Lattice study of conformality in twelve-flavor QCD

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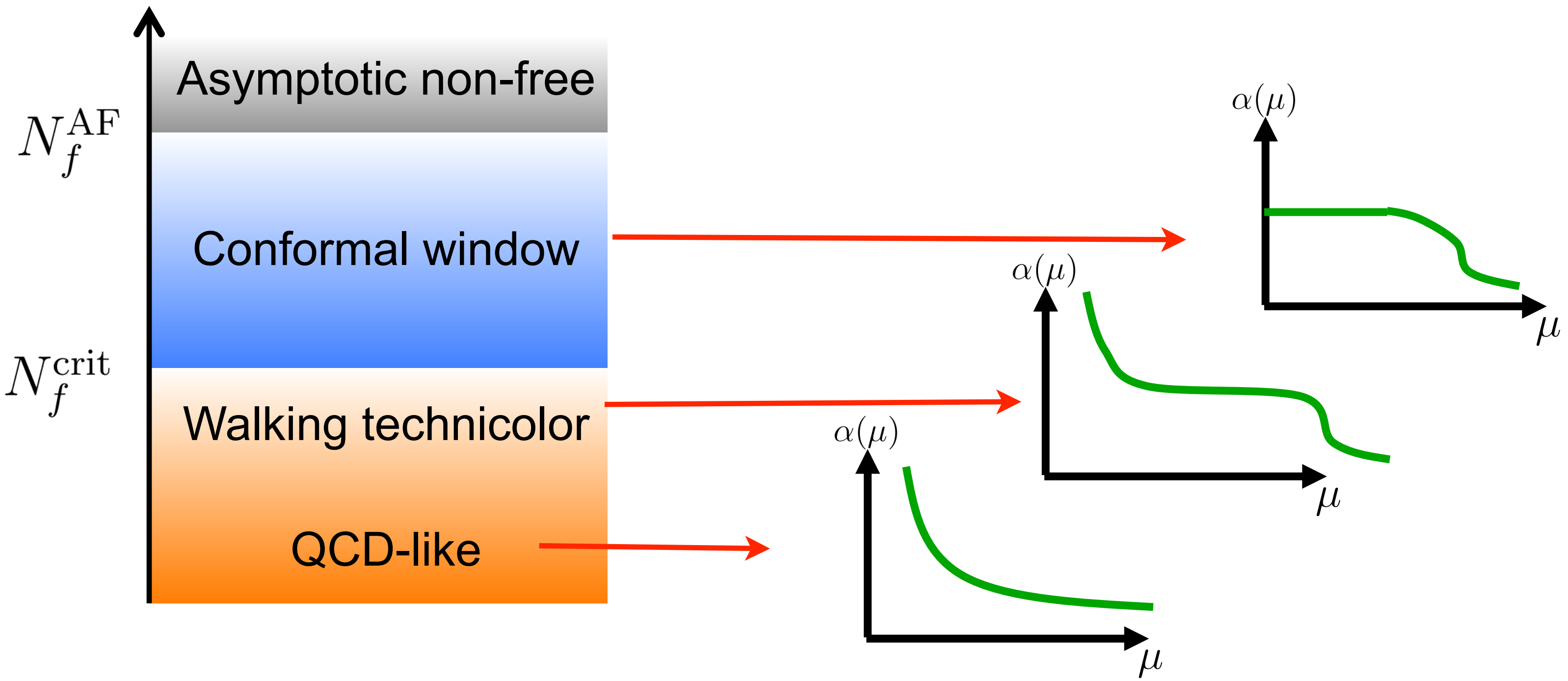
Introduction

Walking and conformal behavior -> non-perturbative dynamics

Many flavor QCD: benchmark test of walking dynamics

N_f : Number of flavor $\alpha(\mu)$: running gauge coupling



- Understanding of the conformal dynamics is important (e.g. critical phenomena)
- Walking technicolor (WTC) could be realized just below conformal window.
- What the value of the anomalous dimensions γ? (γ : critical exponent)
- Rich hadron structures may be observed in LHC.

LatKMI-Nagoya project (since 2011)

Systematic study of flavor dependence in Large Nf QCD using single setup of the lattice simulation

Our goals:

- Understand the flavor dependence of the theory
- Find the conformal window
- Find the walking regime and investigate the anomalous dimension

Status (lattice): → T. Yamazaki (poster)

- Nf=16: likely conformal
- Nf=12: **controversial** → This talk
- Nf=8: controversial, our study suggests walking behavior? → talk by K.-i. Nagai (next)
- Nf=4: chiral broken and enhancement of chiral condensate → M. Kurachi (poster)

Observables:

- pseudoscalar, vector meson -> chiral behavior
- Glueball (O++) and/or flavor-singlet scalar

Is this lighter compared with others? If so, Good candidate of "Higgs" (techni-dilaton).

→ talk by T. Yamazaki
E. Rinaldi for gluonic observables (poster)

Our work

- use of improved staggered action

  Highly improved staggered quark action [HISQ]
- use MILC version of HISQ action

  use tree level Symanzik gauge action

  no $(ma)^2$ improvement (no interest to heavy quarks)= HISQ/tree

Simulation setup

- **SU(3), Nf=12 flavor**

simulation parameters

two bare gauge couplings (β) & four volumes & various fermion masses

- $\beta=6/g^2=3.7$, 3.8, and 4.0
- $V=L^3\mathrm{x}T$: $L/T=3/4$; L=18, 24, 30, 36
- $0.03\leqq m_f\leqq 0.2$ for $\beta=3.7$, $0.04\leqq m_f\leqq 0.2$ for $\beta=4.0$

Statistics ~ 2000 trajectory

- Measurement of meson spectrum

  in particular pseudoscalar (“NG-pion”) mass ($M\pi$), decay constant ($F\pi$)

  vector meson mass ($M\rho$)

Machine: φ @ KMI, CX400 @ Kyushu Univ.

