Study of SU(2) gauge theory with six flavors

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in collaboration with

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Why SU(2)?

- Phenomenologically, N_c=2 and 3 equally interesting

  e.g.) A. Hietanen, R. Lewis, C. Pica, F. Sannino, arXiv:1308.4130

- Different feature from SU(3) gauge theory
  - Deconfine transition is of 2nd order when N_f=0.
  - Pattern of χ-symmetry breaking: $SU(2N_f) \rightarrow Sp(2N_f)$

- Open question: vacuum alignment problem

  $SU_L(2) \times U_Y(1)$ may not be broken.

Goal and How

To see whether this theory is IR conformal or not,

- running coupling and mass
- spectroscopy

are independently studied on the lattice.

Contents

II. Running coupling and mass

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Perturbation Theory

Perturbative predictions for **g^2_{FP}** for SU(2) gauge theory in the $\overline{\mathrm{MS}}$ scheme

N_f	5	6	7	8	9	10
2-loop	-	143.56	35.59	15.79	7.48	2.90
3-loop $\overline{\mathrm{MS}}$	38.10	20.68	13.25	8.65	5.26	2.47
4-loop $\overline{\mathrm{MS}}$	-	30.10	15.21	9.55	5.58	2.52

Far from convergence

Schrödinger Functional (SF) scheme

Luscher, Weisz, Wolff, NPB(1991)

$$e^{-\Gamma[B]} = \int DU D\overline{\psi} D\psi \; e^{-S[U,\overline{\psi},\psi,C,C']}$$

C, C': gauge link at boundaries set by hand

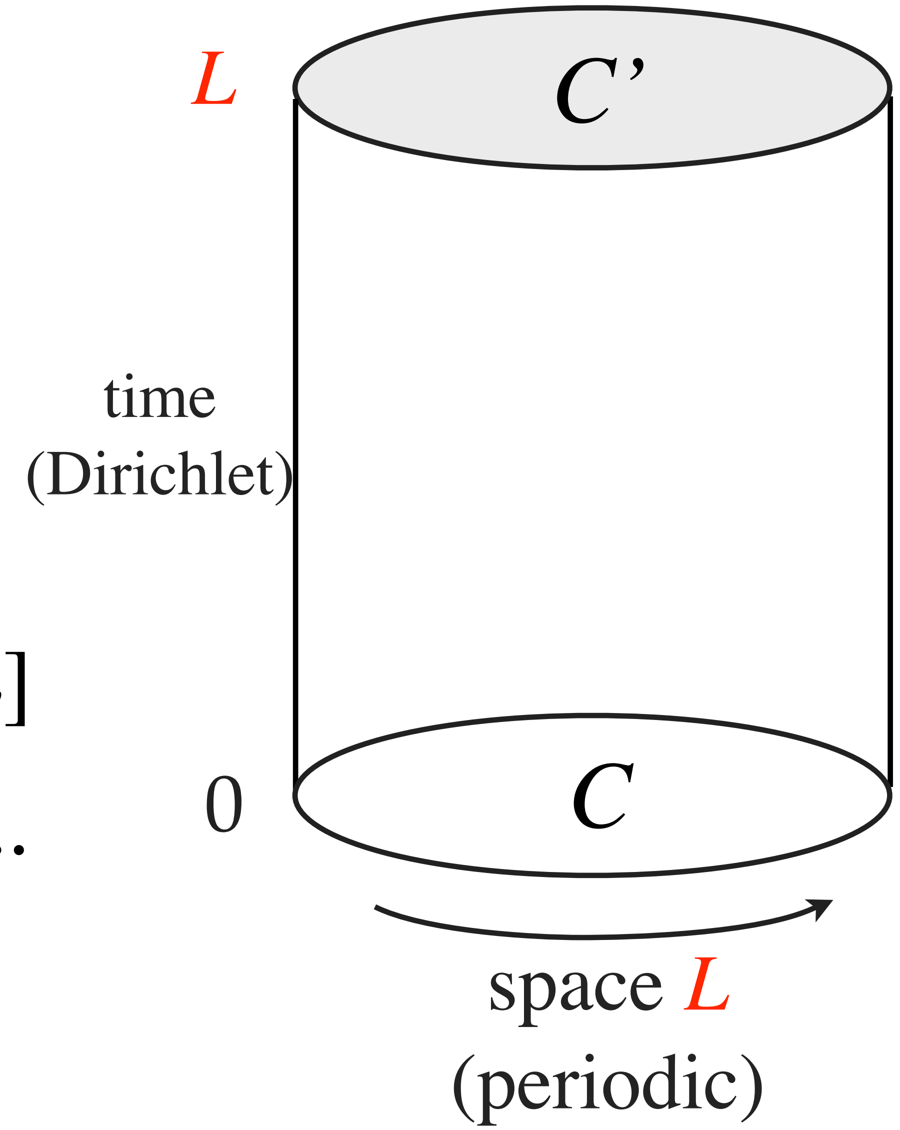
B : background gauge field set by C and C'

$\Gamma[B]$: effective action

Tree level: $\Gamma_0[B] = -1/[2\, g_0^2] \int d^4x \, \mathrm{Tr}[\, B_{\mu\nu} B_{\mu\nu}]$

$\Gamma[B] = -1/[2\, g_{\mathrm{SF}}(L)^2] \int d^4x \, \mathrm{Tr}[\, B_{\mu\nu} B_{\mu\nu}] + \ldots$

By changing L, "running" is measured.



Previous studies with SF

Consistent with IR conformal

- F. Bursa, L. Del Debbio, L. Keegan, C. Pica and T. Pickup, PLB 696, 374 (2011)
  - Unimproved Wilson. Max $V=16^4$
  - g^2(IRFP) ~ 4 or larger
- T. Karavirta, J. Rantaharju, K. Rummukainen and K. Tuominen, JHEP 1205,003(2012)
  - Improved Wilson (clover). Max $V=16^4$ for coupling (20^4 for running mass)
  - g^2(IRFP) ~ 12

No indication of IRFP

- Fleming's talk [arXiv:1311.4889]
  - Unimproved Wilson with smeared links. Max $V=24^4$
  - No evidence for IRFP below g^2 ~ 30

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This work

- Unimproved Wilson, **Max $V=24^4$**
- **Perturbative improvement of scaling violation**

Perturbative improvement

Define Discrete Beta Function $\boldsymbol{B(u,s)}$ by

$B(u,s) = 1/g^2(s\,L) - 1/u$ where $u = g^2\,(L)$, s: step scaling factor

Lattice DBF can be expressed as double expansion in $1/l$ $(=a/L)$ and u;

$$B^{\mathrm{Lat}}(u,s,l) = 1/g^2(s\,L,\,l) - 1/u$$

$$= B(u,s) + u\ (\,a_1/\,l + b_1/\,l^2 + c_1/\,l^3 + \ldots\,)$$

$$+ u^2\,(\,a_2/\,l + b_2/\,l^2 + c_2/\,l^3 + \ldots\,)$$

$$+ u^3\,(\,a_3/\,l + b_3/\,l^2 + c_3/\,l^3 + \ldots\,)$$

$$+ \ldots$$

Perturbative improvement

Define Discrete Beta Function $\boldsymbol{B(u,s)}$ by

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$$
\begin{aligned}
B^{\mathrm{Lat}}(u,s,l) &= 1/g^2(s\,L,\,l) - 1/u \\
&= \boldsymbol{B(u,s)} + \cancel{u\,(\,a_1/\,l + b_1/\,l^2 + c_1/\,l^3 + \ldots\,)} \\
&\quad + u^2\,(\,\boxed{a_2/\,l} + b_2/\,l^2 + c_2/\,l^3 + \ldots\,) \\
&\quad + u^3\,(\,\boxed{a_3/\,l} + b_3/\,l^2 + c_3/\,l^3 + \ldots\,) \\
&\quad + \ldots
\end{aligned}
$$

We explicitly calculated the $O(u)$ error by lattice PT and removed that.

In the continuum limit, we assume that $1/l$ scaling violation dominates.

