Lattice study of the scalar and baryon spectra in many flavor QCD

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(LatKMI collaboration)

@SCGT15

Studies in LatKMI for strong coupling gauge theory

- Lattice study of the SU(3) gauge theory with Nf fundamental fermions
- all calculations are done with same set-up: Highly Improved Staggered Quark (HISQ) type action with Nf=4*n
- Nf=(4),8,(12), generic hadron spectrum properties → Y. Aoki (talk, yesterday)
- Nf=8 spectrum of Dirac operator and topology → K. Nagai (talk, yesterday)
- Nf=8 scalar and baryon for Dark Matter → this talk

Outline

- **Introduction**
- **Scalar analysis**
  mass & decay constant
- **Baryon analysis**
- **Summary**

Introduction

“Discovery of Higgs boson”

- Higgs like particle (125 GeV) has been found at LHC.
- Consistent with the Standard Model Higgs. But true nature is so far unknown.
- Many candidates for beyond the SM

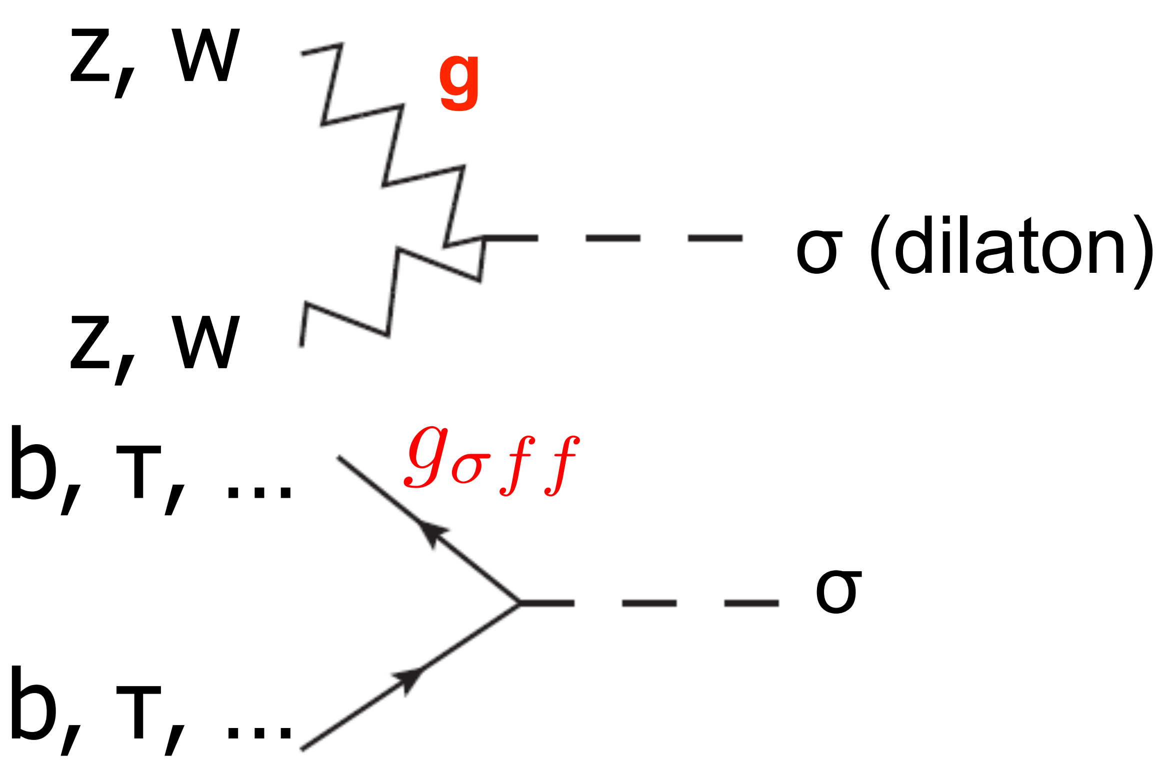
  one interesting possibility

  – **(walking) technicolor**
    - “Higgs” = dilaton (pNGB) due to breaking of the approximate scale invariance

Nf=8 QCD could be a candidate of walking gauge theory.
We find the flavor singlet scalar (σ) is as light as pion.
It may be identified a techni-dilaton (Higgs in the SM), which is a pseudo-Nambu Goldstone boson.
(LatKMI, Phys. Rev. D 89, 111502(R), arXiv: 1403.5000[hep-lat].)

Dilaton decay constant

- It is important to investigate **the decay constant** of the flavor singlet scalar as well as mass, which is useful to study LHC phenomena; the techni-dilaton decay constant governs all the scale of couplings between Higgs and other SM particles.



Fσ: dilaton decay constant

$$\frac{g_{\sigma WW}}{g_{h_{SM} WW}} = \frac{v_{EW}}{F_\sigma}$$

$$\frac{g_{\sigma f f}}{g_{h_{SM} f f}} = \frac{(3-\gamma^*) v_{EW}}{F_\sigma}$$

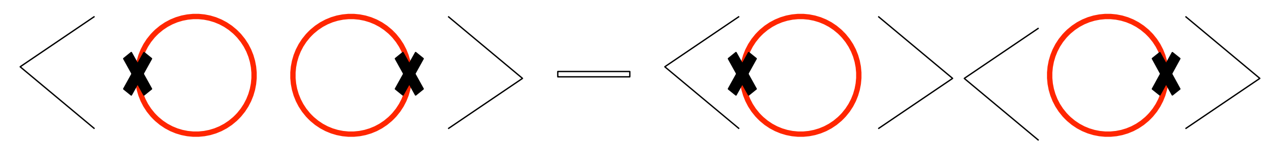
Dilaton effective theory analysis [S. Matsuzaki, K. Yamawaki, PRD86, 039525(2012)]

Lattice calculation of flavor-singlet scalar mass

Flavor singlet scalar from fermion bilinear operator

$$C_\sigma(t) = \frac{1}{V}\sum_x \langle \sum_i^{N_f} \bar{\psi}_i \psi_i(x,t) \sum_j^{N_f} \bar{\psi}_j \psi_j(0) \rangle = (-N_F C(t) + N_F^2 D(t))$$

$$\mathcal{O}_S(t) \equiv \bar{\psi}_i \psi_i(t), \qquad D(t) = \langle \mathcal{O}_S(t)\mathcal{O}_S(0)\rangle - \langle \mathcal{O}_S(t)\rangle\langle \mathcal{O}_S(0)\rangle$$



<u>Staggered fermion case</u>

- Scalar interpolating operator can couple to two states of

$$(\mathbf{1} \otimes \mathbf{1}) \ \& \ (\gamma_4\gamma_5 \otimes \xi_4\xi_5)$$

$$C_\pm(2t) \equiv 2C(2t) \pm C(2t+1) \pm C(2t-1)$$

- Flavor singlet scalar can be evaluated with disconnected diagram.

