalar torsion and the boundary term are non-minimally coupled.

Conclusions and outlooks

•The above analysis covers the most extended gravity models with a generic class of Lagrangian density with higher-order terms in T and R pictures

• Linearized field equations about Minkowski background gives, besides a massless spin-2 field (the GR graviton), also spin-0 and spin-2 massive modes. One gets 5+1=6 polarizatons.

• The detectability of additional polarization modes could be possible, in principle, combining more than 2 interferometers. Possible signal in the GW stochastic background. New polarization modes, in general, can be used to constrain theories beyond GR.

Conclusions and outlooks

….a point has to be discussed in detail !!!

if the source is coherent → The interferometer is directionally sensitive and we also know the orientation of the source

- The massive mode coming from the simplest extension, would induce longitudinal displacements along the direction of propagation which should be detectable and only the amplitude due to the scalar mode would be the detectable, "new" signal

- we could have a second scalar mode inducing a similar effect, coming from the massive ghost, although with a minus sign.

  X. Calmet, S. Capozziello, D. Pryer, EPJC 77 (2017) 589

Conclusions and outlooks

....another point !!!

if the source is not coherent (case of the stochastic background), no directional detection of the gravitational radiation is possible

The background has to be isotropic, the signal should be the same regardless of the orientation of the interferometer, no matter how the plane is rotated, it would always record the characteristic amplitude h_c.

- There is no way to disentangle the modes in the background, being h_c related to the total energy density of the gravitational radiation, which depends on the number of modes available
- Every mode contributes in the same way, at least in the limit where the mass of massive and ghost modes are very small
- It should be the number of available modes that makes the difference, not their origin.

Conclusions and outlooks

- Massive and scalar modes are certainly of interest for direct detection by the Advanced LIGO-VIRGO, KAGRA, LIGO-India experiments

- Massive GW modes could be produced in more significant quantities in cosmological or early astrophysical processes.

- Could they constitute the new frontier of physics also at LHC?

- Are massive GWs natural DM candidates?

WORK IN PROGRESS…..