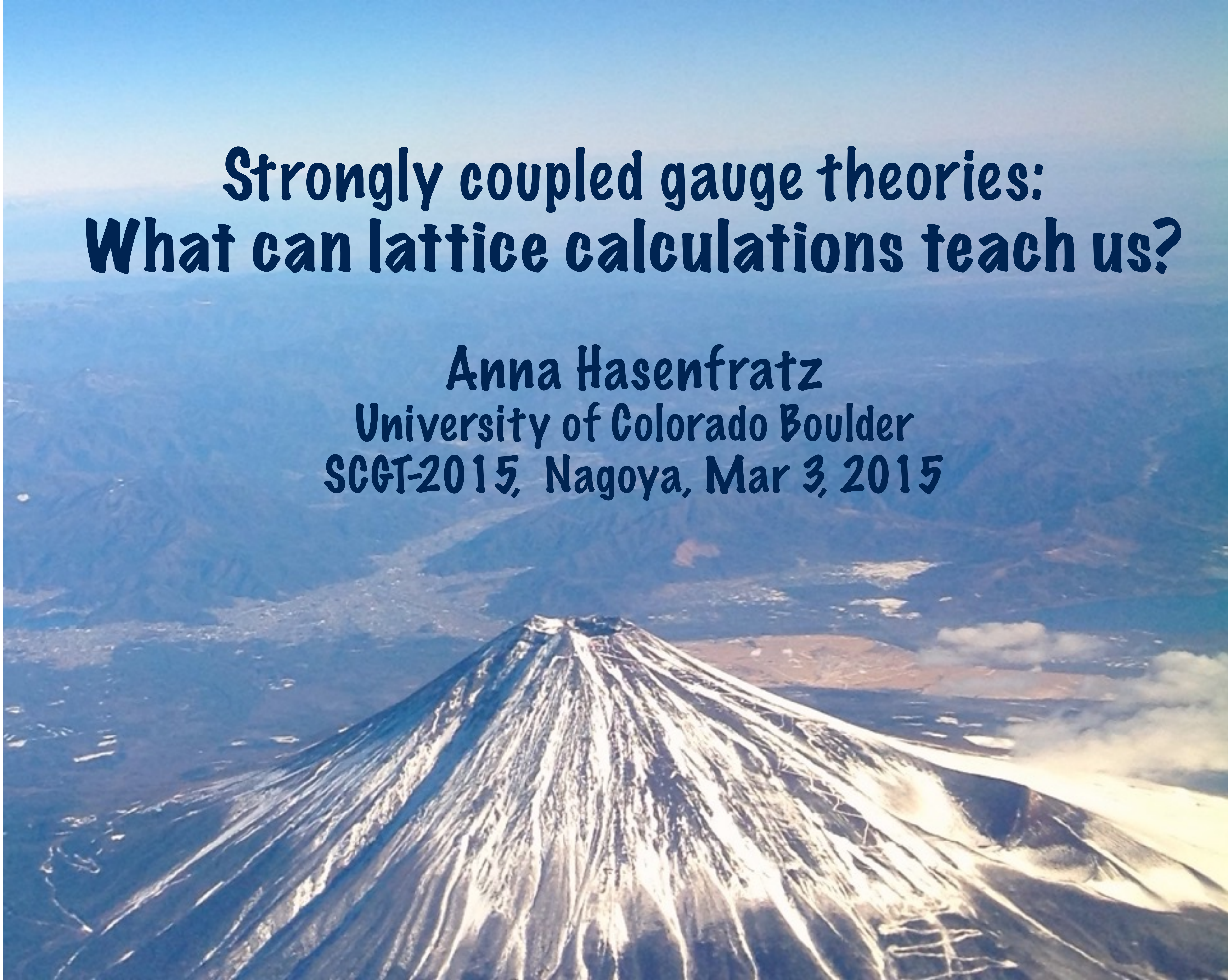
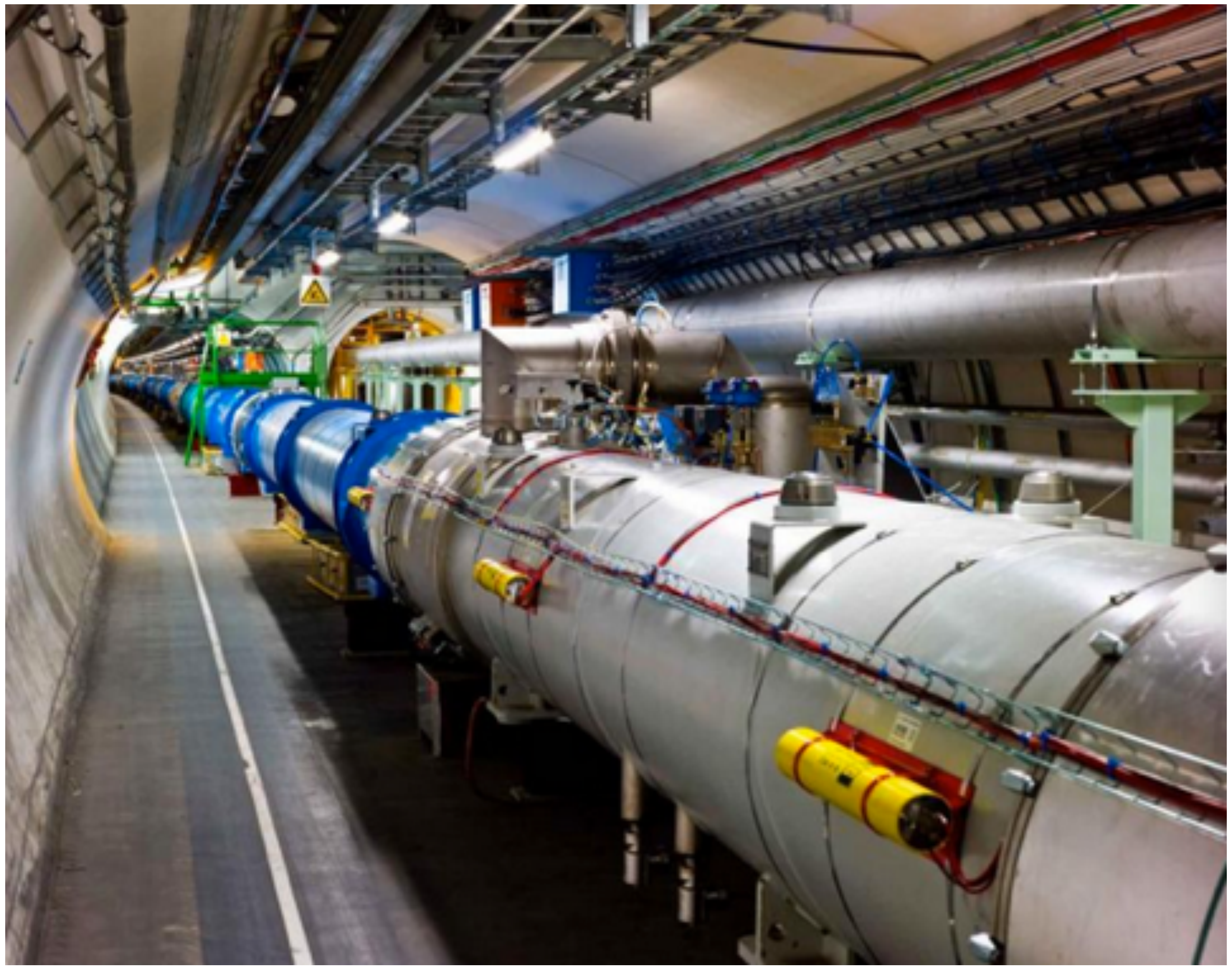


Strongly coupled gauge theories: What can lattice calculations teach us?

Anna Hasenfratz
University of Colorado Boulder

SCGT-2015, Nagoya, Mar 3, 2015

What is Beyond the Standard Model ?



The LHC will restart this month - will it reveal the nature of the Higgs ?

“With this new energy level, the LHC will open new horizons for physics and for future discoveries,” says CERN Director-General Rolf Heuer. “I’m looking forward to seeing what nature has in store for us”. (Feb 2015)

Composite Higgs

is viable possibility:

Higgs is a $\bar{q}q$ bound state (possibly qq)

– What models are compatible with EW data?
  - Most likely strongly coupled

– What are the generic properties of strongly coupled models?
  - is walking necessary ?
  - spectrum : where is M_{0++} compared to M_ρ ?

Strongly coupled models

We need only three Goldstones — 2 massless fermions will do

- N_f = 2 SU(3) : QCD
- N_f = 2 SU(2) adjoint : conformal
- N_f = 2 SU(3) sextet : popular but is it indeed chirally broken?
  - Poster: RG β function with Wilson fermions disagree with staggered
  - discrepancy could be due to rooting (?) or strong coupling effects
    - needs better understanding

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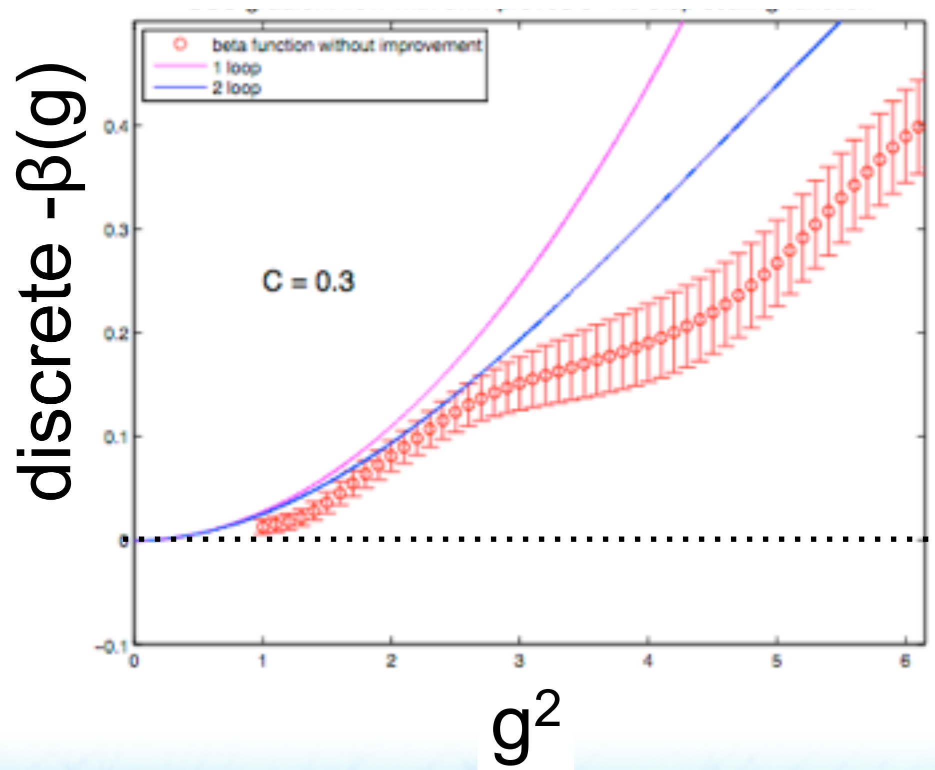
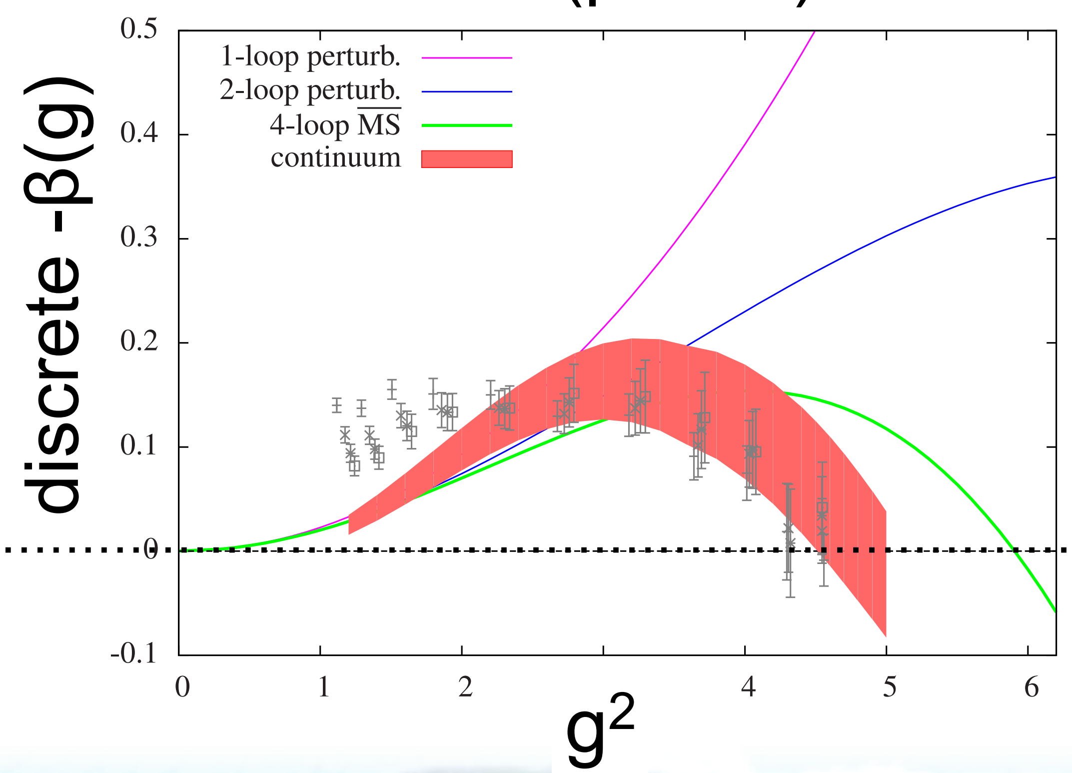
Staggered (1502.00028)



Wilson (poster)



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If not N_f = 2 :

- N_f = 6 SU(2) fundamental (1313.4889 - LSD)
- N_f = 8 SU(3) fundamental : seems to be close to the conformal window : E. Rinaldi talk; D. Schaich finite T poster

We need some mechanism to break flavor

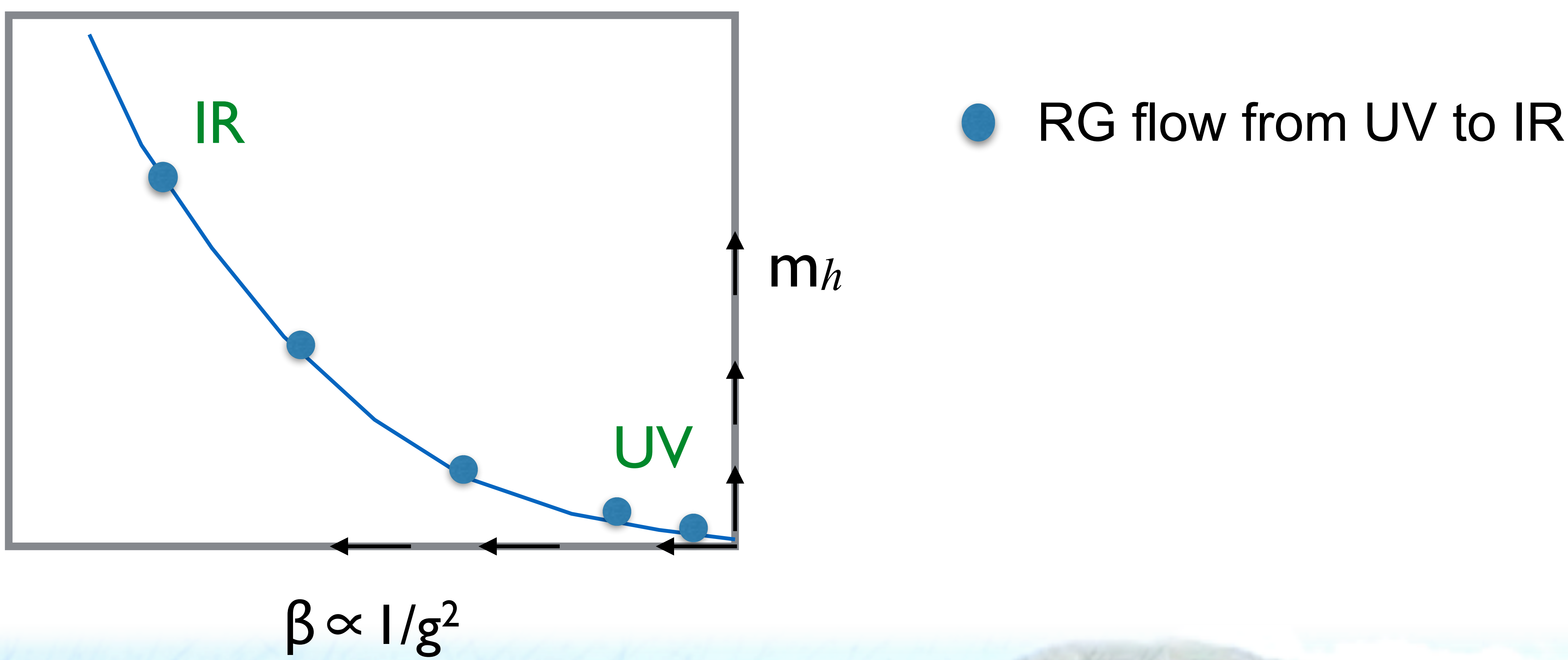
$SU(8)xSU(8) \rightarrow SU(2)xSU(2)$

What is the remnant of the many flavors in the IR?

