Inflationary universe in unimodular $F(T)$ gravity

Reference: Modern Physics Letters A 32, 1750114 (2017) [arXiv:1605.02461 [gr-qc]]

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I. Introduction

- **Recent observations of Type Ia Supernova (SNe Ia) has supported that the current expansion of the universe is accelerating (Dark energy problem).**

[Perlmutter *et al*. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999)]

[Riess *et al*. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998)]

2011 Nobel Prize in Physics

- **Suppose that the universe is strictly homogeneous and isotropic.**

There are two approaches to explain the current accelerated expansion of the universe.

Reviews: E.g.,

[Copeland, Sami and Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006)]

[Nojiri and Odintsov, Phys. Rept. 505, 59 (2011); Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007)]

[Capozziello and Faraoni, *Beyond Einstein Gravity* (Springer, 2010)]

[De Felice and Tsujikawa, Living Rev. Rel. 13, 3 (2010)]

[Clifton, Ferreira, Padilla and Skordis, Phys. Rept. 513, 1 (2012)]

[KB, Capozziello, Nojiri and Odintsov, Astrophys. Space Sci. 342, 155 (2012)]

[Joyce, Jain, Khoury and Trodden, Phys. Rept. 568, 1 (2015)]

[Koyama, Rept. Prog. Phys. 79, 046902 (2016)]

Gravitational field equation

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

Gravity **Matter**

$G_{\mu\nu}$: Einstein tensor, $\kappa^2 \equiv 8\pi / {M_{\mathrm{Pl}}}^2$, $M_{\mathrm{Pl}} = G_{\mathrm{N}}^{-1/2}$

$T_{\mu\nu}$: Energy-momentum tensor : Planck mass

G_{N} : Gravitational constant

(1) General relativistic approach

$\longrightarrow$ **Dark Energy**

(2) Extension of gravitational theory

(1) Candidates for dark energy

Cosmological constant, Scalar field, Fluid

(2) Extension of gravitational theory

- ***F*(*R*) gravity**

$F(R)$: Arbitrary function of the Ricci scalar R

[Capozziello, Carloni and Troisi, Recent Res. Dev. Astron. Astrophys. 1, 625 (2003)]

[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]

[Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]

- **DGP braneworld scenario**

[Dvali, Gabadadze and Porrati, Phys. Lett B 485, 208 (2000)]

[Deffayet, Dvali and Gabadadze, Phys. Rev. D 65, 044023 (2002)]

- **Galileon gravity**

[Nicolis, Rattazzi and Trincherini, Phys. Rev. D 79, 064036 (2009)]

Review: E.g., [Tsujikawa, Lect. Notes Phys. 800, 99 (2010)]

- **Horndeski theory**

[Horndeski, Int. J. Theor. Phys. 10, 363 (1974)]

[Kobayashi, Yamaguchi and Yokoyama, Prog. Theor. Phys. 126, 511 (2011)]

- **Massive gravity**

[van Dam and Veltman, Nucl.Phys. B22, 397 (1970)]

[Zakharov, JETP Lett. 12, 312, (1970)]

[de Rham and Gabadadze, Phys.Rev. D 82, 044020 (2010)]

[de Rham, Gabadadze and Tolley, Phys. Rev. Lett. 106, 231101 (2011)]

- **Bimetric gravity**

[Hassan and Rosen, Phys. Rev. Lett. 108, 041101 (2012)]

[Hassan and Rosen, JHEP 1202, 126 (2012)]

▪ Extended teleparallel gravity (*F*(*T*) gravity)

$F(T)$: Arbitrary function of the torsion scalar T

[Bengochea and Ferraro, Phys. Rev. D 79, 124019 (2009)]

[Linder, Phys. Rev. D 81, 127301 (2010) [Erratum-ibid. D 82, 109902 (2010)]]

Review: E.g.,

[Cai, Capozziello, De Laurentis and Saridakis, Rept. Prog. Phys. 79, 106901 (2016)]

Teleparallel gravity

- $g_{\mu\nu} = \eta_{AB} e^A_\mu e^B_\nu$

η_{AB} : Minkowski metric

$e_A(x^\mu)$: Orthonormal tetrad components

- $T^\rho_{\mu\nu} \equiv \Gamma^{\rho(W)}_{\mu\nu} - \Gamma^{\rho(W)}_{\nu\mu}$

$= e^\rho_A \left(\partial_\mu e^A_\nu - \partial_\nu e^A_\mu\right)$: **Torsion tensor**

$\Gamma^{\rho(W)}_{\mu\nu} \equiv e^\rho_A \partial_\mu e^A_\nu$: Weitzenböck connection

* μ and ν are coordinate indices on the manifold and also run over 0, 1, 2, 3, and $e_A(x^\mu)$ forms the tangent vector of the manifold.

