

The infrared regime of $SU(2)$ with one adjoint Dirac Fermion

Ed Bennett



Swansea University
Prifysgol Abertawe

with

Andreas Athenodorou, Georg Bergner, and Biagio Lucini

Outline

Introduction

- Motivation and background
- Chiral symmetry breaking
- Aims and predictions

Results

- Phase diagram
- Spectrum
- Mass anomalous dimension

Conclusions and outlook

Motivation

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- Can we pin down the end of the conformal window?
- Look at $SU(2) + 1$ adjoint Dirac flavour

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Strong assertions of confinement are not justified.

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- Breaks to $SO(2)$: 2 Goldstones
- Insufficient for EWSB; not a WT candidate

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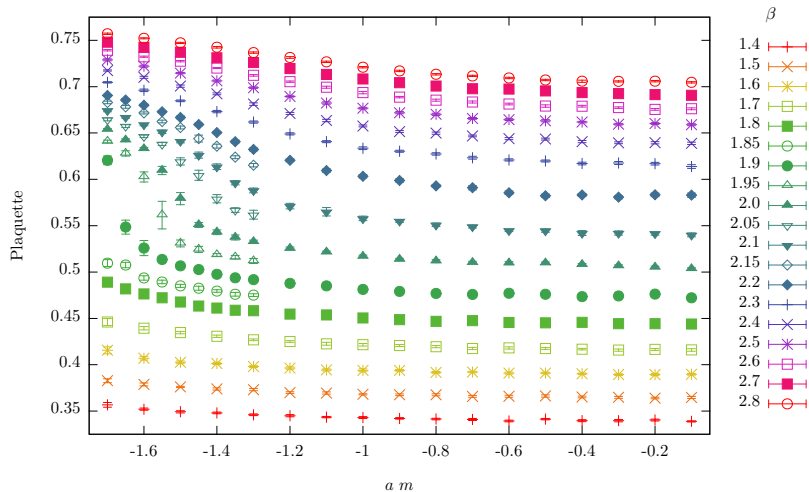
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- Near-conformal: Intermediary conformal-like region, IR confining region
 - Not clearly identifiable, for limited range of masses

