Progress in Massive Gravity

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MOGRA, Nagoya, August 8 2018

Guiding Principle for Modified Gravitational Theories

Theorem: General Relativity (with a c.c.) is the Unique local and Lorentz invariant theory describing an interacting single massless spin two particle that couples to matter

Weinberg, Deser, Wald, Feynman, ...

Locality

Massless

?

Lorentz Invariant

Single Spin 2

Guiding Principle for Modified Gravitational Theories

Corollary: Any theory which preserves Lorentz invariance and Locality leads to new degrees of freedom!

Locality ✓

Massless ✗

?

Lorentz Invariant ✓

Single Spin 2 ✗

Massive Gravity: Hard or Soft?

A generic local, Lorentz invariant theory at the linearized level gives the following interaction between two stress energies

+ spin 0 + massless spin 2 contributions

$$\Delta S \sim \frac{1}{M_{\text{Planck}}^2} \int \frac{d^4k}{(2\pi)^4} T^{\mu\nu}(k) \left[\sum_{\text{pole}} Z_{\text{pole}} \frac{P_{\mu\nu\alpha\beta}(k)}{k^2 + m_{\text{pole}}^2} + \int_0^\infty d\mu \frac{\rho(\mu) P_{\mu\nu\alpha\beta}(k)}{k^2 + \mu} \right] T^{\alpha\beta}(k)$$

Hard

Soft

Soft Massive Graviton is a **resonance** (finite lifetime)

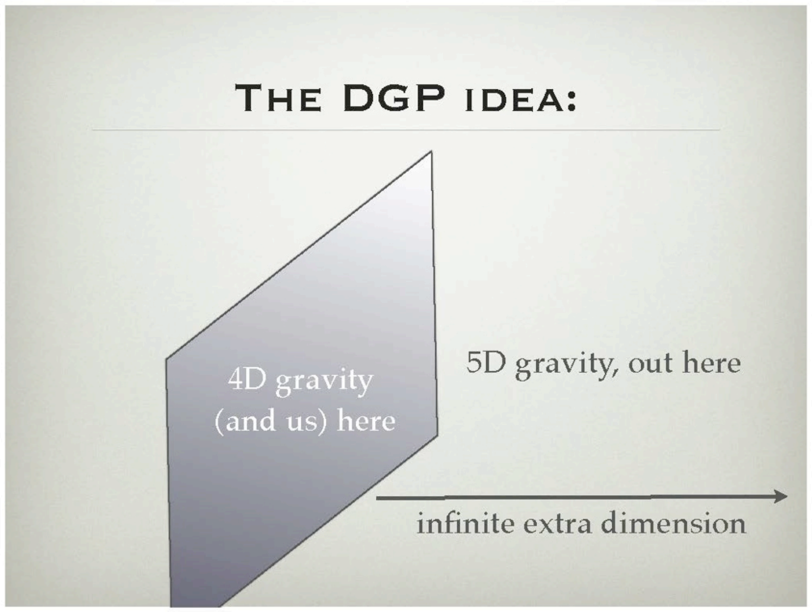
(pole on the second Riemann sheet)

$z = -k^2 + i\epsilon$

Hard Massive Graviton is a **pole** (infinite lifetime)

(pole on the first Riemann sheet)

Soft Massive Gravity: DGP Model



Soft Massive Gravity theories were constructed first! Naturally arise in Braneworld Models: **DGP**, **Cascading Gravity**: Soft Massive Graviton is a Resonance State localized on Brane

$$\Delta S \sim \frac{1}{M_{\text{Planck}}^2} \int \frac{d^4k}{(2\pi)^4} T^{\mu\nu}(k) \left[\int_0^\infty d\mu \frac{\rho(\mu) P_{\mu\nu\alpha\beta}(k)}{k^2+\mu} \right] T^{\alpha\beta}(k)$$

Soft

More irrelevant ⟷ More relevant

$$S = \int d^4x \sqrt{-g_4} \frac{M_4^2}{2} R_4 + \int d^4x \sqrt{-g_4} \mathcal{L}_M + \int d^5x \sqrt{-g_5} \frac{M_5^3}{2} R_5$$

Dominates in UV ⟷ Dominates in IR

What does hard massive gravity mean?

In Standard Model, Electroweak symmetry
is spontaneously broken by the VEV of the Higgs field

$$SU(2) \times U(1)_Y \to U(1)_{\rm EM}$$

Result, W and Z bosons become massive

Would-be-Goldstone-mode in Higgs field becomes
Stuckelberg field which gives boson mass

Higgs Vev   Higgs Boson   Stuckelberg field

$$\Phi = (v + \rho) e^{i\pi}$$

e.g. for Abelian Higgs

$$A_\mu \to A_\mu + \partial_\mu \chi \qquad \pi \to \pi + \chi$$

Symmetry Breaking Pattern

In **Massive Gravity** - Local Diffeomorphism Group and an additional global Poincare group is broken down the diagonal subgroup

$$Diff(M) \times \text{Poincare} \to \text{Poincare}_{\text{diagonal}}$$

In **Bigravity** - Two copies of local Diffeomorphism Group are broken down to a single copy of Diff group

$$Diff(M) \times Diff(M) \to Diff(M)_{\text{diagonal}}$$

Higgs for Gravity

Despite much *blood, sweat and tears* an explicit Higgs mechanism for gravity is not known

However if such a mechanism exists, we DO know how to write down the low energy effective theory in the spontaneously broken phase

For Abelian Higgs this corresponds to integrating out the Higgs boson and working at energy scales lower that the mass of the Higgs boson

Higgs Boson

Stuckelberg field

$E \ll m_\rho$

$$\Phi = (v + \rho)e^{i\pi}$$

→ Stuckelberg formulation of massive vector bosons

Stuckelberg Formulation for Massive Gravity

Arkani-Hamed, Georgi, Schwartz 2002
de Rham, Gabadadze 2009

Diffeomorphism invariance is spontaneously broken but maintained by introducing Stueckelberg fields

Vev of spin 2 Higgs field defines a 'reference metric'

$$f_{\mu\nu} = \langle \hat{O}_{\mu\nu} \rangle$$

reference metric

Stuckelberg fields

Dynamical Metric

$$g_{\mu\nu}(x)$$

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B$$

helicity-1 mode of graviton

$$\phi^a = x^a + \frac{1}{mM_P} A^a + \frac{1}{\Lambda^3}\partial^a \pi$$

$$\Lambda^3 = m^2 M_P$$

helicity-0 mode of graviton

de Rham, Gabadadze, AJT 2010

Λ_3 Massive Gravity

$Diff(M) \times \text{Poincare} \to \text{Poincare}_{\text{diagonal}}$

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2 R[g] - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n\right) + \mathcal{L}_M$$

$$\text{Det}[1 + \lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

Characteristic Polynomials

$K = 1 - \sqrt{g^{-1} f}$ square root designed s.t. $K_{\mu\nu} \to \frac{\partial_\mu \partial_\nu \pi}{\Lambda_3^3}$

Unique low energy EFT where the strong coupling scale is

$$\Lambda_3 = (m^2 M_P)^{1/3}$$

Hassan, Rosen 2011
Hinterbichler, Rosen 2012

Λ_3 Bigravity + Multigravity

$$\mathcal{L} = \frac{1}{2}\left(M_P^2 \sqrt{-g} R[g] + M_f^2 \sqrt{-f} R[f] - m^2 \sum_{n=0}^{d} \beta_n U_n(K) \right) + \mathcal{L}_M$$

$$\mathrm{Det}[1+\lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

$$K = 1 - \sqrt{g^{-1} f}$$

decoupling limit

$M_f \to \infty$

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2 R[g] - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n \right) + \mathcal{L}_M$$

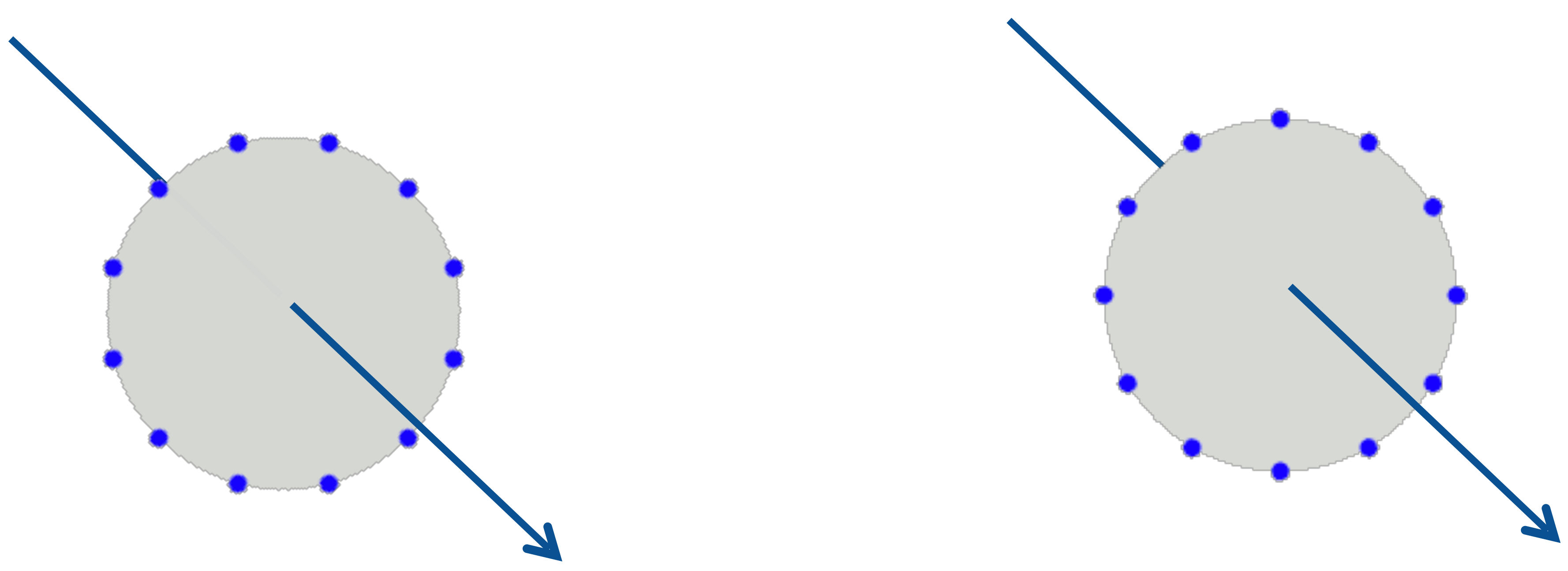
+decoupled massless graviton $f_{\mu\nu}$

Bigravity=
massless graviton (2 d.o.f.)
+ massive graviton (5 d.o.f.)