* An index A runs over 0, 1, 2, 3 for the tangent space at each point x^μ of the manifold.

$$\boxed{T \equiv S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}}$$: **Torsion scalar**

$$S_\rho{}^{\mu\nu} \equiv \frac{1}{2}\left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho \; T^{\alpha\nu}{}_\alpha - \delta^\nu_\rho \; T^{\alpha\mu}{}_\alpha\right)$$

: Super-potential tensor

$$K^{\mu\nu}{}_\rho \equiv -\frac{1}{2}\left(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}\right)$$

: Co-torsion tensor

[Hehl, Von Der Heyde, Kerlick and Nester, Rev. Mod. Phys. 48, 393 (1976)]

[Hayashi and Shirafuji, Phys. Rev. D 19, 3529 (1979)
[Addendum-ibid. D 24, 3312 (1981)]]

Extended teleparallel gravity

Action $S = \int d^4x|e| \left(\frac{F(T)}{2\kappa^2} + \mathcal{L}_{\mathrm{M}} \right)$ **:** ***F*(*T*) gravity**

$|e| = \det\left(e^A_\mu\right) = \sqrt{-g}$ ***** $F(T) = T$ **: Teleparallel gravity**

$\mathcal{L}_{\mathrm{M}}$: Matter Lagrangian

${T^{(\mathrm{M})}}_\rho{}^\nu$: Energy-momentum tensor of matter

Gravitational field equation

$$e^{-1}\partial_\mu(e e^\rho_A S_\rho{}^{\mu\nu})\frac{dF(T)}{dT} + e^\rho_A S_\rho{}^{\mu\nu}\partial_\mu(T)\frac{d^2F(T)}{dT^2} - \frac{dF(T)}{dT}e^\lambda_A T^\rho{}_{\mu\lambda}S_\rho{}^{\nu\mu} + \frac{1}{4}e^\nu_A F(T) = \frac{\kappa^2}{2}e^\rho_A {T^{(\mathrm{M})}}_\rho{}^\nu$$

[Bengochea and Ferraro, Phys. Rev. D 79, 124019 (2009)]

Unimodular gravity

[Anderson and Finkelstein, Am. J. Phys. 39, 901 (1971)]

[Buchmuller and Dragon, Phys. Lett. B 207, 292 (1988)]

[Henneaux and Teitelboim, Phys. Lett. B 222, 195 (1989)]

[Unruh, Phys. Rev. D 40, 1048 (1989)]

[Ng and Dam, J. Math. Phys. 32, 1337 (1991)]

[Finkelstein, Galiautdinov and Baugh, J. Math. Phys. 42, 340 (2001)]

$$\sqrt{-g} = 1$$ **: Unimodular condition**

[Nojiri, Odintsov and Oikonomou, JCAP 1605, 046 (2016); Phys. Rev. D 93, 084050 (2016)]

[Saez-Gomez, Phys. Rev. D 93, 124040 (2016)]

$\longrightarrow$ **The cosmological constant arises as the trace-free part of the gravitational field equations, as long as the determinant of the metric is fixed to a number or a function.**

Unimodular condition

Flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = dt^2 - a^2(t)\left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\right]$$

$$e_\mu^A = \mathrm{diag}(1, a(t), a(t), a(t))$$ $a(t)$: Scale factor

- $$d\tau \equiv a^3(t)dt$$ τ : New time variable

$$\longrightarrow \quad ds^2 = a^{-6}(\tau)d\tau^2 - a^2(\tau)\left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\right]$$

$$a(\tau) \equiv a(t(\tau))$$

$$g_{\mu\nu} = \mathrm{diag}(a^{-6}(\tau), -a^2(\tau), -a^2(\tau), -a^2(\tau))$$

$\Rightarrow$ $|e| = \det\left(e_\mu^A\right) = \sqrt{-g} = 1$: **Unimodular condition is satisfied.**

Lagrange multiplier formulation of unimodular *F*(*T*) gravity

[Lim, Sawicki and Vikman, JCAP 1005, 012 (2010)]

[Capozziello, Matsumoto, Nojiri and Odintsov, Phys. Lett. B 693, 198 (2010)]

[Capozziello, Makarenko and Odintsov, Phys. Rev. D 87, 084037 (2013)]

Action $$S = \int d^4x \left\{ |e| \left[\frac{F(T)}{2\kappa^2} - \lambda \right] + \lambda \right\} + S_{\mathrm{M}}$$

[Nassur, Ainamon, Houndjo and Tossa, arXiv:1602.03172 [gr-qc]]

λ : Lagrange multiplier

S_{M} : Action of matter

* In the following, we set $2\kappa^2 = 1$ for simplicity.