$$C_\sigma(2t) = -C_+(2t) + 2D_+(2t)$$

<u>(8 flavor) = 2 × (one staggered fermion)</u>

N_f=8 Result

Same data as [LatKMI PRD2014]
and
Some updates

Simulation setup

- **SU(3), Nf=8**

- **HISQ (staggered) fermion**
and tree level Symanzik gauge action

Volume (= L^3 x T)
- **L =24, T=32**
- **L =30, T=40**
- **L =36, T=48**
- **L =42, T=56**

Bare coupling constant ($\beta = \frac{6}{g^2}$)
- **beta=3.8**

bare quark mass
- **mf= 0.012-0.06,**
  (5 masses)

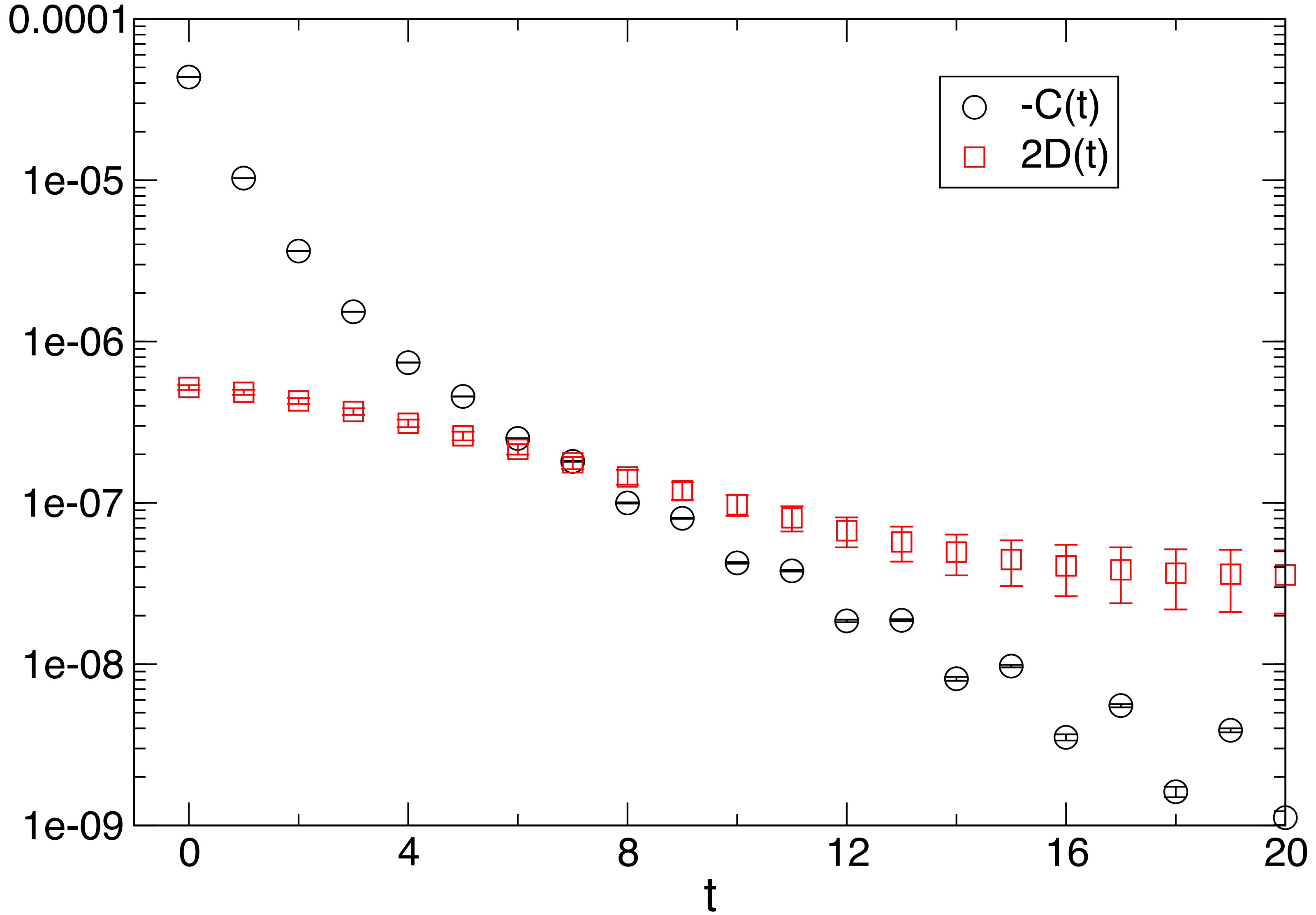
- **high statistics (more than 2,000 configurations)**

m_f	$L^3 \times T$	$N_{\rm cf}[N_{\rm st}]$
0.012	$42^3 \times 56$	2300[2]
0.015	$36^3 \times 48$	5400[2]
0.02	$36^3 \times 48$	5000[1]
0.02	$30^3 \times 40$	8000[1]
0.03	$30^3 \times 40$	16500[1]
0.03	$24^3 \times 32$	36000[2]
0.04	$30^3 \times 40$	12900[3]
0.04	$24^3 \times 32$	50000[2]
0.04	$18^3 \times 24$	9000[1]
0.06	$24^3 \times 32$	18000[1]
0.06	$18^3 \times 24$	9000[1]

- We use a noise reduction technique for disconnected correlator.

(use of Ward-Takahashi identity[Kilcup-Sharpe, ’87, Venkataraman-Kilcup ’97])

