

Light composite flavor-singlet scalar in large N_f QCD

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K.-i. Nagai, H. Ohki, E. Rinaldi, A. Shibata, K. Yamawaki

(LatKMI Collaboration)

Refs. [PRD86\(2012\)054506](#), [PRD87\(2013\)094511](#),

[PRL111\(2013\)162001](#), [PoS\(LATTICE 2013\)070](#); [arXiv:1309.0711](#), and updates

Sakata Memorial KMI Mini-Workshop on

”Strong Coupling Gauge Theories Beyond the Standard Model” (SCGT14Mini)

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 - Recent studies in our project
- Results of flavor-singlet scalar
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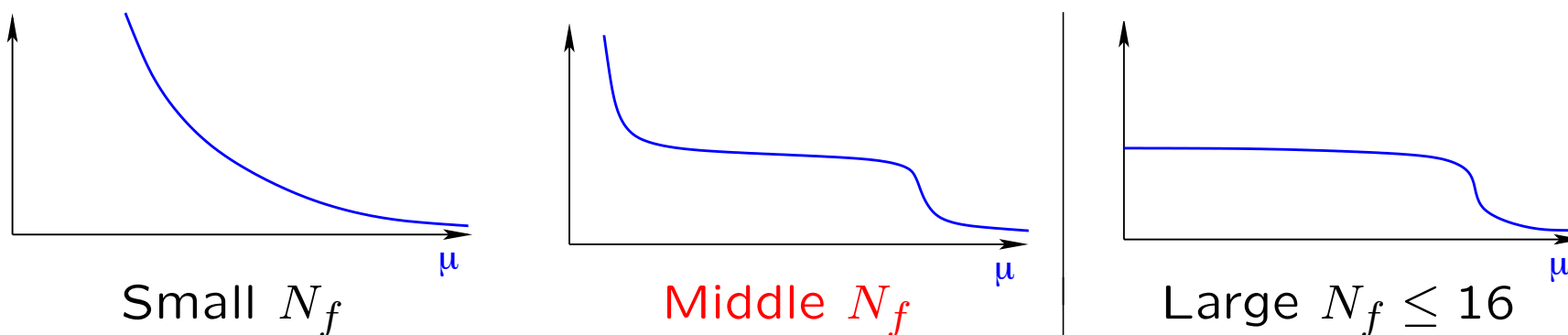
Walking technicolor

N_f massless fermions + $SU(N_{TC})$ gauge at $O(1)$ TeV

Model requirement:

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range

$SU(3)$ gauge theory



Chiral symmetry breaking \leftarrow phase boundary \rightarrow Conformal

- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region

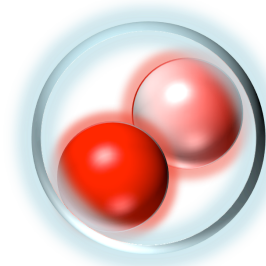
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- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region

- Higgs \approx Light composite scalar
pNGB (technidilaton)
of scale symmetry breaking



$$m_{\text{Higgs}}/v_{\text{EW}} \sim 0.5 = m_{\sigma}/(\sqrt{N_d}F)$$

F : decay constant, N_d : number of weak doublets

usual QCD $m_{\sigma}/F \sim 4-5$

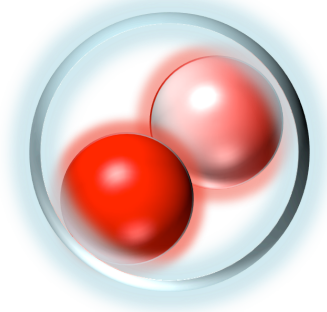
Conditions of walking technicolor

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range
- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region
- Light composite scalar

Question: Such a theory really exists?

Nonperturbative calculation is important.

→ numerical calculation with lattice gauge theory



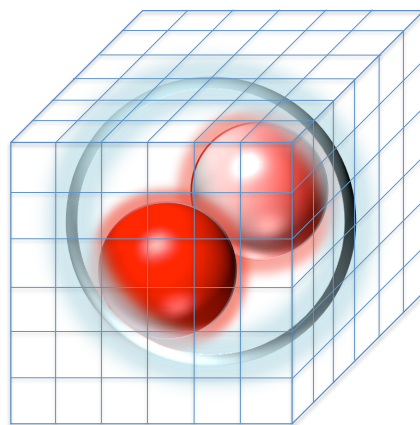
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Recent studies in our project

Purpose in our project

Search for candidate of walking technicolor

Systematic investigation of N_f dependence

SU(3) gauge theory with $N_f = 0, 4, 8, 12, 16$ fermions

Common setup for all N_f : Improved staggered action (HISQ/Tree)

Cheaper calculation cost + small lattice systematic error

HISQ: '07 HPQCD and UKQCD; HISQ/Tree: '12 Bazakov *et al.*

Basic physical quantities: $m_\pi, F_\pi, m_\rho, \langle \bar{\psi}\psi \rangle$

$N_f = 4$: PRD86(2012)054506:PRD87(2013)094511 [Poster: Kurachi]

$N_f = 8$: PRD87(2013)094511 [Talk: Nagai (Thu.)]

$N_f = 12$: PRD86(2012)054506 [Talk: Ohki (Thu.)]

$N_f = 8$ may be candidate of walking theory

some results updated from papers

[Poster]

$N_f = 12$ glueball: [Rinaldi], $N_f = 16$: [Yamazaki]

Recent study of LatKMI Collaboration

Search for candidate of walking technicolor

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511

chiral broken \rightarrow walking \rightarrow conformal increasing N_f

Signal of phase

- Chiral broken phase

Simulations at $m_f \neq 0$

$$m_f \rightarrow 0: m_\pi \rightarrow 0 \text{ and } F_\pi \neq 0 \Rightarrow \frac{F_\pi}{m_\pi} \xrightarrow{m_\pi \rightarrow 0} \infty$$

- Conformal phase

Simulations at $m_f \neq 0$: scale invariance breaking

\rightarrow bound states (mesons)

Hyperscaling with anomalous dimension γ^* at small m_f

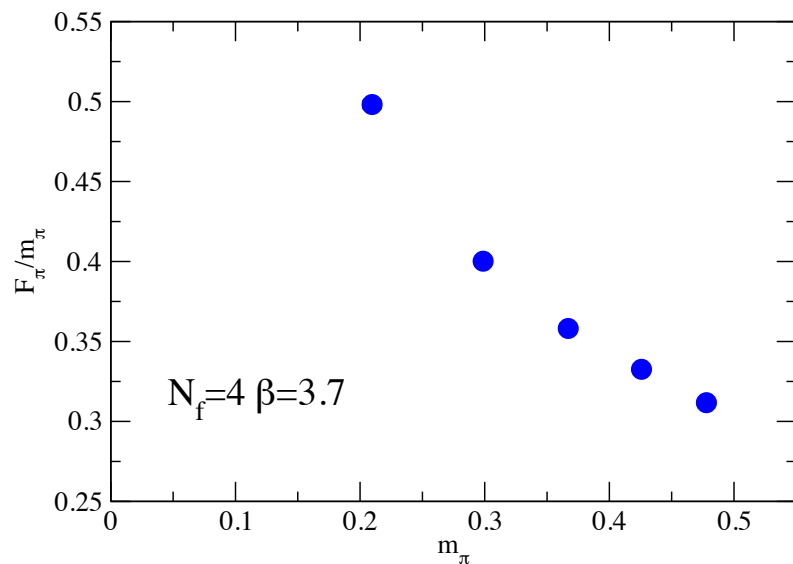
$$\begin{aligned} m_H &= C_H m_f^{1/(1+\gamma^*)} \\ F_\pi &= C_F m_f^{1/(1+\gamma^*)} \end{aligned} \Rightarrow \frac{F_\pi}{m_\pi} \xrightarrow{m_\pi \rightarrow 0} \text{constant}$$

Different $m_f(m_\pi)$ dependence in two phases

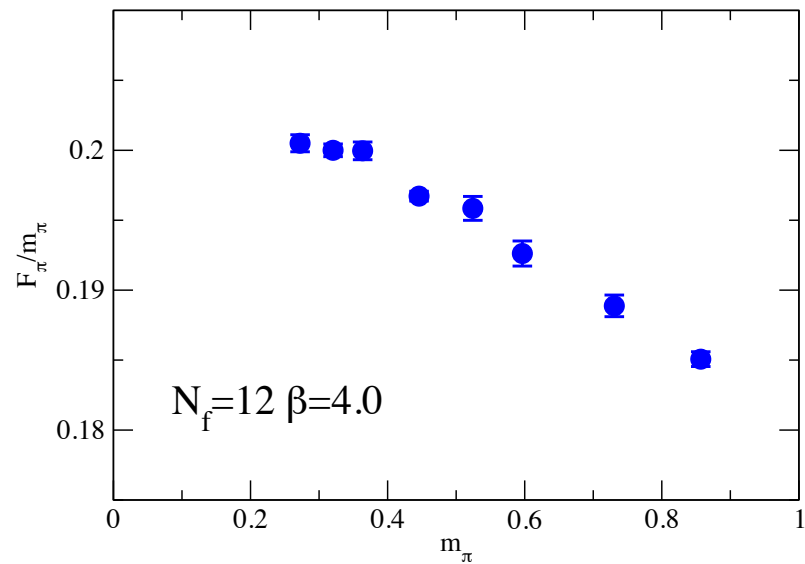
Recent study of LatKMI Collaboration

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511 + updates

$F_\pi/m_\pi \rightarrow \infty$



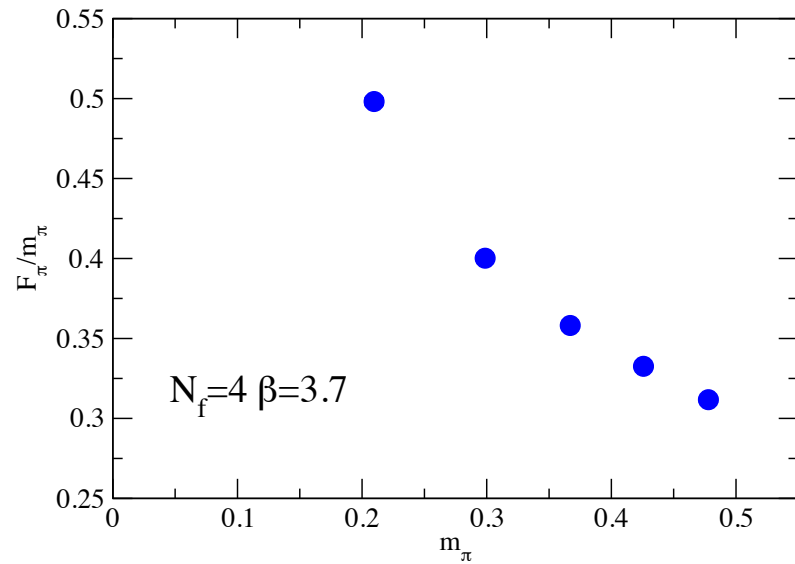
$F_\pi/m_\pi \rightarrow \text{constant}$



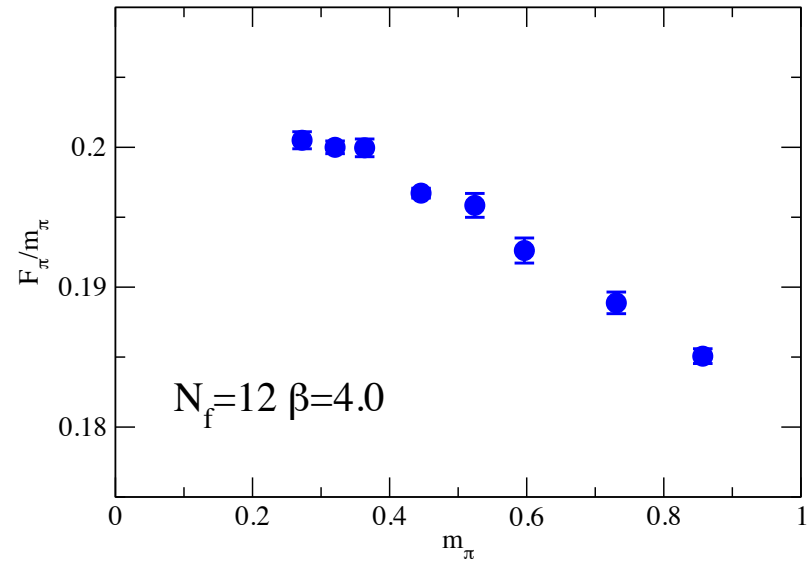
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Chiral broken