Gravitational field equation

$$e^{-1}\partial_\mu(e e_A^\rho S_\rho{}^{\mu\nu})\frac{dF(T)}{dT} + e_A^\rho S_\rho{}^{\mu\nu}\partial_\mu(T)\frac{d^2F(T)}{dT^2}$$

$$-\frac{dF(T)}{dT}e_A^\lambda T^\rho{}_{\mu\lambda}S_\rho{}^{\nu\mu} + \frac{1}{4}e_A^\nu\left[F(T)-\lambda\right] = \frac{1}{4}e_A^\rho\, T^{(\mathrm{M})}{}_\rho{}^\nu$$

$$T^{(\mathrm{M})}{}_\rho{}^\nu \quad = \quad \mathrm{diag}(\rho_\mathrm{M}, -P_\mathrm{M}, -P_\mathrm{M}, -P_\mathrm{M})$$

In the FLRW space-time

$$T = -6H(t)^2 = -6a^6(\tau)\mathcal{H}(\tau)^2$$

$H(t) \equiv \dot{a}(t)/a(t)$: Hubble parameter

* The dot denotes derivatives with respect to *t*.

$$\mathcal{H}(\tau) \equiv \left(da(\tau)/d\tau\right)/a(\tau)$$

Gravitational field equation in the FLRW space-time

$$12a^6(\tau)\mathcal{H}^2\frac{dF(T)}{dT} + [F(T) - \lambda] - \rho_\mathrm{M} = 0$$

$$-48a^{12}(\tau)\mathcal{H}^2\left(3\mathcal{H}^2 + \frac{d\mathcal{H}}{d\tau}\right)\frac{d^2F(T)}{dT^2} + [F(T) - \lambda] + 4a^6(\tau)\left(6\mathcal{H}^2 + \frac{d\mathcal{H}}{d\tau}\right)\frac{dF(T)}{dT} + P_\mathrm{M} = 0$$

Eliminating the term $(F(T) - \lambda)$ from these equations,

$$\longrightarrow \quad 4a^6(\tau)\left(3\mathcal{H}^2 + \frac{d\mathcal{H}}{d\tau}\right)\left[2T\frac{d^2F(T)}{dT^2} + \frac{dF(T)}{dT}\right] + \rho_\mathrm{M} + P_\mathrm{M} = 0$$

Continuity equation for the matter fluid : $$\frac{d\rho_\mathrm{M}}{d\tau} + 3\mathcal{H}\left(\rho_\mathrm{M} + P_\mathrm{M}\right) = 0$$

Reconstruction of unimodular $F(T)$ gravity

- **We present the method of reconstructing the $F(T)$ form that generates a given scale-factor evolution.**

$$\frac{dT}{d\tau} = -12a^6(\tau)\mathcal{H}\left(3\mathcal{H}^2 + d\mathcal{H}/d\tau\right)$$

$$\longrightarrow \quad -\frac{1}{3\mathcal{H}}\left[2T\frac{d}{d\tau}\left(\frac{dF(T)}{dT}\right) + \frac{dF(T)}{d\tau}\right] + \rho_{\mathrm{M}} + P_{\mathrm{M}} = 0$$

- **General power-law form**

$$a(t) = a_* \left(\frac{t}{t_*}\right)^p$$

p : Constant

a_*: Value of a at the time t_*

- **Relation between τ and t :** $\tau = \frac{a_*^3 t_*}{3p+1} \left(\frac{t}{t_*} \right)^{3p+1}$

$$\longrightarrow \quad a(\tau) = \left(\frac{\tau}{\tau_*} \right)^q \qquad q \equiv \frac{p}{3p+1}$$

$$H = p/t \qquad \tau_* \equiv \frac{t_*}{a_*^{1/p} (3p+1)}$$

$$\mathcal{H} = q/\tau$$

$$\Rightarrow \quad T = -6 \left(\frac{q}{\tau_*} \right)^2 \left(\frac{\tau}{\tau_*} \right)^{2(3q-1)}$$

$$d/dT = -\left\{ \tau_*^3 / \left[12q^2 (3q-1) \right] \right\} (\tau/\tau_*)^{-3(2q-1)} d/d\tau$$

