

TWO APPLICATIONS OF THE FACTORIZATION OF INFRARED DYNAMICS

BASED ON U. Kol, MP, R. Javadinezhad
arxiv:1808.xxxx [hep-th]

AND PREVIOUS WORK

M. Mirbabayi MP, PRL 117 (2016) n. 21, 211301 arxiv:
1607.03120 [hep-th]

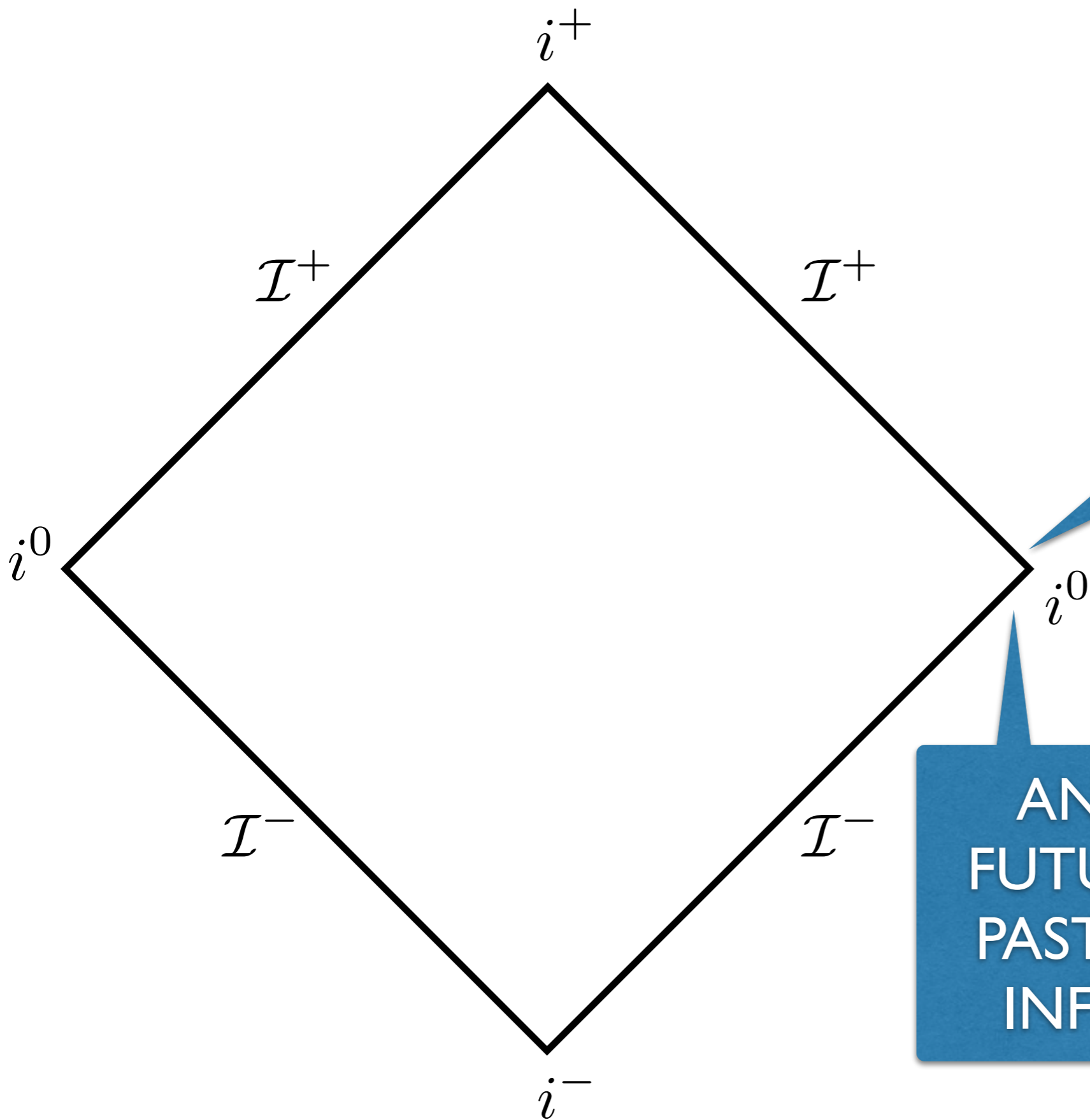
R. Bousso, MP, CQG 34 (2017) n. 20, 204001
arxiv:1706.00436 [hep-th]