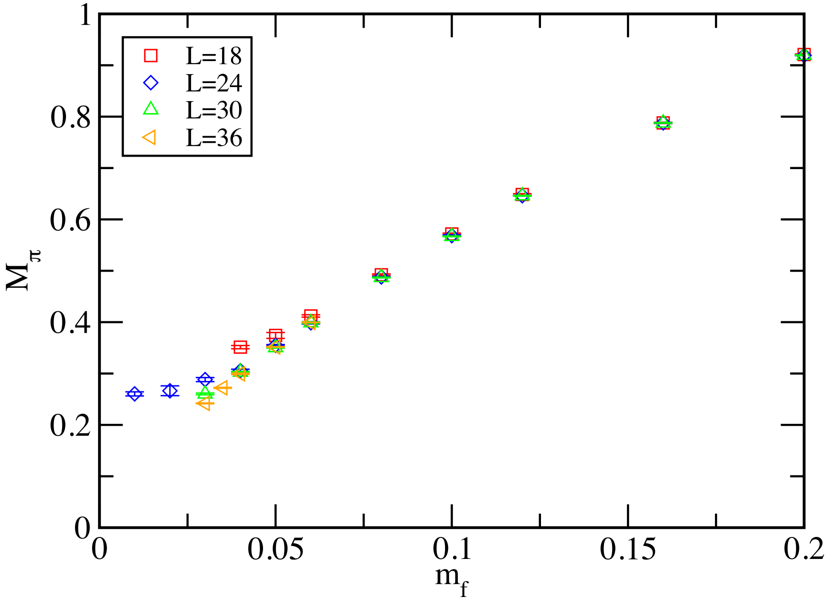
N_f=12 Result

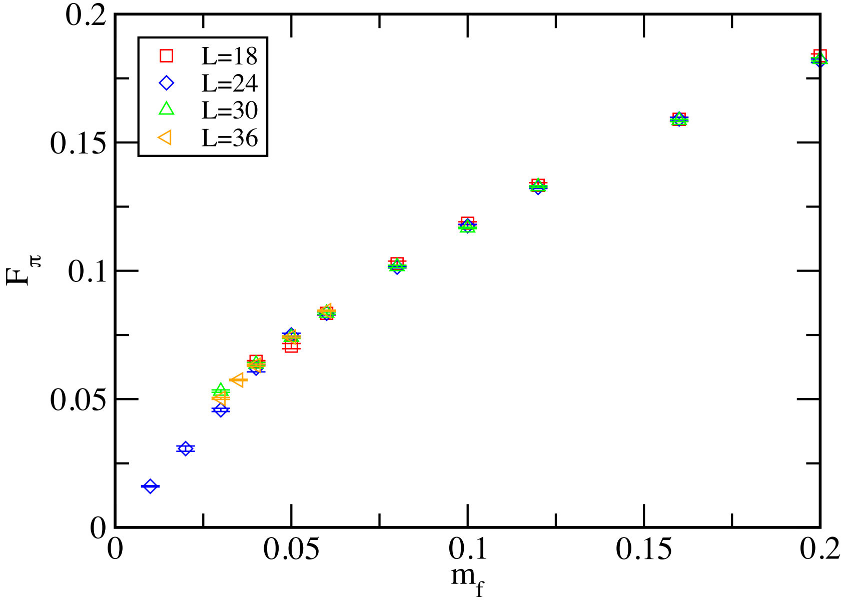
[LatKMI, PRD86 (2012) 054506]
and
Some updates

Preliminary

F_π and M_π

Nf=12





Nf=12 theory:
Conformal phase v.s. Chiral broken phase

From the fermion mass (mf) dependence of the hadron mass, we study the phase structure of the theory.

Conformal hypothesis: critical phenomena near the fixed point

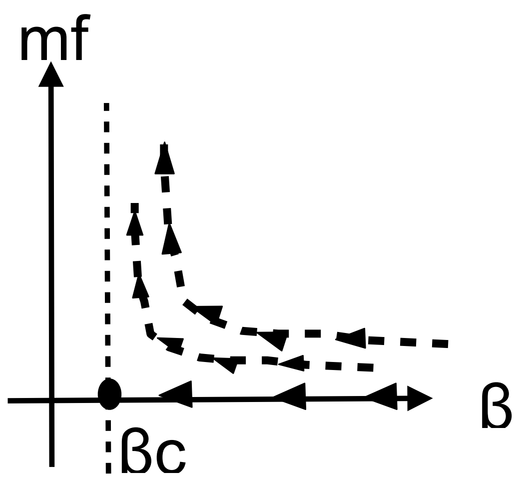
hyper-scaling, γ : mass anomalous dimension at the fixed point

- $M_H \propto mf^{1/(1+\gamma)}$
- $F_\pi \propto mf^{1/(1+\gamma)} + \ldots$ (for small mf)

$\Rightarrow$ $F_\pi/M_\pi \to$ constant (mf$\to$0)

$M_\rho/M_\pi \to$ constant

RG flow in mass-deformed conformal field theory(CFT)



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$\Rightarrow$ $F_\pi/M_\pi \rightarrow$ constant (mf$\rightarrow$0)

