A symmetry breaking mechanism by parity assignment in the noncommutative Higgs model

Arxiv 1503.03888

Masaki J.S. Yang @ KEK

15/03/06 SCGT 15 @ Nagoya U.

A symmetry breaking mechanism by parity assignment in the noncommutative Higgs model

Abstract

SU(5) Orbifold GUT of "composite" gauge theory
in NCG (discretized extra dimension)

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Abstract

Y. Kawamura, PTP(2001) 999
L. Hall & Y. Nomura, PRD(2001)

SU(5) Orbifold GUT

SU(5)

$$\left(\begin{array}{c|c} G_\mu^a & X_\mu^a, Y_\mu^a \\ \hline X_\mu^a, Y_\mu^a & W_\mu^a \end{array}\right) \quad \left(\begin{array}{c} H^c \\ \hline H \end{array}\right)$$

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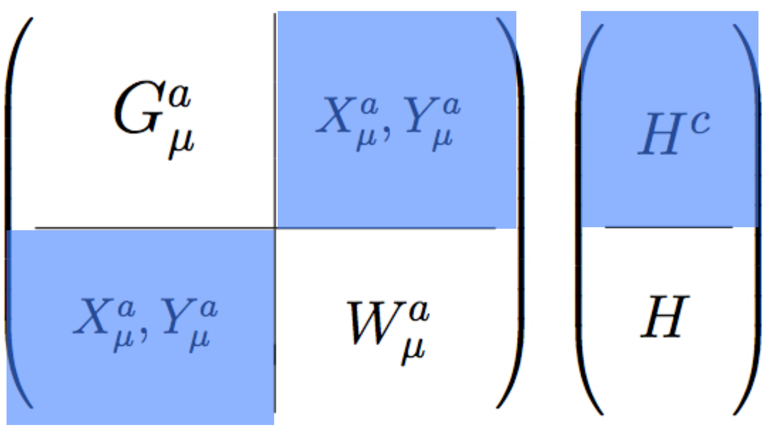
SU(5) Orbifold GUT

~~SU(5)~~ **massive**



$$P = \mathrm{diag}(2, 2, 2, -3, -3)$$

$$A^a_\mu(x, -y) = P A^a_\mu(x, y) P$$

$$H(x, -y) = P H(x, y)$$

Abstract

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A. Chamseddine, G. Felder and
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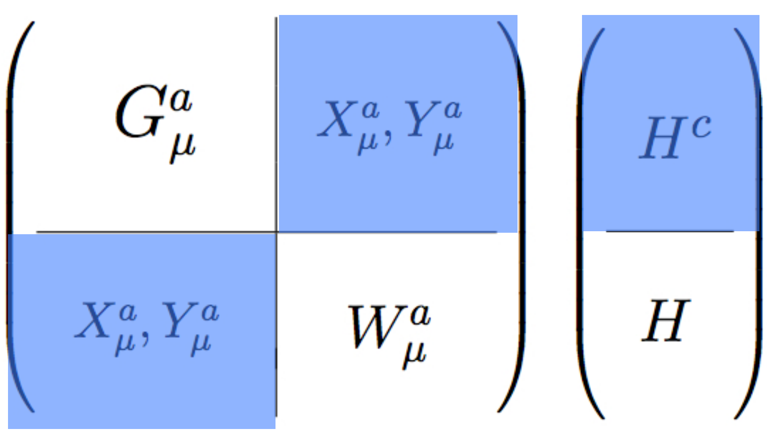

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NCG GUT

(NonCommutative Geometry)

- Discretized extra dimension

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$$P = \mathrm{diag}(2,2,2,-3,-3)$$

$$A^a_\mu(x,-y) = PA^a_\mu(x,y)P$$

$$H(x,-y) = PH(x,y)$$

NCG GUT

(NonCommutative Geometry)

- Discretized extra dimension
- Gauge and Higgs bosons are treated as "composites"

$$A(x,n) = \sum_i a_i^\dagger(x,n) d a_i(x,n)$$

$$H_{nm}(x) = \sum_i a_i^\dagger(x,n) M_{nm} a_i(x,m)$$

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Assignment for "preons"

$$a^i(x,-y) = P a^i(x,y) P$$

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Dependent condition

$$A^a_\mu(x,-y) = P A^a_\mu(x,y) P$$
$$H(x,-y) = P H(x,y)$$

$\Leftarrow$

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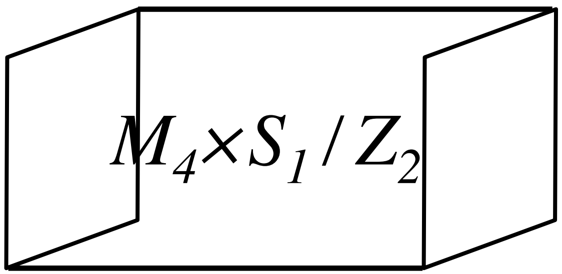
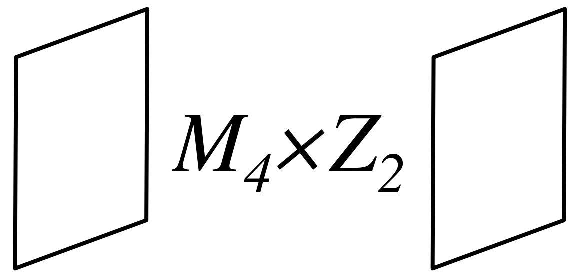
Contents

Background and Motivation

Continuous ex. dim.



Discrete (NCG) ex. dim.



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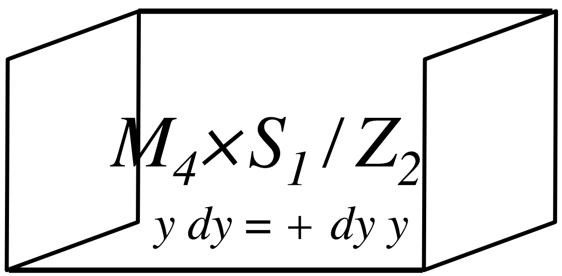

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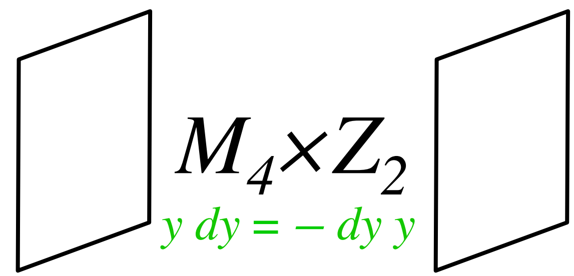

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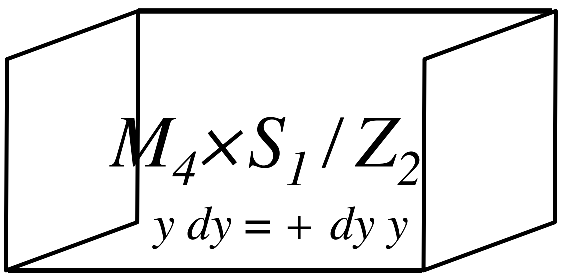

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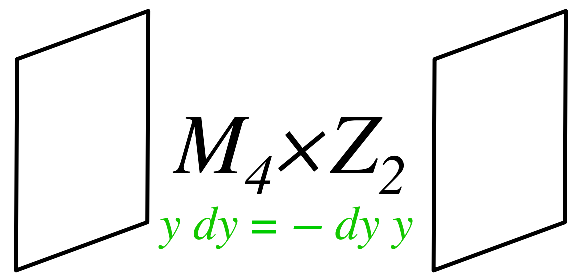

- Gauge-Higgs Unification
- Deconstruction
- Orbifold GUT
  - Family unification
- Holography (⇒CHM)
- Warped (RS) ex. dim.

