Topological insights in many flavor QCD on the lattice

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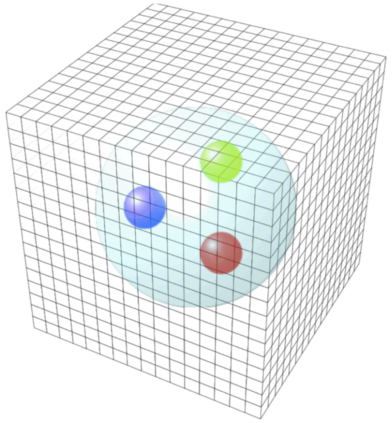
Plan of Talk:



1. Introduction
2. Topological charge and susceptibility

   Mainly, in Nf=8

   ♠ Nf=8 is a candidate for the walking.

3. Eigenvalues and Anomalous dimension
4. Summary, Discussion

1. Introduction

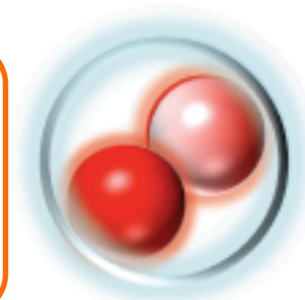
Walking technicolor

N_f massless fermions + SU(N_{TC}) gauge at $O(1)$ TeV

Model requirement:

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range
- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region

- Higgs ≈ Light composite scalar
  pNGB (technidilaton)
  of scale symmetry breaking



$$m_{\mathrm{Higgs}}/v_{\mathrm{EW}} \sim 0.5 = m_\sigma/(\sqrt{N_d}F)$$

F : decay constant, N_d : number of weak doublets

usual QCD $m_\sigma/F \sim 4$–5

In the topological nature,

What happens in the walking/conformal phase?

In hadron phase:

★ Index theorem

fermionic chiral zero mode ⇔ gluonic $F_{\mu\nu}\widetilde{F}_{\mu\nu}$

★ Banks-Casher relation

fermionic zero mode

$$\langle\bar{\psi}\psi\rangle = \lim_{m\to 0}\lim_{V\to\infty}\int d\lambda\rho(\lambda)\frac{2m}{\lambda^2+m^2} = \pi\rho(\lambda=0)$$

★ Leutwyler-Smilga relation $\langle Q_{top}^2\rangle/V = \Sigma m_f/N_f$

★ Eigenvalue distribution

$\rho(\lambda)$ in p- and ε-regime,

$$\nu(\lambda) = \int_{-\lambda}^{\lambda}\rho(\lambda)d\lambda$$

★ Flavor singlet Pseudo-scalar meson ⇐ (famous) U(1) problem

Witten-Veneziano formula

$$m_{\eta'}^2 = \lim_{N_c\to\infty}\frac{2N_f}{F_\pi^2}\chi|_{quenched}\ ,$$

$$m_{\eta'}^2 + m_\eta^2 - 2m_K^2 = \mu_0^2 = \frac{4N_f\chi_T}{f_\pi^2}$$

In Veneziano limit: O(1/Nc) expansion, because of $f_\pi \simeq \sqrt{N_c}$

$$\frac{N_f}{N_c} \ll 1$$

2. Topological charge and susceptibility

Mainly, in Nf=8

- ♠ Nf=8 is a candidate for the walking.

Gradient flow (Wilson flow, Symanzik flow)

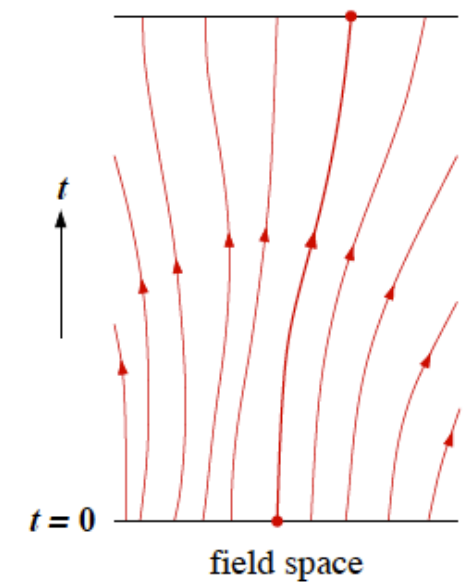
M. Luscher, (2009, 2010)

$$\partial_t V(x,\mu) = -g_0^2\{\partial_{x,\mu} S_g(V_t)\} V_t(x,\mu), \quad V_t(x,\mu)|_{t=0} = U(x,\mu),$$

Example) In the continuum QED,

$$\dot{B}_\mu = D_\nu G_{\nu\mu}, \qquad B_\mu\big|_{t=0} = A_\mu,$$

$$B_\mu(t,x) = \int \mathrm{d}^4 y K_t(x-y) A_\mu(y) + \text{gauge terms}, \qquad K_t(z) = \frac{\mathrm{e}^{-\frac{z^2}{4t}}}{(4\pi t)^2}.$$



In QCD and BSM,

Key technology in the lattice studies, nowadays.

Numerical integration of the Wilson flow (M.Luscher, JHEP1008 (2010) 071)

$$\dot{V}_t = Z(V_t)V_t,$$

4th order Runge-Kutta method

$$W_0 = V_t,$$

$$W_1 = \exp\{\tfrac{1}{4}Z_0\}W_0,$$

$$W_2 = \exp\{\tfrac{8}{9}Z_1 - \tfrac{17}{36}Z_0\}W_1,$$

$$V_{t+\epsilon} = \exp\{\tfrac{3}{4}Z_2 - \tfrac{8}{9}Z_1 + \tfrac{17}{36}Z_0\}W_2,$$

where

$$Z_i = \epsilon Z(W_i), \qquad i = 0, 1, 2.$$

Our case:

Wilson flow (S_{Wilson}) → Symanzik flow ($S_{Symanzik}$)

ε =0.03

(If ε =0.1 for instance, RK doesn't solve correctly or 5th order RK is needed.

We re-use LatKMI configurations generated for the flavor-singlet scalar meson.

Tree-level Symanzik gauge + HISQ fermions

L^3 x (4L/3)

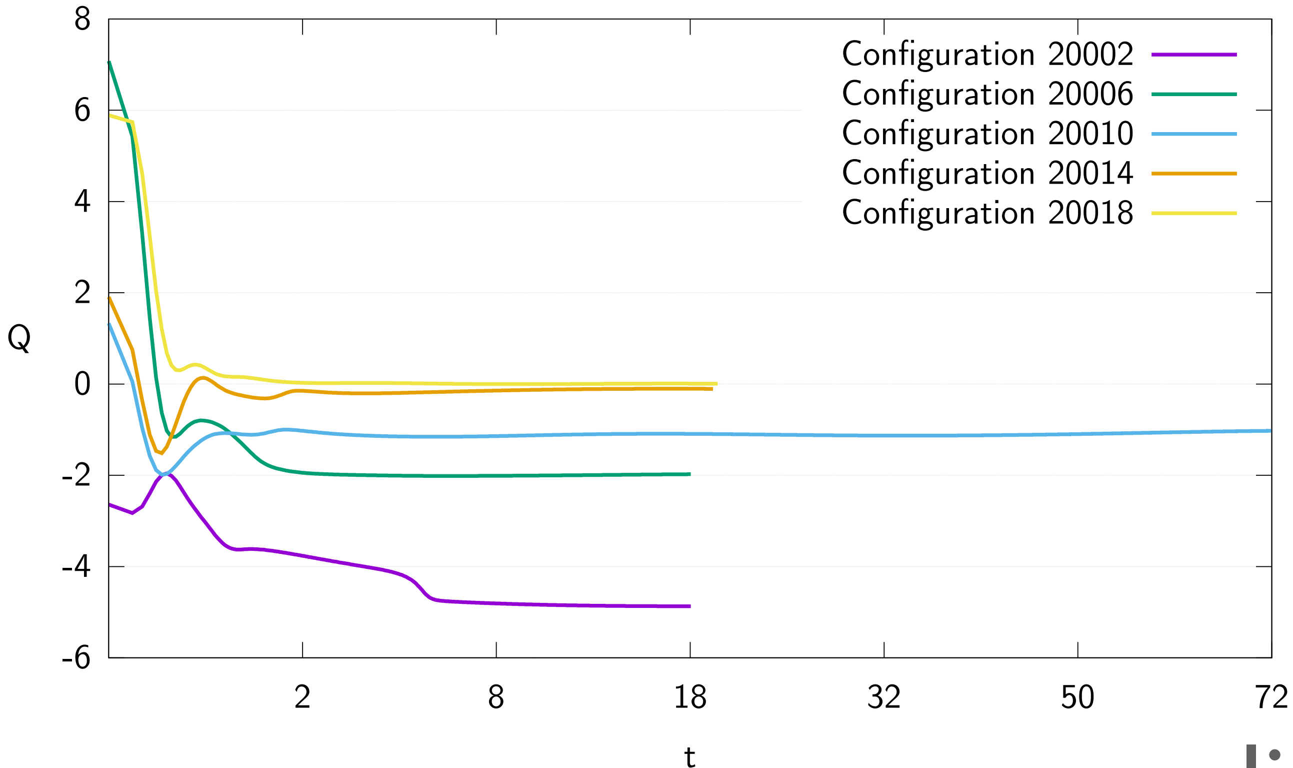
Periodic boundary condition in spatial-dir.

Anti-PBC in time-dir.

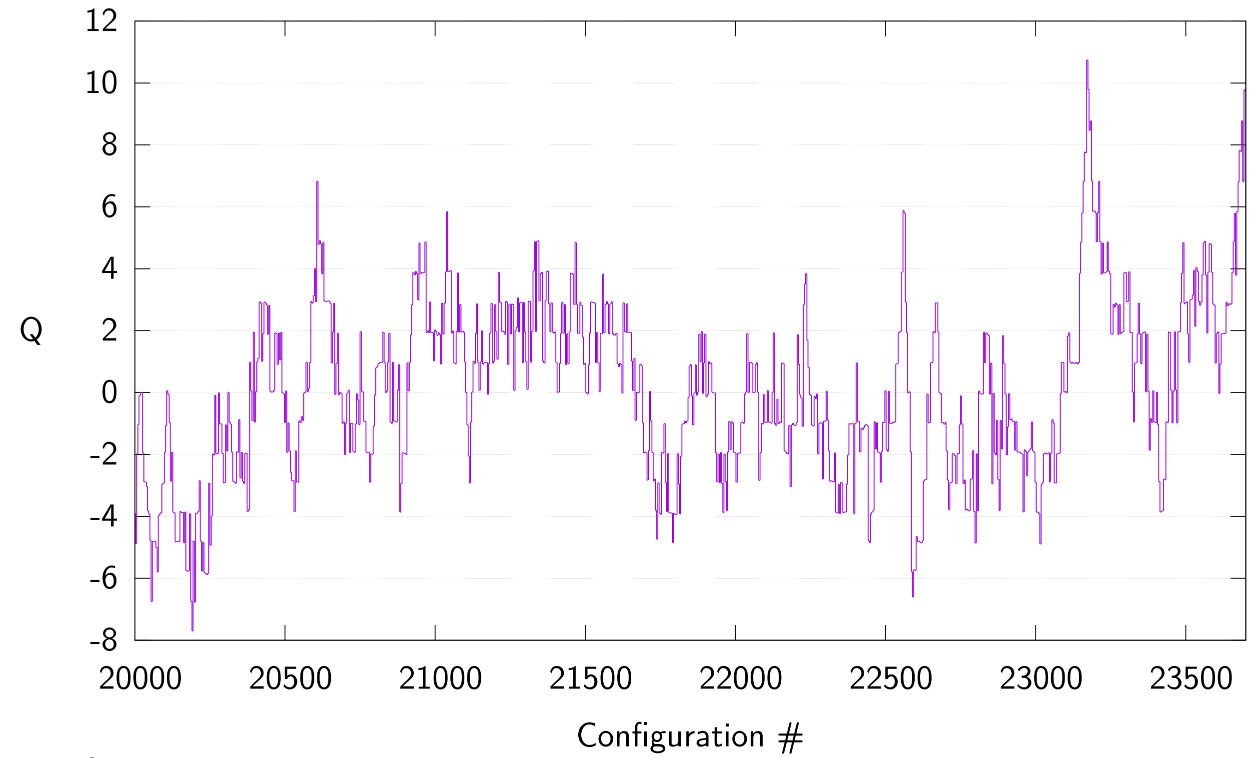
L=12, 18, 24, 30, 36, 42, (48)

