Linking U(2)×U(2) to O(4) via decoupling

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in collaboration with

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SCGT15

Based on "Linking U(2)×U(2) to O(4) model via decoupling", PRD**91**(2015)034025 [arXiv:1412.8026 [hep-lat]]

"RG flow of linear sigma model with $U_A(1)$ anomaly", PoS LATTICE 2014, 191 (2015)[arXiv:1501.06684 [hep-lat]]

"More about vacuum structure of Linear Sigma Model", PoS LATTICE 2013, 430 (2014)[arXiv:1311.4621 [hep-lat]]

Outline

- **Goal**:
  Study **chiral phase transition of massless two-flavor QCD** at finite T and in vanishing μ in terms of ε expansion

- **Assumption**: Non-zero breaking of $U_A(1)$ symmetry remains at T_c

- **Focusing on**:
  i) Whether 2nd order phase transition is possible?
  ii) What is the universality class? ($\Leftarrow$ critical exponents)

- **How**:
  Analyze RG-flow of the 3-d Ginsburg-Landau-Wilson model.
  IRFP $\Leftrightarrow$ 2nd order possible

- **Conclusions**:
  i) No IRFP, but still 2nd order phase transition is possible.
  ii) Exponent differs from those in O(4) due to non-decoupling effects.

Effective theory approach

Pisarski and Wilczek, PRD29, 338 (1984)

Look at RG flow of 3-d linear σ model (LσM)

The nature in PT 2-f QCD depends on fate of $U_A(1)$ at T_c.
($N_f \geq 3 \Rightarrow$ 1st order)

i) Largely broken ⇒ **SU(2)×SU(2)** ⇒ **O(4) LσM**
⇒ Wilson-Fisher FP ⇒ 2nd with O(4) scaling is possible

ii) Fully, effectively restored ⇒ **U(2)×U(2) [or O(2)×O(4)] LσM**
⇒ IRFP? ⇒ 1st or 2nd with U(2)×U(2) scaling

iii) If breaking is small,
⇒ **U(2)×U(2) LσM with ~~$U_A(1)$~~ [$U_A(1)$ broken LσM]**
⇒ **???**

$U_A(1)$ broken LσM

$$\Phi = \sqrt{2}(\phi_0 - i\chi_0)t_0 + \sqrt{2}(\chi_i + i\phi_i)t_i \rightarrow e^{2i\theta_A} L^\dagger \Phi R \quad (L \in SU_L(2),\ R \in SU_R(2),\ \theta_A \in \mathrm{Re})$$

$$\mathcal{L}_{\mathrm{total}} = \mathcal{L}_{U(2)\times U(2)} + \mathcal{L}_{\mathrm{breaking}}$$

$$\mathcal{L}_{U(2)\times U(2)} = \frac{1}{2}\mathrm{tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \frac{1}{2} m_0^2\, \mathrm{tr}\left[\Phi^\dagger \Phi\right] + \frac{\pi^2}{3} g_1 \left(\mathrm{tr}[\Phi^\dagger\Phi]\right)^2 + \frac{\pi^2}{3} g_2 \mathrm{tr}\left[(\Phi^\dagger\Phi)^2\right]$$

$$\mathcal{L}_{\mathrm{breaking}} = -\frac{c_A}{4}(\det\Phi + \det\Phi^\dagger) + \frac{\pi^2}{3} x\, \mathrm{Tr}[\Phi\Phi^\dagger](\det\Phi + \det\Phi^\dagger) + \frac{\pi^2}{3} y\, (\det\Phi + \det\Phi^\dagger)^2 + w\left(\mathrm{tr}\left[\partial_\mu\Phi^\dagger t_2\, \partial_\mu \Phi^* t_2\right] + \mathrm{h.c.}\right)$$