R. Bousso, MP, PRD 96 (2017) n. 8, 086016
arxiv:1706.09280 [hep-th]

- BMS CHARGES AND SOFT BMS CHARGES
- BMS CHARGE CONSERVATION LAWS
- CANONICALLY CONJUGATE IR DEGREES OF FREEDOM
- THE OTHER CONSERVATION LAW
- CANONICAL TRANSFORMATIONS ON IR VARIABLES
- FACTORIZATION OF IR DYNAMICS ON OPERATORS
- A “BETTER LORENTZ TRANSFORMATION”?
- CONSISTENCY OF THE NEW LORENTZ ALGEBRA
- HAWKING RADIATION IN THE PRESENCE OF IR DEGREES OF FREEDOM (SOFT HAIR)

- FACTORIZATION OF IR DYNAMICS ON FIXED BLACK-HOLE BACKGROUND GEOMETRY
- ENTANGLEMENT OF SOFT HAIR WITH THE HAWKING RADIATION: AN EXACT CALCULATION IN THE ZERO-FREQUENCY LIMIT
- ABSENCE OF OBSERVABLE EFFECTS

BMS CHARGES



CHARGES ARE
DEFINED AT
PAST OF
FUTURE NULL
INFINITY

AND AT
FUTURE OF
PAST NULL
INFINITY

BMS CHARGES AT FUTURE INFINITY

$$ds^2 = -du^2 + \frac{2Gm}{r} du^2 - 2dudr + r^2 \gamma_{AB} d\theta^A d\theta^B + r C_{AB}^+ d\theta^A d\theta^B$$

$$N_{AB}^+ = \partial_u C_{AB}^+$$

$$Q_{h\,lm}^+ = \frac{1}{4\pi} \int_{I_{\pm}^+} d^2\theta \sqrt{\gamma} Y_{lm}(\theta) m$$