- **Constant equation-of-state of matter:** $w = P_{\rm M}/\rho_{\rm M}$

$\rho_{\rm M}, P_{\rm M}$: Energy density and pressure of matter

$\rightarrow \rho_{\rm M} = \rho_{\rm M*} (\tau/\tau_*)^{-3q(1+w)}$ $\rho_{\rm M*}$: Value of $\rho_{\rm M}$ at $\tau = \tau_*$

⇨ **Differential equation in terms of τ**

$$\frac{d^2 F(\tau)}{d\tau^2} + \frac{(2-3q)}{\tau}\frac{dF(\tau)}{d\tau} - \frac{3q(3q-1)(1+w)\rho_{\rm M*}}{\tau_*^{-3q(1+w)}}\tau^{-3q(1+w)-2} = 0$$

General solution: $$F(\tau) = c_1 \tau^{3q-1} - \frac{(3q-1)\rho_{\rm M*}}{[3q(2+w)-1]}\left(\frac{\tau}{\tau_*}\right)^{-3q(1+w)} + c_2$$

c_1, c_2 : Constants

$$\longrightarrow \quad F(T) = c_1 \frac{\tau_*^{3q}}{\sqrt{6}q}\sqrt{-T} + c_2 - \frac{(3q-1)\left(6q^2\right)^{\frac{3q(1+w)}{2(3q-1)}} \tau_*^{-\frac{3q(1+w)}{3q-1}} \rho_{M*}}{[3q(2+w)-1]} (-T)^{-\frac{3q(1+w)}{2(3q-1)}}$$

$$\lambda(\tau) = -2\rho_{M*} \left(\frac{\tau}{\tau_*}\right)^{-3q(1+w)} + c_2$$

$$\lambda(T) = -2\rho_{M*} \left(6q^2\right)^{\frac{3q(1+w)}{2(3q-1)}} \tau_*^{-\frac{3q(1+w)}{3q-1}} (-T)^{-\frac{3q(1+w)}{2(3q-1)}} + c_2$$

- **In the vacuum case, i.e., when $\rho_M = 0$ and $P_M = 0$,**

$$F(T) = c_1\sqrt{-T} + c_2$$

$$\lambda = c_2$$

III. Inflationary Cosmology

- **Hubble slow-roll parameters**

$$\epsilon_{n+1} \equiv \frac{d \ln |\epsilon_n|}{dN}$$

n : Positive integer

$N \equiv \ln(\frac{a}{a_{\rm i}})$: e-folding number during inflation

$a_{\rm i}$: Scale factor a at the beginning of inflation $t_{\rm i}$

$H_{\rm i}$: Value of H at $t = t_{\rm i}$

$$\epsilon_0 \equiv \frac{H_{\rm i}}{H}$$

$*$ Inflation ends when $\epsilon_1 = 1$.

Hubble slow-roll parameters

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}$$

$$\epsilon_2 \equiv \frac{\ddot{H}}{H\dot{H}} - \frac{2\dot{H}}{H^2}$$

$$\epsilon_3 \equiv \left(\ddot{H}H - 2\dot{H}^2\right)^{-1}\left[\frac{H\dot{H}\dddot{H} - \ddot{H}(\dot{H}^2 + H\ddot{H})}{H\dot{H}} - \frac{2\dot{H}}{H^2}(H\ddot{H} - 2\dot{H}^2)\right]$$

Planck 2015 results

[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]

(1) Spectral index of power spectrum of the curvature perturbations

$$n_{\mathrm{s}} = 0.968 \pm 0.006 \quad (68\%\,\mathrm{CL})$$

(2) Tensor-to-scalar ratio

$$r < 0.11\,(95\%\,\mathrm{CL})$$

Keck Array and BICEP2 constraints

$$r_{0.05} < 0.09\ (0.07) \quad (95\%\,\mathrm{CL})$$

(Combined results with the Planck analysis)

[Ade *et al*. [Keck Array and BICEP2 Collaborations], arXiv:1510.09217]

Planck 2015 results (2)

[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]

