Phantom crossing dark energy in Horndeski's theory

Jiro Matsumoto
Leung Center for Cosmology and Particle Astrophysics (LeCosPA)
National Taiwan University

Phys. Rev. D **97**, 123538 (2018) [arXiv:1712.10015]

Dark energy and dark matter

Observations of Type Ia supernovae showed that the current universe is acceleratedly expanding.

Einstein equation cannot explain the accelerated expansion of the Universe if we only take into account nonrelativistic matter(s) and radiation(s).
$\Rightarrow$ Introducing dark energy, e.g. cosmological constant Λ.

Observations of cosmic microwave background and baryon acoustic oscillation imply the existence of extra nonrelativistic matter(s) which weakly interact with baryons and photons.

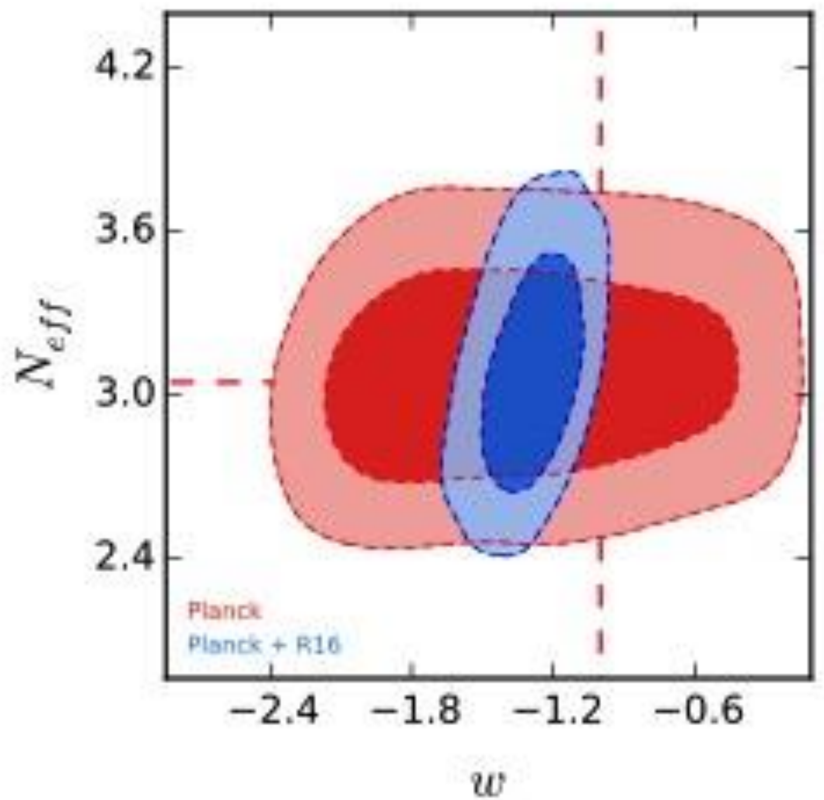
Standard model of cosmology:
Λ cold dark matter (ΛCDM) model

Why phantom DE is necessary?

The ΛCDM model is almost consistent with the observations. However, there is a discrepancy in the value of H_0 between local ($z<0.15$) observations and the PLANCK CMB observation.

The current universe may expand faster than that predicted by the ΛCDM model.

Growing dark energy,
i.e. phantom dark energy,
which satisfies $w<-1$,
is more favored than the standard model.



E. Di Valentino, A. Melchiorri and J. Silk, Phys. Lett. B 761, 242 (2016)

A simple realization of phantom DE

Quintessence model

$$S = \int d^4x \left[\frac{R}{2\kappa^2} - \epsilon \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + V(\phi) \right], \epsilon = \pm 1.$$

Flat FLRW metric $ds^2 = -dt^2 + a^2(t) \sum_{i=1}^{3} (dx^i)^2.$

$\Rightarrow$ $$\epsilon \ddot{\phi} + 3\epsilon H \dot{\phi} + V'(\phi) = 0. \qquad H = \frac{\dot{a}}{a}.$$

$\Leftrightarrow$ $$\dot{\rho}_\phi + 3H(1+w)\rho_\phi = 0, \quad \rho_\phi \equiv \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi). \quad w = \frac{\epsilon\dot{\phi}^2 - 2V}{\epsilon\dot{\phi}^2 + 2V}$$

w < -1 $\Rightarrow$ $\epsilon \dot{\phi}^2 < 0.$

Phantom DE is realized only if kinetic energy is unbounded from below. There is a ghost instability.

Horndeski's theory

In Horndeski's theory, the action is constructed to yield the equations of motion with less than second derivatives in order to avoid ghost instability.

$$S_H = \sum_{i=2}^{5} \int d^4x \sqrt{-g} \mathcal{L}_i ,$$

$$X = -\partial_\mu \phi \partial^\mu \phi / 2$$

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X) \Box \phi,$$

$$\mathcal{L}_4 = \underline{G_4(\phi, X) R} + G_{4X} \left[(\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right],$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5X}}{6} \left[(\Box \phi)^3 - 3(\Box \phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \right].$$

Pure GR: K, G_3, G_5 = 0, G_4 = const.
GR + Cosmological Constant: G_3, G_5 = 0, K = const., G_4 = const.