correlator for Nf=8, beta=3.8, L=36, mf=0.015

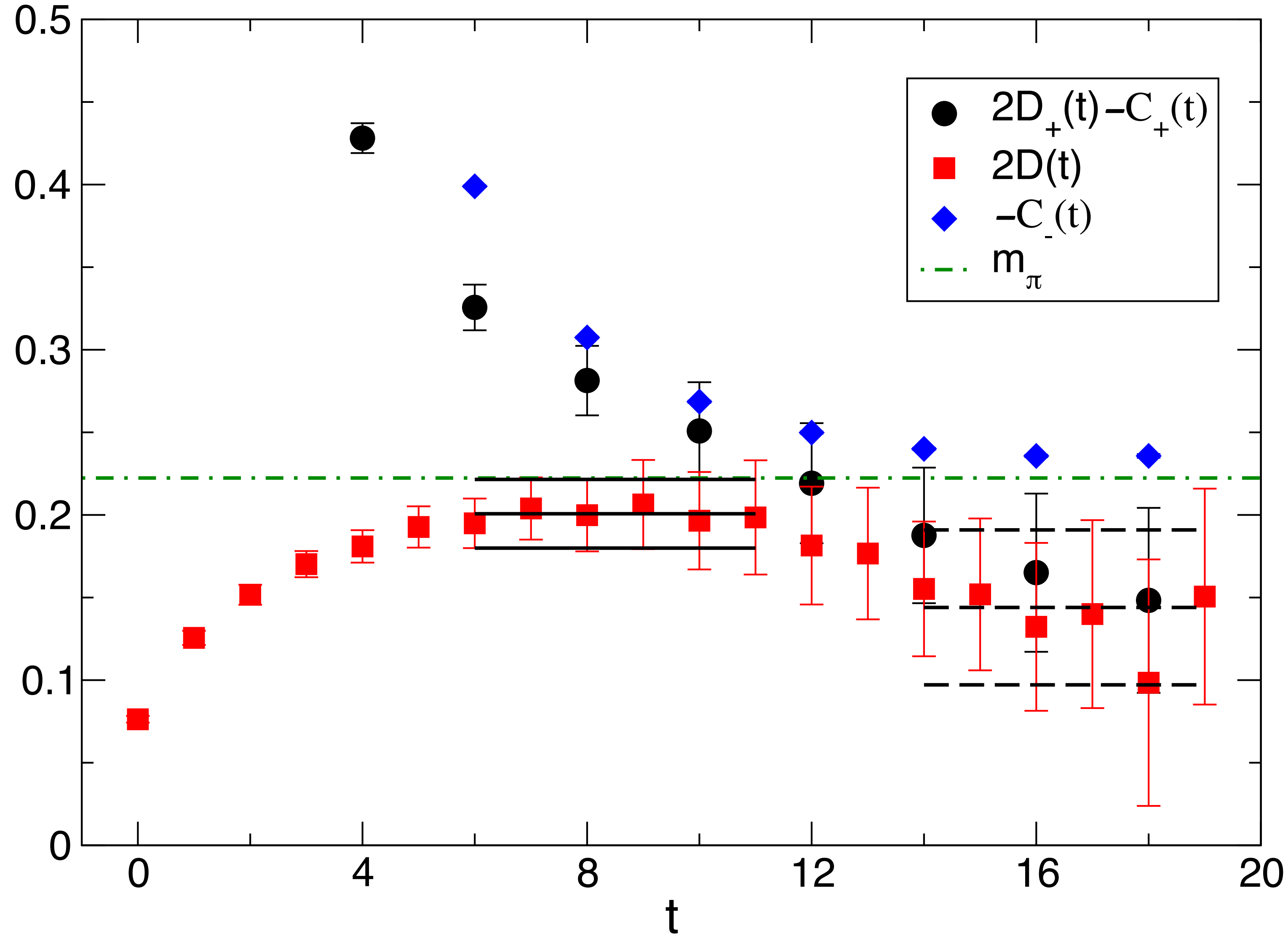


$$C_\sigma(2t) = -C_+(2t) + 2D_+(2t)$$

mσ for Nf=8, beta=3.8, L=36, mf=0.015

(same figure as talk by Y. Aoki, yesterday)

m_σ



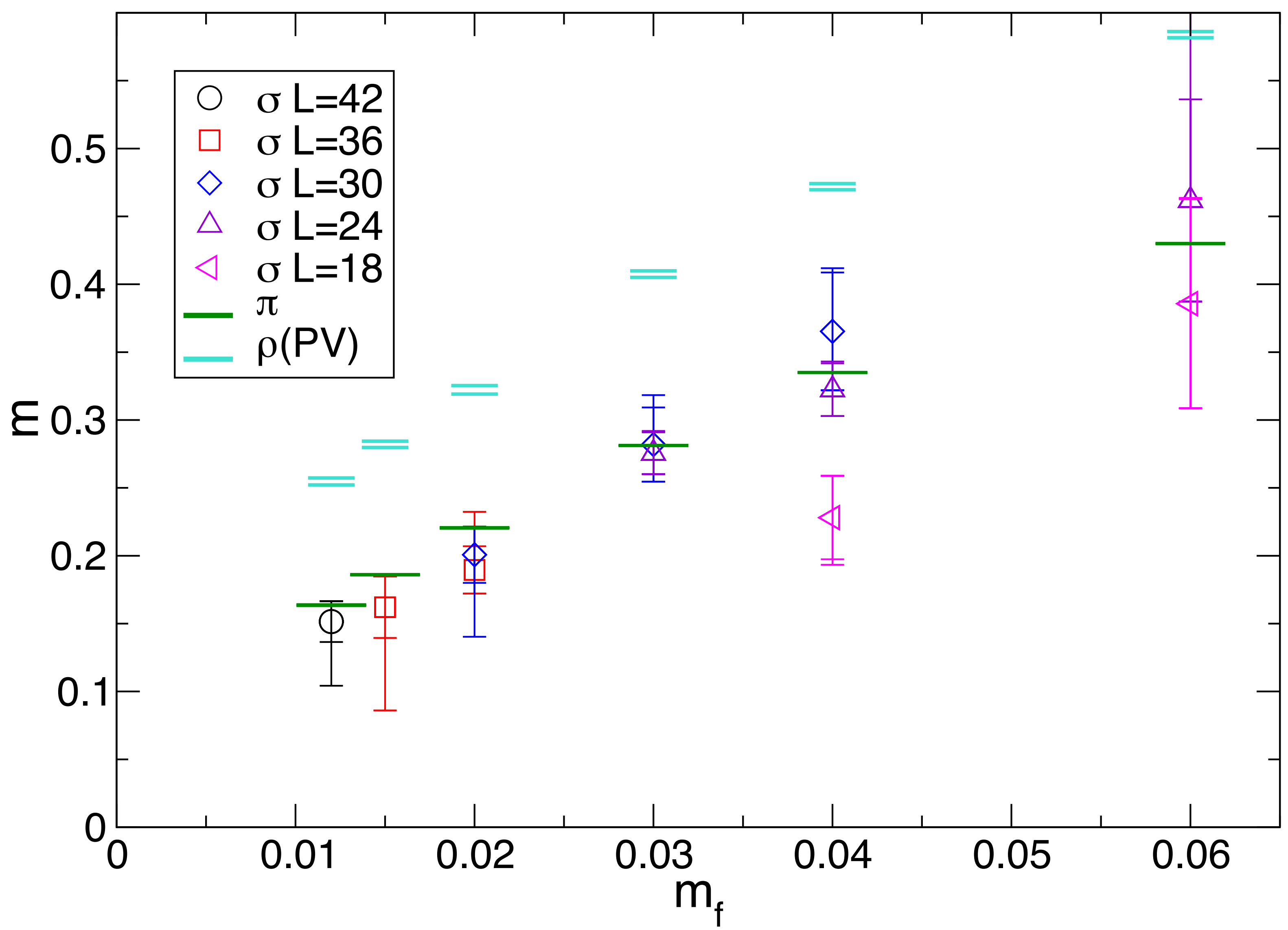
$$2D_+(t) - C_+(t) \rightarrow A_\sigma e^{-m_\sigma 2t}$$

$$D_+(t) = A_\sigma e^{-m_\sigma 2t} + A_{a_0} e^{-m_{a_0} 2t} \rightarrow A_\sigma e^{-m_\sigma 2t}, \quad (\text{if} \;\; m_\sigma < m_{a_0})$$

(in the continuum limit)

mσ for Nf=8, beta=3.8

(same figure as talk by Y. Aoki, yesterday)



σ is as light as π
and clearly lighter than ρ

Scalar decay constant

Preliminary

Two possible decay constants for σ (Fσ and Fs)

1. Fσ: Dilaton decay constant   difficult to calculate

$$\langle 0|\mathcal{D}^\mu(x)|\sigma;p\rangle = iF_\sigma p^\mu e^{-ipx}$$

$\mathcal{D}^\mu$: dilatation current   can couple to the state of σ.