Perturbative improvement

Define Discrete Beta Function $B(u,s)$ by

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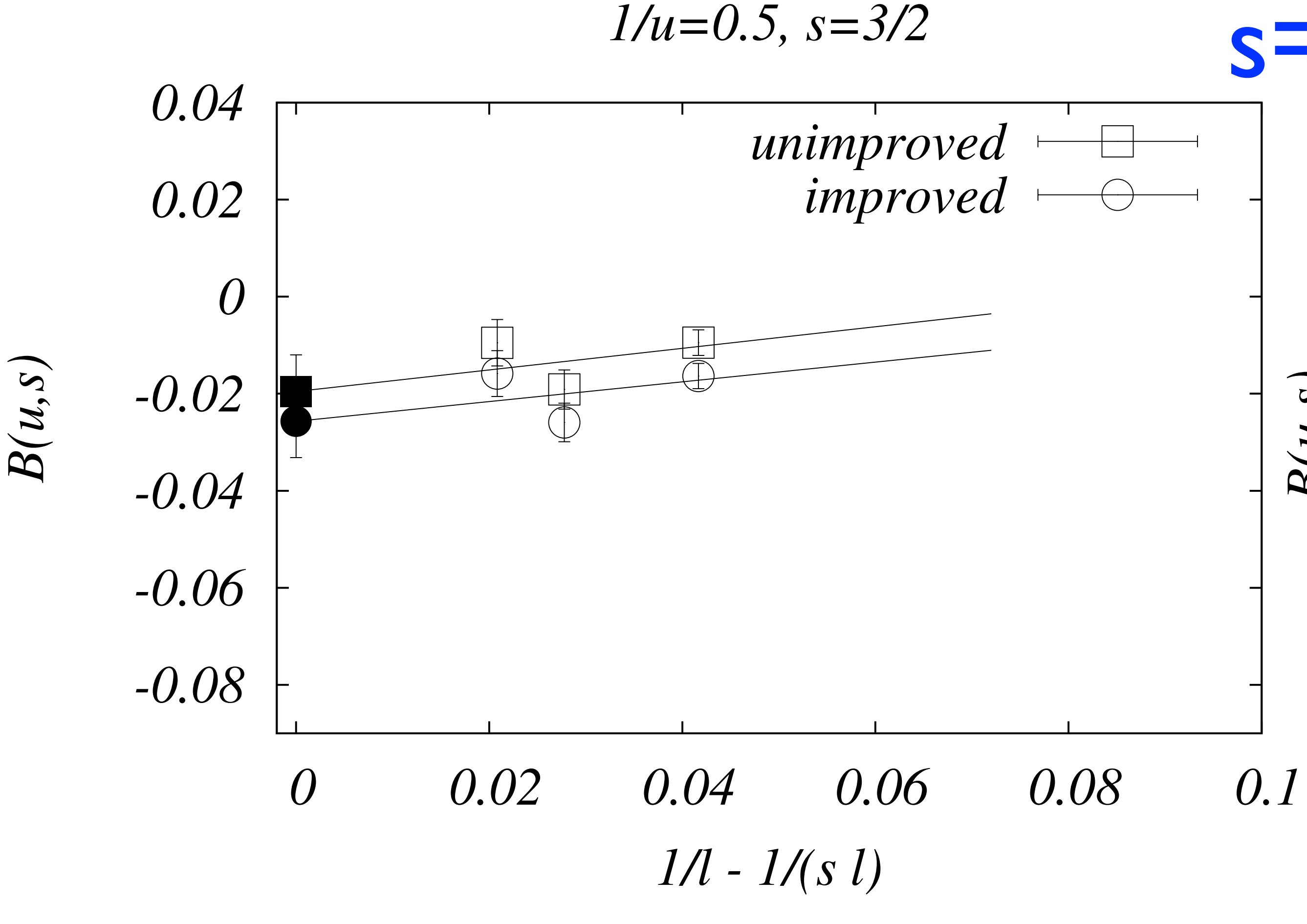
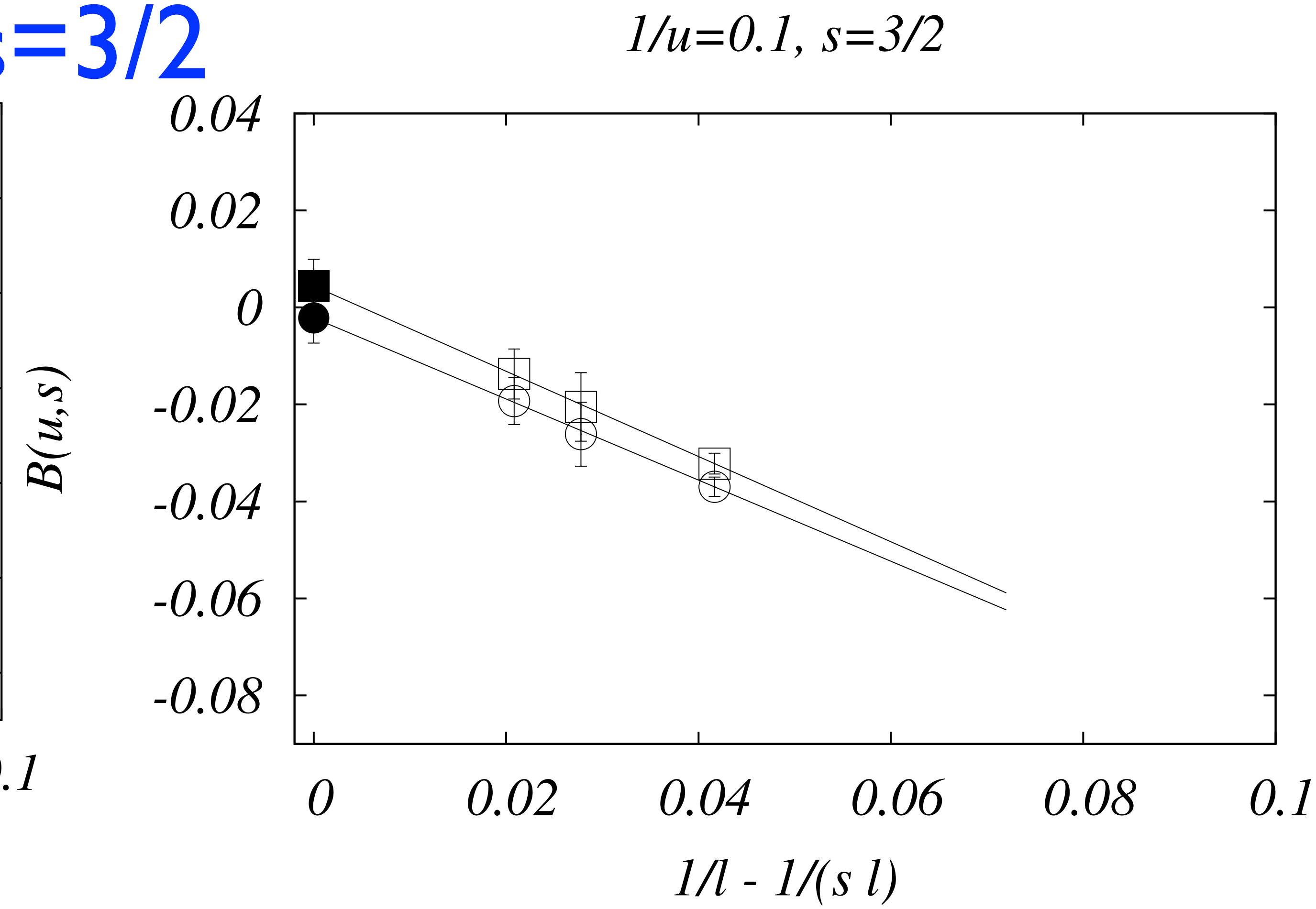
Continuum limit: linear in *1/l*

$\boldsymbol{B(u,s)} = 1/g^2(s\,L) - 1/u$ with $u = g^2(L)$

s=3/2



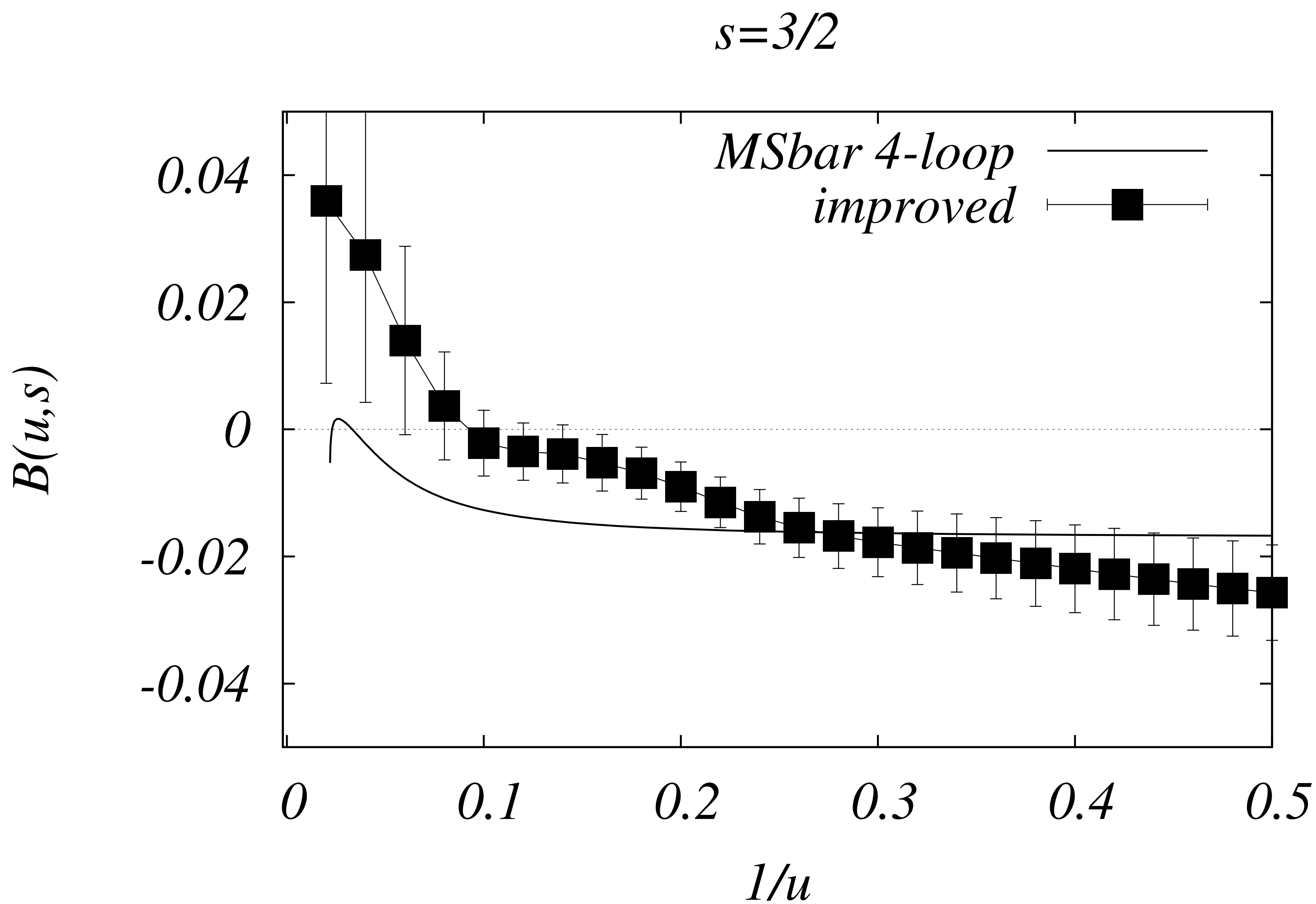


In the weak coupling region, consistent with constant.