(3) Running of the spectral index n_{s}

$$\alpha_{\mathrm{S}} \equiv \frac{dn_{\mathrm{s}}}{d\ln k} = -0.003 \pm 0.007 \ (68\%\,\mathrm{CL})$$

k : Wave number

⇨ **R^2 (Starobinsky) inflation is suggested by the observational values of n_{S} and r .**

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

Inflationary observables

- **Spectral index of the curvature perturbations**

$$n_{\rm s} \approx 1 - 2\epsilon_1 - 2\epsilon_2$$

- **Tensor-to-scalar ratio**

$$r \approx 16\epsilon_1$$

- **Running of the spectral index $n_{\rm s}$**

$$\alpha_{\rm s} \approx -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3$$

- **Spectral index of the tensor perturbations (primordial gravitational waves)**

$$n_{\rm T} \approx -2\epsilon_1$$

Inflation

- **de Sitter inflation**

$$a(t) = e^{H_{\rm inf} t}$$

$H_{\rm inf}$: Constant Hubble parameter at the inflationary stage

$$\longrightarrow \quad \tau = \frac{1}{3H_{\rm inf}} e^{3H_{\rm inf} t}, \qquad a(\tau) = (H_{\rm inf}\tau)^{1/3}$$

- **Power-law inflation**

$$a(t) = a_* \left(\frac{t}{t_*}\right)^p, \qquad a(\tau) = \left(\frac{\tau}{\tau_*}\right)^q$$

$$q = p/(3p+1)$$

$$\tau_* = t_*/[a_*^{1/p}(3p+1)]$$

- **In the limit** $p \to \infty$ **and** $t_* \to \infty$ **with** $p/t_* = H_* = const.$**, the power-law expansion gives de Sitter expansion with** $H_{\rm inf} = 3H_*$**.**

⇨ **FLRW metric under the unimodular condition**

$$ds^2 = \left(\frac{\tau}{\tau_*}\right)^{-6q} d\tau^2 - \left(\frac{\tau}{\tau_*}\right)^{2q} \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right]$$

- $q = 1/3 \longrightarrow$ **de Sitter inflation**
- $1/4 < q < 1/3$ **(i.e.,** $p > 1$ **)**
  $\longrightarrow$ **Power-law inflation**
- $p < 0$ **(i.e.,** $q < 0$ **or** $q > 1/3$ **)**
  $\longrightarrow \dot{H} = -p/t^2 > 0$ **: Super-inflation**

Power-law inflation

- **Hubble slow-roll parameters**

$$\epsilon_1(N) \equiv -\frac{H'(N)}{H(N)}\,, \qquad \epsilon_2(N) \equiv \frac{H''(N)}{H'(N)} - \frac{H'(N)}{H(N)}$$

$$\epsilon_3(N) \equiv \left[\frac{H(N)H'(N)}{H''(N)H(N) - H'(N)^2}\right]\left[\frac{H'''(N)}{H'(N)} - \frac{H''(N)^2}{H'(N)^2} - \frac{H''(N)}{H(N)} + \frac{H'(N)^2}{H(N)^2}\right]$$

$*$ The Prime denotes derivatives with respect to *N*.

$$\longrightarrow \quad \epsilon_1 = \frac{1}{p}, \qquad \epsilon_2 = 0, \qquad \epsilon_3 = \frac{1}{p},$$

$$\Rightarrow \quad n_{\rm s} \approx 1 - \frac{2}{p}, \quad r \approx \frac{16}{p}, \quad \alpha_{\rm s} \approx 0, \quad n_{\rm T} \approx -\frac{2}{p}$$

$$\longrightarrow \quad r = 8(1 - n_{\rm s})$$

$$p = q/(1-3q)$$

Observables for power-law inflation

- **If** $p = 100$**, i.e.,** $q = 0.332$**,**

$$\Rightarrow \quad n_{\rm s} \approx 0.98\,, \quad r \approx 0.16$$

$$\alpha_{\rm s} = 0\,, \quad n_{\rm T} \approx -0.02$$

R^2 (Starobinsky) inflation

[Starobinsky, Phys. Lett. B <u>91</u>, 99 (1980)]

Action

$$S = \int d^4x \sqrt{-g} \frac{1}{2\kappa^2} \left(R \underline{+ \frac{1}{6M^2} R^2} \right)$$

g : Determinant of the metric $g_{\mu\nu}$

R : Scalar curvature

M : Constant with mass dimension

- **Hubble parameter**

$$H = H_* - \frac{M^2}{6}(t - t_*)$$

H_* : Value of H at $t = t_*$

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

[Barrow and Cotsakis, Phys. Lett. B 214, 515 (1988)]