Background and Motivation

Continuous ex. dim. | **Discrete (NCG) ex. dim.**

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Our results

Background and Motivation

Continuous ex. dim. | **Discrete (NCG) ex. dim.**

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- Gauge-Higgs Unification
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- Orbifold GUT — Our results
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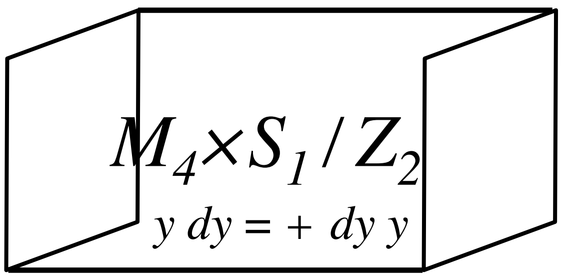
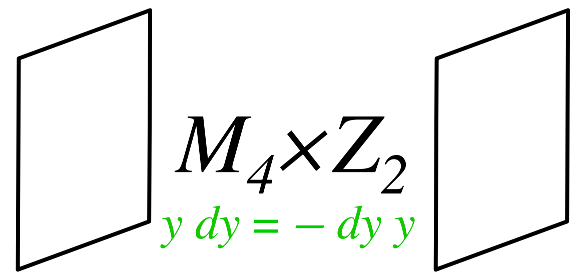
Background and Motivation

Continuous ex. dim.



Recast ⇒

Discrete (NCG) ex. dim.



- Gauge-Higgs Unification

$m_h = 0$ @ tree level
(by gauge symmetry)

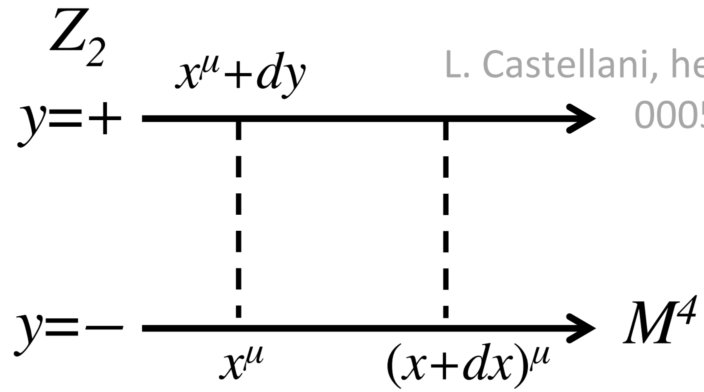
- Holography (⇒CHM)
- Warped (RS) ex. dim.

- Gauge-Higgs Unification

$m_h \neq 0$ @ tree level
(by non commutativity)

Z_2 Discrete geometry and algebra

Coordintes $(x^\mu, y = \pm)$



L. Castellani, hep-th 0005210

Algebra

$x^\mu x^\nu = x^\nu x^\mu, x^\mu y = y x^\mu, y^2 = 1.$

Z_2 Discrete geometry and algebra

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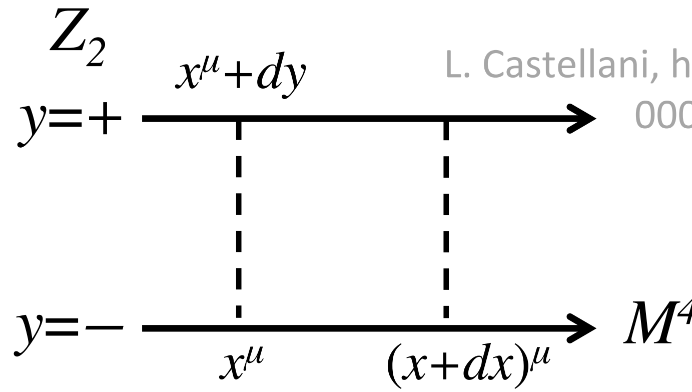

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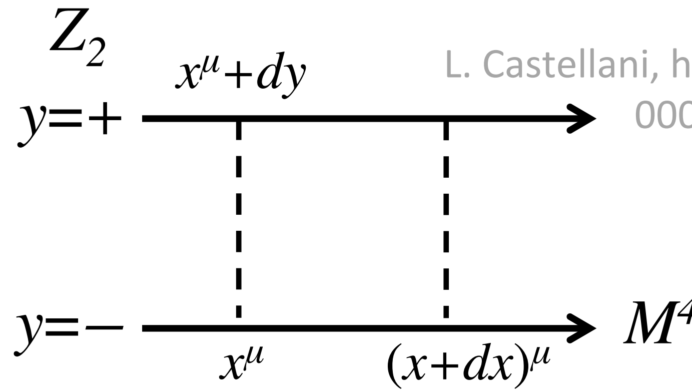
Algebra

$x^\mu x^\nu = x^\nu x^\mu,\ x^\mu y = y\, x^\mu,\ y^2 = 1\,.$

Functions (0-form) $f(x,y) = f_1(x)1 + f_y(x)y$

Z_2 Discrete geometry and algebra



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Functions (0-form) $\quad f(x,y) = f_1(x)1 + f_y(x)y$

Exterior derivative $\quad df = \partial_\mu f dx^\mu + \partial_\bullet f dy$

$$\partial_\bullet f(x,y) \equiv M[f(x,-) - f(x,+)] = yM[f(x,-y) - f(x,y)]$$

Z_2 Discrete geometry and algebra

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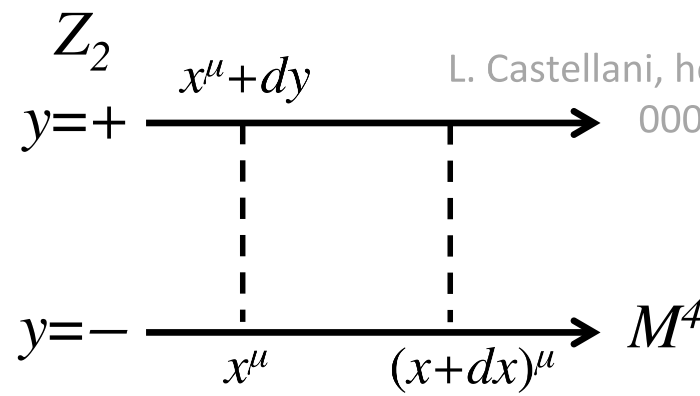

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$$M \sim 1/R$$

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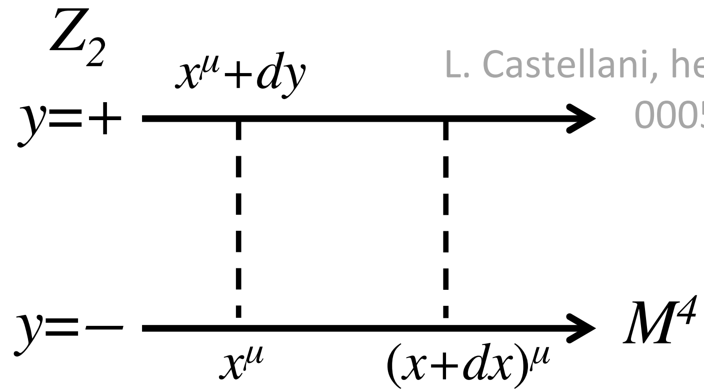