Equations of motion

Friedmann equations

$$\rho_{\mathrm{matter}} + \sum_{i=2}^{5} \mathcal{E}_i = 0,$$

$$\mathcal{E}_2 = 2XK_X - K,$$
$$\mathcal{E}_3 = 6X\dot{\phi}HG_{3X} - 2XG_{3\phi},$$
$$\mathcal{E}_4 = \underline{-6H^2G_4} + 24H^2X(G_{4X} + XG_{4XX}) - 12HX\dot{\phi}G_{4\phi X} - 6H\dot{\phi}G_{4\phi},$$
$$\mathcal{E}_5 = 2H^3X\dot{\phi}(5G_{5X} + 2XG_{5XX}) - 6H^2X(3G_{5\phi} + 2XG_{5\phi X}),$$

$$p_{\mathrm{matter}} + \sum_{i=2}^{5} \mathcal{P}_i = 0,$$

$$\mathcal{P}_2 = K,$$
$$\mathcal{P}_3 = -2X\left(G_{3\phi} + \ddot{\phi}G_{3X}\right),$$
$$\mathcal{P}_4 = \underline{2(3H^2 + 2\dot{H})G_4} - 4H^2X\left(3 + \frac{\dot{X}}{HX} + 2\frac{\dot{H}}{H^2}\right)G_{4X}$$
$$- 8HX\dot{X}G_{4XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4\phi} + 4XG_{4\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4\phi X},$$
$$\mathcal{P}_5 = -2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5X} - 4H^2X^2\ddot{\phi}G_{5XX}$$
$$+ 4HX(\dot{X} - HX)G_{5\phi X} + 2H^2X\left(3 + 2\frac{\dot{X}}{HX} + 2\frac{\dot{H}}{H^2}\right)G_{5\phi} + 4HX\dot{\phi}G_{5\phi\phi},$$

Equation of motion of the scalar field

$$\frac{1}{a^3}\frac{d}{dt}(a^3J) = P_\phi,$$

$$J = \dot{\phi}K_X + 6HXG_{3X} - 2\dot{\phi}G_{3\phi} + 6H^2\dot{\phi}(G_{4X} + 2XG_{4XX}) - 12HXG_{4\phi X}$$
$$+ 2H^3X(3G_{5X} + 2XG_{5XX}) - 6H^2\dot{\phi}(G_{5\phi} + XG_{5\phi X}),$$
$$P_\phi = K_\phi - 2X(G_{3\phi\phi} + \ddot{\phi}G_{3\phi X}) + 6(2H^2 + \dot{H})G_{4\phi} + 6H(\dot{X} + 2HX)G_{4\phi X}$$
$$- 6H^2XG_{5\phi\phi} + 2H^3X\dot{\phi}G_{5\phi X}.$$

Equation of state parameter

$$\rho_\phi \equiv \sum_{i=2}^{5} \mathcal{E}_i + \frac{3H^2}{\kappa^2}, \qquad p_\phi \equiv \sum_{i=2}^{5} \mathcal{P}_i - \frac{1}{\kappa^2}(3H^2 + 2\dot{H}).$$

Friedmann equations

$$3H^2 = \kappa^2(\rho_{\mathrm{matter}} + \rho_\phi),$$
$$-3H^2 - 2\dot{H} = \kappa^2(p_{\mathrm{matter}} + p_\phi),$$

Effective equation of state parameter $w_\phi = p_\phi/\rho_\phi$.

Constraint from GW170817

Observation of the gravitational wave event GW170817 and its electromagnetic counter part GRB 170817A constrain the speed of gravitational wave as

$$|c_T^2 - 1| \lesssim 10^{-15}.$$

If the terms proportional to G_{4X}, $G_{5\phi}$, G_{5X} are relevant for dark energy, then the sound speed for tensor perturbation

$$c_T^2 = \frac{G_4 - XG_{5\phi} - XG_{5X}\ddot{\phi}}{G_4 - 2XG_{4X} - X(G_{5X}\dot{\phi}H - G_{5\phi})}.$$

T. Kobayashi, M. Yamaguchi and J. Yokoyama, Prog. Theor. Phys. 126, 511 (2011).

should be deviated from 1 (because $H^2G_4 \sim H^2/(16\pi G) \sim \rho_{DE}$).

Therefore, it is natural to assume G_{4X}, $G_{5\phi}$, $G_{5X} = 0$.

Horndeski DE after GW170817

In Horndeski's theory, the action is constructed to yield the equations of motion with less than second derivatives in order to avoid ghost instability.

$$S_H = \sum_{i=2}^{5} \int d^4x \sqrt{-g}\mathcal{L}_i,$$

$$X = -\partial_\mu \phi \partial^\mu \phi / 2$$

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + \cancel{G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]},$$

$$\mathcal{L}_5 = \cancel{G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]}.$$

Pure GR: K, G_3, G_5 = 0, G_4 = const.

GR + Cosmological Constant: G_3, G_5 = 0, K = const., G_4 = const.