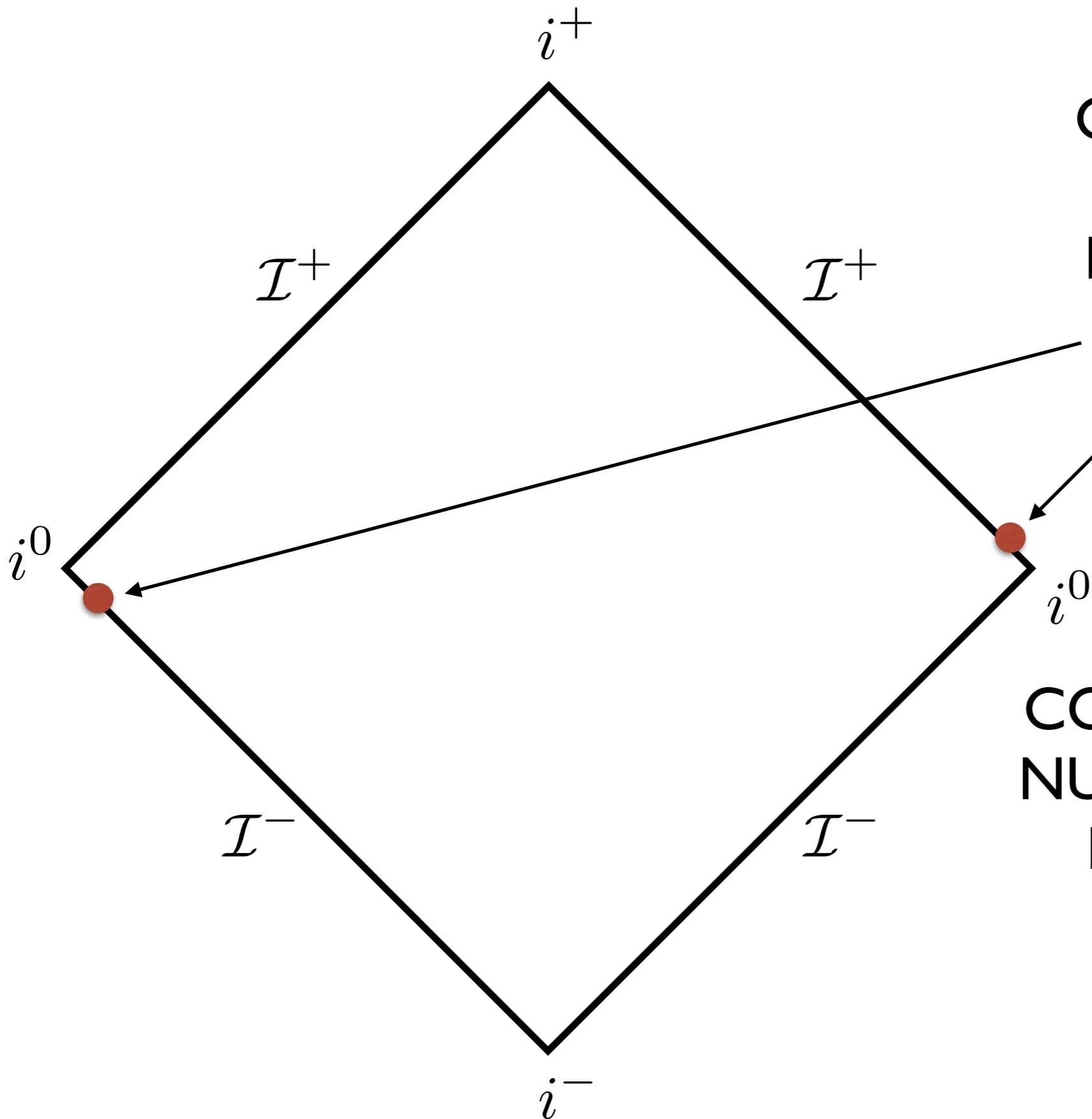
BONDI MASS ASPECT

$$Q_{lm}^+ = Q_{s\,lm}^+ + Q_{h\,lm}^+$$

$$Q_{h\,lm}^+ = \frac{1}{4\pi} \int_{I^+} dud^2\theta \sqrt{\gamma} Y_{lm}(\theta) \left[\lim_{r \rightarrow \infty} r^2 T_{uu} - \frac{1}{8G} N_{AB}^+ N^{+AB} \right]$$

$$Q_{s\,lm}^+ = -\frac{1}{16\pi G} \int_{I^+} dud^2\theta \sqrt{\gamma} Y_{lm}(\theta) D^A D^B N_{AB}^+, \quad l \geq 2$$

BMS CHARGES CONSERVATION



CONFORMAL
COMPACTIFICATION
MAKES THE TWO
POINTS CLOSE AND
IDENTIFIES ANGLES

ANTIPODALLY

$$\phi^- = \phi^+ + 2\pi$$

$$\theta^- = \pi - \theta^+$$

CONTINUITY ALONG
NULL GENERATOR OF
BOUNDARY=BMS
CHARGE
CONSERVATION

$$Q_{lm}^+ = Q_{lm}^-$$

BMS CHARGE CONSERVATION = SOFT THEOREMS, BUT WE CAN DO BETTER!

$$[Q_s^+{}_{lm}, N_{AB}^+(u, \theta)] = 0$$

SOFT CHARGE NEEDS CONJUGATE VARIABLE

(OTHERWISE IT IS A CONSTANT ON IRREPS OF LOCAL OPERATOR ALGEBRA)

ADDITIONAL DEGREE OF FREEDOM: BOUNDARY GRAVITON

$$\lim_{u \rightarrow -\infty} C_{AB}^+(u, \theta) = -2D_A D_B C^+(\theta) + \gamma_{AB} D^2 C^+(\theta)$$

$$[Q_s^+{}_{lm}, C_{kn}^+] = -i\delta_{lk}\delta_{mn} \quad l, k \geq 2$$

CANONICAL COMMUTATION RELATIONS

THE BOUNDARY GRAVITON ALSO OBEYS A SIMPLE MATCHING
CONDITION WHEN ANGLES AT PAST AND FUTURE NULL
INFINITY ARE IDENTIFIED ANTIPODALLY

$$C_{lm}^+ = C_{lm}^-$$

SEVERAL ARGUMENTS SUPPORT THIS MATCHING CONDITION

IN PARTICULAR, IT IS THE ONLY LORENTZ AND CPT
INVARIANT BOUNDARY CONDITION, IT IS USED IN MOST
GENERAL RELATIVITY COMPUTATIONS IN ASYMPTOTICALLY
FLAT SPACETIME AND IT IS VALID TO ALL-ORDER
PERTURBATIVE GRAVITY COMPUTATIONS