$M_\rho/M_\pi \rightarrow$ constant

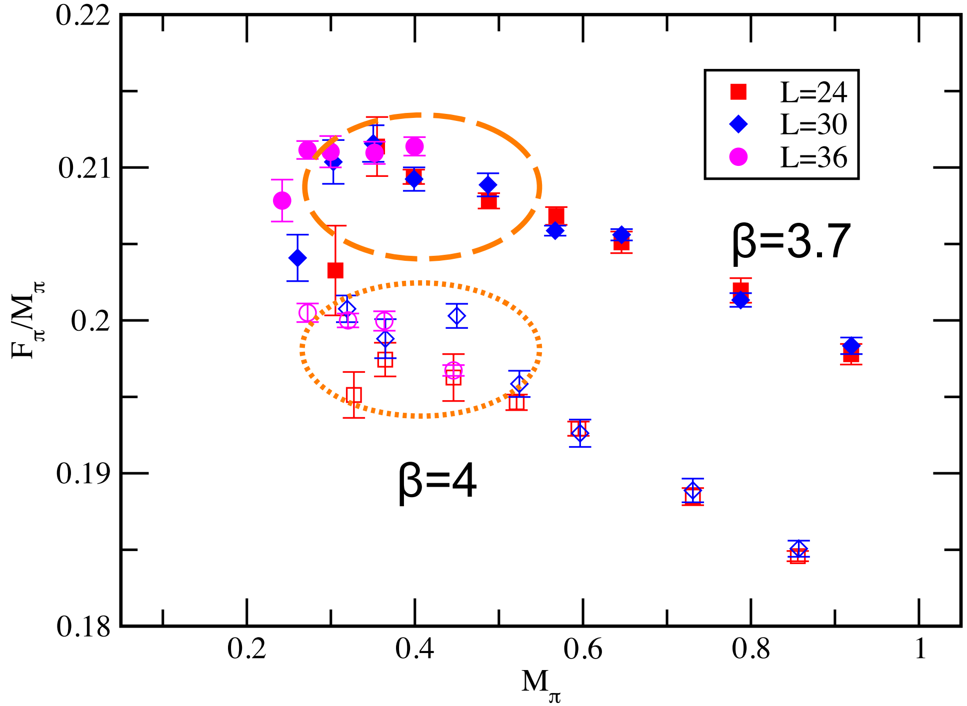
Chiral symmetry breaking hypothesis: π is NG-boson.

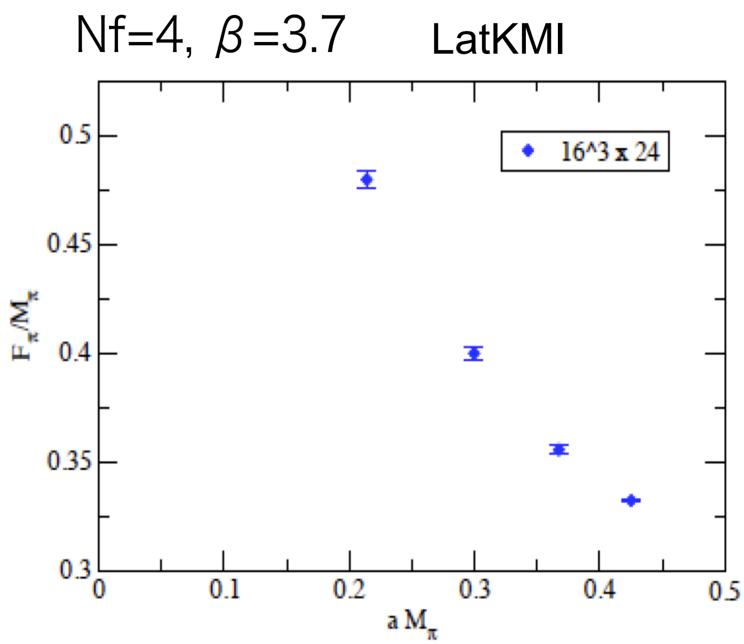
Chiral perturbation theory (ChPT) works.

- $M_\pi^2 \propto$ mf (PCAC relation)
- $F_\pi = F + c\, M_\pi^2 + \ldots$ (for small mf)

$\Rightarrow$ $F_\pi/M_\pi \rightarrow \infty$ (mf $\rightarrow$ 0)

A primary analysis, F_π/M_π vs M_π

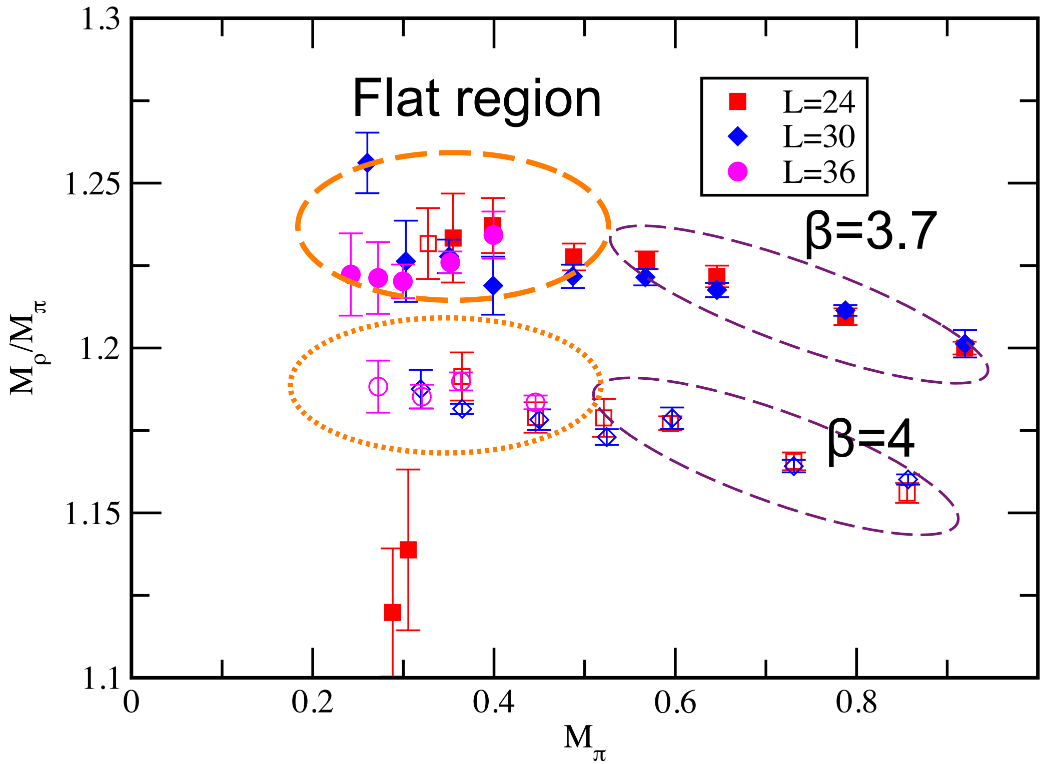




In both of β=3.7 and 4.0, both ratios at L=30 and L=36 seem to be flat in the small mass region, but small volume data (L≦24) shows large finite volume effect.
This behavior is contrast to the result in ordinary QCD system

Mρ/M_π vs M_π

Nf=12



Ratio is almost flat in small mass region (wider than Fπ/Mπ)
-> consistent with hyper scaling
Volume dependence is smaller than Fπ/Mπ.
In the large mass region, large mass effects show up.
Mρ/M_π should be 1, as mf -> infinity.