General Relativity

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2} R$$

2 ‘tensors’ = helicity-2 modes

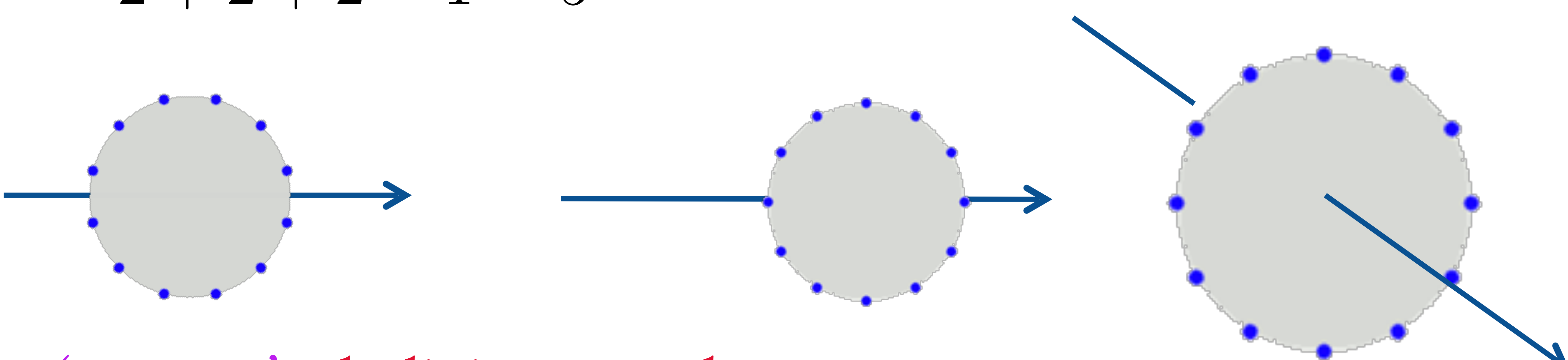


Lorenz Invariant Massive Gravity

$$S = \int \sqrt{-g}\,\frac{M_{\rm Pl}^2}{2}\left(R - \text{Mass Term}\right)$$

Constraint means only one scalar propagates

$$2 + 2 + 2 - 1 = 5$$



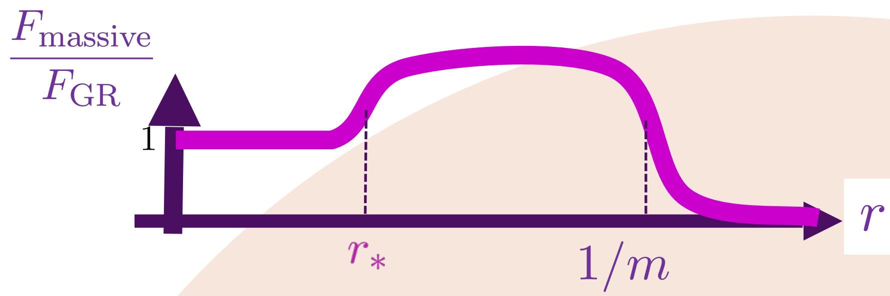
2 'vectors' = helicity-1 modes

2 'scalars' = helicity-0 modes

5 propagating degrees of freedom

5 polarizations of gravitational waves!!!!

Vainshtein effect is strongly scale and density dependent



Yukawa region

$r > m^{-1}$

Strong coupling region

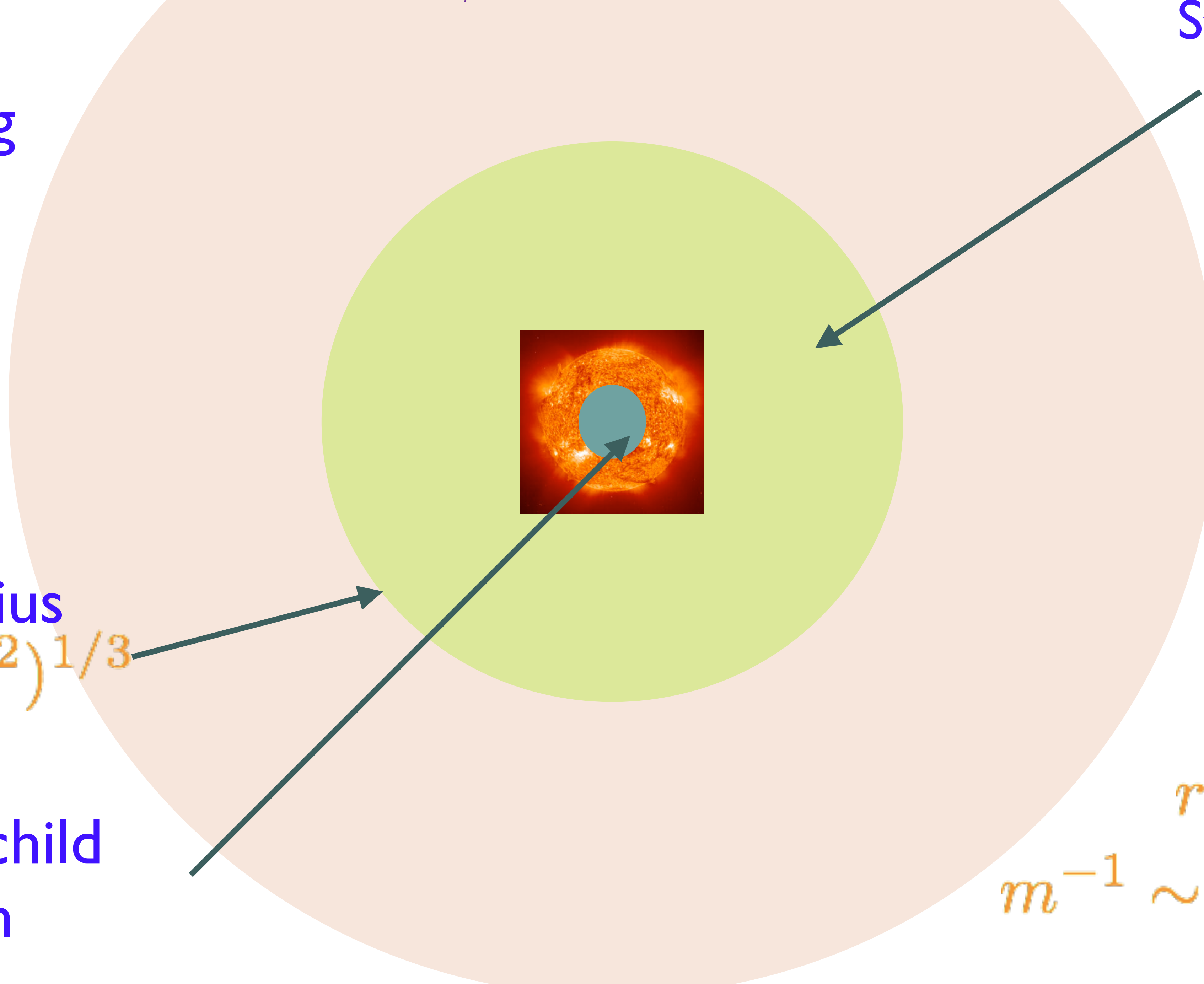
$r \ll r_V$

$Z \gg 1$

Weak coupling region

$r \gg r_V$

$Z \sim 1$



Vainshtein radius

$r_V = (r_s m^{-2})^{1/3}$

Schwarzschild region

$r < r_s$

For Sun

$r_V \sim 250pc$

$m^{-1} \sim 4000Mpc$

$r_s \sim 3km$

Example: Binary Pulsars

Scalar Gravitational Waves: Dominated by Quadrupole radiation

Scalar Gravitational Radiation from Binaries: Vainshtein Mechanism in Time-dependent Systems

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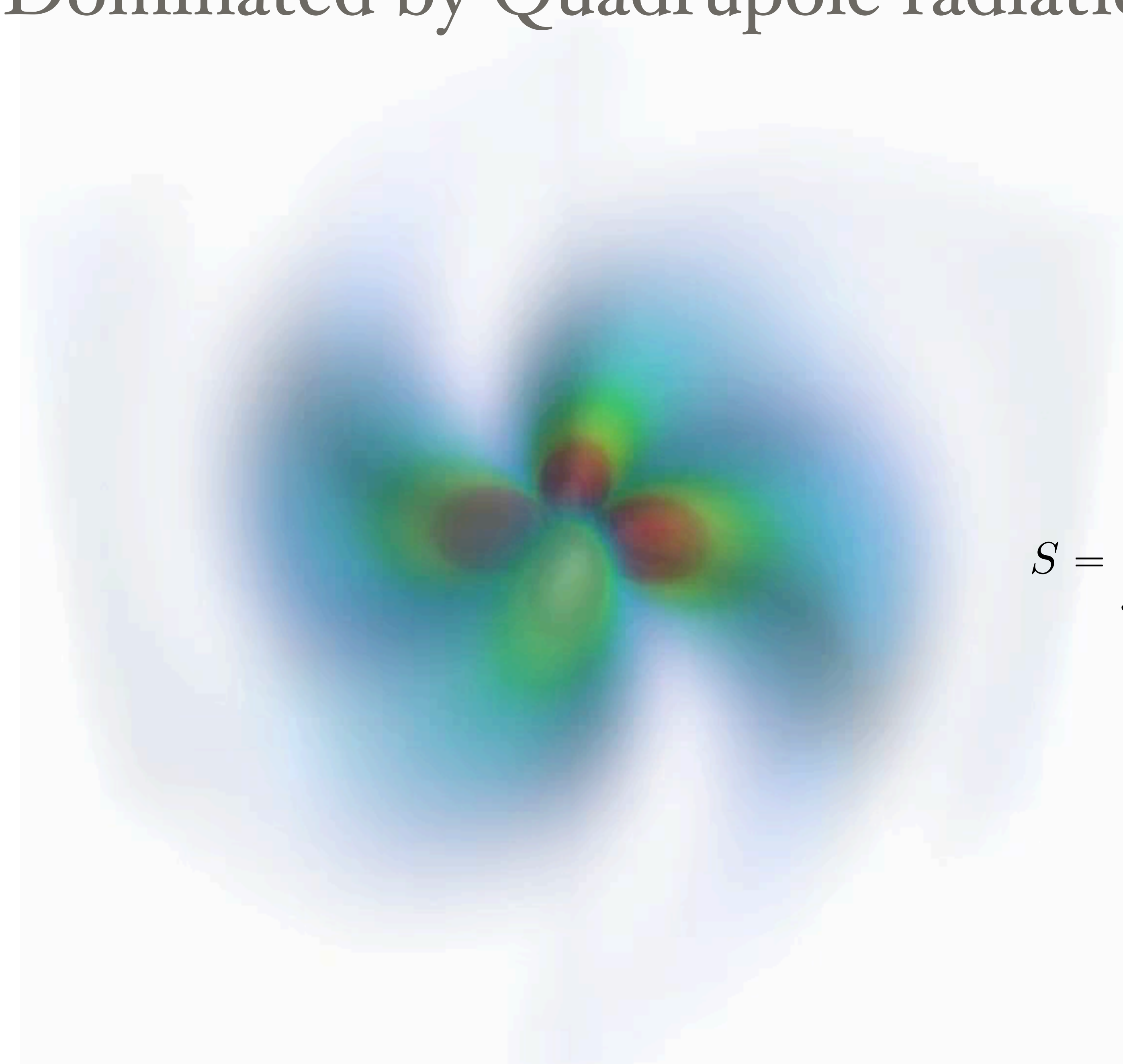
Abstract: We develop a full four-dimensional numerical code to study scalar gravitational radiation emitted from binary systems and probe the Vainshtein mechanism in situations that break the static and spherical symmetry, relevant for binary pulsars as well as black holes and neutron stars binaries. The present study focuses on the cubic Galileon which arises as the decoupling limit of massive theories of gravity. Limitations associated with the numerical methods prevent us from reaching a physically realistic hierarchy of scales; nevertheless, within this context we observe the same power law scaling of the radiated power as previous analytic estimates, and confirm a strong suppression of the power emitted in the monopole and dipole as compared with quadrupole radiation. Following the trend to more physically realistic parameters, we confirm the suppression of the power emitted in scalar gravitational radiation and the recovery of General Relativity with good accuracy. This paves the way for future numerical work, probing more generic, physically relevant situations and sets of interactions that may exhibit the Vainshtein mechanism.