Partially conserved dilatation current relation (PCDC): $\langle 0|\partial_\mu \mathcal{D}^\mu(0)|\sigma;0\rangle = F_\sigma m_\sigma^2$

2. Fs :scalar decay constant   not so difficult

We use scalar density operator $\mathcal{O}(x) = \sum_{i=1}^{N_F} \bar{\psi}_i \psi_i(x)$

which can also couple to the state of σ.
We denote this matrix element as scalar decay constant

$$N_F \langle 0|m_f \bar{\psi}\psi|\sigma\rangle = F_S m_\sigma^2$$

(Fs : RG-invariant quantity)

We study Fs.
We also discuss a relation between Fσ and Fs later.

scalar decay constant from 2pt flavor singlet scalar correlator

$$C_\sigma(t) = \frac{1}{V}\sum_x \langle \sum_i^{N_f} \bar\psi_i \psi_i(x,t) \sum_j^{N_f} \bar\psi_j \psi_j(0)\rangle = (-N_F C(t) + N_F^2 D(t))$$

Insert the complete set (|n><n|)

$$C_\sigma(t) = \frac{N_F^2}{V} |\langle 0|\bar\psi\psi(0)|\sigma;0\rangle|^2 \frac{e^{-m_\sigma t}}{2m_\sigma} + \cdots$$

Asymptotic behavior (large t) of the scalar 2pt correlator Cσ(t)

$$C_\sigma(t) \sim N_F^2 A(e^{-m_\sigma t} + e^{-m_\sigma (T-t)})$$

$$F_S = N_F \frac{m_f \sqrt{2 m_\sigma V A}}{m_\sigma^2}$$

NF: number of flavors
V: L^3
A: amplitude

What is relation between F_s and F_σ?

A relation between Fs and Fσ through the WT id. (in the continuum theory)

the (integrated) WT-identity for dilatation transformation

$$\int d^4x \exp(-iqx)\partial_\mu \langle T\left(\mathcal{D}^\mu(x)\mathcal{O}(0)\right)\rangle$$
$$= \int d^4x \left\{\exp(-iqx)\langle T\left(\partial_\mu \mathcal{D}^\mu(x)\mathcal{O}(0)\right)\rangle + \delta^4(x)\langle \delta_D \mathcal{O}(0)\rangle\right\}$$

Useful relations

$$\partial_\mu \mathcal{D}^\mu = \theta_\mu^\mu$$ (trace anomaly relation)

$$\delta_D \mathcal{O} = [iQ_D, \mathcal{O}] = \Delta_{\mathcal{O}}\mathcal{O}$$ (scale transformation)

$$\Delta_{\mathcal{O}} : \text{ scale dimension of operator } \mathcal{O}$$

Taking the zero momentum limit (q →0), (LHS) is zero.
the WT-identity gives

$$\int d^4x \langle T(\theta_\mu^\mu(x)\mathcal{O}(0))\rangle = -\Delta_{\mathcal{O}}\langle \mathcal{O}\rangle$$

Insert the complete set $\int \frac{d^3p}{(2\pi)^3} \frac{|\sigma(p)\rangle\langle\sigma(p)|}{2E_p} + \cdots$

into $\int d^4x \langle T(\theta^\mu_\mu(x)\mathcal{O}(0))\rangle = -\Delta_{\mathcal{O}} \langle \mathcal{O} \rangle$

and use a scalar density operator $\mathcal{O} = m_f \sum_i^{N_F} \bar{\psi}\psi$

We obtain $$F_S F_\sigma m_\sigma^2 = -\Delta_{\bar{\psi}\psi} N_F m_f \langle \bar{\psi}\psi \rangle$$

(in the dilaton pole dominance approximation)

[Ref: Technidilaton (Bando, Matumoto, Yamawaki, PLB 178, 308-312)]

$$F_\sigma = -\frac{\Delta_{\bar{\psi}\psi} N_F m_f \langle \bar{\psi}\psi \rangle}{\sqrt{2VAm_\sigma}}$$

Recall

$$F_S = N_F \frac{m_f \sqrt{2m_\sigma VA}}{m_\sigma^2}$$

$$\Delta_{\bar{\psi}\psi} = 3 - \gamma_m$$

$$F_\sigma = -\frac{\Delta_{\bar\psi\psi} N_F m_f \langle\bar\psi\psi\rangle}{\sqrt{2VAm_\sigma}}$$

$$F_S F_\sigma m_\sigma^2 = -\Delta_{\bar\psi\psi} N_F m_f \langle\bar\psi\psi\rangle$$

$$\Delta_{\bar\psi\psi} = 3 - \gamma_m$$

(in the dilaton pole dominance approximation)

c.f. PCAC relation

The (integrated) chiral WT-identity tells us that

$$\int d^4x \langle 2mP^a(x)^\dagger P^a(0)\rangle = -2\langle\bar\psi\psi\rangle$$

$$P^a(x) = \bar\psi\gamma_5\tau^a\psi(x)$$

using PCAC relation, this leads to

$$m_\pi^2 F_\pi^2 = -4m_f\langle\bar\psi\psi\rangle$$ (GMOR relation)