In Nf=8 at mf=0.06 on L=24

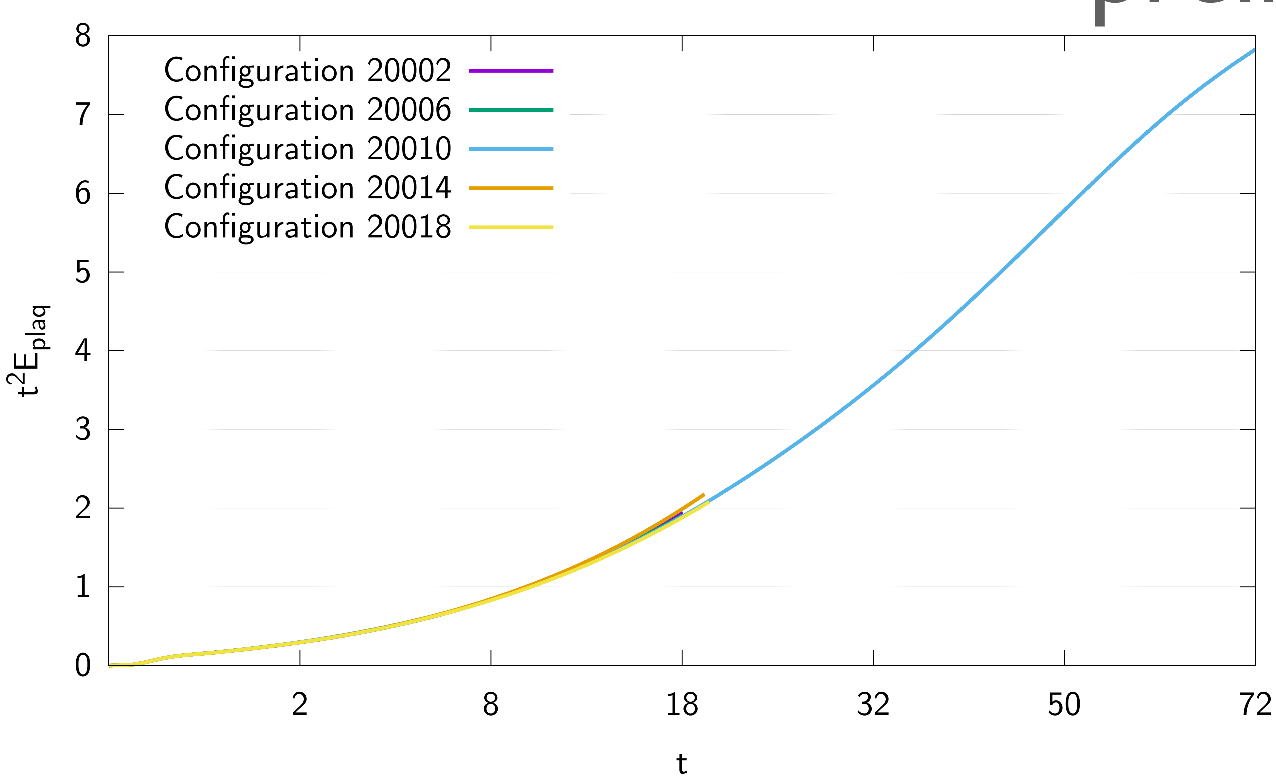
Qtop vs Symanzik flow time



Qtop vs #trajectory

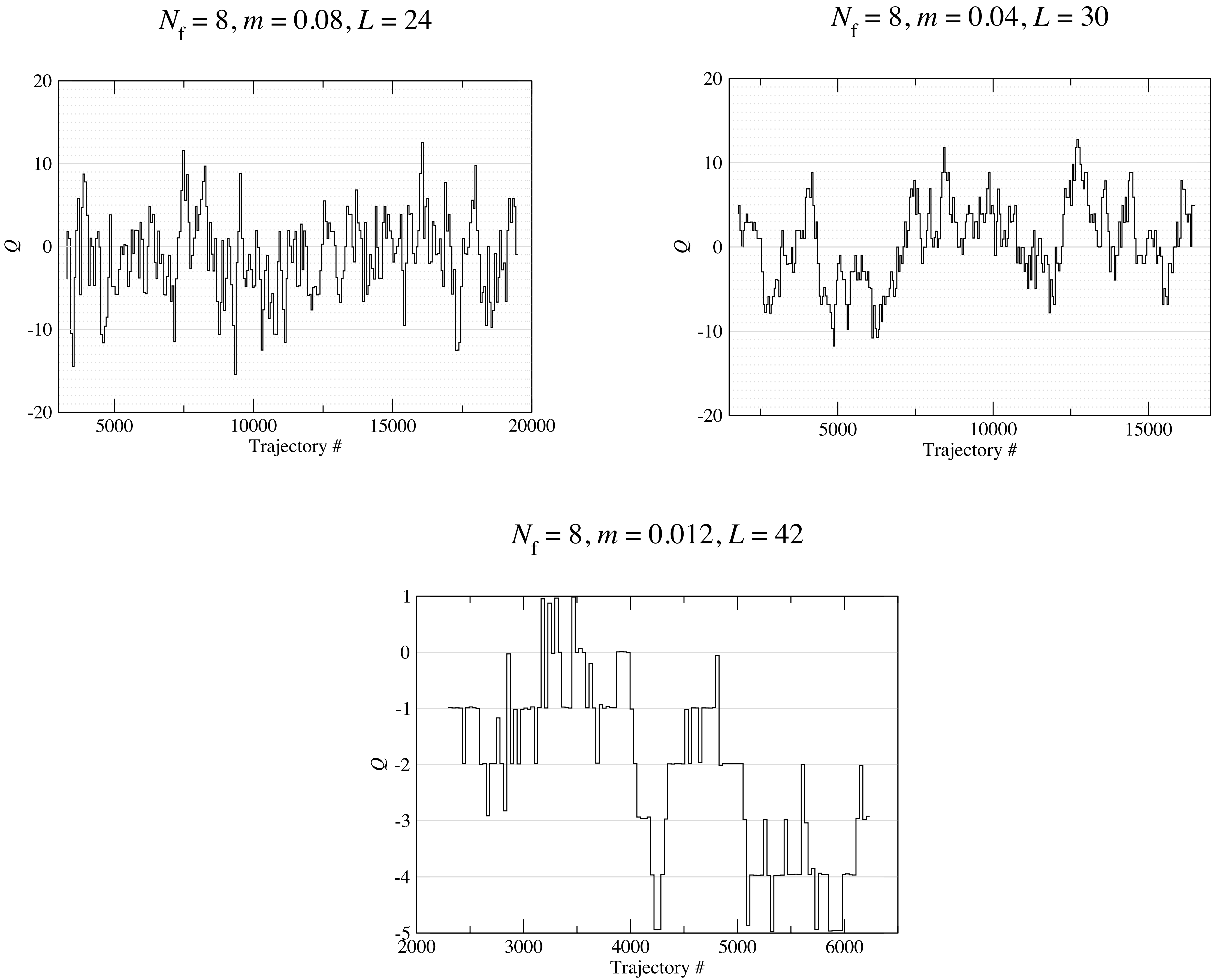


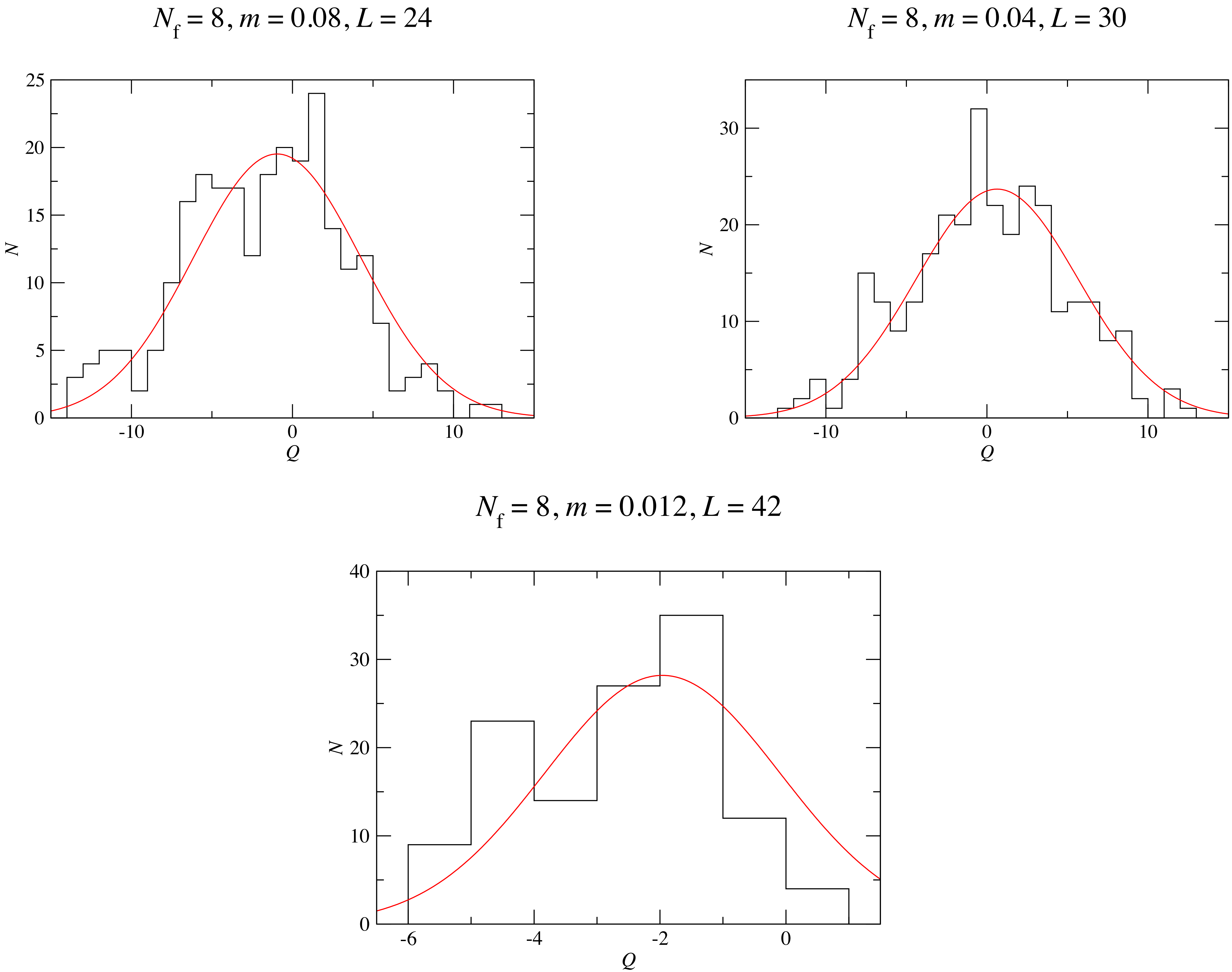
preliminary

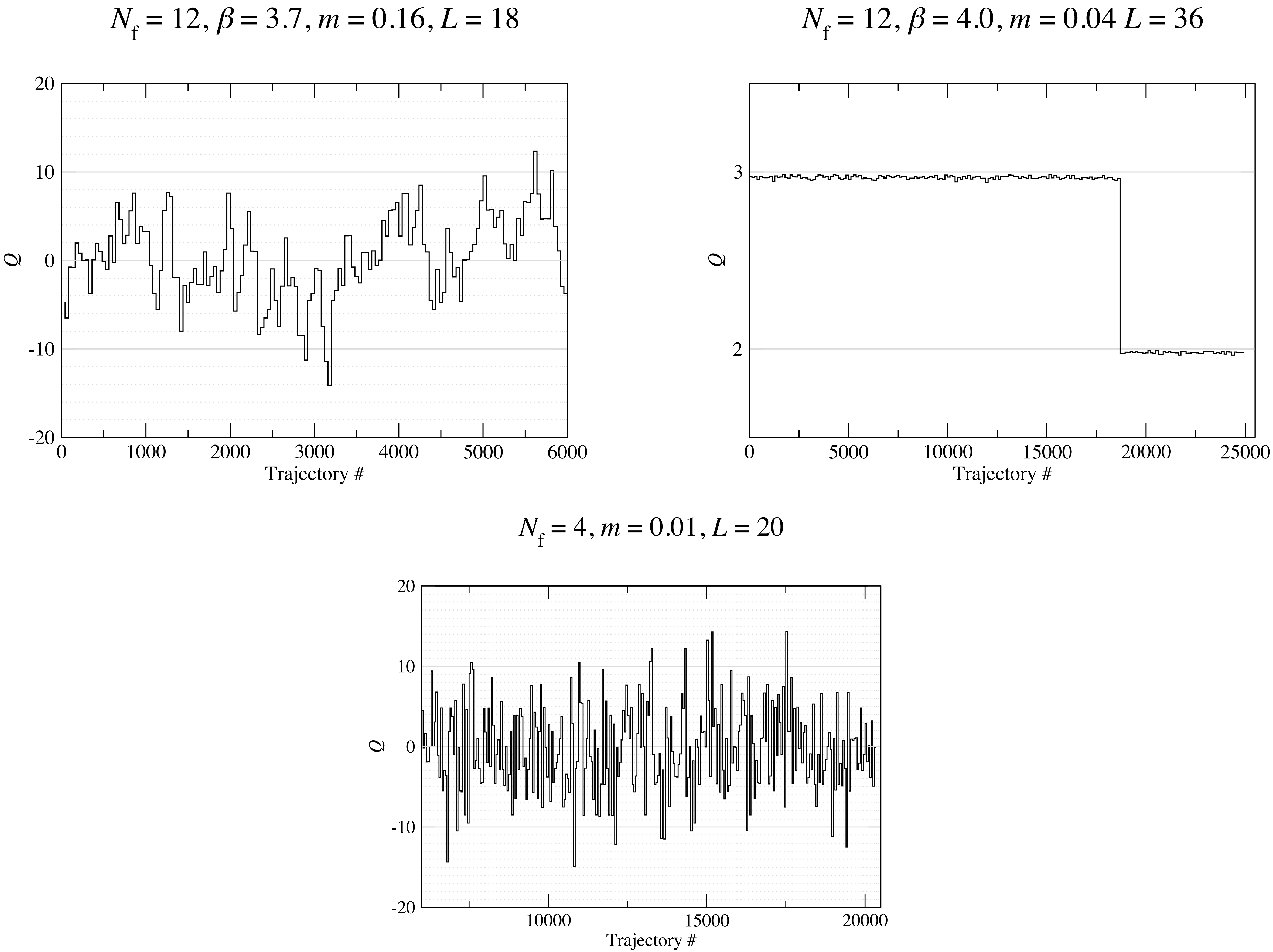


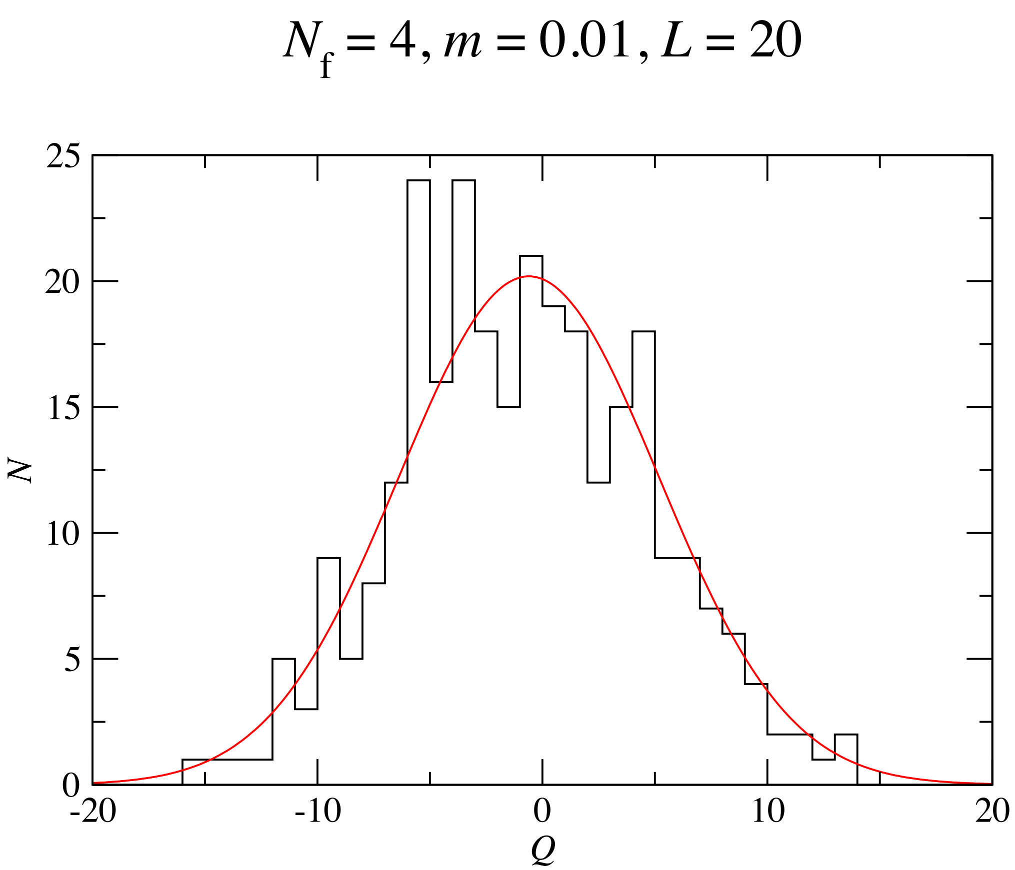
t^2E_{plaq} vs flow time

Scale determination: $t^2\langle E\rangle\big|_{t=t_0} = 0.3$

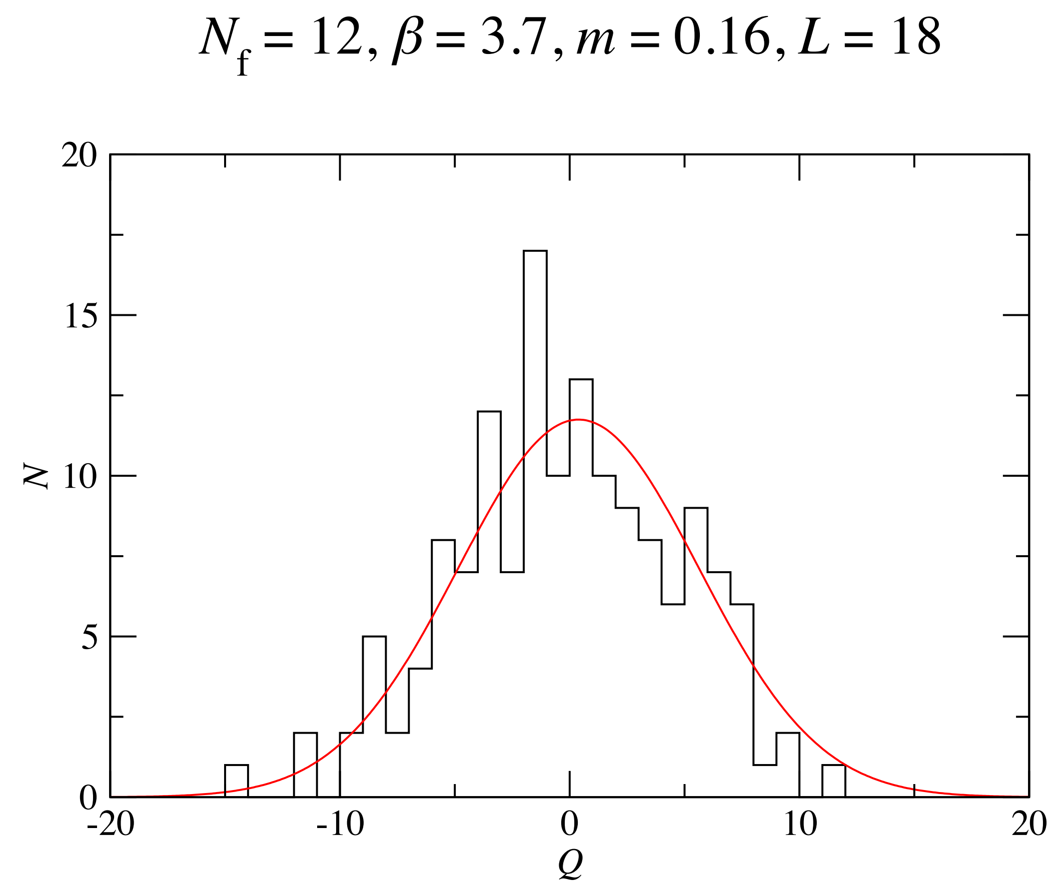

$N_f = 8, m = 0.08, L = 24$
$N_f = 8, m = 0.04, L = 30$
$N_f = 8, m = 0.012, L = 42$
Q
Trajectory #


$N_f = 8, m = 0.08, L = 24$
$N_f = 8, m = 0.04, L = 30$
$N_f = 8, m = 0.012, L = 42$
N
Q
25
20
15
10
5
0
-10
0
10
30
40
-6
-4
-2


$N_f = 12, \beta = 3.7, m = 0.16, L = 18$