Conformal hypothesis in infinite volume & finite volume

- Universal behavior for all hadron masses (hyper-scaling)
- Mass dependence is determined by scaling dimension (mass-deformed CFT.)

$$M_H \propto m_f^{1/(1+\gamma)}, \quad F_\pi \propto m_f^{1/(1+\gamma)}$$ **(infinite volume result)**

Our interest : the same low-energy physics with the one obtained in infinite volume limit

But all the numerical simulations can be done only in finite size system (L).

we use **Finite size scaling hypothesis**

-> Finite size hyper-scaling for hadron mass in L^4 theory
[DeGrand et al. ; Del debbio et. al., '09]

Note: In order to avoid dominant finite volume effect and
to connect with infinite volume limit result,
we focus on the region of L >> ξ (correlation length), (LMπ >>1).

Finite size hyper-scaling

- Universal behavior for all hadron masses
- From RG argument the scaling variable x is determined as a combination of mass and size

$$x = Lm^{1/(1+\gamma_*)}$$

- The universal description for hadron masses are given by the following forms as,

$$L \cdot M_H = f_H(x) \qquad L \cdot F_H = f_F(x)$$

Ref [DeGrand et al. ; Del debbio et. al., '09]

c.f. Finite Size Scaling (FSS) of 2nd order phase transition

$$\xi_L(T) = L f_\xi \left(\frac{L}{\xi_\infty} \right). \qquad \xi_\infty \propto \left| \frac{T_c - T}{T_c} \right|^{-\nu}$$

Test of Finite size hyper-scaling

$$L \cdot M_H = f_H(x) \qquad L \cdot F_H = f_F(x)$$

We test the finite hyper-scaling for our data at L=18, 24, 30, 36.

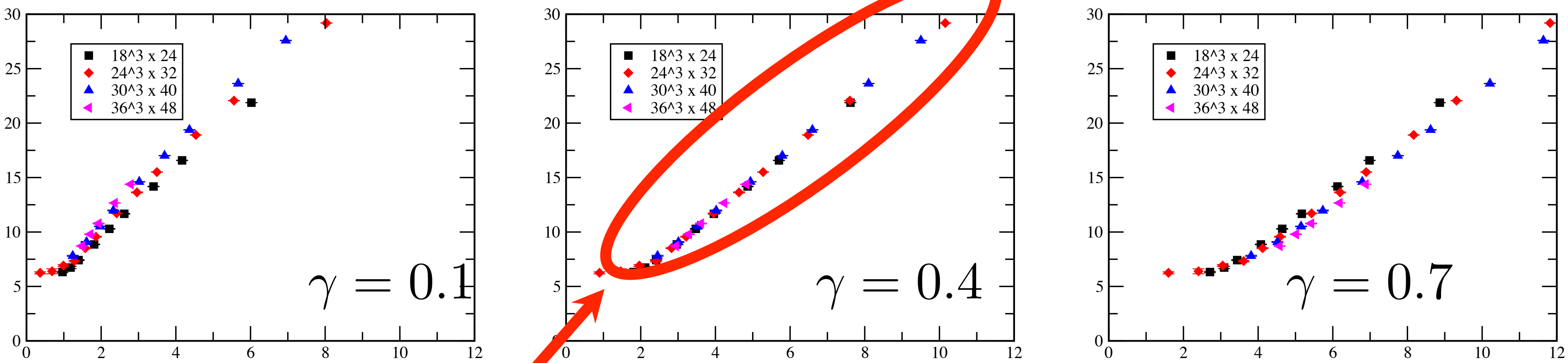
The scaling function f(x) is unknown in general,

But if the theory is inside the conformal window,

the data should be described by one scaling parameter x.