Simple model - I

SU(N_c) gauge with N_ℓ light ($m_\ell \approx 0$) and N_h heavy (m_h) fermions
In the IR the heavy flavors decouple, N_ℓ light remain

$N_\ell + N_h$ =small: gauge coupling runs fast, heavy flavors have limited effect on the IR (QCD)

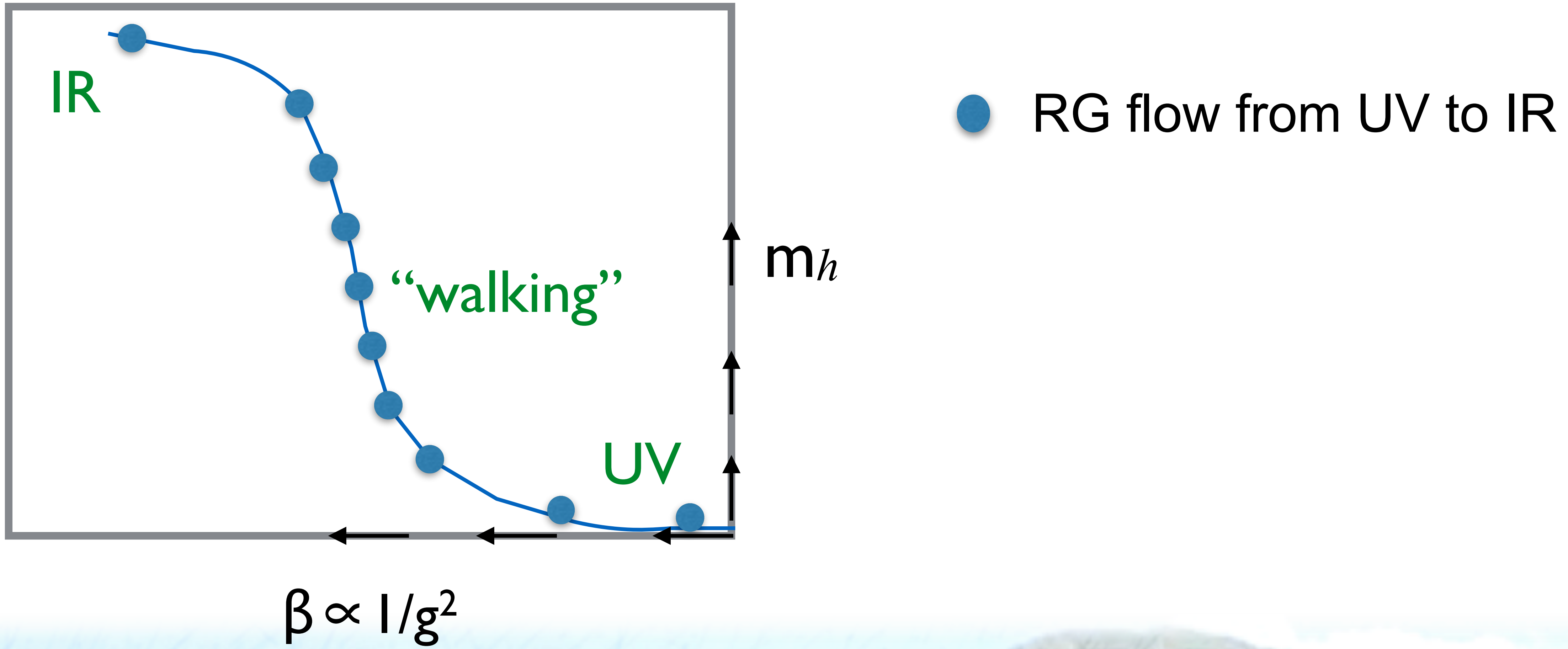


Simple model - II

SU(N_c) gauge with N_ℓ light ($m_\ell \approx 0$) and N_h heavy (m_h) fermions

$N_\ell + N_h$ = near but below the conformal window

IF the gauge coupling is "walking" the IR can be very different

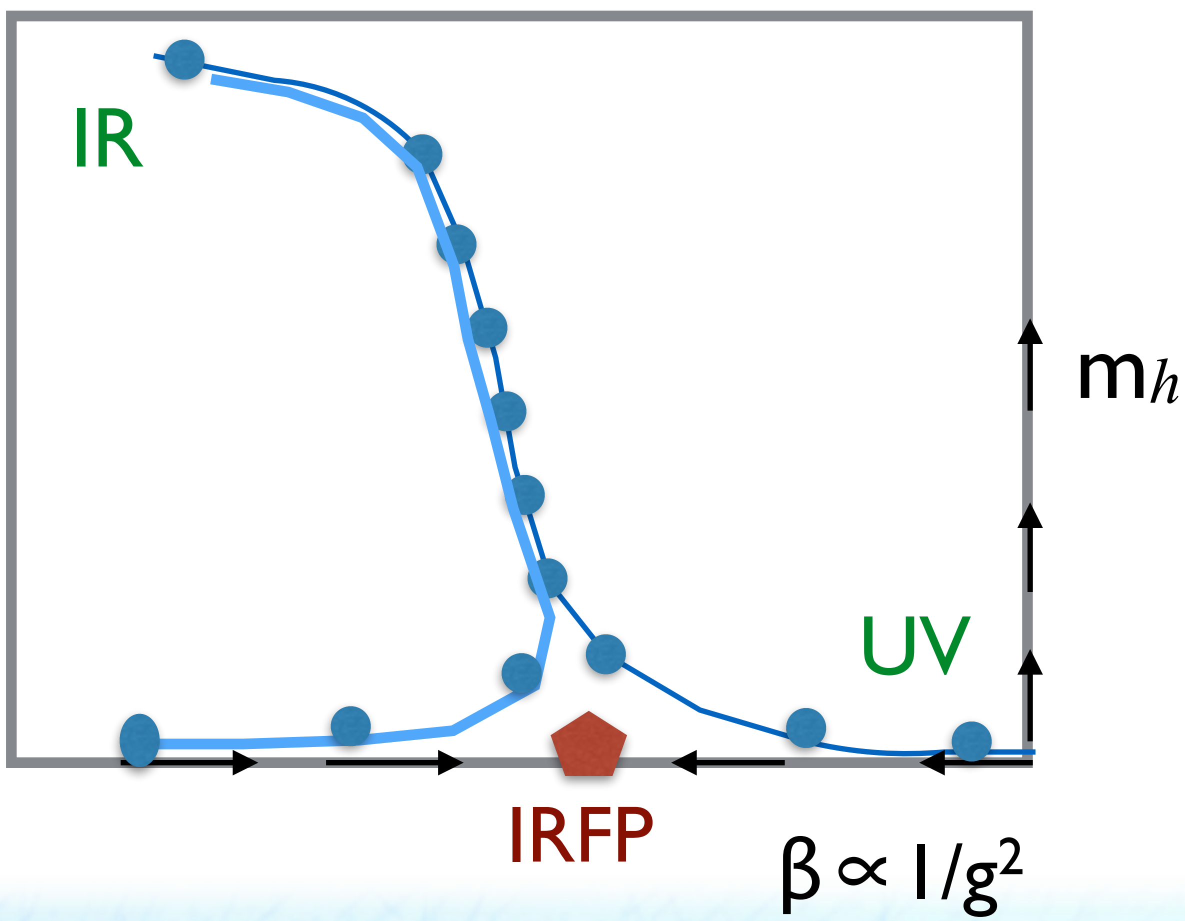


Simple model - III

SU(N_c) gauge with N_ℓ light ($m_\ell \approx 0$) and N_h heavy (m_h) fermions

$N_\ell + N_h$ = above the conformal window

the gauge coupling will be "walking";
the IR will be very different



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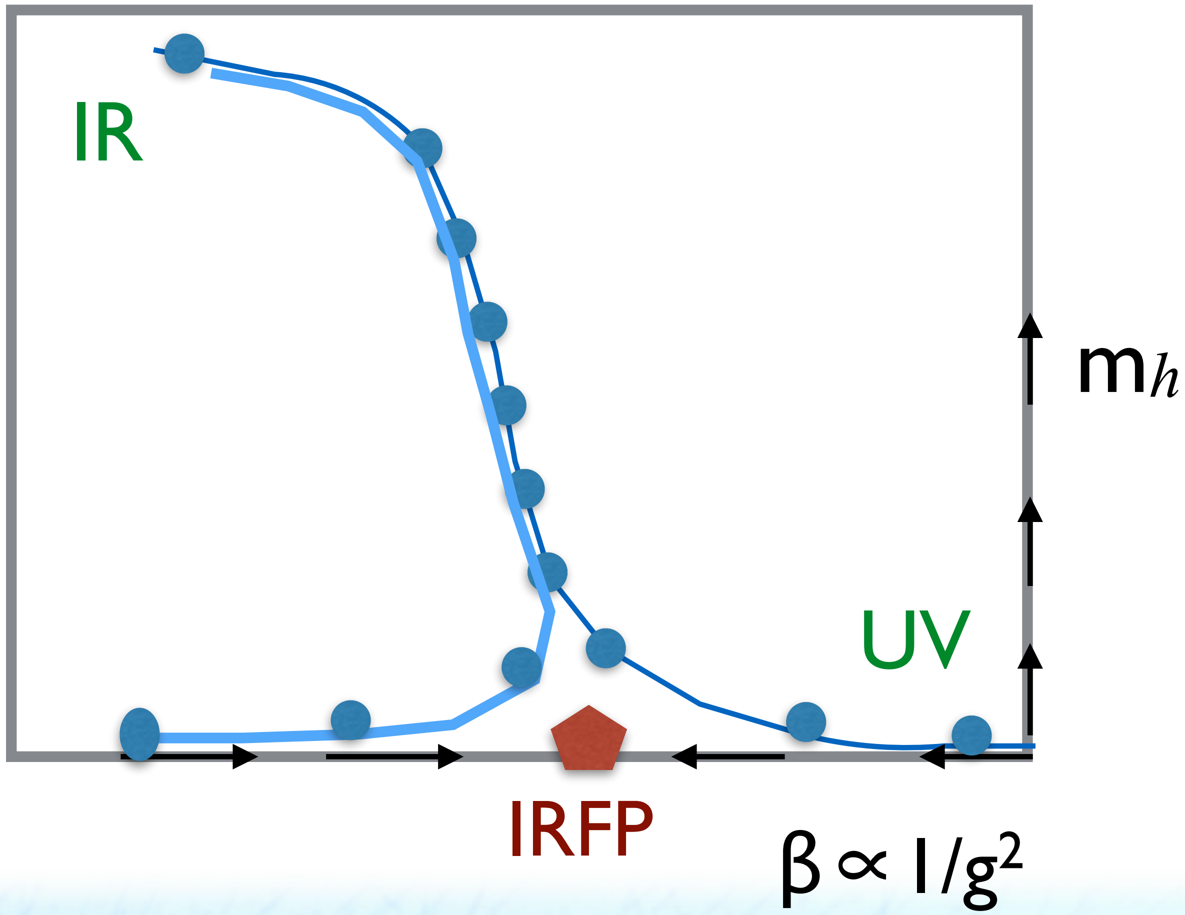


What are the properties of these strongly coupled "walking" systems?

$N_\ell+N_h$ (lattice) models

$N_\ell+N_h = 2 + 6$ if $N_f = 8$ is the UV model

or

$N_\ell+N_h = 2 + 10$ for $N_f = 12$ conformal behavior in the UV

Pilot study:

$N_\ell+N_h = 4 + 8$: conformal in the UV, $N_l=4$ flavor in the IR

in collaboration with R. Brower, C. Rebbi, E. Weinberg, O. Witzel

arXiv:1411.3243

Why 4+8 ? We use staggered fermions:

4 and 8 flavors do not require rooting

(rooting is no-go in a conformal system near IRFP)

$N_\ell + N_h$ = 4+ 8 : The lattice action

Action: nHYP smeared staggered fermions,
fundamental + adjoint gauge plaquette

This action was used in the Boulder 4, 8, and 12 flavor studies
(1106.5293, 111.2317, 1404.0984)

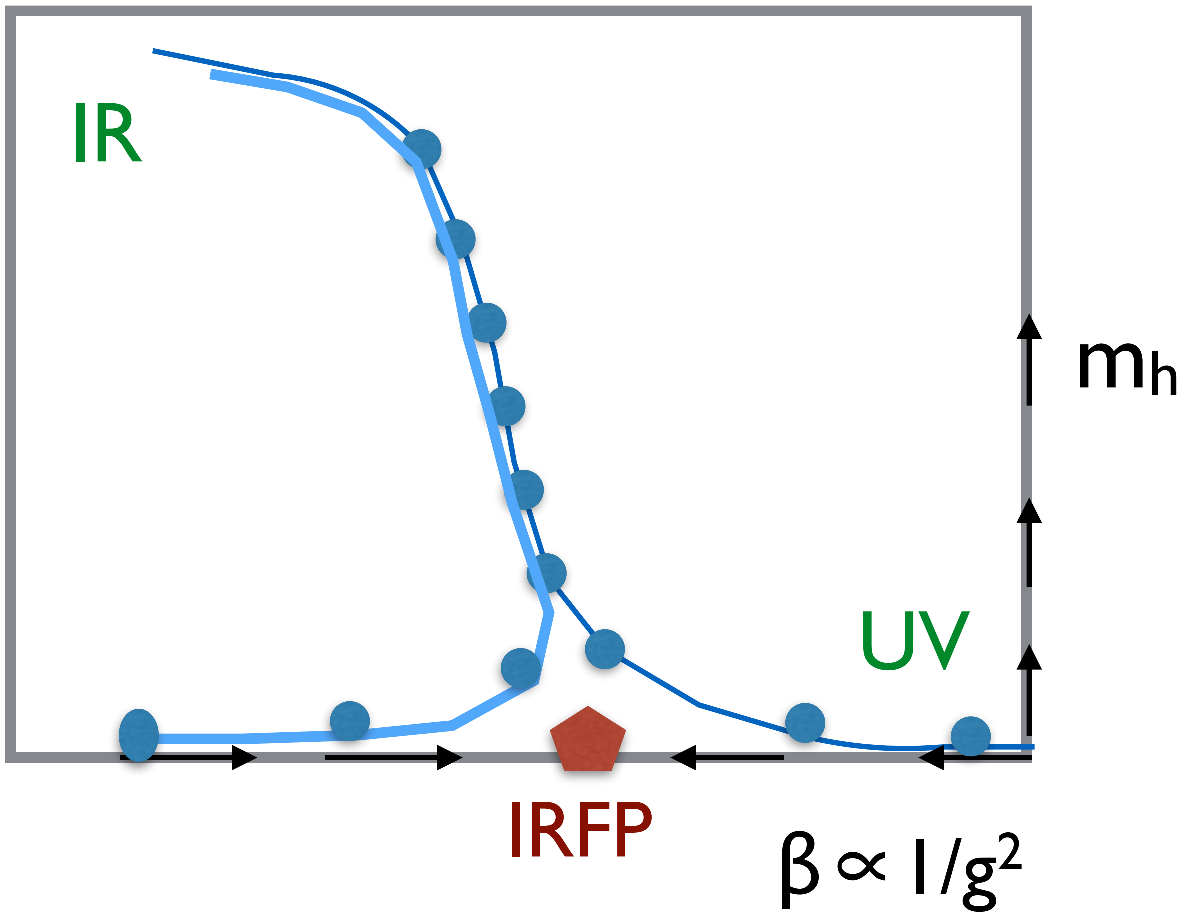
It is the action used in the 8 flavor joint project with LSD
(E. Rinaldi's talk, D. Schaich's poster)

We understand this action well

$N_\ell + N_h = 4 + 8$: Parameter space

3 independent parameters: (g^2, m_ℓ, m_h)

- g^2 does not matter once the flow has reached the RG trajectory
- sufficient to work at g^2 =const, vary m_h only