In the strong coupling region, large scaling violation observed. Scaling violation remains linear.

DBF in the continuum limit

$\boxed{B(u,s) = 1/g^2(s\,L) - 1/u \text{ with } u = g^2(L)}$



$0.06 \le 1/u_{\mathrm{FP}} \le 0.15$

Analysis with s=2 gives consistent result.

Mass of Standard model fermions

S_R^{Lat} : renormalized, iso-singlet scalar bilinear operator

$$\begin{aligned} m_{\mathrm{SM,f}} &= \frac{C_S^{\mathrm{SF}}(1/M_{\mathrm{ETC}})}{M_{\mathrm{ETC}}^2} \frac{Z_S^{\mathrm{SF}}(1/M_{\mathrm{ETC}})}{Z_S^{\mathrm{SF}}(a)} C_S^{\mathrm{SF-Lat}}(a) \langle\ S_R^{\mathrm{Lat}}(a)\ \rangle \\ &= \frac{C_S^{\mathrm{SF}}(1/M_{\mathrm{ETC}})}{M_{\mathrm{ETC}}^2} \frac{Z_S^{\mathrm{SF}}(1/M_{\mathrm{ETC}})}{Z_S^{\mathrm{SF}}(a)} C_S^{\mathrm{SF-Lat}}(a) \frac{\langle\ S_R^{\mathrm{Lat}}(a)\ \rangle}{f_{\pi_{\mathrm{T}}}^3} \times (246\mathrm{GeV})^3 \end{aligned}$$

- $C_s^{\mathrm{SF}}(\mu)$: ETC dependent coefficient
- $C_s^{\mathrm{SF\text{-}Lat}}(\mu)$: finite renormalization connecting Lat to SF
- $\langle S_R^{\mathrm{Lat}}(\mu)\rangle$: chiral condensate
- $Z_S^{\mathrm{SF}}(\mu_1)/Z_S^{\mathrm{SF}}(\mu_2)$: Calculated in this work

Anomalous dimension via Running of Z_P

$$\sigma_P^{\mathrm{SF}}(u,s) \;=\; \frac{Z_P^{\mathrm{SF}}(L)}{Z_P^{\mathrm{SF}}(sL)} = \exp\left(\int_L^{sL} dL' \frac{\gamma_m^{\mathrm{SF}}(u(L'))}{L'}\right)$$

$$\Sigma_{P,0}^{\mathrm{lat}}(u,s,l) = \left.\frac{Z_P^{\mathrm{lat}}(g_0^2,\, l)}{Z_P^{\mathrm{lat}}(g_0^2,\, s\cdot l)}\right|_{u=g_{\mathrm{SF}}^2(g_0^2,l)}$$

$$= \sigma_P{}^{\mathrm{SF}}(u,s) + u\;(\,a_1/\,l + b_1/\,l^2 + c_1/\,l^3 + \ldots)$$

$$+\, u^2(\,a_2/\,l + b_2/\,l^2 + c_2/\,l^3 + \ldots)$$

$$+\, u^3(\,a_3/\,l + b_3/\,l^2 + c_3/\,l^3 + \ldots)$$

$$+ \ldots$$

At u=u_{FP},

$$\gamma_m^* = \frac{\ln \sigma_P^{\mathrm{SF}}(u,s)}{\ln s}$$

Anomalous dimension via Running of Z_P

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$$\Sigma_{P,0}^{\rm lat}(u,s,l) = \left.\frac{Z_P^{\rm lat}(g_0^2,\ l)}{Z_P^{\rm lat}(g_0^2,\ s\cdot l)}\right|_{u=g^2_{\rm SF}(g_0^2,l)}$$

$$= \sigma_P^{\rm SF}(\boldsymbol{u},\boldsymbol{s}) + \cancel{u\ (a_1/l + b_1/l^2 + c_1/l^3 + \ldots)}$$