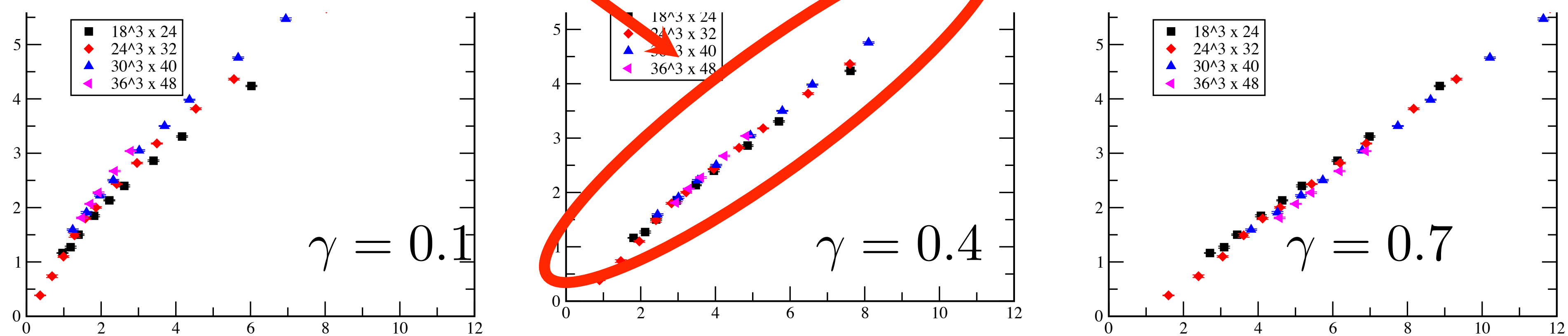
Data alignment at a certain γ

$$\xi = LM_\pi$$



$$LF_\pi$$

good alignment! **How to quantify this situation?**



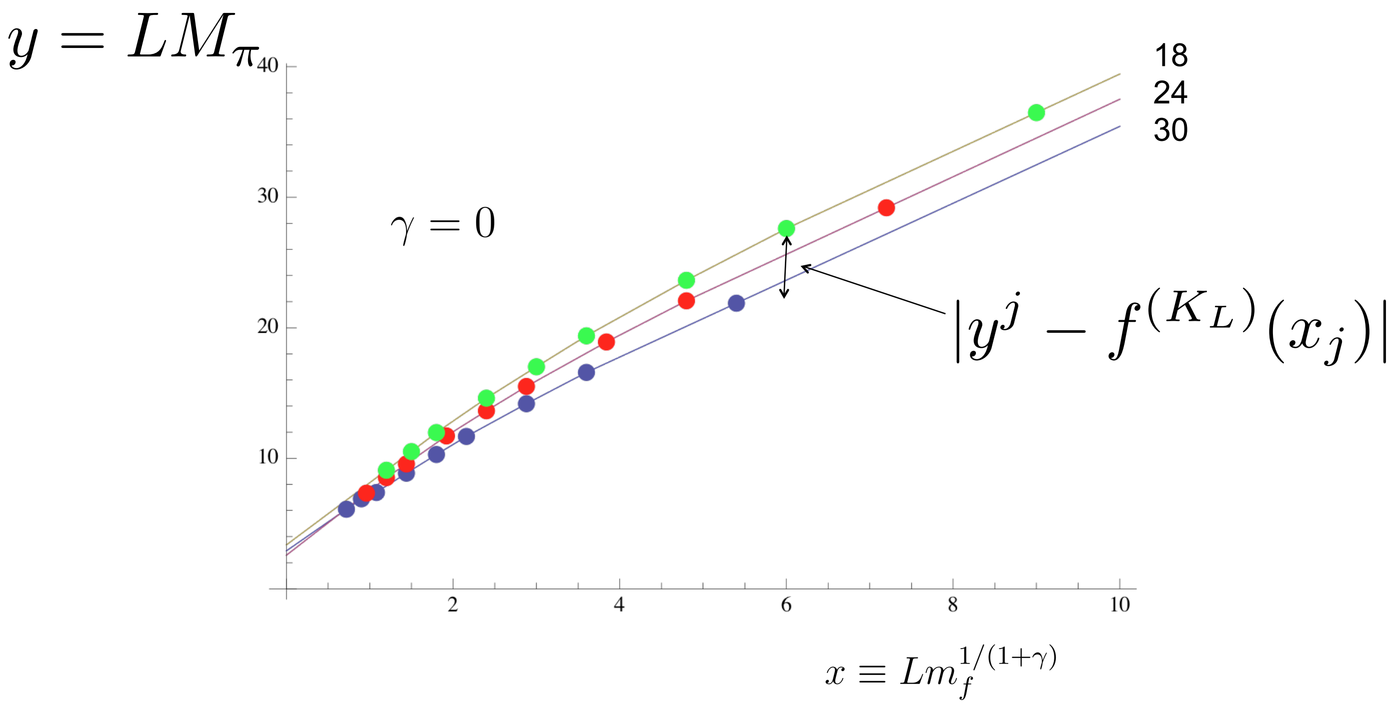
$$x = L \cdot m^{1/(1+\gamma)}$$

■To quantify the alignment and obtain the optimal γ

We define a function P(γ) to quantify how much the data “align” as a function of x.

$$\boxed{P(\gamma) = \frac{1}{\mathcal{N}} \sum_{L} \sum_{j \notin K_L} \frac{|y^j - f(K_L)(x_j)|^2}{|\delta y^j|^2}}$$

[LatKMI, PRD86 (2012) 054506]



■To quantify the alignment and obtain the optimal γ

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[LatKMI, PRD86 (2012) 054506]

Optimal value of γ for alignment will minimize P(γ).

our analysis: three observables of $y_p=LM_p$ for $p=\pi, \rho$; $y_F=LF_\pi$.

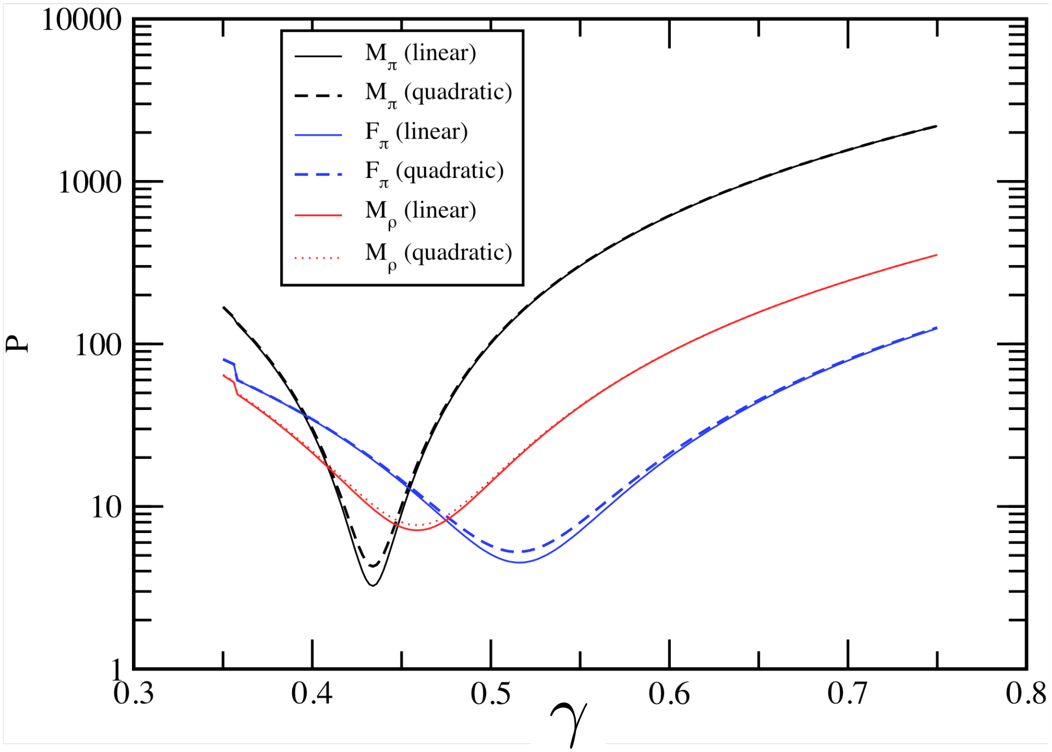
A scaling function f(x) is unknown,

→ f(xj) is obtained by interpolation (spline) with linear ansatz (quadratic for a systematic error).