$N_f = 12, \beta = 4.0, m = 0.04\ L = 36$
$N_f = 4, m = 0.01, L = 20$
Q
Trajectory #
20
10
0
-10
-20
0
1000
2000
3000
4000
5000
6000
3
2
5000
10000
15000
20000
25000



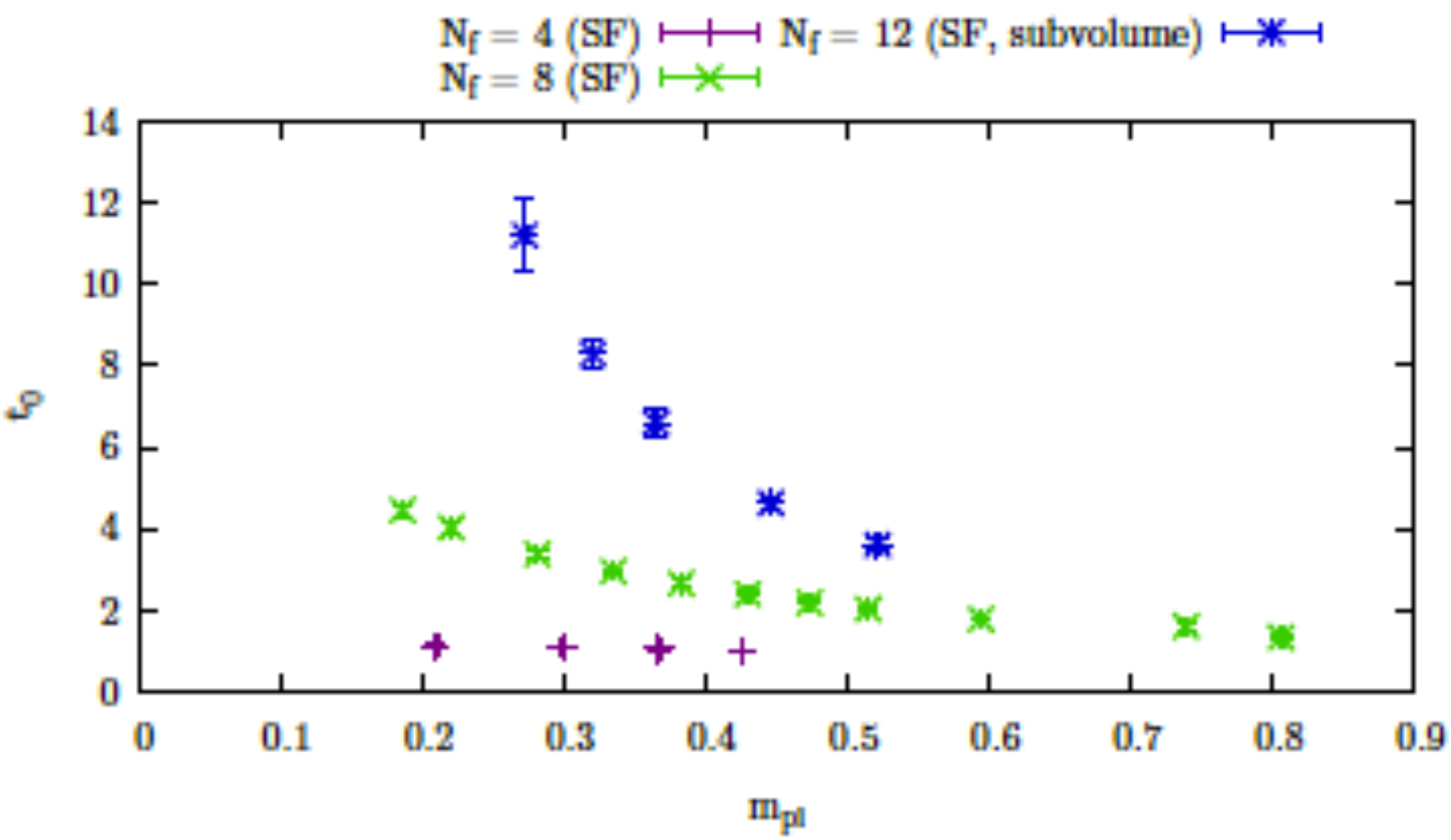
$N_f = 4, m = 0.01, L = 20$
N
Q



$N_f = 12, \beta = 3.7, m = 0.16, L = 18$
N
Q

t_0 vs m_π :

Scale determination: $t^2\langle E\rangle\big|_{t=t_0} = 0.3$



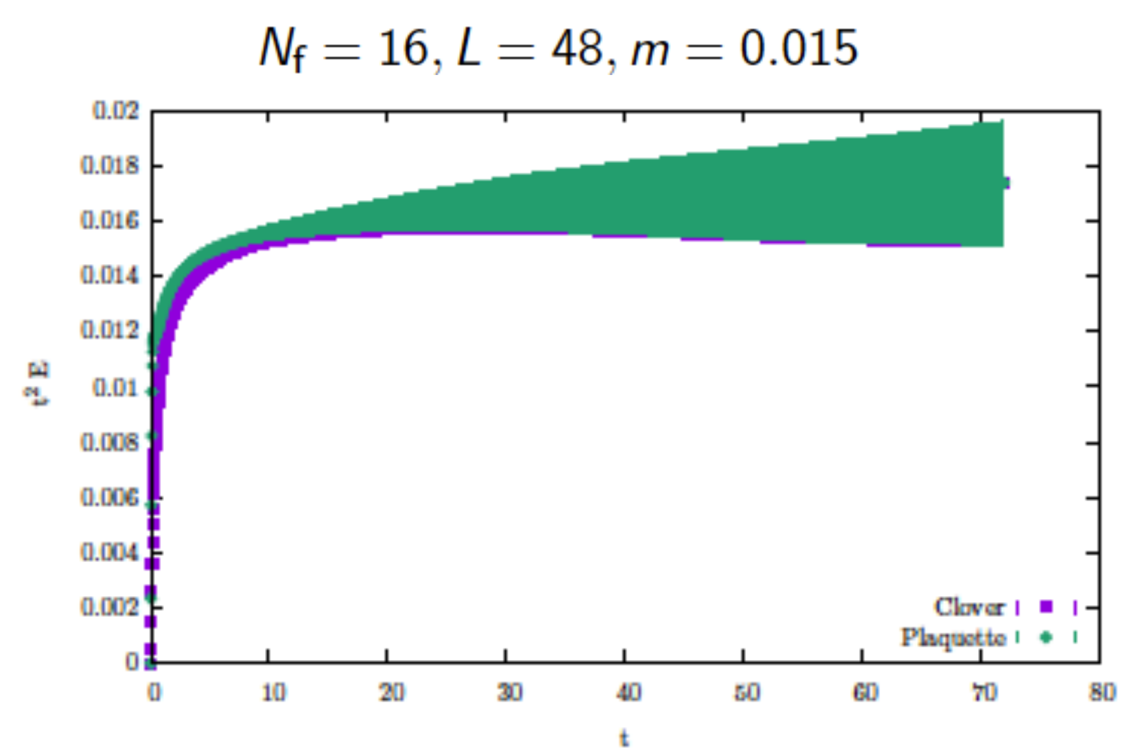
Nf=4; flat in $m_\pi \to 0$

Nf=12; blowup in $m_\pi \to 0$

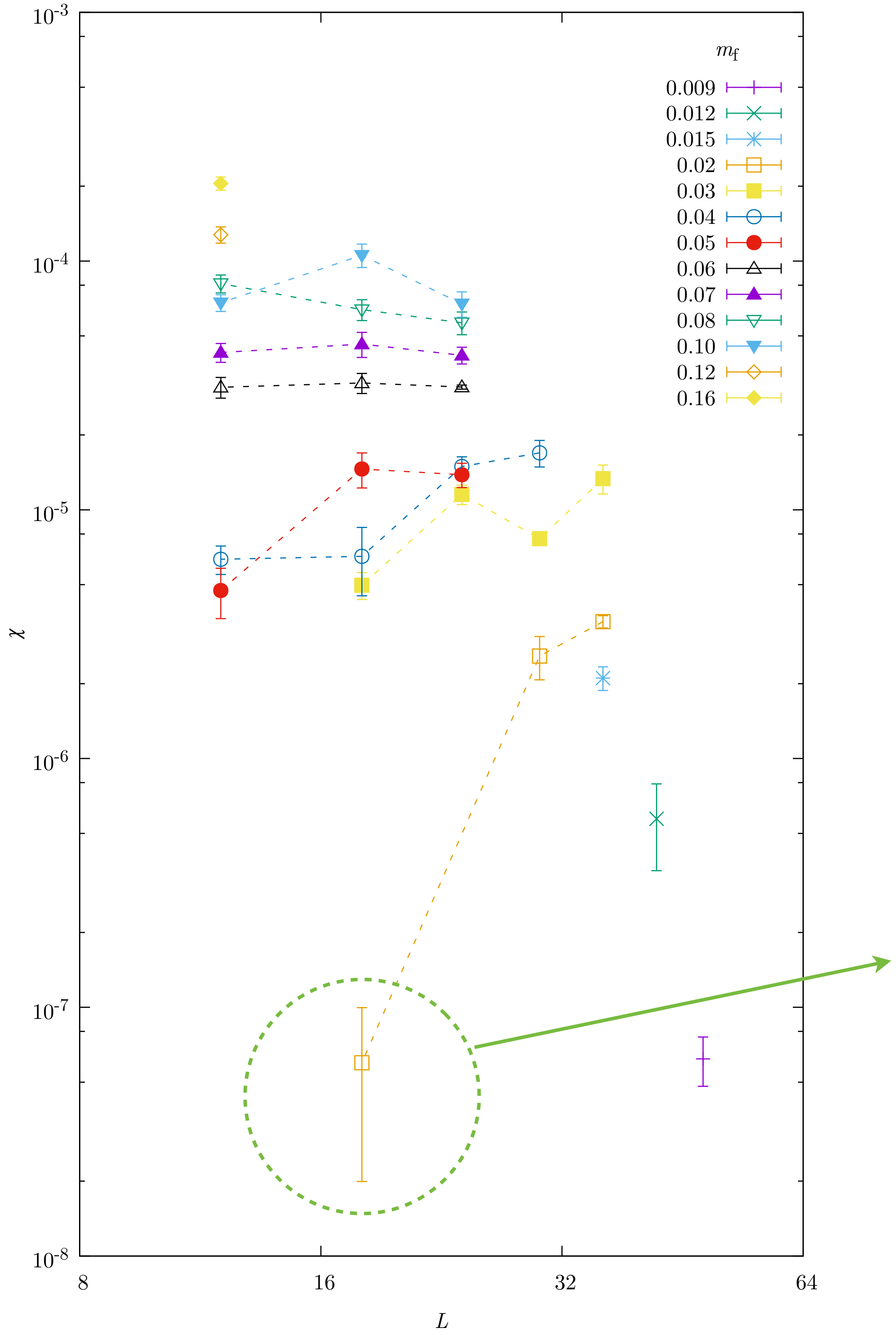
Nf=8; ? in $m_\pi \to 0$

t^2E vs t (flow time) in Nf=16

We can't extract t_0 for the current $N_f = 16$ data, as t^2E flattens off before it reaches 0.3.