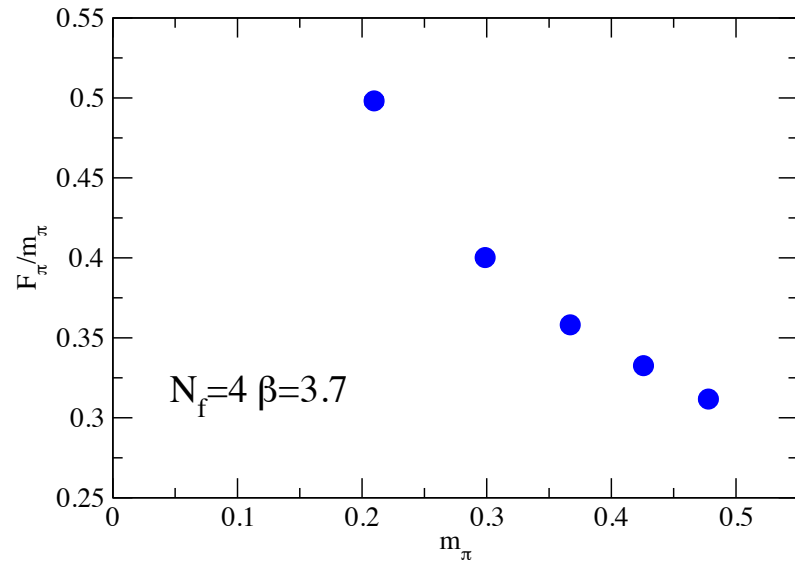
Conformal



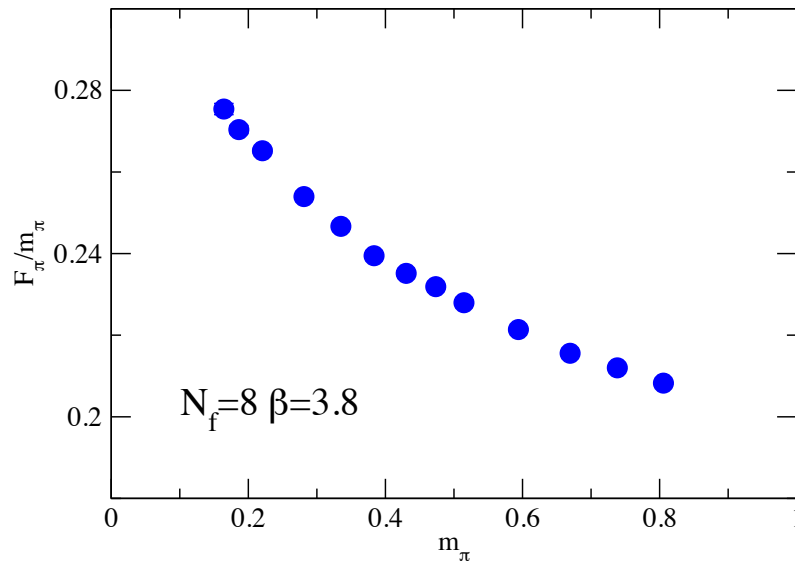
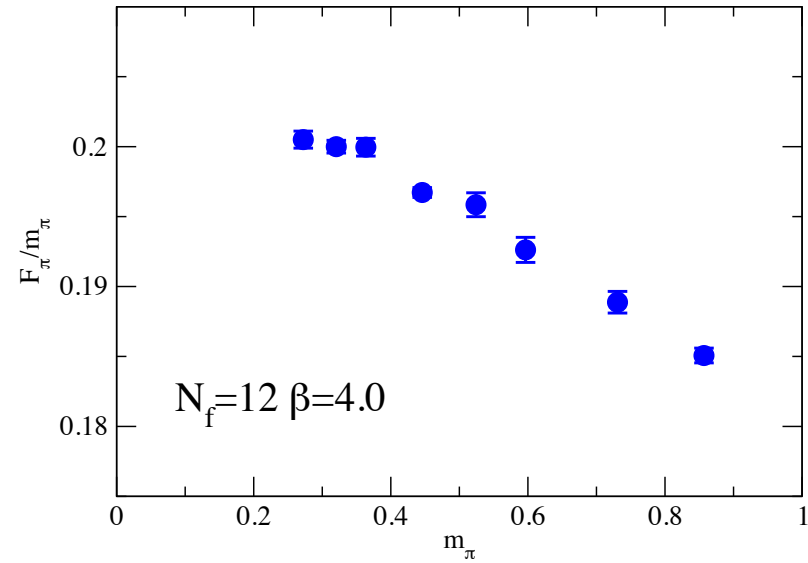
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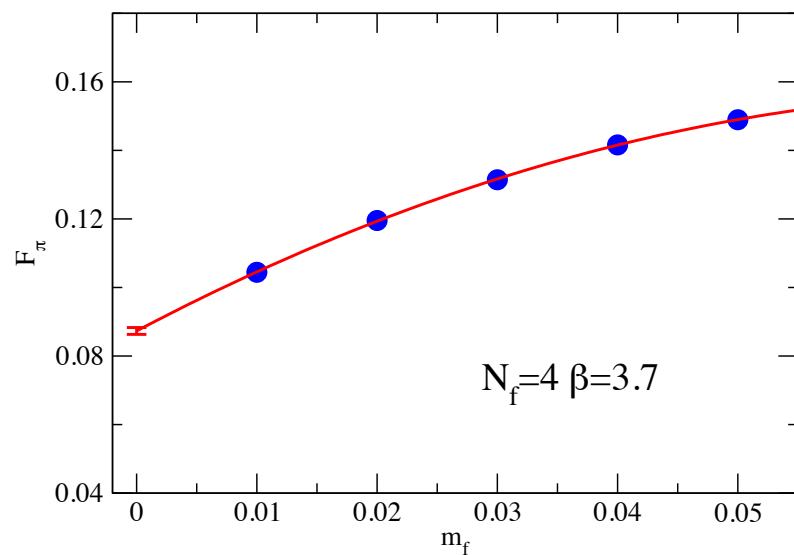
Conformal



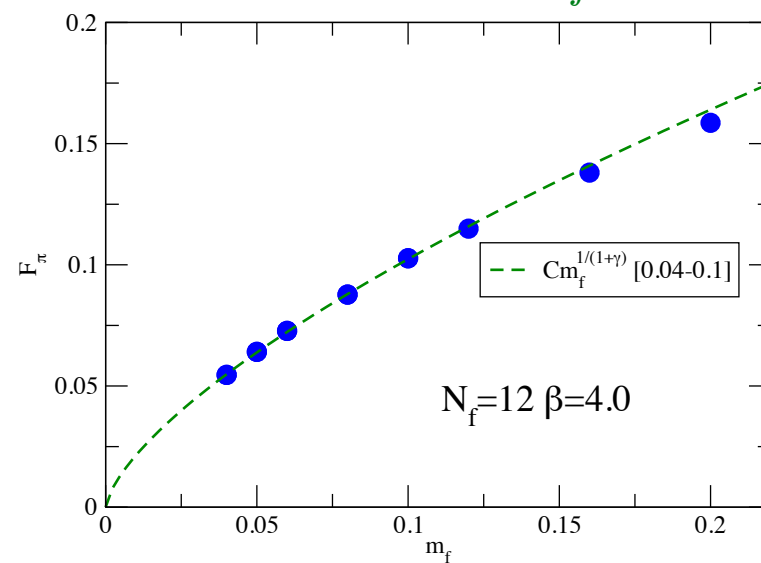
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Chiral broken $F_\pi \rightarrow F \neq 0$



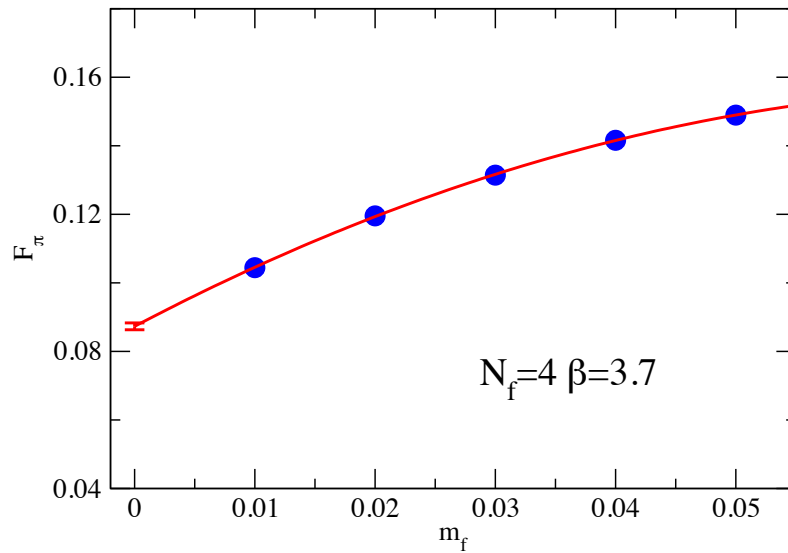
Conformal $F_\pi \rightarrow C m_f^{1/(1+\gamma)}$



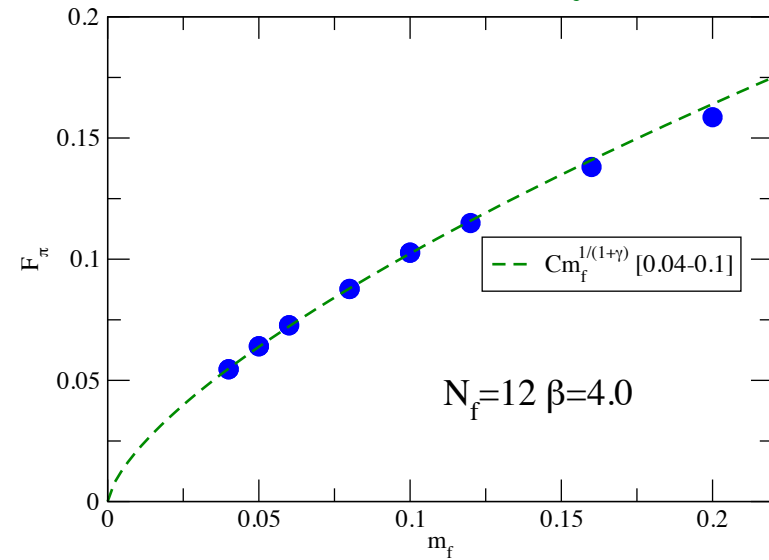
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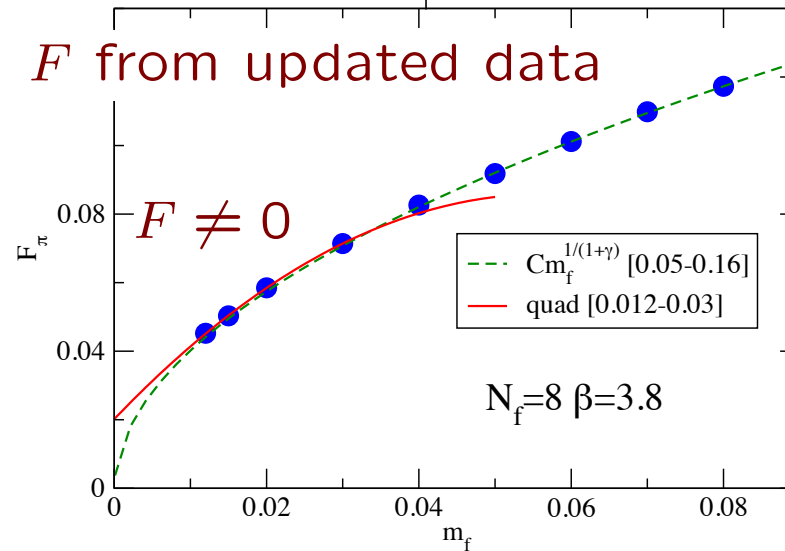
Chiral broken $F_\pi \rightarrow F \neq 0$



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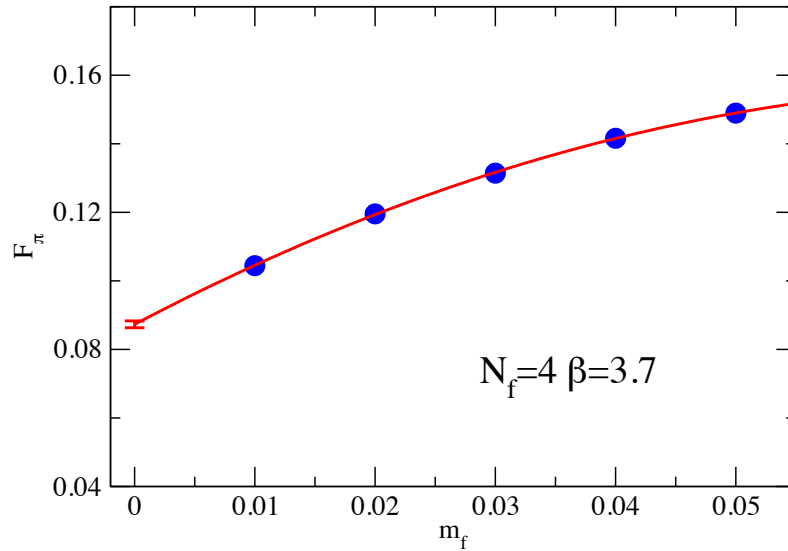
F from updated data