A CANONICAL TRANSFORMATION

DEFINE IT AS FOLLOWS

$$U Q_{s\ lm}^+ U^{-1} = Q_{lm}^+ \quad U C_{lm}^+ U^{-1} = C_{lm}^+$$

EXPLICITLY

$$U = \exp\left[-i \sum_{l=2}^{\infty} \sum_{m=-l}^l Q_{h\ lm}^+ C_{lm}^+\right]$$

DEFINE DRESSED VARIABLES BY THE SAME CANONICAL TRANSFORMATION

$$U N_{AB}^+ U^{-1} \equiv \hat{N}_{AB}^+$$

KEY PROPERTY: THEY COMMUTE WITH THE BMS CHARGES

$$[\hat{N}_{AB}^+, Q_{lm}^+] = U [N_{AB}^+, Q_{s\ lm}^+] U^{-1} = 0$$

SO $\hat{N}_{AB}^+, Q_{lm}^+, C_{lm}^+$ FORM A COMPLETE SET OF CANONICAL VARIABLES WITH CCR

FACTORIZATION OF IR DYNAMICS ON OPERATORS

USE THE HEISENBERG PICTURE: OPERATORS EVOLVE IN TIME, STATES DO NOT

$$O^+ = \Omega^{-1} O^- \Omega$$

BMS CHARGES AND BOUNDARY GRAVITONS COMMUTE WITH TIME EVOLUTION OPERATOR BECAUSE OF MATCHING CONDITIONS

$$Q_{lm}^+ = \Omega^{-1} Q_{lm}^- \Omega = Q_{lm}^- \quad C_{lm}^+ = \Omega^{-1} C_{lm}^- \Omega = C_{lm}^-$$

CCR: $Q_{lm}^\pm = -i \frac{\partial}{\partial C_{lm}^\pm}$

CCR+MATCHING CONDITIONS:

$$[Q_{lm}^\pm, \Omega] = -i \frac{\partial \Omega}{\partial C_{lm}^\pm} = 0 \quad [C_{lm}^\pm, \Omega] = 0 \rightarrow \Omega = \Omega(\hat{N})$$

HEISENBERG TIME EVOLUTION INDEPENDENT OF ALL IR DEGREES OF FREEDOM!

$$[Q_{lm}^{\pm}, \Omega] = -i \frac{\partial \Omega}{\partial C_{lm}^{\pm}} = 0 \quad [C_{lm}^{\pm}, \Omega] = 0 \rightarrow \Omega = \Omega(\hat{N})$$

APPLICATION #1: USE DRESSING TO DEFINE A NEW “BETTER” LORENTZ

LORENTZ DOES NOT COMMUTE WITH BMS SO VACUA RELATED BY GENERAL SUPERTRANSLATIONS ARE NOT SIMULTANEOUSLY LORENTZ INVARIANT.

SOLUTION: DEFINE A NEW LORENTZ THAT DOES COMMUTE WITH BMS

$$\delta_L \hat{N}_{AB}^+ = [\hat{L}^+, \hat{N}_{AB}^+] = U[L, N_{AB}^+]U^{-1}$$
$$\delta_L C_{lm}^+ = \delta_L Q_{lm}^+ = 0 \quad \hat{L}^+ = U L U^{-1}$$

CONSISTENCY CHECKS

I) NEW LORENTZ COMMUTES WITH TIME EVOLUTION

NEW LORENTZ AT PAST NULL INFINITY

$$\delta_L \hat{N}_{AB}^- = [\hat{L}^-, \hat{N}_{AB}^-] = V[L, N_{AB}^-]V^{-1}$$

$$\delta_L C_{lm}^- = \delta_L Q_{lm}^- = 0 \quad \hat{L}^- = VLV^{-1} \quad V = \Omega U \Omega^{-1}$$

LORENTZ TRANSFORM THEN EVOLVE

=

EVOLVE THEN LORENTZ TRANSFORM

$$\Omega^{-1} \delta_L \hat{N}_{AB}^- \Omega = \Omega^{-1} V[L, N_{AB}^+] V^{-1} \Omega = U[L, \Omega^{-1} N_{AB}^- \Omega] U = U[L, N_{AB}^+] U^{-1} = \delta_L \hat{N}_{AB}^+$$