[Odintsov and Oikonomou, Phys. Rev. D 92, 024016 (2015); Phys. Rev. D 92, 124024 (2015)]

- **Scale factor**

$$a(t) = a_* \exp\left[H_* (t - t_*) - \left(M^2/12\right)(t - t_*)^2\right]$$

→ **For** $t \ll t_*$ **,**

$$a(t) = a_* \exp\left\{\left[H_* + \left(M^2/6\right) t_*\right] t - \left[H_* + \left(M^2/12\right) t_*\right] t_*\right\}$$

$$\longrightarrow \quad \tau = \bar{\tau} + \frac{a_*^3}{3H_* + (M^2/2)\,t_*} \exp\left[\left(3H_* + \frac{M^2}{2}t_*\right)t - \left(3H_* + \frac{M^2}{4}t_*\right)t_*\right]$$

$$a(\tau) = \left(3H_* + \frac{M^2}{2}t_*\right)^{1/3} (\tau - \bar{\tau})^{1/3}$$

$\bar{\tau}$: Constant

$$\mathcal{H} = 1/\left[3\left(\tau - \bar{\tau}\right)\right]$$

$$T = -6\left[H_* + \left(M^2/6\right)t_*\right]^2$$

- **e-folding number at the inflationary stage**

$$N \equiv \ln\left(\frac{a_{\mathrm{f}}}{a_{\mathrm{i}}}\right) = -\int_{t_{\mathrm{f}}}^{t_{\mathrm{i}}} H(\tilde{t})d\tilde{t}$$

a_{f} : Value of a at the end of inflation t_{f}

$$\longrightarrow \quad N = -\left(H_* + \frac{M^2}{6} t_*\right)(t_\mathrm{i} - t_\mathrm{f}) + \frac{M^2}{12}\left(t_\mathrm{i}^2 - t_\mathrm{f}^2\right)$$

$$\Rightarrow \quad t_\mathrm{f} = 6\frac{H_*}{M^2} + t_* + M^{-2}\Big\{ \left(6H_* + M^2 t_*\right)^2$$

$$+M^2 \left\{t_\mathrm{i}\left[M^2\left(t_\mathrm{i} - 2t_*\right) - 12H_*\right] - 12N\right\} \Big\}^{\frac{1}{2}}$$

- **Hubble slow-roll parameters**

$$\epsilon_1 = \frac{6M^2}{\left[6H_* - M^2\left(t_\mathrm{f} - t_*\right)\right]^2}$$

$$\longrightarrow \epsilon_1(N) = \frac{6M^2}{\left(6H_* + M^2 t_*\right)^2 + M^2\left\{t_\mathrm{i}\left[M^2\left(t_\mathrm{i} - 2t_*\right) - 12H_*\right] - 12N\right\}}$$

$$\epsilon_2(N) = 2\epsilon_1(N), \qquad \epsilon_3(N) = 2\epsilon_1(N)$$

Observables for R^2 (Starobinsky) inflation

$$n_{\mathrm{s}}(N) = 1 - 6\,\epsilon_1(N), \quad r(N) = 16\,\epsilon_1(N)$$

$$\alpha_{\mathrm{s}}(N) = -8\,\epsilon_1^2(N), \quad n_{\mathrm{T}}(N) = -2\,\epsilon_1(N)$$

$$t_{\mathrm{i}} = 1/H_*, \; t_* = 3/H_*, \; H_*/M = 0.04$$

- **For** $N = 50$ **,**

$$n_{\mathrm{s}} \approx 0.981, \quad r \approx 0.049$$

$$\alpha_{\mathrm{s}} = -7.78 \times 10^{-5}, \quad n_{\mathrm{T}} \approx -0.0062$$