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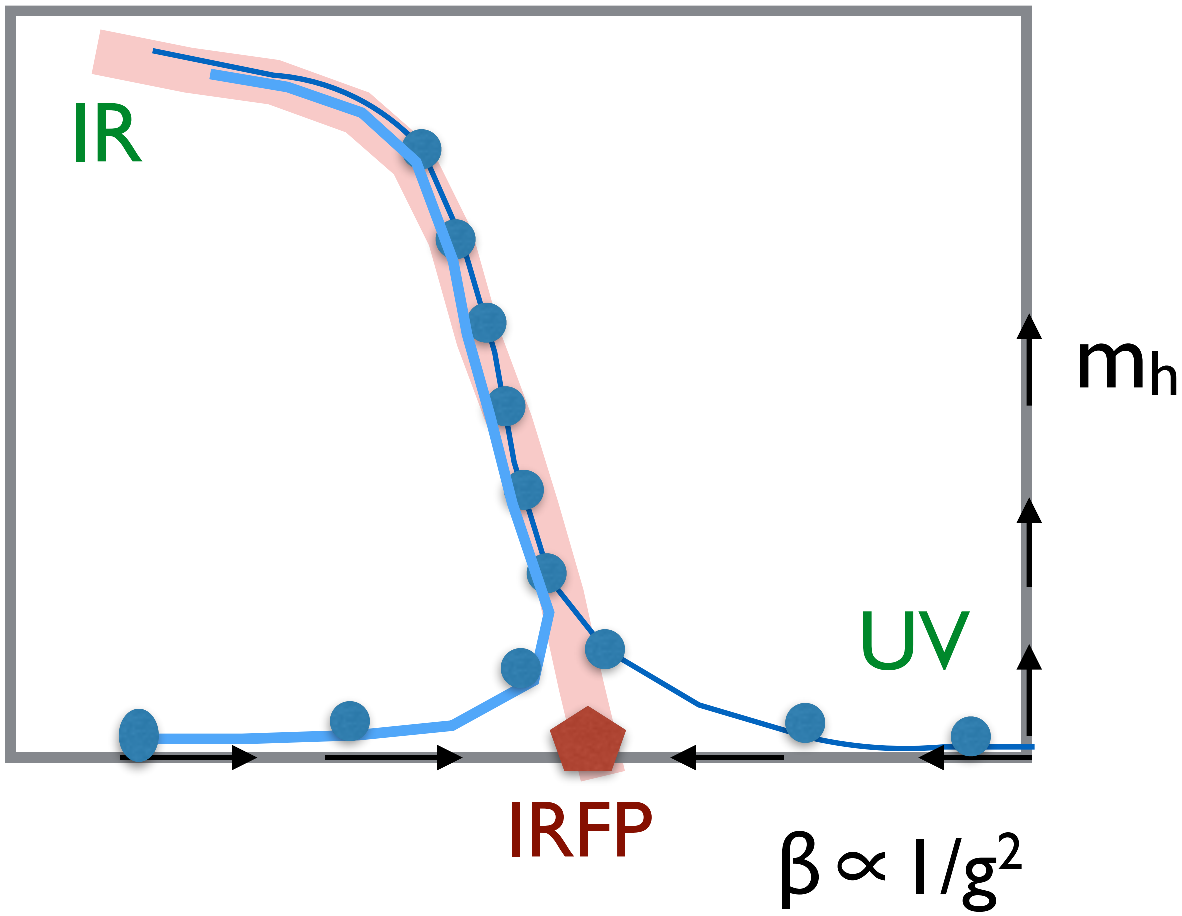

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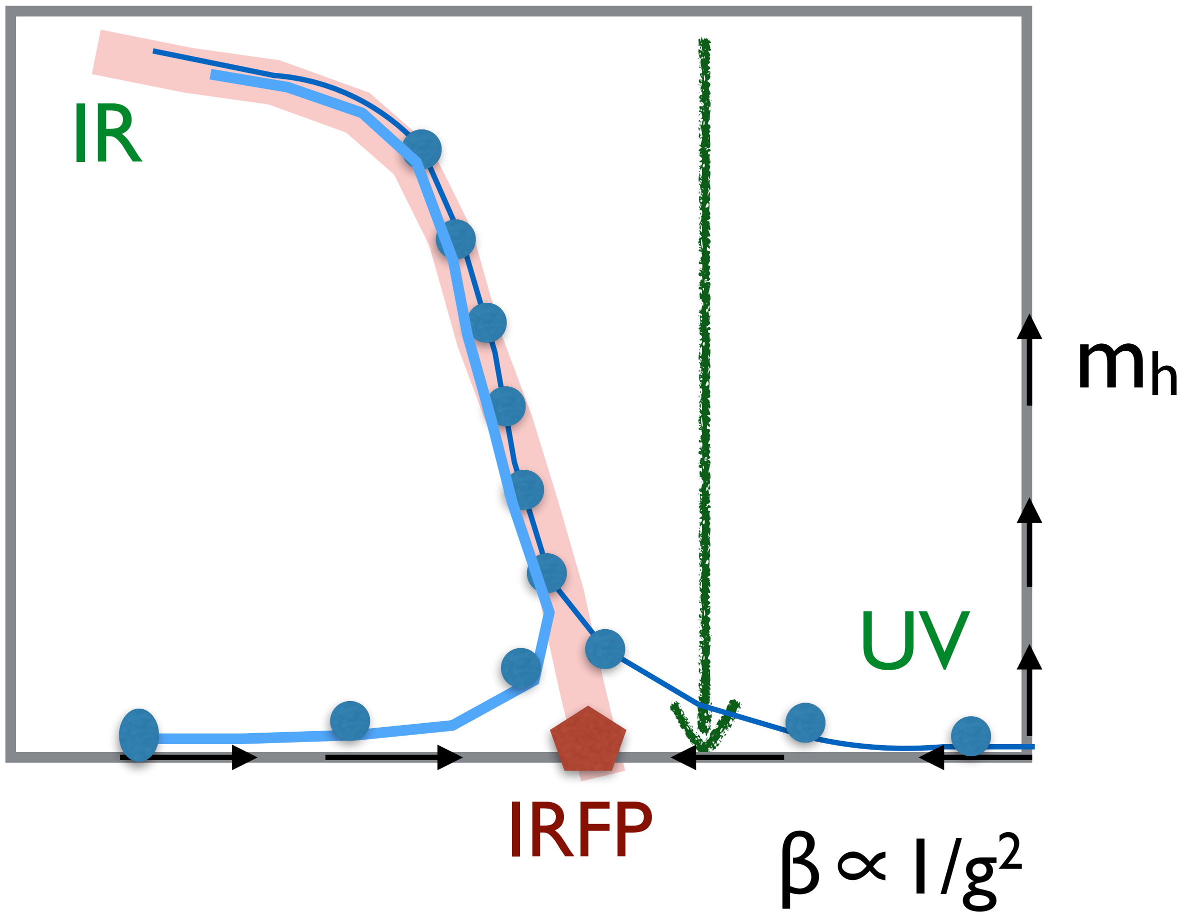
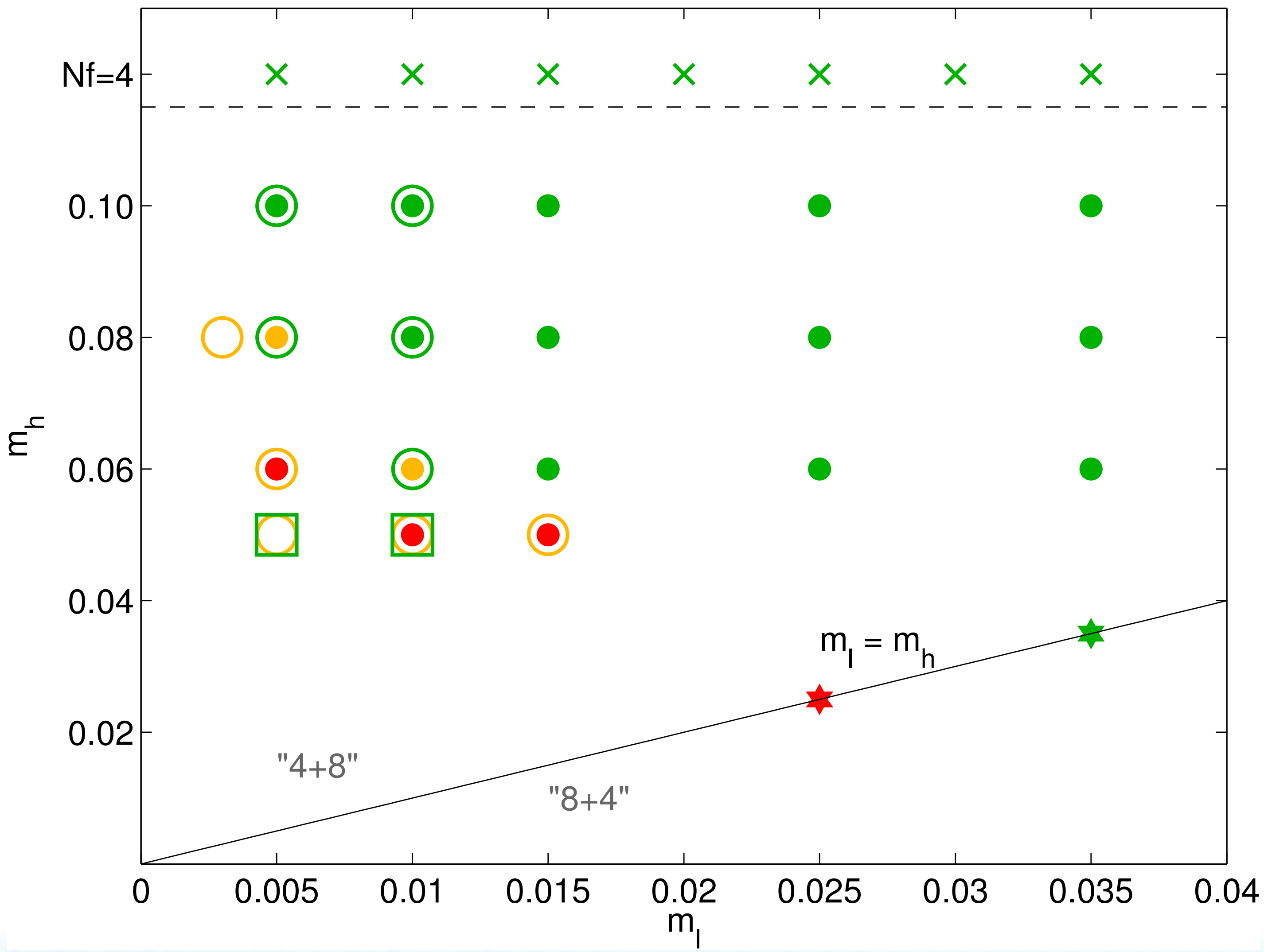


$N_\ell+N_h$ = 4+ 8 : Parameter space

- β=4.0 (close to the 12-flavor IRFP)
- m_h=0.10, 0.08, 0.06, 0.05
- m_ℓ=0.003, 0.005, 0.010, 0.015, 0.025, 0.035

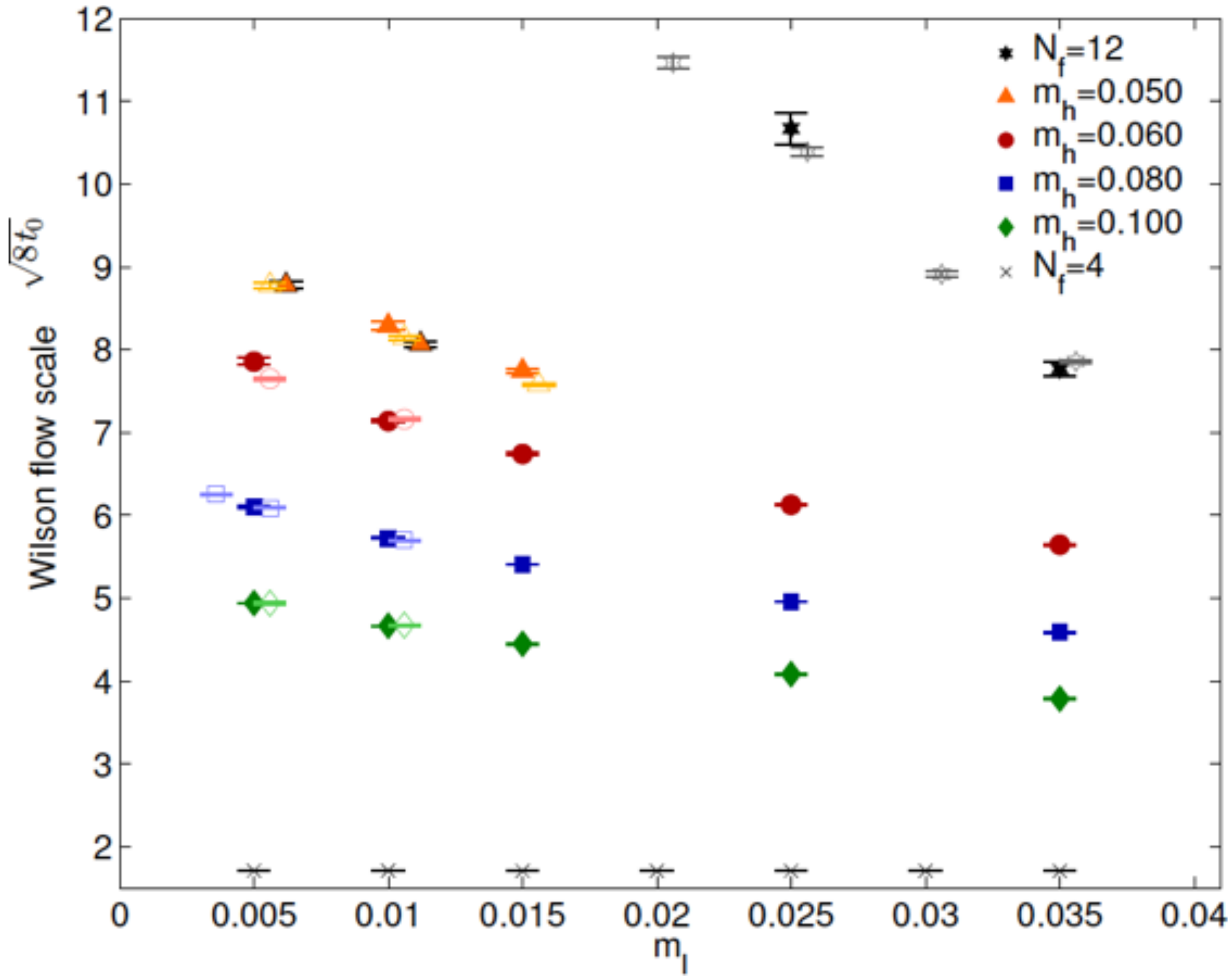


Volumes :
24^3x48, (dots)
$32^3$64 (circle),
48^3x96 (square)
Color: volume OK / marginal/ squeezed

20,000 MDTU, most still in progress

Lattice scale

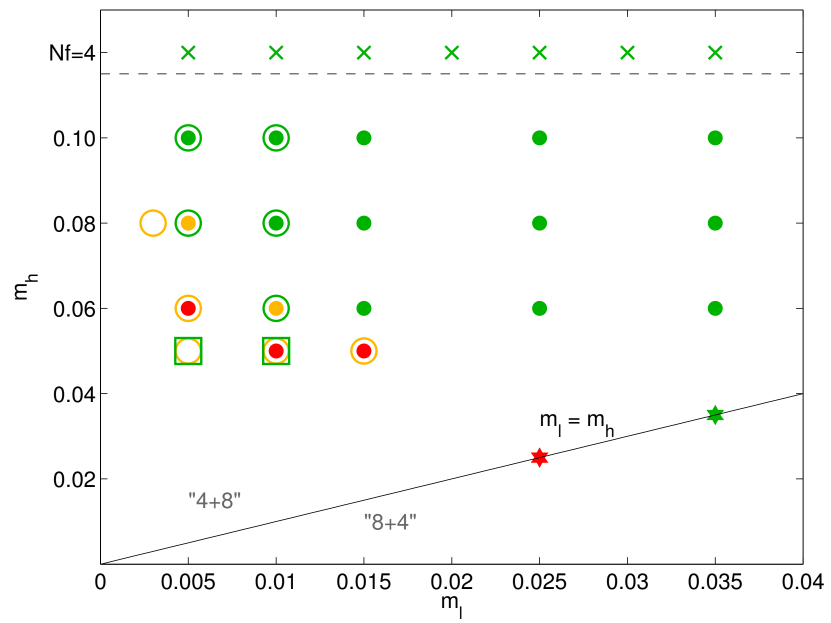
Use Wilson flow to estimate the lattice scale $\sqrt{8t_0}$