FROM DEFINITION OF U AND
L=SYMMETRY OF TIME EVOLUTION

CONSISTENCY CHECKS

2) JACOBI IDENTITIES

$$\delta_\xi \hat{N}_{AB}^+ = U[L, N_{AB}^+]U^{-1} = U \mathcal{L}_\xi N_{AB}^+ U^{-1} = \mathcal{L}_\xi \hat{N}_{AB}^+$$

LIE DERIVATIVE

$$\delta_\xi(\delta_\eta X) - \delta_\eta(\delta_\xi X) = \delta_{[\xi, \eta]} X, \quad X = C_{lm}^+, Q_{lm}^+, N_{AB}^+$$

$$[Y, \delta_\xi X] + \delta_\xi[X, Y] + [\delta_\xi Y, X] = 0, \quad X, Y = Q_{lm}^+, C_{lm}^+ \text{ or } X, Y = N_{AB}^+$$

SATISFIED!

$$[\delta_\xi \hat{N}_{AB}^+, X] + [\hat{N}_{AB}^+, \delta_\xi X] + \delta_\xi[X, \hat{N}_{AB}^+] = [\mathcal{L}_\xi \hat{N}_{AB}^+, X] = 0, \quad X = Q_{lm}^+, C_{lm}^+$$

SATISFIED!

THE NEW LORENTZ IS CONSISTENT AS ISOMORPHISM OF THE
ASYMPTOTIC OPERATOR ALGEBRA.

A CHARGE IS A HERMITIAN OPERATOR DEFINED ON AN
IRREDUCIBLE REPRESENTATION OF THE CCR.

TO PROMOTE THE ISOMORPHISM TO AN OPERATOR WE HAVE
TO DEFINE ITS ACTION ON THE VACUUM, CHECK THAT THE
DOMAIN OF DEFINITION IS DENSE AND THAT IT COINCIDES
WITH THE DOMAIN OF THE ADJOINT.

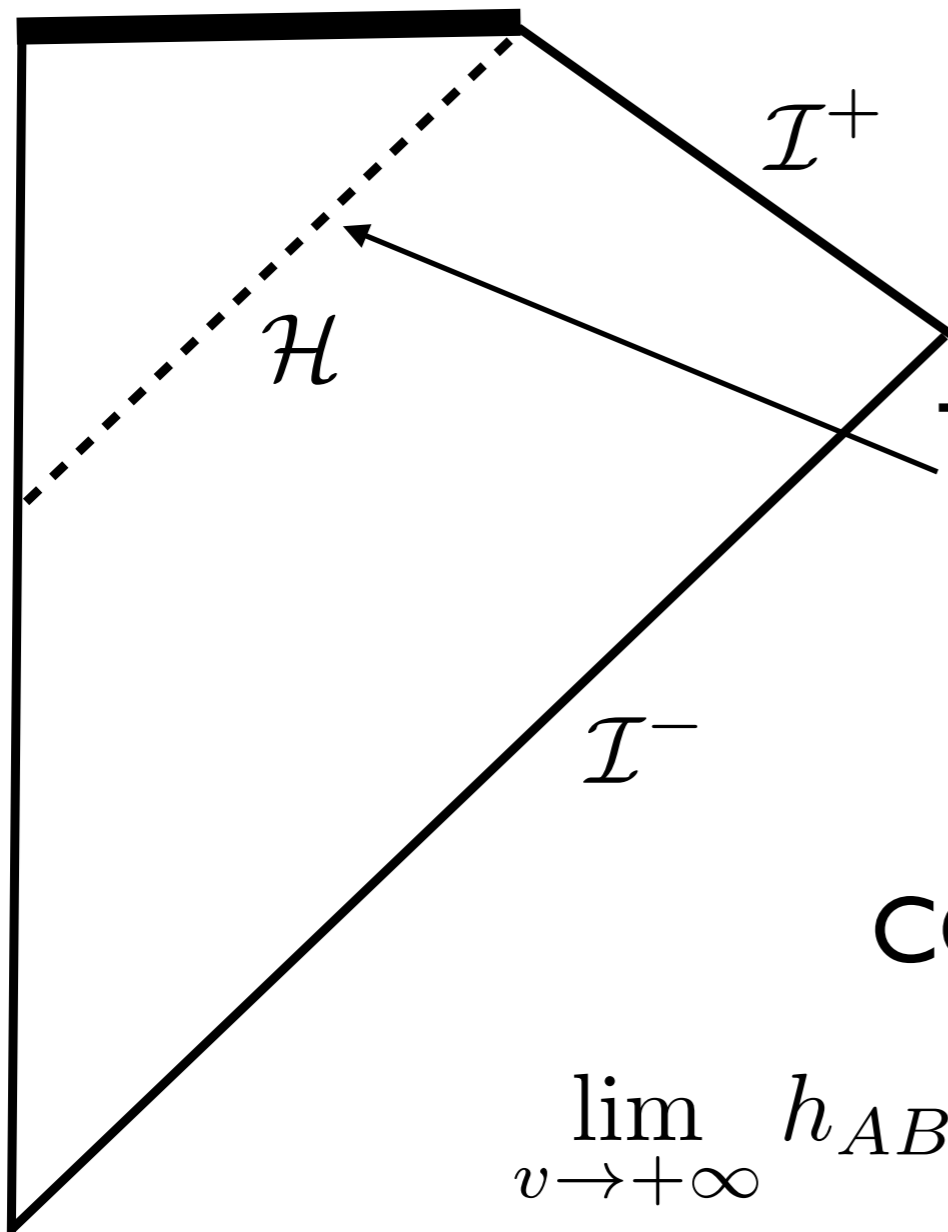
SO THE CONSTRUCTION IS NOT COMPLETE YET, BUT IT IS A
FIRST STEP TOWARDS DEFINING NEW LORENTZ CHARGES
WHICH DO NOT BELONG TO THE BMS ALGEBRA AND THAT
COMMUTE WITH SUPERTRANSLATIONS