Conditions for realizing stable phantom dark energy

Stability conditions

$$G_4 > 0.$$

$$G_4(K_X - 2G_{3\phi} + 2\ddot{\phi}G_{3X} + \dot{\phi}^2 G_{3\phi X} + \dot{\phi}^2\ddot{\phi}G_{3XX} + 4H\dot{\phi}G_{3X}) \\ +3G_{4\phi}^2 - \dot{\phi}^2 G_{3X}G_{4\phi} - \frac{1}{4}\dot{\phi}^4 G_{3X}^2 \geq 0,$$

$$G_4\left[K_X + \dot{\phi}^2 K_{XX} - 2G_{3\phi} - \dot{\phi}^2 G_{3\phi X} + 3H\dot{\phi}(2G_{3X} + \dot{\phi}^2 G_{3XX})\right] \\ +3\left(G_{4\phi} - \frac{1}{2}G_{3X}\dot{\phi}^2\right)^2 > 0,$$

$\rho_\phi > 0$:

$$\dot{\phi}^2 K_X - K - \dot{\phi}^2 G_{3\phi} + 3H\dot{\phi}^3 G_{3X} + 3H^2\left(\frac{1}{\kappa^2} - 2G_4\right) - 6H\dot{\phi}G_{4\phi} > 0,$$

$w_\phi < -1$ and $\rho_\phi > 0$ ($w_\phi = p_\phi/\rho_\phi$):

$$\dot{\phi}^2 K_X - \dot{\phi}^2(2G_{3\phi} + \ddot{\phi}G_{3X} - 3H\dot{\phi}G_{3X}) - 2\dot{H}\left(\frac{1}{\kappa^2} - 2G_4\right) + 2(\ddot{\phi} - H\dot{\phi})G_{4\phi} + 2\dot{\phi}^2 G_{4\phi\phi} < 0.$$

The case $G_3(\phi,X) = G_3(\phi)$ and $G_4(\phi) = 1/(16\pi G)$

$c_s^2 \geqq 0$ and $A > 0$: $$K_X - 2G_{3\phi} \geq 0,$$

$A > 0$: $$K_X + \dot{\phi}^2 K_{XX} - 2G_{3\phi} > 0,$$

$\rho_\phi > 0$: $$\dot{\phi}^2 K_X - K - \dot{\phi}^2 G_{3\phi} > 0,$$

$w_\phi < -1$ and $\rho_\phi > 0$: $$\dot{\phi}^2 K_X - 2\dot{\phi}^2 G_{3\phi} < 0.$$

There is a clear contradiction between the first inequality and the fourth inequality. Therefore, phantom dark energy cannot be realized without instability in this case, and is realized without instability only if there is ϕ dependence in G_4 or X dependence in G_3.

The case G_3 has a X dependence and $G_4(\phi) = 1/(16\pi G)$ I

$c_s^2 \geqq 0$ and $A > 0$: $\underline{K_X - 2G_{3\phi}} + 2\ddot{\phi}G_{3X} + \dot{\phi}^2 G_{3\phi X} + \dot{\phi}^2\ddot{\phi}G_{3XX} + \underline{4H\dot{\phi}G_{3X}} - \frac{\kappa^2}{2}\dot{\phi}^4 G_{3X}^2 \geq 0,$

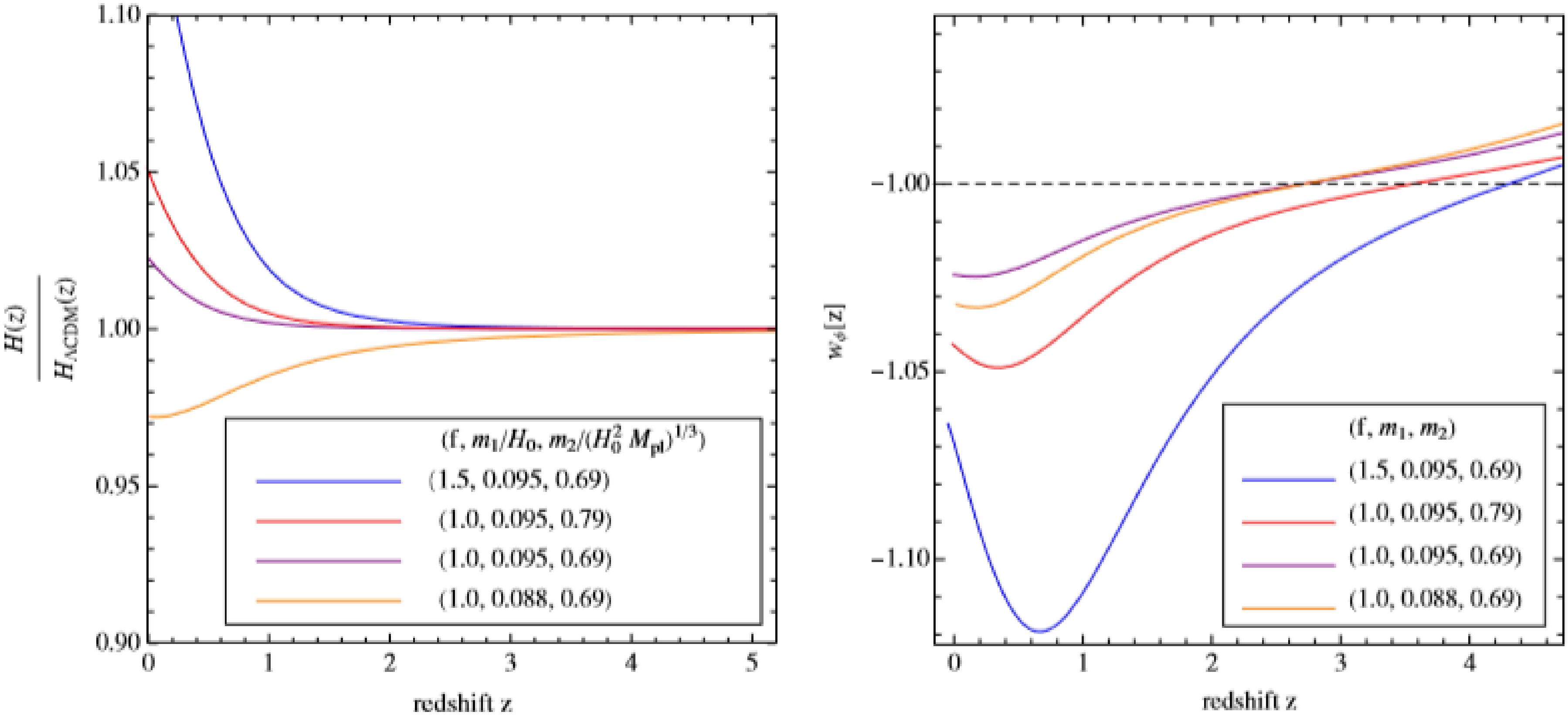
$A > 0$: $\underline{K_X + \dot{\phi}^2 K_{XX} - 2G_{3\phi}} - \dot{\phi}^2 G_{3\phi X} + \underline{3H\dot{\phi}(2G_{3X}} + \dot{\phi}^2 G_{3XX}) + \frac{3\kappa^2}{2}\dot{\phi}^4 G_{3X}^2 > 0,$