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Differential algebra

$$y\, dy = -\, dy\, y,\ \ f(y)\, dy = dy\, f(-y),\ dy \wedge dy = dy \wedge dy \neq 0$$

Z_2 Discrete geometry and algebra

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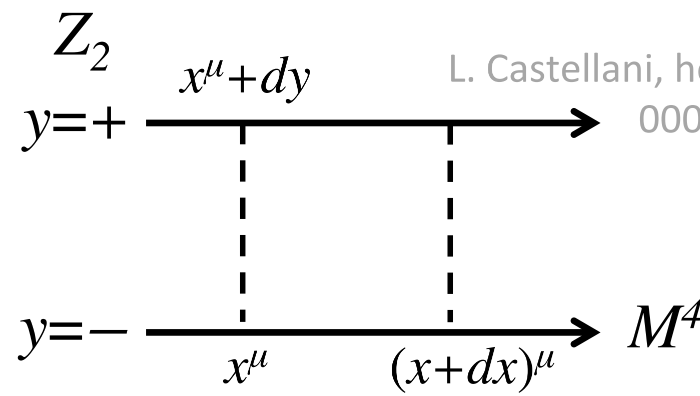

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Non-commutative,
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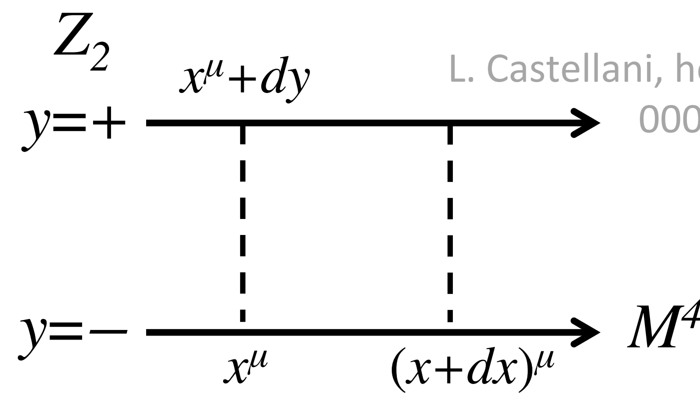

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$$y\, dy = -\, dy\, y,\; f(y)\, dy = dy\, f(-y),\; dy \wedge dy = dy \wedge dy \neq 0$$

Non-commutative,
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⇒ SSB and nonzero m_h.

Gauge theory on discrete space

L. Castellani, hep-th 0005210

Connection $A(x,y) = A_\mu(x,y)dx^\mu + A_\bullet(x,y)dy$ $(A_\bullet \sim \Phi)$

Gauge theory on discrete space

L. Castellani, hep-th 0005210

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The reality condition $A^\dagger = A$

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$$dx^\mu \, A_\mu^\dagger + dy \, A_\bullet^\dagger = A_\mu dx^\mu + A_\bullet dy,$$

$$A_\mu^\dagger(x,y) = A_\mu(x,y), \quad A_\bullet^\dagger(x,y) = A_\bullet(x,-y)$$

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$$\underbrace{A_\mu^\dagger(x,y) = A_\mu(x,y),}_{\text{real}} \quad \underbrace{A_\bullet^\dagger(x,y) = A_\bullet(x,-y)}_{\text{Complex !}}$$

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real　　Complex !

Gauge trf. $A' = -(dG)G^{-1} + GAG^{-1}$ where $G = G(x,y)$.

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$$(\partial_\mu + A_\mu)' = G(x,y)(\partial_\mu + A_\mu)G(x,y)^{-1},$$

$$\Phi(x,y)' \equiv (M + yA_\bullet)' = G(x,y)(M + yA_\bullet)G(x,-y)^{-1},$$

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Higgs field transforms as a bi -fundamental

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$\Rightarrow M^2\Phi^\dagger\Phi$ is not forbidden by gauge symmetry

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Higgs field transforms as a bi -fundamental

$\Rightarrow M^2\Phi^\dagger\Phi$ is not forbidden by gauge symmetry

This model does not solve the hierarchy problem.

Gauge theory on discrete space

L. Castellani, hep-th 0005210

Curvature $F = dA + A \wedge A$

$$= \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu + F_{\mu\bullet}\, dx^\mu \wedge dy + F_{\bullet\bullet}\, dy \wedge dy$$

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$$\mathscr{L} = \sum_{y=\pm} \frac{1}{g^2} F \wedge *F, \quad A \to gA$$

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$$\mathscr{L} = \frac{1}{4} F^{+}_{\mu\nu} F^{+\mu\nu} + \frac{1}{4} F^{-}_{\mu\nu} F^{-\mu\nu} + (D^\mu \Phi)^\dagger D_\mu \Phi + \frac{g^2}{2}\left(\left(\frac{\sqrt{2}M}{g}\right)^2 - \Phi^\dagger \Phi\right)^2,$$

Gauge theory on discrete space

L. C hep-th
05210

Curvature $F = dA + A\wedge A$

$$= \frac{1}{2}F_{\mu\nu}\,dx^\mu \wedge dx^\nu + F_{\mu\bullet}\,dx^\mu \wedge dy$$

$$\mathscr{L} = \sum_{y=\pm}\frac{1}{g^2}F\wedge *F, \quad A \to gA$$

The YMH Lagrangian !!

$$\mathscr{L} = \frac{1}{4}F^{+}_{\mu\nu}F^{+\mu\nu} + \frac{1}{4}F^{-}_{\mu\nu}F^{-\mu\nu} + (D^\mu\Phi)^\dagger D_\mu\Phi + \frac{g^2}{2}((\frac{\sqrt{2}M}{g})^2 - \Phi^\dagger\Phi)^2,$$

Gauge theory on discrete space

L. Castellani, hep-th 0005210

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Then

$$\boxed{v = \frac{2M}{g}, \quad \lambda = \frac{g^2}{2}, \quad m_h = \sqrt{2\lambda} v = gv.}$$

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Mass of the broken gauge boson

$$Z_\mu = \frac{A^+_\mu - A^-_\mu}{\sqrt{2}}$$

$$\boxed{m_Z = \sqrt{2}\, gv = \sqrt{2}\, m_h.}$$

The mass relation (@ tree level) !

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$$\ldots \Phi)^\dagger D_\mu \Phi + \frac{g^2}{2}\left(\left(\frac{\sqrt{2}M}{g}\right)^2 - \Phi^\dagger\Phi\right)^2,$$

We can just interpret Z_μ is the 1st KK excitation of A_μ.

Then

$$\ldots \frac{}{g}, \quad \lambda = \frac{g^2}{2}, \quad m_h = \sqrt{2\lambda}v = gv.$$

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$$Z_\mu = \frac{A_\mu^+ - A_\mu^-}{\sqrt{2}}$$

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The mass relation (@ tree level) !

