MOGRA2018@Nagoya 8/9/2018

Extended Cuscuton : Formulation and Cosmology

Aya Iyonaga (Rikkyo Univ.)

Collaborators:
Kazufumi Takahashi (Rikkyo Univ.)
Tsutomu Kobayashi (Rikkyo Univ.)

Contents

- Scalar-tensor theories
- “Cuscuton” theory
- Extended Cuscuton theories
  - Formulation
  - Cosmological perturbations
- Summary

Scalar-tensor(S-T) theories

S-T theories : GR + new scalar fields

#DOFs of single scalar theories $\longrightarrow$ 2 + 1 = 3

$g_{\mu\nu} + \phi$

Generalized S-T theories :

Horndeski theory Horndeski (1974), Deffayet, et al.(2011) Kobayashi, et al.(2012)

The most general theory whose EOMs are at most 2nd-order

beyond Horndeski (GLPV) theories Gleyzes, et al. (2014)

EOMs are higher-order, but after combining with each other, eqs. are at most 2nd-order

∴Generalized theories have **3DOFs**, but some special subclasses have only 2DOFs

Cuscuton theory (1)

"Cuscuton" theory Afshordi, et al. (2007)

S-T theory with **2DOFs**

on a cosmological background/in the unitary gauge

Features

- Scalar field has a first-order equation
  = Scalar DOF has only a constraint equation
- Cosmological perturbations
  ⟶ the kinetic term of scalar perturbation vanishes

∴ Scalar DOF is **nondynamical, not propagated**

Therefore, propagating number of degrees of freedom is only 2

Origin of the "Cuscuton"

Cuscuta

..The parasitic plant

Cuscuta
coils around
other plants

Why "Cuscuton" ?

Afshordi, et al. (2007)

In the first paper about Cuscuton theory,

..the equation of motion (of a scalar field) does not have any second order time derivatives and the field becomes a nondynamical field, which merely follows the dynamics of the fields that it couples to. Thus we call this field Cuscuton.

Cuscuton theory (2)

Action for Cuscuton theory

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \mu^2\sqrt{|X|} - V(\phi)\right]$$

$$X \equiv \partial_\mu\phi\partial^\mu\phi \qquad \mu = const.$$

- In the unitary gauge $\phi = \phi(t)$,

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \mu^2\dot{\phi} - V(\phi)\right]$$

⟶ ϕ has **a first-order E-L equation**

- Subclass of k-essence

⟶ We can represent Cuscuton theory as a part of Horndeski / beyond Horndeski

Horndeski / beyond Horndeski

(beyond) Horndeski in the unitary gauge

$$S = \int dt d^3x N\sqrt{\gamma}\left[A_2 + A_3 K + A_4\left(K^2 - K_{ij}K^{ij}\right) + B_4 R\right.$$
$$\left. + A_5\left(K^3 - 3KK_{ij}K^{ij} + 2K_{ij}K^{ik}K^j_k\right) + B_5 K^{ij}G_{ij}\right]$$

$$A_i = A_i(t, N) \qquad B_i = B_i(t, N)$$

(Horndeski ⟶ $A_4 = -B_4 - NB_{4N},\ A_5 = \frac{N}{6}B_{5N}$ "Horndeski conditions") $\quad B_N = \frac{\partial B}{\partial N}$

Cuscuton theory in Horndeski :

$$A_2 = -V(t) + \frac{\sigma(t)}{N}, \quad A_3 = 0, \quad A_4 = -\frac{1}{2}, \quad A_5 = 0,$$
$$B_4 = \frac{1}{2}, \quad B_5 = 0$$