$\rho_\phi > 0$: $\dot{\phi}^2 K_X - K - \dot{\phi}^2 G_{3\phi} + 3H\dot{\phi}^3 G_{3X} > 0,$

$w_\phi < -1$ and $\rho_\phi > 0$: $\underline{\dot{\phi}^2 K_X - \dot{\phi}^2(2G_{3\phi}} + \ddot{\phi}G_{3X} \underline{- 3H\dot{\phi}G_{3X}}) < 0.$

In the case of potential driven slow-roll accelerated expansion, i.e. $K(\phi,X) = X-V(\phi)$, $X << V(\phi) \sim 3H^2/(8\pi G)$ and $|d^2\phi/dt^2| << H|d\phi/dt|$, $\rho_\phi > 0$ is satisfied. Then, the other inequalities are completed by fine-tuning the value of G_{3X}.

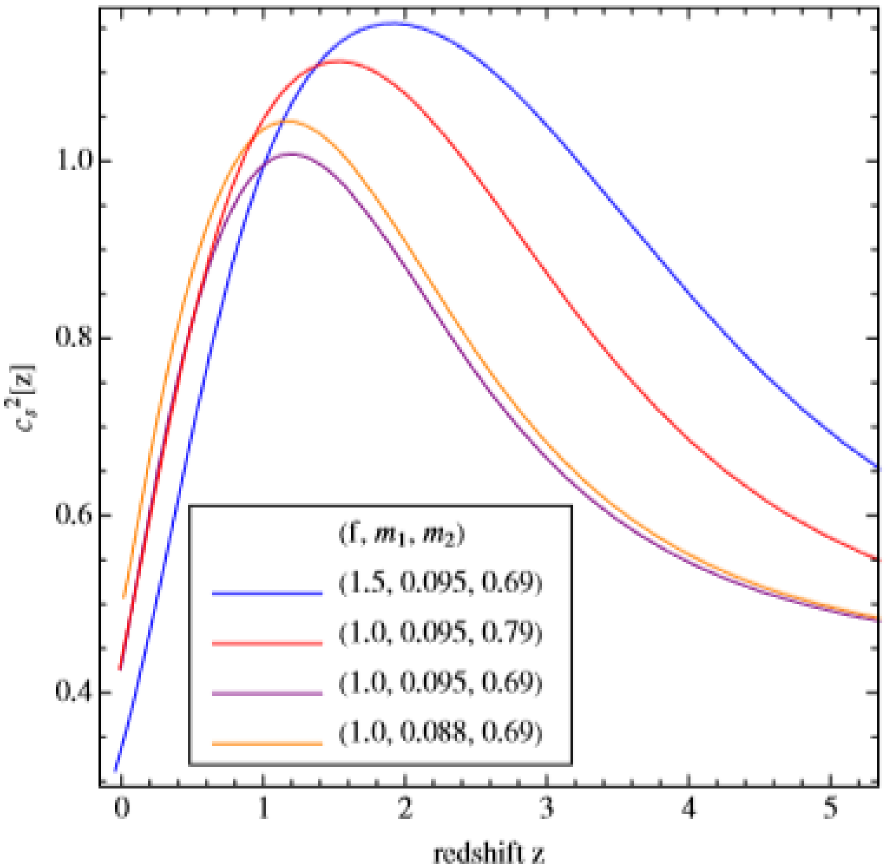
The case G_3 has a X dependence and $G_4(\phi) = 1/(16\pi G)$ II