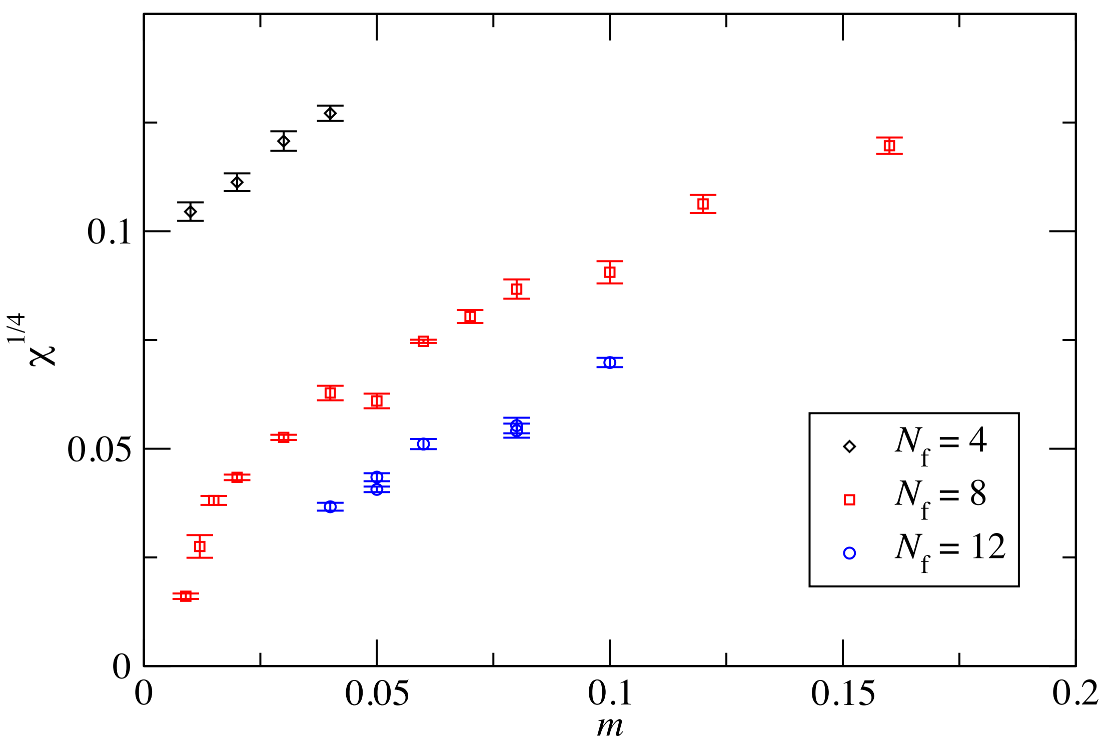
Nf=8 Finite volume study of χ_{top}



At small mf on small Vol., Qtop is almost frozen.

Analysis of χ top -(1)

$\chi^{1/4}$ vs mf



In hadron phase: (ChPT)

$$\chi = Cm_f + f(a)$$

f(a): lattice artifact

$$\chi^{1/4} = (Cm_f + f(a))^{1/4}$$

(blue line)

In conformal phase: (Hyperscaling)

$$\chi^{1/4} = Am_f^{\frac{1}{1+\gamma}}$$

(green line)

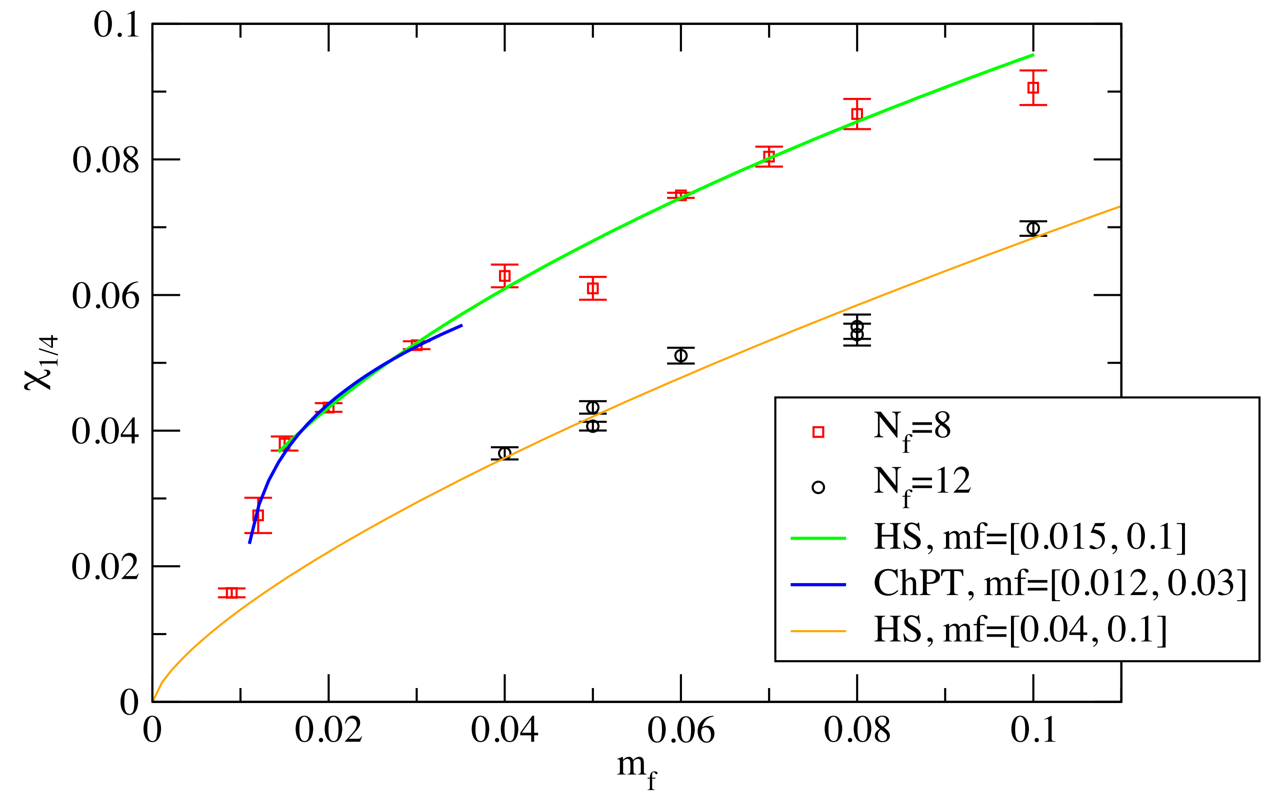
both ChPT and Hyperscaling

→ It seems to be good.

γ =1.04(4) in Nf=8

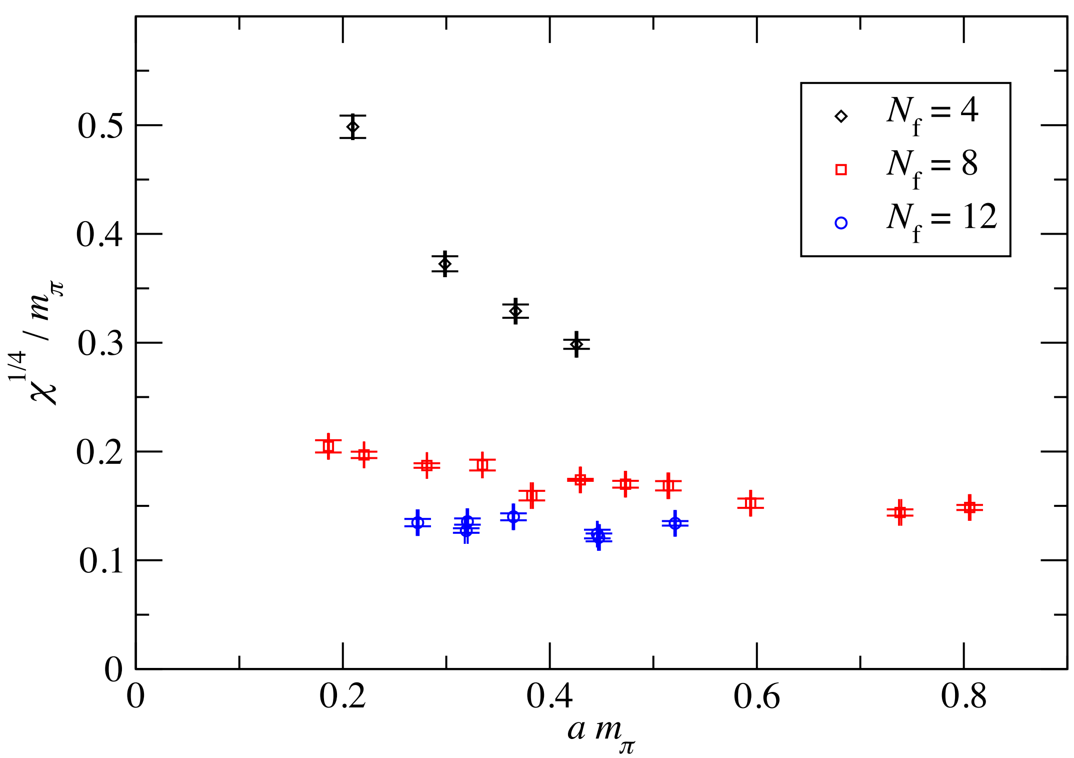
γ =0.43(5) in Nf=12

Nf=8 and 12 data and fit

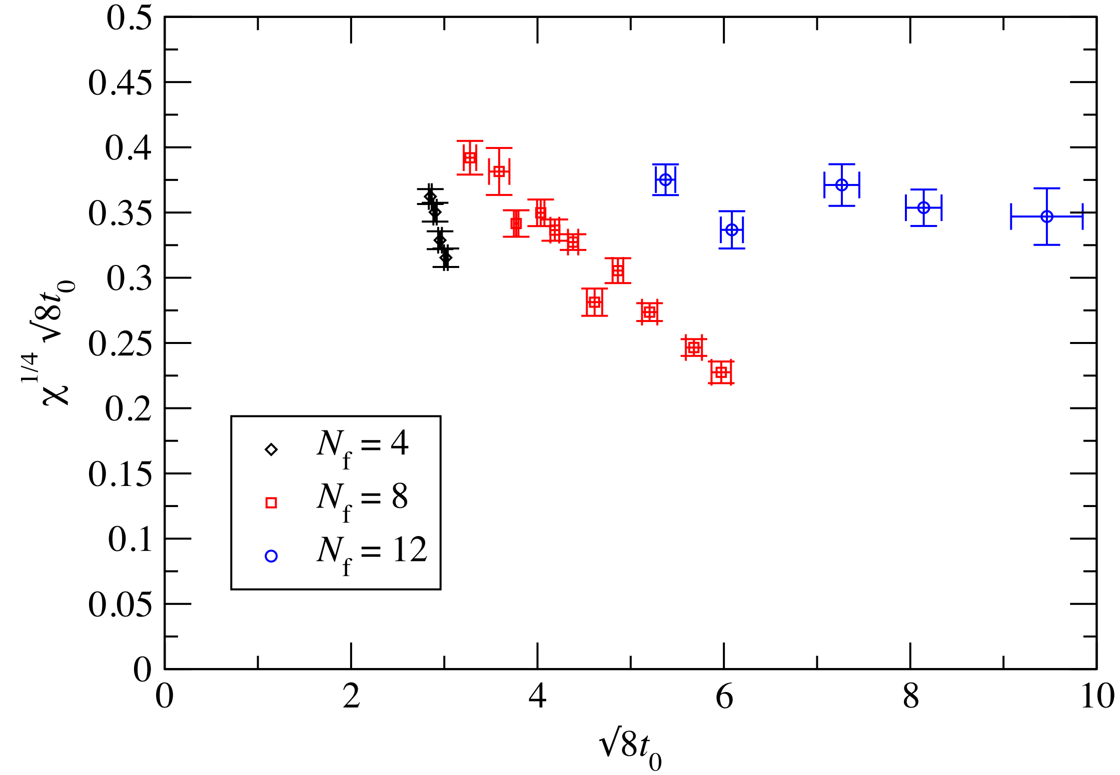


Analysis of $\chi_{\rm top}$ -(2)

$\chi^{1/4}/m_\pi$ vs m_π



$\chi^{1/4}\sqrt{8t_0}$ vs $\sqrt{8t_0}$



Nf=4: $\chi^{1/4}/m_\pi$ & $\chi^{1/4}\sqrt{8t_0}$ for m_π & $\sqrt{8t_0} \to 0$

steep slope

Nf=12: $\chi^{1/4}/m_\pi$ & $\chi^{1/4}\sqrt{8t_0}$ for m_π & $\sqrt{8t_0} \to 0$

flat

Nf=8: between Nf=4 and 12

Summary-(1): Q_{top} and χ_{top}

From Q_{top} and χ_{top},

it is difficult with current data to determine whether Nf=8 is confining/walking/conformal.