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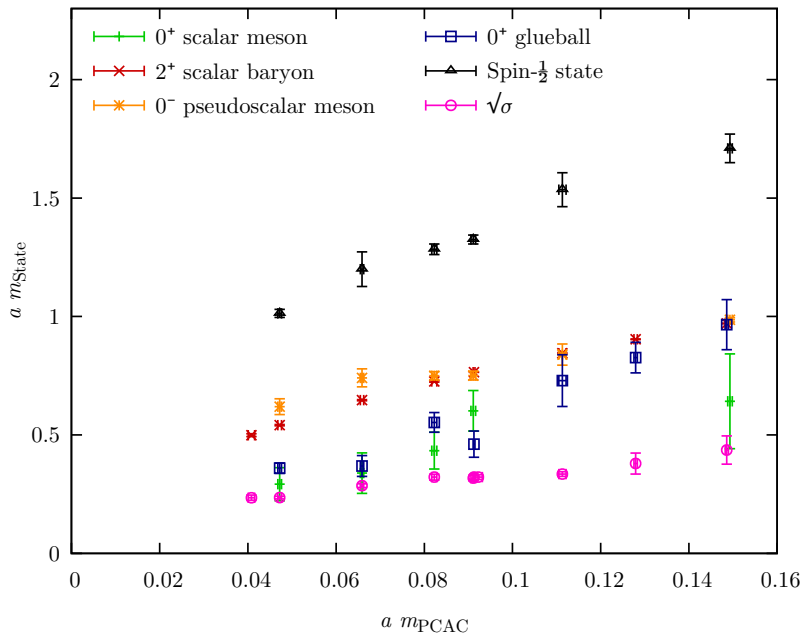
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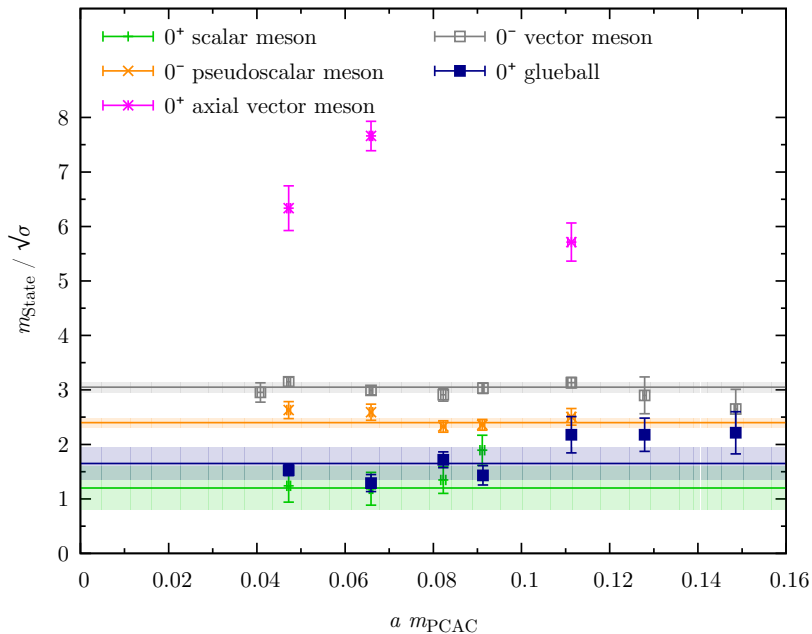
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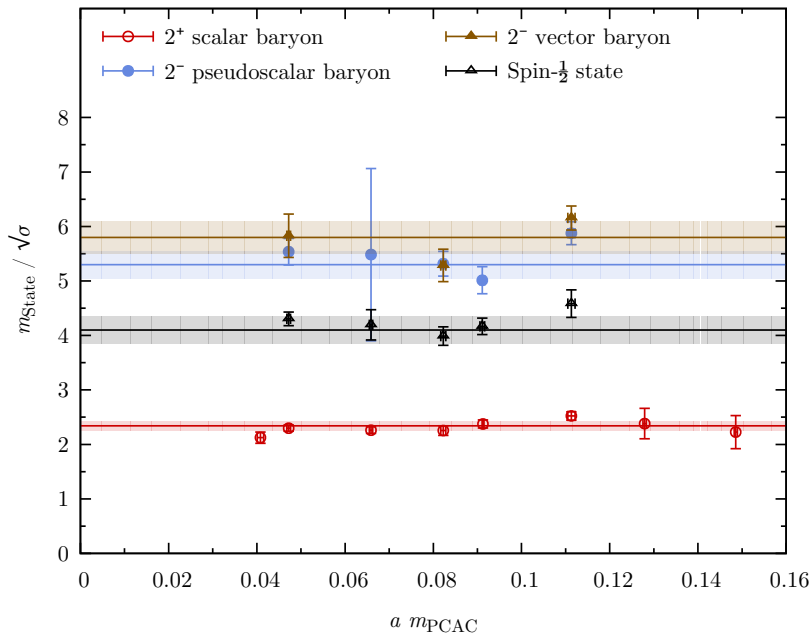
$\beta = 2.05$ spectrum



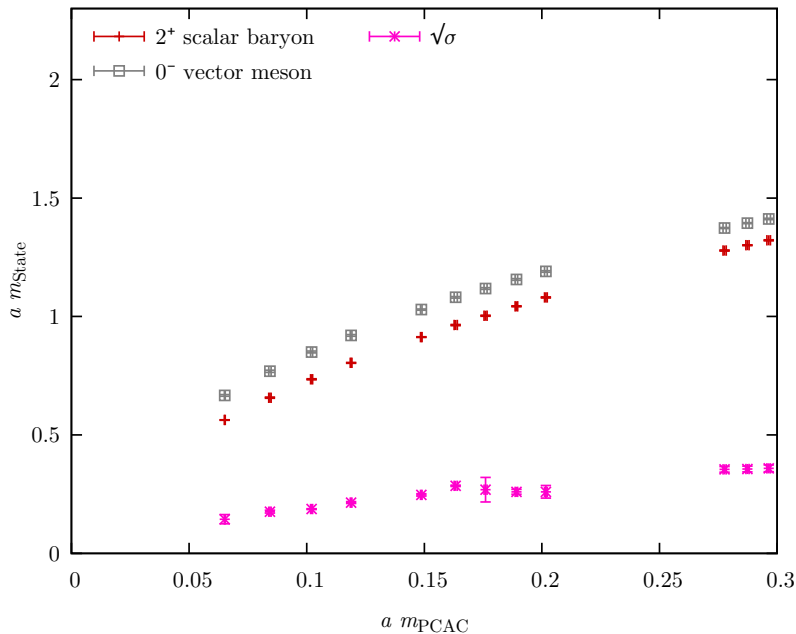
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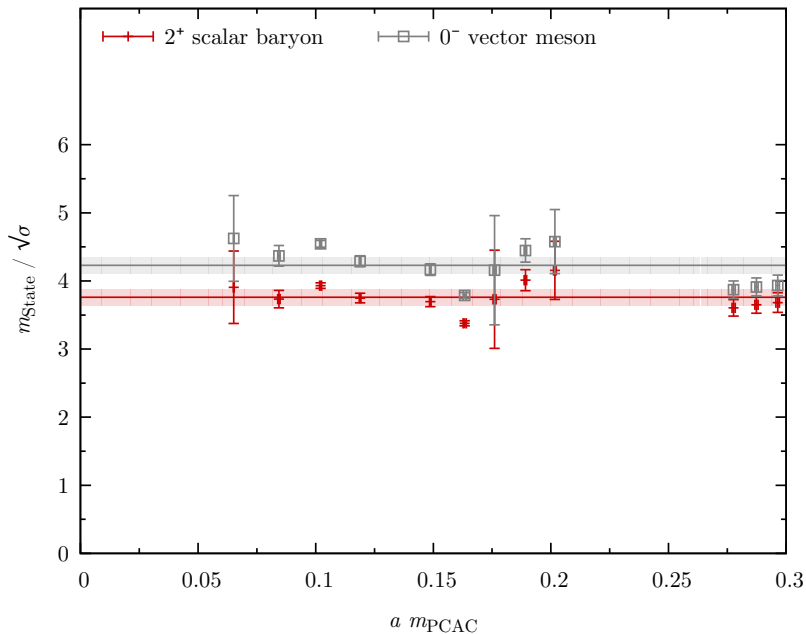
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$\beta = 2.2$ spectrum—provisional



