Quasi-local Energy and universal horizon thermodynamics

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Gravitational Energy

- Tatal → (quasi-)local level
- Black hole thermodynamics (internal energy, entropy, angular momentum)
- Penrose inequality
- Numerical

Gravitational Energy

- known that these quantities cannot be given by a local density.
- Modern understanding:
  - Quasi-local (associated with a closed 2-surface),
  - they have no unique formula
  - they have no reference frame independent description
- GR pseudo-tensors: Einstein 1915, Hilbert 1916, Lorentz 1916, Klein 1918 Papapetrou '48, Bergmann-Thompson '53, Møller '58, Landau-Lifshitz '62, Weinberg '72(MTW)
- Quasi-local ideas: Goldberg '58, Møller '61, Witten spinor '83, Brown & York '93, Bicak & Katz & Lynden-Bell '95, Chen & Nester & Tung '95, Epp '00, Petrov-Katz '02, Kijiowski '97, Liu-Yau '03, Wang-Yau '09
- Wald formalism (generalize BY to any diffeomorphism covariant theory)

[L.B. Szabados, Living Rev. Relativ. 12 (2009) 4]

Brown-York mass for the first Law of BH thermodynamics (GR)

PHYSICAL REVIEW D VOLUME 47, NUMBER 4 15 FEBRUARY 1993

Microcanonical functional integral for the gravitational field

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The gravitational field in a spatially finite region is described as a microcanonical system. The density of states ν is expressed formally as a functional integral over Lorentzian metrics and is a functional of the geometrical boundary data that are fixed in the corresponding action. These boundary data are the thermodynamical extensive variables, including the energy and angular momentum of the system. When the boundary data are chosen such that the system is described semiclassically by *any* real stationary axisymmetric black hole, then in this same approximation $\ln \nu$ is shown to equal $\frac{1}{4}$ the area of the black-hole event horizon. The canonical and grand canonical partition functions are obtained by integral transforms of ν that lead to "imaginary-time" functional integrals. A general form of the first law of thermodynamics for stationary black holes is derived. For the simpler case of nonrelativistic mechanics, the density of states is expressed as a real-time functional integral and then used to deduce Feynman's imaginary-time functional integral for the canonical partition function.

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$$\delta \mathcal{S}[\varepsilon, j, \sigma] \approx \delta(A_H/4)$$
$$= \int_B d^2x \left[\beta\delta(\sqrt{\sigma}\,\varepsilon) - \beta\omega\delta(\sqrt{\sigma}\, j_a \phi^a) + \beta(\sqrt{\sigma}\, p^{ab}/2)\delta\sigma_{ab}\right] .$$

$$d\mathcal{S} = \text{“}\beta\, dE - \beta\omega\, dJ + \beta p\, dV\text{,”}$$

Wald Formalim for the first Law of BH thermodynamics (diffeomorphism covariant)

PHYSICAL REVIEW D VOLUME 50, NUMBER 2 15 JULY 1994

Some properties of the Noether charge and a proposal for dynamical black hole entropy

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We consider a general, classical theory of gravity with arbitrary matter fields in n dimensions, arising from a diffeomorphism-invariant Lagrangian $\mathbf{L}$. We first show that $\mathbf{L}$ always can be written in a "manifestly covariant" form. We then show that the symplectic potential current $(n-1)$-form $\mathbf{\Theta}$ and the symplectic current $(n-1)$-form $\boldsymbol{\omega}$ for the theory always can be globally defined in a covariant manner. Associated with any infinitesimal diffeomorphism is a Noether current $(n-1)$-form $\mathbf{J}$ and corresponding Noether charge $(n-2)$-form $\mathbf{Q}$. We derive a general "decomposition formula" for $\mathbf{Q}$. Using this formula for the Noether charge, we prove that the first law of black hole mechanics holds for arbitrary perturbations of a stationary black hole. (For higher derivative theories, previous arguments had established this law only for stationary perturbations.) Finally, we propose a local, geometrical prescription for the entropy $S_{\rm dyn}$ of a dynamical black hole. This prescription agrees with the Noether charge formula for stationary black holes and their perturbations, and is independent of all ambiguities associated with the choices of $\mathbf{L}$, $\mathbf{\Theta}$, and $\mathbf{Q}$. However, the issue of whether this dynamical entropy in general obeys a "second law" of black hole mechanics remains open. In an appendix, we apply some of our results to theories with a nondynamical metric and also briefly develop the theory of stress-energy pseudotensors.

