Walking dynamics from gauge/gravity duality

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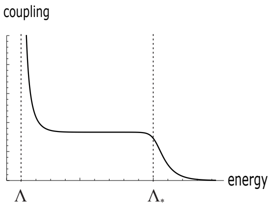
A light scalar from deformations of Klebanov-Strassler
(DE, arXiv:1401.3412 [hep-th])
On the glueball spectrum of walking backgrounds from wrapped-D5 gravity duals
(DE, Maurizio Piai, arXiv:1212.2600 [hep-th])
Towards multi-scale dynamics on the baryonic branch of Klebanov-Strassler
(DE, Jerome Gaillard, Carlos Nunez, Maurizio Piai, arXiv:1104.3963 [hep-th])
Light scalars from a compact fifth dimension
(DE, Maurizio Piai, arXiv:1010.1964 [hep-th])
A light scalar from walking solutions in gauge-string duality
(DE, Maurizio Piai, Carlos Nunez, arXiv:0908.2808 [hep-th])

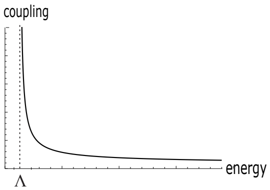
Introduction

Can we find walking dynamics using gauge/gravity duality?

- IR dynamics governed by an approximate fixed point:



- Contrast with QCD:



Motivation

Theoretical:

- Natural to consider strongly coupled field theories with more than one scale
- Potential for richer dynamics

Electro-Weak Symmetry Breaking:

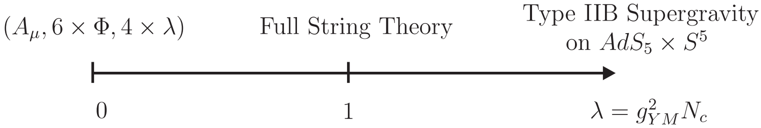
- Strongly coupled theories could solve hierarchy problem
- Simple Technicolor models are ruled out experimentally
- Walking offers a way out: $(\Lambda/\Lambda_*)^\gamma$, large anomalous dimension γ

Techni-dilaton

- Spontaneously broken approximate scale invariance
- Could this lead to a light scalar, the dilaton (pseudo-Goldstone of dilatations), in the spectrum?
- Such a light scalar would couple to the Standard Model fields in a way that mimicks the Higgs

How to compute at strong coupling?

- AdS/CFT is a duality between $\mathcal{N}=4$ SYM and Type IIB String Theory on $\mathrm{AdS}_5 \times S^5$:



- Allows to study strongly coupled dynamics in field theory
- The extra bulk dimension (the radial coordinate r) is related to energy scale in the field theory, and thus the bulk is in a sense a geometrical representation of the RG flow of the dual theory

Gauge/gravity duality

What does this have to do with walking dynamics?

- The walking region can be thought of as the theory flowing near an IR fixed point
- This near conformality means that we can apply ideas from AdS/CFT

These fall into two classes:

- Phenomenological bottom-up models where the matter content in the bulk is put in by hand
  (Example: $\int d^4x dr \sqrt{-g} \left[R - \frac{1}{2} (\partial\Phi)^2 - V(\Phi) \right]$)
- Top-down models which have their origin in string theory constructions, and therefore are on firmer ground

Gauge/gravity duality

We will focus on top-down approaches

Questions:

- Can we build top-down models from string theory with walking dynamics?
- Do they contain a light scalar in the spectrum?

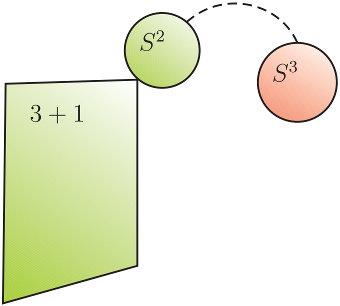
Outline of talk

- Walking dynamics from wrapped D5-branes
- Spectrum
- Deformations of Klebanov-Strassler
- Conclusions and open questions

Walking dynamics from wrapped D5-branes

Let us consider a top-down model obtained from string theory

- D5 system:



- D5-branes wrapped on S^2
- This gives us an $\mathcal{N} = 1$ SUSY field theory