$\sqrt{8t_0} \lesssim L/5$

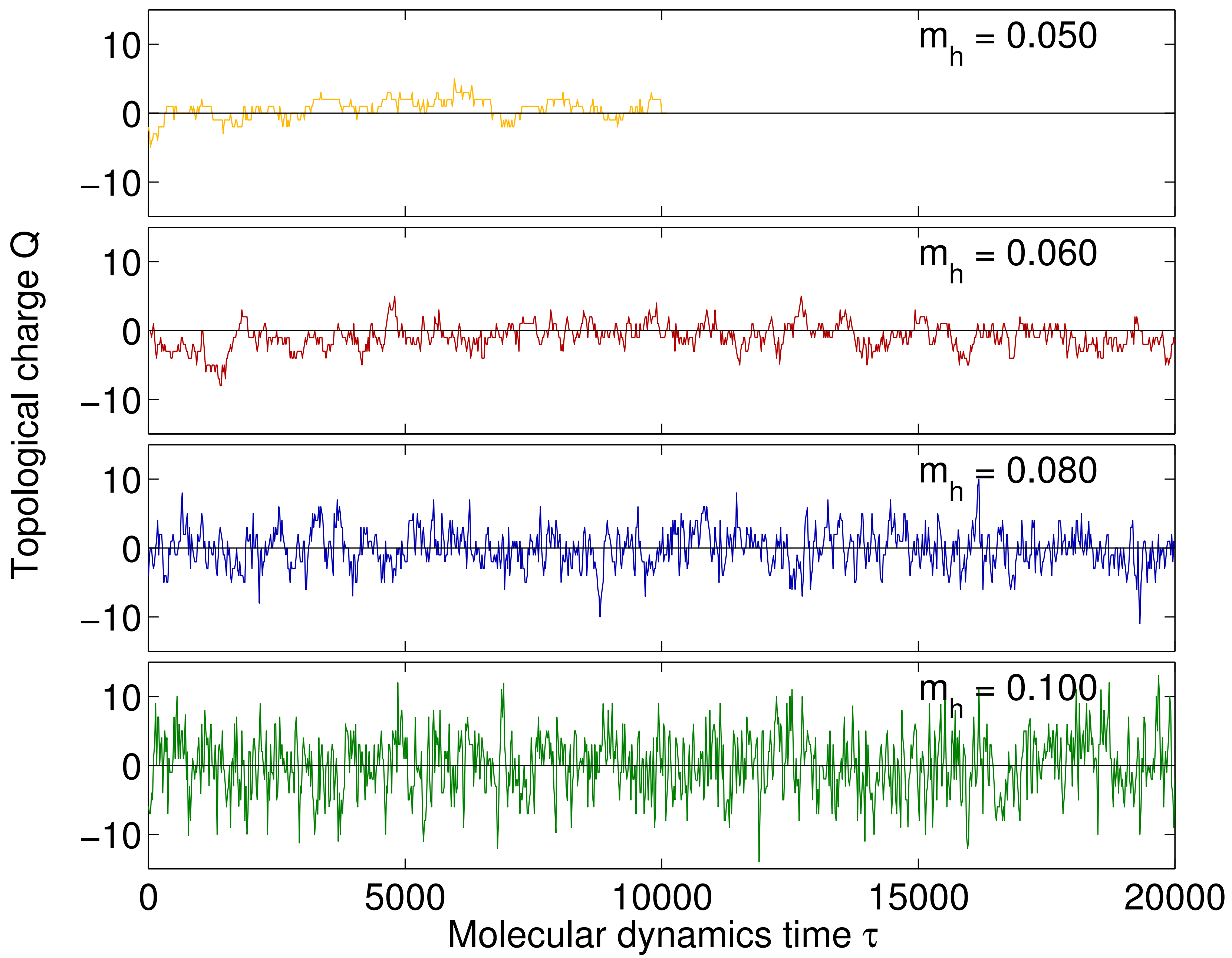
is usually sufficient

→ color coding



Topology evolution

Topology is moving well even with the lightest mass



m_ℓ =0.010,
24^3x48 volume

Running coupling

Gradient flow transformation defines a renormalized coupling

arXiv:1006.4518

$$g_{GF}^2\left(\mu=\frac{1}{\sqrt{8t}}\right)=\frac{1}{\mathcal{N}}t^2\langle E(t)\rangle$$

t: flow time;
E(t):energy density

g_{GF}^2 is used for scale setting as

$$g_{GF}^2(t=t_0)=\frac{0.3}{\mathcal{N}}$$

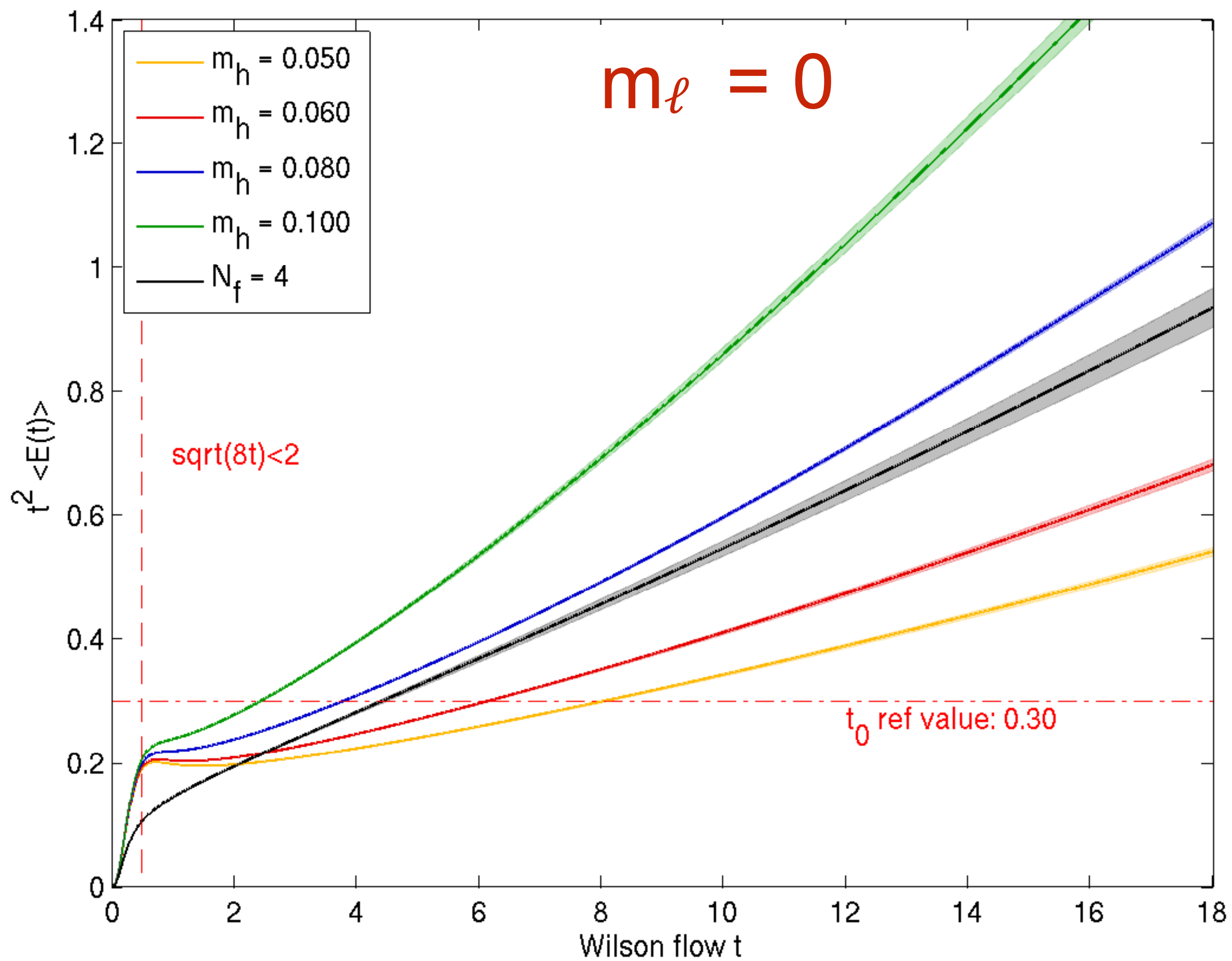
Is it appropriate for renormalized running coupling?

Yes,

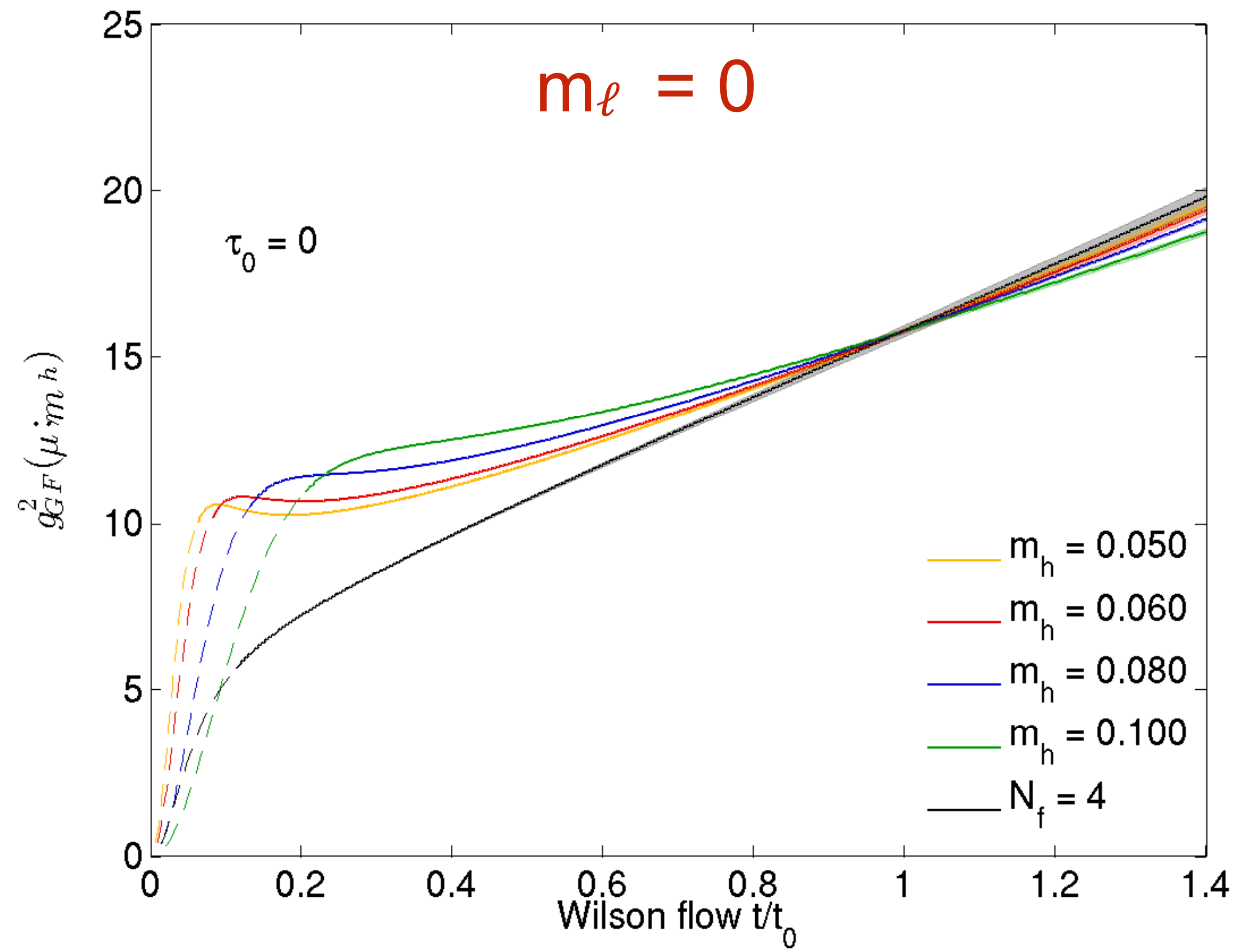
- on large enough volumes
- at large enough flow time
- in the continuum limit

Running coupling

$t^2\langle E(t)\rangle$ in the chiral limit at various m_h values



$g^2_{GF}(t/t_0)$ rescaled by t_0 at various m_h values



Rescaling forces the renormalized couplings to agree at t_0

Fan-out before and after are due to cut-off lattice artifacts

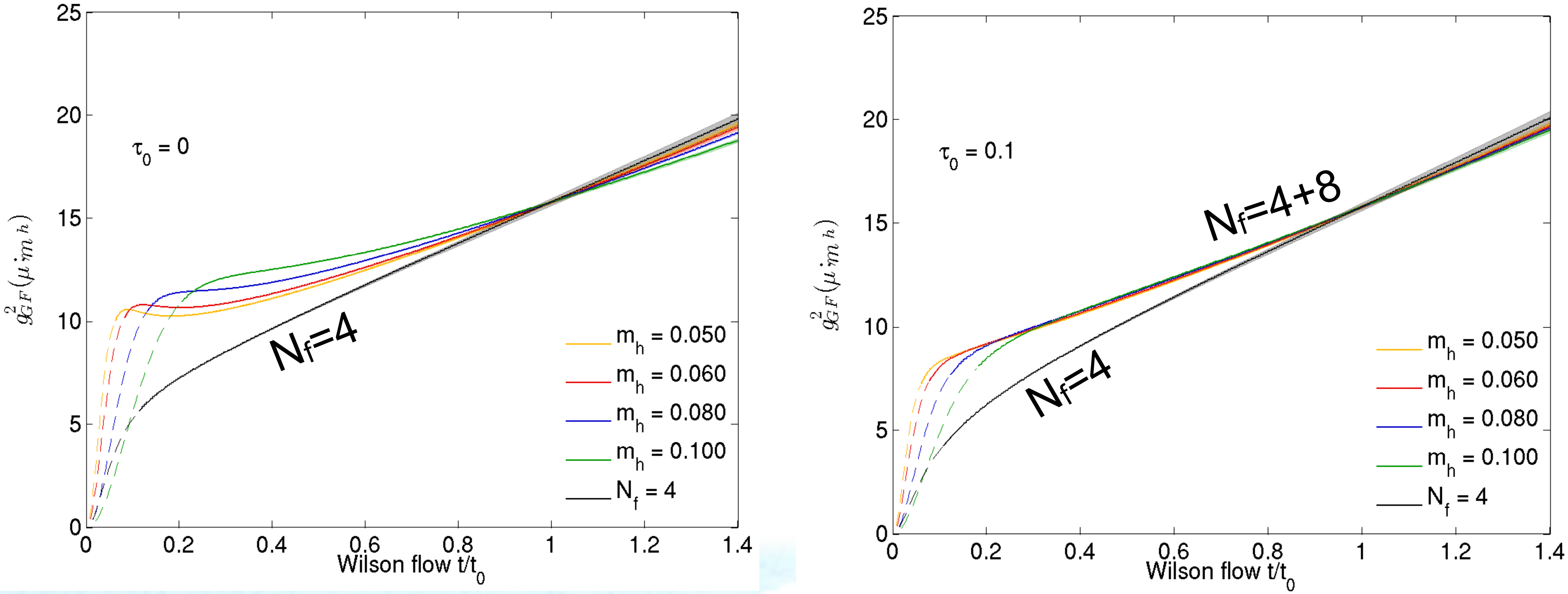
Improved running coupling

t-shift improved running coupling

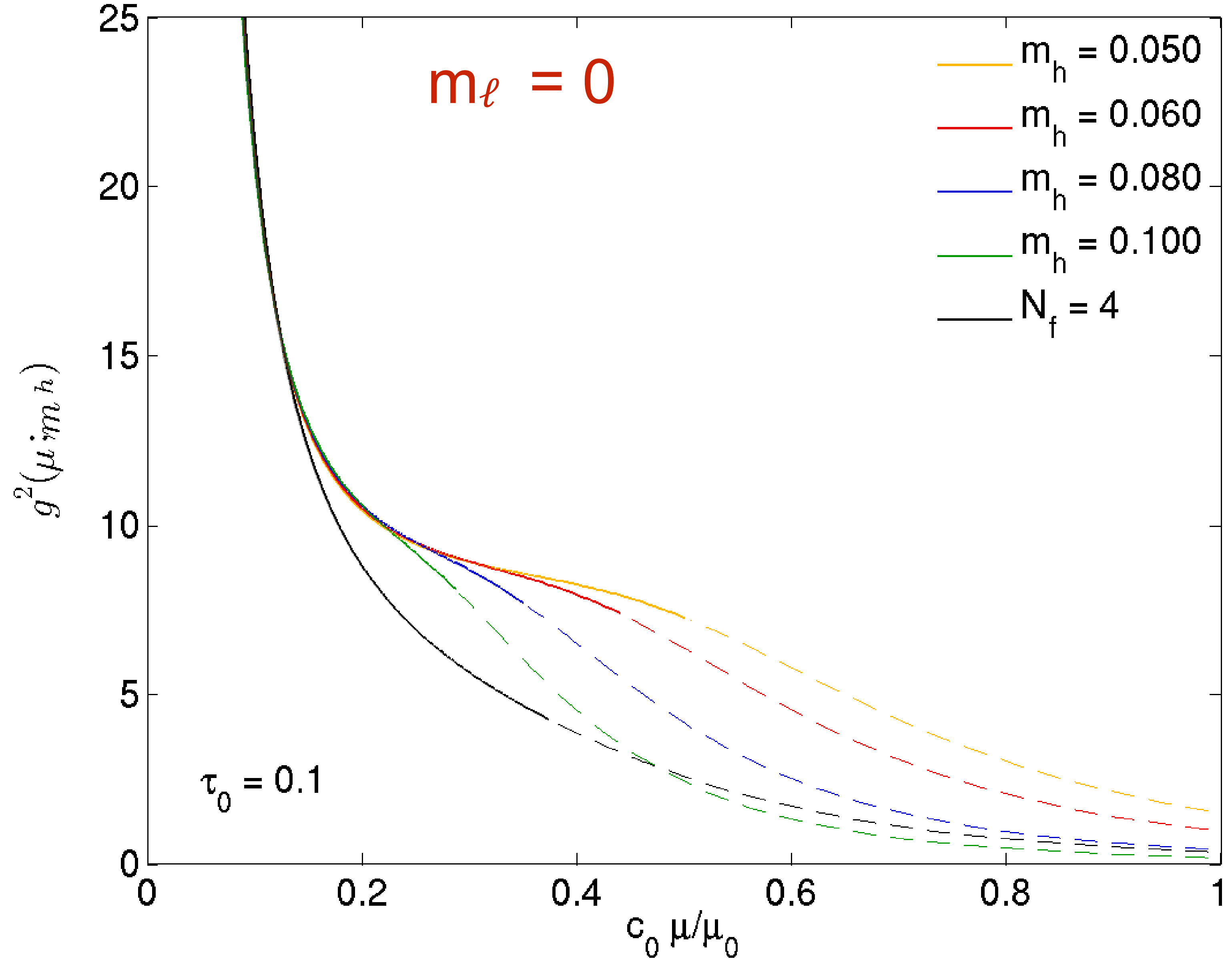
$$\tilde{g}^2_{GF}\left(\mu=\frac{1}{\sqrt{8t}}\right)=\frac{1}{\mathcal{N}}t^2\langle E(t+\tau_0)\rangle$$

by adjusting τ_0 most cut-off effects can be removed

(1404.0984, 1501.07848)