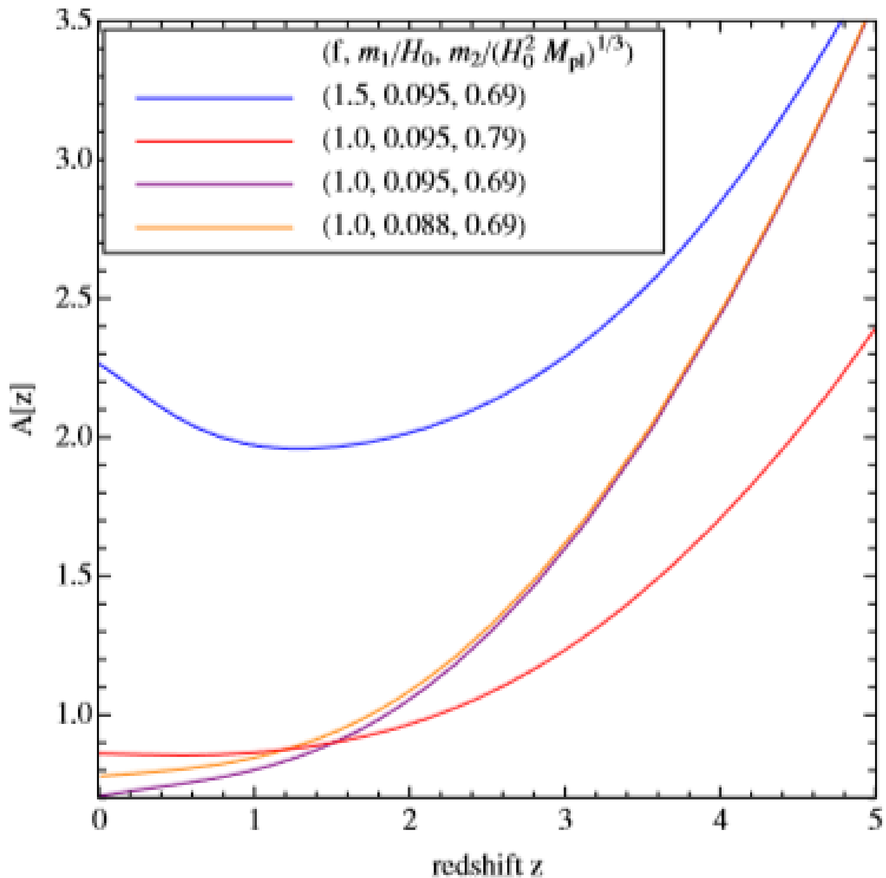


$K(\phi,X) = X - m_1^2\phi^2$, $G_3(\phi,X) = f\phi + X/m_2^3$. $X = -\partial_\mu\phi\partial^\mu\phi/2$

Initial conditions: $\phi(z=10) = 3M_{pl}$ and $d\phi(z=10)/dt = 0.04 M_{pl} H_0$.

The case G_3 has a X dependence and $G_4(\phi) = 1/(16\pi G)$ III





$K(\phi,X) = X - m_1^2\phi^2$, $G_3(\phi,X) = f\phi + X/m_2^3$.

Initial conditions: $\phi(z=10) = 3M_{pl}$ and $d\phi(z=10)/dt = 0.04 M_{pl} H_0$.

The case $G_3 = 0$ and G_4 has a ϕ dependence I

$$G_4 > 0,$$

$c_s^2 \geqq 0$ and A > 0:

$$G_4(K_X G_4 + 3G_{4\phi}^2) \geq 0,$$

A > 0:

$$G_4[G_4(K_X + \dot{\phi}^2 K_{XX}) + 3G_{4\phi}^2] > 0,$$

$\rho_\phi > 0$:

$$3\left(\frac{1}{\kappa^2} - 2G_4\right)H^2 + \dot{\phi}^2 K_X - K - 6H\dot{\phi}G_{4\phi} > 0,$$

$w_\phi < -1$ and $\rho_\phi > 0$:

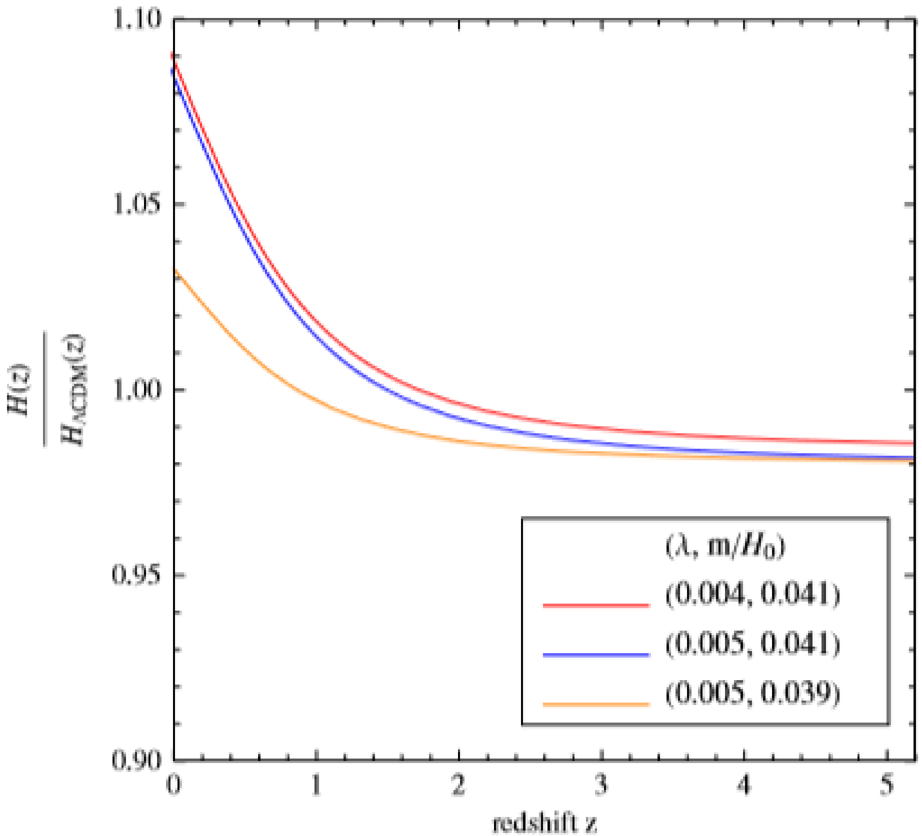
$$-2\left(\frac{1}{\kappa^2} - 2G_4\right)\dot{H} + \dot{\phi}^2 K_X + 2(\ddot{\phi} - H\dot{\phi})G_{4\phi} + 2\dot{\phi}^2 G_{4\phi\phi} < 0.$$