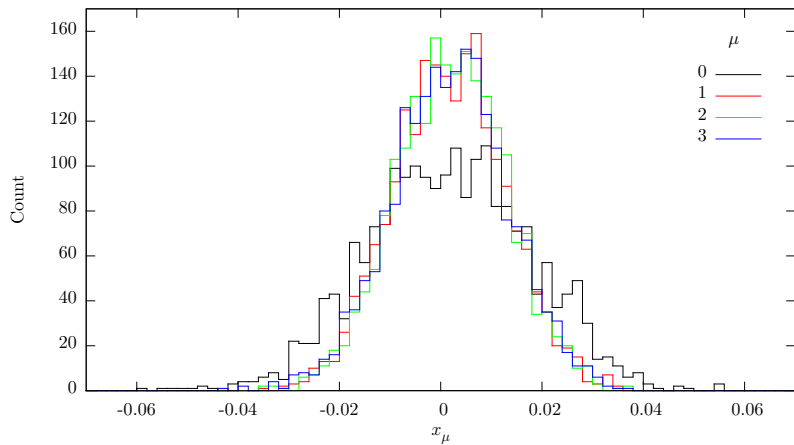
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- Spectral ratios roughly constant—consistent with conformality

Center symmetry

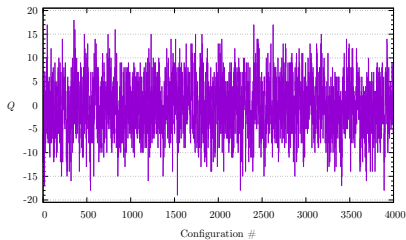


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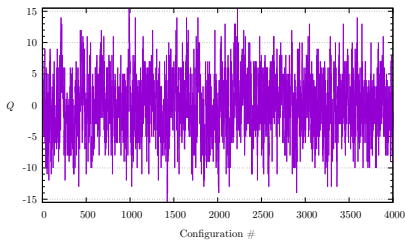
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Sample topological charge histories

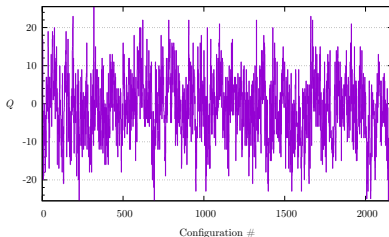
$$\beta = 2.05, m = -1.475, L = 32$$



$$\beta = 2.05, m = -1.510, L = 32$$



$$\beta = 2.05, m = -1.523, L = 48$$



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- $V = 2L \times L^3$, $L = 8, 12, 16, 24, 32$
- Spectral ratios roughly constant—consistent with conformality
- Wilson loop $\sigma \equiv$ torelon σ
- Center unbroken
- Good sampling of topological sectors