Walking dynamics from wrapped D5-branes

Type IIB supergravity ansatz (ds^2, F_3, ϕ):

$$
\begin{aligned}
ds^2 &= e^{2p-x} ds_5^2 + (e^{x+g} + a^2 e^{x-g})(e_1^2 + e_2^2) + e^{x-g}\left(e_3^2 + e_4^2 + 2a(e_1 e_3 + e_2 e_4)\right) + e^{-6p-x} e_5^2,\\
ds_5^2 &= dr^2 + e^{2A} dx_{1,3}^2,\\
F_3 &= N\left[-e_5 \wedge (e_4 \wedge e_3 + e_2 \wedge e_1 + b(e_4 \wedge e_1 - e_3 \wedge e_2)) + dr \wedge \left(\partial_r b(e_4 \wedge e_2 + e_3 \wedge e_1)\right)\right],
\end{aligned}
$$

where

$$
\begin{aligned}
e_1 &= -\sin\theta \, d\phi ,\\
e_2 &= d\theta ,\\
e_3 &= \cos\psi \sin\tilde\theta \, d\tilde\phi - \sin\psi \, d\tilde\theta ,\\
e_4 &= \sin\psi \sin\tilde\theta \, d\tilde\phi + \cos\psi \, d\tilde\theta ,\\
e_5 &= d\psi + \cos\tilde\theta \, d\tilde\phi + \cos\theta \, d\phi
\end{aligned}
$$

Walking dynamics from wrapped D5-branes

Finding solutions:

- Write down BPS equations for the background fields
- These can be repackaged into a single second order differential equation (Hoyos, Nunez, Papadimitriou 2008):

$$P'' + P'\left[\frac{P' + Q'}{P - Q} + \frac{P' - Q'}{P + Q} - 4\coth(2\rho)\right] = 0,$$

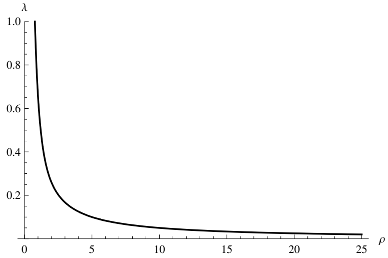
$$Q(\rho) = N_c(2\rho\coth(2\rho) - 1)$$

- Map from P to solutions in Type IIB supergravity: $P \to \{p, x, g, \phi, A, a, b\}$

Walking dynamics from wrapped D5-branes

Example: Maldacena-Nunez ($P = 2N_c\rho$)

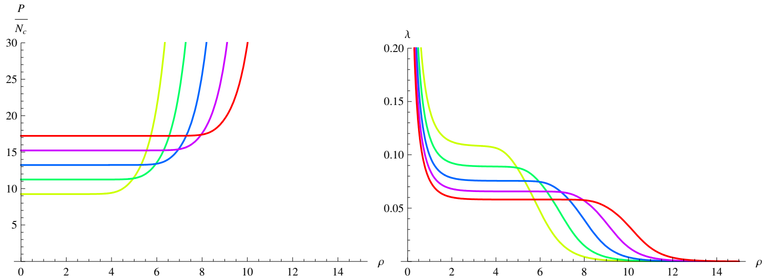
- Non-singular: in the IR, S^2 shrinks to zero size, while the size of S^3 stays finite (deformed conifold)
- 4d gauge coupling constant $\lambda = \frac{g^2 N_c}{8\pi^2} = \frac{N_c \coth\rho}{P}$



- One scale: confinement scale

Walking dynamics from wrapped D5-branes

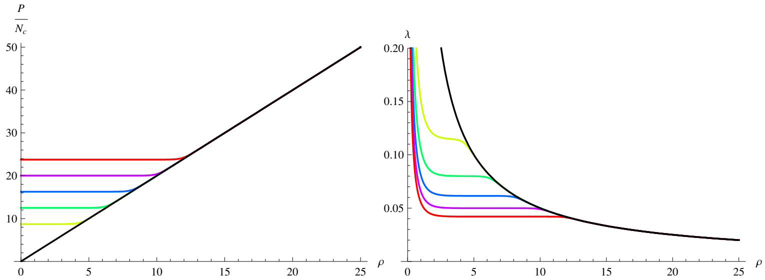
Walking backgrounds (UV behaviour: $P \sim e^{4\rho/3}$):
(Nunez, Papadimitriou, Piai 2008)