Wald Formalim for the first Law of BH thermodynamics

Phys. Rew. D **48** 3427 ('93)

- *0th Law*: $\chi^a \nabla_a \chi^b = \kappa \chi^b.$

  (in an arbitrary theory of gravity, a BH with constant surface gravity will "***Hawking radiate***" at $T = \kappa/2\pi.$)

- *1st Law*: $\psi : M \to M$, we have

  $$\mathbf{L}[\psi^*(\phi)] = \psi^* \mathbf{L}[\phi].$$ D-form

  $$\mathbf{J} = \Theta(\phi, \mathcal{L}_\chi \phi) - \chi \cdot \mathbf{L}$$ (D-1)-form

  $$\mathrm{d}\mathbf{J} = -\mathrm{E}\mathcal{L}_\chi \phi = 0, \quad \Rightarrow \quad \mathbf{J} = \mathrm{d}\mathbf{Q}$$

  (So **Q** as the Noether charge (D-2)-form relative to, local symmetry, vector filed χ^a.)

- *Noether charge* :

  $$\mathbf{Q}_{ab} = -\frac{1}{16\pi} \epsilon_{abcd} \nabla^c \xi^d.$$

  $$\mathbf{Q}_{a_1 \cdots a_{n-2}} = -\epsilon_{dea_1 \cdots a_{n-2}} \left(\frac{1}{16\pi} \nabla^d \xi^e + 2\alpha (R \nabla^d \xi^e + 4 \nabla^{[f} \xi^{d]} R^e{}_f + R^{defh} \nabla_f \xi_h) \right).$$

Einstein-Aether Theory

PHYSICAL REVIEW D, VOLUME 64, 024028

Gravity with a dynamical preferred frame

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(Received 7 September 2000; published 26 June 2001)

We study a generally covariant model in which local Lorentz invariance is broken by a dynamical unit timelike vector field u^a—the ''aether.'' Such a model makes it possible to study the gravitational and cosmological consequences of preferred frame effects, such as ''variable speed of light'' or high frequency dispersion, while preserving a generally covariant metric theory of gravity. In this paper we restrict attention to an action for an effective theory of the aether which involves only the antisymmetrized derivative $\nabla_{[a}u_{b]}$. Without matter this theory is equivalent to a sector of the Einstein-Maxwell-charged dust system. The aether has two massless transverse excitations, and the solutions of the model include all vacuum solutions of general relativity (as well as other solutions). However, the aether generally develops gradient singularities which signal a breakdown of this effective theory. Including the symmetrized derivative in the action for the aether field may cure this problem.

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PHYSICAL REVIEW D **70**, 024003 (2004)

Einstein-aether waves

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Local Lorentz invariance violation can be realized by introducing extra tensor fields in the action that couple to matter. If the Lorentz violation is rotationally invariant in some frame, then it is characterized by an "aether," i.e., a unit timelike vector field. General covariance requires that the aether field be dynamical. In this paper we study the linearized theory of such an aether coupled to gravity and find the speeds and polarizations of all the wave modes in terms of the four constants appearing in the most general action at second order in derivatives. We find that in addition to the usual two transverse traceless metric modes, there are three coupled aether-metric modes.

DOI: 10.1103/PhysRevD.70.024003 PACS number(s): 04.50.+h, 04.30.Nk, 04.80.Cc

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R + \mathcal{L}_{ae});$$

$$\mathcal{L}_{ae} = -Z^{ab}{}_{cd}(\nabla_a u^c)(\nabla_b u^d) + \lambda(u^2 + 1).$$

$$Z^{ab}{}_{cd} = c_1 g^{ab}g_{cd} + c_2\delta^a{}_c\delta^b{}_d + c_3\delta^a{}_d\delta^b{}_c - c_4 u^a u^b g_{cd}.$$