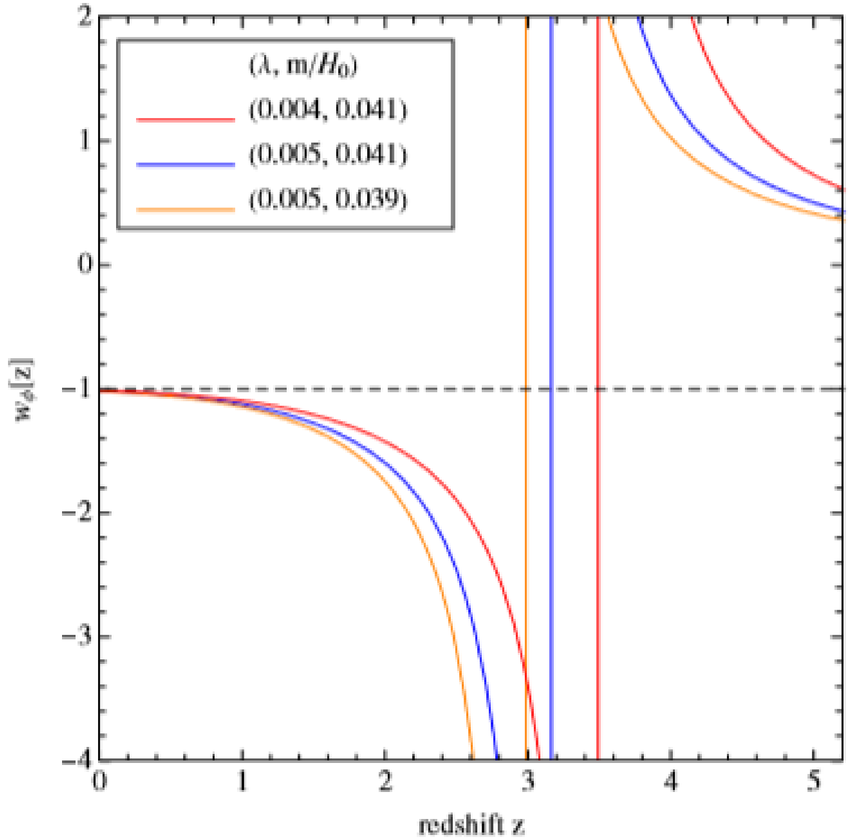
If $G_4 > 0$, $K_X > 0$, and $K_{XX} > 0$, then the conditions $c_s^2 \geqq 0$, A > 0 are satisfied. In particular, $K(\phi,X) = X-V(\phi)$ gives $c_s^2 = 1$ and A >0. To find a consistent model which satisfies $\rho_\phi > 0$, $w_\phi < -1$, we can adjust two arbitrary functions $G_4(\phi) > 0$ and $V(\phi)$.