Gauge theory on discrete space

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Curvature $F = dA + A \wedge A$

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$$\mathscr{L} = \sum_{y=\pm} \frac{1}{g^2} F \wedge *F, \quad A$$

We can just interpret Z_μ is the 1[st] KK excitation of A_μ.

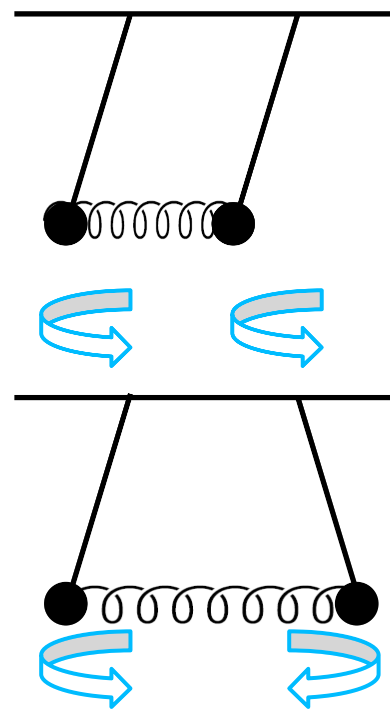
Then

$$\frac{}{g}, \quad \lambda =$$

Mass of the broken gauge boson $Z_\mu = \frac{A_\mu^+ - A_\mu^-}{\sqrt{2}}$

Physical origin of SSB

Ex) Coupled harmonic oscillators



0 mode

$$A_\mu = \frac{A_\mu^+ + A_\mu^-}{\sqrt{2}}$$

1 mode

$$Z_\mu = \frac{A_\mu^+ - A_\mu^-}{\sqrt{2}}$$

The mass relation (@ tree level) !

Gauge theory on discrete space

L. Castellani, hep-th 0005210

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$$\mathscr{L} = \frac{1}{4} F^+_{\mu\nu} F^{+\mu\nu} + \frac{1}{4} F^-_{\mu\nu} F^{-\mu\nu} + (D^\mu \Phi)^\dagger D_\mu \Phi + \frac{g^2}{2} ((\frac{\sqrt{2}M}{g})^2 - \Phi^\dagger \Phi)^2,$$

Then

$$v = \frac{2M}{g}, \quad \lambda = \frac{g^2}{2}, \quad m_h = \sqrt{2\lambda} v = gv.$$

Mass of the broken gauge boson $Z_\mu = \frac{A^+_\mu - A^-_\mu}{\sqrt{2}}$

$$m_Z = \sqrt{2} gv = \sqrt{2} m_h.$$

The mass relation (@ tree level) !

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$$= \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu + F_{\mu\bullet}\, dx^\mu \wedge dy + F_{\bullet\bullet}\, dy \wedge dy$$

The Lagrangian is equivalent to YMH theory with SSB !

$$\mathscr{L} = \frac{1}{4} F^+_{\mu\nu} F^{+\mu\nu} + \frac{1}{4} F^-_{\mu\nu} F^{-\mu\nu} + (D^\mu \Phi)^\dagger D_\mu \Phi + \frac{g^2}{2} \left(\left(\frac{\sqrt{2} M}{g} \right)^2 - \Phi^\dagger \Phi \right)^2,$$

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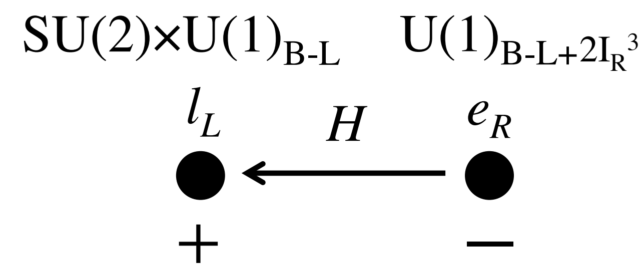
The mass relation (@ tree level) !

The toy standard model

A. Connes and J. Lott,
Nucl. Phys. B18(1990) 29-47,

R. Coquereaux, G. Esposito-Farese, G. Vaillant, NPB353 (1991) 689

- Fermionic Lagrangian



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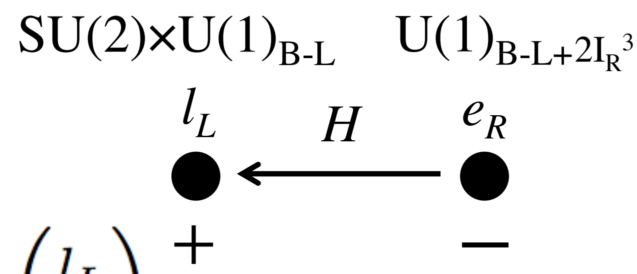

$$\mathcal{L}_{\text{Dirac}} = \bar{\Psi} i \gamma^M D_M \Psi =$$

$$\begin{pmatrix} \bar{l}_L & \bar{e}_R \end{pmatrix} g \begin{pmatrix} i(\partial_\mu + A^L_\mu)\gamma^\mu & \mu + h \\ \mu + h^\dagger & i(\partial_\mu + A^R_\mu)\gamma^\mu \end{pmatrix} \begin{pmatrix} l_L \\ e_R \end{pmatrix},$$

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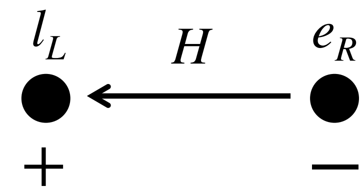
$SU(2)\times U(1)_{B-L}$ $\quad$ $U(1)_{B-L+2I_R^3}$



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$$\Rightarrow \quad y = g,$$ (Not bad for 3rd generation?)

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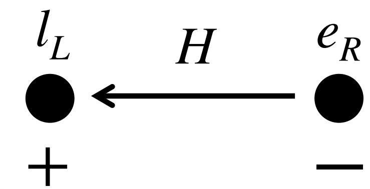

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Chamseddine et. al. introduce flavor dependent "distance"

Chamseddine, G. Felder, J. Frohlich, ph/9209224,

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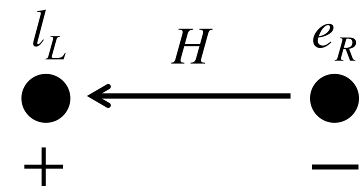

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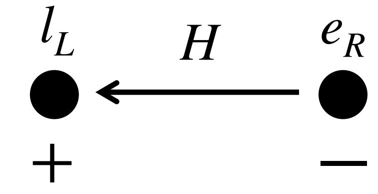

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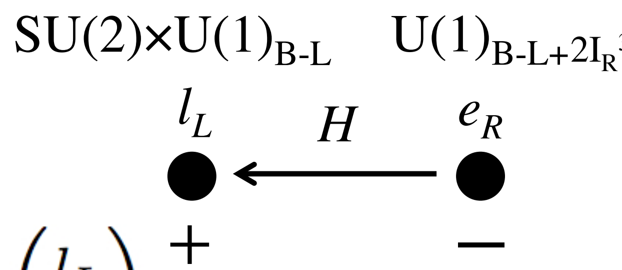

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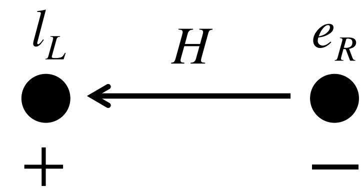