- **For** $N = 60$ **,**

$$n_s \approx 0.98, \quad r \approx 0.053$$

$$\alpha_s = -8.85 \times 10^{-5}, \quad n_T \approx -0.0067$$

⇨ **These results are consistent with the Planck analysis.**

$$r = \frac{8}{3}(1-n_s), \quad \alpha_s = -\frac{2}{9}(1-n_s)^2$$

A specific model

$$F(T) = \alpha_1 \sqrt{T} + \alpha_2 T^n$$

$$\lambda(T) = \alpha_3 T^n$$

$\alpha_1,\ \alpha_2,\ \alpha_3,$ and $n(\neq 0,\ 1/2)$: Constants

* We set $\alpha_1 = 1.$

(i) $n = 2,\ \alpha_2 = 1,\ \alpha_3 = -3.0006$ **: Black solid curve**

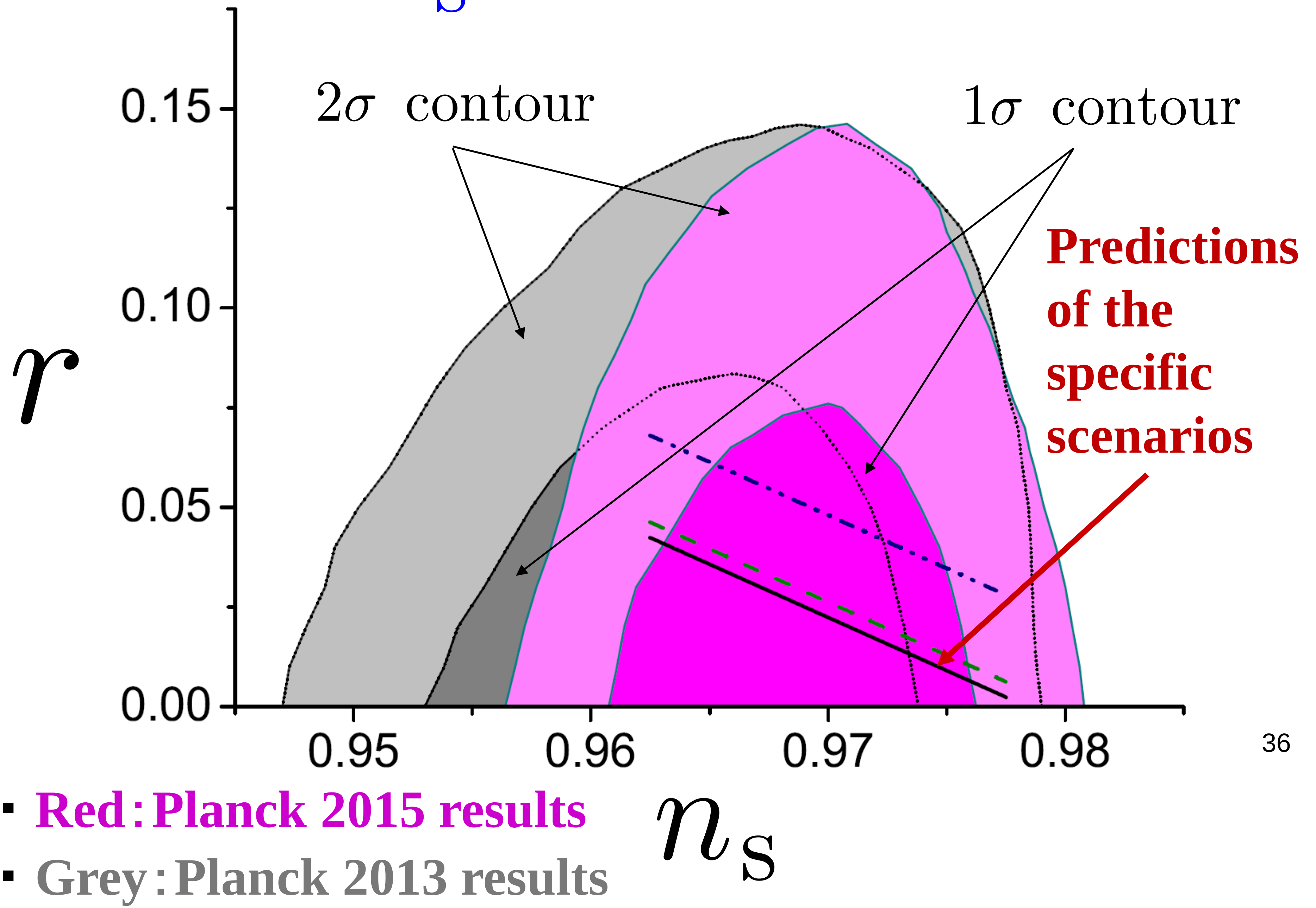
(ii) $n = 4,\ \alpha_2 = 1,\ \alpha_3 = -7.00012$