Rewriting in terms of components

$$\mathcal{L}_{\mathrm{total}} = (1+w)\frac{1}{2}(\partial_\mu\phi_a)^2 + \frac{m_\phi^2}{2}{\phi_a}^2 + \frac{\pi^2}{3}\lambda({\phi_a}^2)^2 + (1-w)\frac{1}{2}(\partial_\mu\chi_a)^2 + \frac{m_\chi^2}{2}{\chi_a}^2 + \frac{\pi^2}{3}\left[(\lambda - 2x)({\chi_a}^2)^2 + 2(\lambda + g_2 - z){\phi_a}^2{\chi_b}^2 - 2g_2(\phi_a\chi_a)^2\right]$$

← O(4) LσM

Setting $T=T_c \Leftrightarrow m_\phi^2=0 \Leftrightarrow$ **$m_\chi^2 = c_A > 0$ (Thus χ is massive)**

At T_c, calculate β-functions in d=4-ε dims with ε=1.

β functions

1-loop calc. with dim reg., a mass-dep. scheme and w=0 yields

$$\beta_{\hat\lambda} = -\epsilon\hat\lambda + 2\hat\lambda^2 + \frac{1}{6}f(\hat\mu)\left(4\hat\lambda^2 + 6\hat\lambda\hat g_2 + 3\hat g_2^2 - 8\hat\lambda\hat z - 6\hat g_2\hat z + 4\hat z^2\right),$$

$\Leftarrow$ O(4) LSM

$$\beta_{\hat g_2} = -\epsilon\hat g_2 + \frac{1}{3}\hat\lambda\hat g_2 + \frac{1}{3}f(\hat\mu)\hat g_2\left(\hat\lambda - 2\hat x\right) + \frac{1}{3}h(\hat\mu)\hat g_2\left(4\hat\lambda + \hat g_2 - 4\hat z\right),$$

$$\beta_{\hat x} = -\epsilon\hat x + 4f(\hat\mu)\left(\hat\lambda\hat x - \hat x^2\right) + \frac{1}{12}(1 - f(\hat\mu))\left(8\hat\lambda^2 - 6\hat\lambda\hat g_2 - 3\hat g_2^2 + 8\hat\lambda\hat z + 6\hat g_2\hat z - 4\hat z^2\right),$$