Improved running coupling : 4+8 flavors

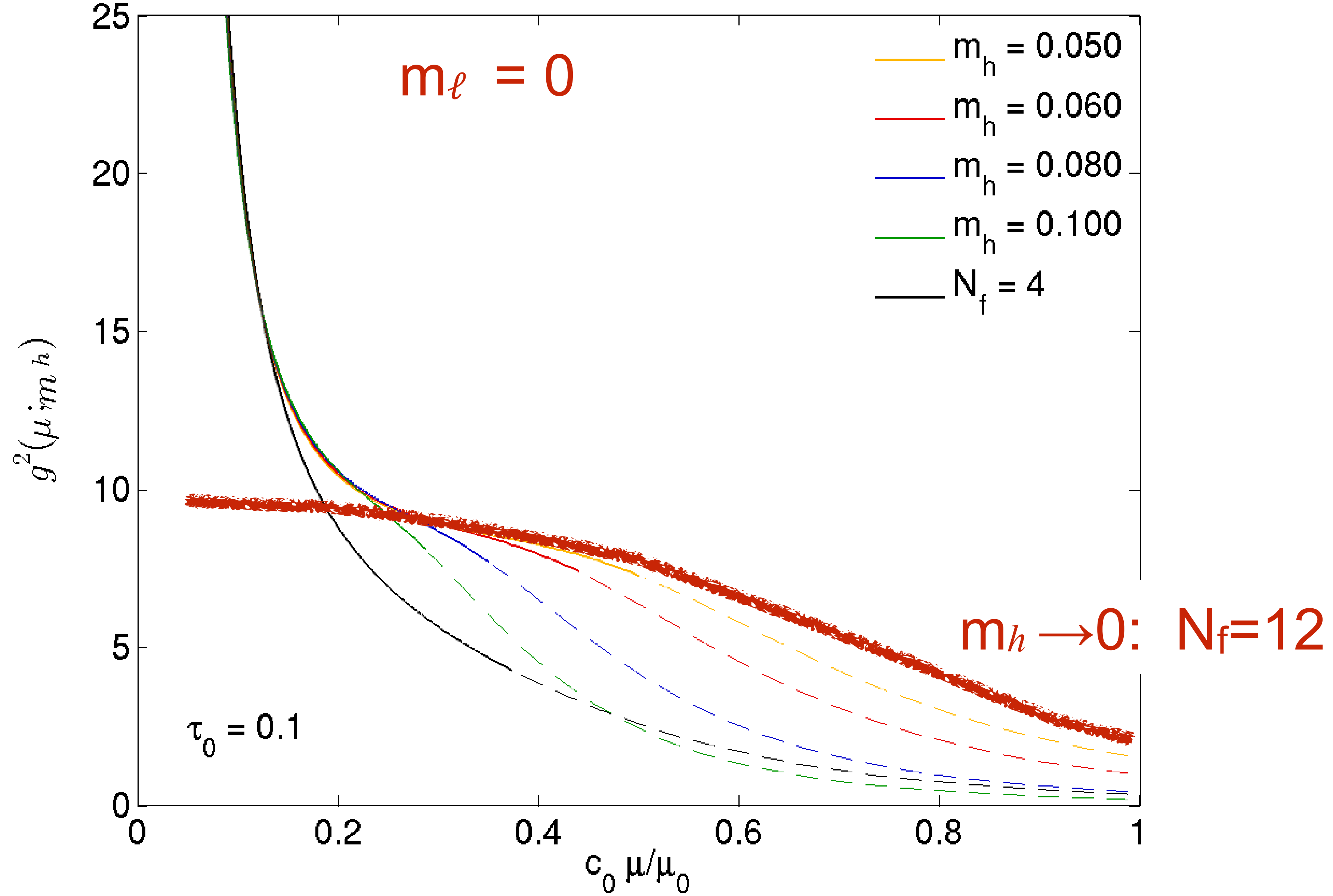


N_f=4 : running fast

$g^2_{GF}(\mu)$ develops a "shoulder" as $m_h \rightarrow 0$: this is walking !

Walking range can be tuned arbitrarily with m_h

Improved running coupling : 4+8 flavors



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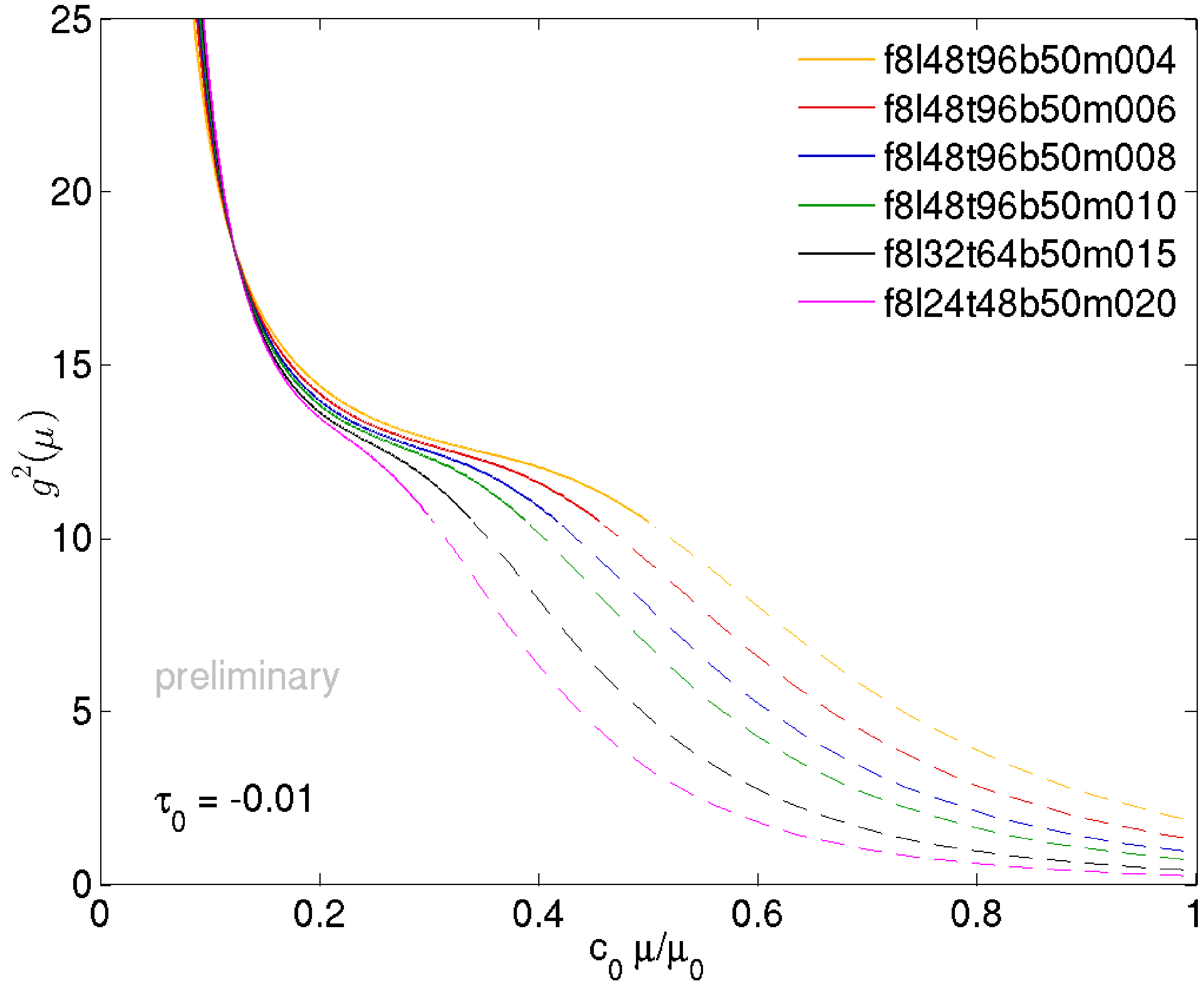
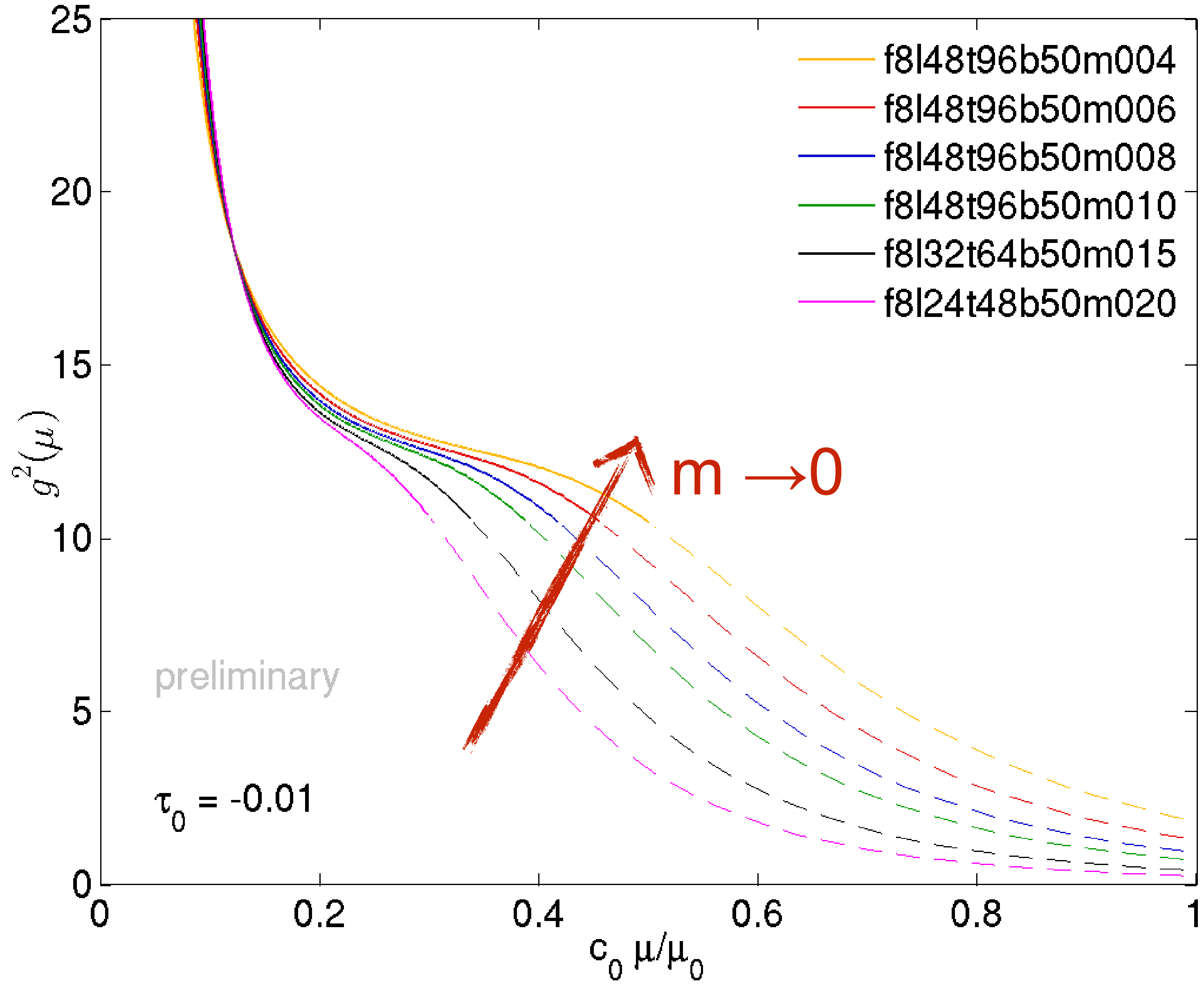


Is this walking?

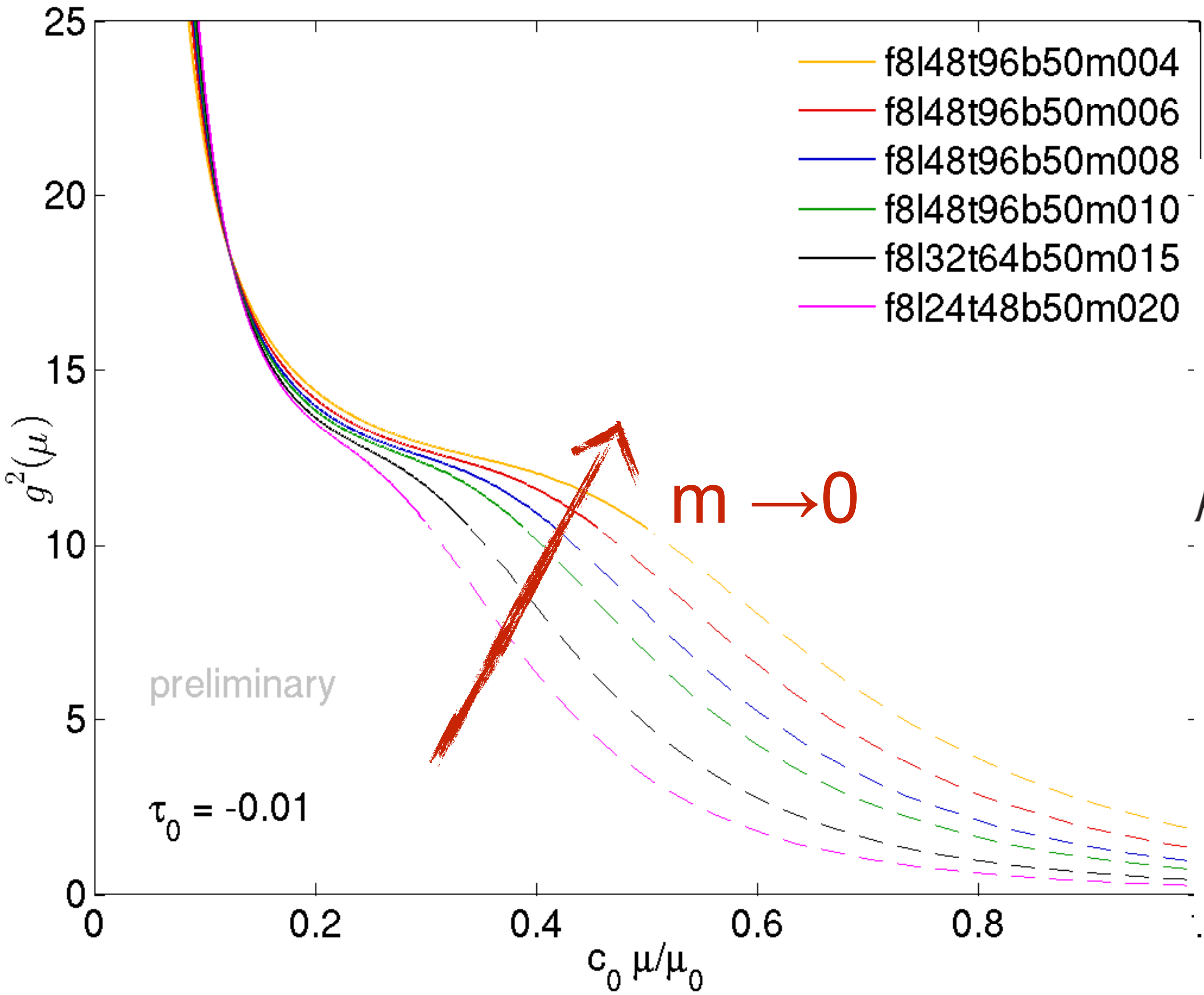
Improved running coupling : **8 flavors**



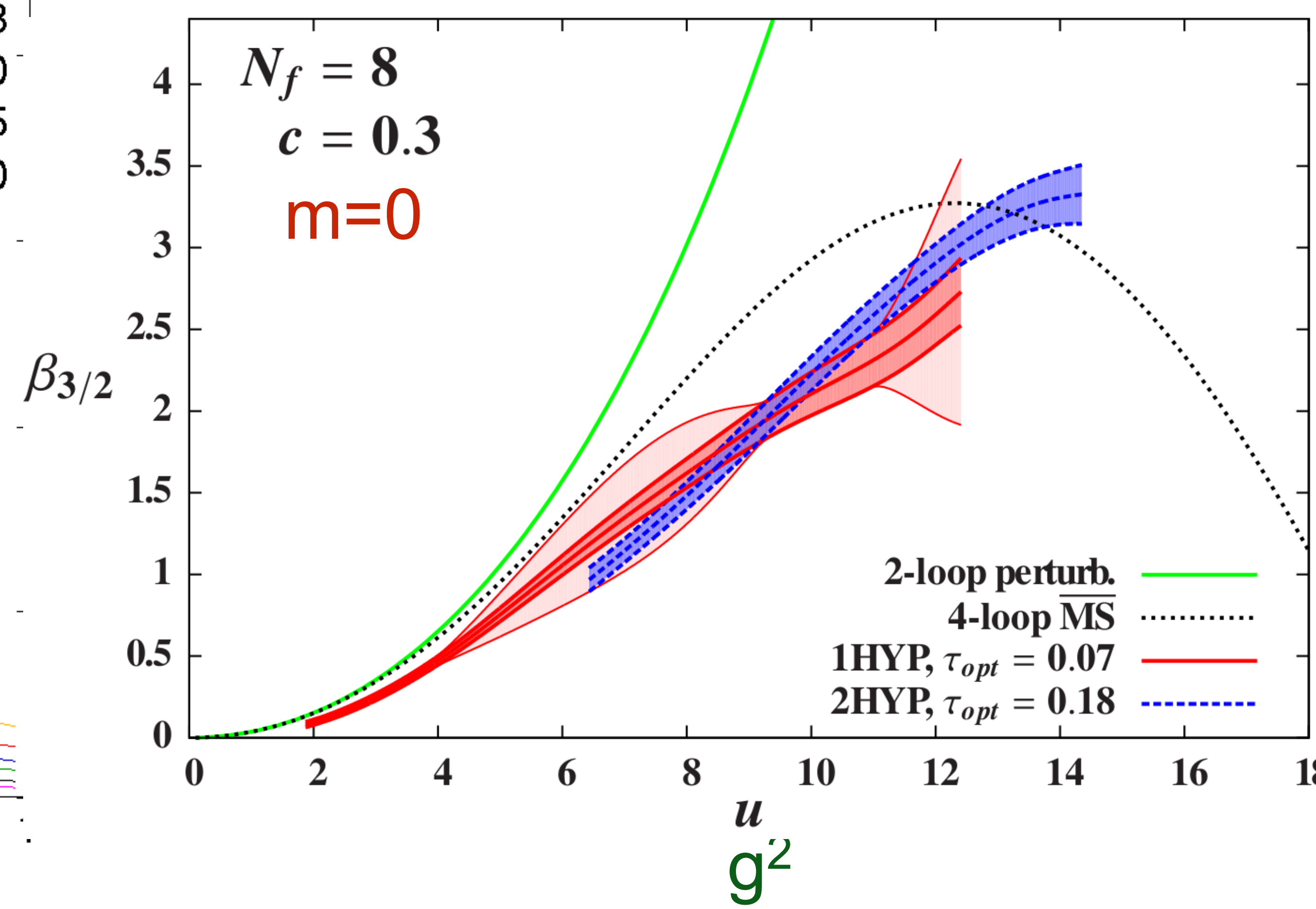
Is this walking?