$\sigma(t)$: arbitrary function

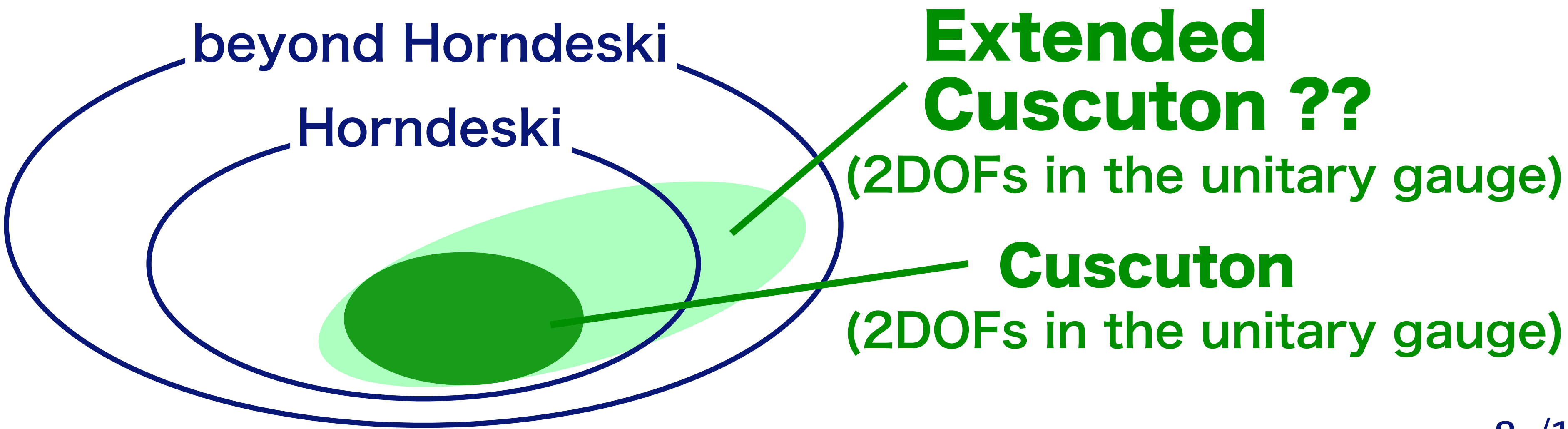
Motivation of Our Study

Are there **more generalized theories** with 2DOFs in the unitary gauge?

↓

We considered "Extended Cuscuton" theory from Horndeski / beyond Horndeski

Inclusion relation



Extended Cuscuton

Definition of Extended Cuscuton

Scalar perturbation is not propagated on a cosmological background / the unitary gauge

Homogeneous / isotropic spacetime :

$$ds^2 = -N^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

Quadratic scalar action in the unitary gauge :

$$S^{(2)} = \int dt d^3x N a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\partial_i \zeta)^2 \right]$$

ζ : Curvature perturbation

where $\mathcal{G}_S = \sum_{n=0}^{4} \alpha_n(t, N) H^n$

H : Hubble parameter

$\alpha_n = 0 \quad (n = 0 \sim 4) \longrightarrow$ **scalar DOF is not propagated**

Extended Cuscuton

$\alpha_n = 0$ **are satisfied if the coefficient functions are**

$$\begin{cases} A_2 = \mu_2 + \dfrac{\nu_2}{N} + \dfrac{2(\mu_4 N + \nu_4)^3}{9N(\mu_5 N + \nu_5)^2} \\ A_3 = \mu_3 + \dfrac{2(\mu_4 N + \nu_4)^2}{3(\mu_5 N + \nu_5)^2} \\ A_4 = \dfrac{N(\mu_4 N + \nu_4)}{(\mu_5 N + \nu_5)^2}, \quad A_5 = \dfrac{N^2}{(\mu_5 N + \nu_5)^2} \\ B_4, B_5 \text{ are arbitrary} \end{cases}$$

$$u_i = u_i(t)$$
$$v_i = v_i(t)$$

$\Uparrow$ **"Extended Cuscuton" theories**

- **Horndeski conditions are not satisfied**

$\longrightarrow$ **Subclass of the beyond Horndeski**

Cosmological Perturbations

The full Extended Cuscuton with matter

$$\mathcal{L} = \mathcal{L}_{bH} + \mathcal{L}_\chi \quad \textbf{with} \quad \mathcal{G}_S = \sum_{n=0}^{4} \alpha_n(t, N) H^n = 0$$

matter χ **sector :** $\mathcal{L}_\chi = P(Y), \quad Y \equiv -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi$