If ξ^j is away from $f(x_i)$ by $\delta\, \xi^j$ as average → P=1.

P(γ) analysis

- P(γ) has minimum at a certain value of γ, from which we evaluate the optimal value of γ.
- At minimum, P(γ) is close to 1.



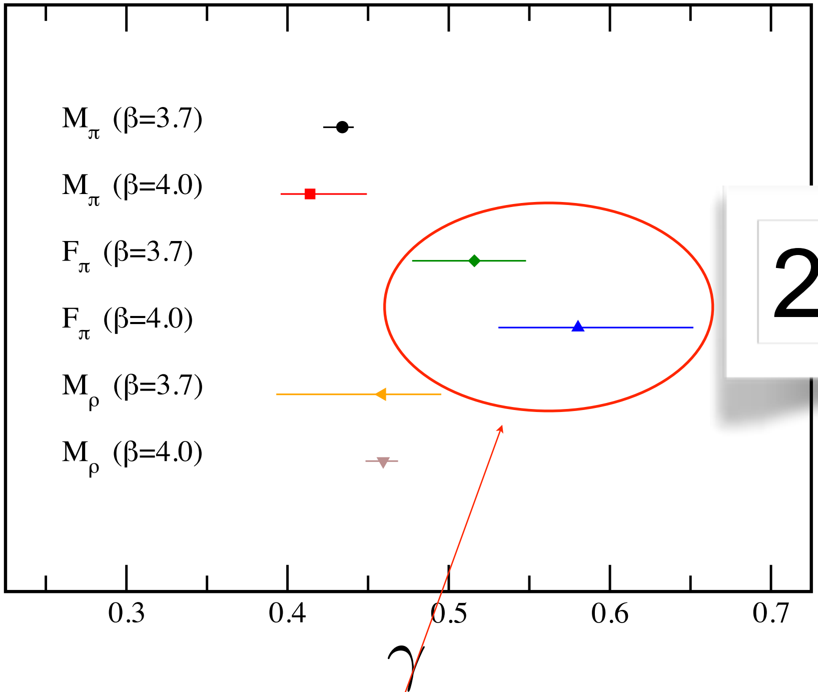
Results for data for L=18, 24, 30 at β=3.7

L > ξ is satisfied in our analysis.

$$(LM_\pi > 8.5 \text{ for our simulation parameter region})$$

■Result of gamma (data L=18,24,30)

[LatKMI, PRD86 (2012) 054506]

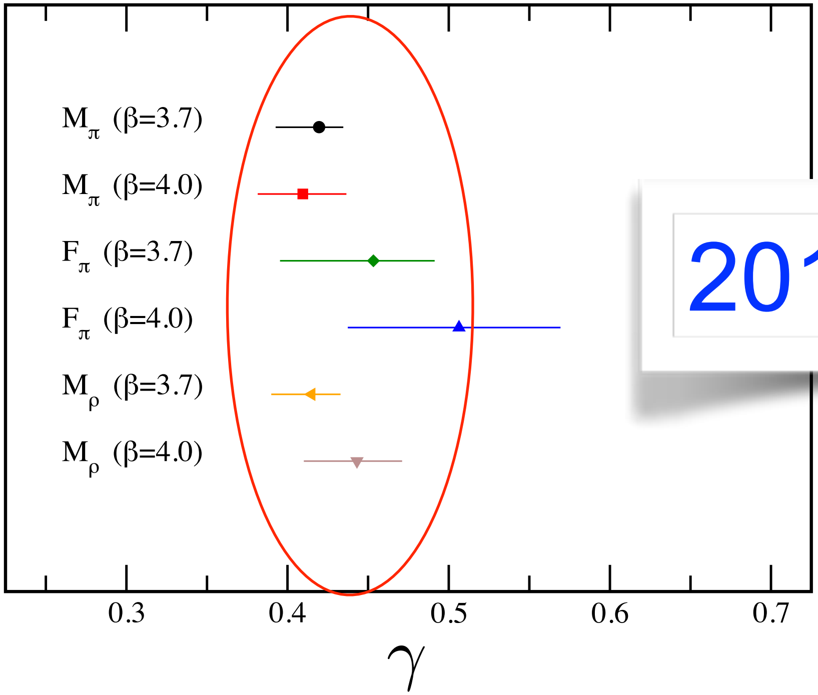


2012 Result

- The error -> both statistical & systematic errors
  <- estimation by changing x range of the analysis

•Remember: <u>Fπ data seems to be out of scaling region due to finite mass & volume corrections. Flat range is smaller than Mρ/Mπ.</u>

■Result of gamma (data L=24,30,36 with lighter mass region)



2013-14 Update

- γ(Mπ) is stable against the change of the mass (x) and β .
- smaller mass with larger volume (18,24,30 ->24,30,36)

→closer value to γ(Mπ)

The universal scaling is obtained for both values of β =3.7 & 4.0
γ=0.4-0.5.

Further corrections
to
the hyperscaling

▪Possible corrections to the finite size hyper scaling

We consider simultaneous fit for the three quantities of

$$\xi = LM_\pi,\ LF_\pi,\ LM_\rho$$

with finite mass (volume) correction.

We consider following possibilities by adding different mass dependence as

$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)} \cdots \text{(no correction)}$$

$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)} + c_2 L m_f^{\alpha}$$

$$\xi = (c_0 + c_1 L m_f^{1/(1+\gamma)})(1 + c_2 m^{\omega})$$

ɑ. ω … unknown exponent

e.g.