$$+ u^2 (\boxed{a_2/l} + b_2/l^2 + c_2/l^3 + \ldots)$$

$$+ u^3 (\boxed{a_3/l} + b_3/l^2 + c_3/l^3 + \ldots)$$

$$+ \ldots$$

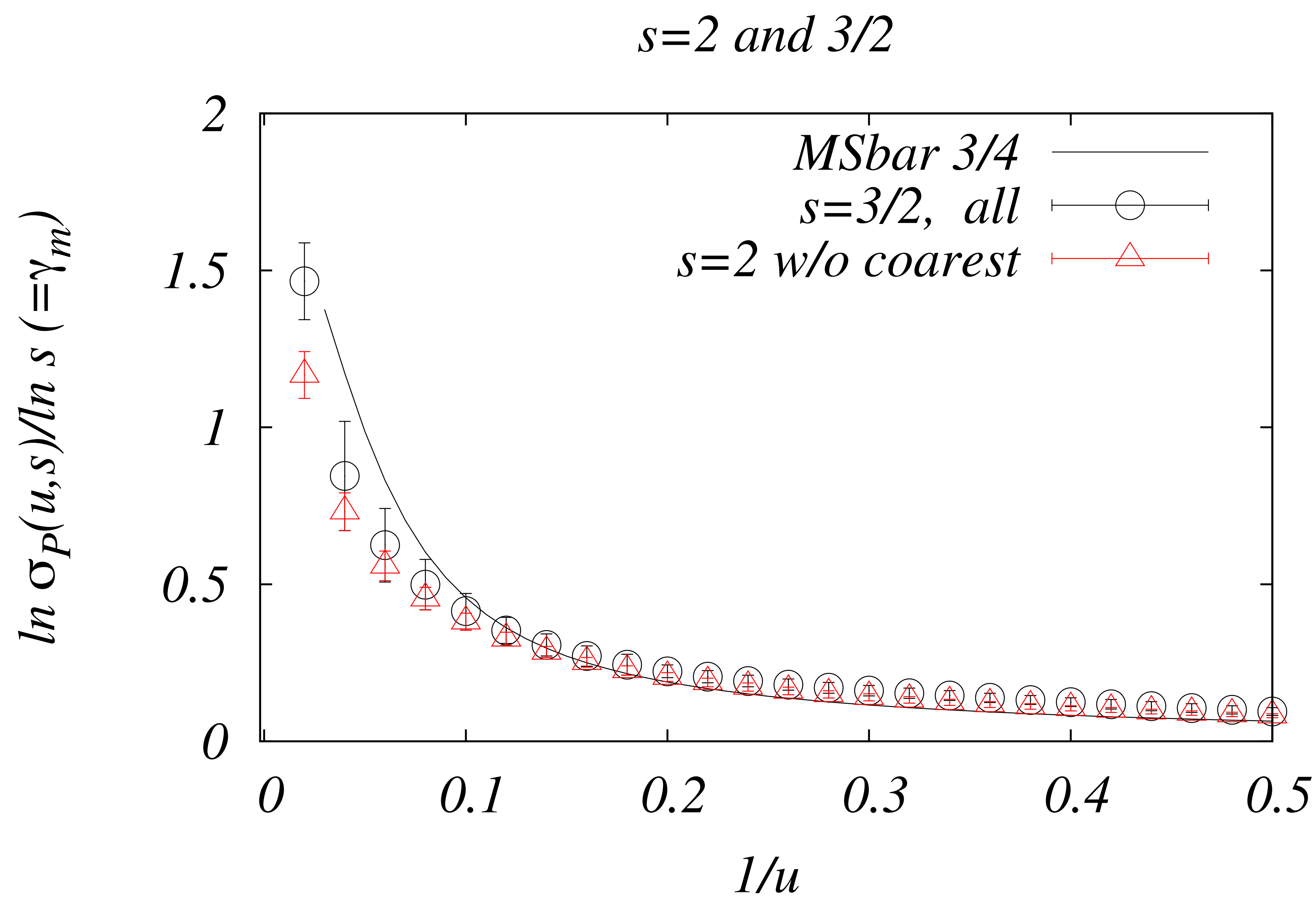
At $u=u_{\rm FP}$,

$$\gamma_m^* = \frac{\ln \sigma_P^{\rm SF}(u,s)}{\ln s}$$

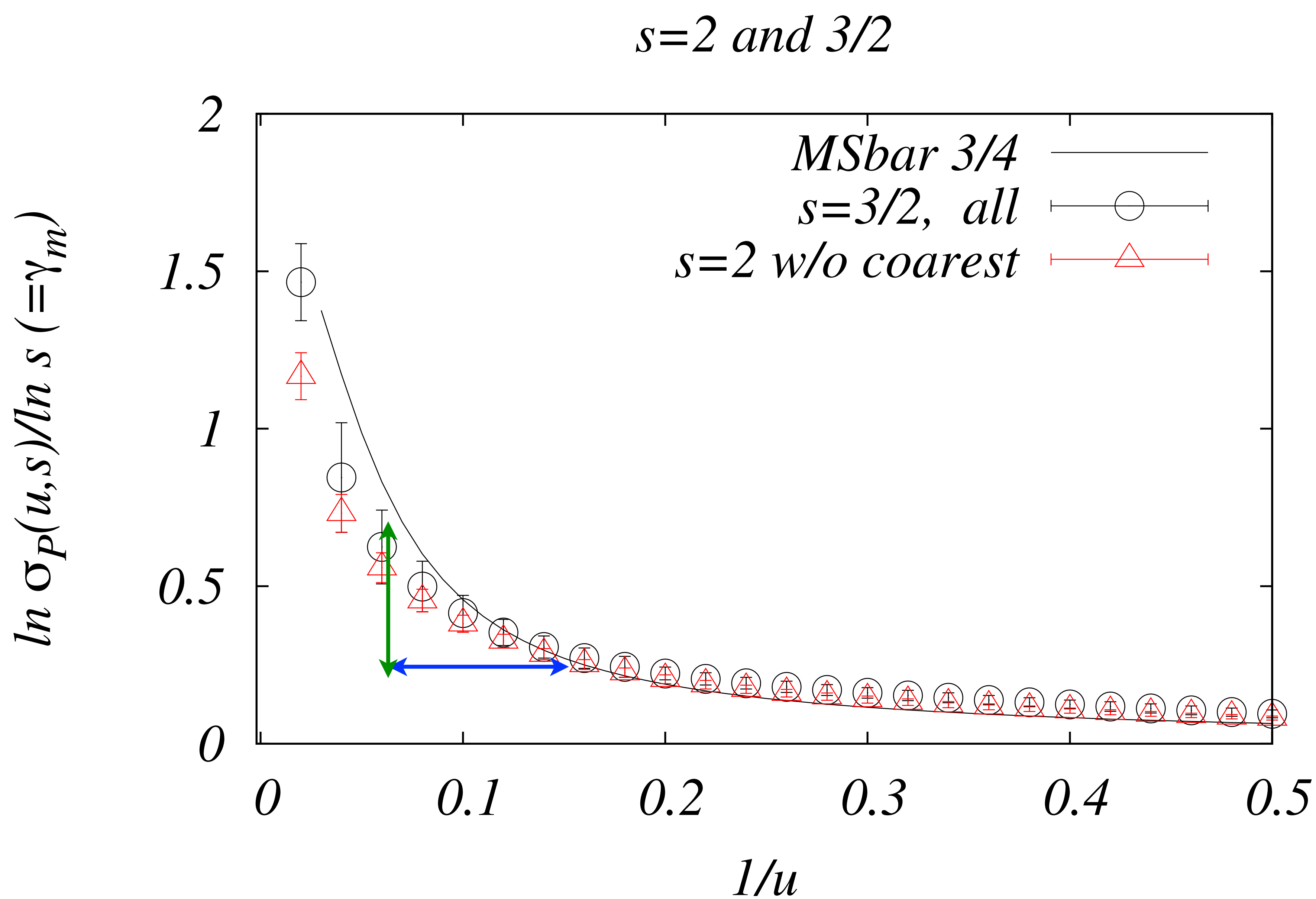
We numerically determined the $O(u)$ error.

In the continuum limit, we assume that $1/l$ scaling violation dominates.

Continuum limit of γ_m



Continuum limit of γ_m



$0.06 \leq 1/u_{\mathrm{FP}} \leq 0.15 \Rightarrow 0.26 \leq \gamma_m \leq 0.74$

III. Spectroscopy

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Simulation Parameters:

Unimproved Wilson fermions + Wilson plaquette

3 Volumes: 16^3x32, 24^3x48, 32^3x64

Single lattice spacing: $\beta = 2.0$

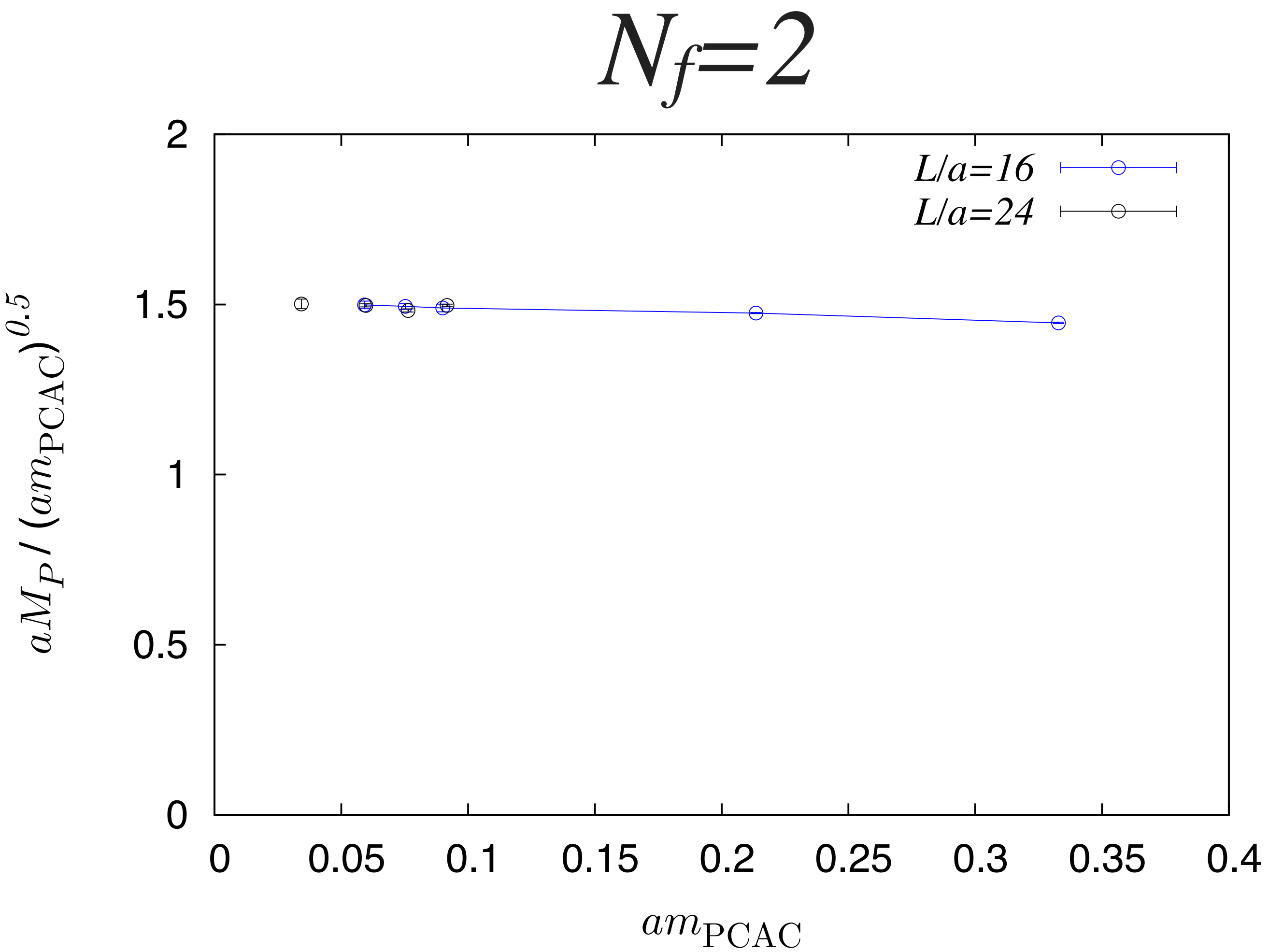
N_f=6 and N_f=2 ← to compare with chirally broken theory

Special care on FSE

Quark mass dependence of various quantities is carefully examined in the FSE-free region. (FSE=Finite Size Effect)