Einstein-Aether Theory

$$g_{ab} = -u_a u_b + s_a s_b + \hat{g}_{ab},$$

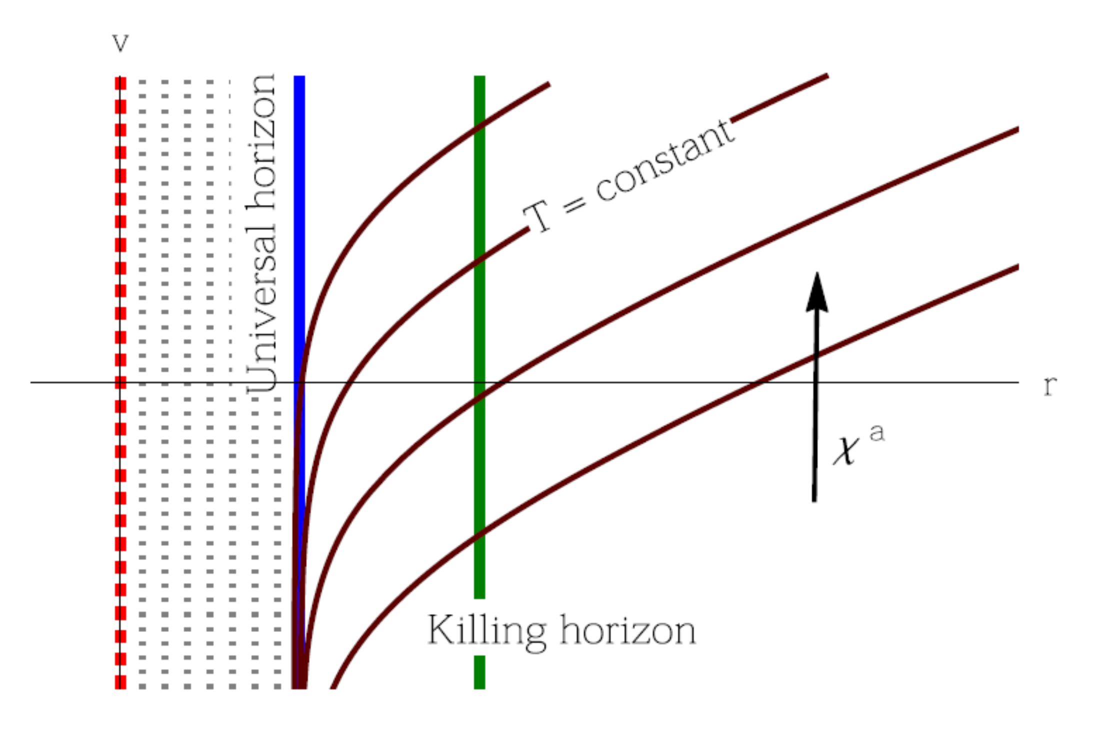
$$u^2 \equiv u^a u_a = -1 \qquad a^a = \nabla_u u^a \;.\; a^a = (a \cdot s) s^a \quad (u \cdot a = 0)$$

- Eddington-Finkelstein: $\mathrm{d}s^2 = -e(r)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2.$
- The Killing and aether voctor is: $\chi^a = \partial_t, \quad u^a = \{\alpha(r), \beta(r), 0, 0\};$
  $u_a = \{\beta(r) - e(r)\alpha(r), \alpha(r), 0, 0\}.$
- From the renormalization condition: $\beta(r) = \dfrac{e(r)\alpha(r)^2 - 1}{2\alpha(r)}.$
- Define a spacelike vector: $s^a u_a = 0; \qquad s^2 = 1.$

$$s^a = \{\alpha(r), e(r)\alpha(r) - \beta(r), 0, 0\}$$
$$= \left\{\alpha(r), \frac{e(r)\alpha(r)^2 + 1}{2\alpha(r)}, 0, 0\right\},$$