γ=1.04(4)

However,

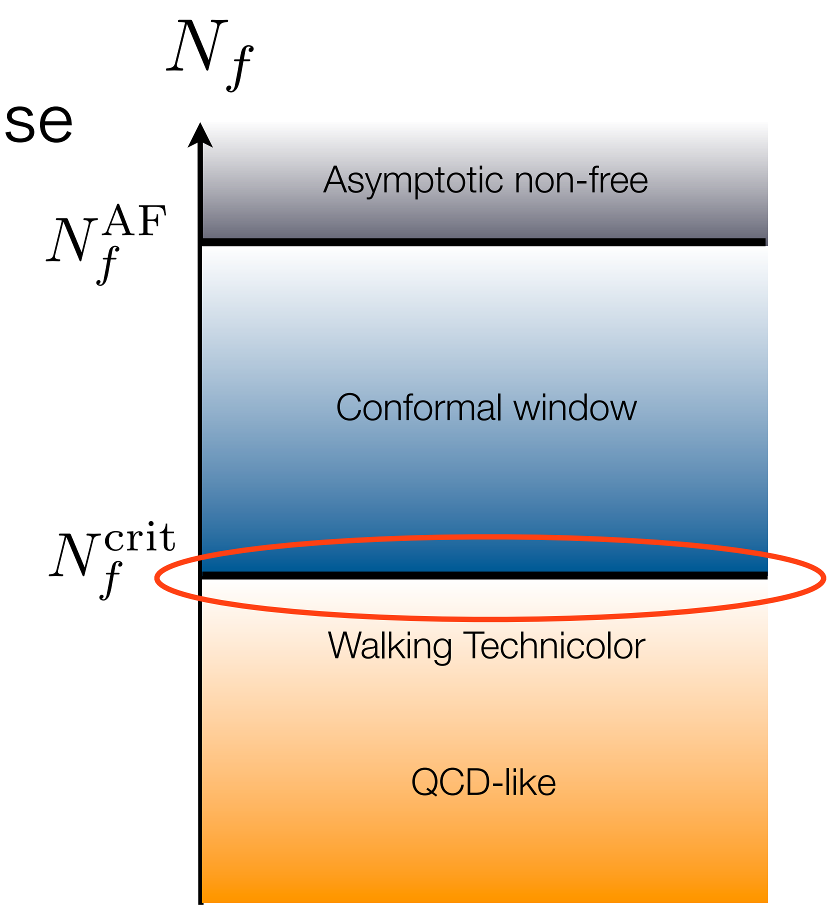
χ_{top} in Nf=8 is just between Nf=4 and 12.

{ If χ_{top} in Nf=4 is regarded as in the hadron phase
If χ_{top} in Nf=12 is regarded as in the conformal phase

γ(Nf=12)=0.43(5)

⇒ Nf=8 is in the near-conformal/walking phase.
(near the conformal edge)

We have to confirm this conjecture.



3. Eigenvalues and Anomalous dimension

★ Contribution of λ~0 region (IR)

$$\langle\bar{\psi}\psi\rangle = \lim_{m\to 0}\lim_{V\to\infty}\int d\lambda\rho(\lambda)\frac{2m}{\lambda^2+m^2} = \pi\rho(\lambda=0)$$ Banks-Casher relation

$\rho(\lambda)$ is measured in the dynamical gauge background.

$$\Rightarrow \quad \rho_{m_f}(\lambda) = \langle\rho(\lambda)\rangle_{m_f}$$

$$\langle\mathcal{O}\rangle_{m_f} = \int dU\mathcal{O}(U)\det(i\lambda(U)+m_f)\exp(-S_g(U))$$

$$\nu(\lambda, m_f) = \int_0^\lambda \rho_{m_f}(\omega)d\omega$$

, $\lambda=\sqrt{EV(D^\dagger_{HISQ}D_{HISQ})}$, for $D^\dagger_{HISQ}D_{HISQ}+m_f^2$

If the system is in the conformal,

$$\nu(\lambda, m_f) = d_1\lambda^{\alpha+1}$$

$$\alpha+1 = \frac{4}{1+\gamma}$$

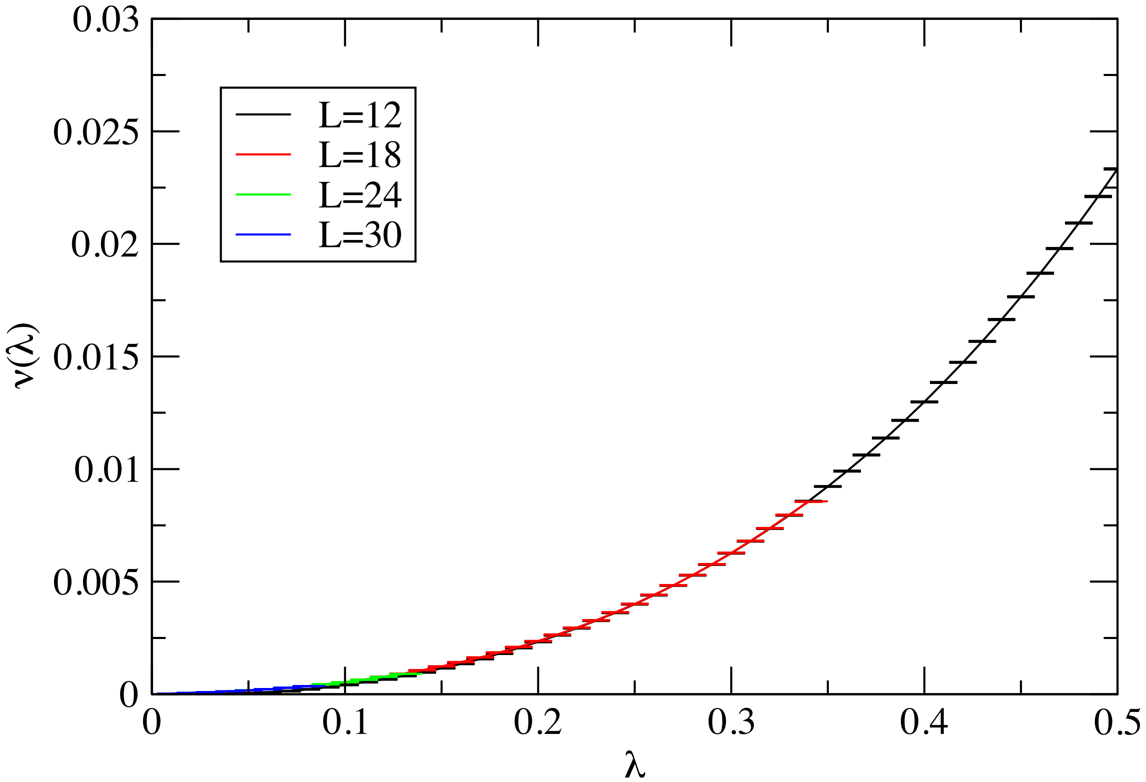
γ is a function of λ and m_f; $\gamma=\gamma(\lambda, m_f)$

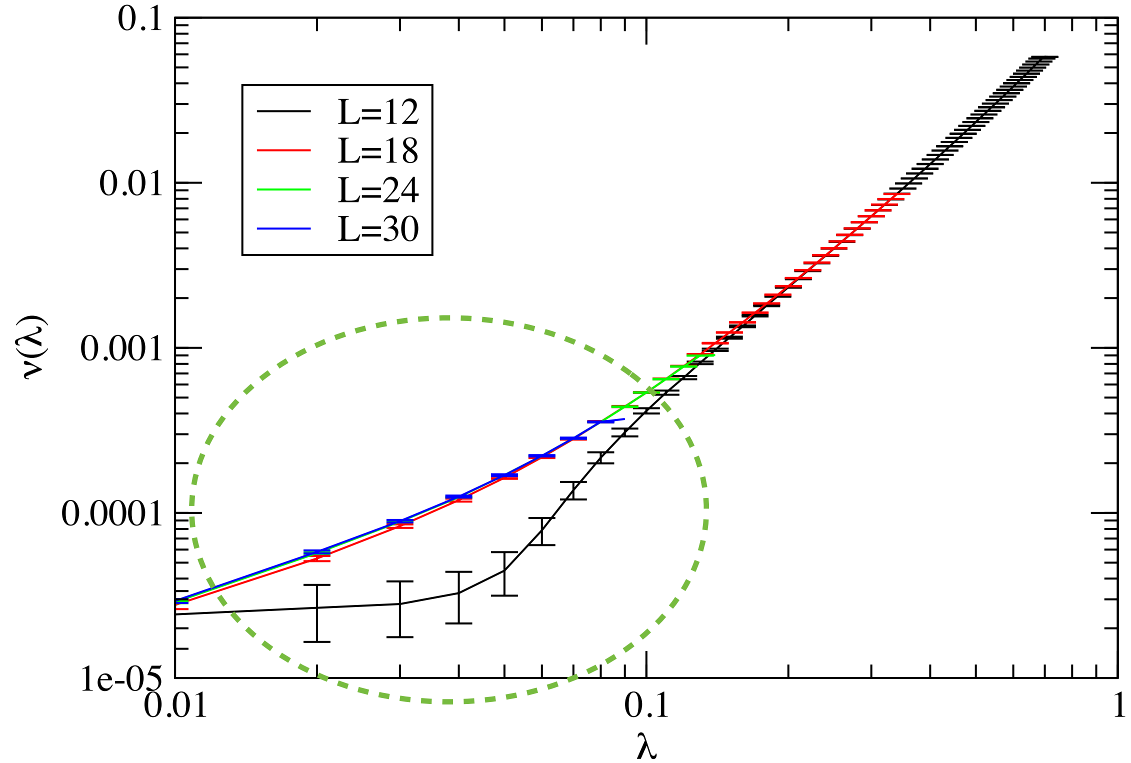
$$\alpha+1 = \frac{\ln\nu_2-\ln\nu_1}{\ln\lambda_2-\ln\lambda_1}$$ where $$\lambda_2 = \lambda_1+\Delta$$

⇒ local γ (γ_{eff})