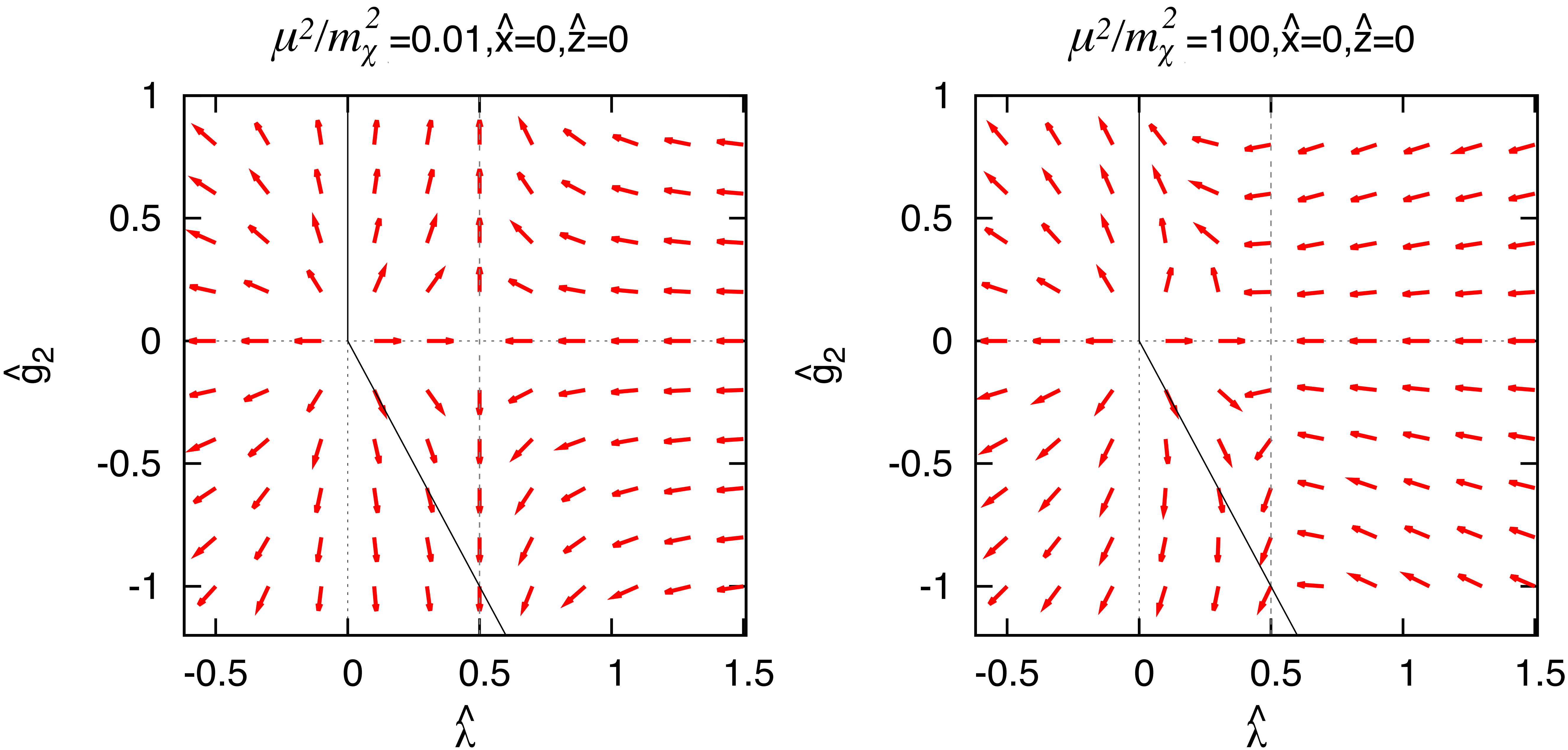
$$\beta_{\hat z} = -\epsilon\hat z + \frac{1}{2}\left(2\hat\lambda^2 - \hat\lambda\hat g_2 + 2\hat\lambda\hat z\right) - \frac{1}{6}h(\hat\mu)\left(4\,\hat\lambda^2 + 3\,\hat g_2^2 - 8\,\hat\lambda\,\hat z + 4\,\hat z^2\right) + \frac{1}{6}f(\hat\mu)\left(-2\hat\lambda^2 + 3\hat\lambda\hat g_2 + 3\hat g_2^2 - 2\hat\lambda\hat z - 6\hat g_2\hat z + 12\hat\lambda\hat x + 6\hat g_2 x - 12\hat x\hat z + 4\hat z^2\right),$$

$$\hat\mu = \mu/m_\chi,\quad \lim_{\hat\mu\to 0} f(\hat\mu) = \lim_{\hat\mu\to 0} h(\hat\mu) = O(\hat\mu^2),\quad \lim_{\hat\mu\to\infty} f(\hat\mu) = \lim_{\hat\mu\to\infty} h(\hat\mu) = 1$$

- $\mu \to 0$ (IR limit) with m_χ fixed $\rightarrow$???