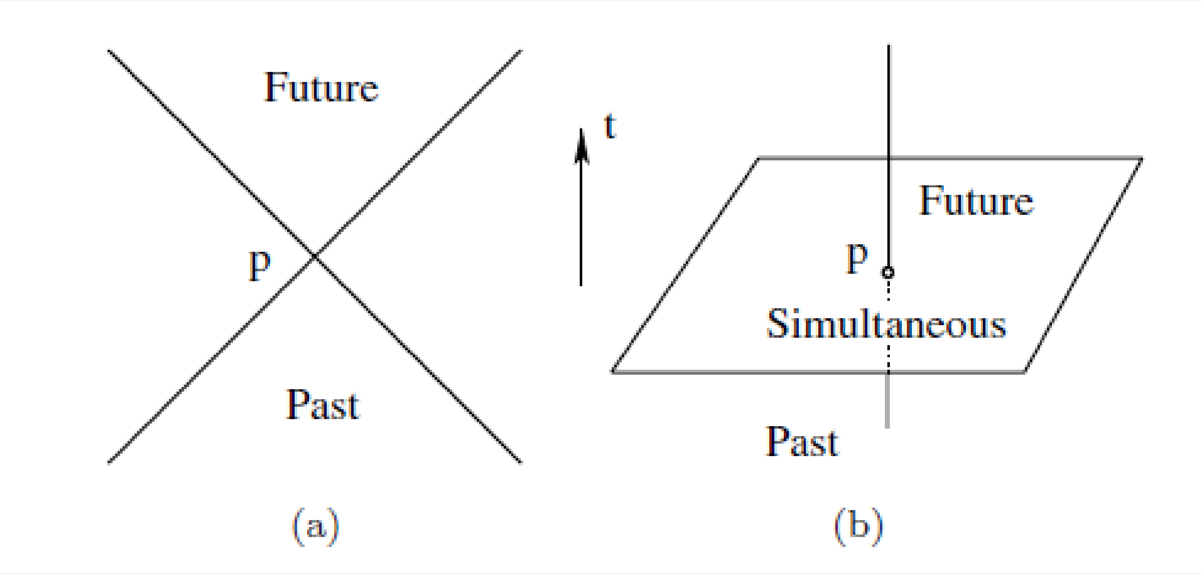
Universal horizon

[Blas & Sibiryakov, *Phys. Rev. D* **84** (2011) 124043.]



- KH: $\chi_a \chi^a = 0$
- UH: $u_a \chi^a = 0,$
- [Blas & Sibiryakov, *Phys. Rev. D* **84** (2011) 124043]
- [K. Lin, O. Goldoni, MF da Silva, A. Wang, *Phys. Rev. D.* **91** 024047]

Causal structure with broken Lorentz Symmetry



- [Kai Lin, Elcio Abdalla, Rong-Gen Cai & Anzhong Wang, IJMPD 23, No. 13 (2014) 1443004]

Einstein-Maxwell-Aether Theory

PHYSICAL REVIEW D **92**, 084055 (2015)

Charged Einstein-aether black holes and Smarr formula

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In the framework of the Einstein-Maxwell-aether theory, we present two new classes of exact charged black hole solutions, which are asymptotically flat and possess the universal as well as Killing horizons. We also construct the Smarr formulas and calculate the temperatures of the horizons, using the Smarr mass-area relation. We find that, in contrast to the neutral case, a temperature obtained this way is not proportional to its surface gravity at either of the two types of horizons. Einstein-Maxwell-aether black holes with the cosmological constant and their topological cousins are also presented.

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$$\mathcal{L}_M = -\frac{1}{16\pi G_{æ}}\mathcal{F}_{ab}\mathcal{F}^{ab}, \qquad \mathcal{F}_{ab} = \nabla_a\mathcal{A}_b - \nabla_b\mathcal{A}_a,$$

Killing horizon: $e(r) = 0$

Universal horizon: $(u\cdot\chi) = 0.$

A. Exact solution for $c_{14} = 0$

$$e(r) = 1 - \frac{r_0}{r} + \frac{Q^2}{r^2} - \frac{c_{13}r_{æ}^4}{r^4}, \qquad f(r) = 1,$$

$$(s\cdot\chi) = \frac{r_{æ}^2}{r^2},$$

$$(u\cdot\chi) = -\sqrt{1 - \frac{r_0}{r} + \frac{Q^2}{r^2} + \frac{(1-c_{13})r_{æ}^4}{r^4}},$$

$$r_{æ}^4 = \frac{1}{1-c_{13}}\left(r_{\mathrm{UH}}^4 - \frac{1}{2}r_0 r_{\mathrm{UH}}^3\right), \qquad r_{\mathrm{UH}} = \frac{r_0}{2}\left(\frac{3}{4} + \sqrt{\frac{9}{16} - 2\frac{Q^2}{r_0^2}}\right),$$

B. Exact solution for $c_{123} = 0$

$$e(r) = 1 - \frac{r_0}{r} - \frac{r_u(r_0 + r_u)}{r^2}, \qquad f(r) = 1,$$

$$(s\cdot\chi) = \frac{r_0 + 2r_u}{2r}, \qquad (u\cdot\chi) = -1 + \frac{r_0}{2r},$$

$$r_u = \frac{r_0}{2}\left(\sqrt{\frac{2-c_{14}}{2(1-c_{13})} - \frac{4Q^2}{(1-c_{13})r_0^2}} - 1\right).$$

$$r_{\mathrm{KH}} = r_0 + r_u, \qquad r_{\mathrm{UH}} = \frac{r_0}{2},$$

Surface gravity: (T)emperature

Surface gravities for non-Killing horizons

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Abstract
There are many logically and computationally distinct characterizations of the surface gravity of a horizon, just as there are many logically rather distinct notions of horizon. Fortunately, in standard general relativity, for stationary horizons, these characterizations are degenerate. However, in modified gravity, or in analogue spacetimes, horizons may be non-Killing or even non-null, and hence these degeneracies can be lifted. We present a brief overview of the key issues, specifically focusing on horizons in analogue spacetimes and universal horizons in modified gravity.