The “shoulder” is the gauge dynamics : slow evolution

Improved running coupling : **8 flavors**



arXiv:1410.5886



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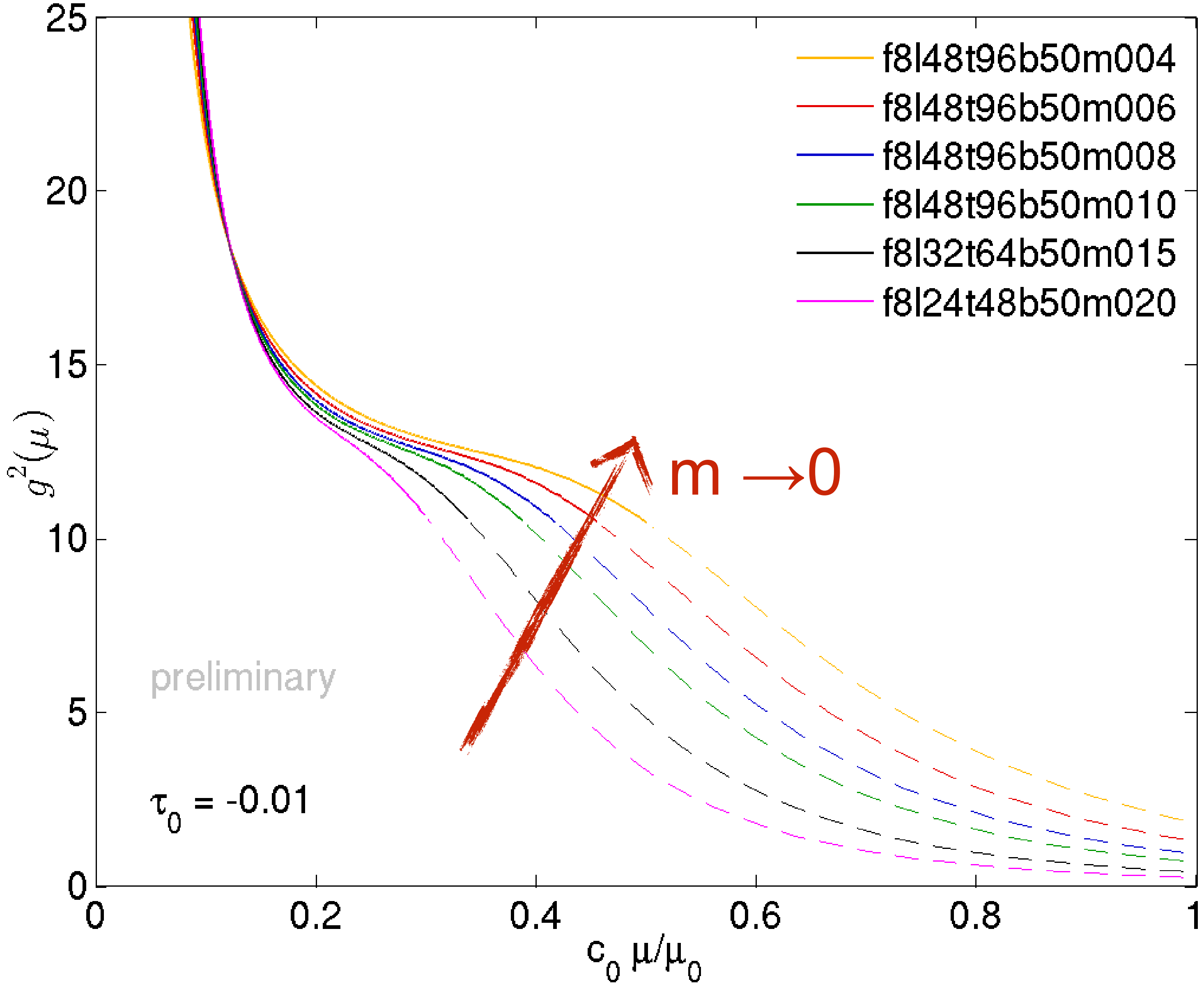
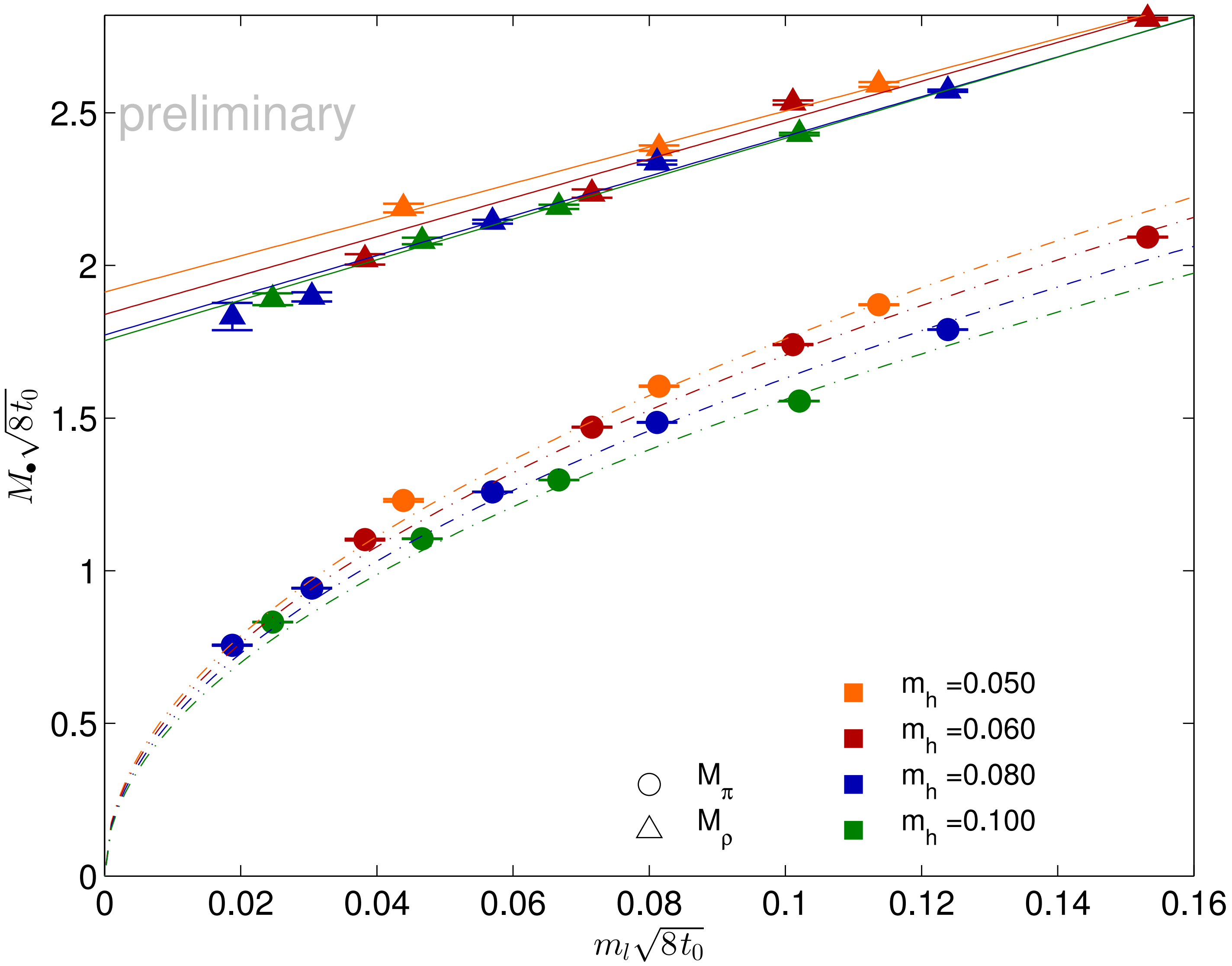


The "shoulder" is the gauge dynamics : slow evolution
The fast rise is due to the fermion mass running
What is the consequence of the two separate regimes?

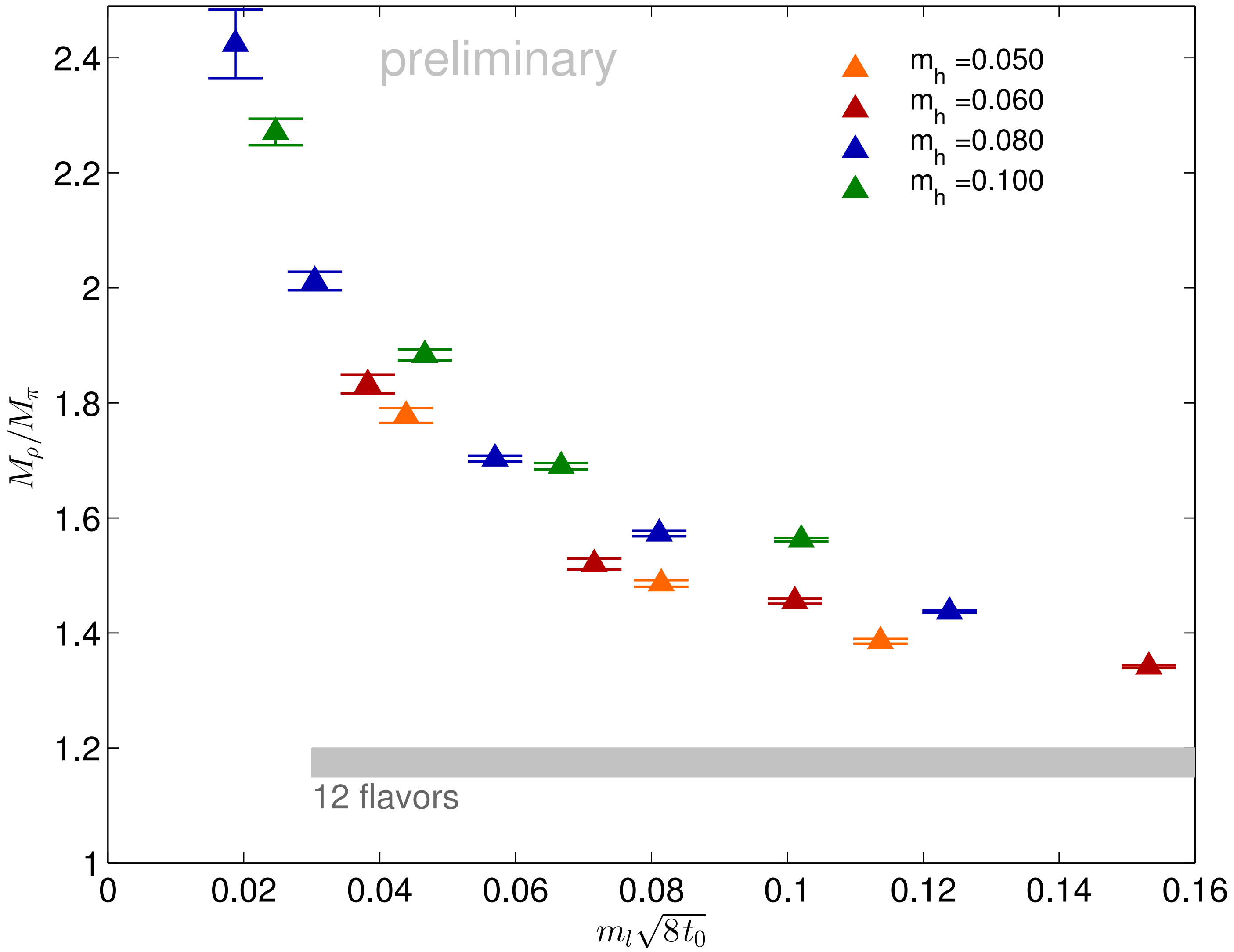
Connected spectrum, **4+8** flavors



- > M_π, M_ρ vs m_ℓ (rescaled by the gradient flow scale $\sqrt{8t_0}$)
  - – little variation with m_h

Chiral limit ?

M_ρ/M_π shows that we approach the chiral regime

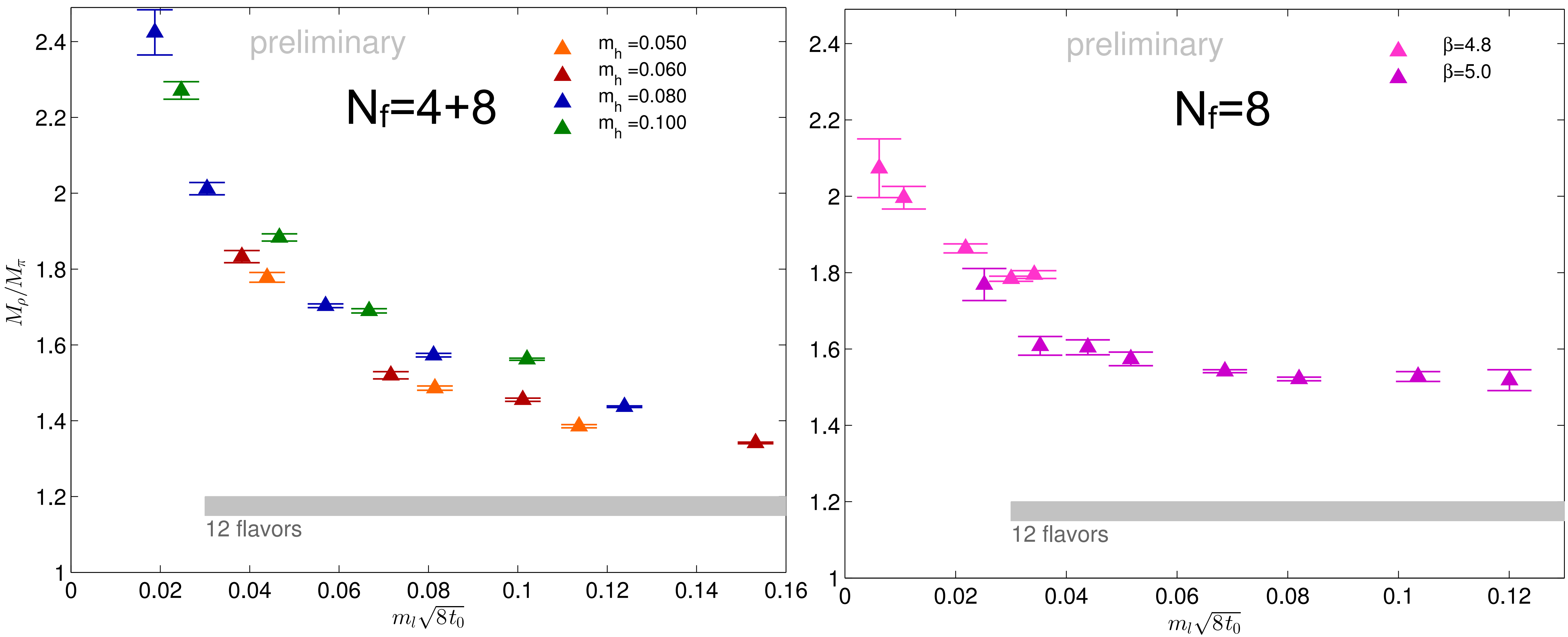


< N_f=12 predicts an almost constant ratio (as should be in a conformal system)

(arXiv:1401.0195)

Chiral limit ?