Mass anomalous dimension

- Mass anomalous dimension $\gamma_* \sim 1$ important for WTC

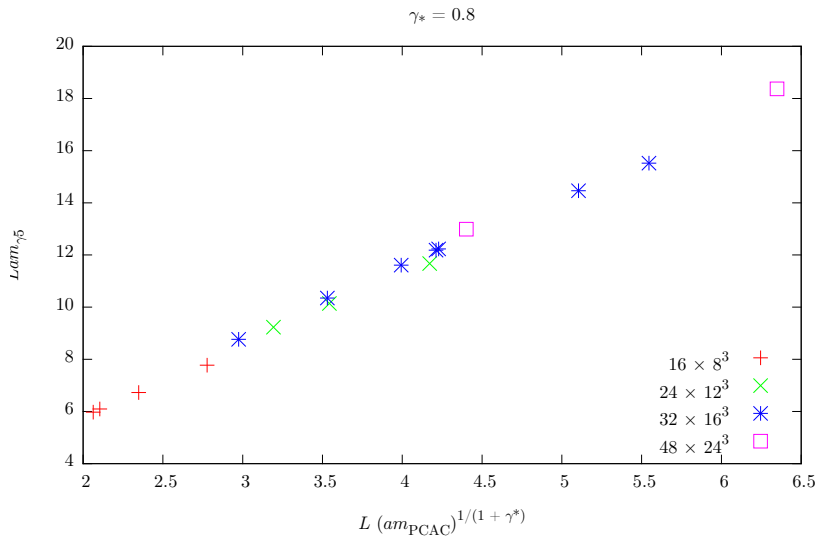
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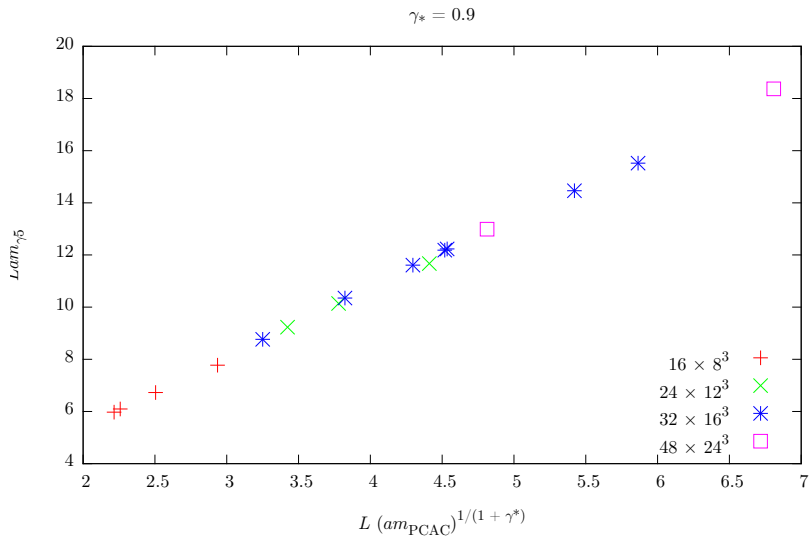
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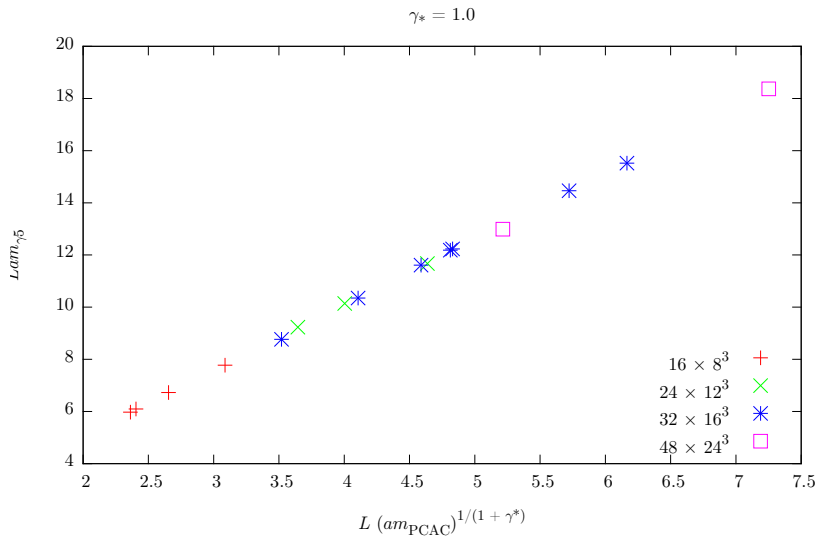
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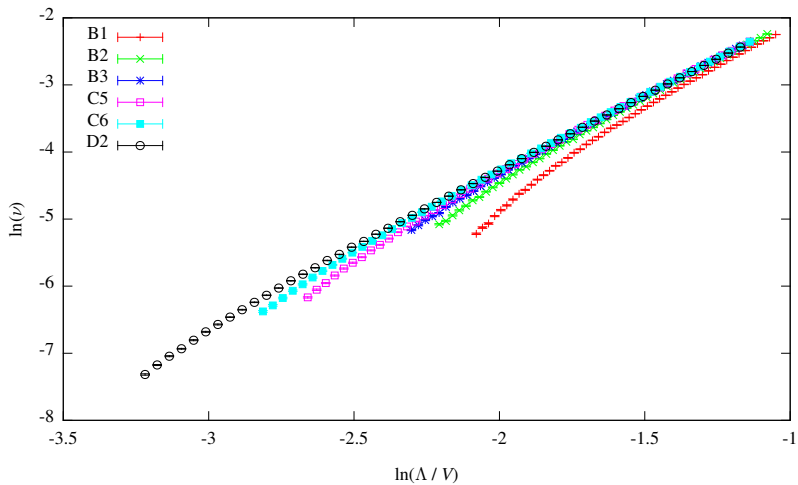
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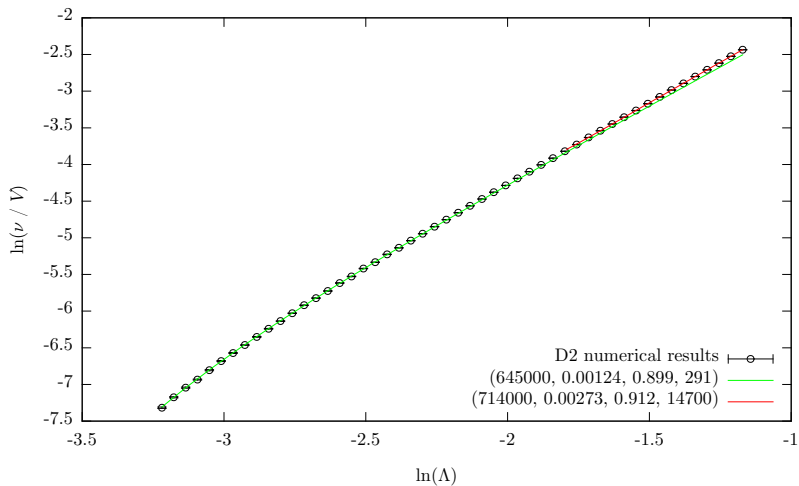
$$a^{-4}\bar{\nu}(\Omega) \approx a^{-4}\nu_0(m) + A [(a\Omega)^2 - (am)^2]^{\frac{2}{1+\gamma_*}}$$