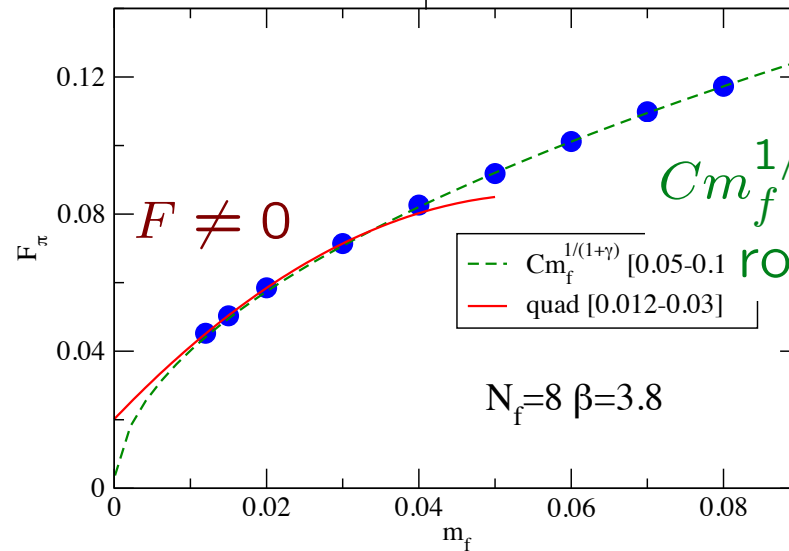
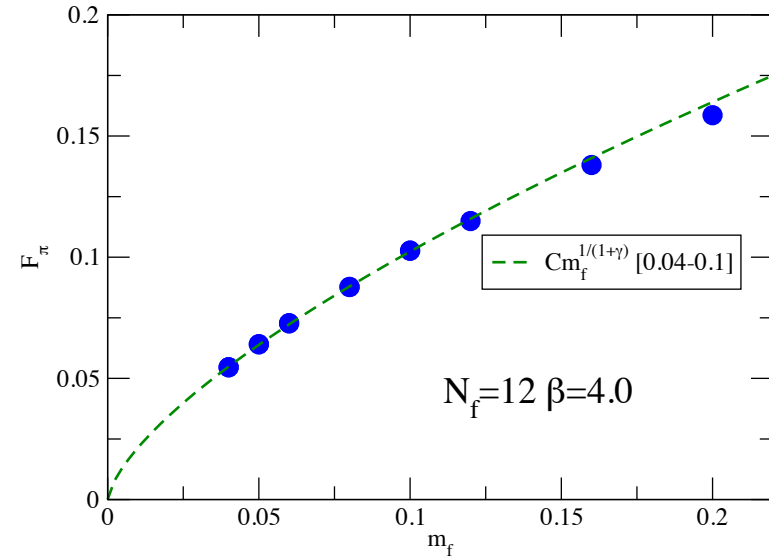
Recent study of LatKMI Collaboration

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Chiral broken $F_\pi \rightarrow F \neq 0$



Conformal $F_\pi \rightarrow C m_f^{1/(1+\gamma)}$



$C m_f^{1/(1+\gamma)}$

rough estimate of γ

Recent study of LatKMI Collaboration

Search for candidate of walking technicolor

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511 + updates

$N_f = 4$ QCD: Spontaneous chiral symmetry breaking

$N_f = 12$ QCD: Consistent with conformal phase

$N_f = 8$ QCD may be a candidate of Walking technicolor

- Spontaneous chiral symmetry breaking

$F_\pi/m_\pi \rightarrow \infty$ and $F_\pi \neq 0$ towards $m_f \rightarrow 0$

- Slow running (walking) coupling in wide scale range

Approximate hyperscaling in F_π

- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region

$\gamma = 0.6-1.0$: Hyperscaling-like behavior of m_π , F_π , m_ρ

- Light composite scalar \Leftarrow Important to check!

Next: Flavor-singlet scalar in (approximate) conformal theory

Recent study of LatKMI Collaboration

Search for candidate of walking technicolor

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511 + updates

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Next: Flavor-singlet scalar in (approximate) conformal theory

Composite flavor-singlet scalar

in $N_f = 12$ and 8 QCD

Difficulty of flavor-singlet scalar meson

- Flavor non-singlet scalar meson $S_{NS}(t) = \sum_{\vec{x}} \bar{\psi}_a(\vec{x}, t) \psi_b(\vec{x}, t)$ ($a \neq b$)

$$\langle 0 | S_{NS}(t) S_{NS}^\dagger(0) | 0 \rangle = \left\langle \text{Diagram} \right\rangle = -C(t)$$

c.f. m_π, F_π from non-singlet pseudoscalar

$$O(100) \text{ configurations} \times O(1) D^{-1}[U](x, y)$$

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$$\langle 0 | S(t) S^\dagger(0) | 0 \rangle = -C(t) + (N_f/4) D(t) \text{ (disconnected)}$$

$$D(t) = \left\langle \text{Diagram 1} \quad \text{Diagram 2} \right\rangle - \left\langle \text{Diagram 3} \right\rangle^2$$

Much harder but essential for flavor-singlet

$$O(10000) \text{ configurations} \times O(100) D^{-1}[U](x, x)$$

Difficulty of flavor-singlet scalar meson

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Much harder but essential for flavor-singlet

$O(10000)$ configurations $\times O(10) D^{-1}[U](x, x)$
 using noise reduction method

'97 Venkataraman and Kilcup

used in $N_f = 2 + 1 \eta'$: Gregory *et al.*; $N_f = 12 \sigma$: Jin and Mawhinney

Composite flavor-singlet scalar in $N_f = 12$ QCD

Purpose of $N_f = 12$ QCD calculation

Why $N_f = 12$

- Investigated by many groups

'08,'09 Appelquist *et al.*, '10 Deuzeman *et al.*, '10,'12 Hasenfratz,
'11 Fodor *et al.*, '11 Appelquist *et al.*, '11 DeGrand, '11 Ogawa *et al.*,
'12 Lin *et al.*, '12,'13 Iwasaki *et al.*, '12,'13 Itou, '12 Jin and Mawhinney, and ...

In our work PRD86(2012)054506

[Talk: Ohki (Thu.)]

consistent behavior with conformal phase

- A few studies of flavor-singlet scalar in conformal theory

1. SU(2) Adjoint $N_f = 2$ glueball: '09 Del Debbio *et al.*

2. SU(3) $N_f = 12$ meson: '12 Jin and Mawhinney

c.f. SU(3) $N_f = 12$ meson: '13 LH Collaboration

Purpose of this work

Understand properties of flavor-singlet scalar in $N_f = 12$

regarded as pilot study of $N_f = 8$ theory

Flavor-singlet scalar in $N_f = 12$ QCD

PRL111(2013)162001

Simulation parameters

- $\beta = 4$ HISQ/Tree action
calculation of m_σ
- Huge number of configurations
measuring every 2 tarj.
- Four m_f on more than two volumes
- Noise reduction method with $N_r = 64$
- Local meson operator of $(1 \otimes 1)$

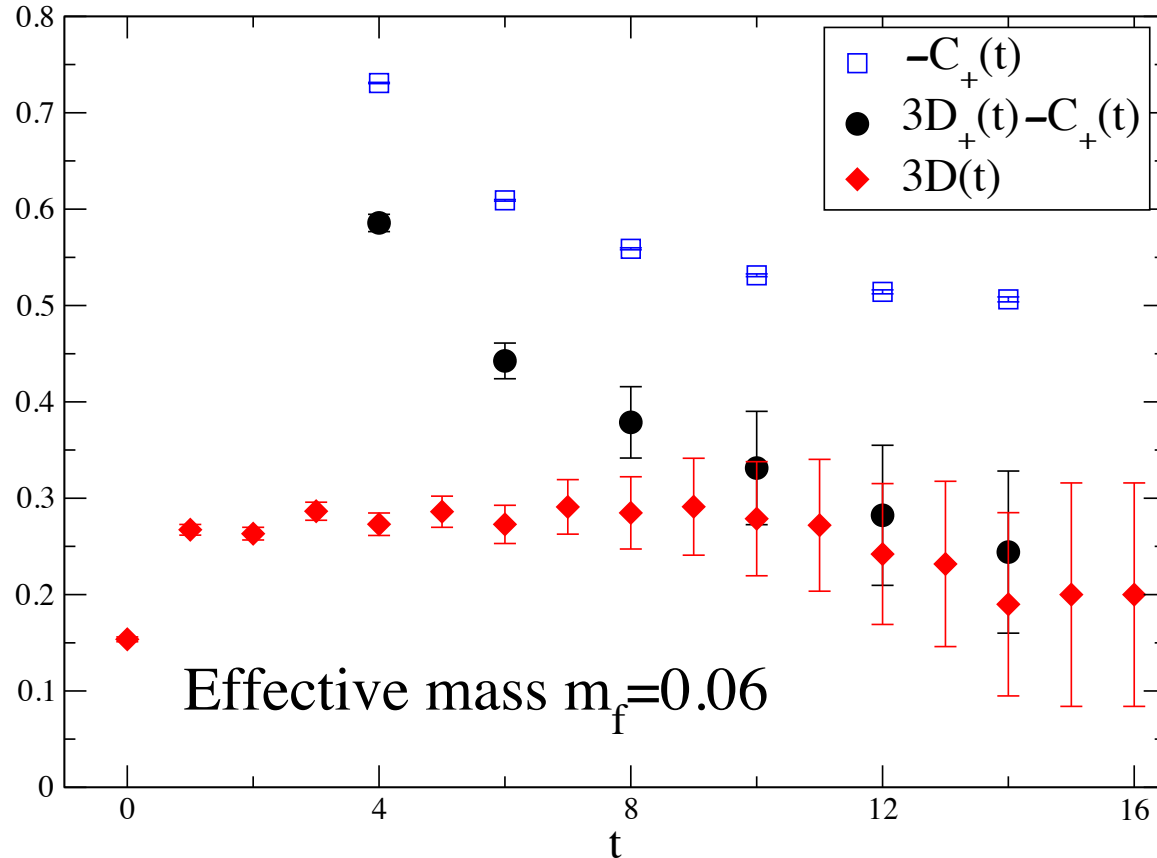
L, T	m_f	confs
24,32	0.05	11000
	0.06	14000
	0.08	15000
	0.10	9000
30,40	0.05	10000
	0.06	15000
	0.08	15000
	0.10	4000
36,48	0.05	5000
	0.06	6000

Machines: φ at KMI, CX400 at Kyushu Univ.