Two scales: confinement scale and end of walking region ρ_*

Walking dynamics from wrapped D5-branes

Walking backgrounds (UV behaviour: Maldacena-Nunez):
(DE, Nunez, Piai 2009)



Two scales: confinement scale and end of walking region ρ_*

Spectrum

Is there a light scalar in the spectrum of glueballs? Compute spectrum holographically:

- Expand EOMs to linear order in fluctuations around the background
- Impose appropriate BCs on fluctuations in the IR and UV
- The values of momenta $K^2(= -M^2)$ for which solutions exist give us the spectrum

Spectrum

In practice, it is easier to work in five dimensions:

- Consistent truncation to a 5d non-linear sigma model with fields $\Phi = (g, p, x, \phi, a, b)$:
  (Berg, Haack, Mück, 2005)

$$S_{5d} = \int d^4x dr \sqrt{g} \left[\frac{R}{4} - \frac{1}{2} G_{ab}(\Phi) \partial_M \Phi^a \partial^M \Phi^b - V(\Phi) \right]$$

- Metric with warp factor A:

$$ds^2 = dr^2 + e^{2A} dx_{1,3}^2 \qquad (1)$$

- Any solution of 5d system solves 10d EOMs

Spectrum

$$V = \frac{e^{-2(g+2(p+x))}}{128}\Bigg[16\left(a^4 + 2\left(\left(e^g - e^{6p+2x}\right)^2 - 1\right)a^2 + e^{4g} - 4e^{g+6p+2x}\left(1+e^{2g}\right) + 1\right) + e^{12p+2x+\phi}\left(2e^{2g}(a-b)^2 + e^{4g} + \left(a^2 - 2ba + 1\right)^2\right)N_c^2\Bigg],$$

$$G_{ab} = \mathrm{diag}\left(\frac{1}{2}, 6, 1, \frac{1}{4}, \frac{e^{-2g}}{2}, \frac{e^{-2x+\phi}N_c^2}{32}\right)$$

Spectrum

- ADM-formalism: write the metric as (lapse function n and shift vector n^μ)

$$ds^2 = (n^\mu n_\mu + n^2)dr^2 + 2n_\mu dx^\mu dr + g_{\mu\nu}dx^\mu dx^\nu \tag{2}$$

- Expand to linear order in fluctuations $\{\varphi^a, \nu, \nu^\mu, h^{TT\mu}{}_\nu, h, H, \epsilon^\mu\}$ around the background:

$$\begin{aligned}
\Phi^a &= \bar{\Phi}^a + \varphi^a, \\
n &= 1 + \nu, \\
n^\mu &= \nu^\mu, \\
g_{\mu\nu} &= e^{2A}(\eta_{\mu\nu} + h_{\mu\nu}),
\end{aligned}$$

with

$$h^\mu{}_\nu = h^{TT\mu}{}_\nu + \partial^\mu \epsilon_\nu + \partial_\nu \epsilon^\mu + \frac{\partial^\mu \partial_\nu}{\Box} H + \frac{1}{3}\delta^\mu{}_\nu h$$

Spectrum

Linearized equations of motion for spin-0 sector:

$$\left[D_r^2 + 4A'D_r - e^{-2A}K^2\right]\mathfrak{a}^a -$$
$$\left[V^a{}_{|c} - \mathcal{R}^a{}_{bcd}\bar{\Phi}'^b\bar{\Phi}'^d + \frac{4(\bar{\Phi}'^a V_c + V^a\bar{\Phi}'_c)}{3A'} + \frac{16V\bar{\Phi}'^a\bar{\Phi}'_c}{9A'^2}\right]\mathfrak{a}^c = 0$$

with

$$\mathfrak{a}^a = \varphi^a - \frac{\bar{\Phi}'^{\,a}}{6A'}h$$

Spectrum

What boundary conditions should we impose?