- **Sound speed of** χ **:** $c_\chi^2 = \dfrac{P_Y}{P_Y + 2Y P_{YY}}$ $\qquad P_Y = \dfrac{\partial P}{\partial Y}$
- **Energy density of** χ **:** $\rho_\chi = 2YP_Y - P$

$\longrightarrow$ **Null energy condition :** $\rho_\chi + P > 0$

Cosmological Perturbations

The full Extended Cuscuton with matter

$$\mathcal{L} = \mathcal{L}_{bH} + \mathcal{L}_\chi \quad \text{with} \quad \mathcal{G}_S = \sum_{n=0}^{4} \alpha_n(t, N) H^n = 0$$

matter χ sector : $\mathcal{L}_\chi = P(Y), \quad Y \equiv -\frac{1}{2}\partial_\mu \chi \partial^\mu \chi$

- Scalar perturbations :

$$N = 1 + \delta N, \quad N_i = \partial_i \psi, \quad \gamma_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

$$\chi(x^\mu) \to \chi(t) + \delta\chi(t, \vec{x})$$

- Field redefinition : $\tilde{\zeta} \equiv \zeta - \dfrac{\Theta}{\mathcal{G}_T} \dfrac{\delta\chi}{\dot{\chi}}$

$\mathcal{G}_T, \ \Theta$: functions of A_i, B_i, H

Cosmological Perturbations

In Fourier space,

$$\mathcal{L}^{(2)} = a^3 \left[\mathcal{A}(t,k) \left| \dot{\tilde{\zeta}} \right|^2 - \mathcal{B}(t,k) \frac{k^2}{a^2} \left| \tilde{\zeta} \right|^2 \right]$$

where $$\mathcal{A} \equiv \frac{\mathcal{G}_T^2 (\rho_\chi + P)}{2 c_\chi^2 \Theta^2} \frac{k^2/a^2 + \alpha_1}{k^2/a^2 + \alpha_2},$$

$$\mathcal{B} \equiv \Upsilon \frac{\mathcal{G}_T^2 (\rho_\chi + P)}{2\Theta^2} \frac{k^4/a^4 + \beta_1 k^2/a^2 + \beta_2}{(k^2/a^2 + \alpha_2)^2}$$

$\mathcal{G}_T,\ \Theta$: functions of A_i, B_i, H

$\alpha_i, \beta_i, \Upsilon$: functions of A_i, B_i, H, Y

$$\Upsilon \equiv \frac{\mathcal{F}_S \Theta^2 - \bar{\mathcal{G}}_T^2 Y P_Y}{\mathcal{F}_S \Theta^2 - \mathcal{G}_T (2\bar{\mathcal{G}}_T - \mathcal{G}_T) Y P_Y}$$

Ghost/Gradient Instability

In Fourier space,

$$\mathcal{L}^{(2)} = a^3 \left[\mathcal{A}(t,k) \left| \dot{\tilde{\zeta}} \right|^2 - \mathcal{B}(t,k) \frac{k^2}{a^2} \left| \tilde{\zeta} \right|^2 \right]$$

UV regime (k is sufficiently large)

$$\mathcal{A} \equiv \frac{\mathcal{G}_T^2(\rho_\chi + P)}{2c_\chi^2 \Theta^2} \frac{k^2/a^2 + \alpha_1}{k^2/a^2 + \alpha_2},$$

$$\mathcal{B} \equiv \Upsilon \frac{\mathcal{G}_T^2(\rho_\chi + P)}{2\Theta^2} \frac{k^4/a^4 + \beta_1 k^2/a^2 + \beta_2}{(k^2/a^2 + \alpha_2)^2}$$