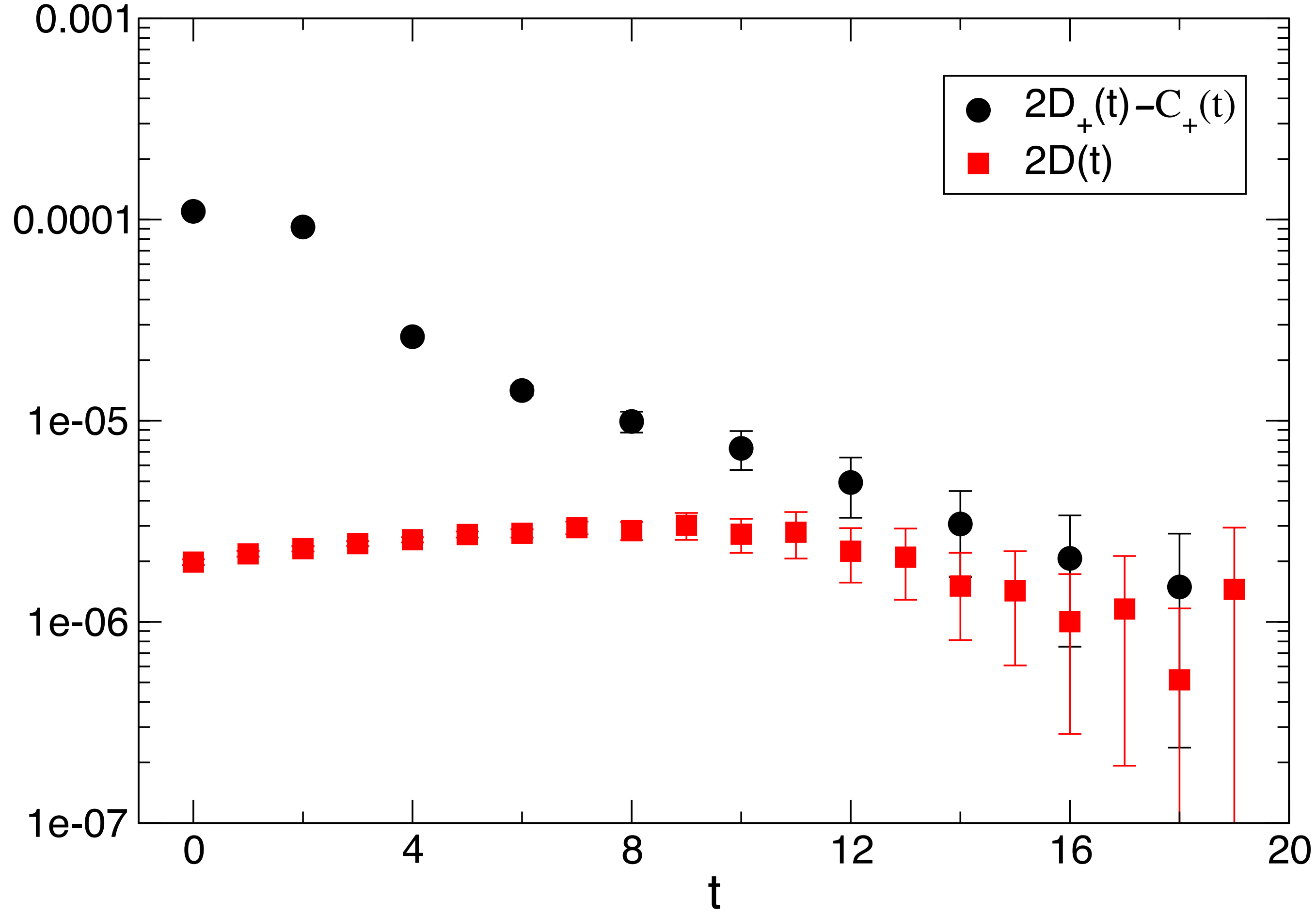
(in the pion pole dominance approximation)

N_f=8 Result

Effective amplitude for Nf=8, beta=3.8

L=30, T=40, mf=0.02

A



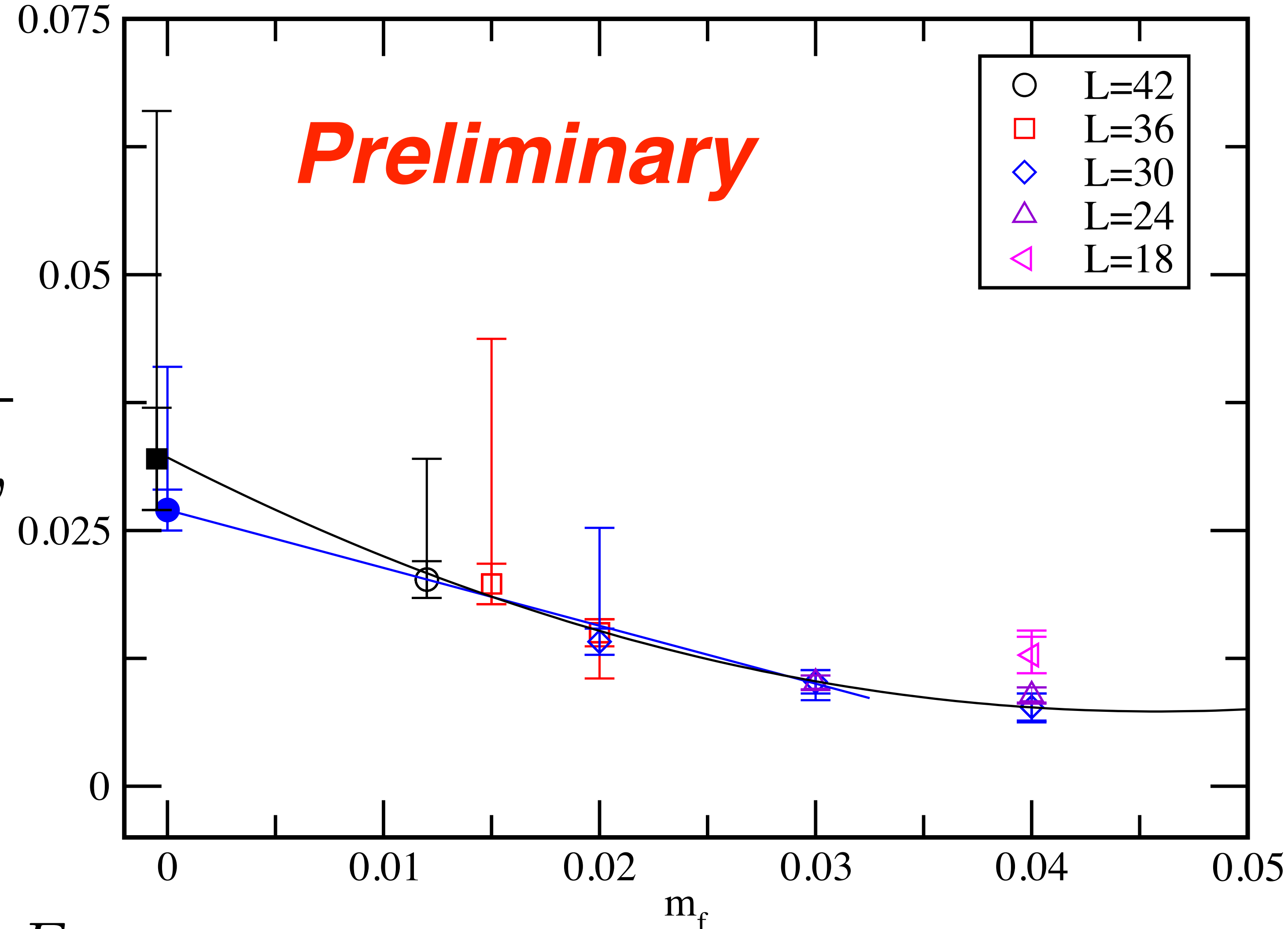
$$C = A(e^{-mt} + e^{-m(T-t)})$$

Fσ for Nf=8, beta=3.8

$$F_\sigma = -\frac{\Delta_{\bar\psi\psi} N_F m_f \langle\bar\psi\psi\rangle}{\sqrt{2VAm_\sigma}}$$

with $\langle\bar\psi\psi\rangle \to \langle\bar\psi\psi\rangle_0$ chiral limit

$\frac{F_\sigma}{\Delta_{\bar\psi\psi}}$



Chiral extrapolation fit

Blue (mf=0.012-0.03)

$$F_\sigma = c_0 + c_1 m_f$$

Black (mf=0.015-0.04)

$$F_\sigma = c_0 + c_1 m_f + c_2 m_f^2$$

$\frac{F_\sigma}{F_\pi} \sim 1.5\Delta_{\bar\psi\psi} \sim 3$ in the chiral limit

with assumption of $\gamma \sim 1, \quad (\Delta_{\bar\psi\psi} = 3 - \gamma \sim 2)$

c.f. Another estimate via the scalar mass in the dilaton ChPT (DChPT).