Nf=8, mf=0.04, mode number counting, $\nu(\lambda)$

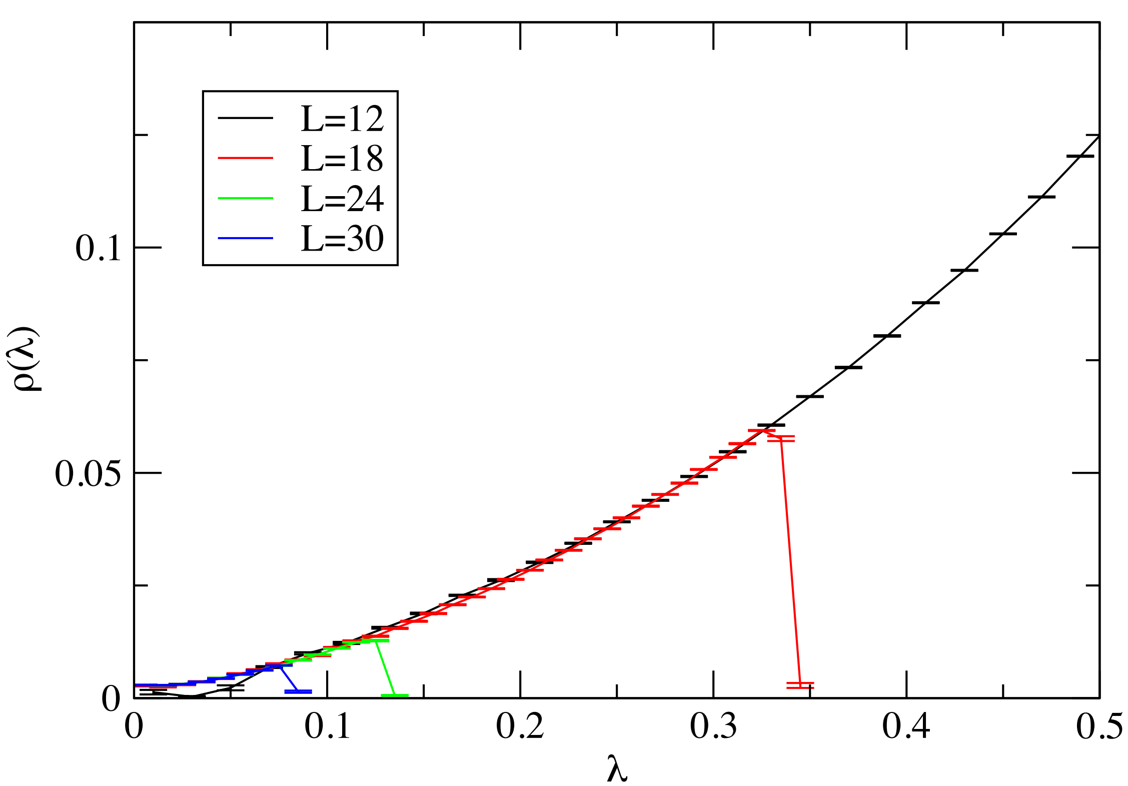
log-log plot

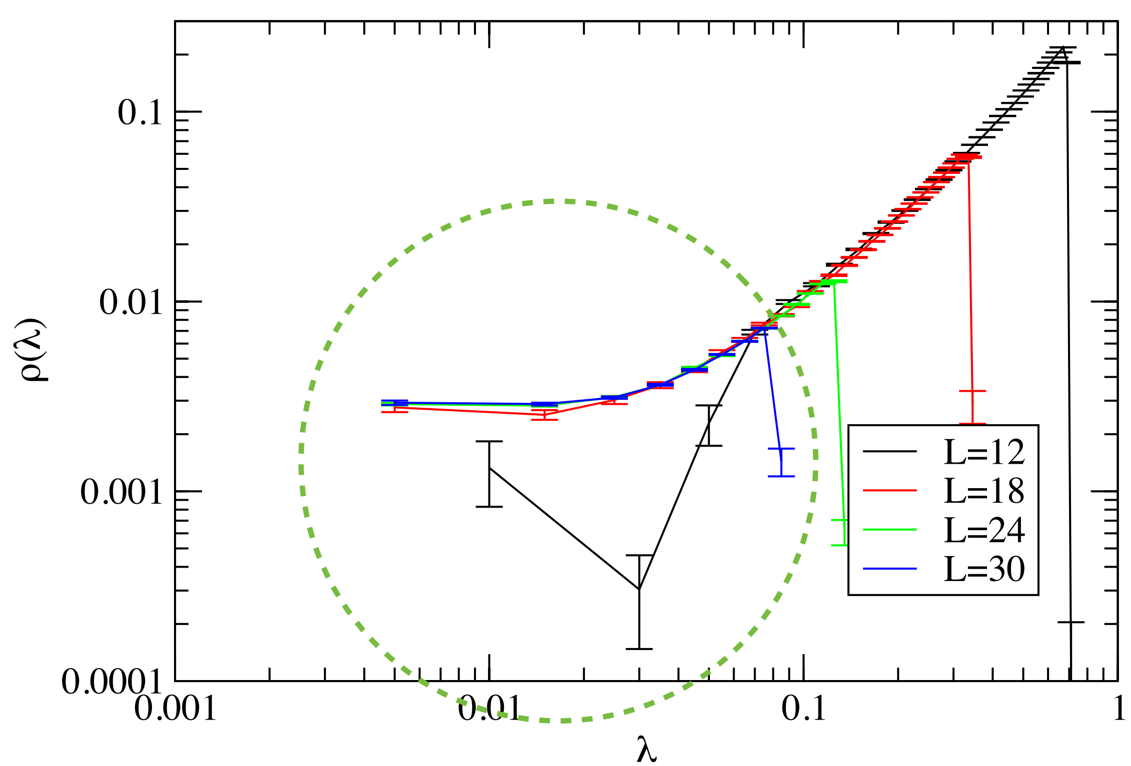




Nf=8, mf=0.04, spectral density, $\rho_{mf}(\lambda)$

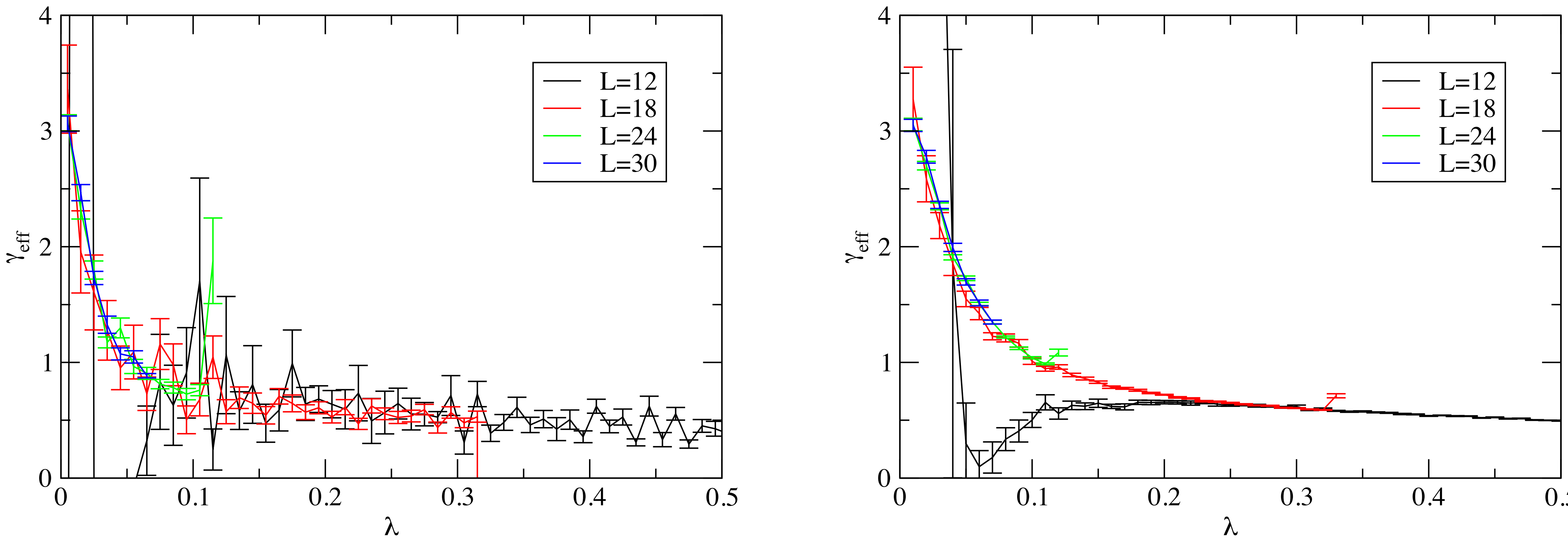
log-log plot





Effective γ eff from $\rho(\lambda)$ and $\nu(\lambda)$

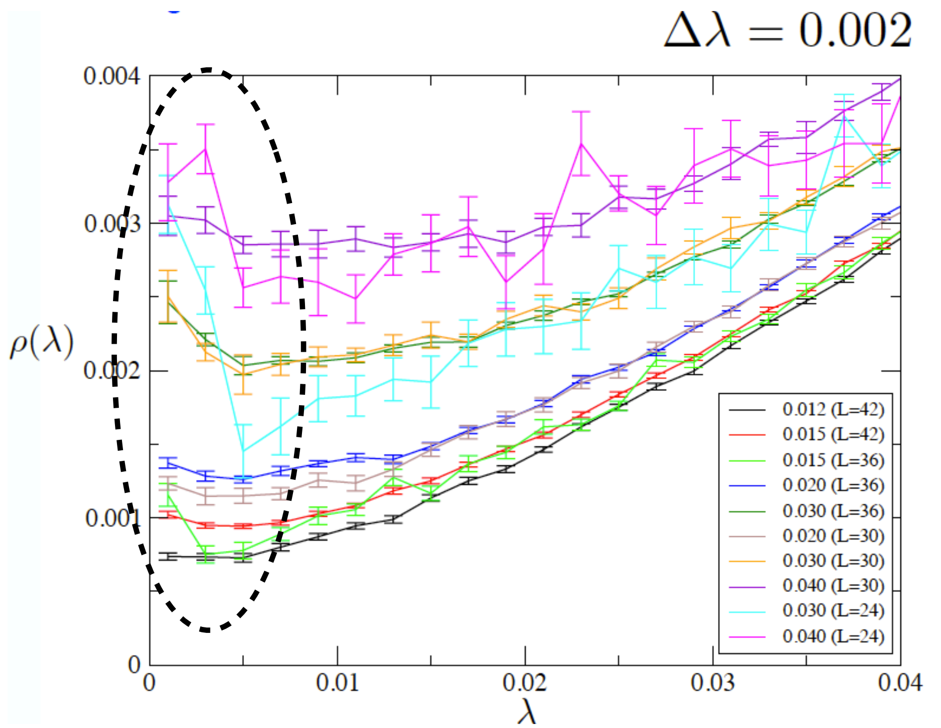
from $\rho_{mf}(\lambda)$ **Nf=8, m=0.04** from $\nu(\lambda)$

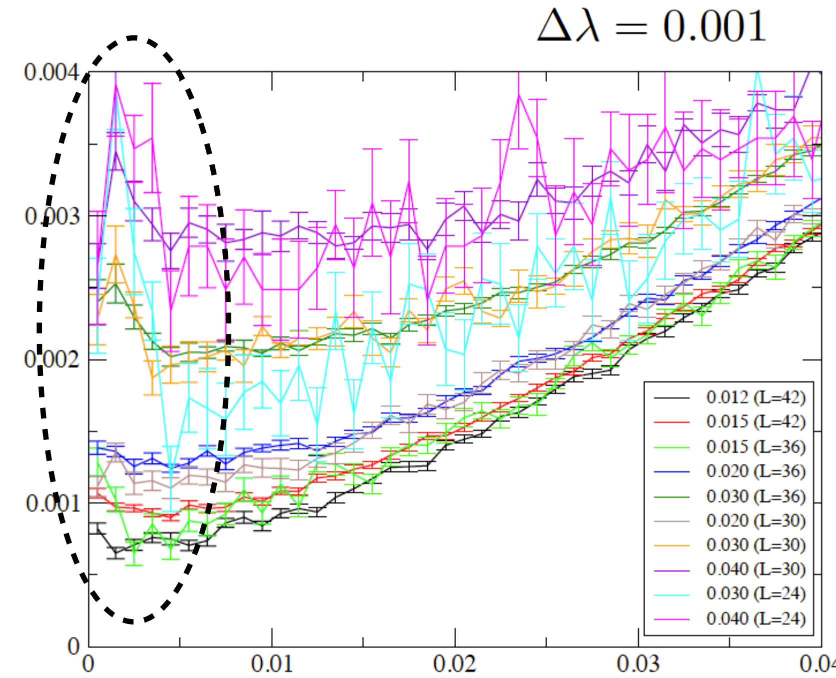


γ_{eff} =0.5~0.7 for λ>0.1~0.2 (>>m_f)

This value is smaller than γ from the spectroscopy.

Behavior and Care in λ~0 region:





At λ~0, $\rho(\lambda)$ has the strong dependence of $\Delta\lambda$ (=bin size of histogram).

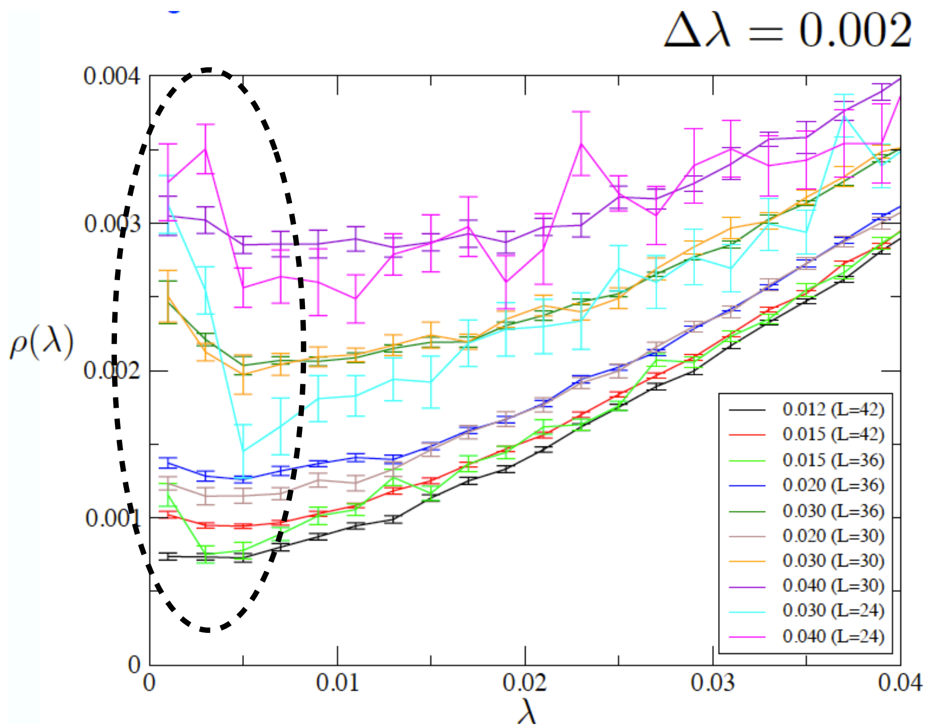
For finer $\Delta\lambda$, $\rho(\lambda$~0) has a peak. → mass deformed effect?

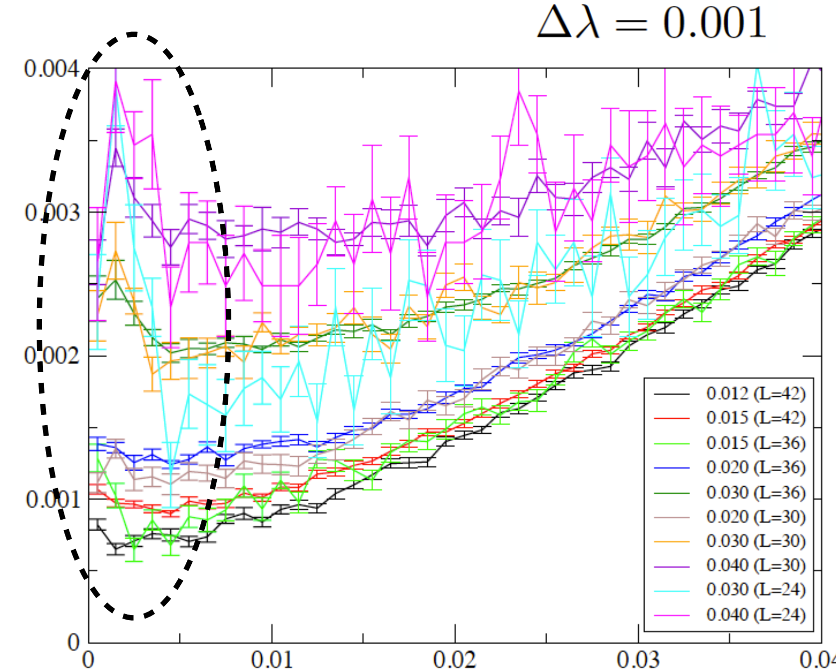
$$\rho_{m_f}(\lambda) = c_0 + c_1\lambda^\alpha$$ is not valid.

$$\nu(\lambda, m_f) = \int_0^\lambda \rho_{m_f}(\omega)d\omega$$

$$\nu(\lambda, m_f) = d_0\lambda + d_1\lambda^{\alpha+1}$$ doesn't describe exactly this behavior.