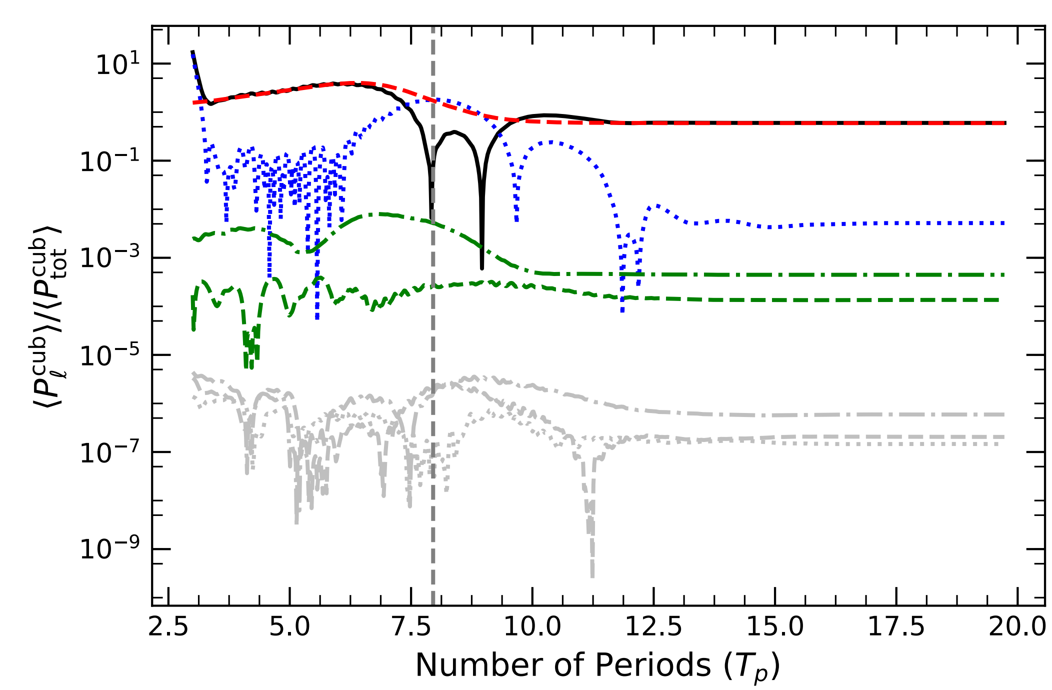
Numerical Simulations:
Dar, de Rham, Deskins, Giblin, AJT
1808.02165!



Energy density of the cubic Galileon field after the simulation has relaxed

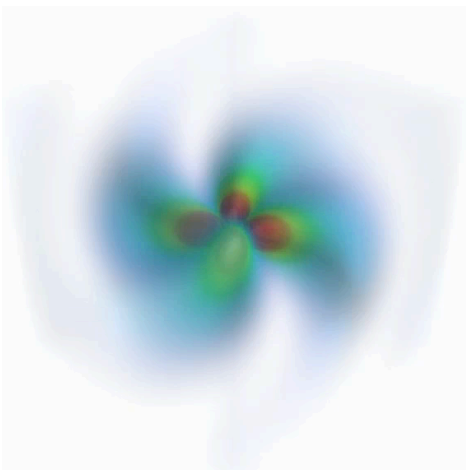
Red is higher energy density and blue lower.

$$S = \int \mathrm{d}^4x \left(-\frac{3}{4}(\partial\pi)^2 - \frac{1}{4\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{2M_{\rm Pl}}\pi T \right)$$



Dar, de Rham, Deskins, Giblin, AJT
1808.02165!

Scalar Gravitational Wave Power



$$\frac{P_2^{\rm cubic}}{P_2^{\rm KG}} = \frac{25\times 3^{17/4}\pi^{3/2}}{1024\Gamma\left(\frac{9}{4}\right)}(\Omega_{\rm p}\bar{r})^{-1}(\Omega_{\rm p}\bar{r}_{\rm v})^{-3/2}$$

Previous Analytic Work
de Rham, AJT, Wesley 2013
de Rham, Matas, AJT 2013

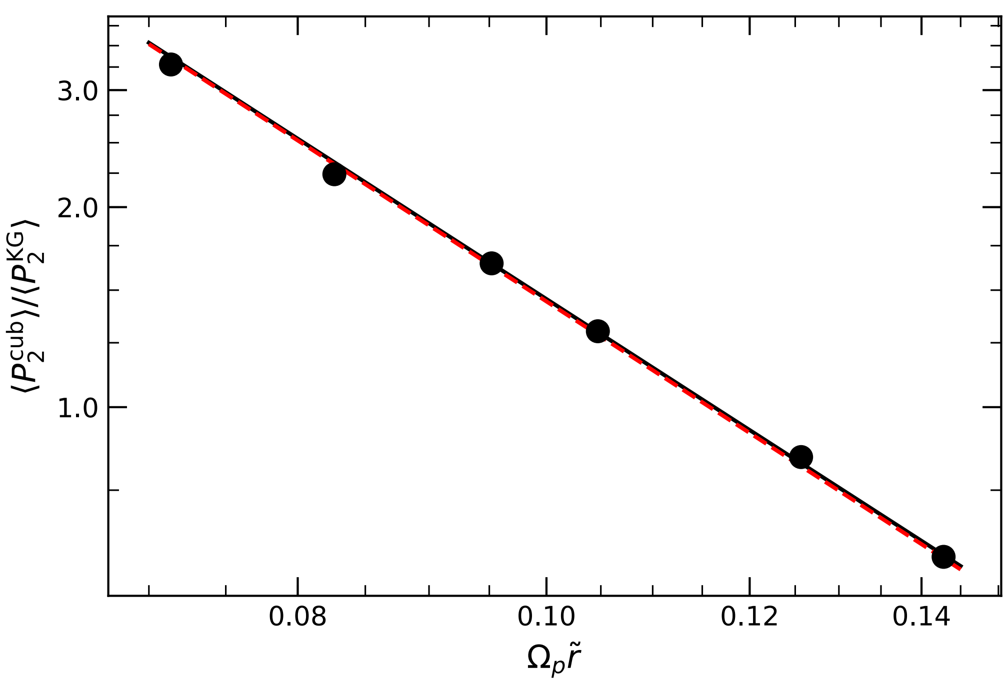
$$P = \frac{3r^2}{2}\left(1+\frac{4}{3\Lambda^3}\frac{E}{r}\right)\int {\rm d}\Omega\ \partial_t\pi\partial_r\pi$$

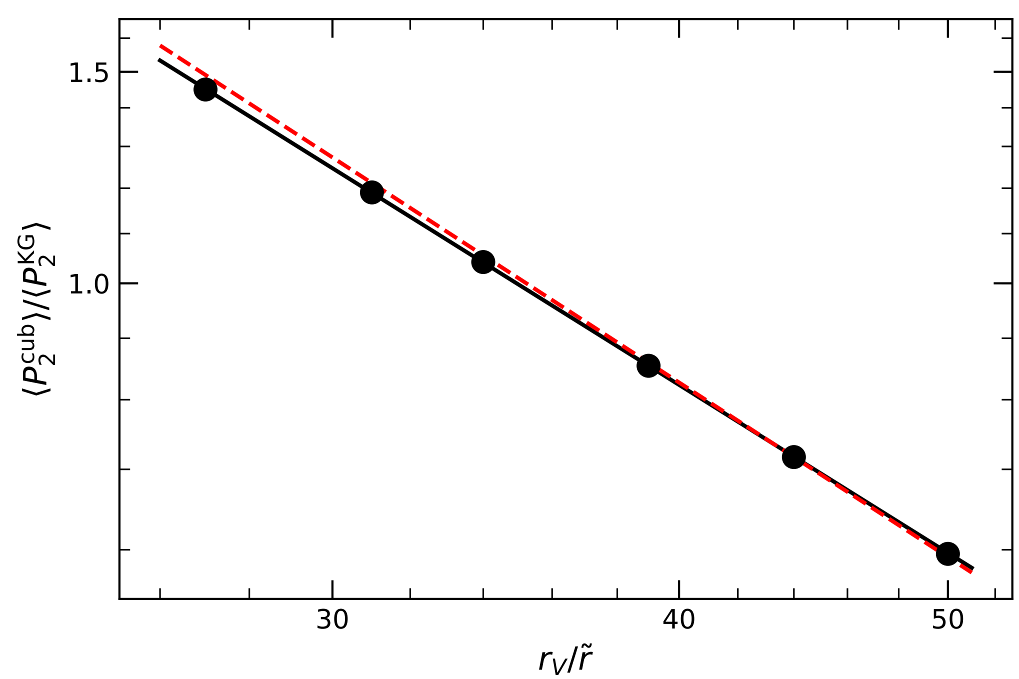
Static Source suppression

Time-dependent enhancement

$$(\bar{r}/r_{\rm v})^{3/2}$$

$$(\Omega_p\bar{r})^{-5/2} = 1/v^{5/2}$$



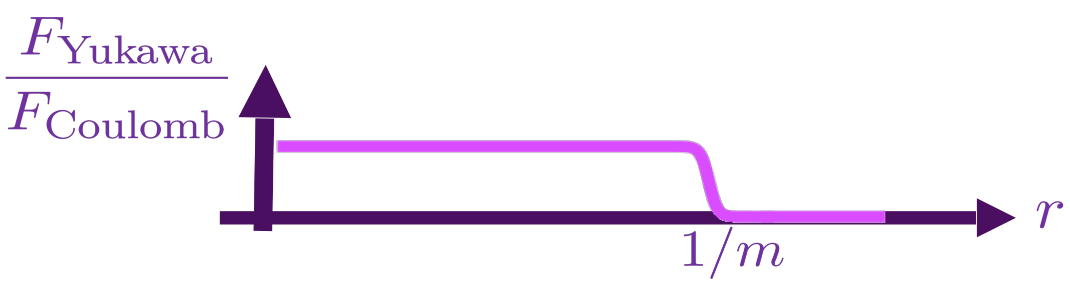


Constraints on the Graviton Mass

de Rham, Deskins, AJT, Zhou, 1606.08462

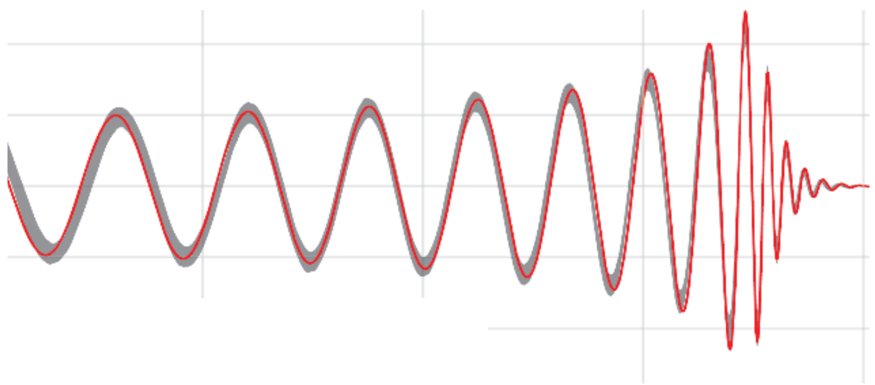
Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-32}	10^{21}	Weak lensing
10^{-29}	10^{19}	Bound clusters