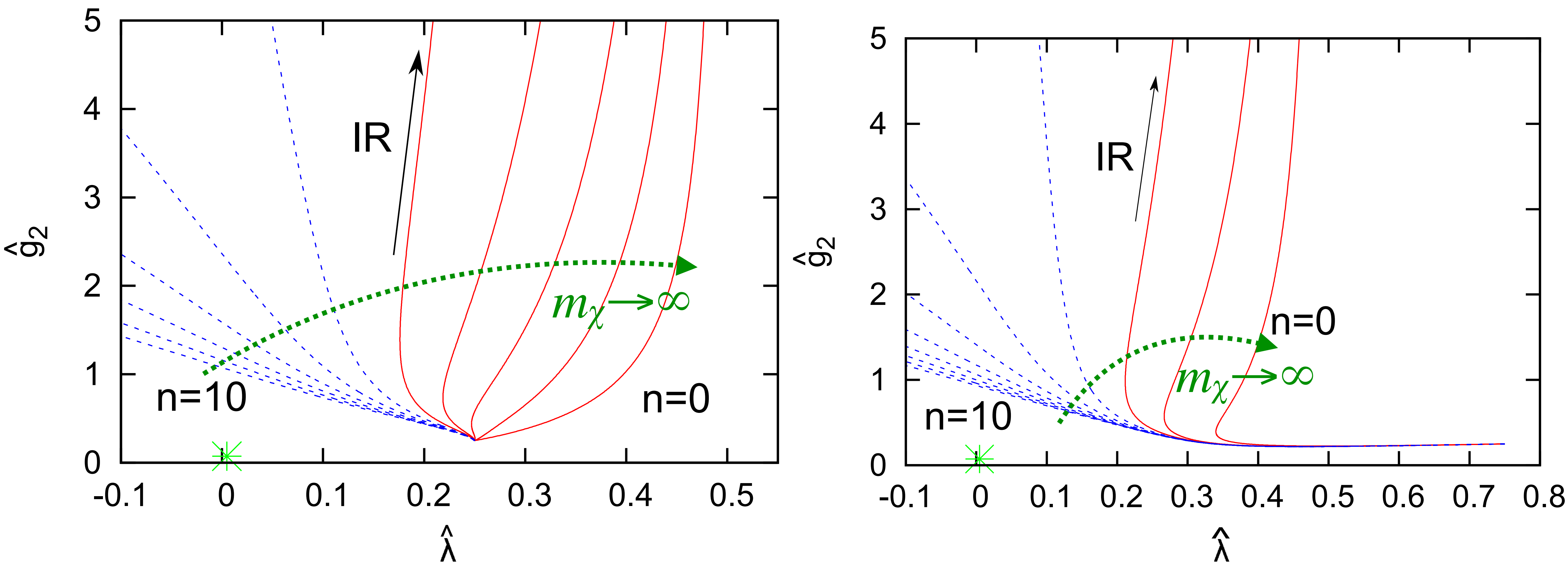
  ($m_\chi \to \infty$ with μ fixed $\rightarrow$ O(4) LσM)

RG-flow (1)



No IRFP

RG-flow (2)



RG flows can be classified by its IR behavior into two types:

1. All couplings diverge ⇒ 1st order

2. $\hat{\lambda}$ approaches $\varepsilon/2$ and others diverge ⇒ 2nd order?

How to interpret **no IRFP** but $\hat{\lambda} \to \hat{\lambda}_{FP}$

Usually, no IRFP $\Rightarrow$ 1^{st} order

But, in the present case, we infer that

the system undergoes 2^{nd} order.

Reason: **$U_A(1)$ broken LσM = O(4) LσM** in IR limit

If, in IR limit, massive χ decouples and arbitrary n-point functions of ϕ_i agree btw two theories:

$$\langle \phi_i(x_1)\phi_j(x_2)\cdots\rangle|_{O(4)} = \langle \phi_i(x_1)\phi_j(x_2)\cdots\rangle|_{U_A(1)\text{broken}}$$

Confirmed for arbitrary 4-point functions to 1-loop.

O(4) LσM has Wilson-Fisher FP in IR limit.