1. ladder Schwinger-Dyson eq. analysis: $\alpha = (3-2\gamma)/(1+\gamma)$
2. lattice (am)^2 artifact : $\alpha = 2$

[LatKMI PRD85(2012)074502]

3. exponent of the gauge coupling $\omega = -y_0/(1+\gamma)$

[c.f. A. Cheng, et al. '14] y0= -0.36 (2-loop perturbation theory)

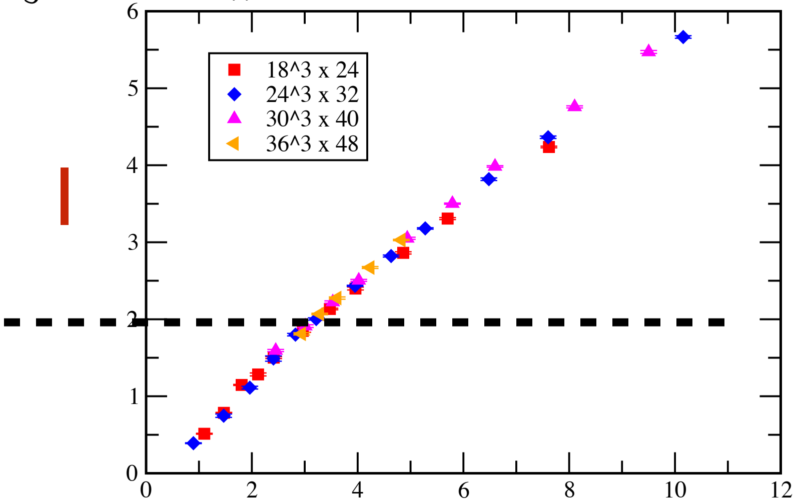
We demonstrate simultaneous fit for three observables of Mπ, Fπ, Mρ using following functions.

$$LM_H = (1 + c_H m^{\omega}) f_H(x)$$
$$f_H(x) = a_H + b_H x$$
$$x = L m_f^{1/(1+\gamma)}$$
$$\gamma, \omega \cdots \text{universal}$$

10 fit parameters
(ω is fixed to some specific value)

$$\gamma, a_{M_\pi}, a_{F_\pi}, a_{M_\rho},$$
$$b_{M_\pi}, b_{F_\pi}, b_{M_\rho},$$
$$c_{M_\pi}, c_{F_\pi}, c_{M_\rho}$$

$$\xi = LF_\pi$$

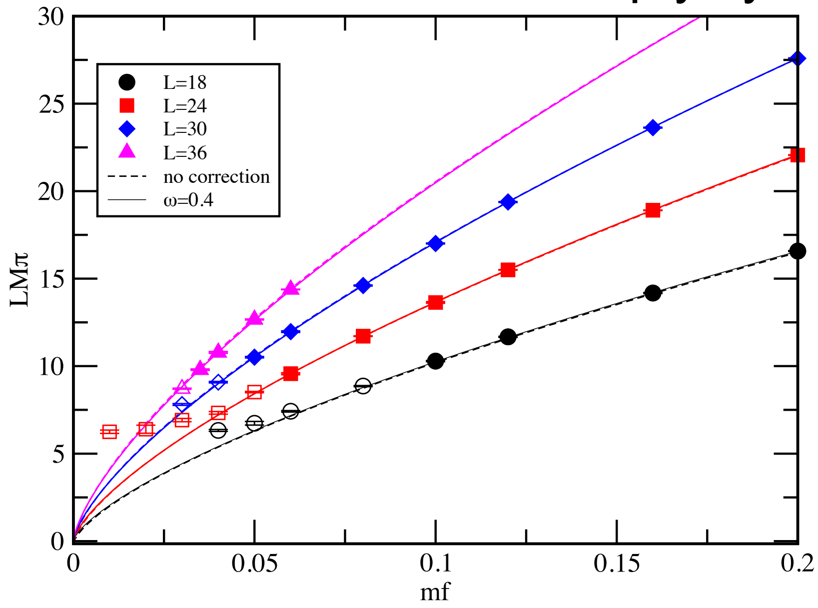


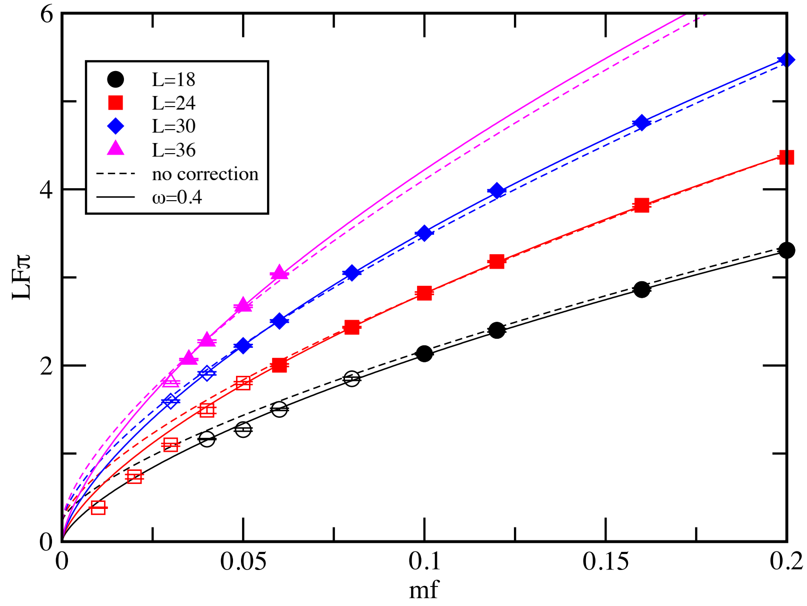
We consider following fit region I … LFπ > 2
(LMπ >8)

Result for "region I"

preliminary

- The data with empty symbols are not used in the fit





Fit result with L=18, 24, 30, 36

ω [fixed]	γ	χ^2/dof
0 (no correction)	0.457(1)	15
0.4	0.398(5)	2.6
0.8	0.425(2)	2.0

L=24, 30, 36

ω [fixed]	γ	χ^2/dof
0 (no correction)	0.459(2)	12
0.4	0.406(5)	2.4
0.8	0.430(4)	2.0

In various trials of this analysis: γ=0.2—0.45

Short summary in Nf=12

- β=3.7-4.0: Mπ, Fπ, Mρ show conformal hyper scaling
- Fπ : large mass corrections in our whole mass parameters, likely too heavy mf to be neglect.