The case $G_3 = 0$ and G_4 has a ϕ dependence II

$$\rho_\phi \equiv \sum_{i=2}^{5} \mathcal{E}_i + \frac{3H^2}{\kappa^2},$$

$w_\phi = p_\phi/\rho_\phi$

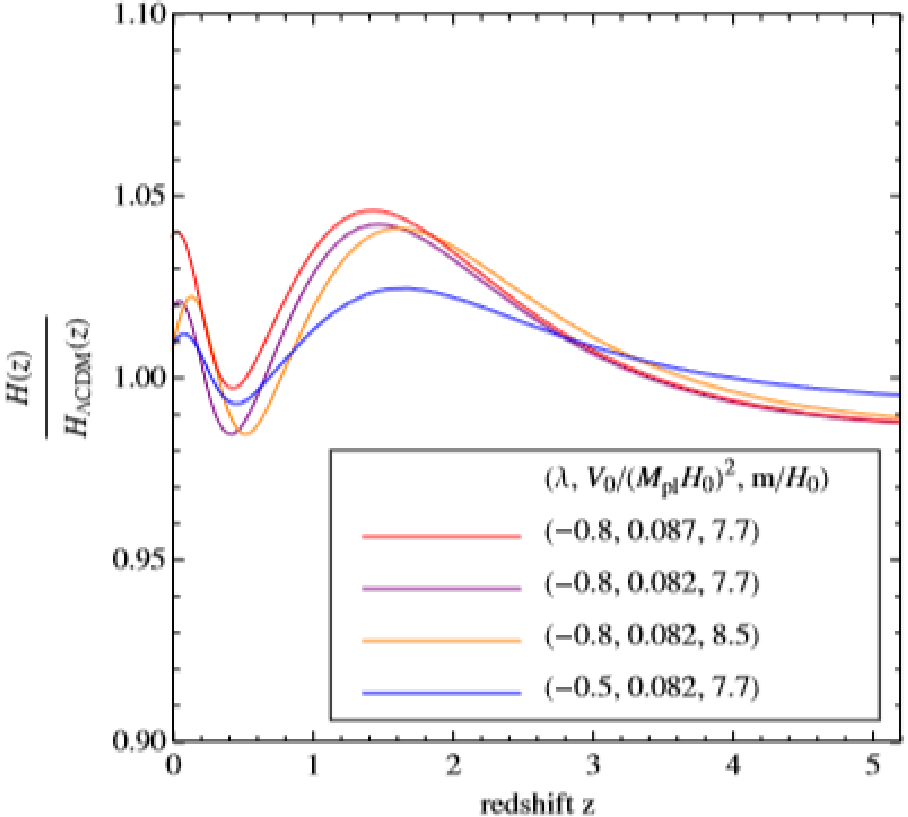


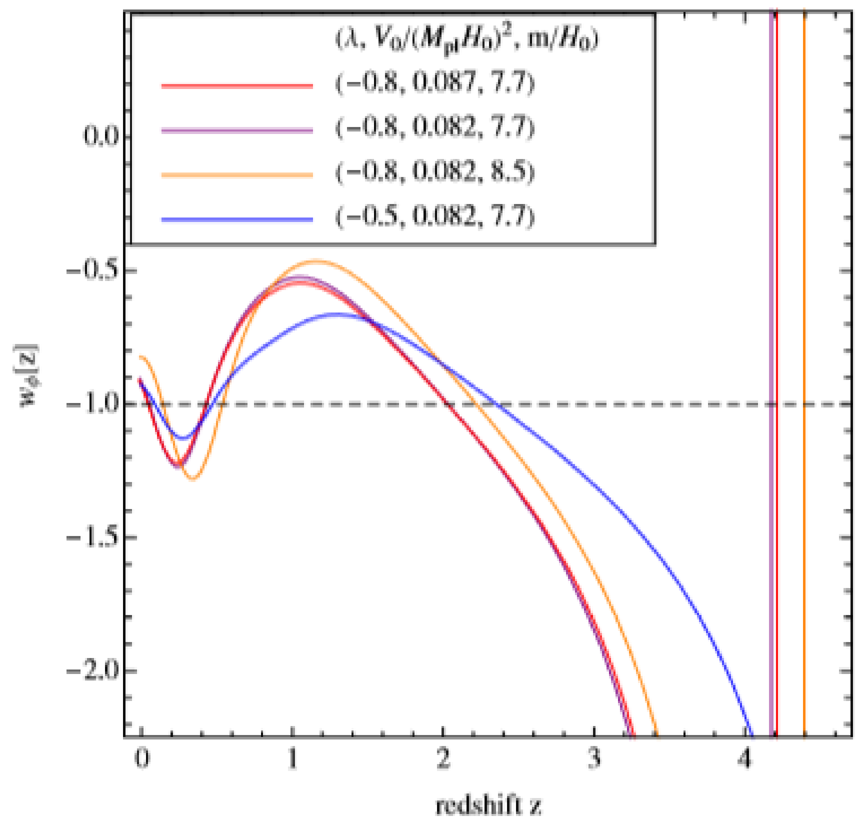


$K(\phi,X) = X - m^2\phi^2$, $G_4(\phi) = \mathrm{Exp}[\lambda\phi/M_{pl}]/16\pi G$. $X = -\partial_\mu\phi\partial^\mu\phi/2$

Initial conditions: $\phi(z=10) = 8M_{pl}$ and $d\phi(z=10)/dt = 0.04 M_{pl} H_0$.