$\Rightarrow$ **IR limit of $U_A(1)$ broken LσM should have it, too.**

Critical exponents

Critical exponents $\Rightarrow$ ***Universality class***

Powerful tool to analyze various critical phenomena

$$\nu,\ \eta,\ \alpha,\ \beta,\ \gamma,\ \delta,\ \omega$$

ν : correlation length $\sim |t|^{-\nu}$ (t: reduced temperature)

η : $\langle \phi(x)\, \phi(0) \rangle \sim |x|^{-d+2-\eta}$

ω : scaling dim. of the leading irrelevant op.

Critical exponents in O(4) model

Model	ν	η	ω
O(4) (a few % error.)	0.750	0.0360	0.774
O(4) ε-exp (at leading order)	$2/(4-\varepsilon)$	0	ε

←Hasenbusch and Vicari, PRB84, 125136 (2011)

Critical exponents in $U_A(1)$ model

Model	ν	η	ω
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O(4) ε-exp (at leading order)	$2/(4-\varepsilon)$	0	ε
$U_A(1)$ ε-exp (at leading order)	$2/(4-\varepsilon)$	0	$2-5\varepsilon/3$

←Hasenbusch and Vicari, PRB84, 125136 (2011)

⇐ This work

At least, one of the critical exponents, ω, is different from O(4).

Two-loop calculation is underway.

Reason for different ω

$$\omega = \frac{d\beta_{\hat{\lambda}}}{d\hat{\lambda}}\Big|_{\hat{\lambda}=\hat{\lambda}_{\rm IRFP}}$$

ω : determined by RG dimension of leading irrelevant op.

In O(4) LσM,

$$\mathcal{L}_{O(4)} = \frac{1}{2}(\partial_\mu \phi_a)^2 + \frac{\pi^2}{3}\lambda(\phi_a{}^2)^2$$

$$\Rightarrow \quad \omega_{O(4)} = \epsilon$$

Reason for different ω

In $U_A(1)$ broken LσM,

$$\mathcal{L}_{\rm total} = (1+w)\frac{1}{2}(\partial_\mu \phi_a)^2 + \frac{m_\phi^2}{2}{\phi_a}^2 + \frac{\pi^2}{3}\lambda({\phi_a}^2)^2$$
$$+(1-w)\frac{1}{2}(\partial_\mu \chi_a)^2 + \frac{m_\chi^2}{2}{\chi_a}^2 + \frac{\pi^2}{3}\left[(\lambda-2x)({\chi_a}^2)^2 + 2(\lambda+g_2-z){\phi_a}^2{\chi_b}^2 - 2g_2(\phi_a\chi_a)^2\right]$$

IR limit

Leading irrelevant op.