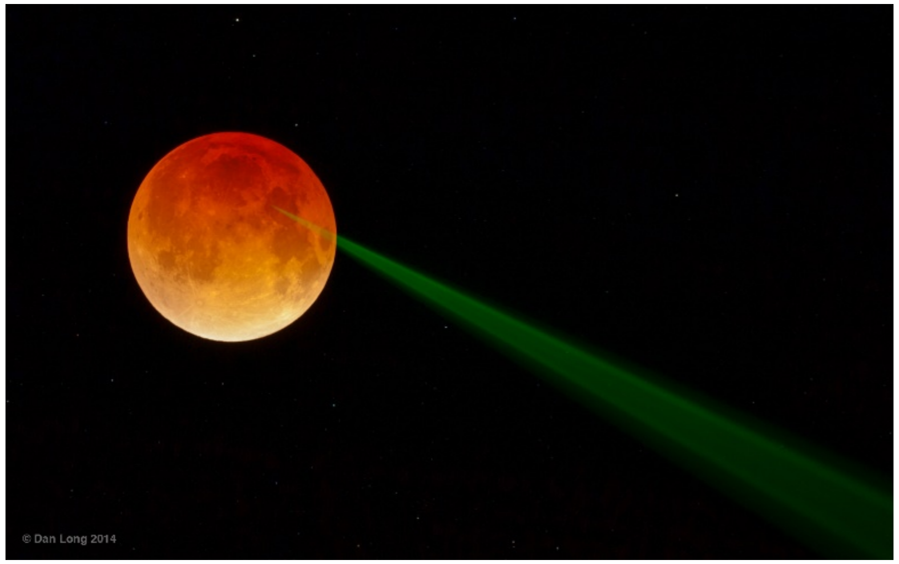
Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB



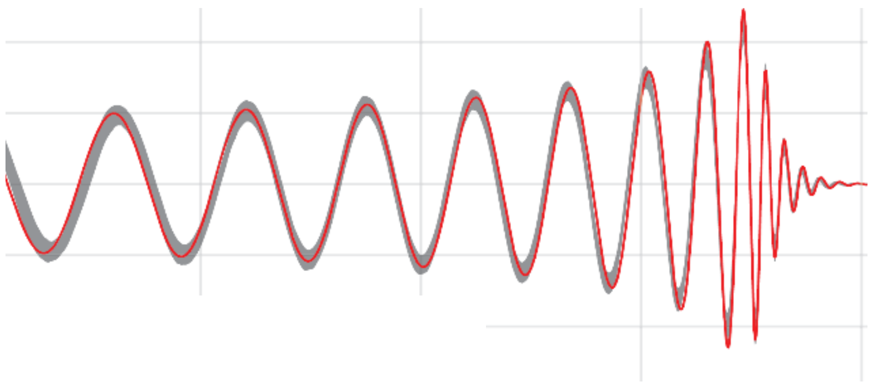
Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation



m_g (eV)	λ_g (km)	Dispersion Relation
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

Direct Detection of GW

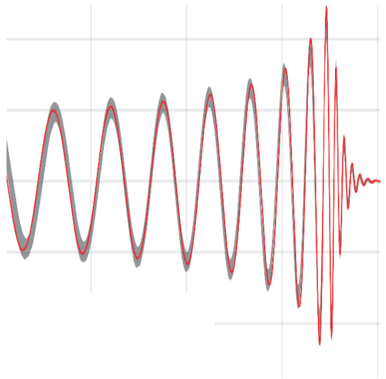


Constraints modifications of the dispersion relation

$$E^2 = \mathbf{k}^2 + m_g^2$$

Generic for the helicity-2 modes of any Lorentz invariant model of massive gravity

GW signal would be more squeezed than in GR



Speed increases with frequency

$$v_g/c \approx 1 - \frac{1}{2}(c/\Lambda_g f)^2$$

$$1 - \frac{v_g}{c} = 5 \times 10^{-17} \left(\frac{200\text{Mpc}}{D}\right)\left(\frac{\Delta t}{1\text{s}}\right)$$

$$m_g \lesssim 4 \times 10^{-22}\text{eV} \left(f\Delta t \frac{f}{100\text{Hz}} \frac{200\text{Mpc}}{D}\right)^{1/2}$$

For GW150914,

$D \sim 400\text{Mpc}, \; f \sim 100\text{Hz}, \; \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\text{eV}$

Will 1998
Abbott et al., 2016

Does we know all the constraints on graviton mass from aLIGO??

No! Many other effects to consider

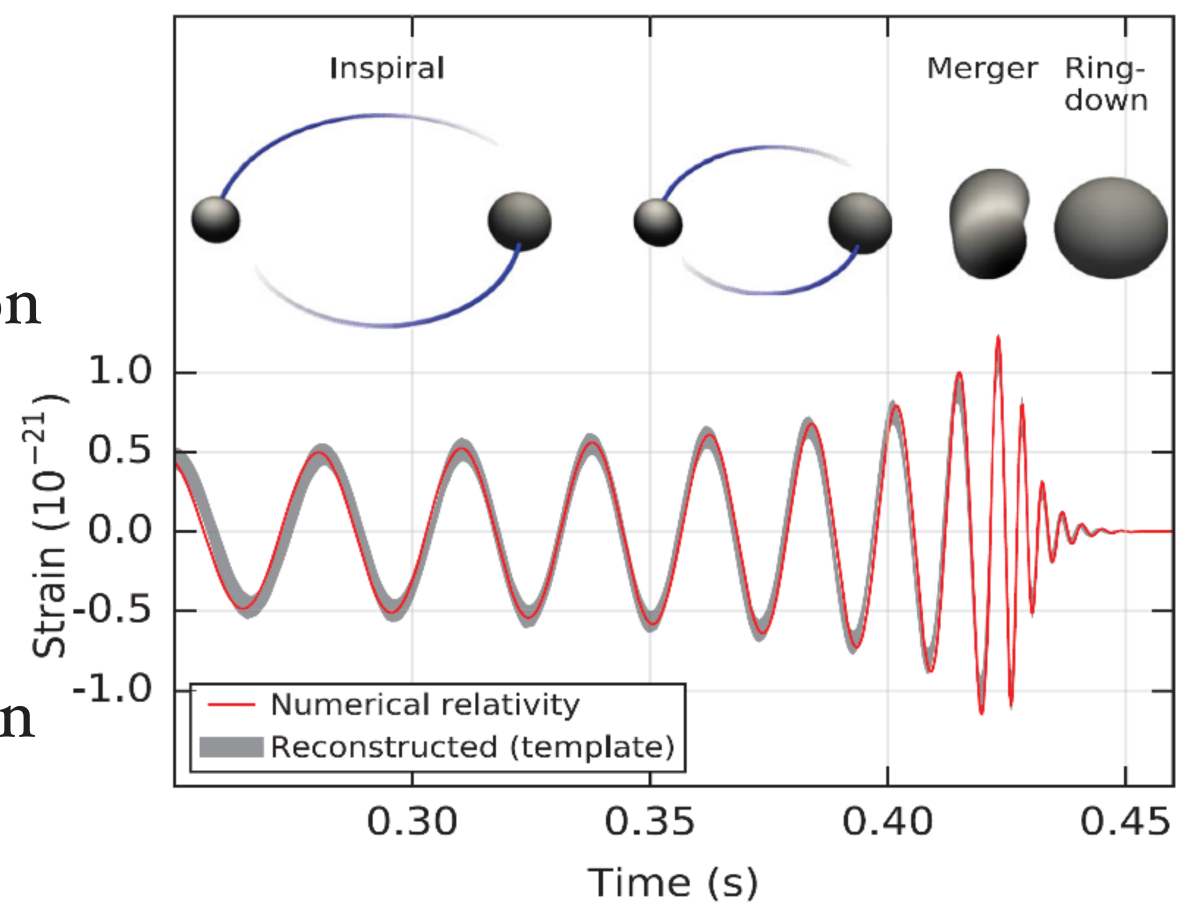
regime and, bound, for the first time several high-order post-Newtonian coefficients. We constrain the graviton Compton wavelength in a hypothetical theory of gravity in which the graviton is massive and place a 90%-confidence lower bound of 10^{13} km. Within our statistical uncertainties, we find no evidence for violations of

LIGO & VIRGO, PRL116, 221101 (2016)

$$m_{\text{graviton}} < 10^{-22}\text{eV}$$

GW150914

- Graviton Mass *depends on environment,* for instance it *depends on distance to black holes*
- Graviton Mass likely to vary non-adiabatically during merger creating additional non-adiabatic effects in the waveform
- Additional scalar (and vector) gravitational radiation. Scalar radiation may dominate effects on tensors.
- Black hole/NS solution modified, in particular quasi-normal modes may be different
- Vainshtein suppression may not be active in merger region - needs proper numerical simulation
- PN expansion almost certainly doesn't work in Vainshtein region



AJT Conjecture: Likely real constraints on LI MG are stronger!

What about Black hole solution, is horizon modified?

Many attempts to construct Black Hole solutions of massive (bi) gravity have focused on special symmetric solutions many in non-standard branches.

Babichev, Brito, Volkov, Comelli, Pilo… many more

There should be a solution with

Yukawa asymptotics!