Effective mass in $N_f = 12$

PRL111(2013)162001

$m_f = 0.06, L = 24$ with $N_{\text{conf}} = 14000$



Non-singlet scalar

$a_0: -C_+(t)$

Singlet scalar

$\sigma: 3D_+(t) - C_+(t)$

$m_\sigma < m_{a_0}$

$\sigma: D(t)$

Consistent m_σ

with smaller error

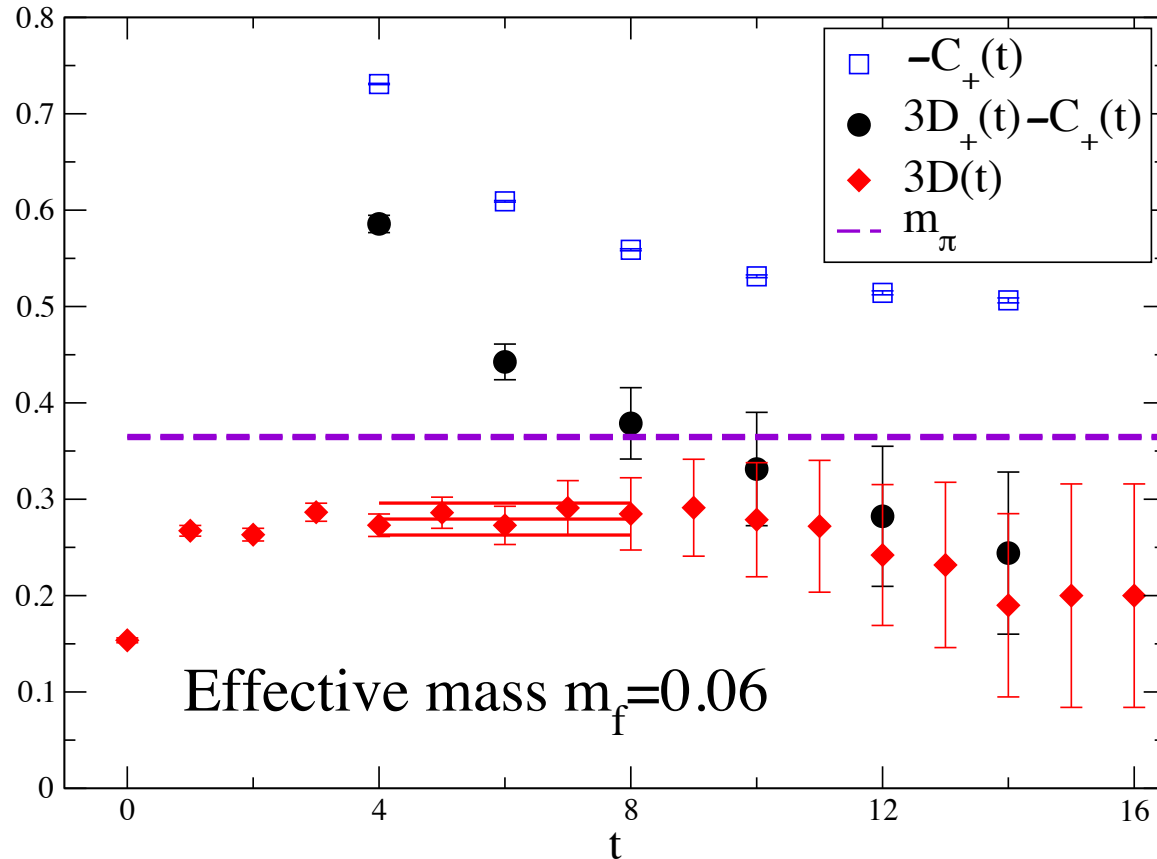
$$X_+(t) = 2X(t) + X(t+1) + X(t-1)$$

Good signal of m_σ from $D(t)$

Effective mass in $N_f = 12$

PRL111(2013)162001

$m_f = 0.06, L = 24$ with $N_{\text{conf}} = 14000$



Non-singlet scalar

$a_0: -C_+(t)$

Singlet scalar

$\sigma: 3D_+(t) - C_+(t)$

$m_\sigma < m_{a_0}$

$\sigma: D(t)$

Consistent m_σ

with smaller error

$m_\sigma < m_{a_0}$

Effective mass $m_f = 0.06$

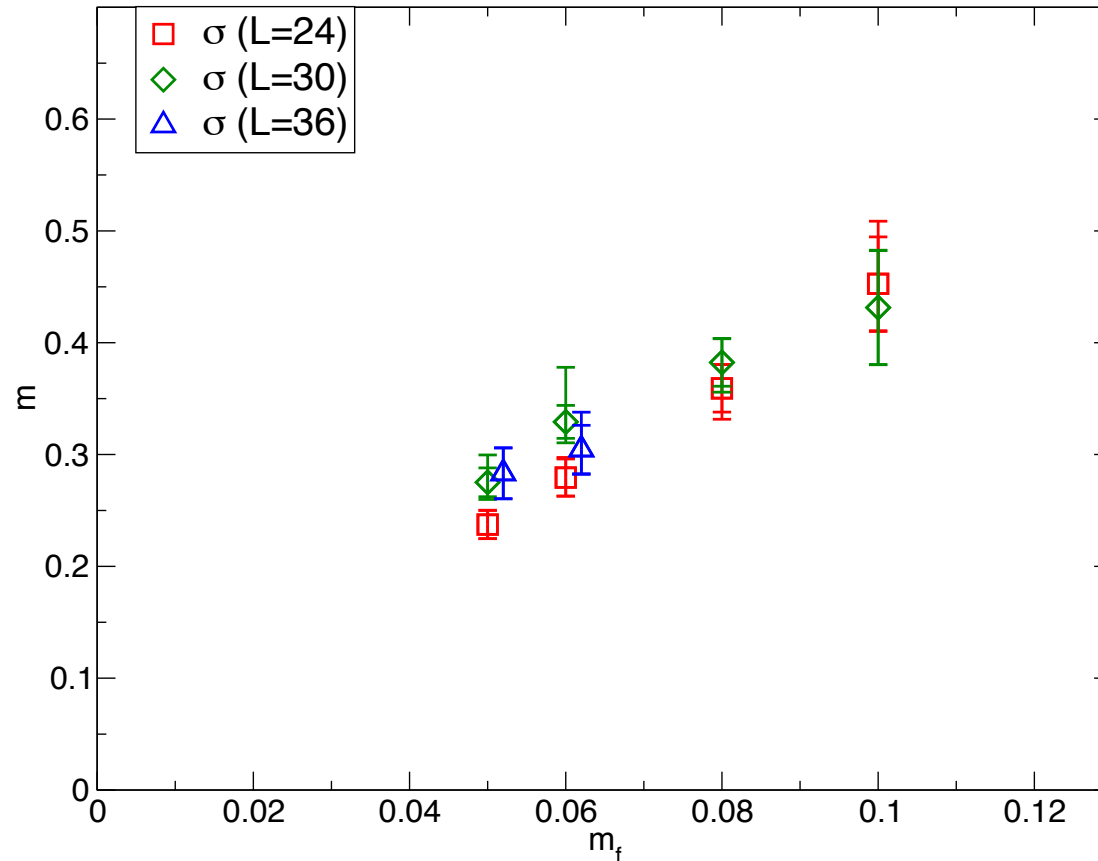
$$X_+(t) = 2X(t) + X(t+1) + X(t-1)$$

Good signal of m_σ from $D(t)$

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4-8$



Reasonable signals with almost 10% statistical error

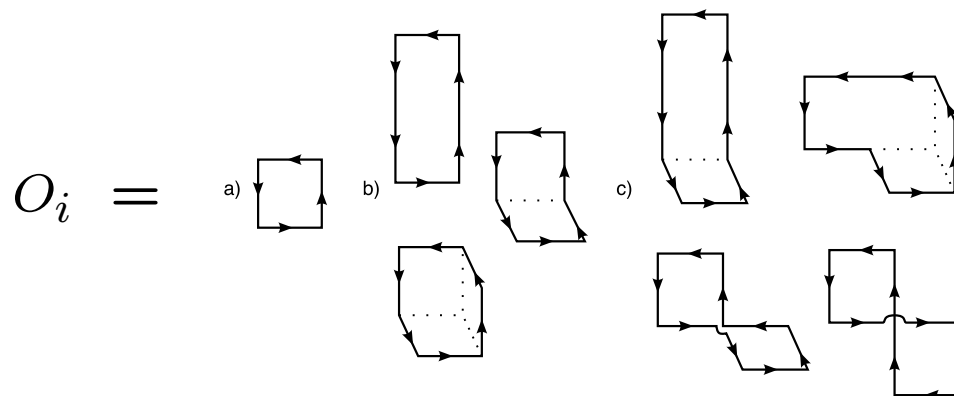
Systematic error from fit range dependence of m_σ

Finite volume effect under control \leftarrow 2 larger volumes agree

Flavor-singlet state from Glueball operator

[Poster: Rinaldi]

Flavor-singlet scalar (0^{++} glueball) operator from U



$$\langle 0|O_i(t)O_j^\dagger(0)|0\rangle - \langle 0|O_i|0\rangle\langle 0|O_j^\dagger|0\rangle, \quad i, j = a, b, c$$

Same difficulty as meson operator \rightarrow Huge statistical noise

Noise reduction techniques (Lucini, Rago, Rinaldi; JHEP08(2010)119)

- Fattening link
- Large size operator
- Diagonalization of correlation function matrix

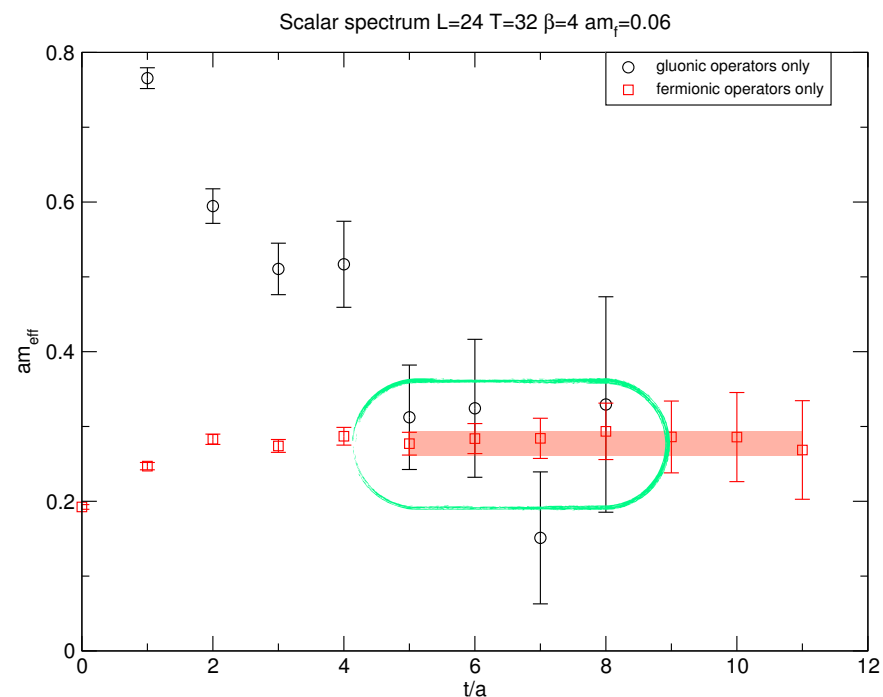
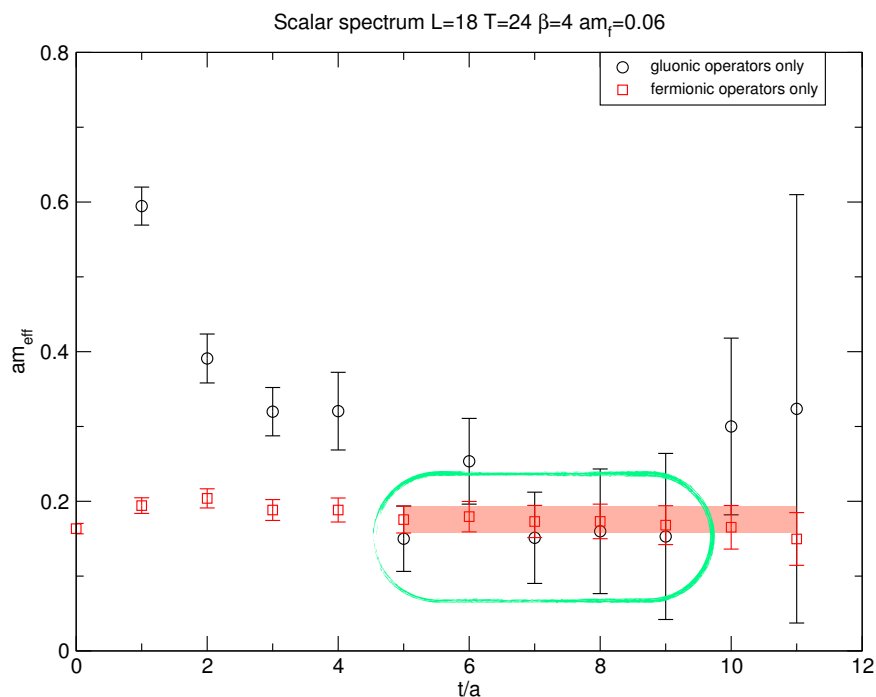
Same m_σ is obtained from meson and glueball correlators, in principle.