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Connes said on 1004.0464

$$m_h \sim 170\text{GeV}$$

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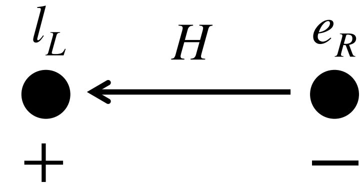

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Connes said on 1004.0464		on 1208.1030
$m_h \sim 170\text{GeV}$	$\Rightarrow$	125GeV

3. (Orbifold) SU(5) GUT in NCG

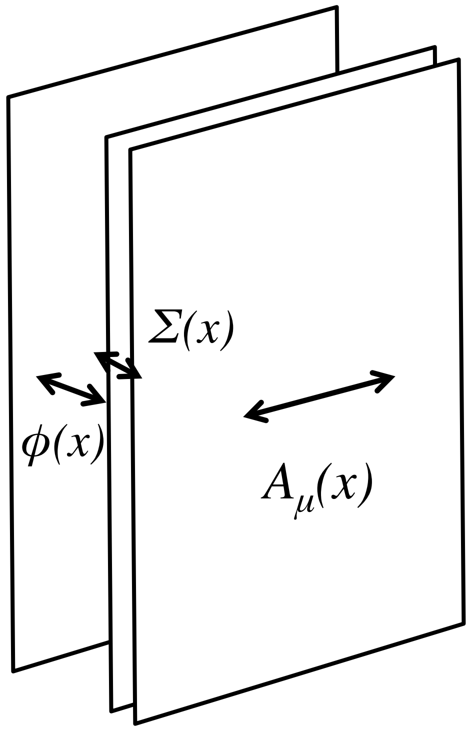
Grand Unified Theory (GUT)

- $\mathrm{SU}(5) : M_4 \times Z_3$

Chamseddine, G. Felder and J. Frohlich, ph/9209224,
Chamseddine and J. Frohlich, ph/9304023

- $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B\text{-}L} : M_4 \times Z_2 \times Z_2$
- $\mathrm{SO}(10) : M_4 \times Z_3 \; , M_4 \times Z_6$

- GUT requires several SSB scales
  ⇒ several discrete spaces !



SU(5) GUT

Chamseddine, G. Felder and J. Frohlich, ph/9209224,
K. Morita and Y. Okumura, Prog.Theor.Phys. 91 (1994) 975

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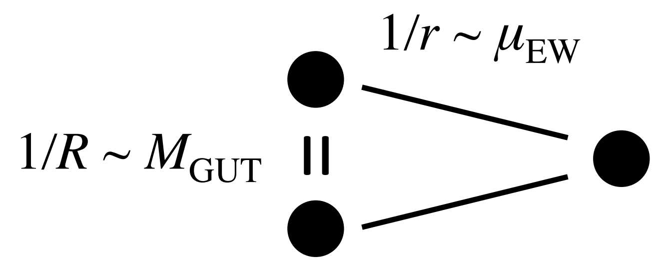

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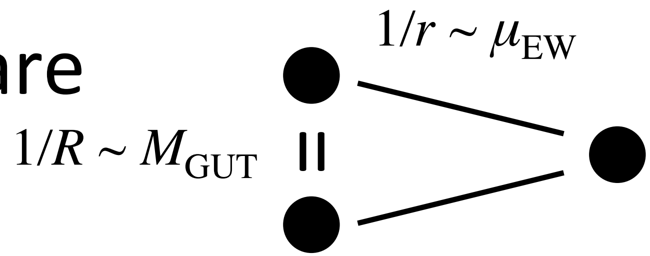

$$\boldsymbol{A}(x,n) = \sum_i a_n^{i\dagger}(x)\boldsymbol{d}a_n^i(x), \quad \boldsymbol{d} = d + d_\chi,$$

$$A(x,n) = \sum_i a_n^{i\dagger}(x) d a_n^i(x), \quad H_{nm}(x) = \sum_i a_n^{i\dagger}(x) M_{nm} a_m^i(x).$$

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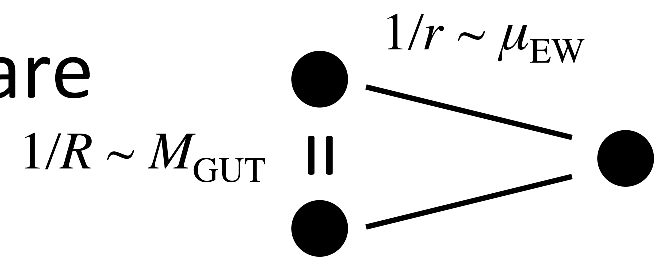

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where $a_1^i(x) = a_2^i(x) = 5 \times 5$ complex matrix

$a_3^i(x) = $ real function

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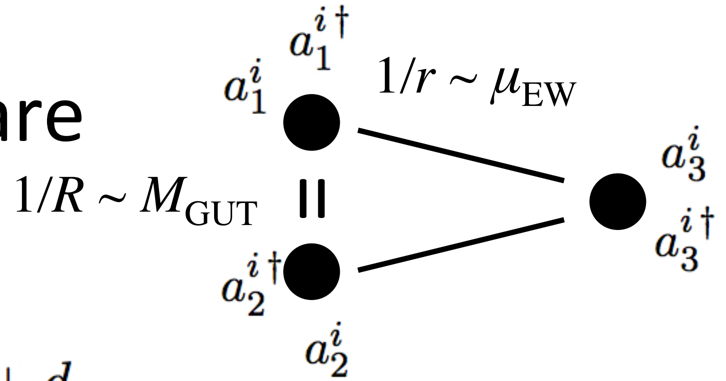

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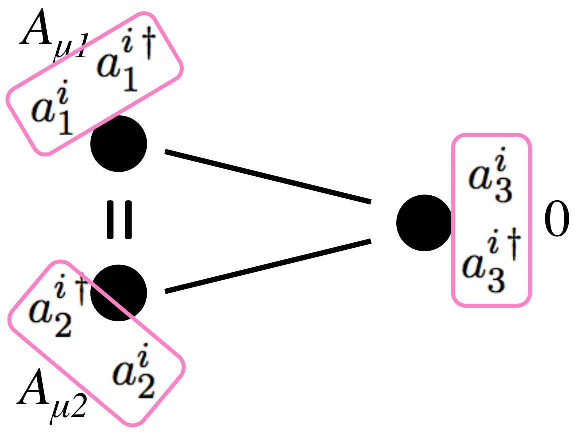

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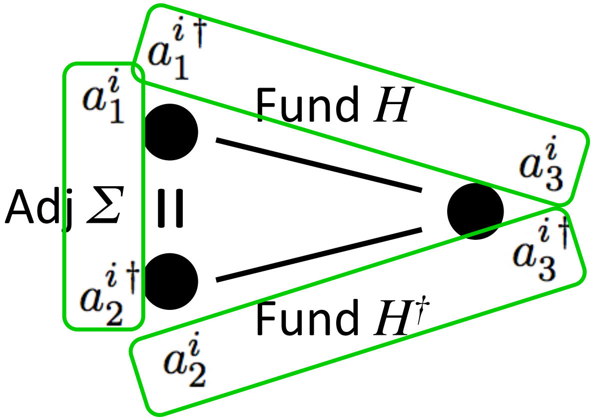

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SU(5) GUT

Chamseddine, G. Felder and J. Frohlich, ph/9209224,
K. Morita and Y. Okumura, Prog.Theor.Phys. 91 (1994) 975