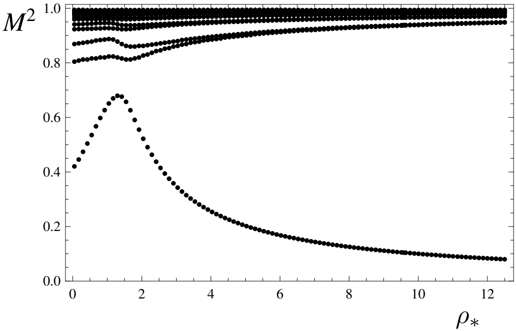
- Usually, one picks the normalizable modes in the UV, and imposes regularity in the IR
- We put $\varphi^a = 0$ in the IR and UV, which corresponds to

$$-\frac{2\Phi'^a \Phi'_b}{3A'} D_r \mathfrak{a}^b \Big|_{IR,UV} = \left[e^{-2A} q^2 + \frac{A'}{2} \partial_r \left(\frac{A''}{A'^2} \right) \right] \mathfrak{a}^a \Big|_{IR,UV}$$

- This automatically picks the normalizable mode in simple examples (e.g. Goldberger-Wise) and reproduces the same spectrum in all examples we have tried (e.g. Maldacena-Nunez, Klebanov-Strassler)

Spectrum

Spectrum of scalar glueballs for different values of ρ_*:



Light scalar whose mass is suppressed by the length of the walking region

Deformations of Klebanov-Strassler

UV behaviour of the walking backgrounds discussed so far:

- Not asymptotically AdS (either MN or dim-8 operator)
- The dictionary is less well-defined
- It is not easy to identify the QFT that is dual to a particular geometry

Deformations of Klebanov-Strassler

- The baryonic branch of Klebanov-Strassler is parameterized by a dim-2 VEV $\langle \mathrm{Tr}(A\bar{A} - B\bar{B}) \rangle$
- Maldacena-Nunez can be thought of as taking the limit $\langle \mathrm{Tr}(A\bar{A} - B\bar{B}) \rangle \to \infty$
- Can we find walking dynamics on the baryonic branch of Klebanov-Strassler?
- This would lead to better UV asymptotics (well-known duality cascade)

Deformations of Klebanov-Strassler

Solution generating technique (Maldacena, Martelli 2009):

- Start with a solution to the D5 system (ds^2, F_3, ϕ)
- Generate a new (rotated) solution $(ds^{(r)2}, F_3^{(r)}, H_3^{(r)}, F_5^{(r)}, \phi^{(r)})$:

$$
\begin{aligned}
ds^{(r)2} &= e^{\phi/2}\Big[\left(1-\kappa^2 e^{2\phi}\right)^{-1/2} dx_{1,3}^2 + \left(1-\kappa^2 e^{2\phi}\right)^{1/2} ds_6^2\Big],\\
\phi^{(r)} &= \phi,\\
F_3^{(r)} &= F_3,\\
H_3^{(r)} &= -\kappa e^{2\phi} *_6 F_3,\\
F_5^{(r)} &= -\kappa(1+*_{10})\mathrm{vol}_{(4)} \wedge d\left(e^{-2\phi}-\kappa^2\right)^{-1}
\end{aligned}
$$

- D5 system $\rightarrow$ D3/D5 system
- Preserves SUSY

Deformations of Klebanov-Strassler

- Apply to the walking backgrounds with $P \sim e^{4\rho/3}$ in the UV (dim-8 operator)
- For $\kappa = e^{-\phi_{UV}}$, the rotated backgrounds behave asymptotically like Klebanov-Strassler in the UV
- The field theory dual to KS is more well-understood: $SU(N+M) \times SU(N)$ gauge group, bifundamental matter A_i and B_i $(i = 1, 2)$ in representations $(N + M, \overline{N})$ and $(\overline{N + M}, N)$, superpotential $\mathcal{W} = \lambda_1 \mathrm{Tr}(A_i B_j A_k B_l)\epsilon^{ik}\epsilon^{jl}$

Deformations of Klebanov-Strassler

Properties of the rotated solutions $(\kappa = e^{-\phi_{UV}})$:

- The dim-8 operator is no longer present, making the UV well-defined
- There is a dim-3 VEV, the gaugino condensate
- There is a dim-2 VEV, $\langle \mathrm{Tr}(A\bar{A} - B\bar{B}) \rangle \neq 0$, signalling that we are on the baryonic branch of Klebanov-Strassler
- There is a dim-6 VEV, $\langle \mathrm{Tr} W^2 \bar{W}^2 \rangle \neq 0$
- Compute Wilson loops $\Rightarrow$ confinement

Deformations of Klebanov-Strassler

A particularly simple case, turn off dim-2 VEV:

$$
\begin{aligned}
ds_{10}^2 &= h(\rho)^{-1/2} dx_{1,3}^2 + h(\rho)^{1/2} ds_6^2, \qquad (\textit{string frame}) \\
ds_6^2 &= \frac{\epsilon^{4/3} K(\rho)}{4} \left[\frac{2}{3K(\rho)^3} (4d\rho^2 + e_5^2) + 2(e_1 e_3 + e_2 e_4) + \cosh(2\rho) \sum_{i=1}^{4} e_i^2 \right], \\
\phi &= \phi_0, \\
B_2 &= f(\rho) \Big[e_1 \wedge e_2 + e_3 \wedge e_4 + \mathrm{sech}(2\rho)(e_1 \wedge e_4 - e_2 \wedge e_3) \Big], \\
F_3 &= N \Big[-e_5 \wedge (e_4 \wedge e_3 + e_2 \wedge e_1 + b(\rho)(e_4 \wedge e_1 - e_3 \wedge e_2)) + b'(\rho) d\rho \wedge (e_4 \wedge e_2 + e_3 \wedge e_1) \Big], \\
F_5 &= (1 + *_{10}) f_5, \quad f_5 = dc_4, \quad c_4 = e^{-\phi_0} h(\rho)^{-1} \mathrm{vol}_{1,3},
\end{aligned}
$$

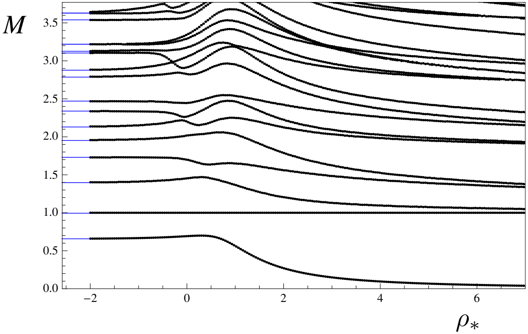
where

$$
\begin{aligned}
b(\rho) &= \frac{2\rho}{\sinh(2\rho)}, \quad K(\rho) = \frac{(f_0 - 4\rho + \sinh(4\rho))^{1/3}}{2^{1/3} \sinh(2\rho)}, \\
h(\rho) &= \frac{64 N e^{\phi_0}}{\epsilon^{8/3}} \int_\rho^\infty d\tilde\rho \, \frac{f(\tilde\rho)}{\sinh^2(2\tilde\rho) K(\tilde\rho)^2} \left(\frac{4\tilde\rho}{\sinh(4\tilde\rho)} - 1 \right), \\
f(\rho) &= N e^{\phi_0} \coth(2\rho)(1 - 2\rho \coth(2\rho))
\end{aligned}
$$

The dim-6 VEV is parameterized by $f_0 \equiv e^{4\rho_*}$ (KS corresponds to $f_0 = 0$)

Deformations of Klebanov-Strassler

Spectrum of scalar glueballs for different values of ρ_*:



Light scalar whose mass is suppressed by the length of the walking region

Conclusions and open questions

Conclusions:

- We constructed top-down models with walking dynamics
- We found a light state suggestive of being a techni-dilaton

Open Questions:

- The backgrounds have a mild singularity in the IR: Wilson loop makes sense, R and $R_{\mu\nu}R^{\mu\nu}$ are finite, but $R_{\mu\nu\delta\sigma}R^{\mu\nu\delta\sigma}$ diverges. Can we construct non-singular walking backgrounds?
- Spectrum when dim-2 VEV is finite?
- How to incorporate EWSB? Can we find U-shaped D-brane embeddings?