Dependence in χ-broken theory ≠ Dependence in conformal theory

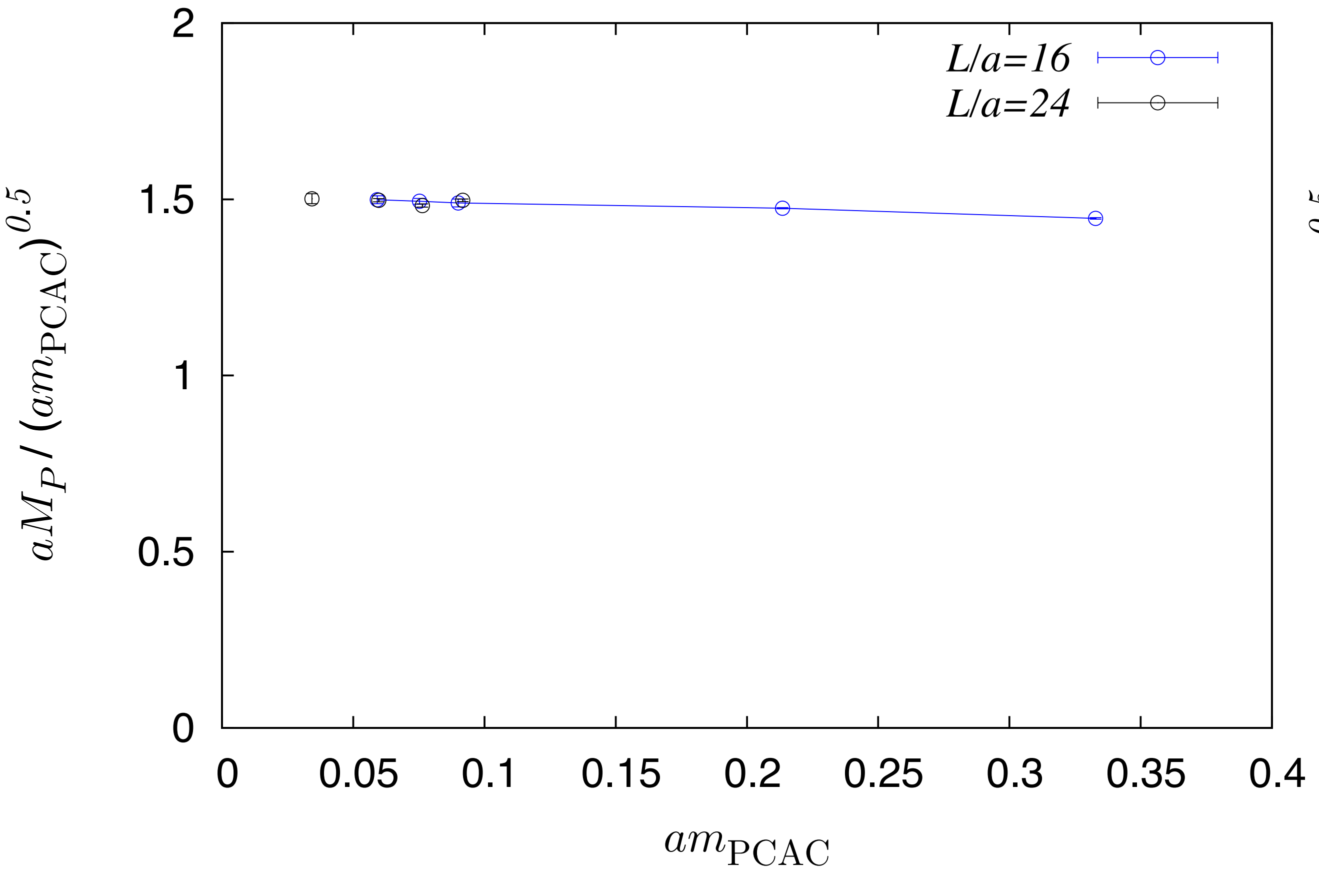
$aM_P/(am_q)^{1/2}$



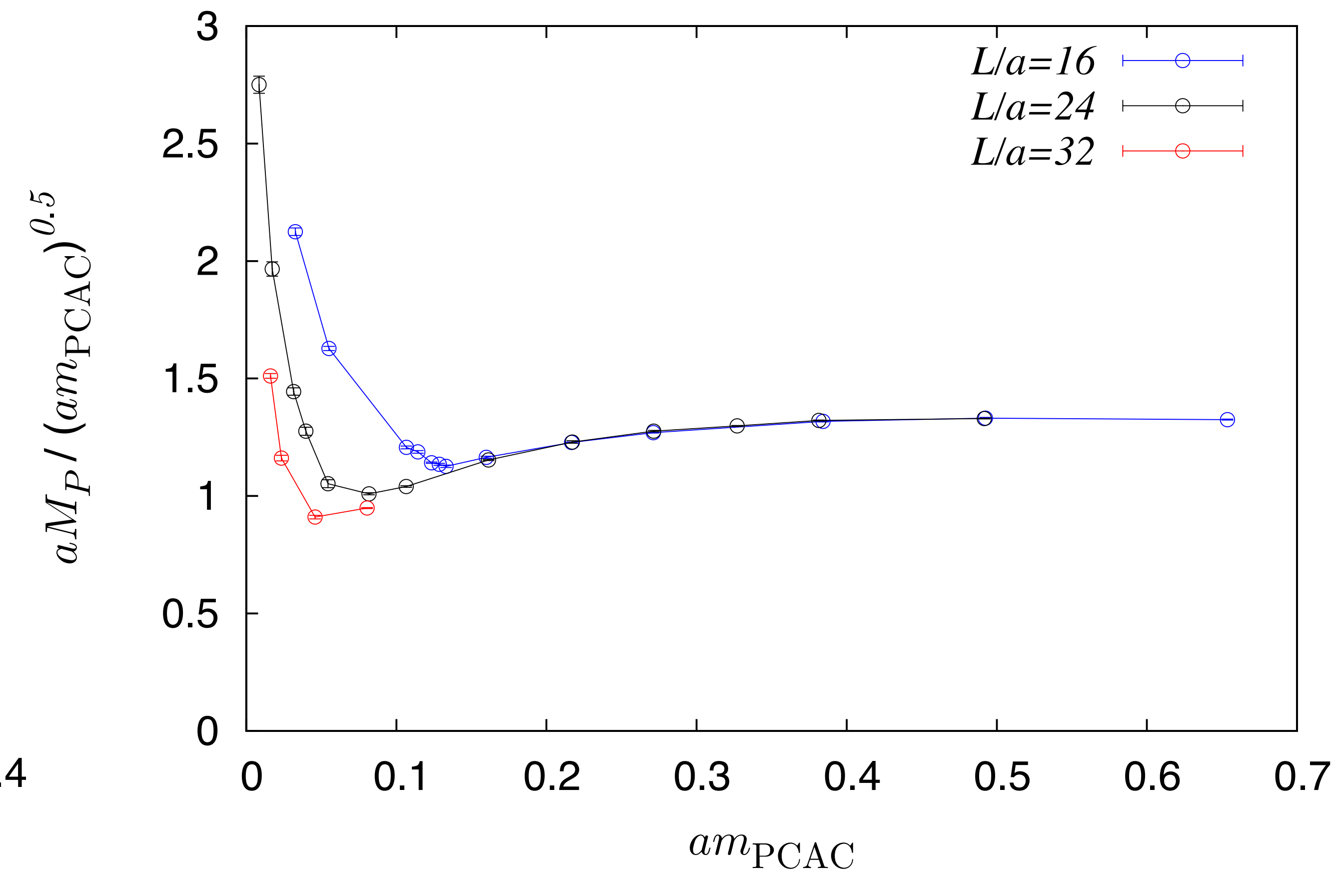
Nf=2
L/a=16
L/a=24
aMP / (amPCAC)0.5
amPCAC
0
0.5
1
1.5
2
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

$aM_P/(am_q)^{1/2}$

N_f=2



N_f=6



The way to approach to the chiral limit is different.

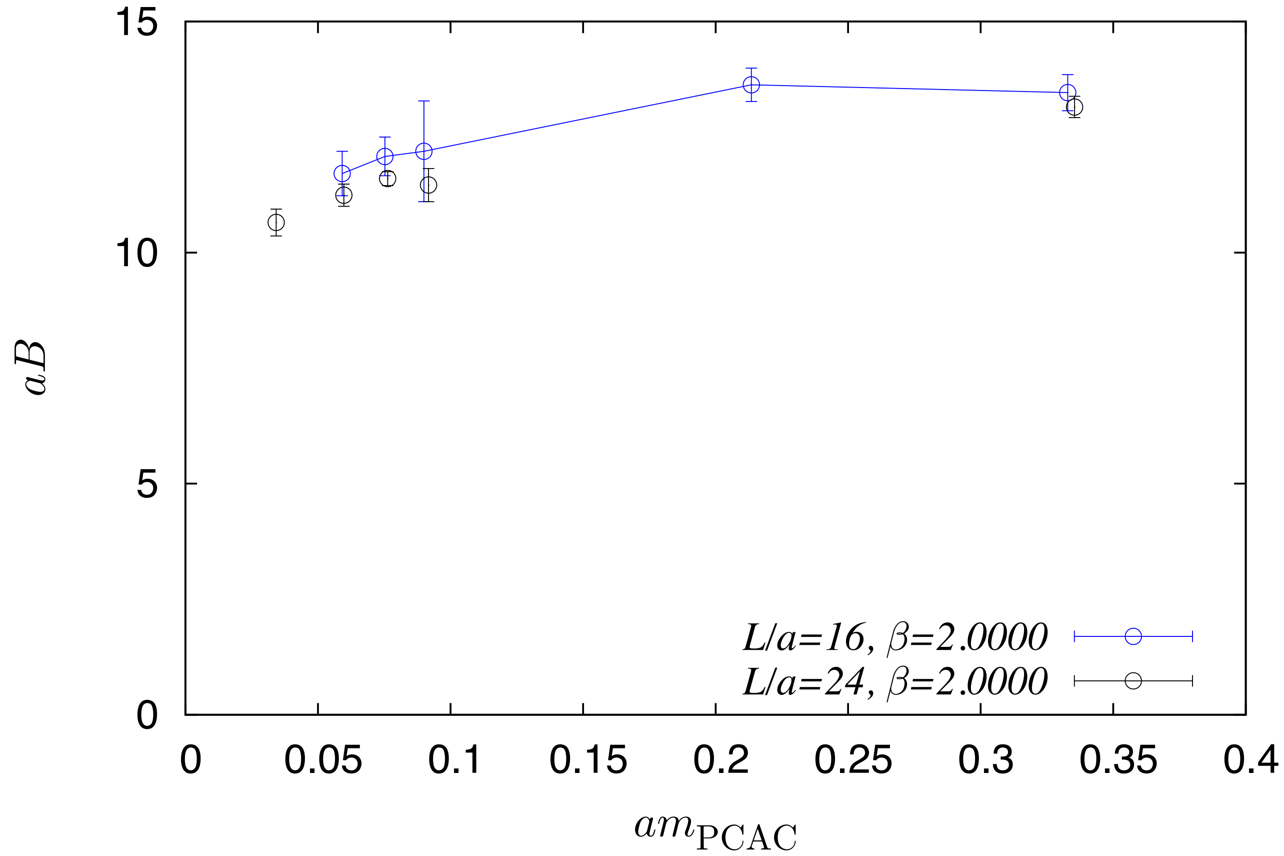
In Nf=6, $aM_P \propto (am_q)^{\alpha}$ with $\alpha > 1/2$ near the chiral limit

IRFP⇒$M_P \propto (m_q)^{\alpha^*}$with $\alpha^*=1/(1+\gamma^*)$

α>1/2 indicates $\gamma^* < 1$.