APPLICATION #2: HAWKING RADIATION IN THE PRESENCE OF SOFT HAIR

WE WORK IN THE LIMIT

$$G \rightarrow 0, \quad GM = \text{constant}$$

(NOTHING TO SAY ABOUT INFORMATION PUZZLE)



METRIC: VAIDYA

TO FIRST ORDER IN FLUCTUATIONS
CHARGES CAN BE DEFINED HERE

$$Q_{lm}^H = Q_{s\,lm}^H + Q_{h\,lm}^H$$

CONJUGATE VARIABLE TO $Q_{s\,lm}^H$

$$\lim_{v \rightarrow +\infty} h_{AB} \Big|_{\mathcal{H}} = (4MD_A D_B - 2M\gamma_{AB} D^2) F$$

CHARGES AND BOUNDARY GRAVITONS AT NULL INFINITY
CAN BE DEFINED EXACTLY AS BEFORE.

WE WILL NEED ONLY THE MATCHING CONDITIONS

$$Q_{s\,lm}^H + Q_{h\,lm}^H + Q_{s\,lm}^+ + Q_{h\,lm}^+ = Q_{s\,lm}^- + Q_{h\,lm}^-$$

$$C^+ = C^-$$

DEFINE TWO CANONICAL TRANSFORMATIONS

$$U_- Q_{s\,lm}^- U_-^{-1} = Q_{lm}^- \quad U_+ (Q_{s\,lm}^H + Q_{s\,lm}^+) U_+^{-1} = Q_{lm}^H + Q_{lm}^+$$

$$U_- = \exp\left[-i \sum_{lm} Q_{h\,lm}^- C_{lm}^-\right]$$

$$U_+ = \exp\left[-i \sum_{lm} Q_{h\,lm}^+ C_{lm}^+\right] \exp\left[-i \sum_{lm} Q_{h\,lm}^H F_{lm}\right]$$

DRESSED OPERATORS AT PAST AND FUTURE NULL INFINITY

$$\hat{\phi}^+ = U_+ \phi^+ U_+^{-1} \quad \hat{\phi}^- = U_- \phi^- U_-^{-1}$$

DRESSED OPERATORS AT PAST NULL INFINITY COMMUTE WITH

$$C_{lm}^-, Q_{lm}^-$$

SINCE VARIABLES AT THE HORIZON COMMUTE WITH VARIABLES AT FUTURE NULL INFINITY BY LOCALITY, DRESSED OPERATORS AT FUTURE NULL INFINITY COMMUTE WITH

$$C_{lm}^+, F_{lm}, Q_{lm}^H, Q_{lm}^+$$

DRESSING CANCELS THE EFFECT OF SUPERTRANSLATIONS. SUPERTRANSLATIONS ARE ANGLE-DEPENDENT SHIFTS OF NULL COORDINATES, HENCE:

$$\hat{\phi}^+(u, \theta) = \phi^+(u + C^+(\theta), \theta) \quad \hat{\phi}^- = \phi^-(v + C^-(\theta), \theta)$$