Behavior and Care in λ~0 region:



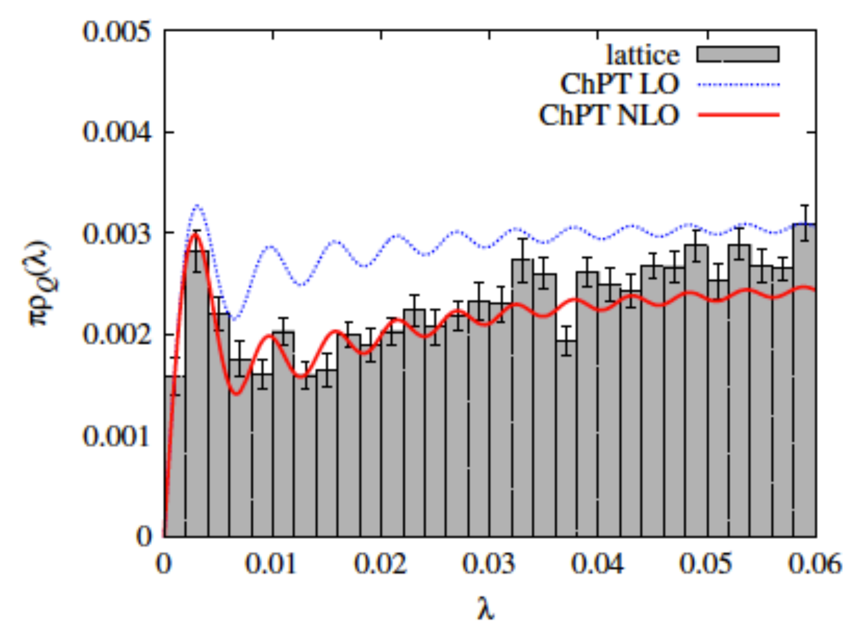


At λ~0, $\rho(\lambda)$ has the strong dependence of $\Delta\lambda$ (=bin size of histogram).

For finer $\Delta\lambda$, $\rho(\lambda$~0) has a peak. → mass deformed effect?

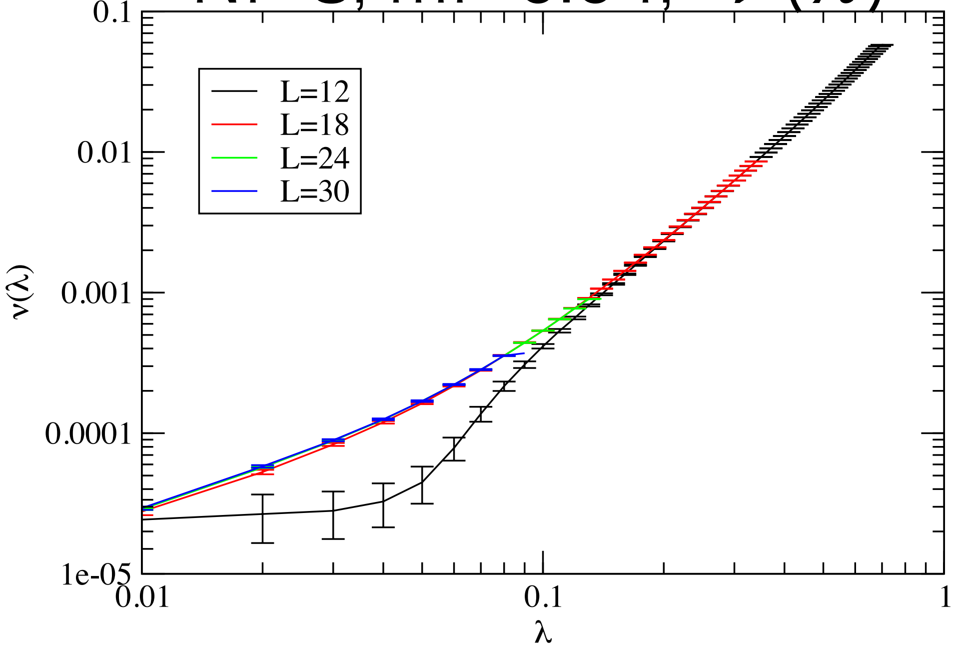
Is it similar to the ε-regime ChPT?

c.f. H.Fukaya et al., PRL104(2010)122002



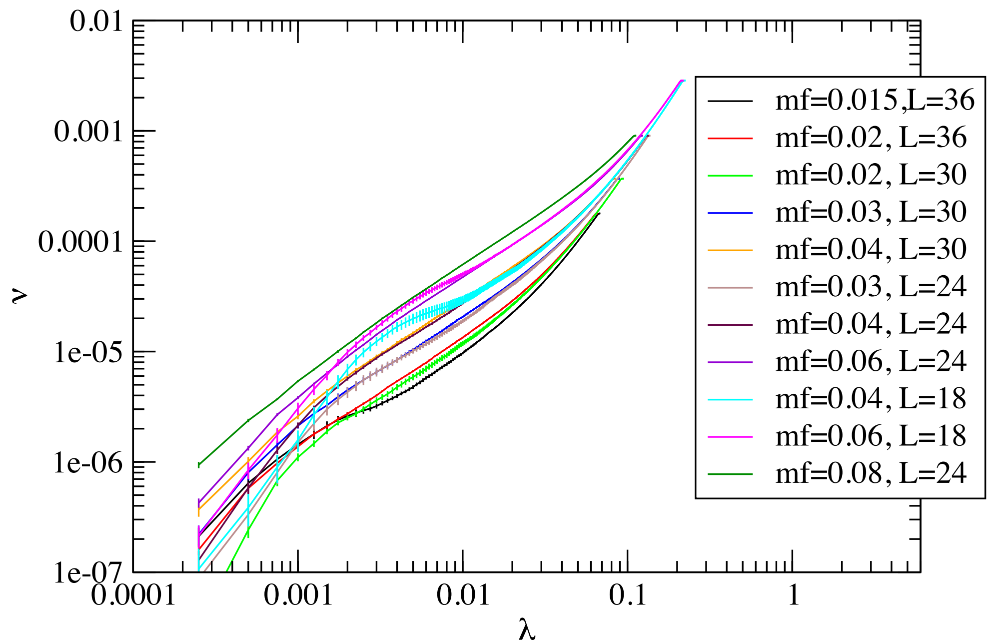
Furthermore; to consider λ~0 (IR) region

Nf=8, mf=0.04, $\nu(\lambda)$



Size independent for λ>0.2 >> mf

Nf=8, ν(λ, mf, L) Δ=0.00025



It seems that $\nu(\lambda)$ is independent of mf
for λ>0.2 >> mf

For λ<0.1, $\nu(\lambda)$ is not simple power behavior.

⇒ γ obtained from λ>0.1~0.2 ≡ the anomalous dimension?

Summary-(2): EV and γ

From EV distribution and the mode number counting,

γ =0.5~0.7 (Nf=8) for λ>0.1~0.2 (>>m_f)

smaller than that from spectroscopy (γ=0.7~1.0)

Why? ⇒ We should make clear.

We estimate γ from large λ region. (λ>>mf)

⇒ near UV λ? (very far from IR?)

At λ~0, the peak in $\rho(\lambda)$ appears. Is this similar to the ε-regime ChPT?

(Due to mass deformed, mf≠0 effect ?)

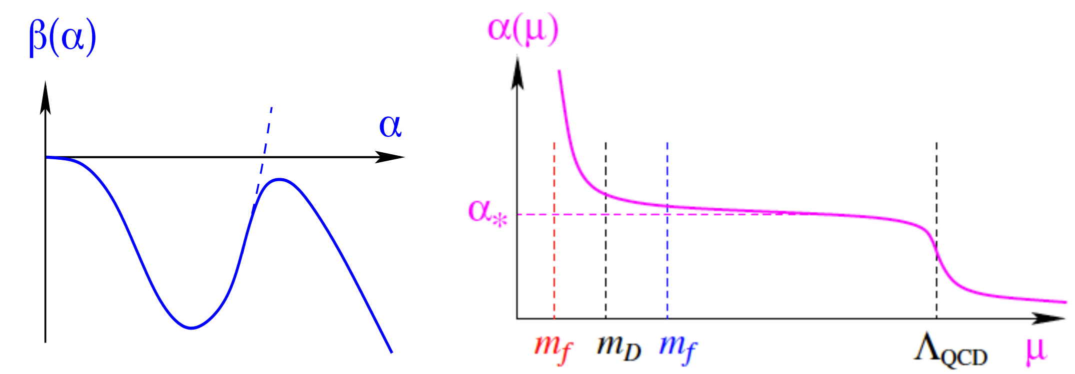
Our simulation is done in mf≠0 dynamical gauge background.

⇒ How to take mf→0 ?

⇒ How to treat λ~0 region ?

Summary

- SU(3) gauge theories with 8 HISQ quarks.
  Preliminary (data updated: 2013→2014→2015)



- $F_{\mu\nu}\widetilde{F}_{\mu\nu}$ + Gradient flow
- Topological charge & susceptibility ⇒ γ =1.04(4) in Nf=8

  it is difficult with current data to determine whether Nf=8 is confining/walking/conformal.

- Dirac Eigenvalue Distribution (Mode Number Counting)

  ⇒ Anomalous dimension ⇒ but, small value

  Care of λ=0 region and mf→0

We have a lot of issues to understand a large Nf QCD

Furthermore,

What we are doing, What we are planning.

Discussion:

In conformal/walking phase

(0) In each Qtop sector

(1) Index theorem

(2) Banks-Casher relation

(3) Leutwyler-Smilga relation

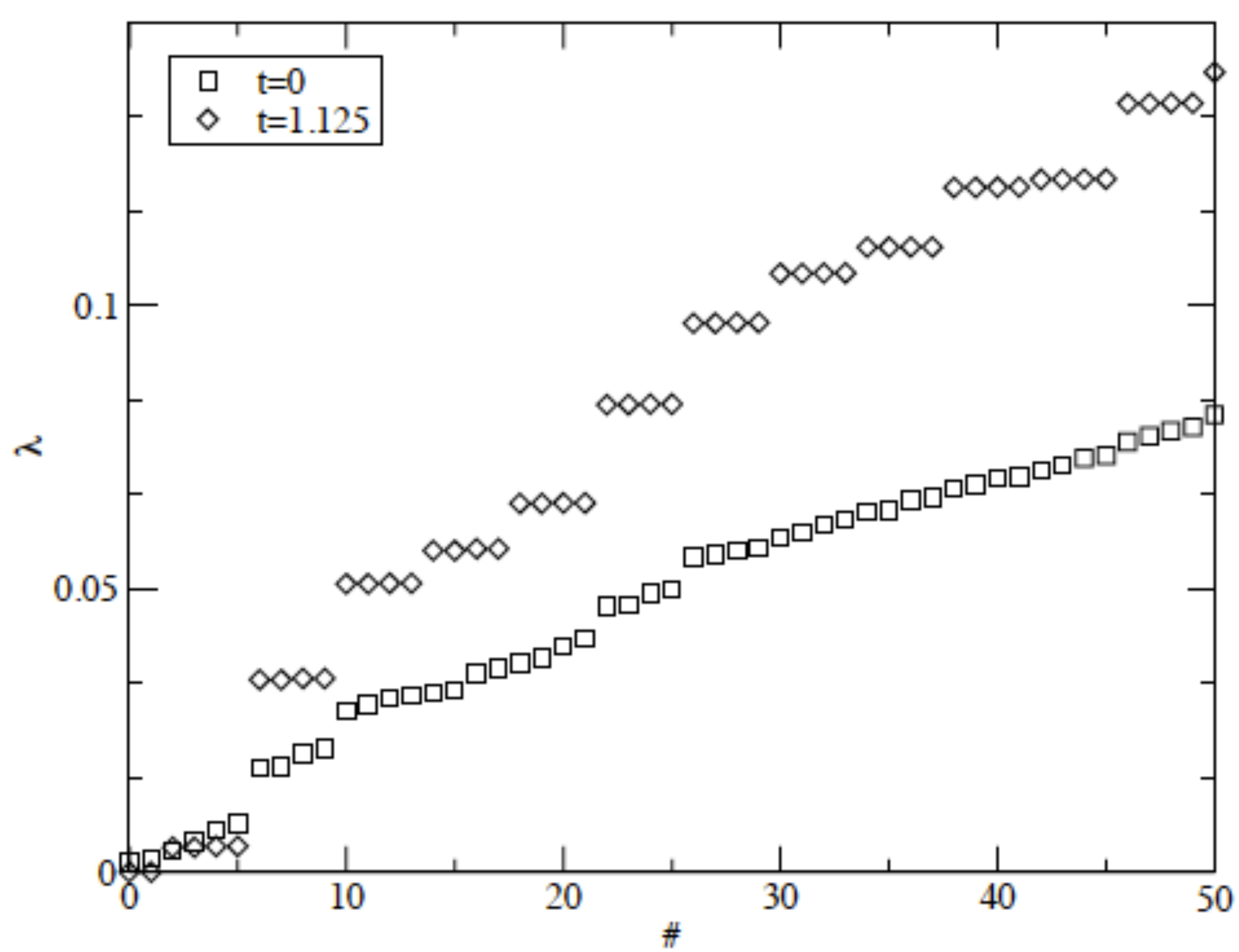
(4) Flow ⇒ smearing (smoothing)

(5) Glueball, String tension (Wilson loop), Polyakov loop

(6) Flavor singlet scalar meson (re-calc.), Cross Corr. with G-ball

(7) Flavor singlet Pseudo-scalar meson ⇒ probe of $U(1)_A$

Eigenvalue in gradient flow configuration
t: flow time (t=0: original configuration)