\rightarrow Reliability test of result

Comparison of effective mass in $N_f = 12$

$$m_{\text{eff}}(t) = \log(C_H(t)/C_H(t+1)) \xrightarrow{t \gg 1} m_H$$

Glueball correlator and meson $D(t)$



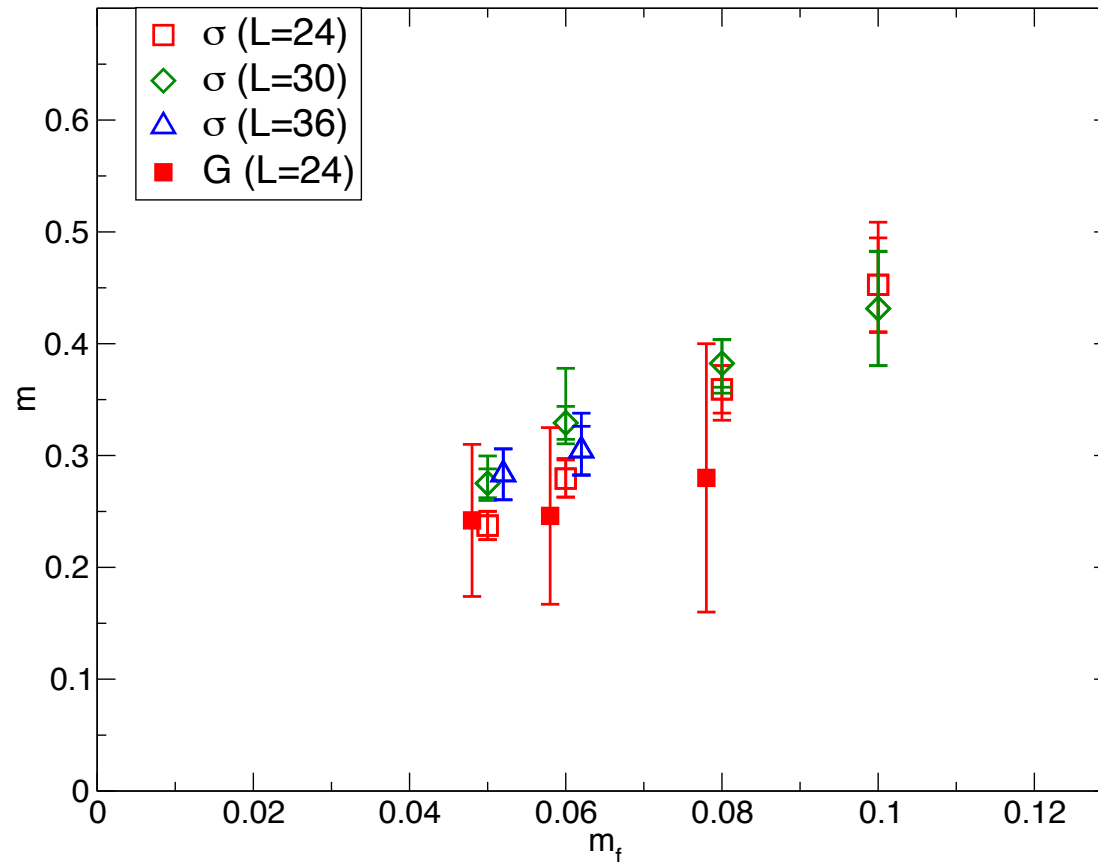
Larger error in glueball correlator

Reasonably consistent in large t

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4-8$



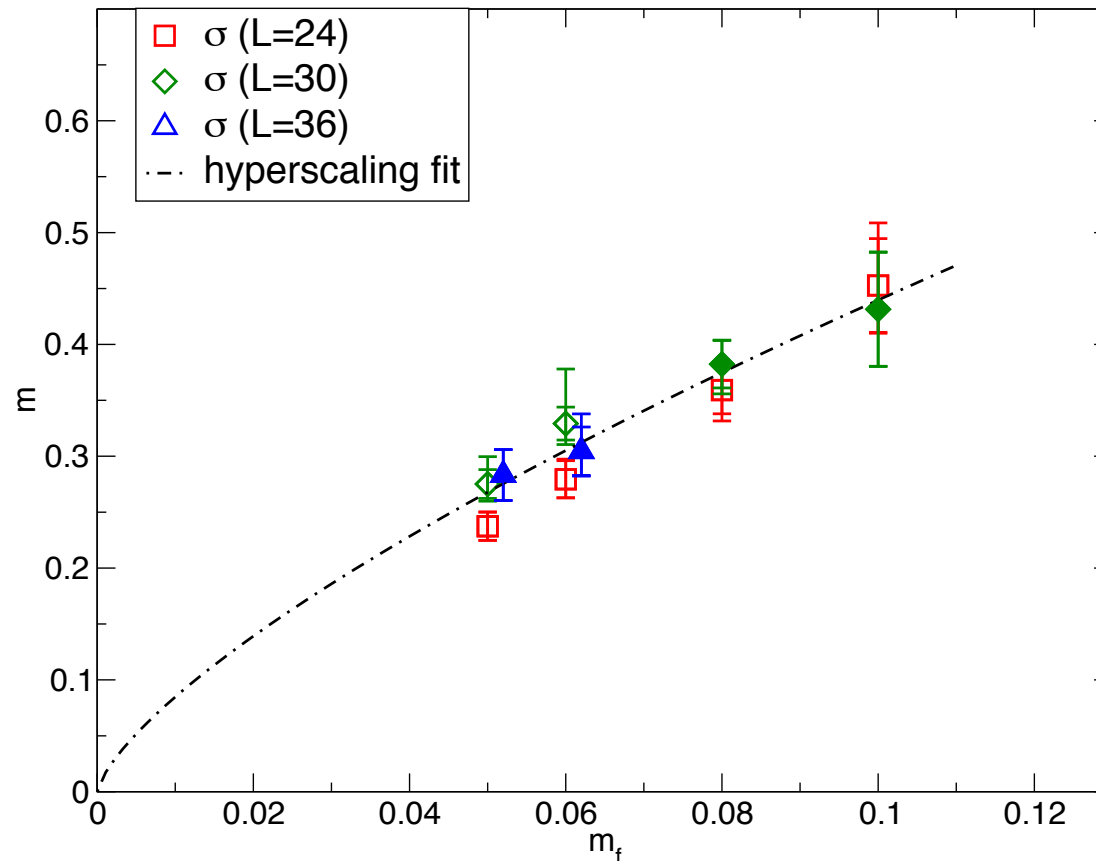
Consistent mass from glueball operator calculation

→ show only meson results in the following pages

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4-8$



Hyperscaling test with fixed γ using target volume at each m_f

$$m_\sigma = C m_f^{1/(1+\gamma)} \text{ with } \gamma = 0.414 \text{ from hyperscaling of } m_\pi$$

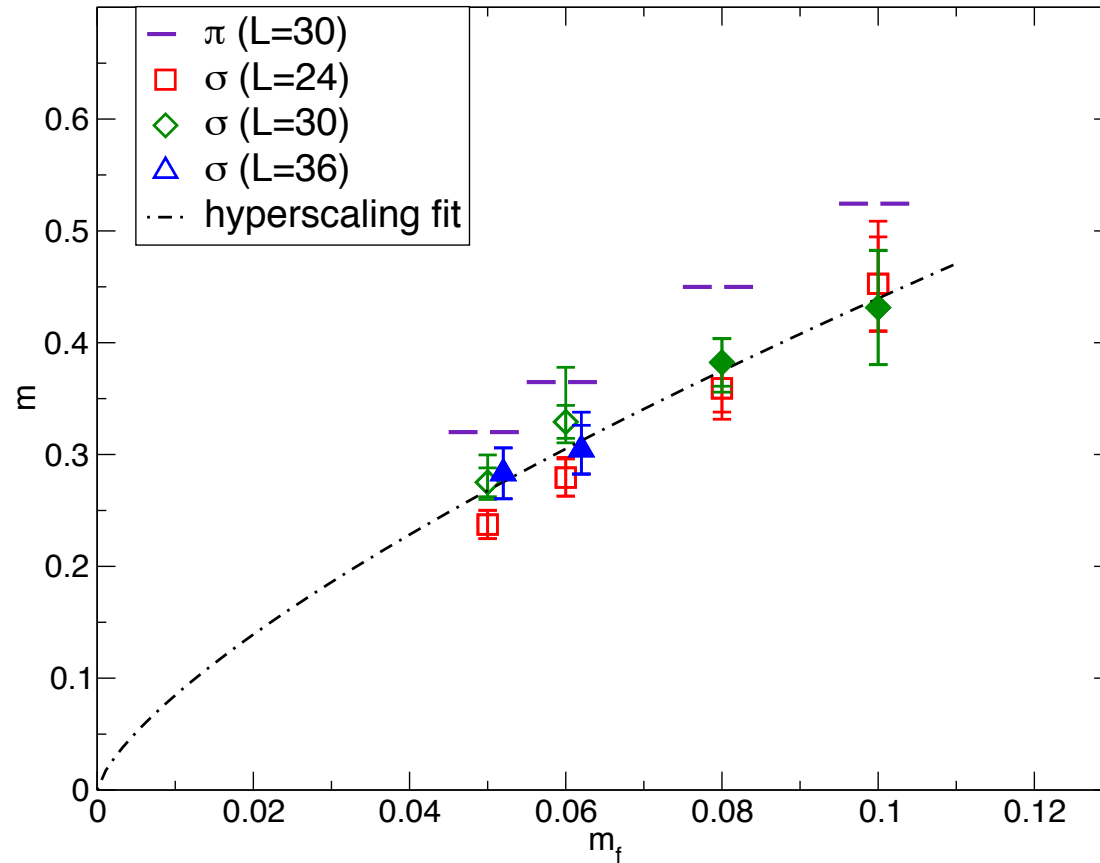
PRD86(2012)054506

Consistent hyperscaling as m_π

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4-8$

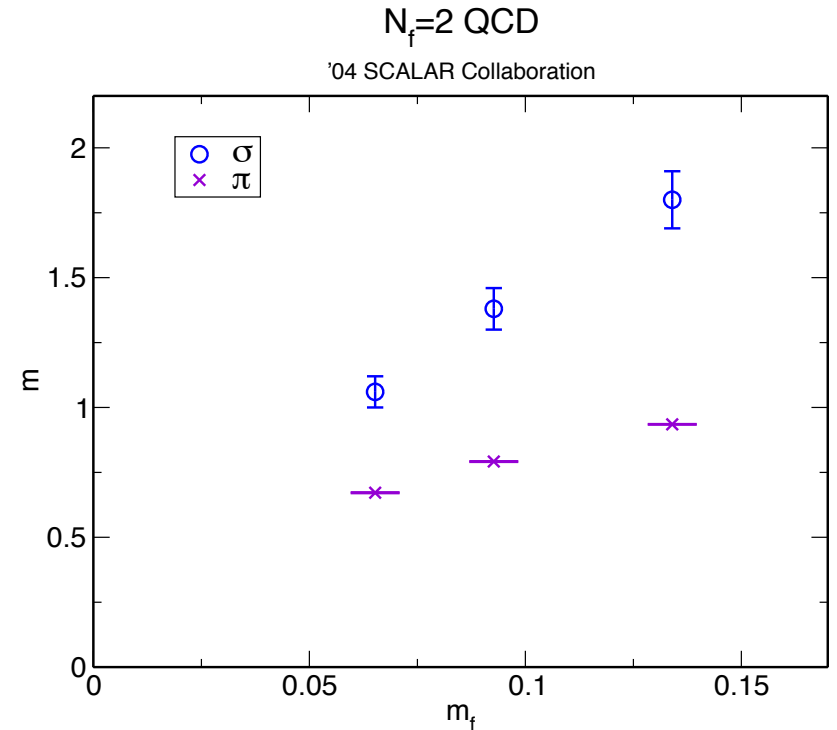
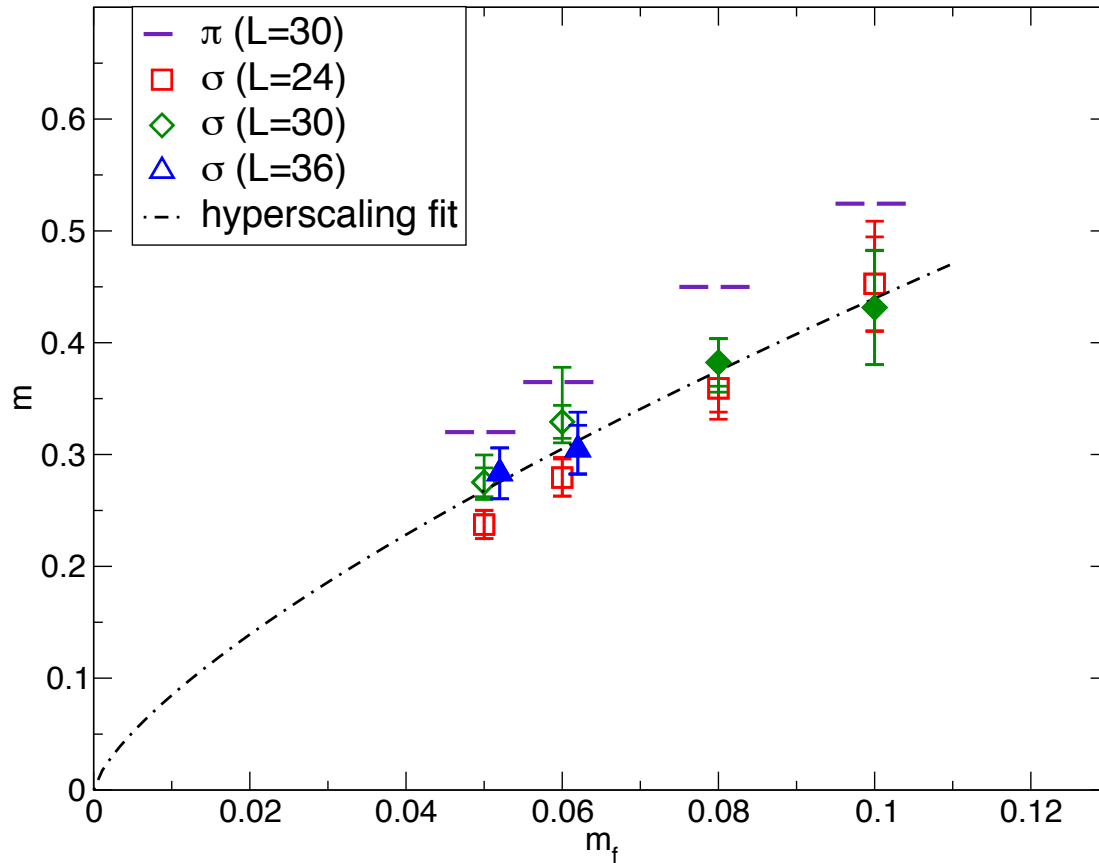