DChPT: $m_\sigma^2 \sim d_0 + d_1 m_\pi^2$

$$d_1 = \frac{(1+\gamma)\Delta_{\bar\psi\psi}}{4}\frac{N_F F_\pi^2}{F_\sigma^2} \sim 1$$

$$\frac{F_\sigma}{F_\pi} \sim \sqrt{N_F} = 2\sqrt{2}$$

Technibaryon Dark Matter

Technibaryon

- The lightest baryon is stable due to the technibaryon number conservation

- Good candidate of the dark matter (DM)

- Boson or fermion? (depend on the #TC)
  our case: DM is fermion (#TC=3).

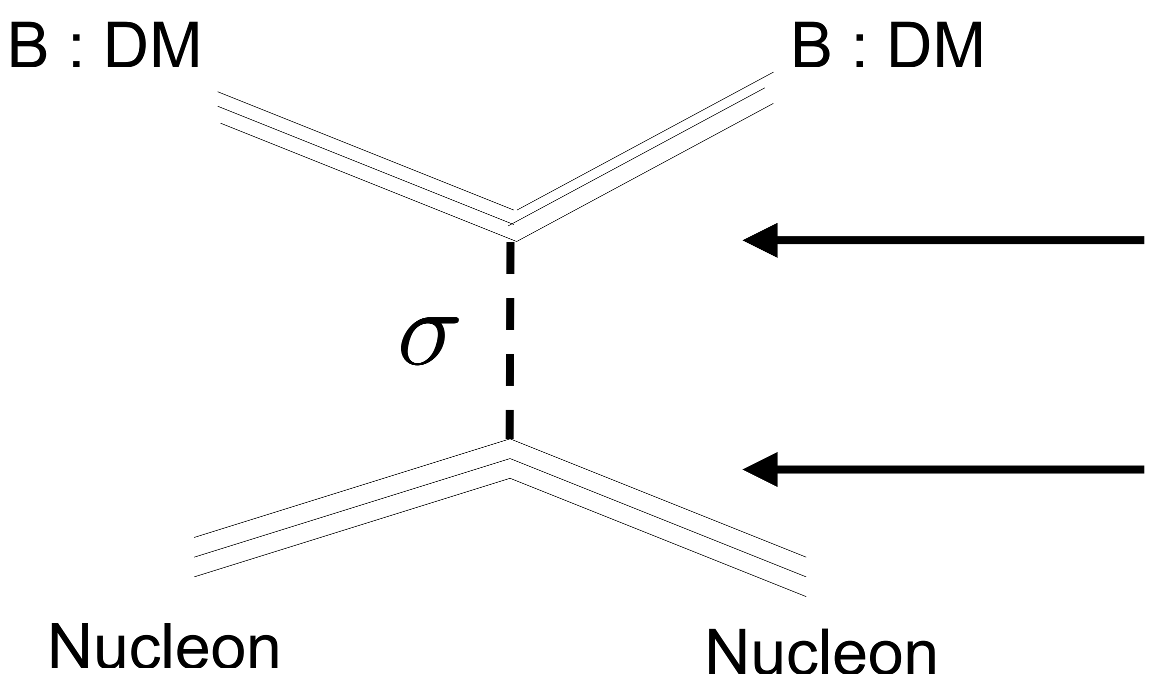
- Direct detection of the dark matter is possible.

DM effective theory

Technibaryon(B) interacts with quark(q), gluon in standard model

$$\mathcal{L}_{eff} = c\underline{\bar{B}B\bar{q}q} + c\bar{B}BG^{a}_{\mu\nu}G^{a\mu\nu} + \frac{1}{M}\bar{B}i\partial_\mu\gamma_\nu B\mathcal{O}^{\mu\nu} + \cdots$$

One of the dominant contributions in spin-independent interactions comes from the microscopic Higgs (technidilaton σ) mediated process (below diagram)



$y_{\bar{B}B\sigma}$ Technibaryon-scalar effective coupling

$y_{\bar{n}n\sigma}$ nucleon-scalar effective coupling

(Techni)baryon Chiral perturbation theory with **dilaton**

leading order of BChPT

$$\mathcal{L} = \bar{B}(i\gamma^\mu \partial_\mu - \underline{m_B} + \frac{g_A}{2}\gamma_5 \gamma^\mu u_\mu)B$$

$$u^\mu = i\left(u^\dagger(\partial^\mu - i\, r^\mu)u - u(\partial^\mu - i\, l^\mu)u^\dagger\right)$$

$$U = u^2 = e^{2\pi i/F_\pi}$$

$$\mathcal{L} = \bar{B}(i\gamma^\mu \partial_\mu - e^{\sigma/F_\sigma} m_B + \frac{g_A}{2}\gamma_5 \gamma^\mu u_\mu)B$$

$$\chi = e^{\sigma/F_\sigma}$$

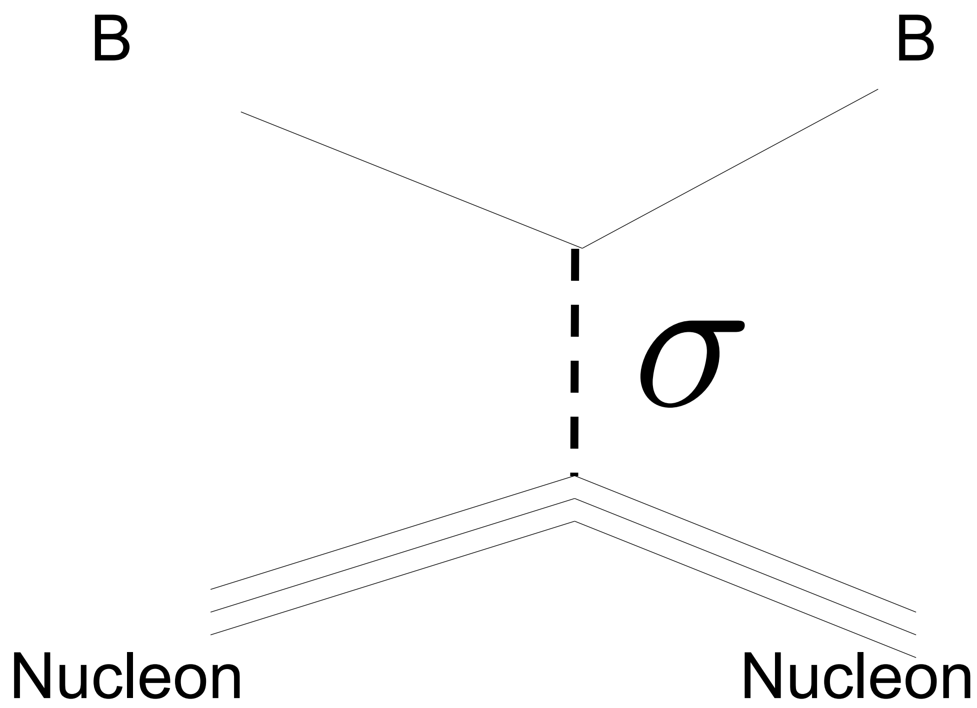
The dilaton-baryon effective coupling (leading order) is uniquely determined as

$$y_{\bar{B}B\sigma} = m_B/F_\sigma$$

DM Direct detection

Spin-independent cross section with nucleus

$$\sigma_{SI}(\chi, N) = \frac{M_R^2}{\pi}(Zf_p + (A-Z)f_n)^2$$



Note: Yukawa coupling is different from the SM : $\frac{g_{\sigma ff}}{g_{h_{SM}ff}} = \frac{(3-\gamma^*)v_{EW}}{F_\sigma}$