= Schwarzschild as $m \to 0$

Nonsingular Black Holes in Massive Gravity: Time-Dependent Solutions

Rachel A. Rosen

coordinate-invariant singularities at the horizon. In this work we investigate the possibility of black hole solutions which can accommodate both a nonsingular horizon and Yukawa asymptotics. In particular, by adopting a time-dependent ansatz, we derive perturbative analytic solutions which possess nonsingular horizons. These black hole solutions are indistinguishable from Schwarzschild black holes in the massless limit. At finite mass, they depend explicitly on time. However, we demonstrate that the location of the apparent horizon is not necessarily time-dependent, indicating that these black holes are not necessarily accreting or evaporating (classically). In deriving these

Black Hole Mechanics for Massive Gravitons

Rachel A. Rosen[1]

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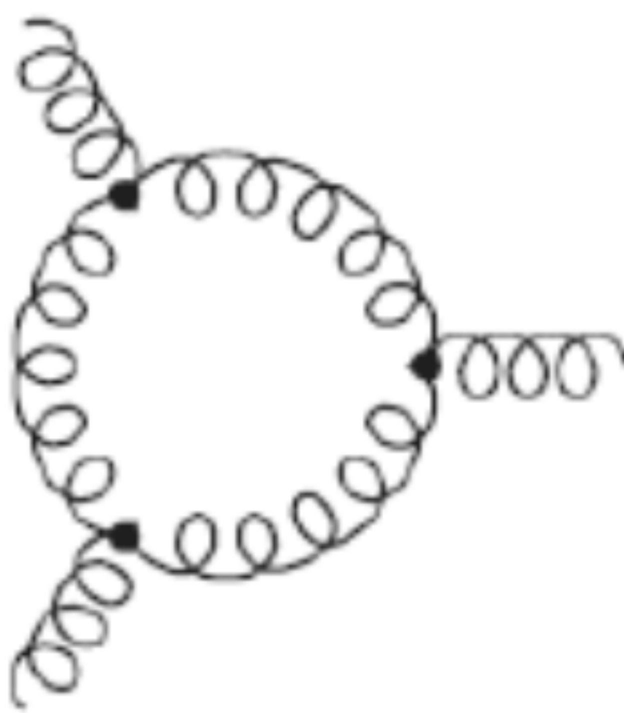
It has been argued that black hole solutions become unavoidably time-dependent when the graviton has a mass. In this work we show that, if the apparent horizon of the black hole is a null surface with respect to a fiducial Minkowski reference metric, then the location of the horizon is necessarily time-*independent*, despite the dynamical metric possessing no time-like Killing vector. This result is non-perturbative and model-independent. We derive a second law of black hole mechanics for these black holes and determine their surface gravity. An additional assumption establishes a zeroth and a first law of black hole mechanics. We apply these results to the specific model of dRGT ghost-free massive gravity and show that consistent solutions exist which obey the required assumptions. We determine the time-dependent scalar curvature at the horizon of these black holes.

Massive Gravity as an EFT

Ghost free massive gravity, bigravity and multigravity are Effective Field Theories (EFT), which breaks down at the scale $\Lambda_3 = (m^2 M_{\text{Planck}})^{1/3}$

Generic one-loop Graviton diagram needs counter-terms at the scale (principally due to helicity zero mode interactions)

$$\Lambda_3 = (m^2 M_{\text{Planck}})^{1/3}$$



Counter-terms which are not needed in GR!

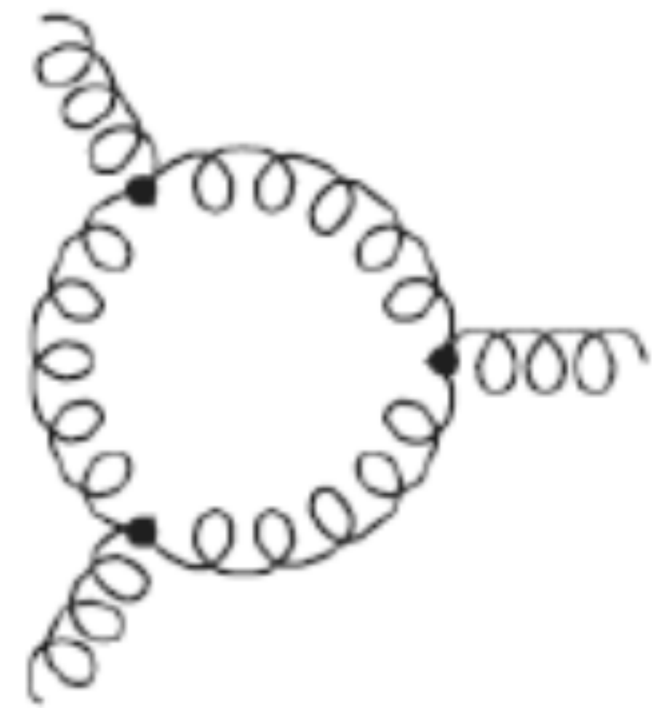
Vainshtein radius LARGER than Schwarzschild radius

Massive Gravity as an EFT

One-loop Graviton diagram needs counter-terms at the scale

$$\Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$

$$K = 1 - \sqrt{g^{-1} f}$$



In decoupling limit: $M_{\rm PLanck} \to \infty, m \to 0$

$$K_{\mu\nu} \to \frac{\partial_\mu \partial_\nu \pi}{\Lambda_3^3}$$

EFT corrections then take the form

(even away from the decoupling limit)

de Rham, Melville, Tolley 2017

$$\Lambda^4 L_0 = \left[\frac{M^2}{2} R - \Lambda^3 M \sum_n \alpha_n \mathcal{E}\mathcal{E} g^{4-n} K^n\right] + \Lambda^4 \sum \beta_{p,q,r} \left(\frac{\nabla}{\Lambda}\right)^p K^q_{\mu\nu} \left(\frac{R_{\mu\nu\rho\sigma}}{\Lambda^2}\right)^r$$

Infinite number of derivative suppressed operators

Existential Crisis of MG, does a UV completion exist?

Can I describe theories of massive gravity/multi-gravity at energy scales higher than Λ_3 ?

Is there a UV completion?

Is there a Lorentz Invariant Higgs mechanism for gravity?

If not, what do we give up? Lorentz invariance? Locality?

Part of a larger question:

Are all EFTs allowed?

aka Swampland!

With typical assumption that:
UV completion is Local, Causal, Poincare Invariant and Unitary

Answer: NO! Certain low energy effective theories do not admit well defined UV completions

Recent Recognition: Positivity Bounds!

'Older' work by Adams et al 2006, recent related work by Cheung, Remmen, Hinterbichler, Rosen, Joyce, Bonifacio, de Rham, Tolley, Melville, Zhou 2016/2017/2018

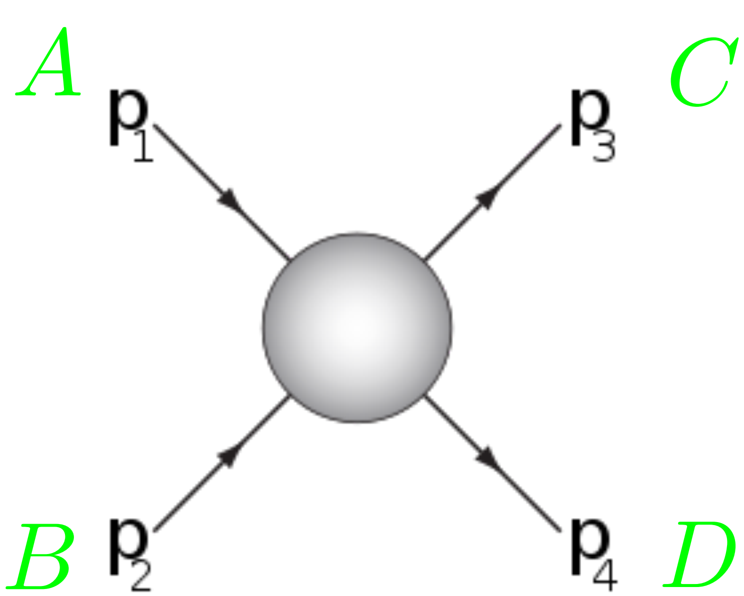
1960's S-matrix assumptions

$|A(k)| < \alpha e^{\beta |k|}$

1. Unitarity $S^\dagger S = 1$
2. Locality: Scattering Amplitude Polynomially (Exponentially) Bounded
3. Causality: Analytic Function of Mandelstam variables (modulo poles+cuts)
4. Poincare Invariance
5. Crossing Symmetry: Follows from above assumptions
6. Mass Gap: Existence of Mandelstam Triangle and Validity of Froissart Bound