$aB = a\,\langle\overline{\psi}\psi\rangle_{\text{subt}} \,/\, f_P{}^2$

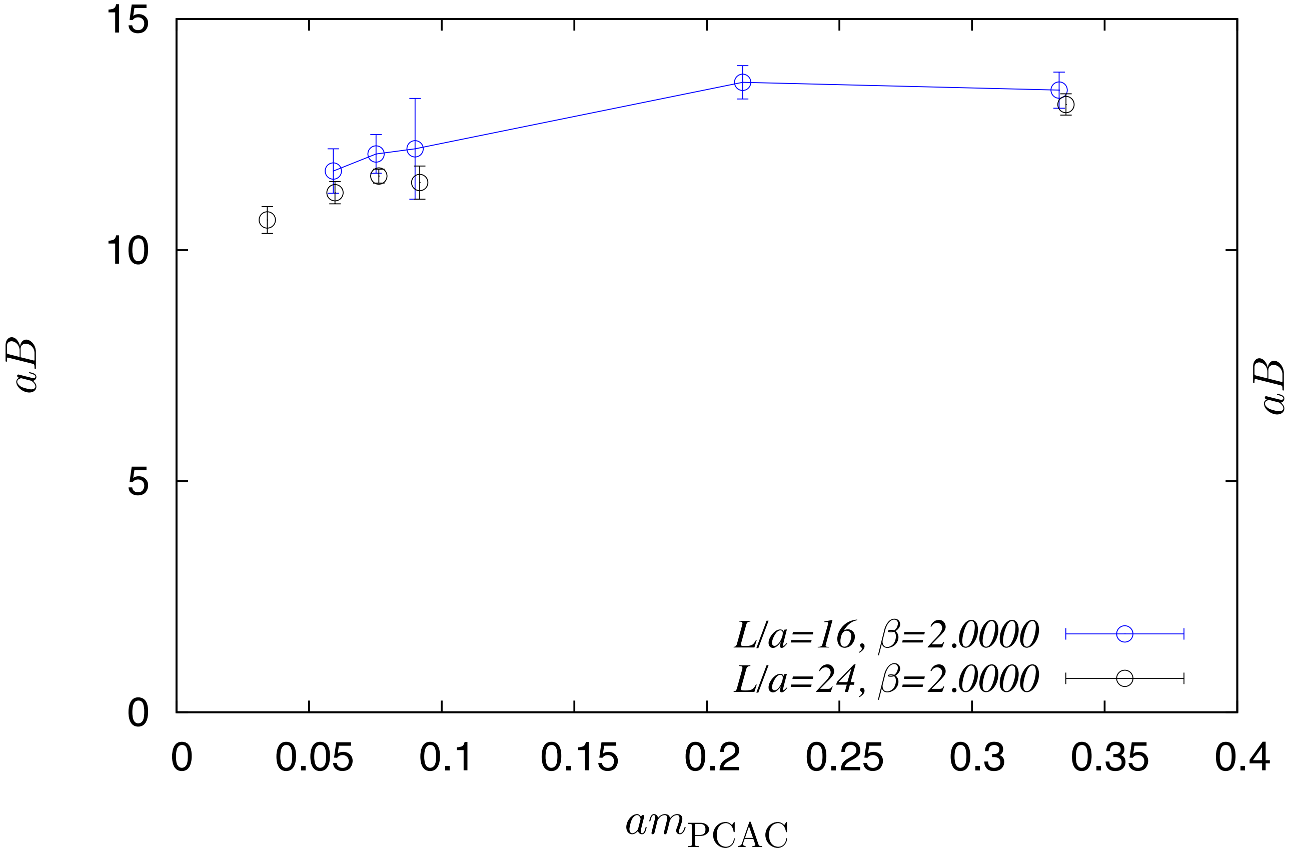
N_f=2



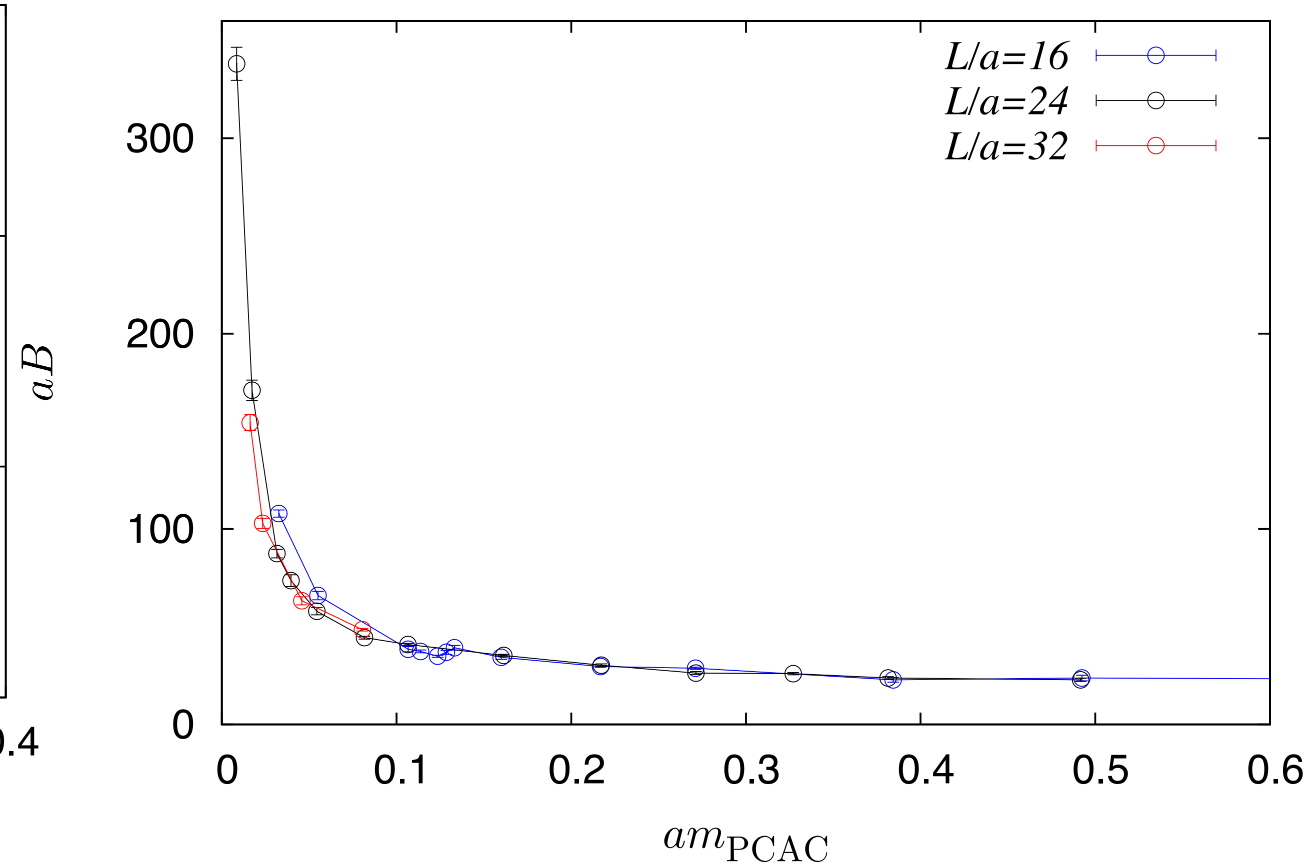
$$\delta^{ab}\cdot\left\langle\overline{\psi}\psi\right\rangle_{\text{subt}}\left(m_{\text{PCAC}},\,L/a\right) = 2m_{\text{PCAC}}\cdot(2\kappa)^2\sum_n\left\langle P^a(n)P^b(0)\right\rangle$$