SCATTERING IN FIXED BACKGROUND (HAWKING 1974)

$$\phi^+(u, \theta) = \phi^-(v, \theta)|_{v=f(u)}, \quad f(u) = v_0 - C \exp(-\kappa u) + \dots$$

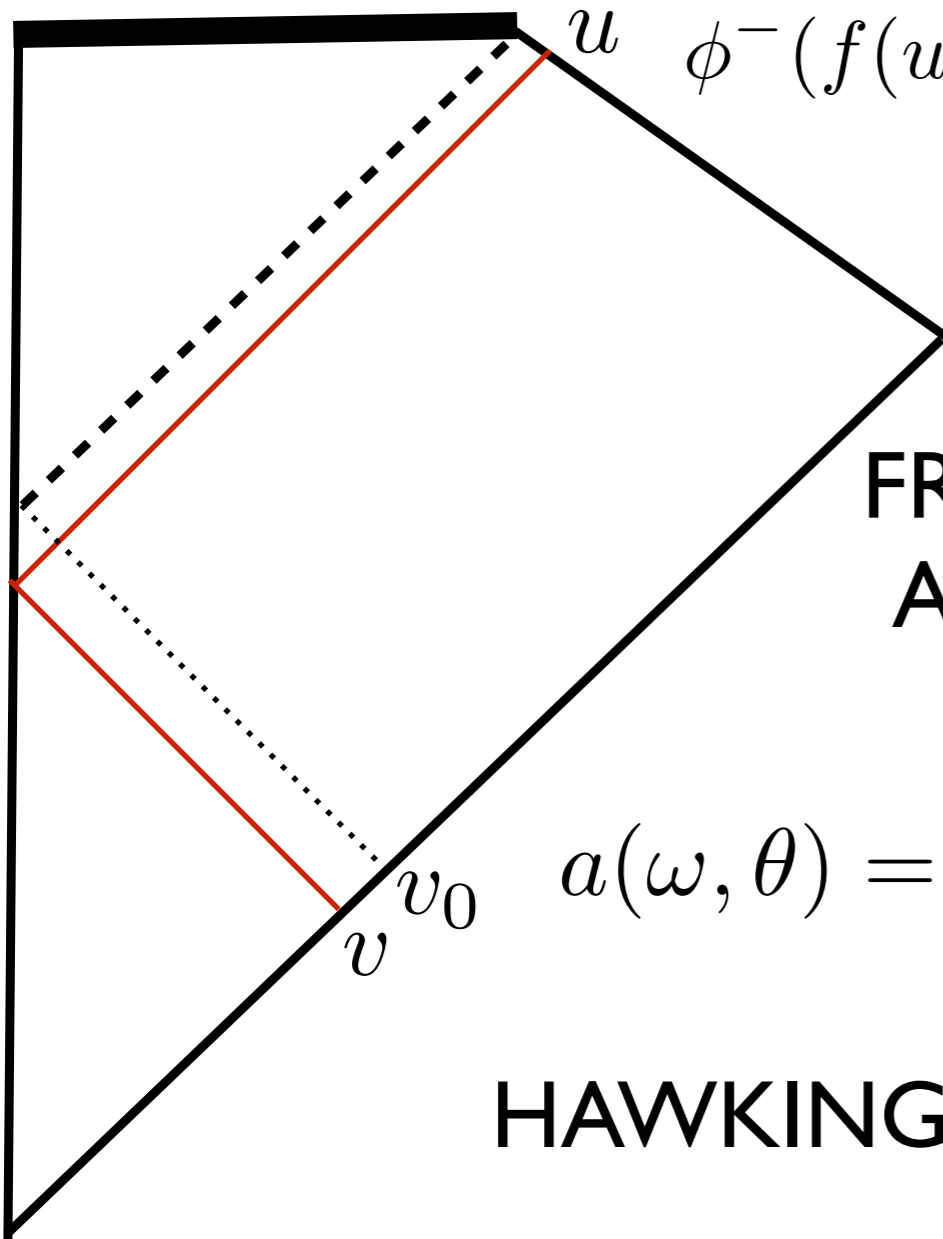
$$a(\omega, \theta) = \int du e^{i\omega u} \phi^+(u, \theta) = \int du e^{i\omega u} \phi^-(f(u), \theta)$$

$$\phi^-(f(u), \theta) = \int_0^\infty \frac{d\nu}{2\nu} \left[e^{-i\nu f(u)} b(\nu, \theta) + e^{i\nu f(u)} b^\dagger(\nu, \theta) \right]$$