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$$\chi^a \nabla_a \chi^b = \kappa_{\text{inaffinity}}\ \chi^b .$$

$$\left.\frac{\mathrm{d}r}{\mathrm{d}v}\right|_{\text{out}} = \left.\frac{\mathrm{d}r}{\mathrm{d}v}\right|_{\text{kh}} + \left.\frac{\mathrm{d}}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}v}\right|_{\text{kh}} (r - r_{\text{kh}}) + \mathcal{O}(r - r_{\text{kh}})^2 .$$

$$\begin{aligned}\kappa_{\text{inaffinity}} &\equiv \sqrt{-\frac{1}{2}(\nabla_a \chi_b)(\nabla^a \chi^b)} \\ &= |(u\cdot\chi)(a\cdot s) - (s\cdot\chi)K_0| \\ &= (s\cdot\chi)K_0|_{UH} .\end{aligned}$$

$$\begin{aligned}\kappa_{\text{peeling}} &\equiv \frac{1}{2}\nabla_u\,(u\cdot\chi)\Big|_{UH} \\ &= \frac{1}{2}\,(a\cdot s)\,(s\cdot\chi)\Big|_{UH} ,\end{aligned}$$

Smarr Formula

- D=4, the *Noether charge*:

$$\mathbf{Q}[\chi] - \chi\cdot\mathbf{A} = \frac{1}{8\pi G}\left[-\left(1-\frac{c_{14}}{2}\right)(u\cdot\chi)(u\cdot\chi)' + \left(1+\frac{c_{13}}{2}-c_{123}\right)(s\cdot\chi)(s\cdot\chi)' + \frac{2Q^2}{r^3}(u\cdot\chi)\right]\bar{\epsilon}$$

- The *Smarr formula* at UH:

$$0 = \int_{\partial\Sigma}[\mathbf{Q}[\chi]-\chi\cdot\mathbf{A}] = \int_{S_{\mathrm{UH}}}[\mathbf{Q}[\chi]-\chi\cdot\mathbf{A}] - \int_{S_\infty}[\mathbf{Q}[\chi]-\chi\cdot\mathbf{A}]$$

Smarr Formula

- $c_{14}=0$:

$$\int_{S_{\rm UH}}[\mathbf{Q}[\chi]-\chi\cdot\mathbf{A}] = \frac{1}{2G}(1+\frac{c_{13}}{2}-c_{123})(s\cdot\chi)(s\cdot\chi)'r^2$$
$$= (1+\frac{c_{13}}{2}-c_{123})\frac{\kappa}{2\pi}(\pi r^2) = (1+\frac{c_{13}}{2}-c_{123})TS$$

$$\int_{S_\infty}[\mathbf{Q}[\chi]-\chi\cdot\mathbf{A}] = \frac{1}{2G}[-M+VQ]$$

$$M = (1+\frac{c_{13}}{2}-c_{123})2TS+VQ$$

Smarr Formula

- c_{123} =0 :

$$\int_{S_{\mathrm{UH}}}[\mathbf{Q}[\chi]-\chi\cdot\mathbf{A}] = \frac{1}{2G}(1-\frac{c_{13}}{2})(s\cdot\chi)(s\cdot\chi)'r^2$$
$$= (1-\frac{c_{13}}{2})\frac{\kappa}{2\pi}(\pi r^2) = (1-\frac{c_{13}}{2})TS$$

$$\int_{S_\infty}[\mathbf{Q}[\chi]-\chi\cdot\mathbf{A}] = \frac{1}{2G}[-(1-\frac{c_{14}}{2})M+VQ]$$

$$(1-\frac{c_{14}}{2})M = (1-\frac{c_{13}}{2})2TS+VQ$$

Discussion

- We apply ***quasi-local energy*** idea to some alternative gravity theory
- We can investigate quasi-local energy in ***horizon thermodynamics***.
- For entropy, $S = \frac{A_{UH}}{4}$?
- Integral formula to differential formula

Thank you