$aB = a\,\langle\overline{\psi}\psi\rangle_{\text{subt}}\,/\,f_P{}^2$

N_f=2



N_f=6

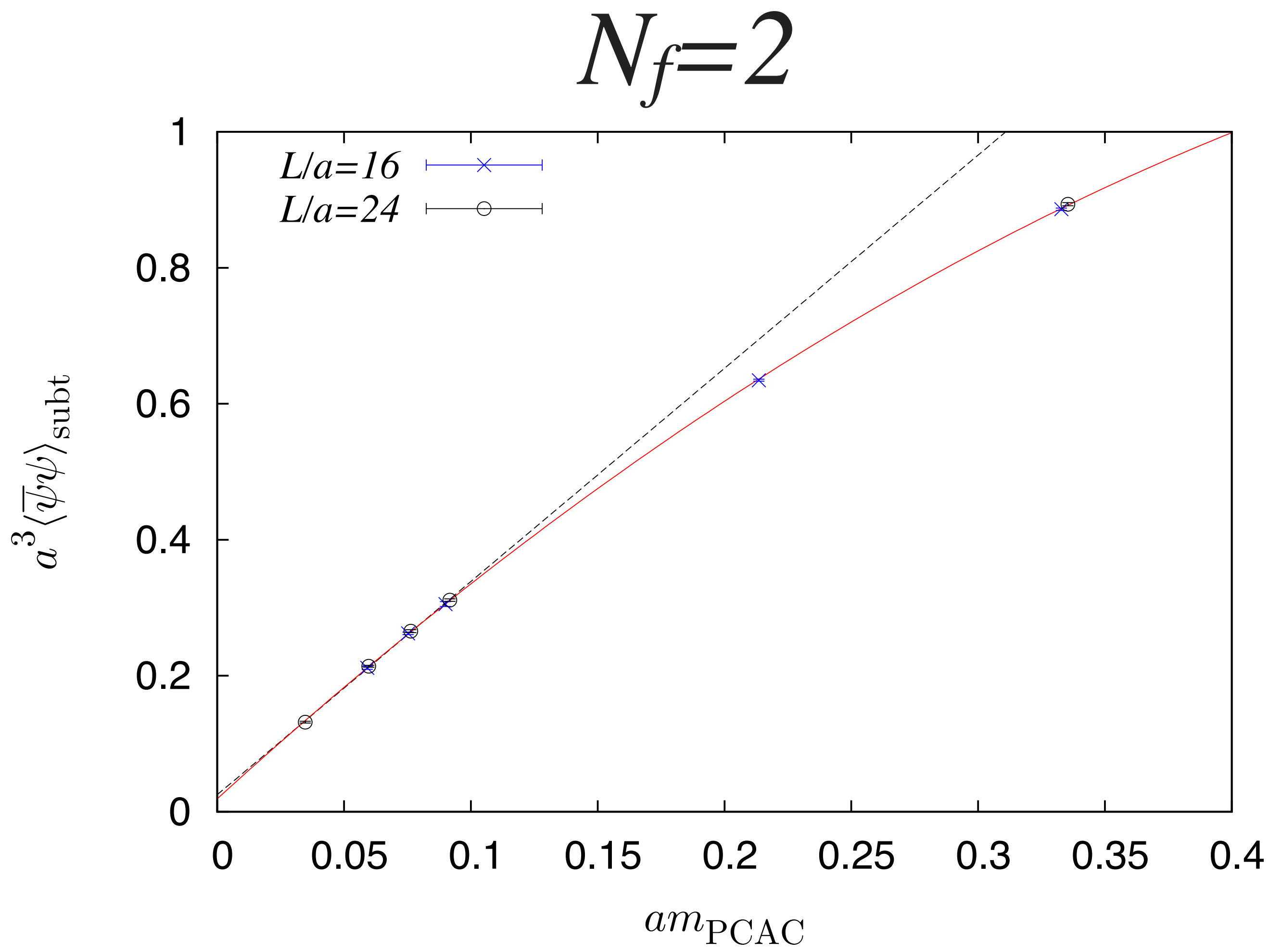


$$\delta^{ab}\cdot\langle\overline{\psi}\psi\rangle_{\rm subt}(m_{\rm PCAC},\,L/a) = 2m_{\rm PCAC}\cdot(2\kappa)^2\sum_n\langle P^a(n)P^b(0)\rangle$$

Assuming IRFP and Hyper-scaling⇒ $aB\propto(m_q)^{-|1-\gamma^*|/(1+\gamma^*)}$

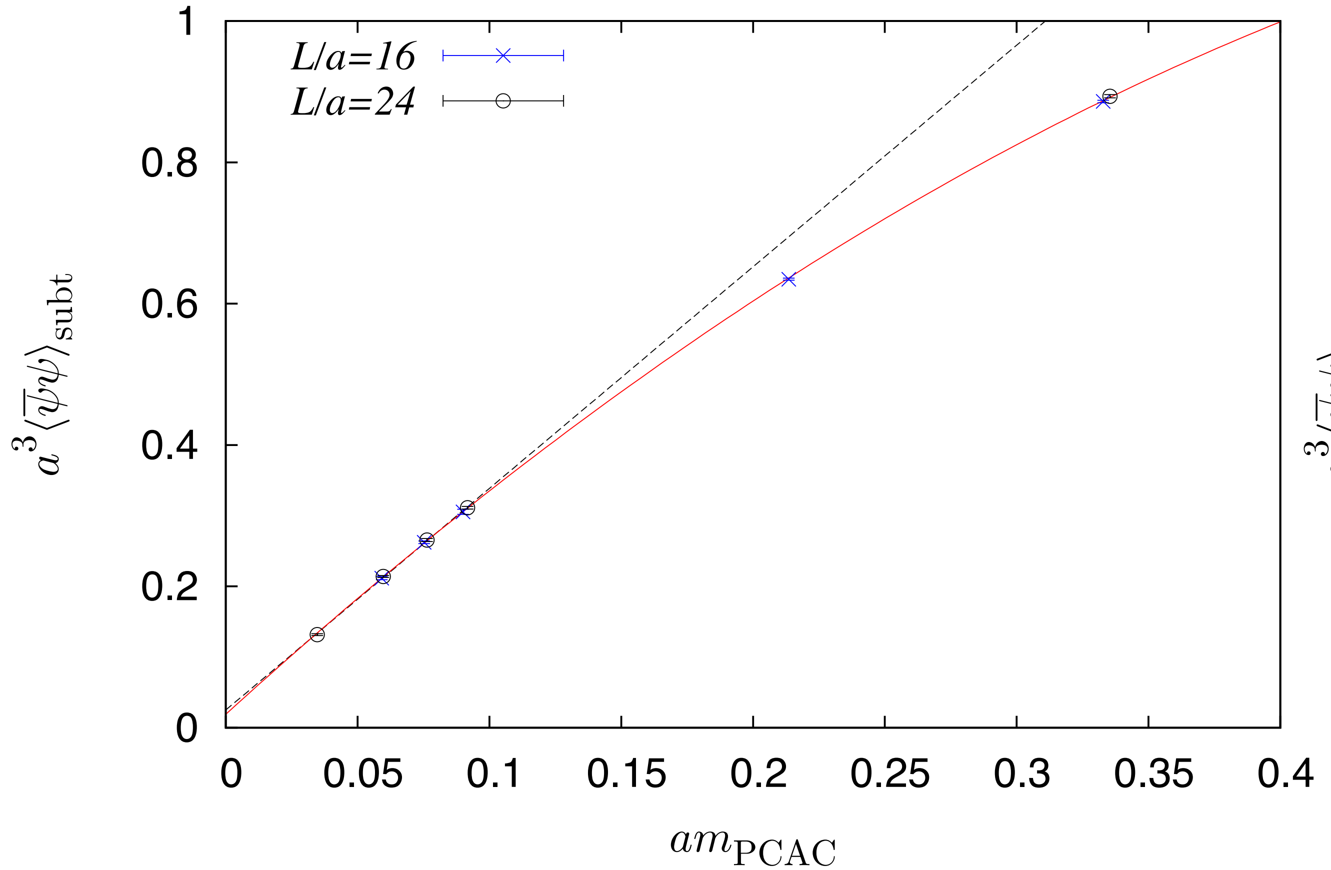
Increasing toward the chiral limit

$a^3\langle\overline{\psi}\psi\rangle_{\rm subt}$

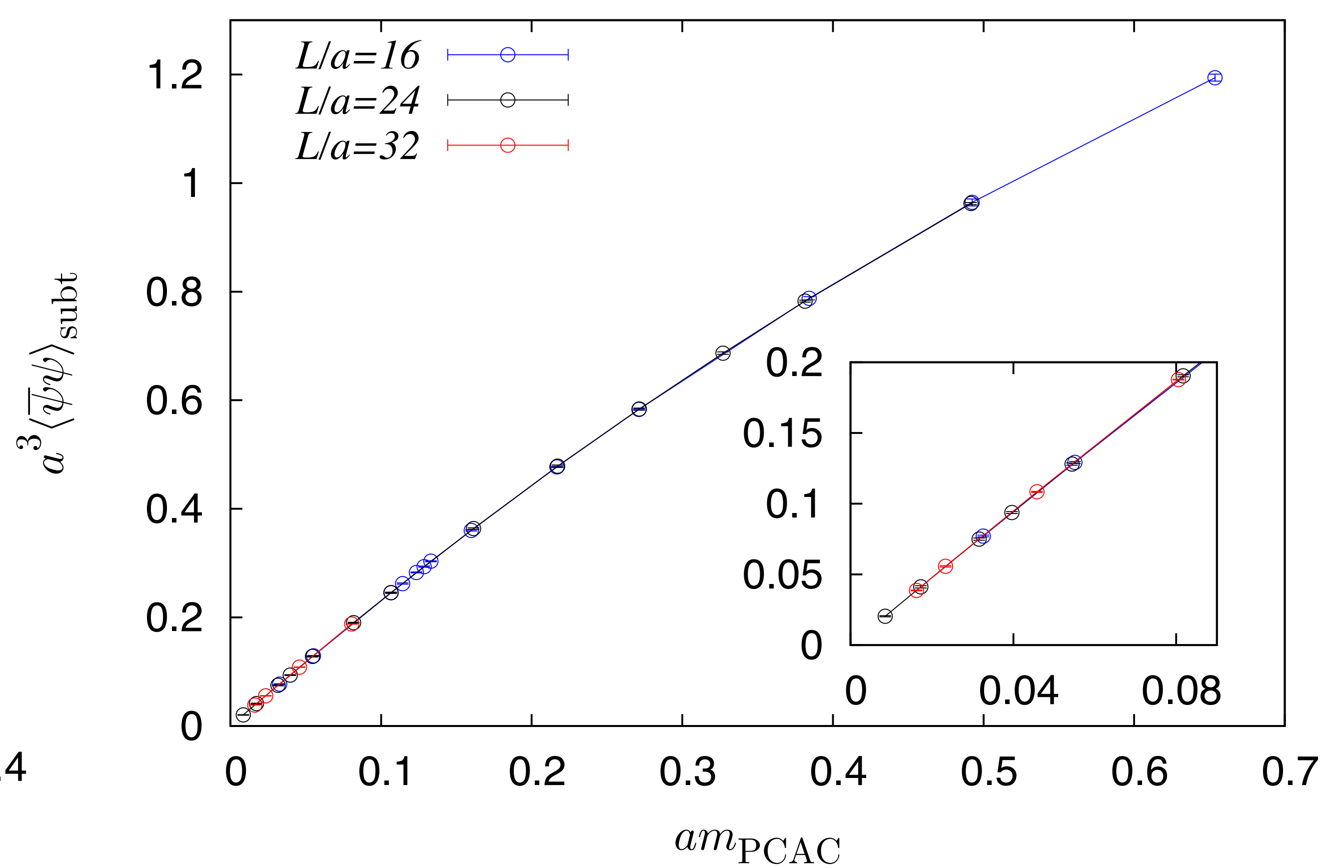


$a^3\langle\overline{\psi}\psi\rangle_{\text{subt}}$

N_f=2



N_f=6



IRFP $\Rightarrow a^3 \langle\overline{\psi}\psi\rangle_{\text{subt}} \propto (am_q) + (am_q)^{(3-\gamma^*)/(1+\gamma^*)} + \ldots$