Lighter than π in all m_f

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4-8$

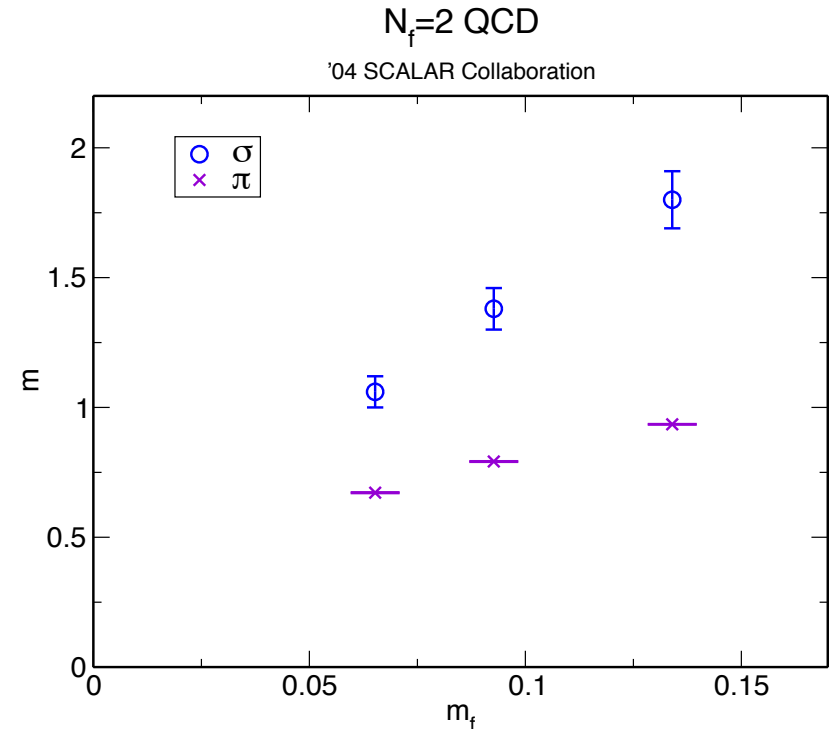
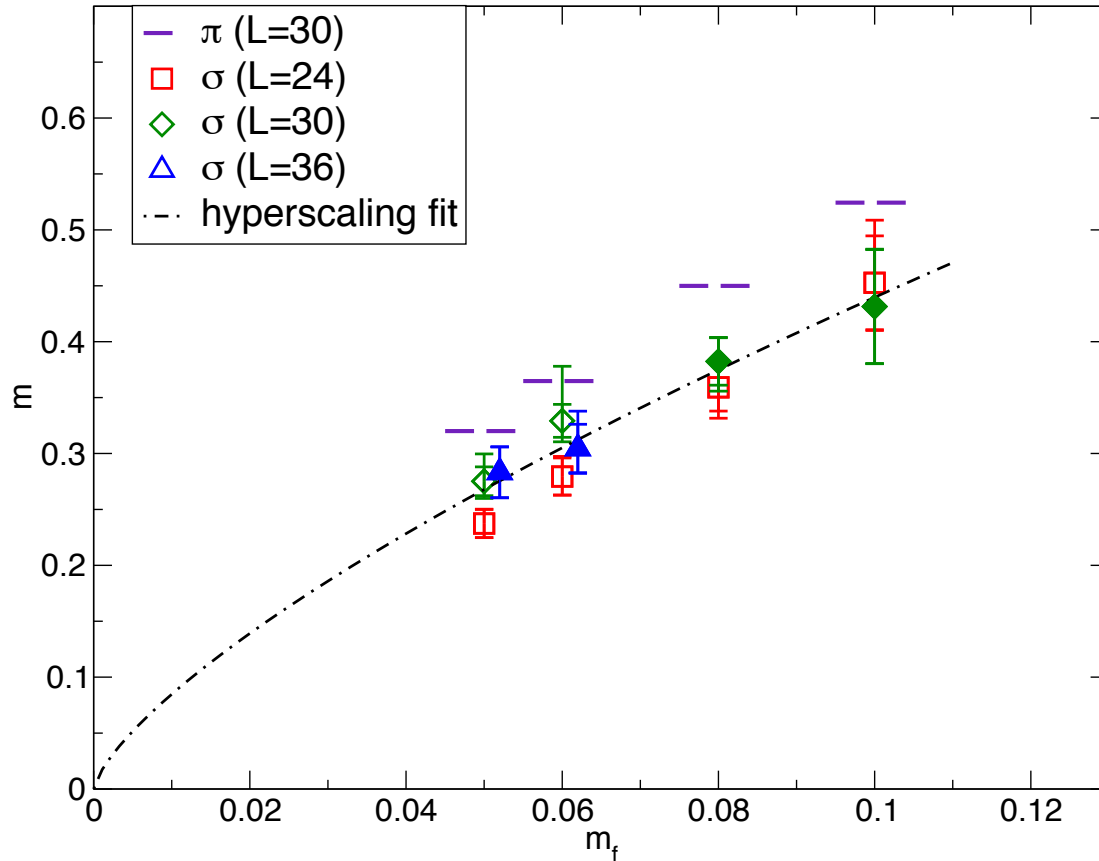


Lighter than π in all m_f
Much different from usual QCD

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4-8$



Conformal symmetry may make σ light

Encouraging for observing light scalar
in approximate conformal theory

Composite flavor-singlet scalar in $N_f = 8$ QCD

Flavor-singlet scalar in $N_f = 8$ QCD

$N_f = 8$ QCD may be candidate of walking theory; PRD87(2013)094511
[Talk: Nagai (Thu.)]

If flavor-singlet scalar is light

→ Possibility of composite Higgs (technidilaton)

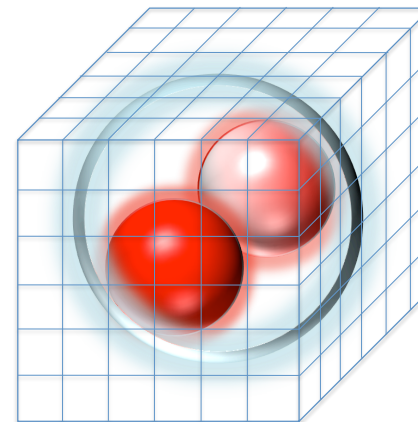
Required condition to explain $m_{\text{Higgs}}/v_{\text{EW}} \sim 0.5$

$$m_{\sigma}/F \sim 1 \text{ in } m_f = 0 \text{ limit}$$

c.f. usual QCD $m_{\sigma}/F \sim 4-5$

Purpose

1. Different from usual QCD?
2. Estimate m_{σ}/F in $m_f = 0$ limit



Flavor-singlet scalar in $N_f = 8$ QCD

report of preliminary results arXiv:1309.0711

Maybe candidate of walking theory; PRD87(2013)094511

Simulation parameters

- $\beta = 3.8$ HISQ/Tree action
calculation of m_σ
- Huge number of configurations
measuring every 2 tarj.
- Five m_f with three volumes
- Noise reduction method with $N_r = 64$
- Local meson operator of $(1 \otimes 1)$

L, T	m_f	confs
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30,40	0.02	8000
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	0.04	12900
36,48	0.02	5000
	0.015	3200

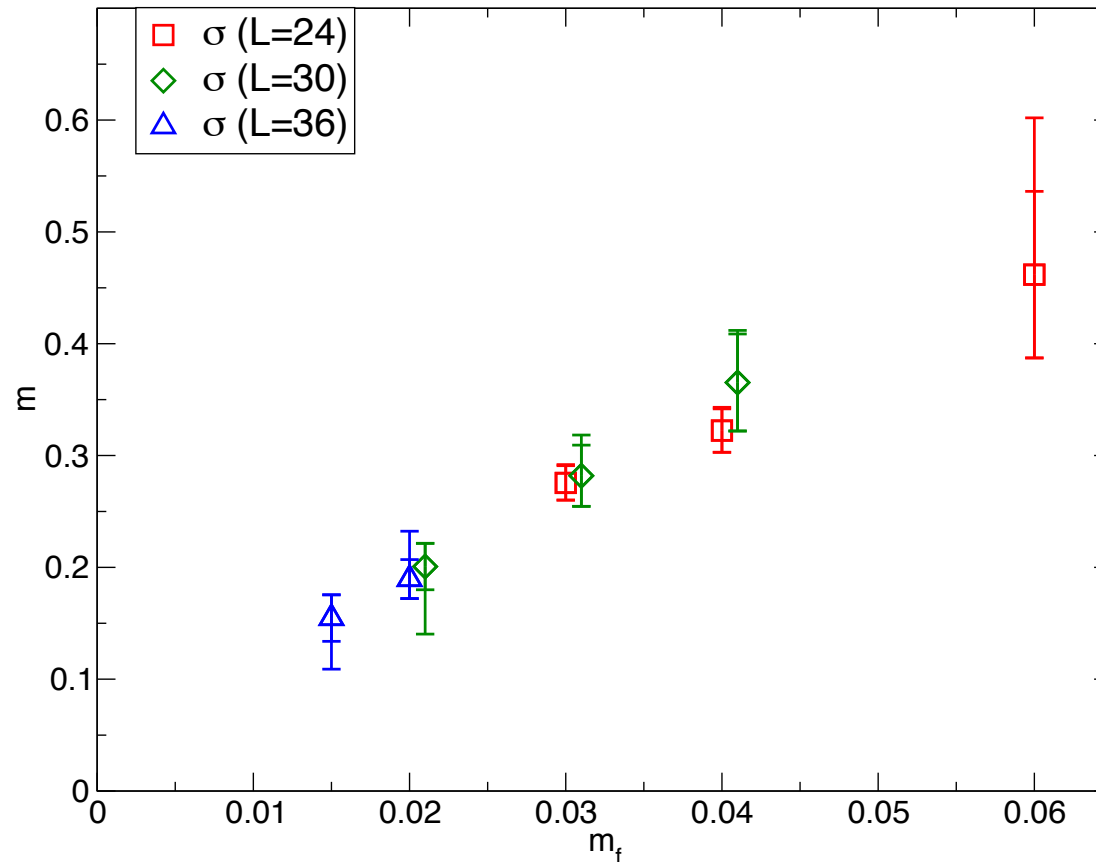
All results are preliminary.