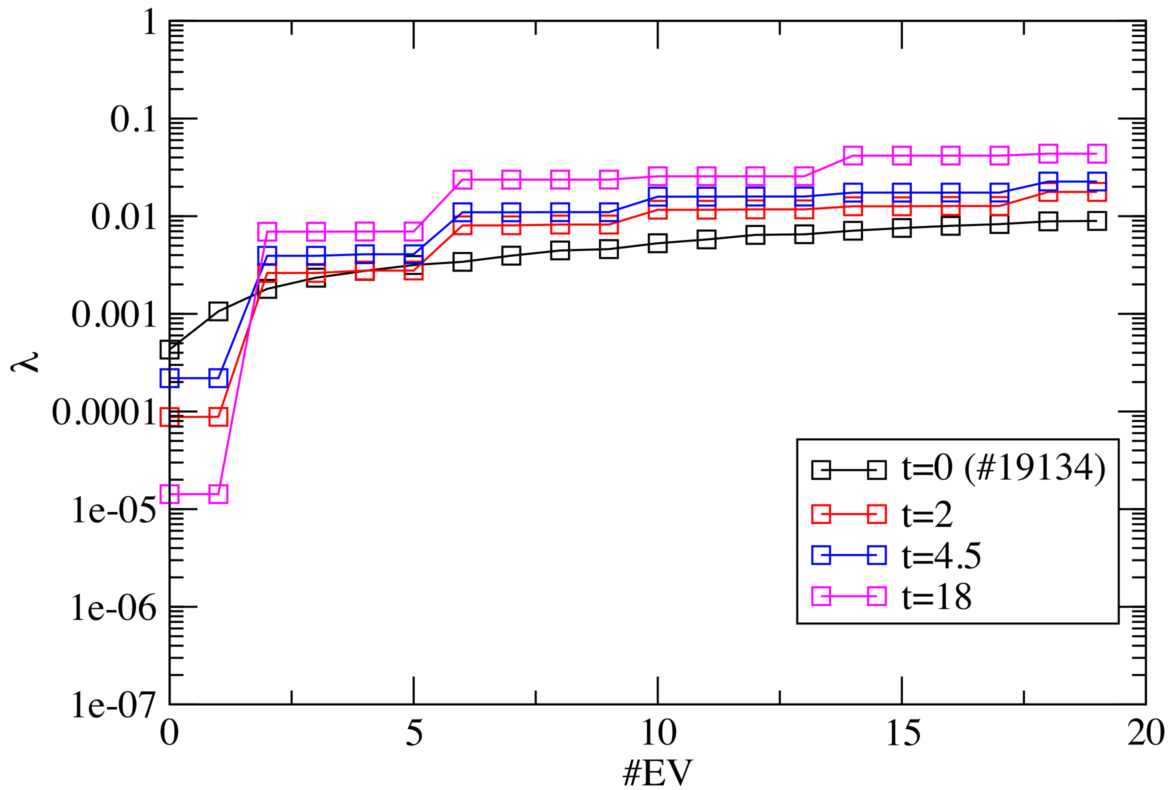
← flowed, t=1.125

← original, t=0

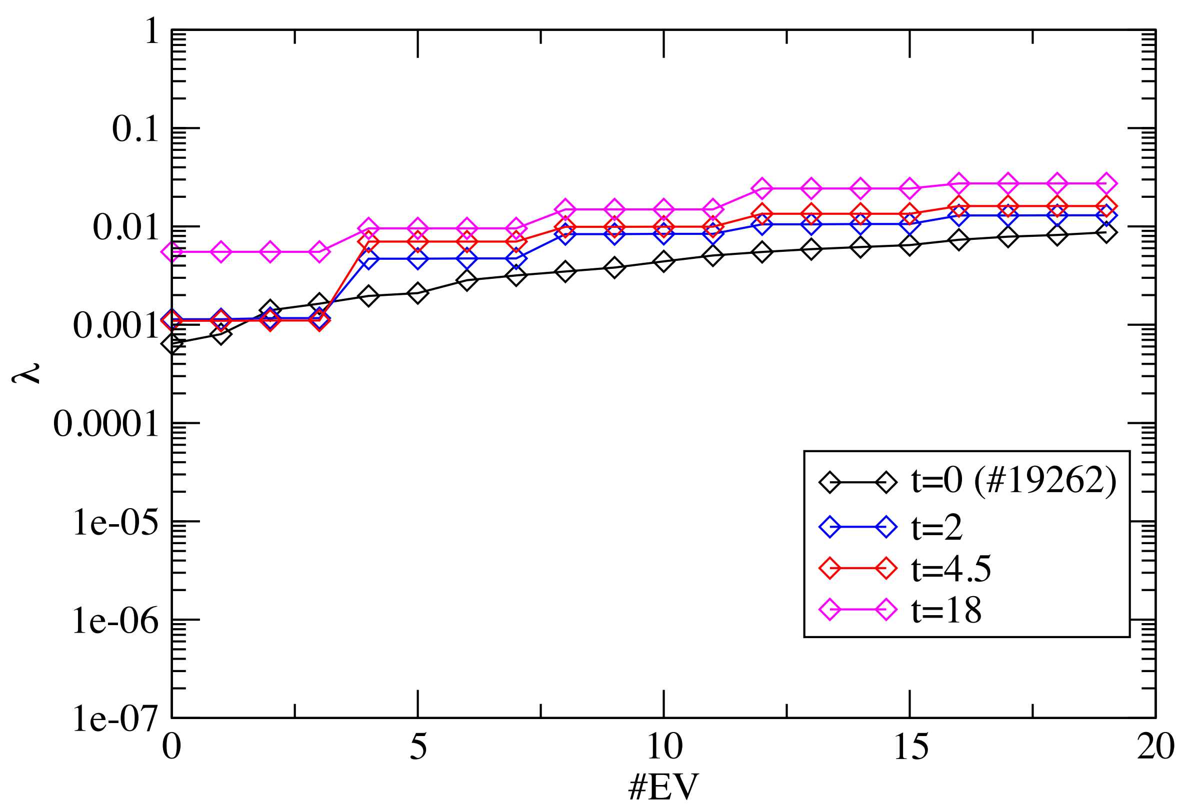
After the flow,
it is possible to obtain the clear signal of EVs of staggered fermions.

Nf=8, L=24, mf=0.06

EV behavior after the flow = spectral-“flow”

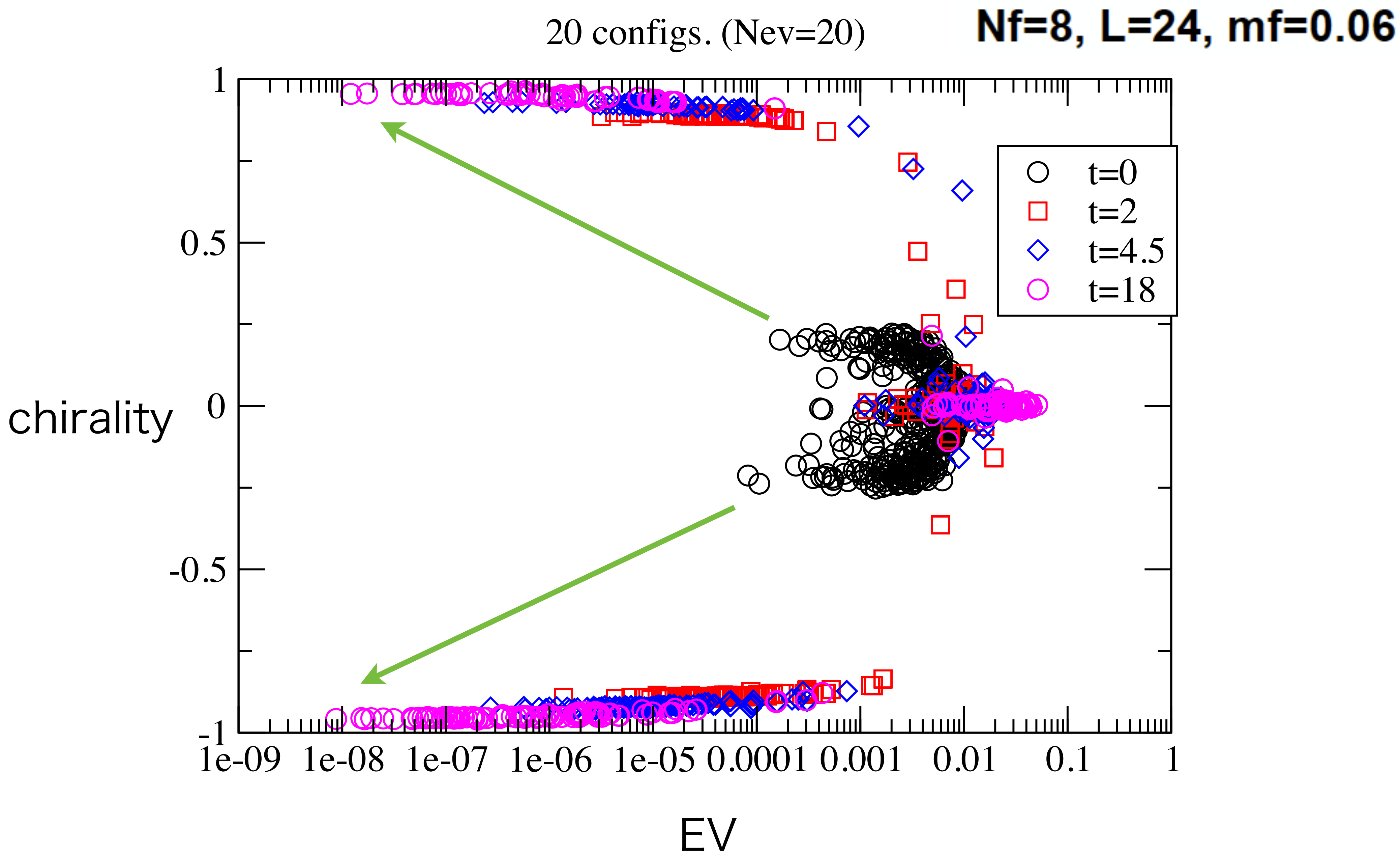


The cases that zero mode exists.



The case of no zero mode

Chirality in the spectral-"flow"



⇒ After the flow, the (would-be) zero mode appears with definite chiralities=±1.

⇒ good application to the topological insights

Discussion:

In conformal/walking phase

(0) In each Qtop sector

(1) Index theorem

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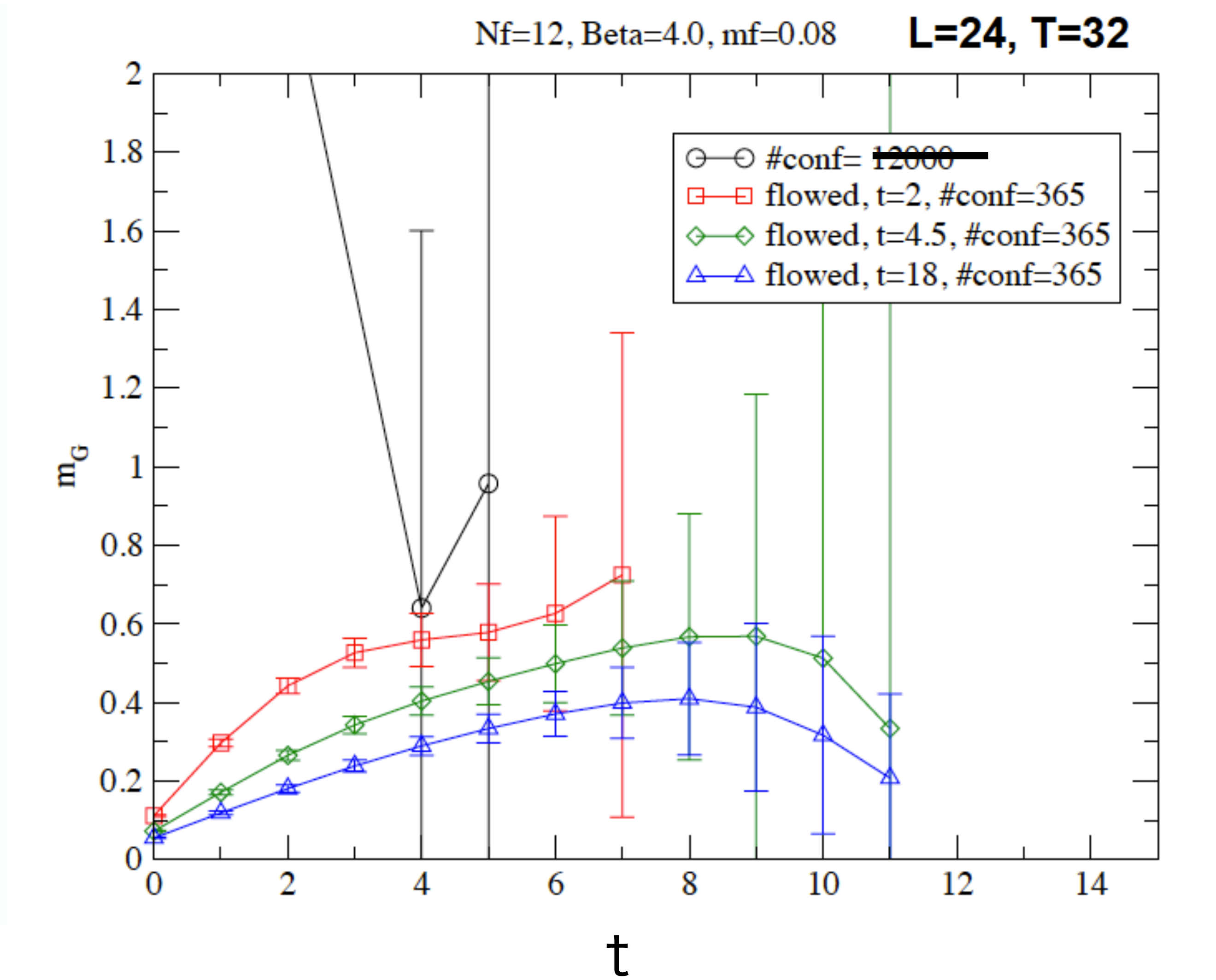
Glueball mass (0^{++})

effective mass, $m_G = \log \frac{C_H(t)}{C_H(t+1)}$



t=0: no plateau → cannot extract m_G

t>0: plateau (?), better signal than t=0 case → m_G or $m_G\sqrt{t_0}$

Discussion:

In conformal/walking phase

(0) In each Qtop sector

(1) Index theorem

(2) Banks-Casher relation

(3) Leutwyler-Smilga relation

(4) Flow ⇒ smearing (smoothing)

(5) Glueball, String tension (Wilson loop), Polyakov loop

(6) Flavor singlet scalar meson (re-calc.), Cross Corr. with G-ball

(7) Flavor singlet Pseudo-scalar meson ⇒ probe of $U(1)_A$

η' mass

$m_{\eta'}$ in $N_c<N_f<N_f^{cr}$ and $N_f>N_f^{cr}$?

In the region of N_f/N_c <<1 or >>1 ?

Witten-Veneziano formula, Veneziano limit ?

$U(1)_A$ in conformal phase ?

$$\langle\bar{\psi}\psi\rangle = \lim_{m\to 0}\lim_{V\to\infty}\int d\lambda\rho(\lambda)\frac{2m}{\lambda^2+m^2} = \pi\rho(\lambda=0) \quad =0$$

? Instanton , $\langle\partial_\mu j^5_\mu\rangle$, Q_{top} , χ_{top}, $\rho(\lambda)$?

$m_{\eta'}$?

In Progress & In the near future

(0) In each Qtop sector

(1) Index theorem

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Thank you