Fit to this form by assuming IRFP and $\gamma^* < 1 \Rightarrow \gamma^* \sim 0.51$

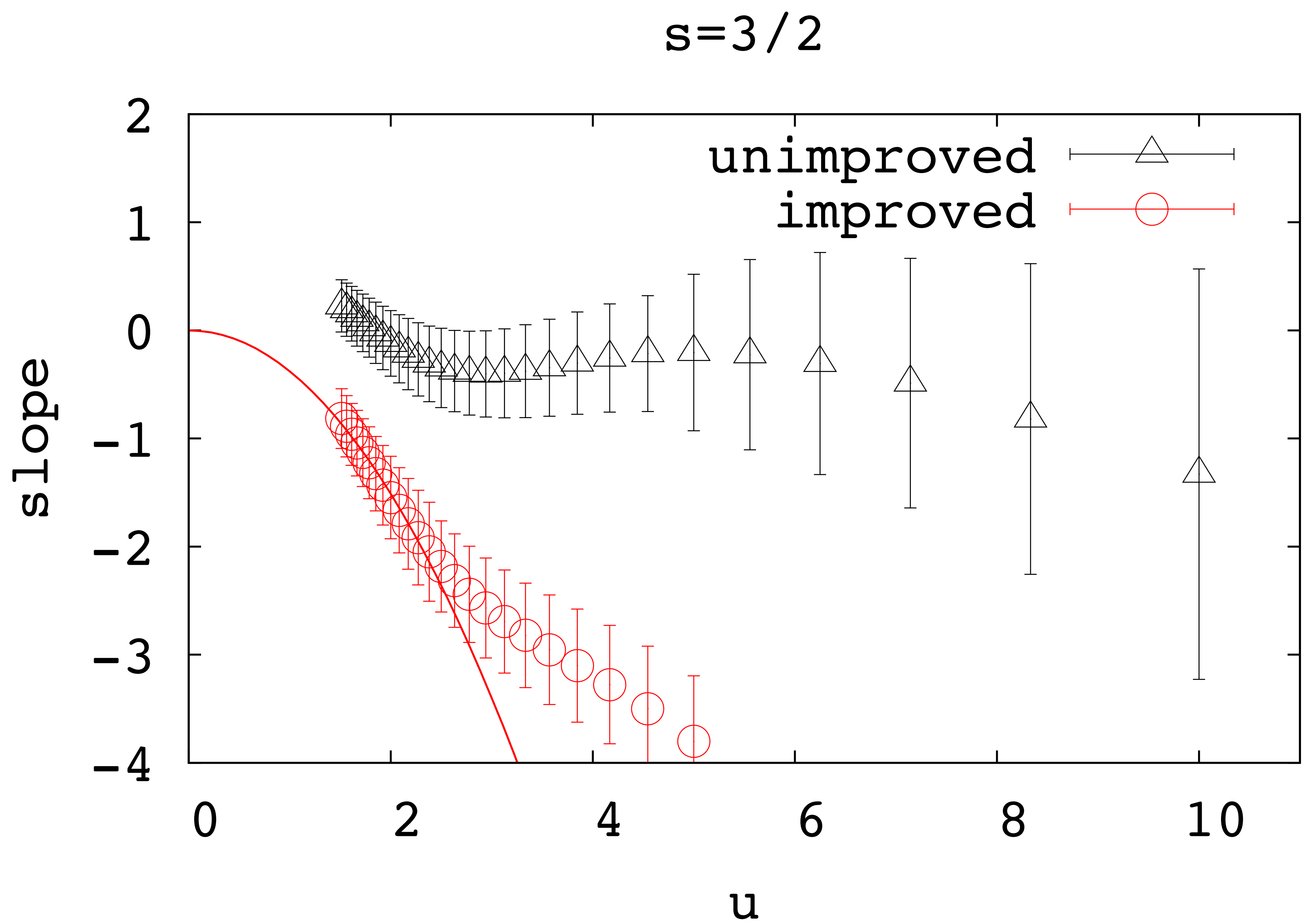
Compatible with the SF result, $0.26 \leq \gamma_m \leq 0.74$

IV. Summary and outlook

Summary and Outlook

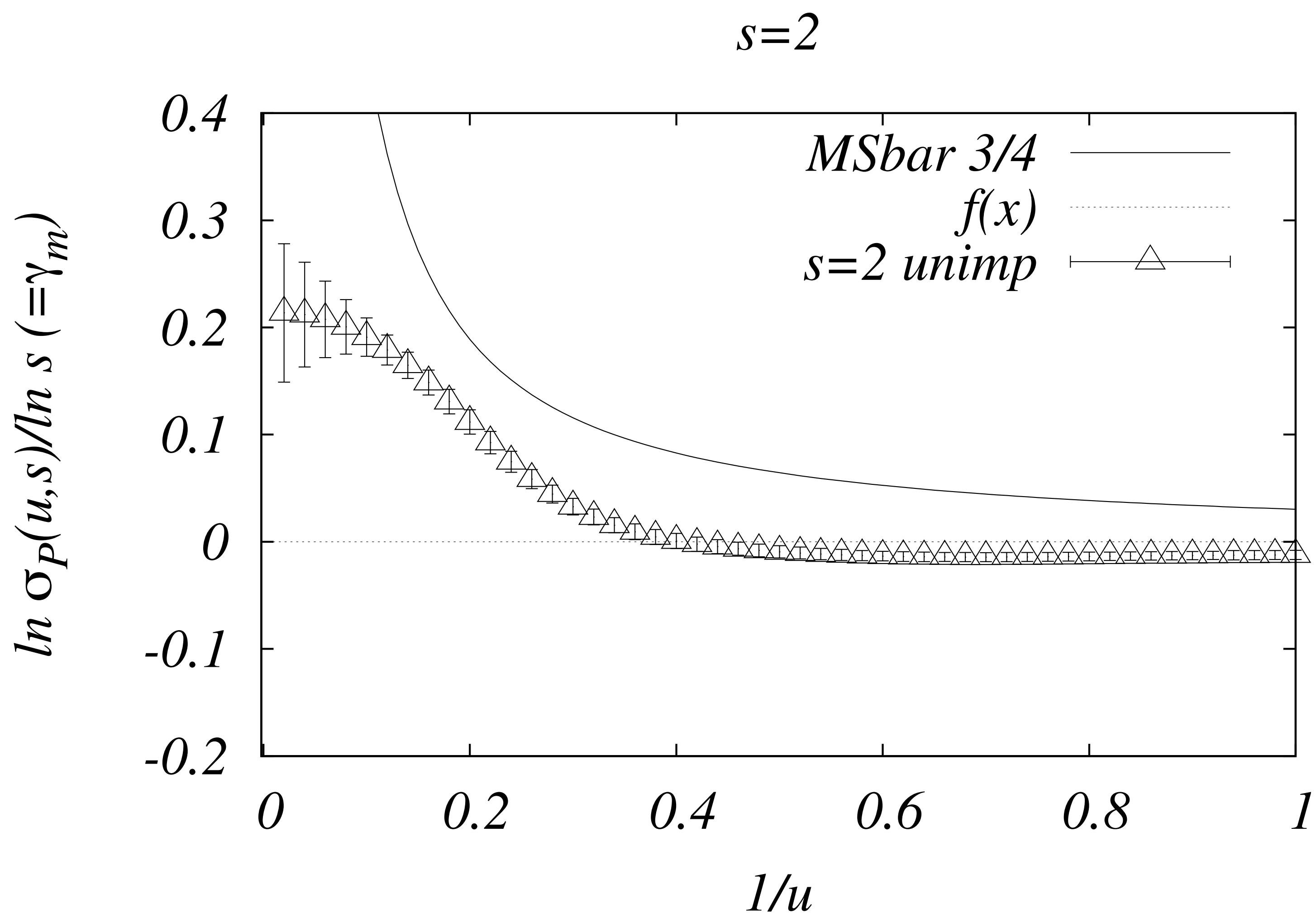
- Running coupling: consistent with the IRFP.
- Mass anomalous dimension: $0.26 \leq \gamma_m \leq 0.74$.
- Quark mass dependence of several quantities are different from those in 2-flavor theory, and γ_m extracted is consistent with $0.26 \leq \gamma_m \leq 0.74$.
- In order to establish IRFP, simulations with improved actions are on-going.

Slope in the extrapolation



Slope ~ O(u^2) as expected

Improvement really works?



Without improvement, the continuum limit clearly undershoot even in perturbative regime.

⇒ improvement is necessary