$\mathcal{G}_T, \ \Theta$: functions of A_i, B_i, H

$\alpha_i, \beta_i, \Upsilon$: functions of A_i, B_i, H, Y

→ If $\rho_\chi + P > 0, \quad c_\chi^2 > 0, \quad \Upsilon > 0$,

(null energy condition)

there is no ghost/gradient instability

Ghost/Gradient Instability

In Fourier space,

$$\mathcal{L}^{(2)} = a^3 \left[\mathcal{A}(t,k) \left| \dot{\tilde{\zeta}} \right|^2 - \mathcal{B}(t,k) \frac{k^2}{a^2} \left| \tilde{\zeta} \right|^2 \right]$$

IR regime (*k* is sufficiently small)

$$\mathcal{A} \equiv \frac{\mathcal{G}_T^2(\rho_\chi + P)}{2c_\chi^2 \Theta^2} \frac{k^2/a^2 + \alpha_1}{k^2/a^2 + \alpha_2},$$

$$\frac{\mathcal{G}_T^2(\rho_\chi + P)}{2c_\chi^2 \Theta^2} > 0$$

$$\mathcal{B} \equiv \Upsilon \frac{\mathcal{G}_T^2(\rho_\chi + P)}{2\Theta^2} \frac{k^4/a^4 + \beta_1 k^2/a^2 + \beta_2}{(k^2/a^2 + \alpha_2)^2}$$

$\mathcal{G}_T,\ \Theta$: functions of A_i, B_i, H

$\alpha_i, \beta_i, \Upsilon$: functions of A_i, B_i, H, Y

→ **If** $\frac{\alpha_1}{\alpha_2} = \frac{3c_\chi^2 \Theta^2}{3c_\chi^2 H \Theta \mathcal{G}_T + \mathcal{G}_T (Y P_Y + \dot{\Theta}) - \Theta \dot{\mathcal{G}}_T} > 0$,

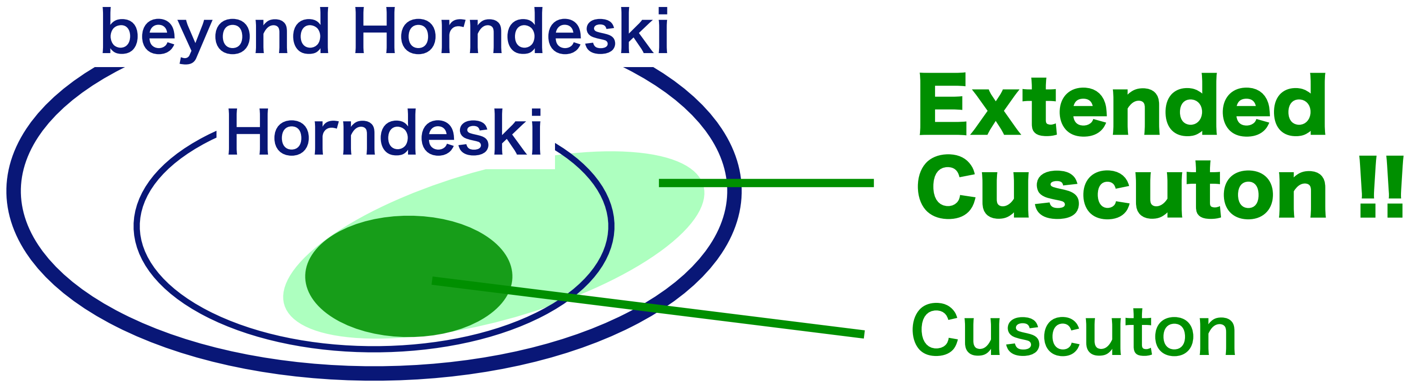
there are no ghost instability

Summary

Cuscuton theory : Scalar-tensor theory with **2DOFs**
on a cosmological background
/in the unitary gauge

↓

We determined the "**Extended Cuscuton**" theories
in the Horndeski / beyond Horndeski



- Cosmological perturbations with matter sector

⟶ If $\rho_\chi + P > 0, \quad c_\chi^2 > 0, \quad \Upsilon > 0, \quad \frac{\alpha_1}{\alpha_2} > 0$

are satisfied, scalar perturbation is stable