M_ρ/M_π : compare to 8 flavors



Finally : the 0^{++} scalar state

We use the same method to construct and fit the correlators as with N_f = 8 joint LSD project:

- Disconnected correlators:
  - 6 U(1) sources
  - diluted on each timeslice, color, even/odd spatial
  - variance reduced $\langle\bar{\psi}\psi\rangle$
- Fit:
  - correlated fits to both parity (staggered) states
  - the vacuum subtraction introduces very large uncertainties
    - it is advantageous to add a (free) constant to the fit

$$C(t) = c_{0^{++}} \cosh\left(M_{0^{++}}\left(N_T/2 - t\right)\right) + c_{\pi_{\overline{sc}}}(-1)^t \cosh\left(M_{\pi_{\overline{sc}}}\left(N_T/2 - t\right)\right) + v$$

–this is equivalent to fitting the finite difference of the correlator

$$C(t+1) - C(t)$$

Mixing in the 0^{++} channel

There is one major difference between $N_f = 4 + 8$ and 8 :

- with non-degenerate masses the 0^{++} splits to light and heavy states
- there is mixing the heavy and light species

This is similar to η - η' mixing in QCD

→ need to diagonalize the correlator matrix

$$C(t) = \begin{pmatrix} D_{ll}(t) - C_{ll}(t) & \sqrt{2} D_{lh}(t) \\ \sqrt{2} D_{hl}(t) & 2D_{hh}(t) - C_{hh}(t) \end{pmatrix}$$

Normalization: even though we we describe 4 and 8 flavors, on the lattice they correspond to 1 and 2 staggered species

Mixing in the 0^{++} channel

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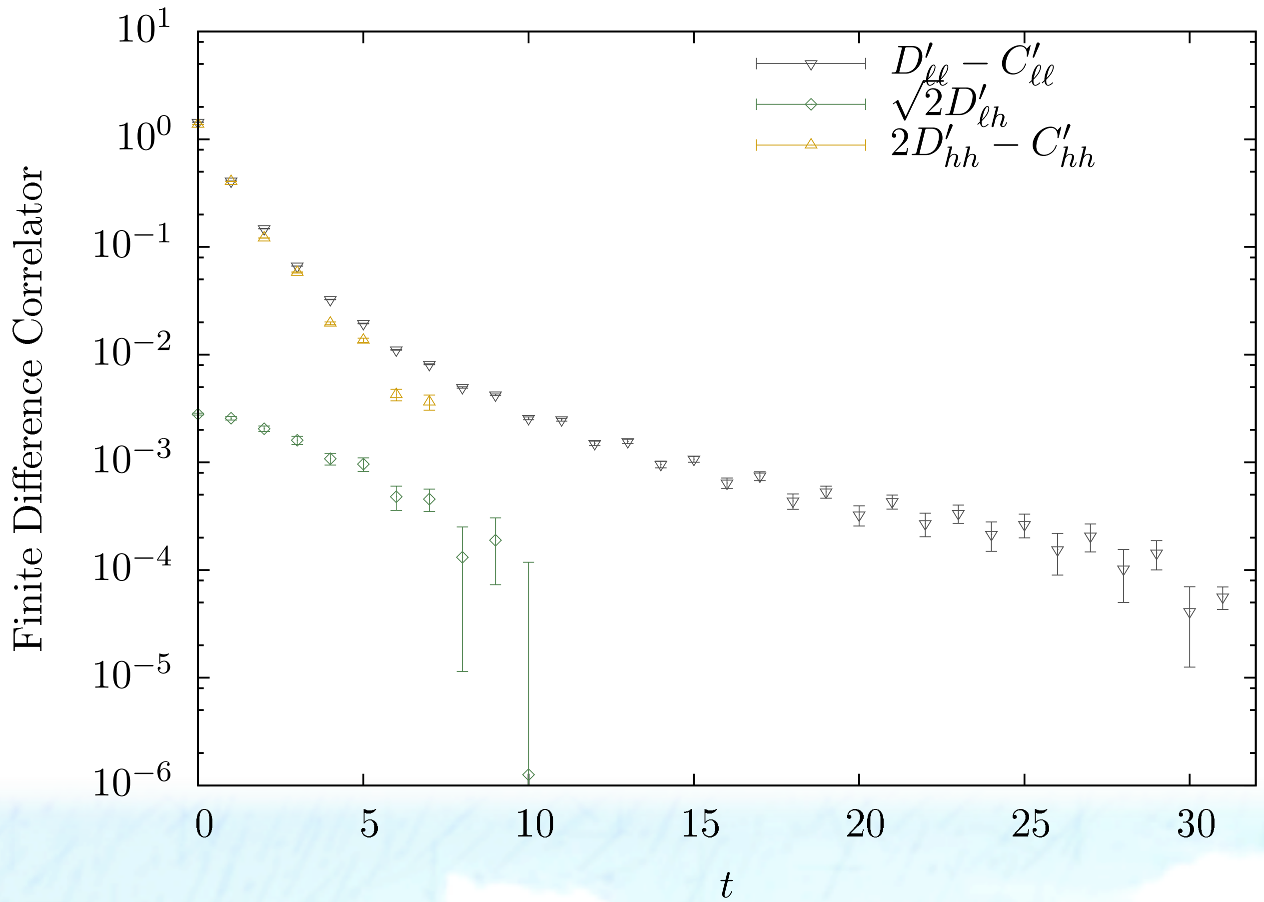
Diagonalizing C(t) could lead to very large statistical errors.

Fortunately: $D_{\ell h}$ << diagonal terms for almost all parameter values



Finite difference correlators at $m_h = 0.05$, $m_\ell = 0.005$

Mixing in the 0^{++} channel

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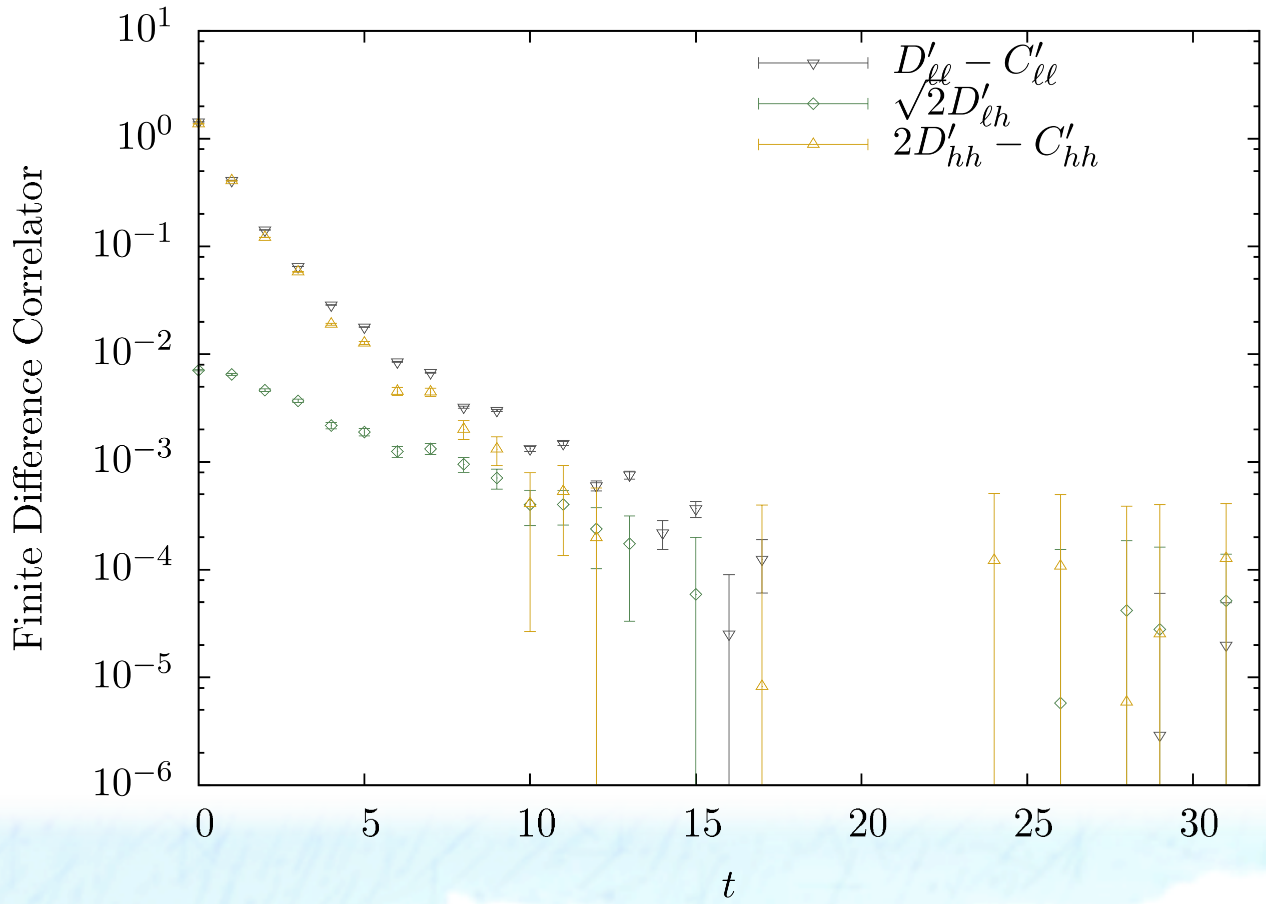
Diagonalizing C(t) could lead to very large statistical errors.

Fortunately: $D_{\ell h}$ << diagonal terms for almost all parameter values

but not always!



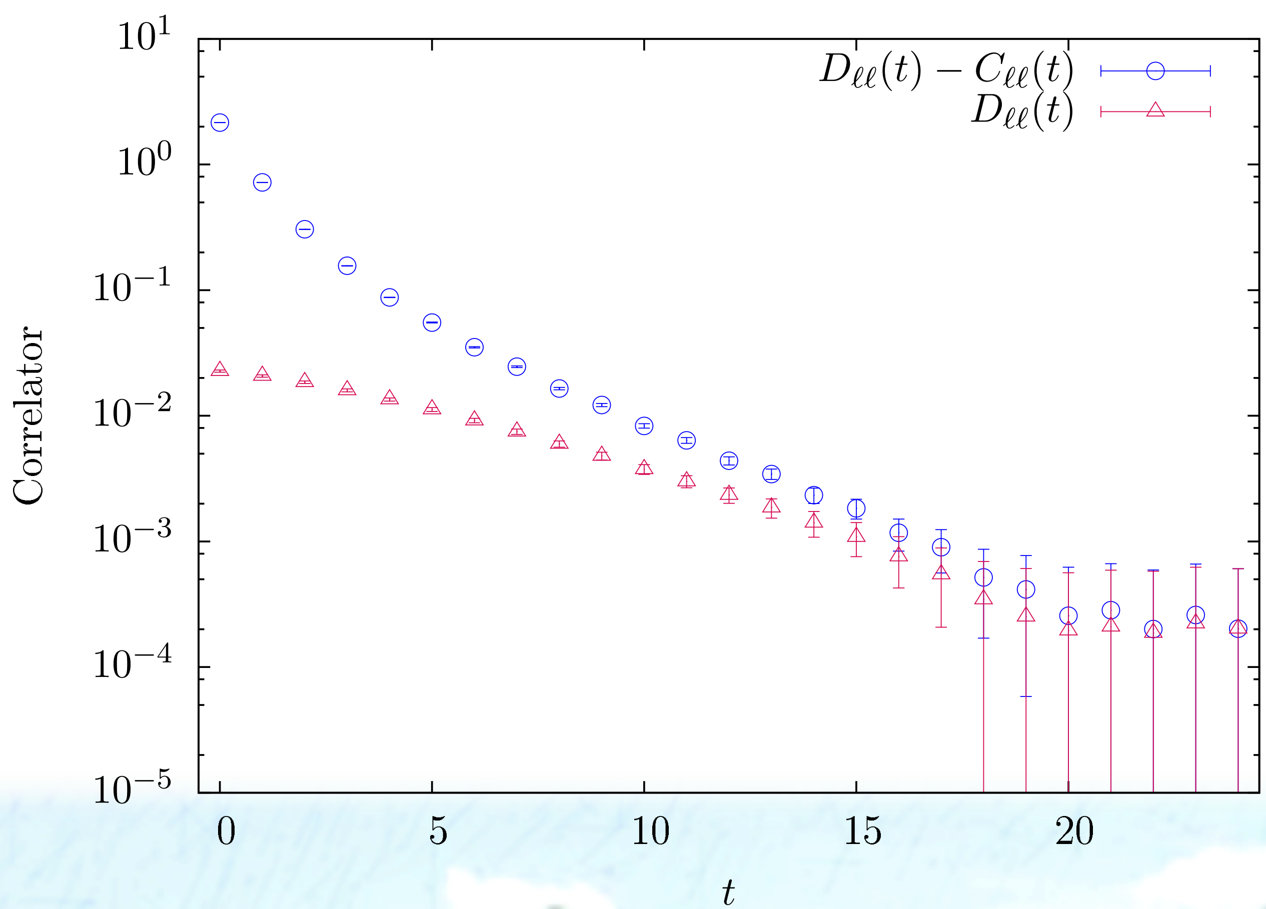
Derivative correlators at $m_h = 0.05$, $m_\ell = 0.015$

Mixing in the 0^{++} channel

$$C(t) = \begin{pmatrix} D_{ll}(t) - C_{ll}(t) & \sqrt{2} D_{lh}(t) \\ \sqrt{2} D_{hl}(t) & 2D_{hh}(t) - C_{hh}(t) \end{pmatrix}$$

Diagonalizing C(t) could lead to very large statistical errors.

Fortunately: the lightest excitation in $D_{\ell\ell}$ (and $D_{\ell h}$, D_{hh}) is the 0^{++}



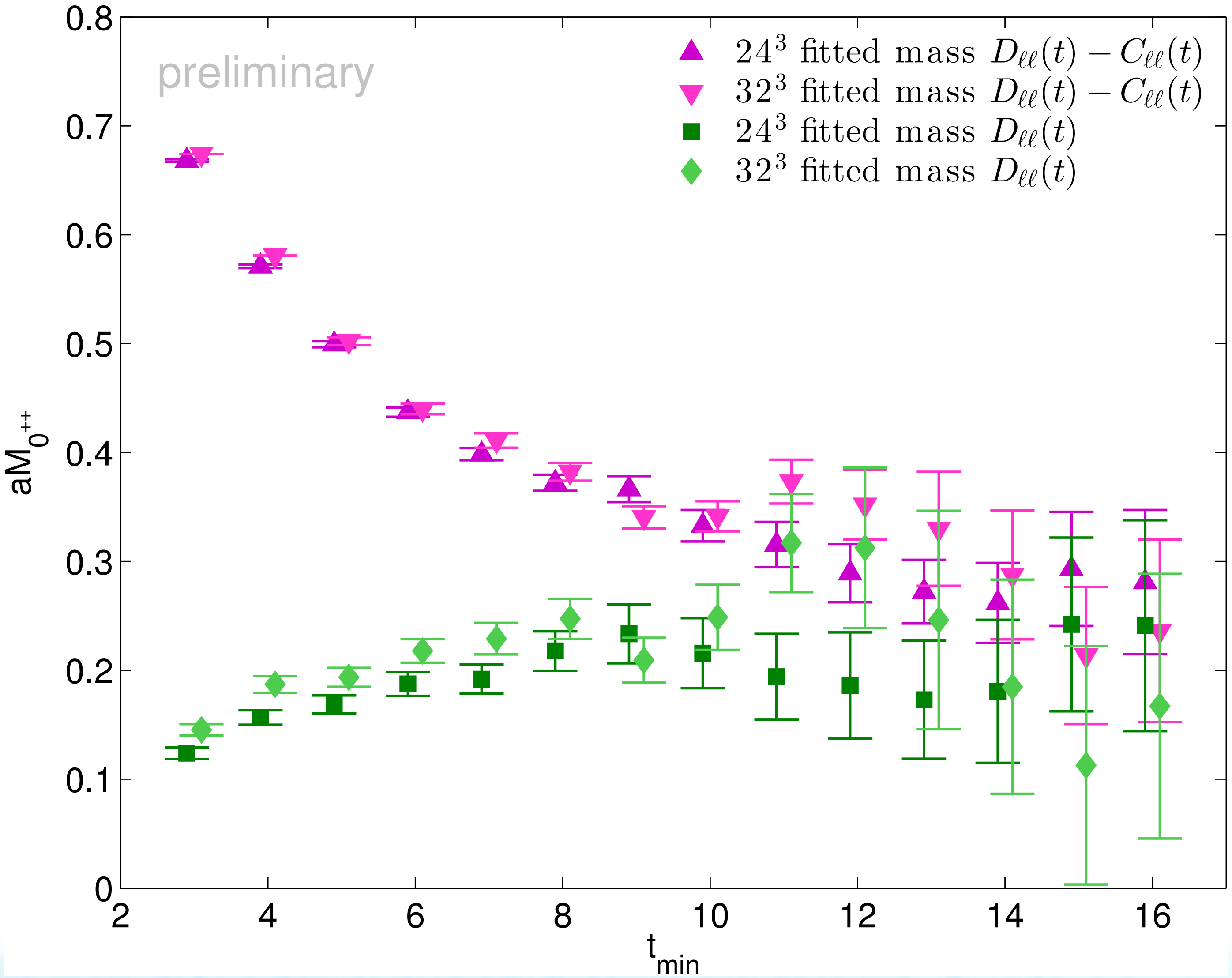
Derivative correlators at $m_h = 0.06$, $m_\ell = 0.010$: $D_{\ell\ell}$ and $D_{\ell\ell}$ - $C_{\ell\ell}$