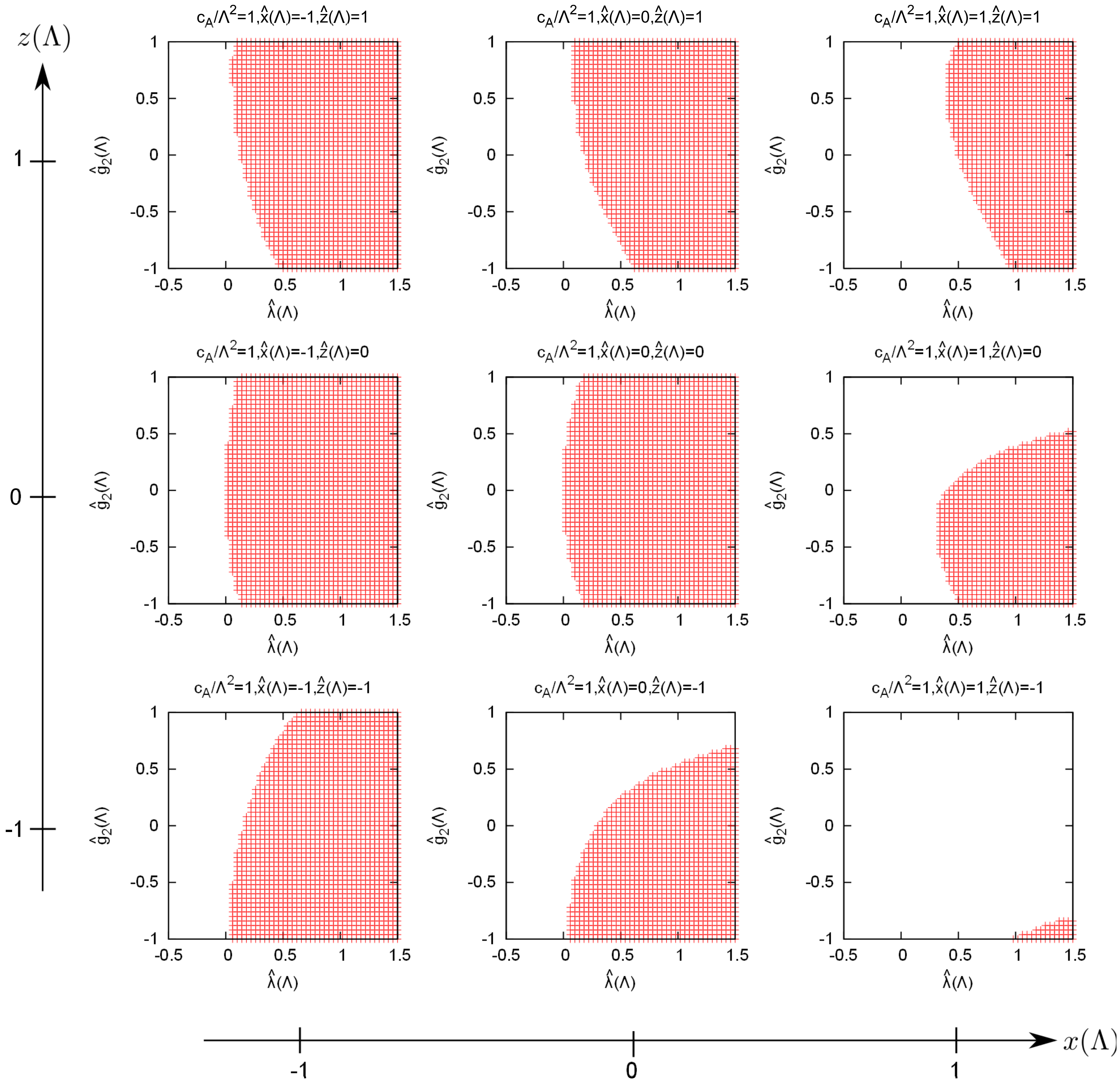
$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\partial_\mu \phi_a)^2 + \frac{\pi^2}{3}\mu^\epsilon\left[\hat{\lambda}({\phi_a}^2)^2 + \; ? \;\right]$$