The case $G_3 = 0$ and G_4 has a ϕ dependence III

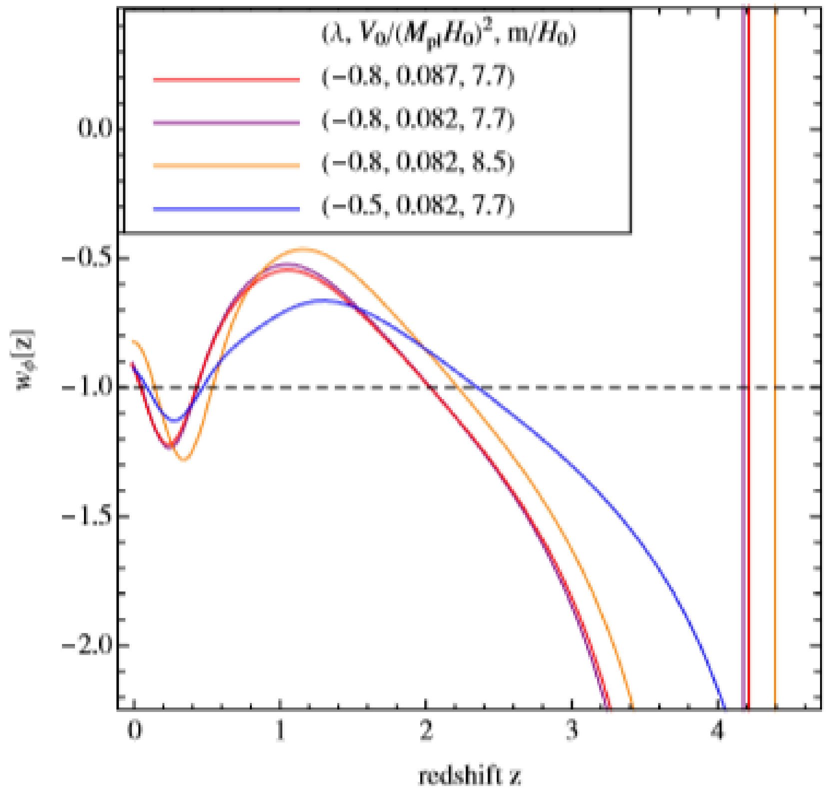




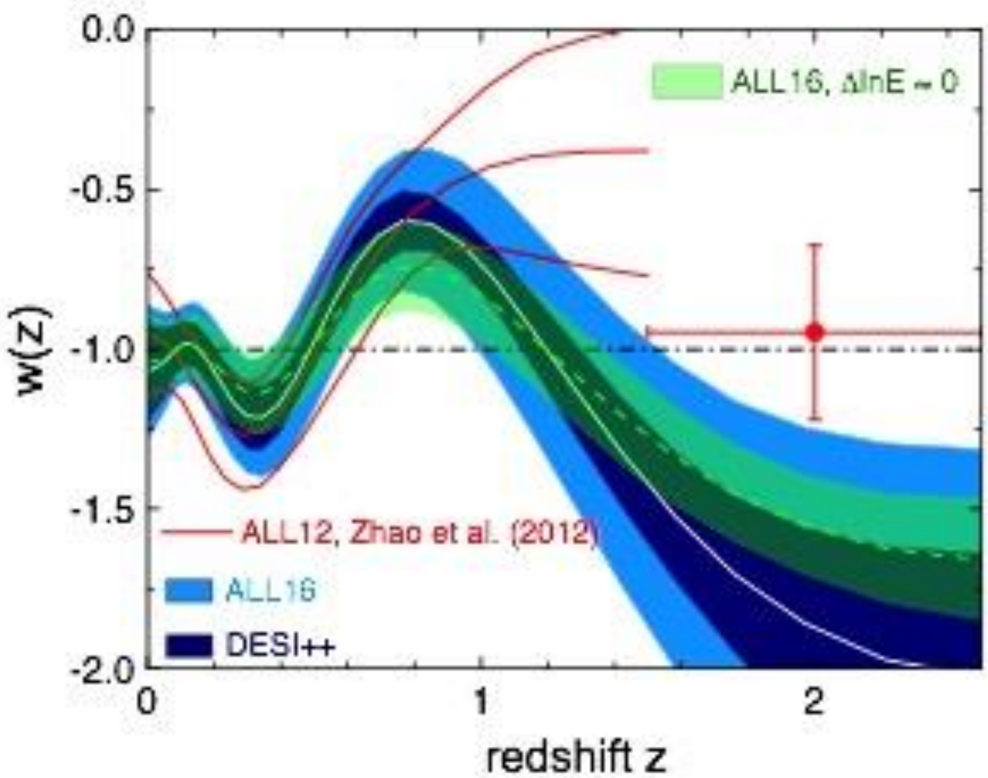
$K(\phi,X) = X - (V_0 + m^2\phi^2)$, $G_4(\phi) = \mathrm{Exp}[\lambda\phi/M_{pl}]/16\pi G$.

Initial conditions: $\phi(z=10) = -0.03M_{pl}$ and $d\phi(z=10)/dt = 0.04M_{pl}H_0$.

The reconstructed EoS parameter



G. B. Zhao et al., Nat. Astron. 1, 627 (2017).



$K(\phi,X) = X - (V_0 + m^2\phi^2)$, $G_4(\phi) = \mathrm{Exp}[\lambda\phi/M_{pl}]/16\pi G$.

Initial conditions: $\phi(z=10) = -0.03M_{pl}$ and $d\phi(z=10)/dt = 0.04M_{pl}H_0$.

Summary

We have investigated the conditions for realizing phantom dark energy without instability in Horndeski's theory. By constrain the theory with the help of the recent observation of gravitational wave and its counter part, we have shown the following results.

- Phantom dark energy is realized without instability only if there is a X dependence in G_3 term or a ϕ dependence in G_4 term.

- In both cases, slow-roll accelerated expansion with a canonical kinetic term and ϕ^2 potential can cause a phantom crossing without instability.

- The case $K(\phi,X) = X- (V_0 + m^2\phi^2)$, $G_4(\phi) = \mathrm{Exp}[\lambda\phi/M_{pl}]/16\pi G$, the evolution of w is similar to that obtained form observations.