from Patella [arxiv:1204.4432]

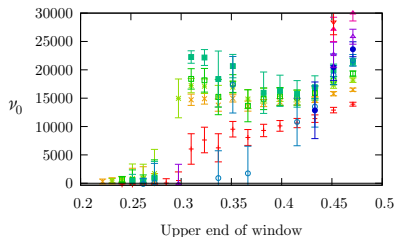
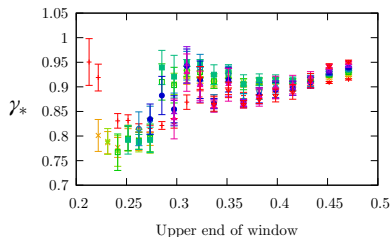
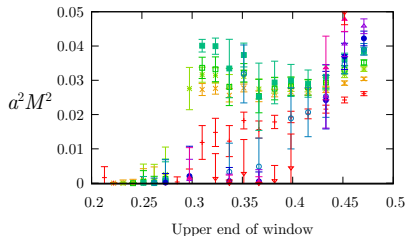
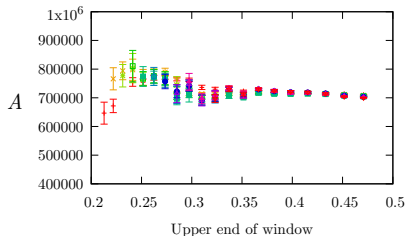
Mode number results



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γ_* mode number fit



Lower end of window:

0.100309		0.128894		0.165625		0.212824		0.273473	
0.104589		0.134394		0.172693		0.221906		0.285144	
0.109053		0.140130		0.180063		0.231376		0.297312	
0.113707		0.146110		0.187747		0.241250		0.310000	
0.118559		0.152345		0.195759		0.251546		0.323229	
0.123618		0.158846		0.204113		0.262280			

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- Potentially walking or conformal
- Could form part of a slightly larger technicolor sector (e.g. $SU(2) + 1$ adjoint + 1 fundamental Dirac flavour)

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- Look to running of coupling via Wilson flow