Machines: φ at KMI, CX400 at Nagoya Univ.,

CX400 and HA8000 at Kyushu Univ.

m_f dependence in $N_f = 8$

m_σ from fit of $2D(t)$ with $t = 6-11$



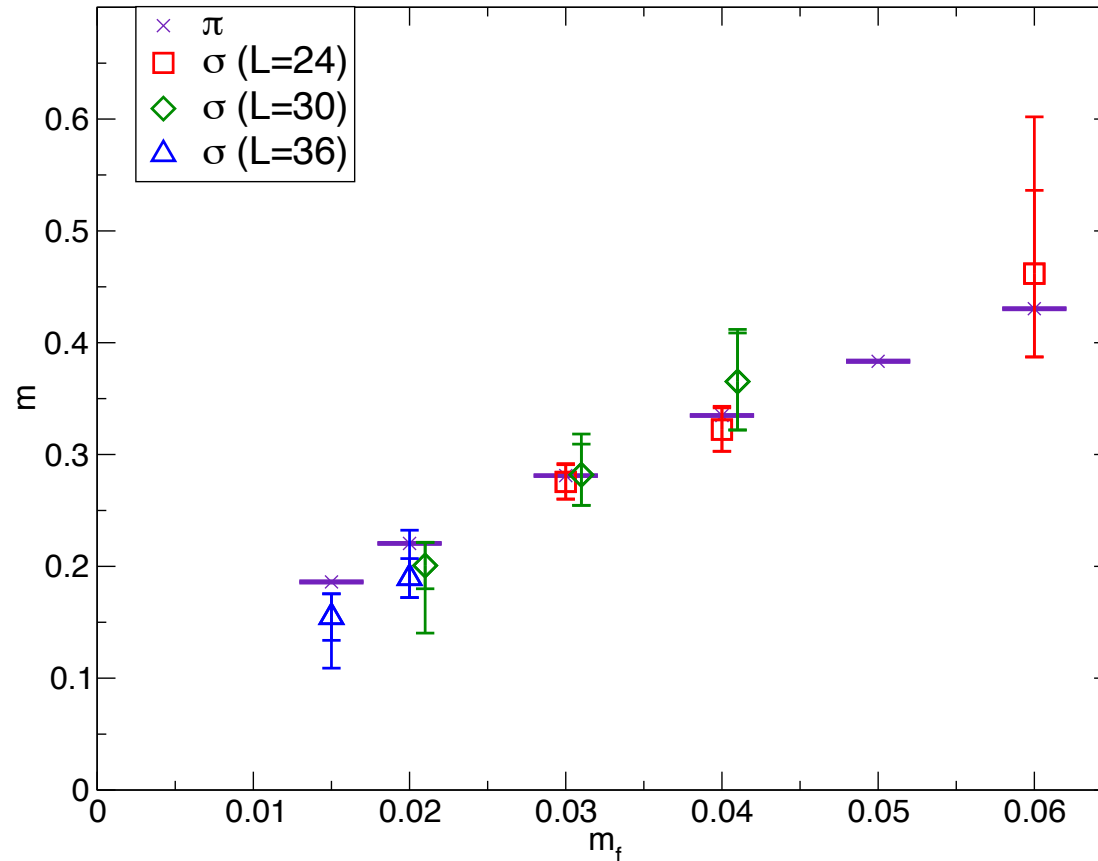
Reasonable signals with statistical error $< 20\%$

Systematic error from fit range dependence of m_σ

Finite volume effect seems under control

m_f dependence in $N_f = 8$

m_σ from fit of $2D(t)$ with $t = 6-11$



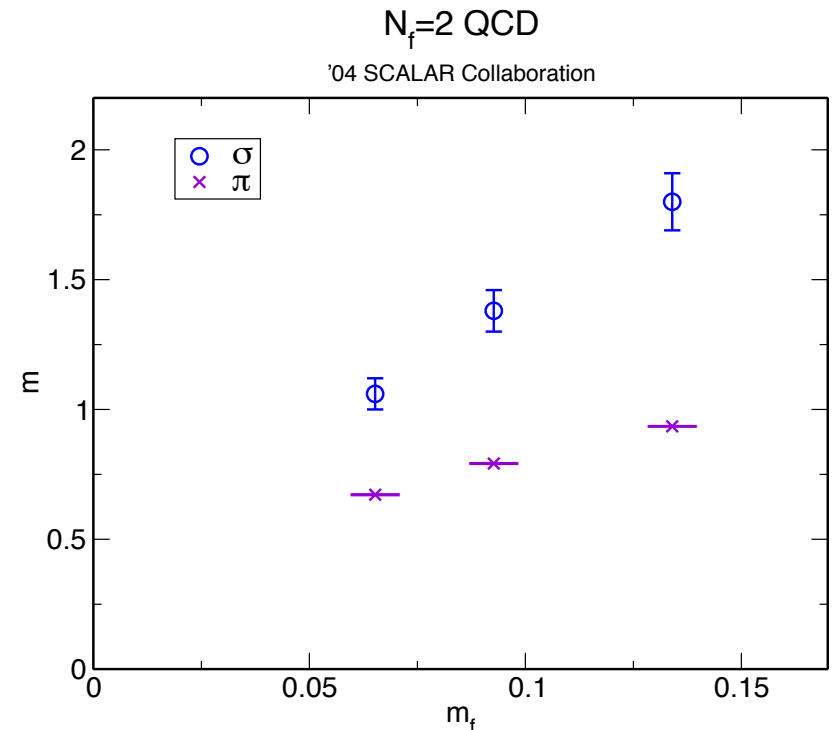
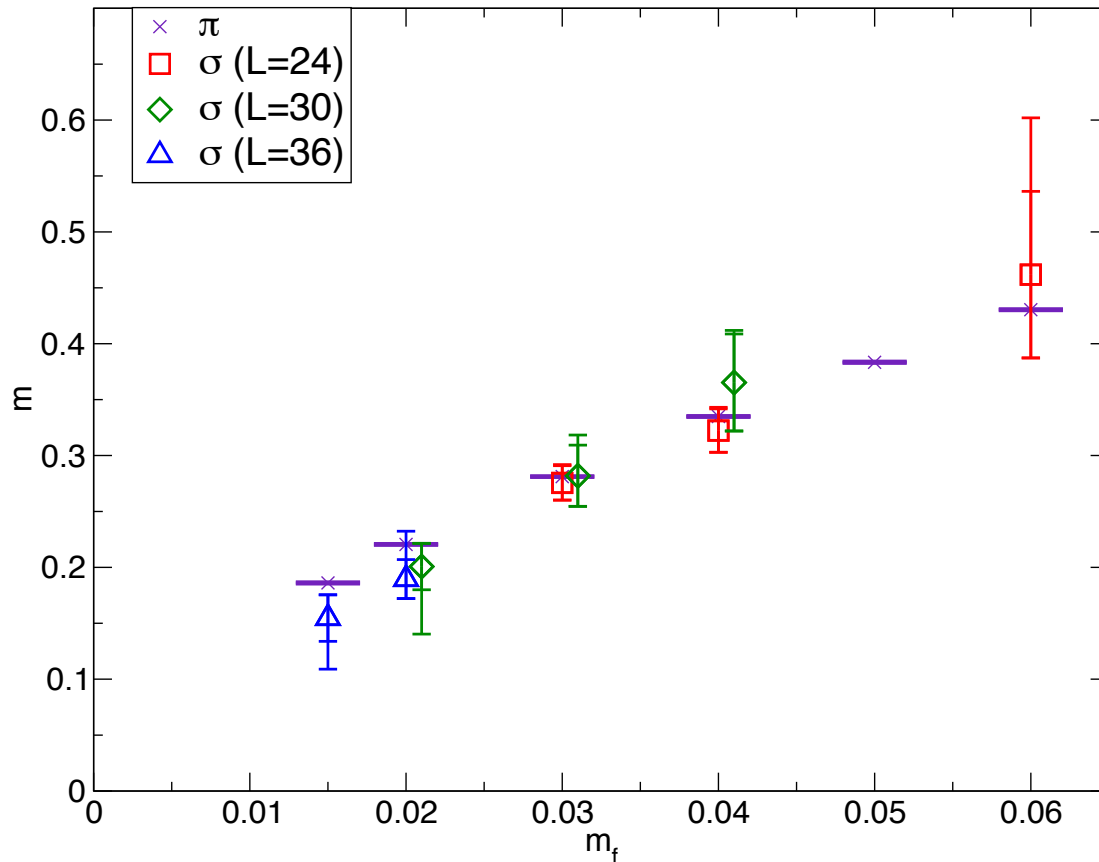
Reasonable signals with statistical error $< 20\%$

Systematic error from fit range dependence of m_σ

$$m_\sigma \sim m_\pi \text{ in all } m_f$$

m_f dependence in $N_f = 8$

m_σ from fit of $2D(t)$ with $t = 6-11$

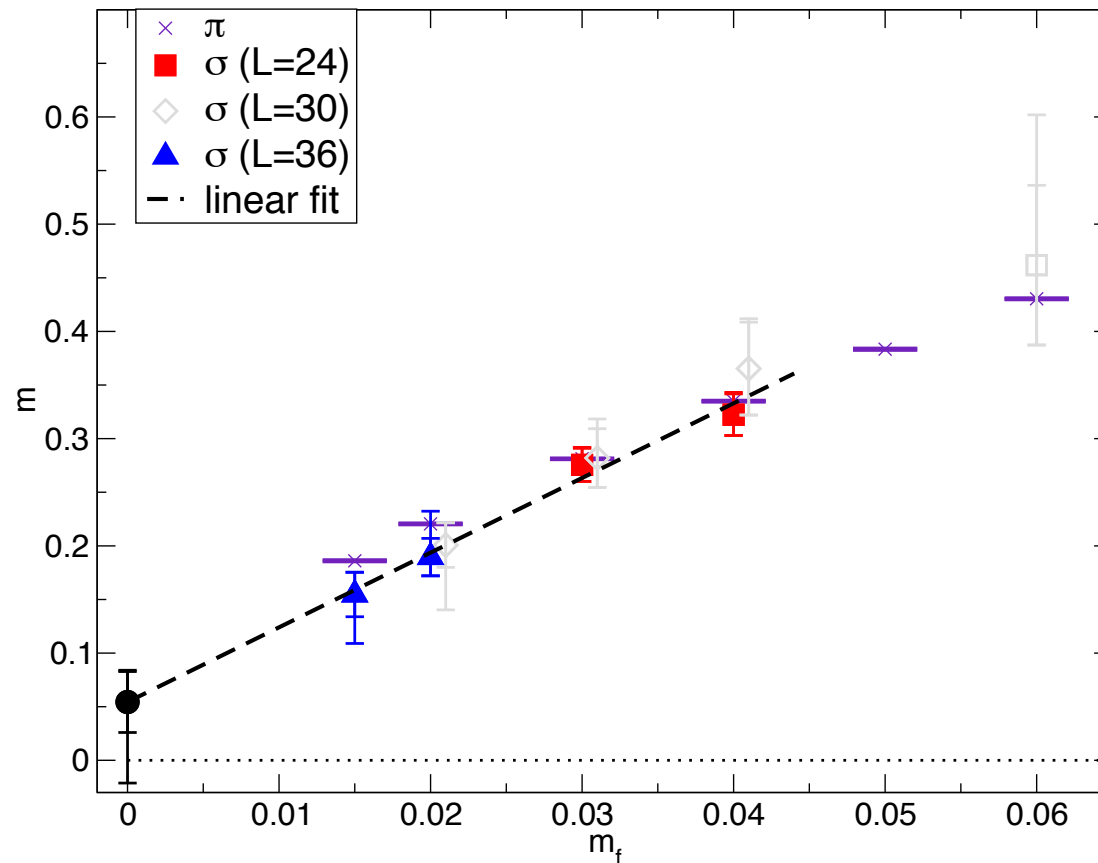


Reasonable signals with statistical error $< 20\%$

Systematic error from fit range dependence of m_σ

$m_\sigma \sim m_\pi$ in all m_f , much different from $N_f = 2$ QCD

Chiral extrapolation (1) in $N_f = 8$



$$m_\sigma = m_0 + Am_f: m_0 = 0.054(28)_{(70)}^8 \rightarrow \frac{m_\sigma}{F/\sqrt{2}} = 3.8(2.0)_{(5.0)}^{1.4}$$

$$F = 0.0202(13)_{(67)}^{54} \text{ updated from PRD87(2013)094511}$$

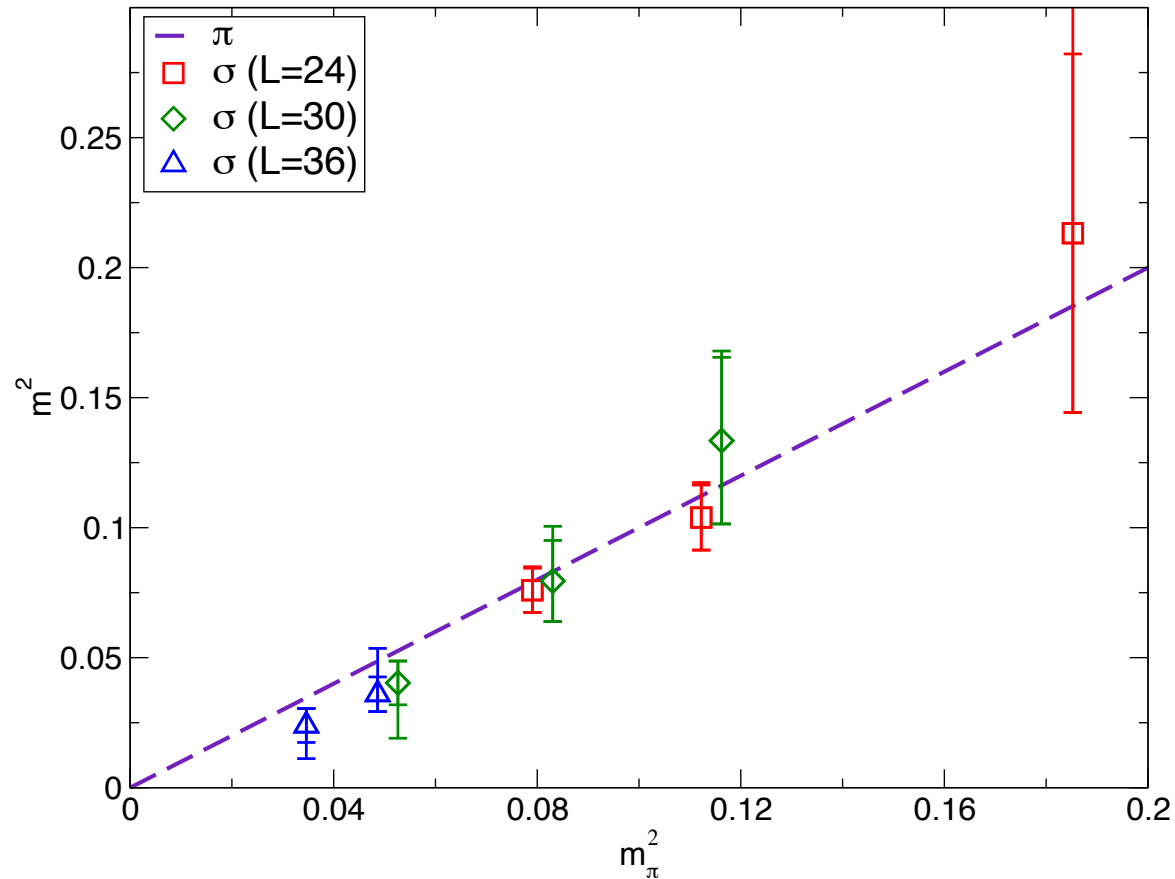
$$F_\pi/\sqrt{2} = 93 \text{ MeV in usual QCD}$$