THE FOURIER TRANSFORM IN **u**
 NOW CONTAINS BOTH POSITIVE
 FREQUENCY ANNIHILATION OPERATORS
 AND NEGATIVE FREQUENCY CREATION
 OPERATORS

$$a(\omega, \theta) = \int_0^\infty \frac{d\nu}{2\nu} \left[\alpha(\omega, \nu) b(\nu, \theta) + \beta(\omega, \nu) b^\dagger(\nu, \theta) \right]$$

HAWKING FLUX $\propto \int_0^\infty \frac{d\nu}{2\nu} |\beta(\omega, \nu)|^2$



INTRODUCING SOFT HAIR

$$\phi^\pm \rightarrow \hat{\phi}^\pm$$

$$\hat{a}(\omega, \theta) = \int du e^{i\omega u} \phi^+(u + C^+, \theta) = \int du e^{i\omega u} \phi^-(f(u) + C^-, \theta)$$

$$\phi^-(f(u) + C^-, \theta) = \int_0^\infty \frac{d\nu}{2\nu} \left[e^{-i\nu f(u)} e^{-i\nu C^-} b(\nu, \theta) + e^{i\nu f(u)} e^{i\nu C^-} b^\dagger(\nu, \theta) \right]$$

$$\hat{a}(\omega, \theta) = e^{-i\omega C^+} a(\omega, \theta)$$

SO THE BOGOLIUBOV COEFFICIENTS CHANGE BY A FREQUENCY-DEPENDENT PHASE

$$\beta(\omega, \nu) \rightarrow \beta'(\omega, \nu) = e^{i\omega C^+ + i\nu C^-} \beta(\omega, \nu)$$

THE HAWKING FLUX IS UNCHANGED

$$\int_0^\infty \frac{d\nu}{2\nu} |\beta(\omega, \nu)|^2 \rightarrow \int_0^\infty \frac{d\nu}{2\nu} |\beta'(\omega, \nu)|^2 = \int_0^\infty \frac{d\nu}{2\nu} |\beta(\omega, \nu)|^2$$

CONCLUSIONS

- THE ZERO-FREQUENCY SECTOR OF GRAVITY IN ASYMPTOTICALLY FLAT SPACE FACTORIZES UNDER VERY GENERAL ASSUMPTIONS, INDEPENDENTLY OF THE DETAILED DYNAMICS IN THE INTERIOR OF THE SPACETIME
- FACTORIZATION IS PARTICULARLY CLEAR WHEN THE CANONICAL FORMALISM IS USED AND FACTORIZATION IS APPLIED TO OPERATORS INSTEAD OF STATES
- FACTORIZATION OF IR DYNAMICS ON OPERATORS IS ACHIEVED BY A CANONICAL TRANSFORMATION, WHICH WE CALLED “DRESSING.”
- DRESSING MAY BE USED TO DEFINE A “BETTER LORENTZ” SYMMETRY THAT COMMUTES WITH SUPERTRANSLATIONS

- DRESSING ALLOWS TO COMPUTE IN A SIMPLE MANNER THE EFFECT OF INFRARED HAIR ON HAWKING RADIATION IN THE NO-BACKREACTION APPROXIMATION
- THERE IS AN EFFECT ON BOGOLIUBOV COEFFICIENTS BUT SUCH EFFECT DISAPPEARS FROM THE HAWKING FLUX (AND FROM ANY CORRELATOR OF LOCAL OBSERVABLES COMPUTED AT FUTURE NULL INFINITY)

LAUNDRY LIST

- PROMOTE NEW LORENTZ TO UNITARY OPERATOR
- GO BEYOND THE STRICT ZERO-FREQUENCY LIMIT
- INCORPORATE THE EFFECT OF BACKREACTION ON HAWKING RADIATION COMPUTATION