- SU(5) : $M_4 \times Z_3$
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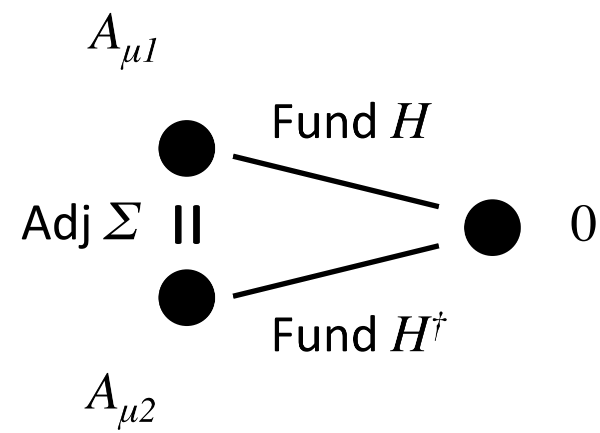

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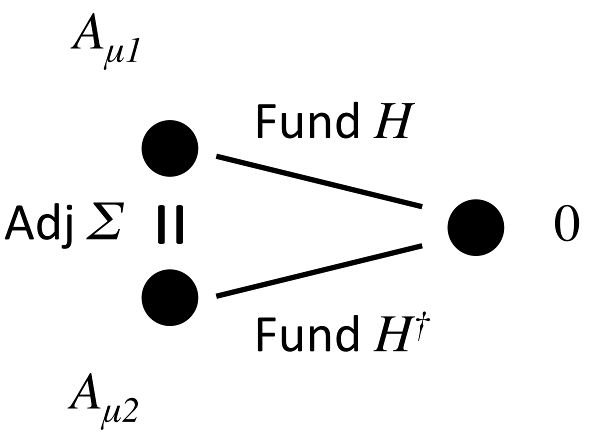

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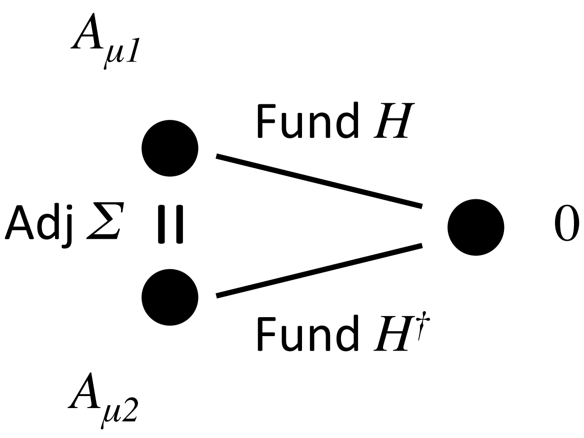

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I'm so sorry, but confining dynamics is not considered.
(we can consider this type of model from NJL theory.)

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Algebra of $M^4 \times Z_N$ (for insurance)

- The algebra is modified in $M^4 \times Z_N$.

$$\boldsymbol{d} f_n = (d + d_\chi) f_n,$$

$$d_\chi f = \sum_{m \neq n} d_{\chi_m} f = \sum_{m \neq n} [M_{nm} f_m - f_n M_{nm}] \chi_m, \qquad n, m = 1 - N.$$

- $d_\chi^{\,2} = 0$ (nilpotency) under a new NC algebra.

Graded Leibniz rule $\quad d_{\chi_l}(M_{nm} f(x,m)) = (d_{\chi_l} M_{nm}) f(x,m) - M_{nm}(d_{\chi_l} f(x,m)),$

Derivative of M_{nm} $\quad d_\chi M_{nm} = \sum_{l \neq m} M_{nm} M_{ml} \chi_l$

Index shifting rules
(source of noncommutativity)

$$F_{\cdot m} \chi_m M_{nm} = F_{\cdot m} M_{mn} \chi_n$$

$$F_{\cdot n} \chi_n M_{ml} = F_{\cdot n} M_{nl} \chi_n$$

$$\chi_k \wedge \chi_l = \chi_l \wedge \chi_k$$

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2 vevs (chosen by hand)

$$M_{12}=\Sigma_0=M\,\mathrm{diag}(2,2,2,-3,-3)\qquad M_{13}=M_{23}=H_0=(0,0,0,0,\mu)^T$$

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Higgs potential

$$-V(\Sigma,H)=-\frac{\alpha^4}{2g^2}\mathrm{tr}|(\Sigma(x)+\Sigma_0)^2+M(\Sigma(x)+\Sigma_0)-\frac{1}{5}\mathrm{tr}\Sigma_0^2|^2$$

$$-\frac{\alpha^2\beta^2}{2g'^2}||H(x)+H_0|^2-\mu^2|^2-\frac{\alpha^2\beta^2}{2g^2}|(\Sigma(x)+\Sigma_0+3M)(H(x)+H_0)|^2$$

$$\langle\chi_{1,2},\chi_{1,2}\rangle=-\alpha^2$$
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Higgs potential

$n = l$, Self interactions (determines the mass)

$$-V(\Sigma, H) = -\frac{\alpha^4}{2g^2} \mathrm{tr} |(\Sigma(x) + \Sigma_0)^2 + M(\Sigma(x) + \Sigma_0) - \frac{1}{5} \mathrm{tr} \Sigma_0^2|^2$$

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Higgs potential

$n \neq l$, Higgs interactions (2-3 splitting is solved)

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(2-3 splitting is not solved)

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$-V$ ... α^4 ... 1 ... $\langle \chi_{1,2}, \chi_{1,2} \rangle = -\alpha^2$

... $2g^2$ ... $2g^2$ ...

Composite scheme is necessary for successful GUT.

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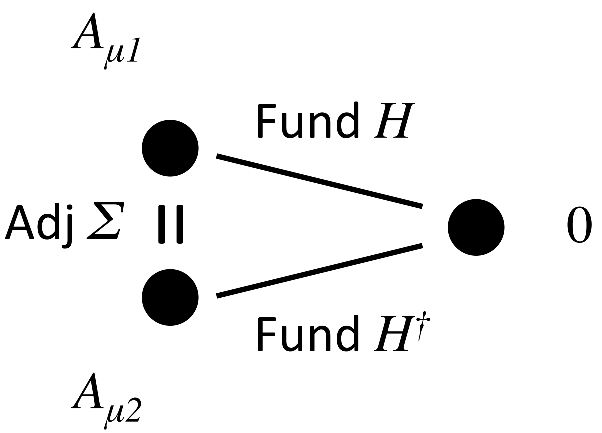

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SU(5) Orbifold GUT

MJSY, 1501.03888

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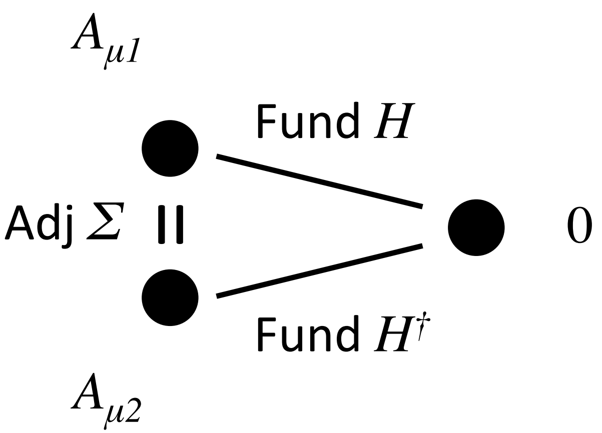