Chiral extrapolation (2) in $N_f = 8$

ChPT with scale symmetry breaking

'13 Matsuzaki and Yamawaki

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$



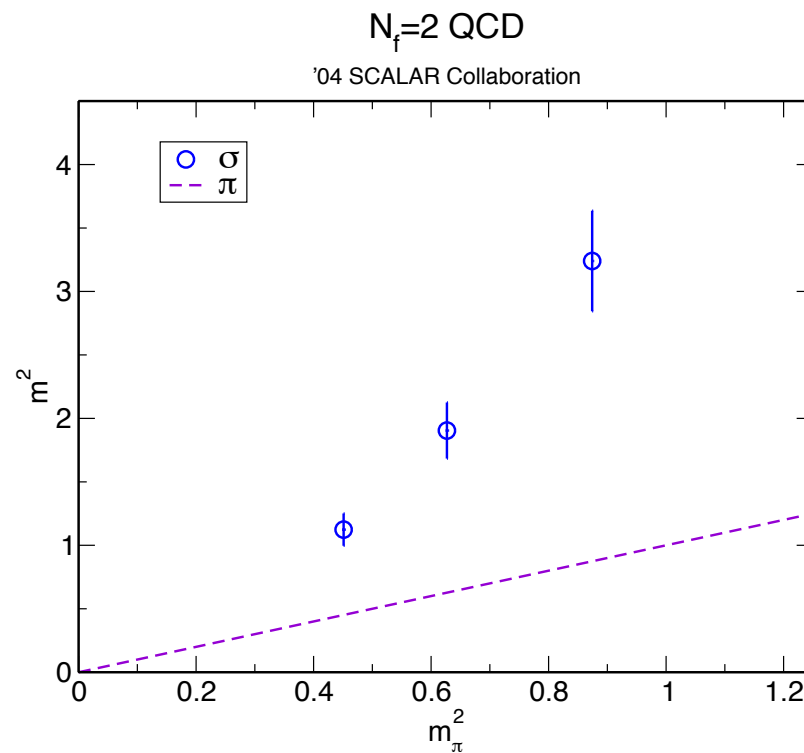
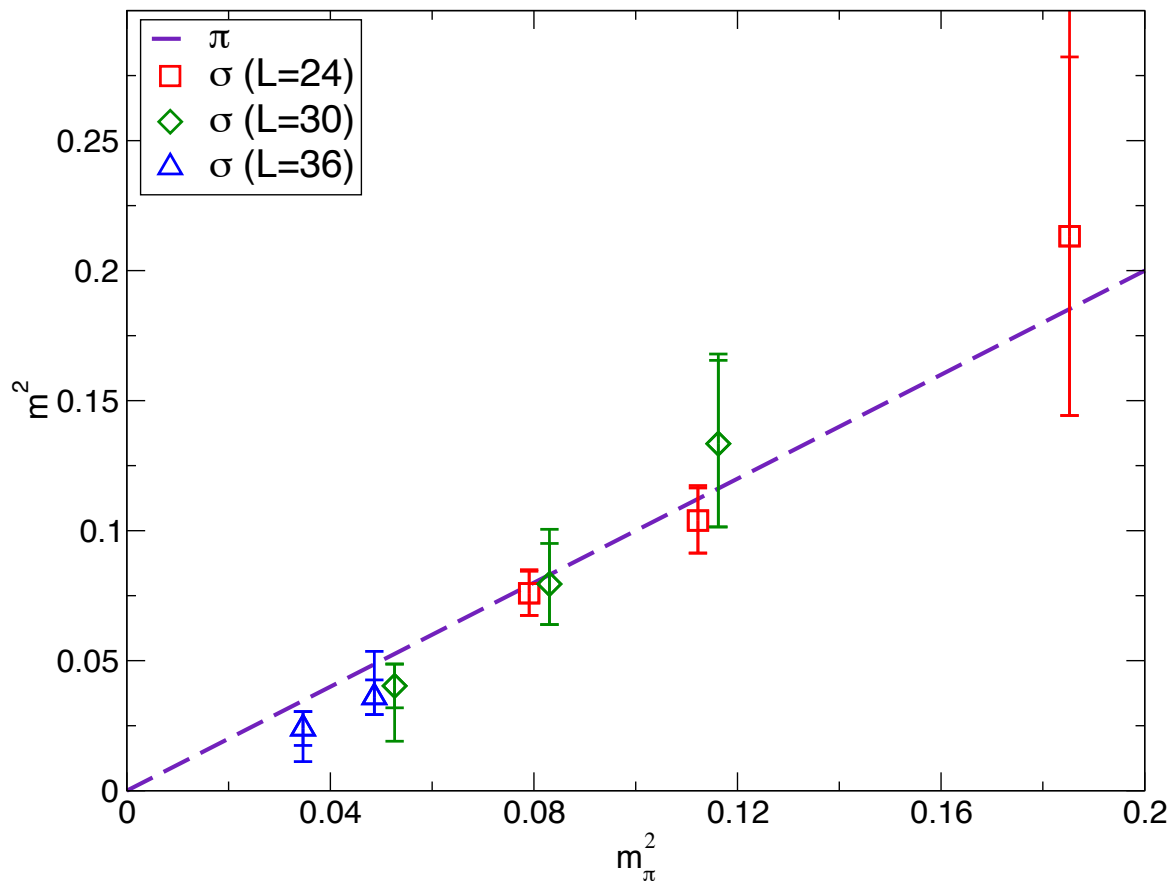
$$m_\sigma \sim m_\pi \rightarrow C \sim 1$$

Chiral extrapolation (2) in $N_f = 8$

ChPT with scale symmetry breaking

'13 Matsuzaki and Yamawaki

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$



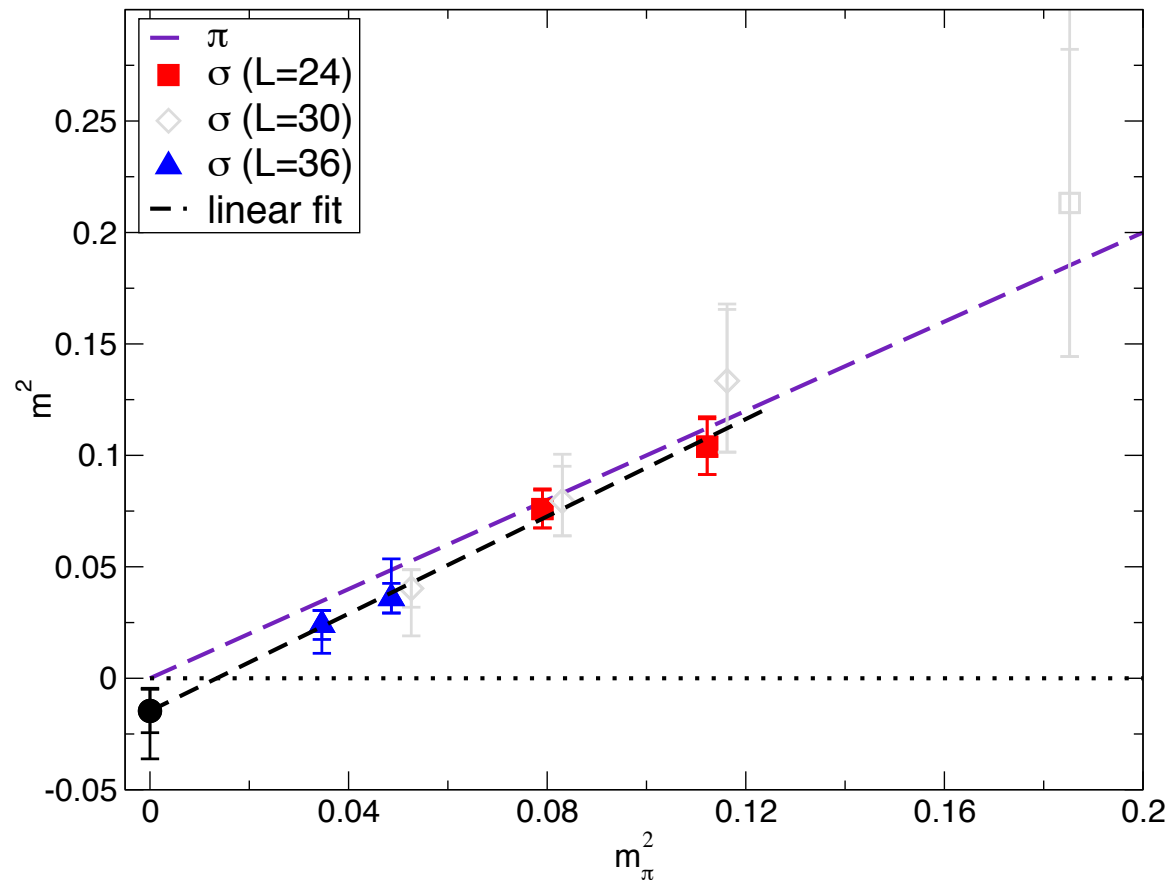
$m_\sigma \sim m_\pi \rightarrow C \sim 1$: different from $N_f = 2$ QCD

Chiral extrapolation (2) in $N_f = 8$

ChPT with scale symmetry breaking

'13 Matsuzaki and Yamawaki

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$



$m_0^2 < 0$: data not in $m_\sigma > m_\pi$ region

Need to check $m_\sigma > m_\pi$ at smaller m_f as in usual QCD

Comparison of m_σ in $N_f = 8$ with m_{Higgs}

$F/\sqrt{2} = 123$ GeV; One-family model (four-doublet fermions)

- Simple linear fit

$$\frac{m_\sigma}{F/\sqrt{2}} = 3.8(2.0)_{(1.4)}^{(5.0)}$$

consistent with $m_{\text{Higgs}} = 125$ GeV $\sim F/\sqrt{2}$ within lower error

- ChPT with spontaneous scale symmetry breaking

$$m_\sigma^2 = -0.015(10)_{(19)}^{(3)}$$

consistent with $m_{\text{Higgs}}^2 \sim F^2/2$ within 1.6 standard deviations

- Several other fits, e.g., $m_\sigma^2/(F_\pi/\sqrt{2})^2 = d_0 + d_1 m_\pi^2$
reasonably consistent results with above

Possibility to reproduce m_{Higgs}

Summary

Flavor-singlet scalar is important in walking technicolor theory.

Difficult due to huge noise in lattice simulation

⇒ Noise reduction method and large $N_{\text{conf}} \mathcal{O}(10000)$

Results of $N_f = 12$ QCD (consistent behaviors with conformal phase)

- $m_\sigma < m_\pi$; much different from small N_f QCD
- Conformal symmetry may make σ light

Results of $N_f = 8$ QCD (maybe candidate of walking technicolor)

- $m_\sigma \sim m_\pi$; much different from small N_f QCD
- Might be reflection of approximate conformal symmetry
- Need more data at smaller m_f for reliable chiral extrapolation
- Several fit results suggest

Possibility of light composite scalar $\rightarrow m_{\text{Higgs}} \sim v_{EW}$
(technidilaton)