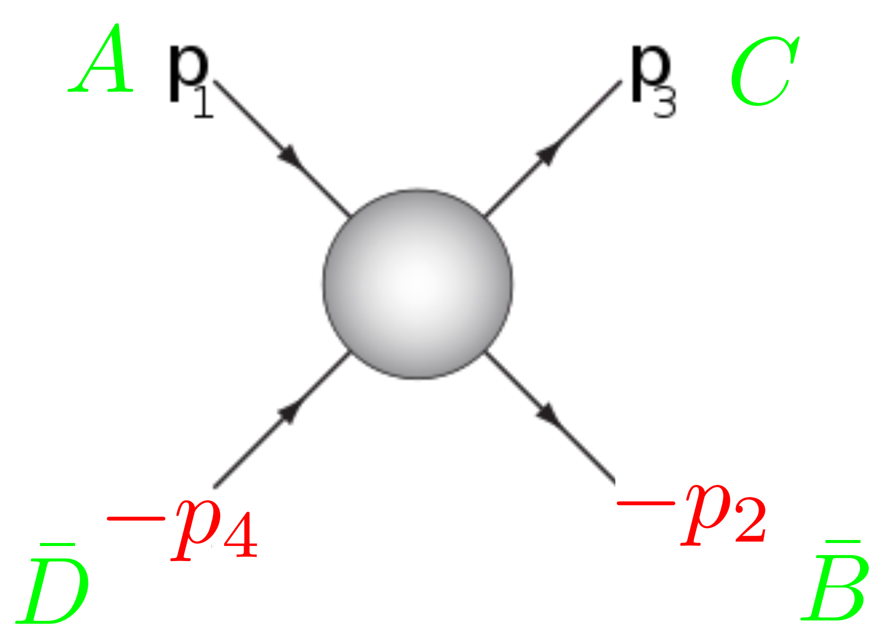
s-channel

$$A + B \to C + D$$



u-channel

$$A + \bar{D} \to C + \bar{B}$$



$$s + t + u = 4m^2$$

$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

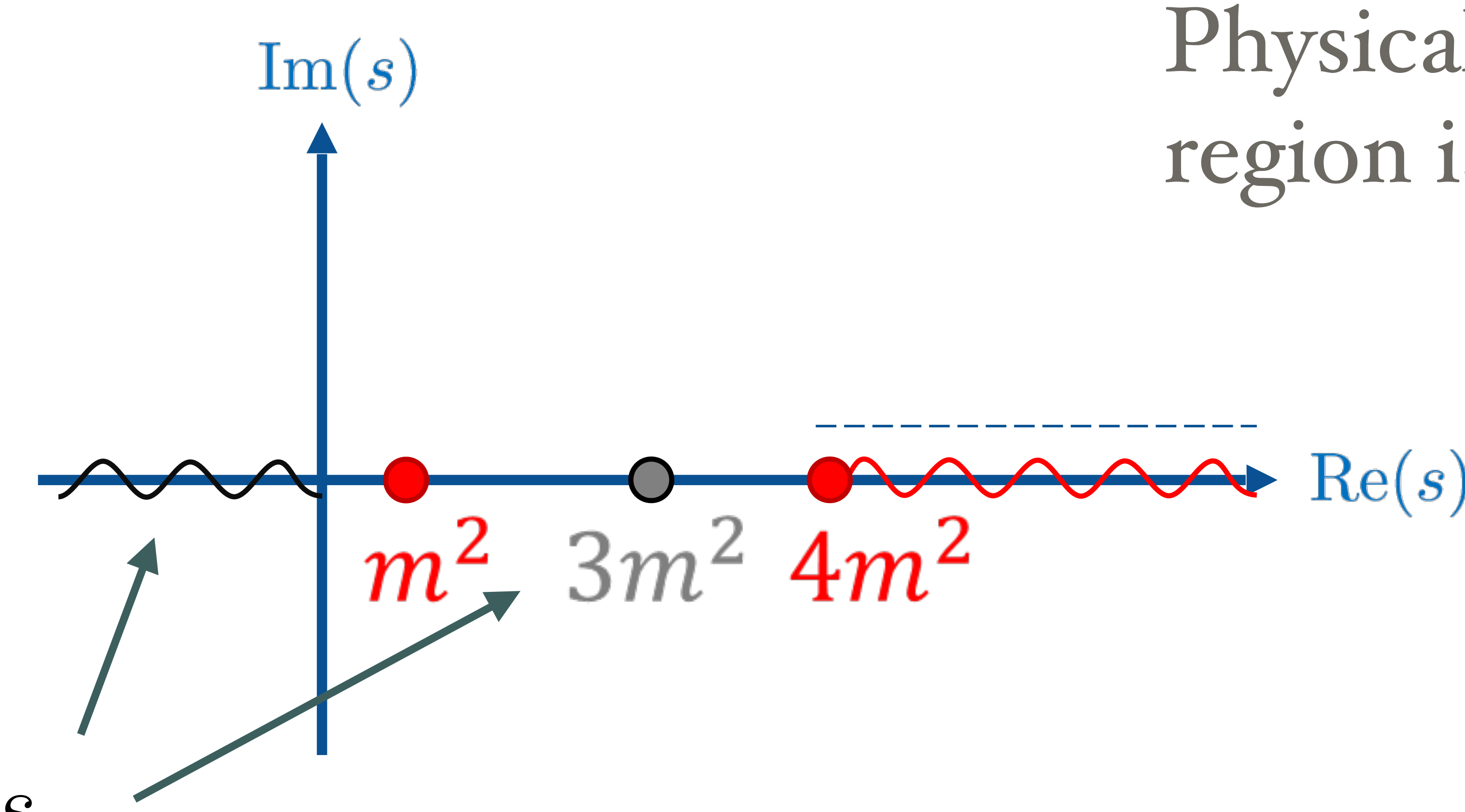
$$s \leftrightarrow u$$

Forward Scattering Limit Dispersion relation

$t = 0$

Complex s plane

Physical scattering region is $s \geq 4m^2$



crossing:

$u = 4m^2 - s$

$$\mathcal{A}_s(s,0) = \frac{\lambda_s}{m^2 - s} + \frac{\lambda_u}{m^2 - u} + (a + bs) + s^2 \int_{4m^2}^{\infty} \frac{\rho_s(\mu)}{\mu^2(\mu - s)} + u^2 \int_{4m^2}^{\infty} \frac{\rho_u(\mu)}{\mu^2(\mu - u)}$$

Positivity/Unitarity

$$\rho(s) = \frac{1}{\pi} Im[A(s,0)] = \frac{\sqrt{s(s-4m^2)}\sigma(s)}{\pi} > 0$$

No. of subtractions =2

$$\sigma(s) < \frac{c}{m^2}(\log(s/s_0))^2$$

Forward Limit Positivity Bounds

Recipe: Subtract pole, differentiate to remove subtraction constants

$$\mathcal{A}'_s(s,t) = A_s(s,t) - \frac{\lambda_s}{m^2 - s} - \frac{\lambda_u}{m^2 - u}$$

$$\frac{1}{M!}\frac{d^M}{ds^M}\mathcal{A}'_s(2m^2,0) = \int_{4m^2}^{\infty} \frac{\rho_s(\mu)}{(\mu - 2m^2)^{M+1}} + \int_{4m^2}^{\infty} \frac{\rho_u(\mu)}{(\mu - 2m^2)^{M+1}} > 0$$

RH Cut — LH Cut

$M \geq 2$

Adams et. al. 2006

Assume Weak Coupling

$$\frac{1}{M!}\frac{d^M}{ds^M}\mathcal{A}'^{\text{tree}}_s(2m^2,0) = \int_{\Lambda^2}^{\infty} \frac{\rho_s^{\text{tree}}(\mu)}{(\mu - 2m^2)^{M+1}} + \int_{\Lambda^2}^{\infty} \frac{\rho_u^{\text{tree}}(\mu)}{(\mu - 2m^2)^{M+1}} > 0$$

Directly translates into constraints on Wilsonian action

Extension away from forward scattering limit

de Rham, Melville, AJT, Zhou 1702.06134

$$\mathcal{A}(s,t) = 16\pi\sqrt{\frac{s}{s-4m^2}}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)a_\ell(s)$$

Unitarity

$$\mathrm{Im}\, a_\ell(s) = |a_\ell(s)|^2 + \cdots$$

$$\mathrm{Im}\, a_l(s) > 0\,, \quad s \geq 4m^2$$

$$\frac{d^n}{dt^n}\mathrm{Im} A(s,t)\Big|_{t=0} > 0$$

using $\frac{d^n}{dx^n}P_l(x)\Big|_{x=1} > 0$

$$\mathrm{Im}\, A(s,t) > 0\,, \quad 0 \leq t < 4m^2\,,\ s \geq 4m^2$$

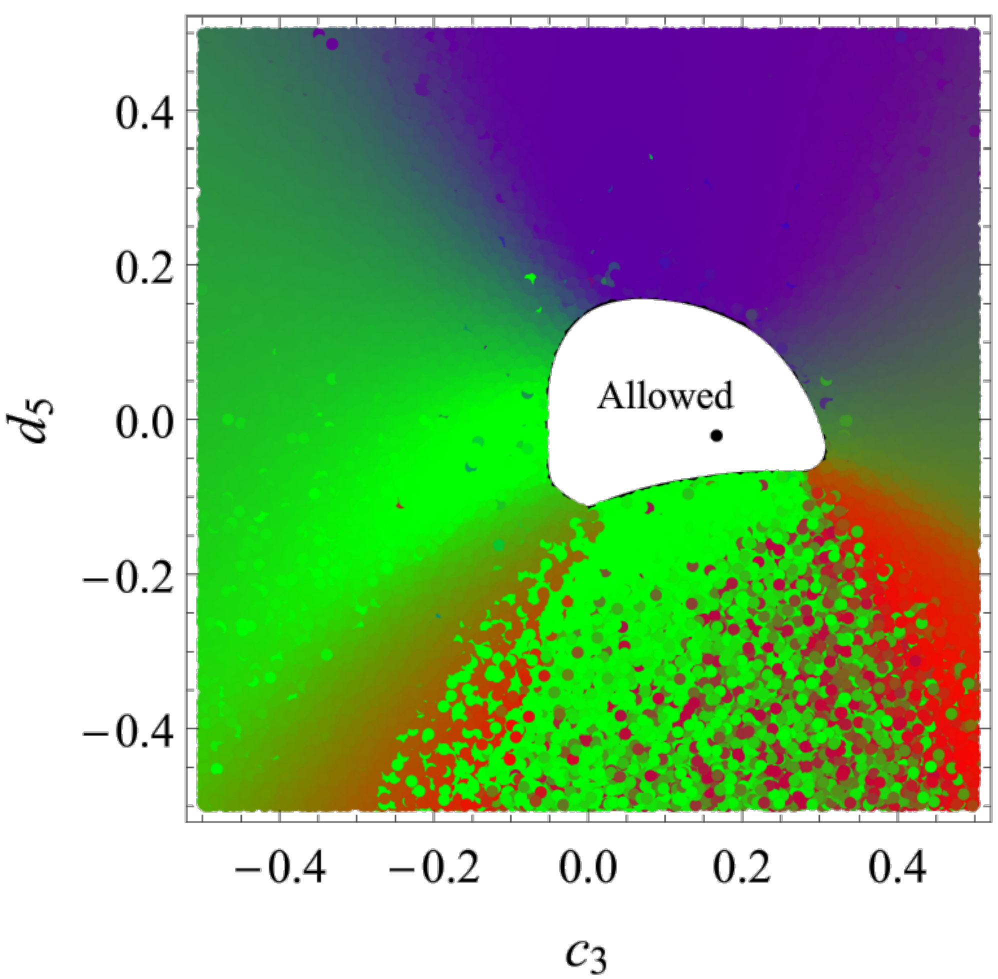
$$M \geq 2$$

$$\frac{1}{M!}\frac{d^M}{ds^M}\mathcal{A}'_s(2m^2,t) = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{ImA_s(\mu,t)}{(\mu-2m^2+t/2)^{M+1}} + \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{ImA_u(\mu,t)}{(\mu-2m^2+t/2)^{M+1}} > 0$$

What about general spins, e.g. spin 2 = massive gravity?

In forward limit, dispersion relation holds for helicity amplitudes

$A_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,0)$ has dispersion relation with 2 subtractions



Helicity: $\frac{\mathbf{J}\cdot\mathbf{p}}{|\mathbf{p}|}|\mathbf{p},S,\lambda\rangle = \lambda|\mathbf{p},S,\lambda\rangle$

Also applies to INDEFINITE helicity

Cheung Remmen 2016 have used this to place constrains on the mass parameters in massive gravity

Cheung & Remmen (2016)