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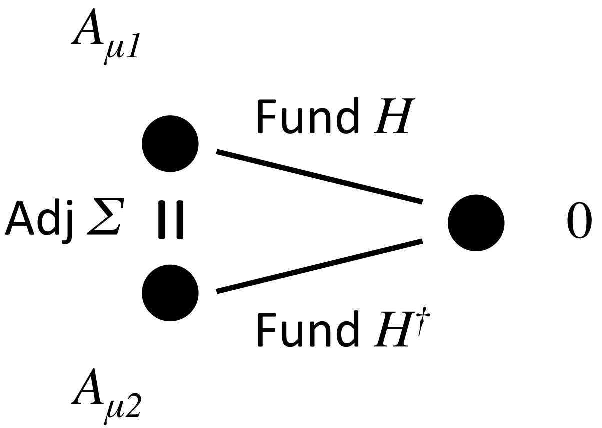

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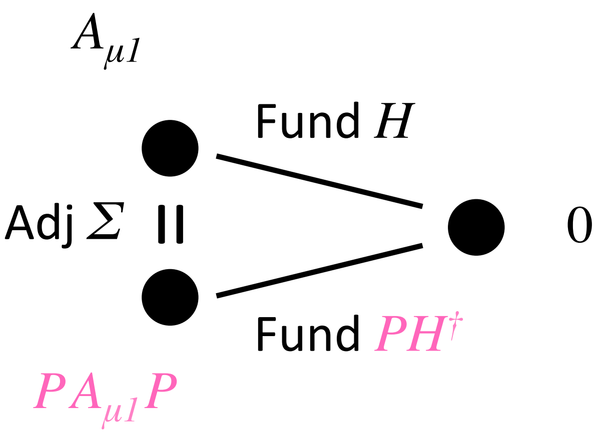

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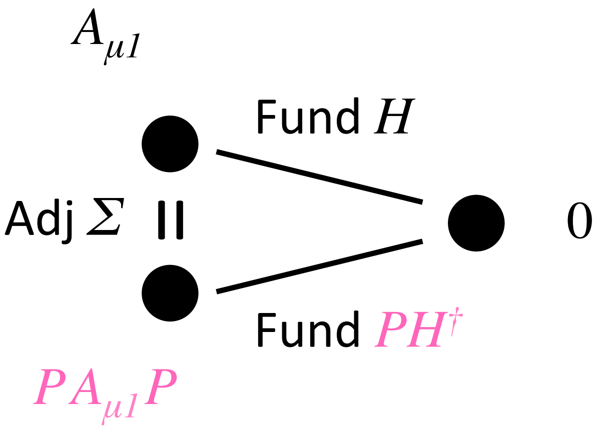

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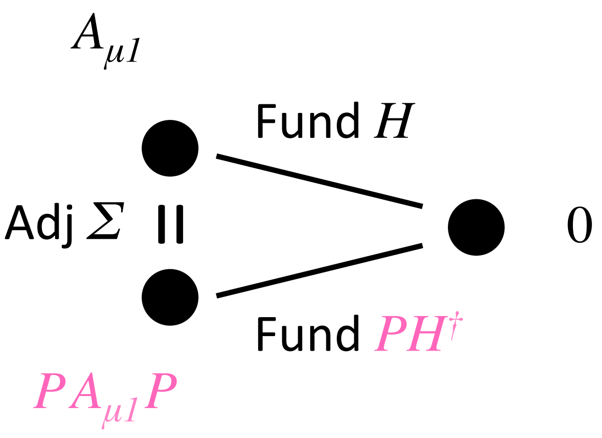

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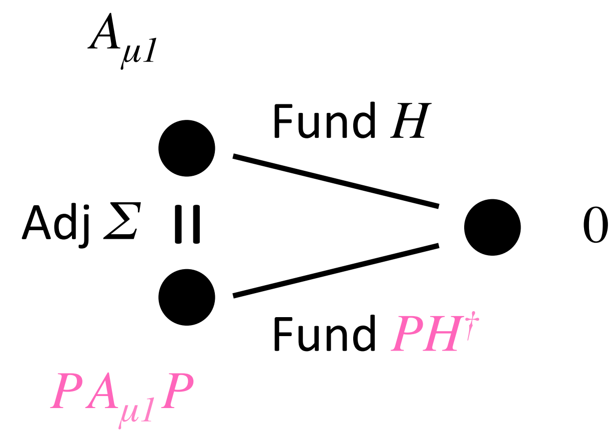
SU(5) Orbifold GUT

MJSY, 1501.03888

- SU(5) : $M_4 \times Z_3$



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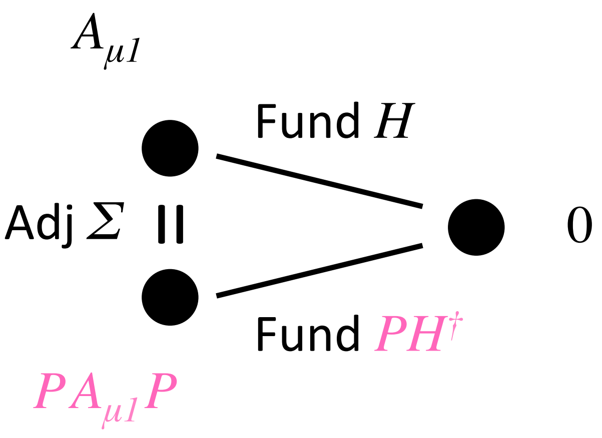

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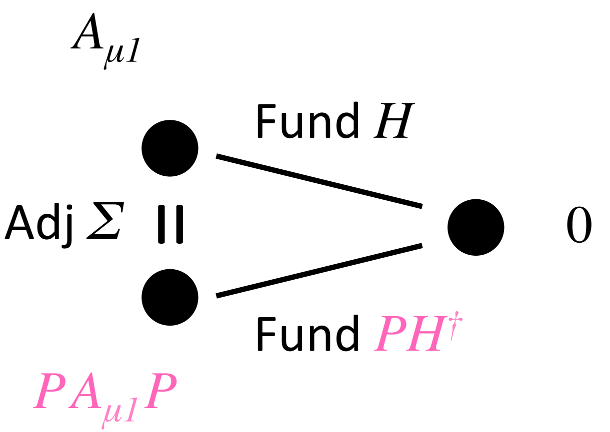

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Higgs potential Higgs-Higgs interactions (2-3 splitting is solved)

$$-V(\Sigma, H) \quad m_H^2 = |(MP - M)H|^2 = M^2 \text{diag}(4,4,4,0,0)\, H^\dagger H.$$

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Discussions

- Adjoint Σ field
  - The orbifold GUT mechanism and elimination of Σ field are incompatible
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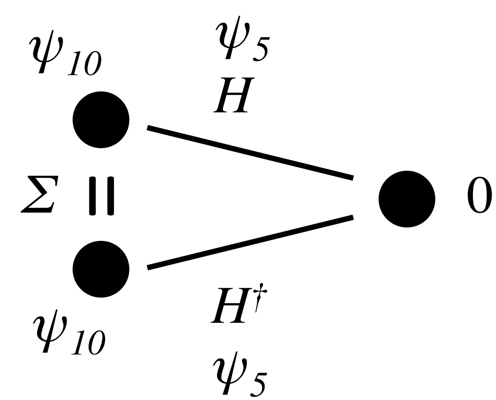
  - The orbifold GUT mechanism and elimination of Σ field are incompatible
  - The property is similar to the usual GUT
- Proton decay
  - Parity assignment can forbid the coupling to the fermions and X_μ , Y_μ , and H^c bosons.
  - We leave the construction of this sector as a future work.