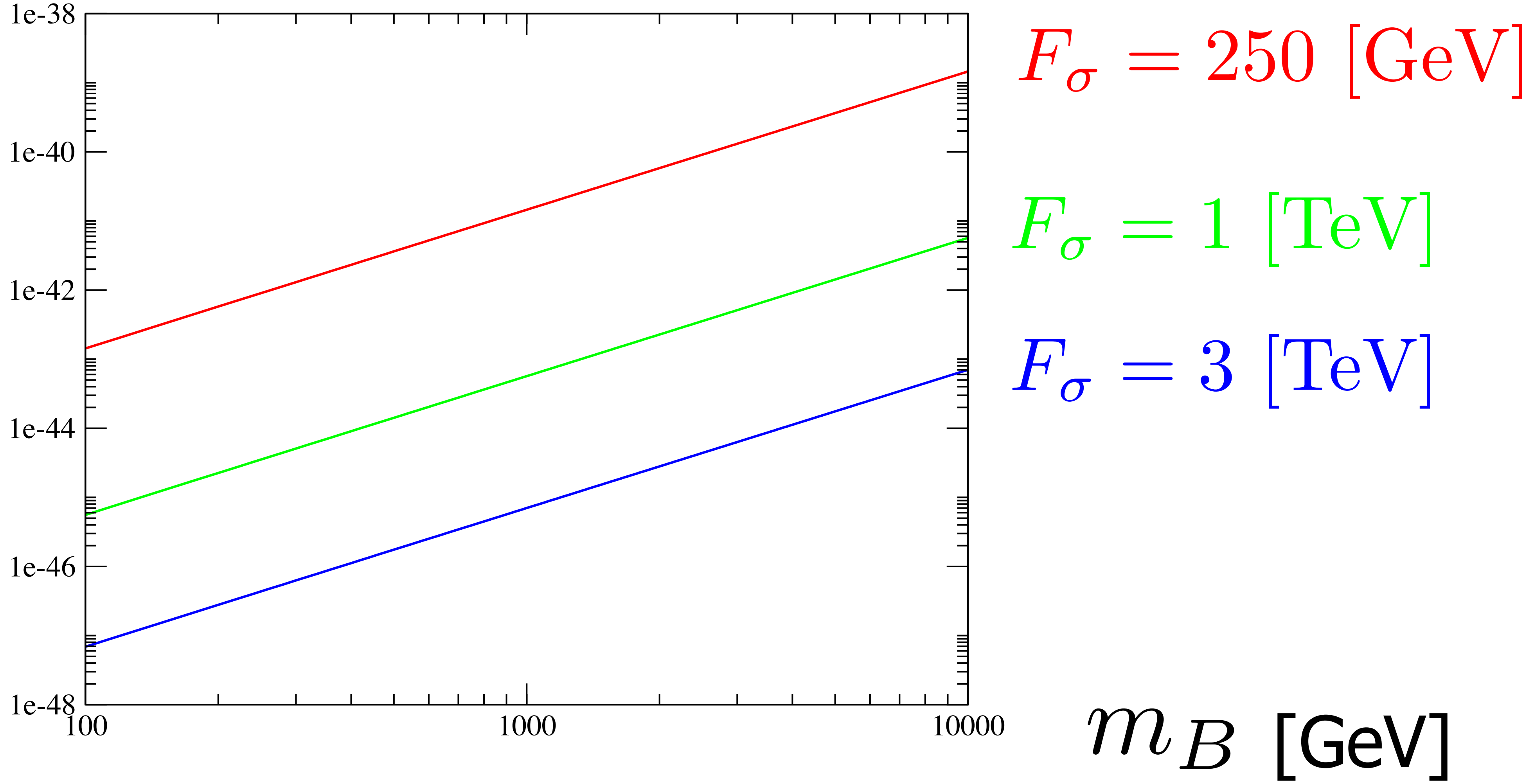
$$f_{T_q}^{(N)} \equiv \langle N|m_q \bar{q} q|N\rangle / m_N$$

Nucleon sigma term in QCD

Nucleon matrix element non-perturbatively determined by lattice QCD calculation

Lattice calculation for both nucleon and technibaryon interactions

An illustrative example of DM cross section



Sample input values

$f_{T_u}^{(p)}$	0.019	$f_{T_u}^{(n)}$	0.013
$f_{T_d}^{(p)}$	0.027	$f_{T_d}^{(n)}$	0.040
$f_{T_s}^{(p)}$	0.009	$f_{T_s}^{(n)}$	0.009

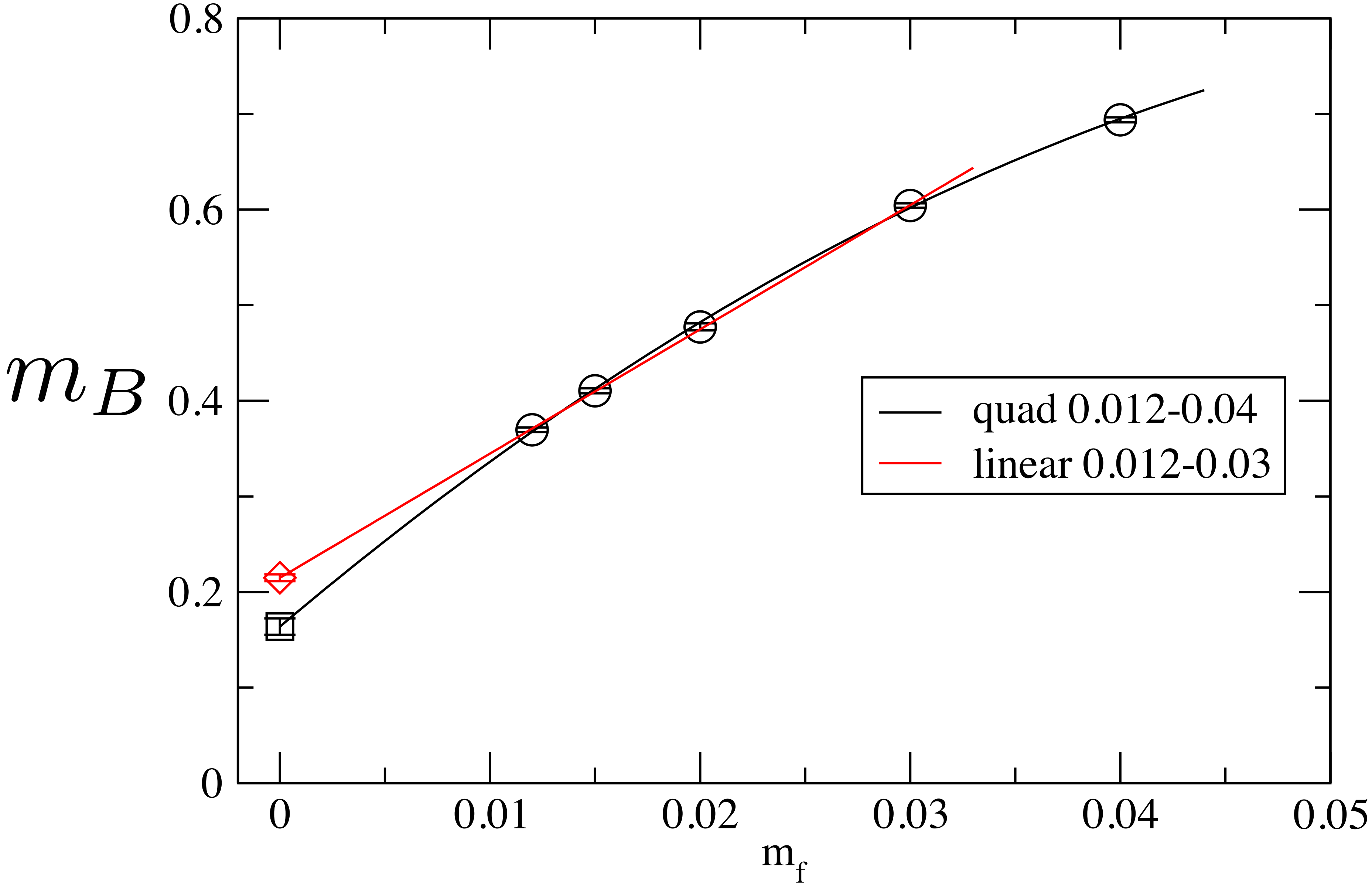
$m_\sigma = 125$ [GeV]