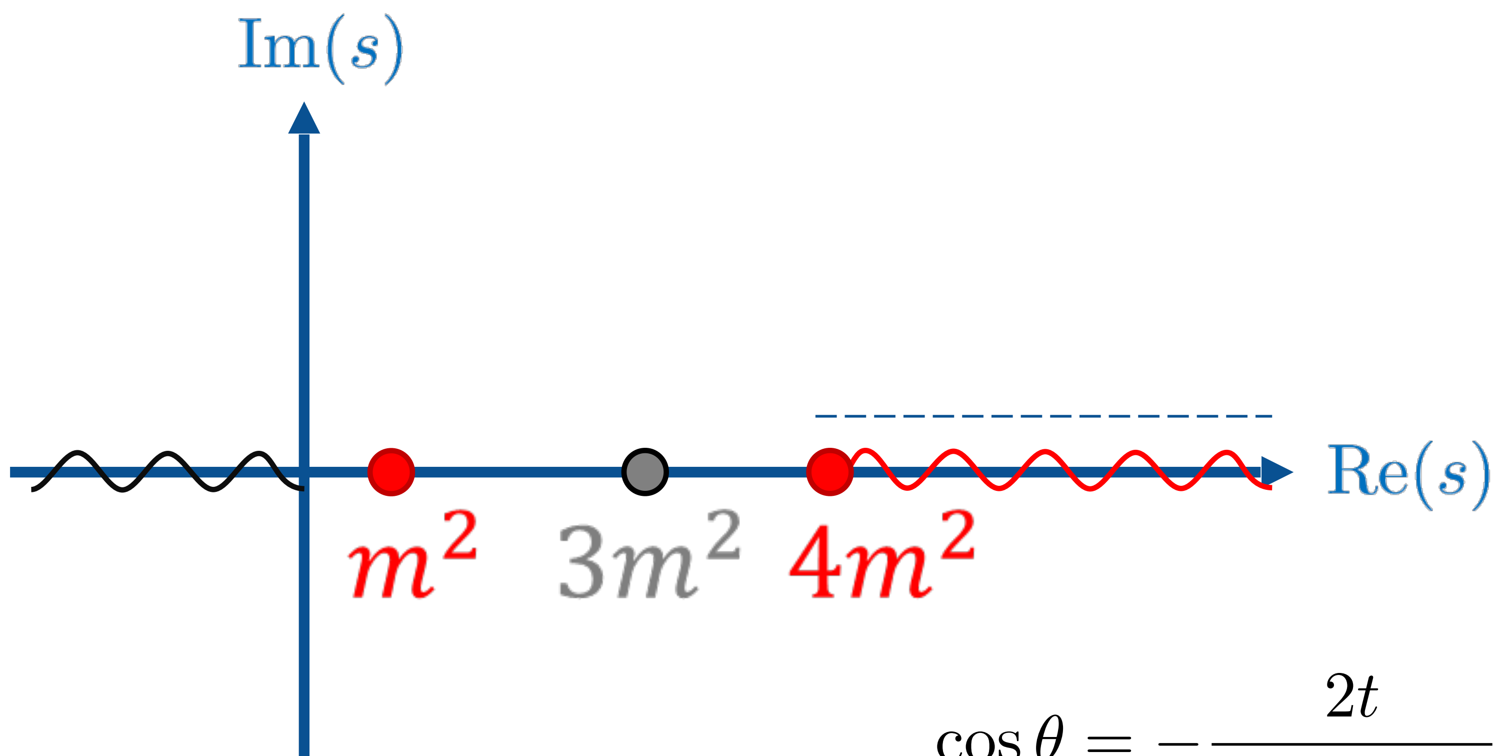
And for spin -1 Proca field, see Bonifacio, Hinterbichler & Rosen (2016)

both in the forward scattering limit

see also Bellazzini 2017

Analyticity for Spins

In addition to usual scalar poles and branch cuts we have ....



$$\cos\theta = -\frac{2t}{(s-4m^2)}$$

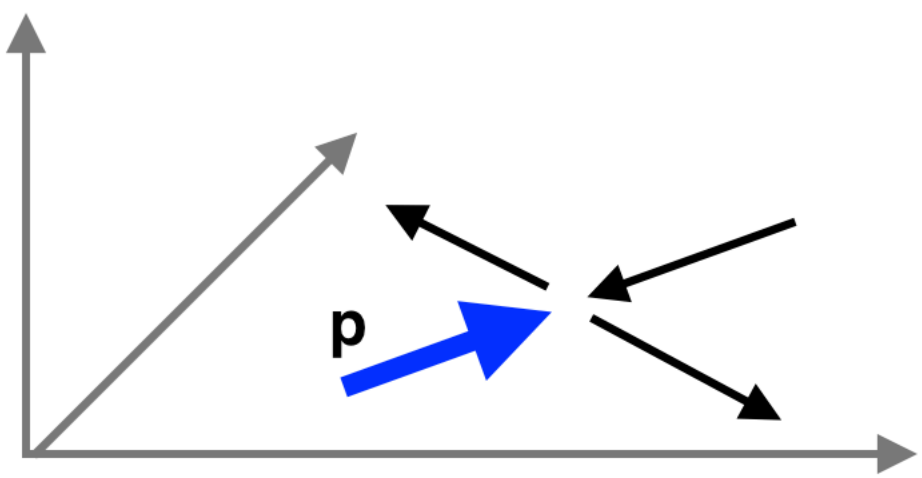
1. Kinematic (unphysical) poles at $s = 4m^2$
2. $\sqrt{stu}$ branch cuts
3. For Boson-Fermion scattering $\sqrt{-su}$ branch cuts

Origin: non-analyticities of polarization vectors/spinors

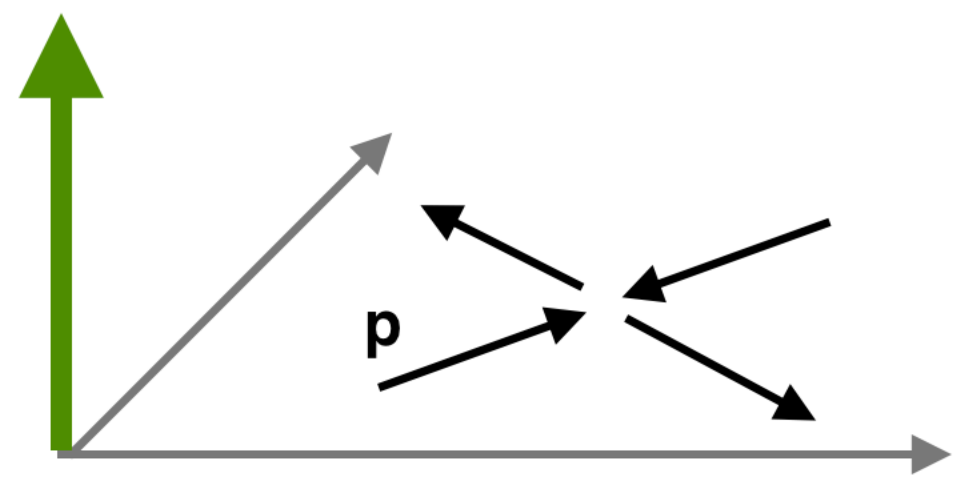
Transversitas, Transversitatum, et omnia Transversitas

Kotanski, 1965

Helicity



Transversity



$$\mathcal{T}_{\tau_1\tau_2\tau_3\tau_4} = \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} u^{S_1}_{\lambda_1\tau_1} u^{S_2}_{\lambda_2\tau_2} u^{S_1*}_{\tau_3\lambda_3} u^{S_2*}_{\tau_4\lambda_4} \mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}$$

Change of Basis $u^S_{\lambda\tau} = \langle S, \lambda | e^{-i\frac{\pi}{2}\hat{J}_z} e^{-i\frac{\pi}{2}\hat{J}_y} e^{i\frac{\pi}{2}\hat{J}_z} | S, \tau\rangle$

$$T^s_{\tau_1\tau_2\tau_3\tau_4}(s,t,u) = e^{-i\sum_i \tau_i \chi} T^u_{-\tau_1-\tau_4-\tau_3-\tau_2}(u,t,s)$$

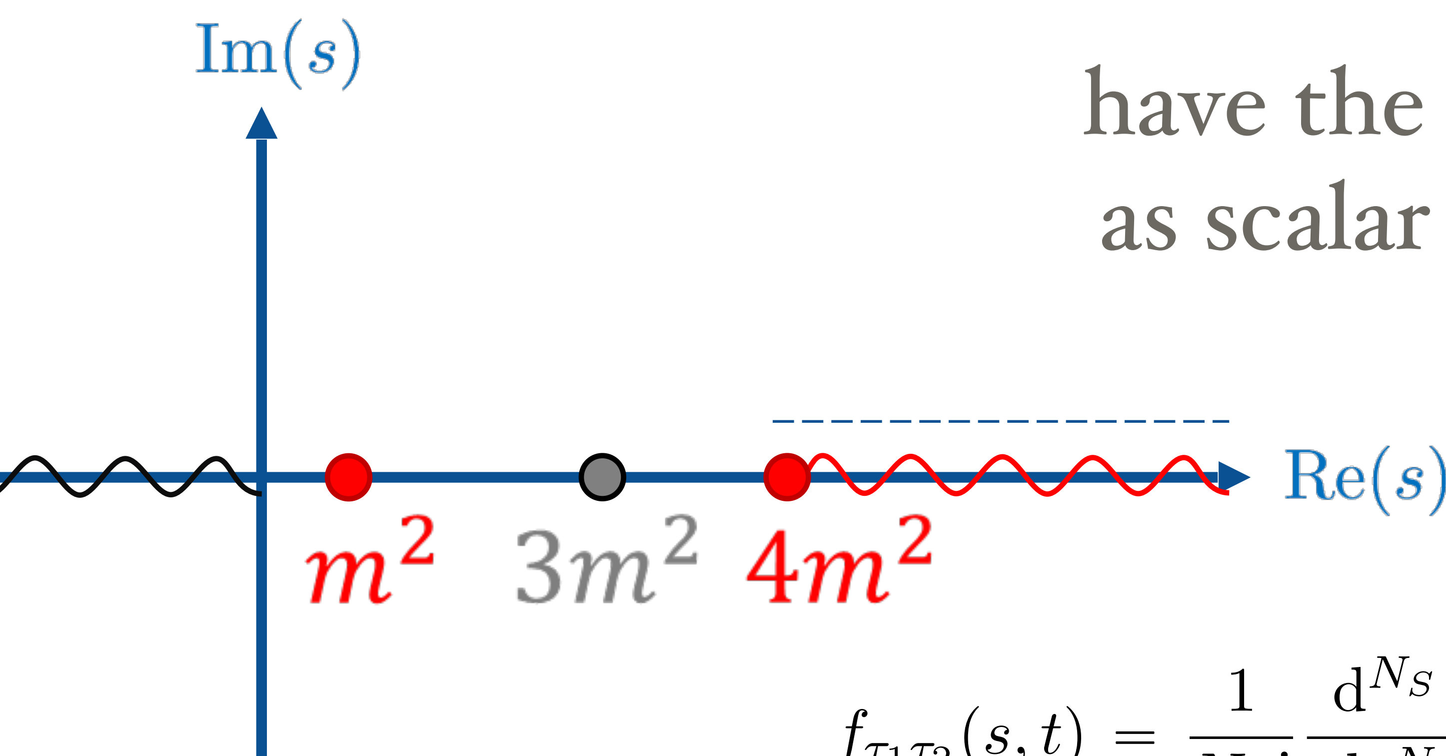
Crossing is Simple!!

Dispersion Relation with Positivity along BOTH cuts

de Rham, Melville, AJT, Zhou 1706.02712

Punch line: The specific combinations:

$$\mathcal{T}^{+}_{\tau_1\tau_2\tau_3\tau_4}(s,\theta) = \left(\sqrt{-su}\right)^{\xi}\mathcal{S}^{S_1+S_2}\left(\mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}(s,\theta) + \mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}(s,-\theta)\right)$$



have the same analyticity structure as scalar scattering amplitudes!!!!!!!