: Green dashed curve

(iii) $n = 2,\ \alpha_2 = 0.333,\ \alpha_3 = -1$

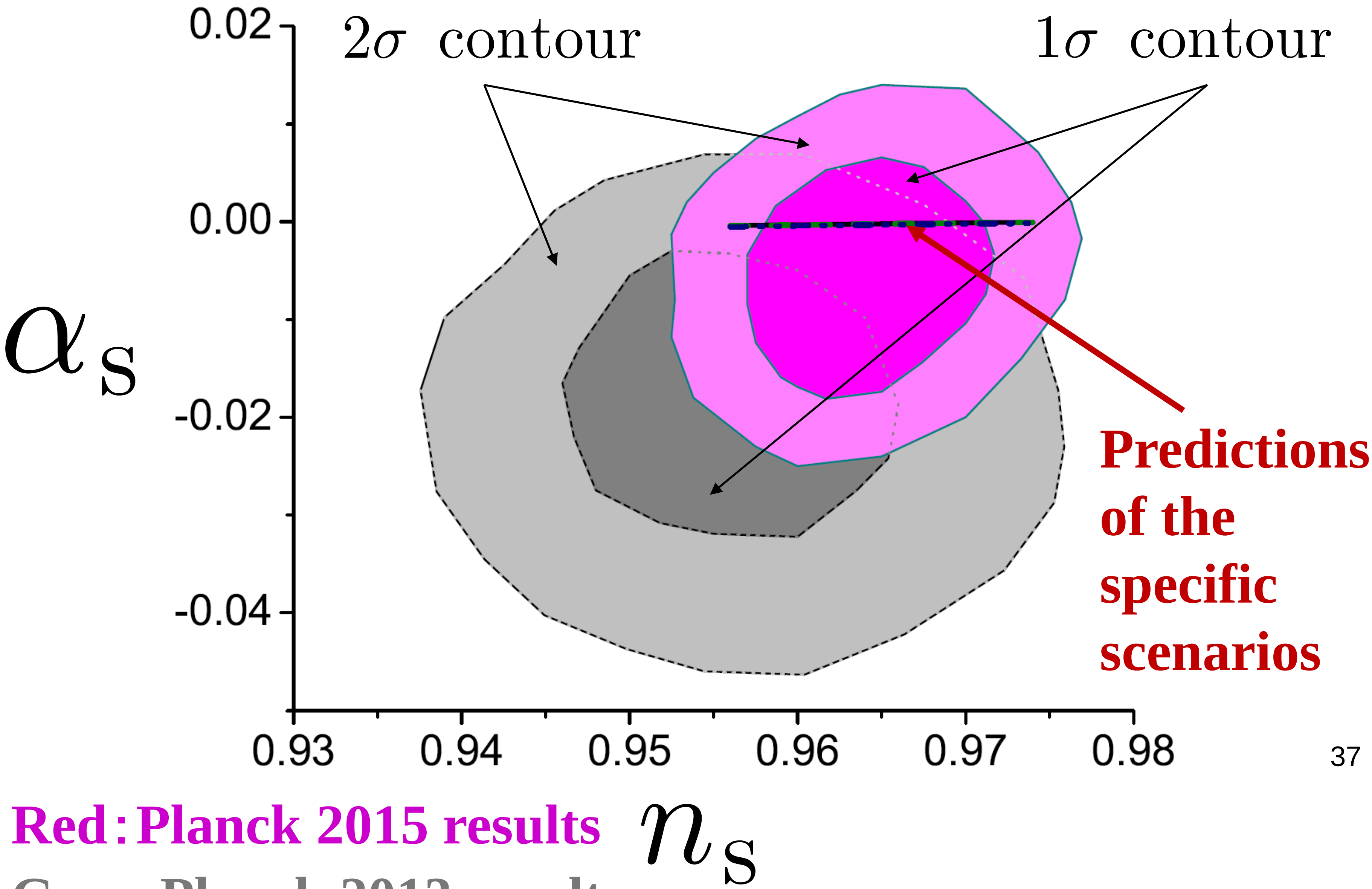
: Blue dashed-dotted curve

$n_{\rm S} - r$ Contours



- **Red : Planck 2015 results**
- **Grey : Planck 2013 results**

$\alpha_S - r$ Contours



- **Red : Planck 2015 results**
- **Grey : Planck 2013 results**

VI. Conclusions

- **We have considered inflationary cosmology in the context of unimodular *F*(*T*) gravity.**
- **We have analyzed the power-law inflation and R^2 (Starobinsky) inflation under the unimodular condition.**
- **We have shown that for R^2 (Starobinsky) inflation, the spectral index n_{s}, the tensor-to-scalar ratio r, and the running α_{s} of the spectral index n_{s} can be consistent with the recent Planck results.**