$\gamma = 1$

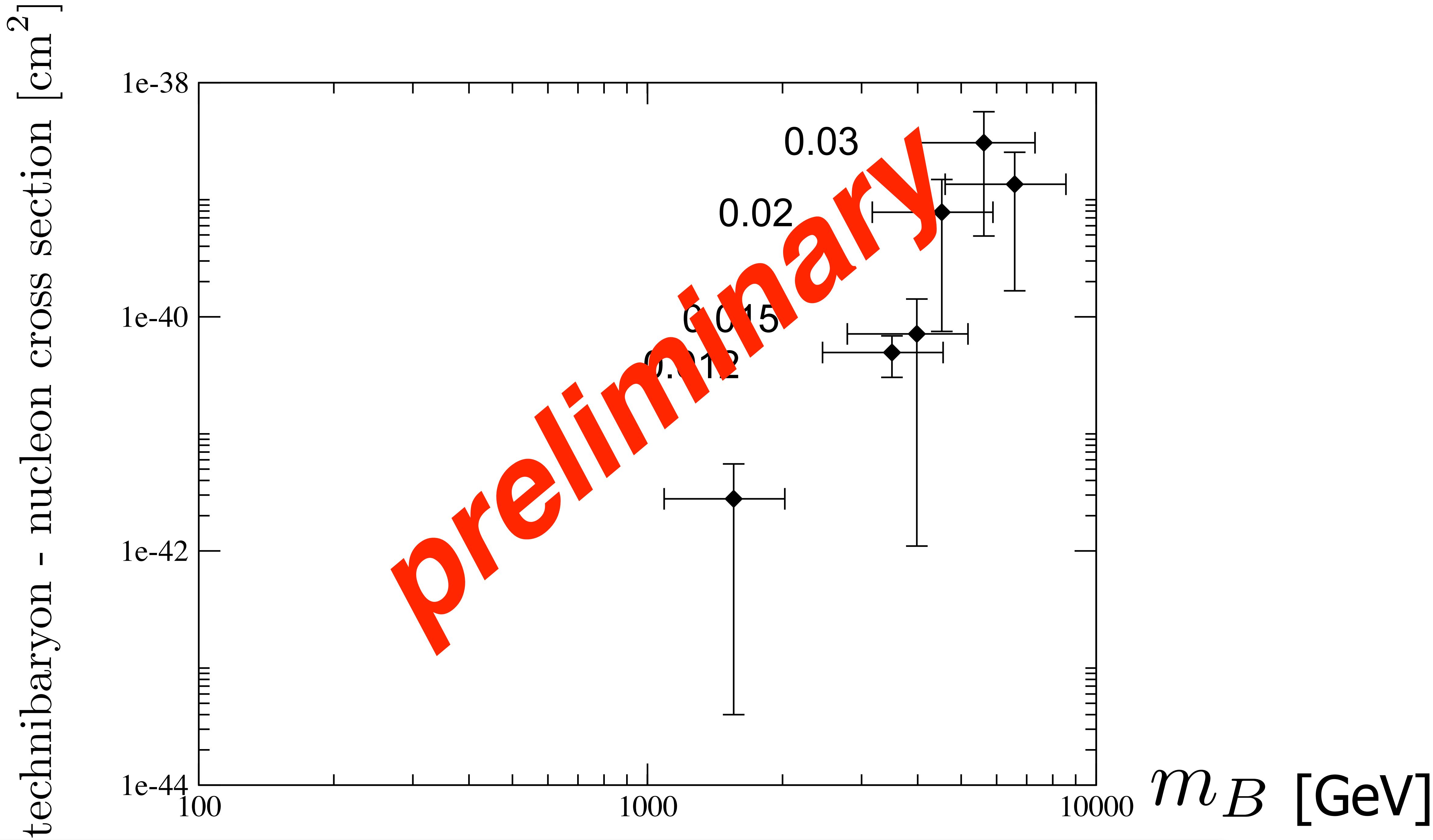
Lattice calculation of the nucleon sigma term (fTq)
Ref [R.D. Young, and A. W. Thomas,'10, HO et al. JLQCD '13,]

LatKMI result

Baryon mass in Nf=8 QCD



Lattice result in Nf=8



scale setting : $\sqrt{N_d}F_\pi/\sqrt{2} = 246[\text{GeV}], \quad (\mathrm{N_d} = 4)$

$F_\pi = 0.0212(12)(^{+49}_{-70})$

Errors come from Fπ & Fσ

Summary

Scalar channel

- Using the flavor singlet scalar correlator, we calculated decay constant as well as mass.
- Signal of Fs is as good as mσ.
- Fσ is related Fs through the WT id.
- Accuracy of the data is not enough to take the chiral limit in Nf=8.
- Very rough estimate suggests Fσ/Fπ ~1.5 Δ , in rough agreement with other measurement (LatKMI, Phys. Rev. D 89, 111502(R) (2014), arXiv:1403.5000)

Baryon channel

- Baryon mass is calculated in Nf=8 QCD
- Combining the result of the dilaton decay constant, we can estimate the dark matter cross section.
- Allowed region for the technibaryon dark matter is severely constrained by current dark matter direct detection.

Thank you