Implies Dispersion Relation

$$f_{\tau_1\tau_2}(s,t) = \frac{1}{N_S!}\frac{\mathrm{d}^{N_S}}{\mathrm{d}s^{N_S}}\tilde{\mathcal{T}}^{+}_{\tau_1\tau_2\tau_1\tau_2}(s,t)$$

$$f_{\tau_1\tau_2}(v,t) = \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}\mu\frac{\mathrm{Abs}_s\mathcal{T}^{+}_{\tau_1\tau_2\tau_1\tau_2}(\mu,t)}{(\mu-2m^2+t/2-v)^{N_S+1}} + \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}\mu\frac{\mathrm{Abs}_u\mathcal{T}^{+}_{\tau_1\tau_2\tau_1\tau_2}(4m^2-t-\mu,t)}{(\mu-2m^2+t/2+v)^{N_S+1}}$$

Application to Massive Gravity

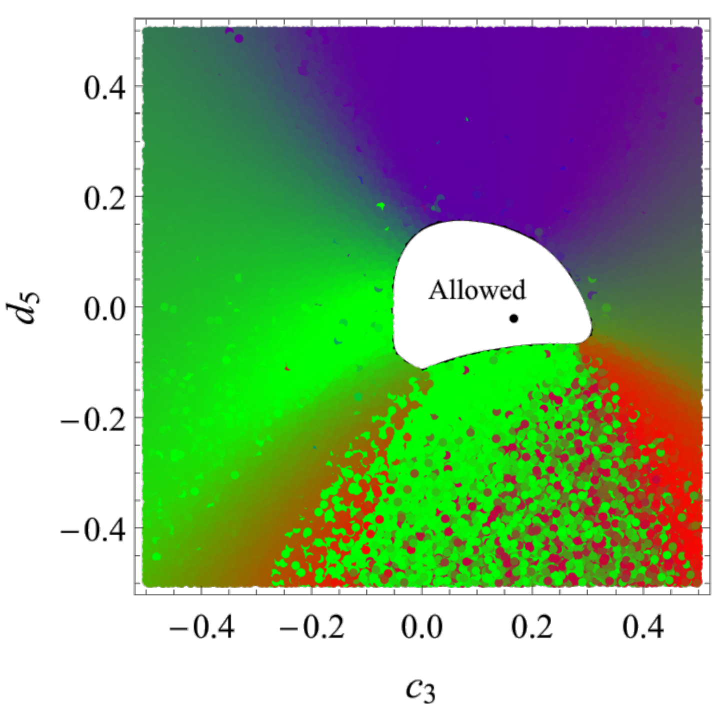
Unitary Gauge Massive Gravity

Einstein-Hilbert   Mass Term

$$\mathcal{L} \supset \frac{M_{\rm Pl}^2}{2}\left(R\left[g\right] - \frac{m^2}{4}V(g,h)\right)$$

Parameterize generic mass term (without dRGT tuning) as

$$V(g,h) \supset [h^2] - [h]^2 + (c_1 - 2)[h^3] + (c_2 + \frac{5}{2})[h^2][h] + (d_1 + 3 - 3c_1)[h^4] + (d_3 - \frac{5}{4} - c_2)[h^2]^2 + \dots .$$



where $[h] = \eta^{\mu\nu}h_{\mu\nu},\ [h^2] = \eta^{\mu\nu}h_{\mu\alpha}\eta^{\alpha\beta}h_{\beta\nu},$

$$d_3 = -d_1/2 + 3/32 + \Delta d, \quad c_2 = -3c_1/2 + 1/4 + \Delta c$$

Application to Massive Gravity

Forward Limit

$$2M_{\rm Pl}^2 m^6 \frac{\partial^2}{\partial v^2} f_{\alpha\beta}|_{t=0} = \frac{352}{9}|\alpha_S\beta_S|^2\left(\Delta c\left(-6+9c_1-4\Delta c\right)-6\Delta d\right) + \frac{176}{3}\alpha_S^*\beta_S^*(\alpha_{V_1}\beta_{V_1}-\alpha_{V_2}\beta_{V_2})\,\Delta c\left(3-3c_1+4\Delta c\right)$$

Positivity for general helicity implies: $\Delta c = 0$

Beyond forward

$$\frac{\partial}{\partial t} f_{\tau_1\tau_2}(v,t) \;\propto\; \frac{v}{\Lambda_5^{10}}\Delta d + \mathcal{O}\left(\frac{m^2}{\Lambda_5^{10}}\right) \;>\; 0$$

$$\Delta d = 0$$

These are precisely the tunings that raise the cutoff from

$$\Lambda_5 = (m^4 M_{\rm Planck})^{1/5} \longrightarrow \Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$

Raising the Cutoff- the Third Way?

$$\Delta S \sim \frac{1}{M_{\text{Planck}}^2} \int \frac{d^4 k}{(2\pi)^4} T^{\mu\nu}(k) \left[\sum_{\text{pole}} Z_{\text{pole}} \frac{P_{\mu\nu\alpha\beta}(k)}{k^2 + m_{\text{pole}}^2} + \int_0^\infty d\mu \frac{\rho(\mu) P_{\mu\nu\alpha\beta}(k)}{k^2 + \mu} \right] T^{\alpha\beta}(k)$$

Hard

Hard and Soft

Soft

No vDVZ discontinuity on AdS

Its an old result, that on AdS you can take the massless limit of massive gravity and recover GR plus a decoupled sector

= NO vDVZ discontinuity!

$$K^{\mu}_{\alpha} K^{\alpha}_{\nu} = \delta^{\mu}_{\mu} - g^{\mu\alpha} f_{ab}(\phi) \partial_\alpha \phi^a \partial_\nu \phi^b$$

$$\mathcal{L} = \frac{1}{2} M^{d-2}_{\text{Planck}} R - \frac{1}{2} m^2 M^{d-2}_{\text{Planck}} (K^2_{\mu\nu} - K^2)$$

$\frac{d(d-3)}{2}$ d.o.f. $(d-2)+1$ d.o.f.

On AdS $f_{ab}(\phi) = \frac{L^2}{\phi_d^2} \eta_{ab}$ we can take

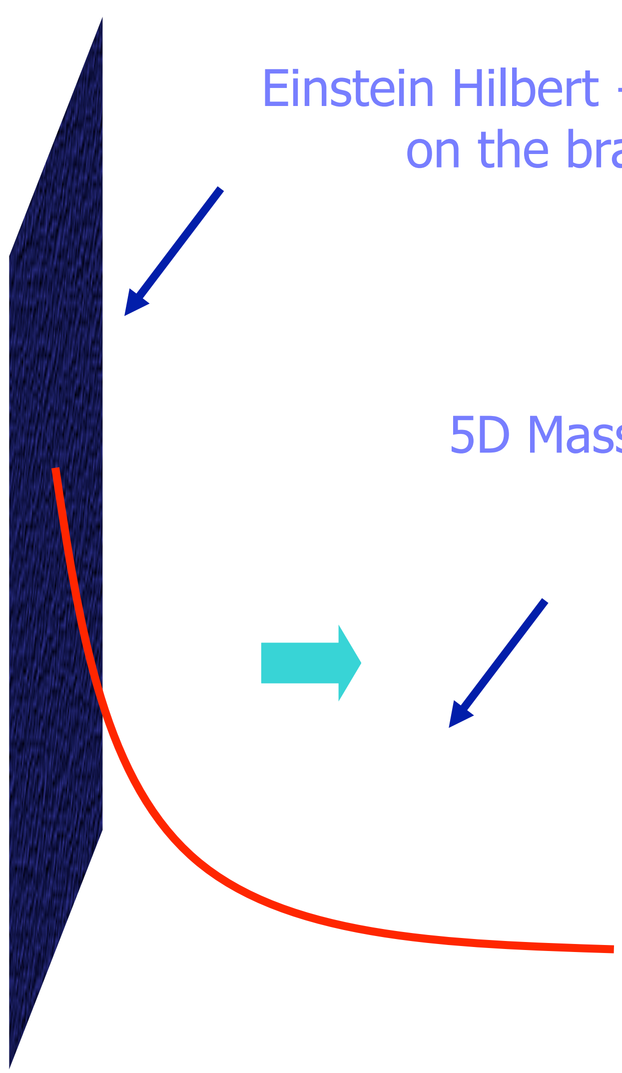
$M_{\text{Planck}} \to \infty$ $\Lambda = (m^2 M^{d-2}_{\text{Planck}})^{1/d}$ fixed

$\Lambda_2 \gg \Lambda_3$ Only Problem: We don't live in AdS!!!!!!

Warped Massive Gravity

Gabadadze 2017

Solution: Do AdS Massive gravity in 5 dimensions, with our universe localized on a 3+1 brane



$$\mathcal{L}_{\rm brane} = \frac{1}{2} M_4^2 R_4 - \frac{\alpha}{2} m^2 M_4^2 (k_{\mu\nu}^2 - k^2)$$

$$f_{ab}(\phi) = \frac{L^2}{\phi_5^2} \eta_{ab}$$

$$\mathcal{L}_{\rm bulk} = \frac{1}{2} M_5^3 R_5 - \frac{1}{2} m^2 M_{\rm Planck}^3 (K_{\mu\nu}^2 - K^2)$$

Soft and Hard (nonlocal) massive gravity

Gabadadze 2017

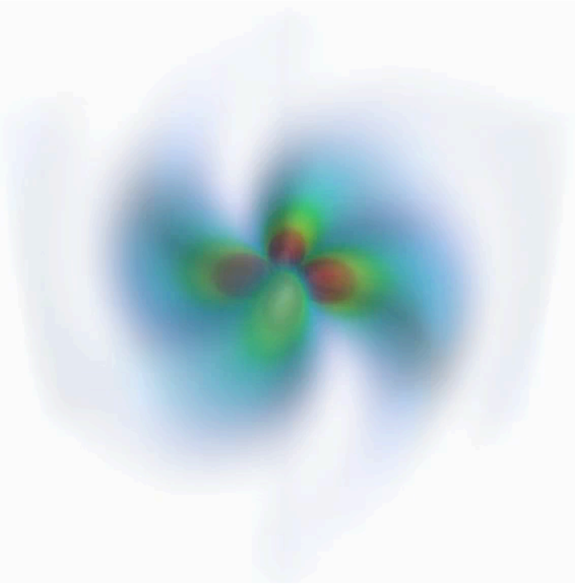
Cutoff is raise to

$$\Lambda_2 = (m^2 M_4^2)^{1/4} \sim \frac{1}{L} \sim (M_5^3 m^2)^{1/5} \gg \Lambda_3$$

This is achieved because of a continuum/resonance of soft gravitons whose masses are smaller than usual hard mass graviton

Result: Low energy effective theory is more non-local (although full theory is completely local)

Summary



Massive Gravity theories come in several types:

Soft and Hard

Vainshtein mechanism works for static sources and time-dependent like binary pulsars

- time-dependence can suppress screening

Full understanding of dynamics, e.g. even for black hole solutions is far from understood due to necessary time dependence of the additional degrees of freedom, however some progress being made ...

Recent Recognition: S-matrix Positivity Bounds applied to massive gravity automatically raise the cutoff from

$$\Lambda_5 = (m^4 M_{\rm Planck})^{1/5} \longrightarrow \Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$

In the context of AdS braneworlds, cutoff of 4d theory can potentially be raised to $\Lambda_2 = (m^2 M_4^2)^{1/4}$ while maintaining Lorentz invariance