$$\Rightarrow \quad \omega_{U_A(1)\text{broken}} = 2 - 5\epsilon/3$$

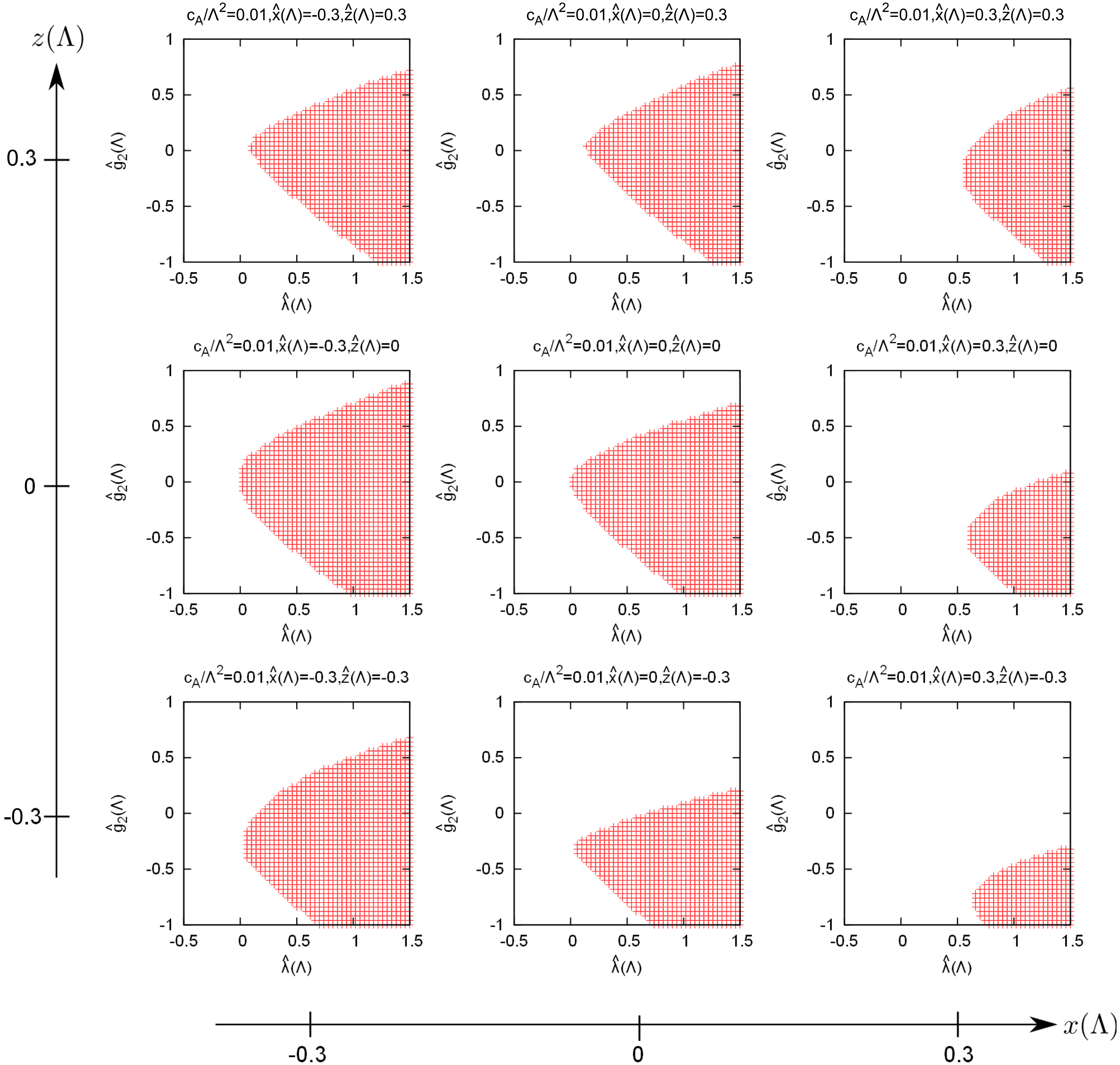
Non-decoupling causes non-universality.

Attractive Basin (large $U_A(1)$ breaking)

$m_\chi^2/\Lambda^2=1$



Attractive Basin (small $U_A(1)$ breaking)



m_χ^2/Λ^2=0.01

For smaller m_χ^2, attractive basin shrinks in vertical (g_2) direction.

⇒ In order to realize 2nd order transition, g_2 has to be tuned.

Impact on the nature of SχSB

So far, three possibilities have been discussed for the nature of transition:

i) 1st order

ii) 2nd order with O(4) scaling

iii) 2nd order with U(2)×U(2) scaling

Our study suggests fourth possibility:

iv) 2nd order with **O(4)-like scaling**

Summary

- U(2)×U(2) LSM with a finite $U_A(1)$ breaking is studied in ε-expansion.
- A novel possibility for the nature of chiral phase transition of massless two-flavor QCD, 2nd order with a scaling different from O(4).
- Difference from O(4) comes from non-decoupling.
- Non-decoupling effects induced non-universality.