The 0^{++} mass

We strive to compare predictions from $D_{\ell\ell}$ and $D_{\ell\ell}$ - $C_{\ell\ell}$ correlators

– in the $t \to \infty$ limit they should agree

m_h= 0.06 , m_ℓ = 0.010:



M_{0++} predicted from non-linear range fits (t_{min} - $N_T/2$)

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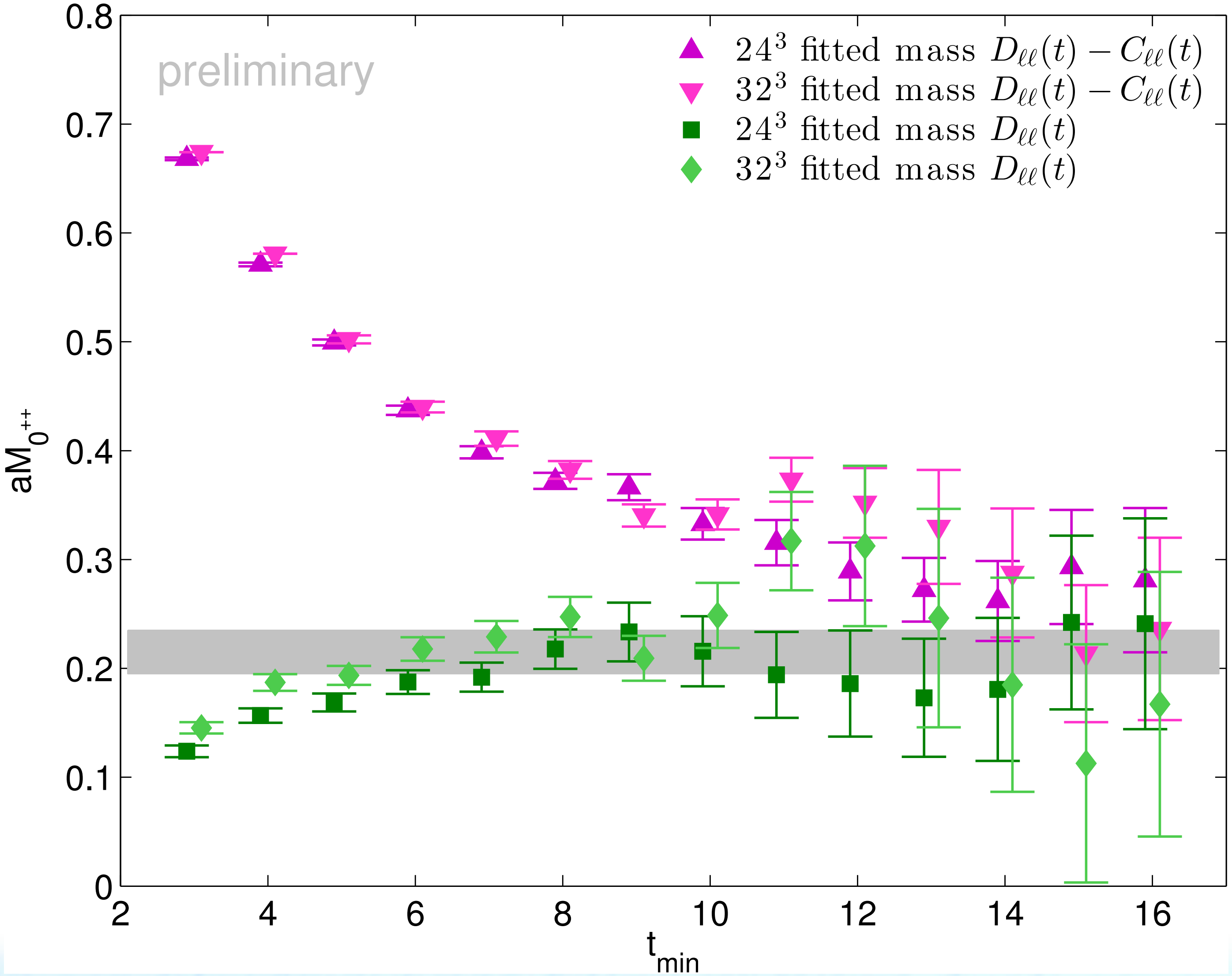

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both volumes, both correlators predict a consistent value

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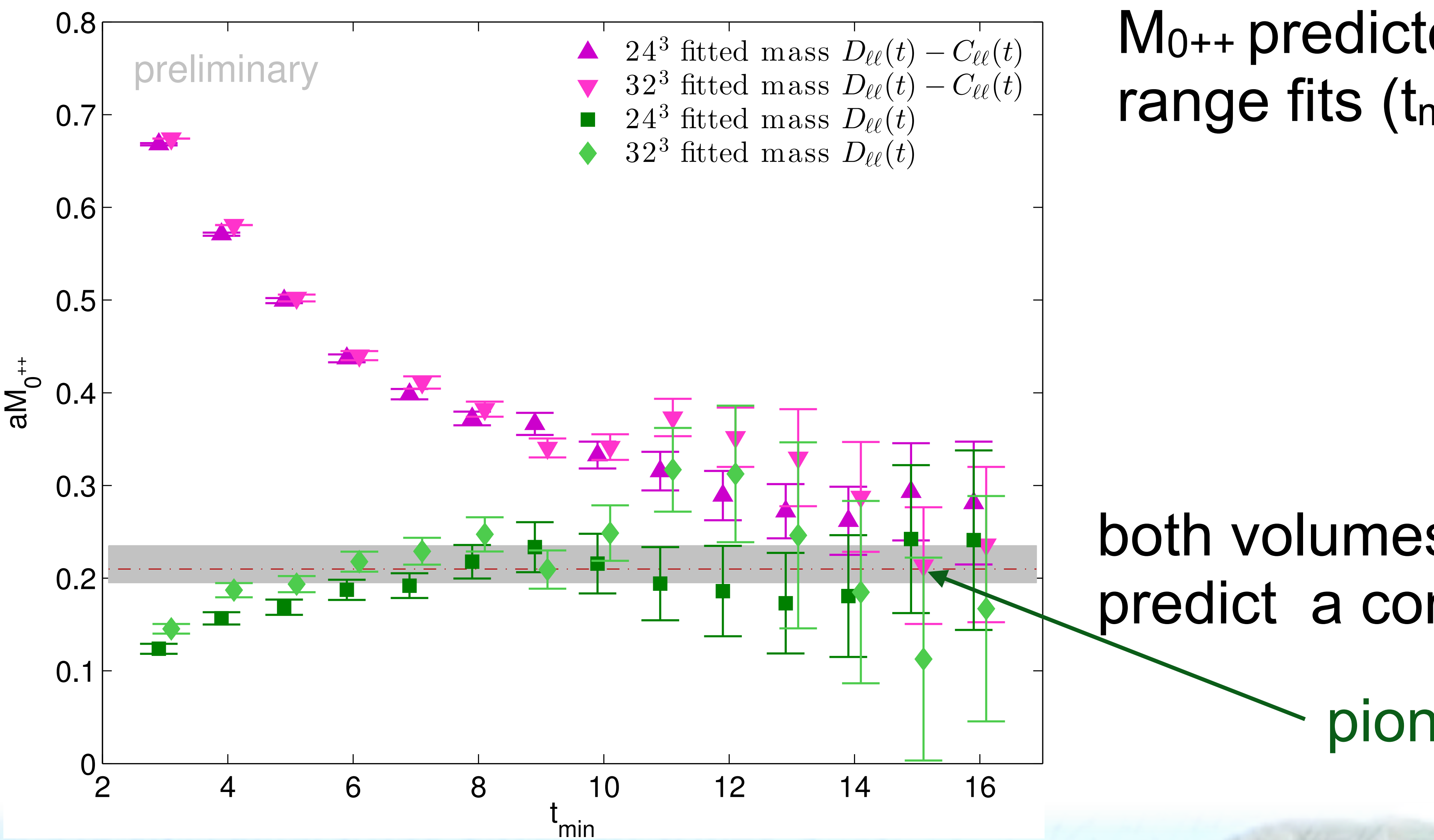

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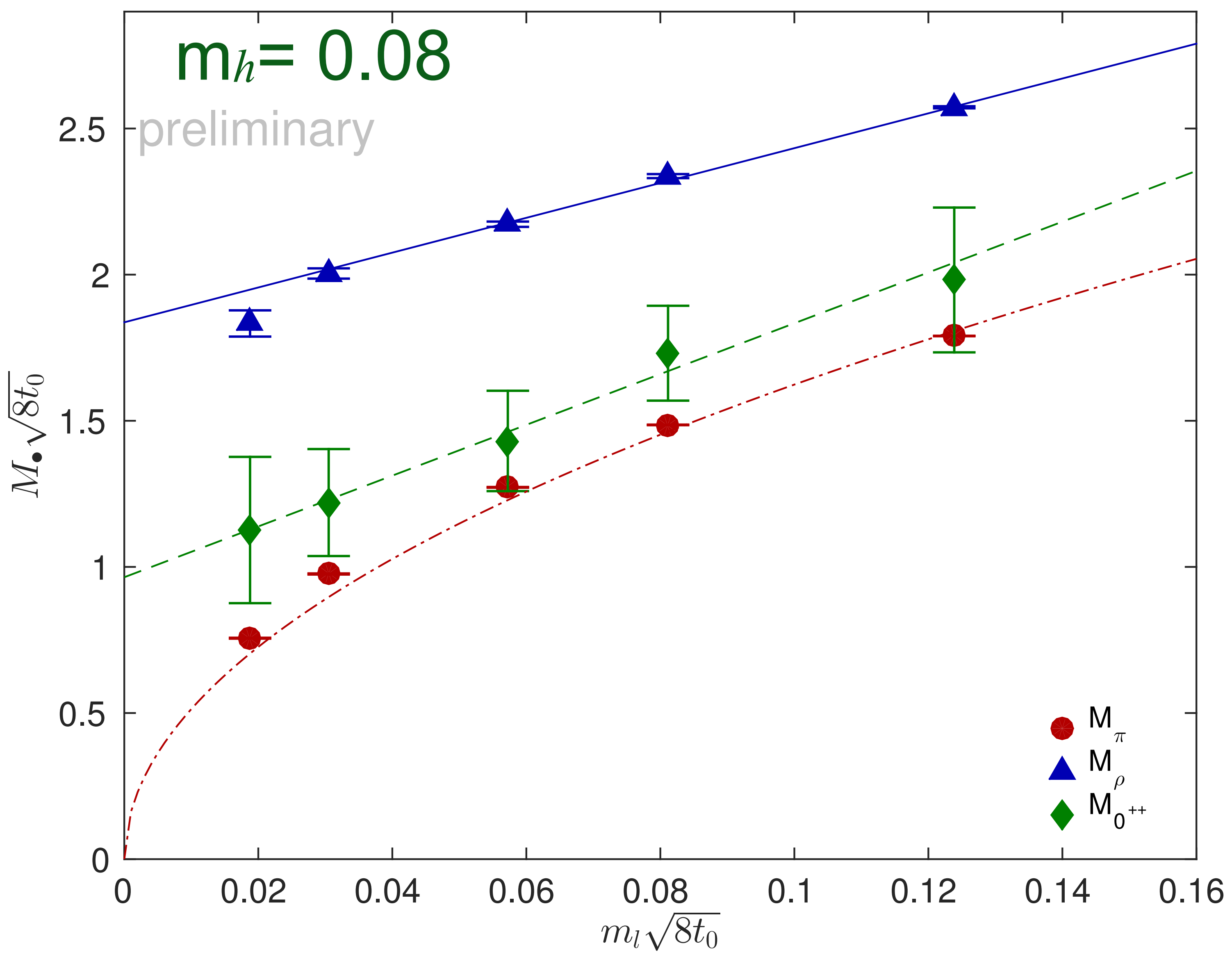
Spectrum

Compare the pion, rho and 0^{++} masses:

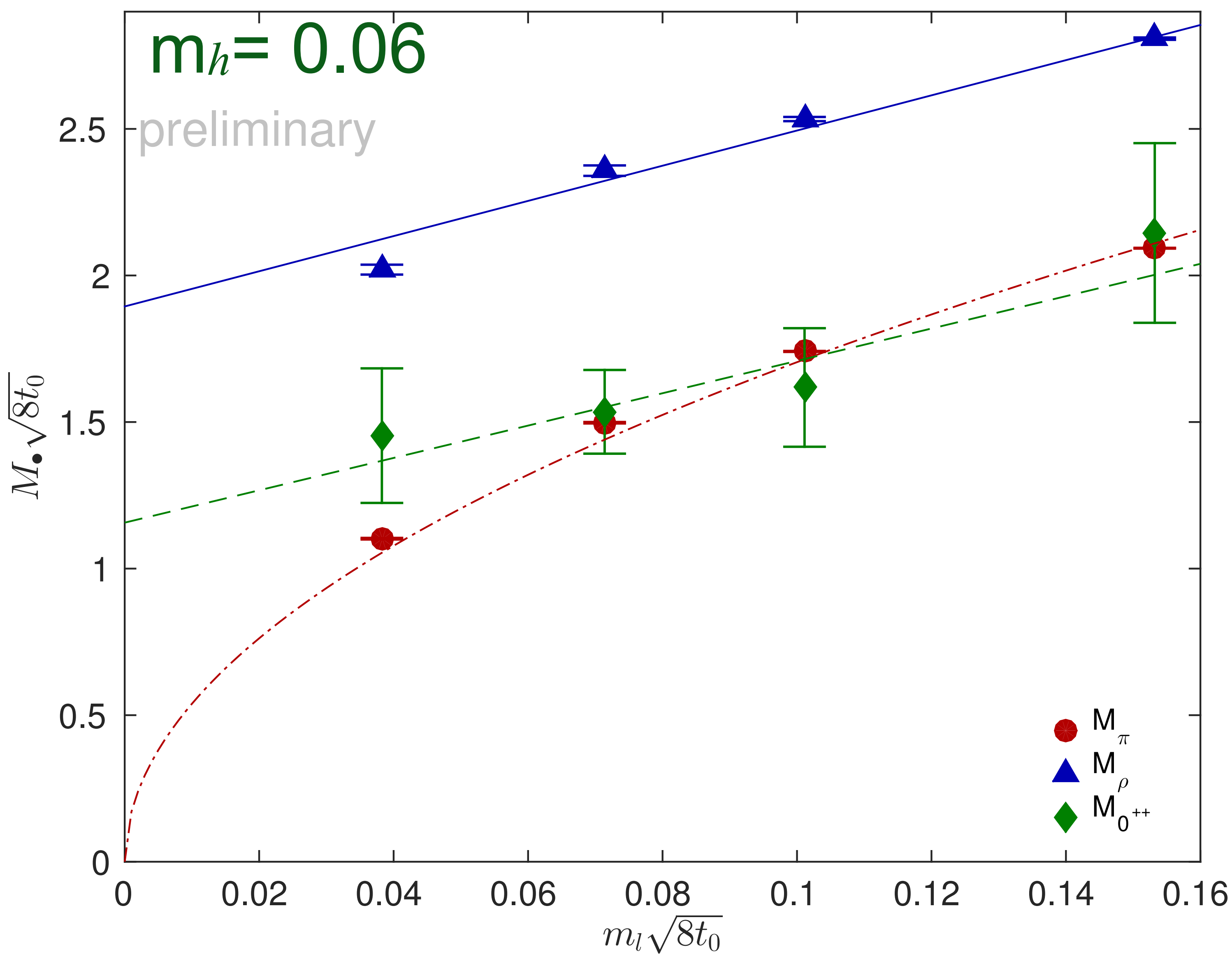


m_h = 0.08: the 0++

- is just above the pion,
- not Goldstone
- well below the rho

Spectrum

Compare the pion, rho and 0^{++} masses:



m_h = 0.06: the 0++

- is degenerate with pion at heavier m_ℓ
- need larger volumes, more statistics to resolve the small m_ℓ region

Conclusion & Summary

Lots of interesting possibilities ….

Lattice studies are needed to investigate strongly coupled systems

Even those without apparent phenomenological importance can teach us :

- understand universality
  - Wilson vs staggered vs rooted staggered vs domain wall fermions
- understand general properties of strongly coupled systems
  - walking near the conformal window
  - 0^{++} near the conformal window

Models with split fermion masses, like the 4+8 flavor model, help us navigate the landscape

Special thanks to my collaborators

Rich Brower, Claudio Rebbi,
Evan Weinberg and Oliver Witzel