An example of
Fermion configuration



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$$A(x,n) = \sum_i a_i^\dagger(x,n) d a_i(x,n)$$

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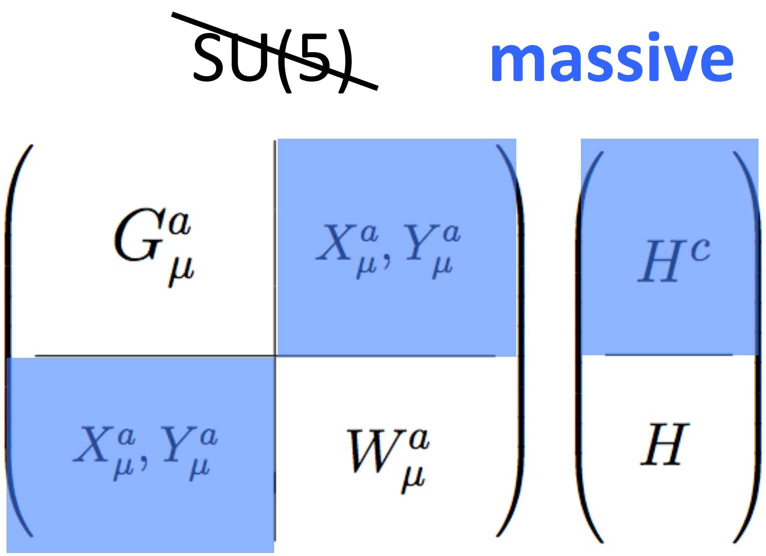

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Fermionic sector, proton decay

⇒ Future work …

That’s all. Thank you!

Back up

Auxiliary fields

- The curvature has an auxiliary fields X_{nml}.

$$\boldsymbol{F}_n = F_{n\,\mu\nu} dx^\mu \wedge dx^\nu + \sum_{m \neq n} D_\mu H_{nm}\, dx^\mu \wedge \chi_m + \sum_{m\neq n, l\neq m} (H_{nm} H_{ml} - X'_{nml}) \chi_m \wedge \chi_l,$$

where $X'_{nml} = \sum_i a_n^{i\,\dagger} M_{nm} M_{ml} a_l^i$. If H&$A_\mu$ are elemental, $X'_{nml} = M_{nm} M_{ml}$

If $X'_{nml} = f(H_{nm})$, $\chi_m \wedge \chi_l$ terms are treated as Higgs interactions. If not, X'_{nml} is an auxiliary field, and eliminated by $\partial\mathcal{L}/\partial X'_{nml} = 0$.

Ex)

$$X'_{123} = \sum_i a_1^{i\dagger} M_{12} M_{23} a_3^i = \sum_i a_1^{i\dagger} \Sigma_0 H_0 a_3^i = -3M(H(x) + H_0).$$

Then,

$$H_{12}H_{23} - X'_{123} = (\Sigma(x) + \Sigma_0)(H(x) + H_0) + 3M(H(x) + H_0).$$

170 GeV → 126 GeV, 1208.1030

From Pierre Martinetti

The new field σ and the Higgs mass

Spectral action requires a unique unification scale. With $\Lambda = 10^{17}$GeV, the running of the Higgs quartic selfcoupling λ_H under the big desert hypothesis yields

$$\lambda_0(\Lambda) = \frac{16}{3}\pi\alpha_3(\Lambda) = 0.356 \quad \Rightarrow \quad m_H \simeq 170 \text{ GeV}.$$

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A new scalar field σ - that lives at high energy and gives mass to the neutrinos - has been introduced by phenomenologists[†] to solve some instability due to the low mass of the Higgs (radiative corrections may drive λ_H negative and destabilize the electroweak vacuum):

$$V(H, \sigma) = \frac{1}{4}(\lambda_H H^4 + \lambda_\sigma \sigma^4 + 2\lambda_{H\sigma} H^2 \sigma^2).$$

As a bonus, it pulls m_H back to 126 GeV.

Resilience of the spectral SM, Chamseddine, Connes 2012

Is σ natural in NCG, or is it just an artifact for solving the model ?

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$$V(H,\sigma) = \frac{1}{4}(\lambda_H H^4 + \lambda_\sigma \ldots$$

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Actually, even this result Depends several assumptions

Is σ natural in NCG, or is it just an artifact for solving the model ?

Worries, concerns, issues

Less discussed problems

- Quantum corrections E. Alvarez, Jose M. Gracia-Bondia, C.P. Martin, Phys.Lett. B306 (1993) 55-58
  - No RGE inv. mass relation
  - RGE analyses $m_h(\mu) = g(\mu)\,\lambda(\mu)$ are done.
- Quadratic Divergence
  - I can't found discussion about this problem.

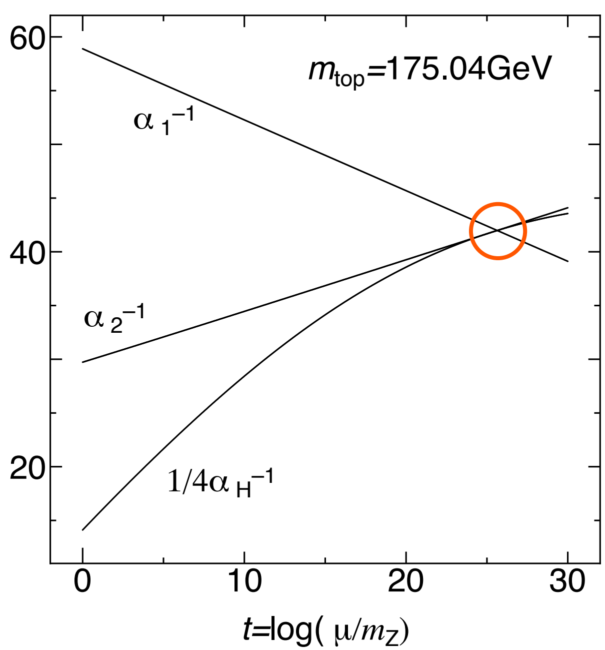
Renormalization Group Equation analysis

Y. Okumura, hep-ph/9707350

- We can search unification scale from observed parameter (g_i, $m_{Z,W,t}$) and SM RGEs

$$\lambda(\Lambda) = \frac{g_2^2(\Lambda)}{4} = \frac{g_1^2(\Lambda)}{4}, \quad \Rightarrow \quad m_h = \sqrt{2\lambda(m_h)}v(m_h).$$

Turn back the scale



With the 2-loop SM RGEs,

$$\mu = 3.37 \times 10^{13}\text{GeV}.$$

$$m_{\rm H} = 158.18\text{GeV for } m_{\rm top} = 175.04\text{GeV}$$

Perhaps M ∼ v doesn't hold?
(Connes treat Λ is GUT scale = 10^{17} GeV)