  → Approaching small mass region, we obtain hyper-scaling behavior.
- The hyper-scaling is realized in larger volume region together with smaller mass region.

- We consider possible corrections to the finite size hyper scaling, to understand both the outsides of the scaling region.

→ The large fermion mass region can be described by such a correction.

The value of γ could be smaller as γ ~ 0.2-0.45.

•ChPT expansion is not valid, expansion parameter is much larger than 1.
(Not yet exclude chiral broken scenario (very small Fπ))

β dependence (UV cutoff) effect

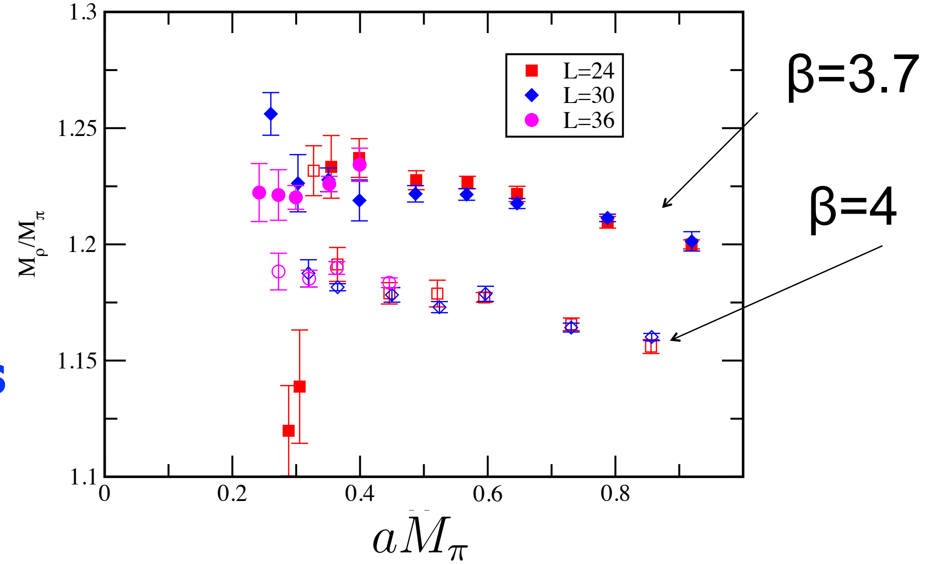
- β dependence is important to study the lattice phase structure (existence of bulk transition, asymptotic free or non-free)

  and to obtain the continuum limit physics

- In the conformal phase, we demonstrate some scaling matching analyses,

1. Matching of the dimension-less ratio
2. Matching of hyper scaling curves for L M

■scale (β) dependence



Why is there difference in the ratio between β=3.7 and 4.0?

Note: This ratio is dimension-less quantity.

To study more about β dependence,
we use the hyper scaling relation in infinite volume limit for simplicity.

continuum theory

$$M_\pi = c_\pi m_f^{1/(1+\gamma)} + \cdots$$

$$M_\rho = c_\rho m_f^{1/(1+\gamma)} + \cdots$$

$$M_\rho/M_\pi \to \frac{c_\rho}{c_\pi} + \cdots$$

a possible discretization (cutoff) effect

$$c_\pi = c_\pi + a^2 \tilde{c}_\pi$$

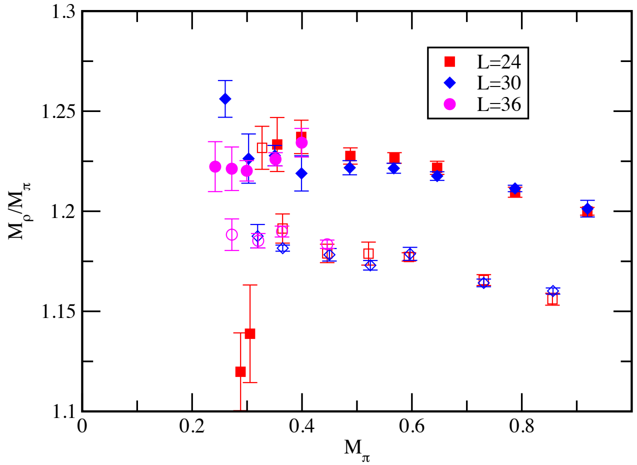
$$c_\rho = c_\rho + a^2 \tilde{c}_\rho$$

$$M_\rho/M_\pi \to \frac{c_\rho}{c_\pi}\left[1 + a^2\left(\frac{\tilde{c}_\rho}{c_\rho} - \frac{\tilde{c}_\pi}{c_\pi}\right)\right] + \cdots$$

The discretization error appears in the overall factor.
This can make the difference of the ratio.

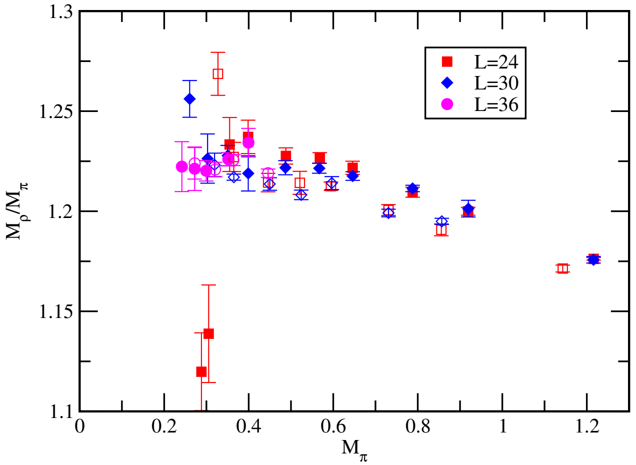
■How to match the scale

1. Matching the factor of the ratio (which come from the disc. effects) by introducing a factor R to multiply Mρ/Mπ for β=4.



$$\frac{M_\rho}{M_\pi} \to R\frac{M_\rho}{M_\pi}$$

for β=4

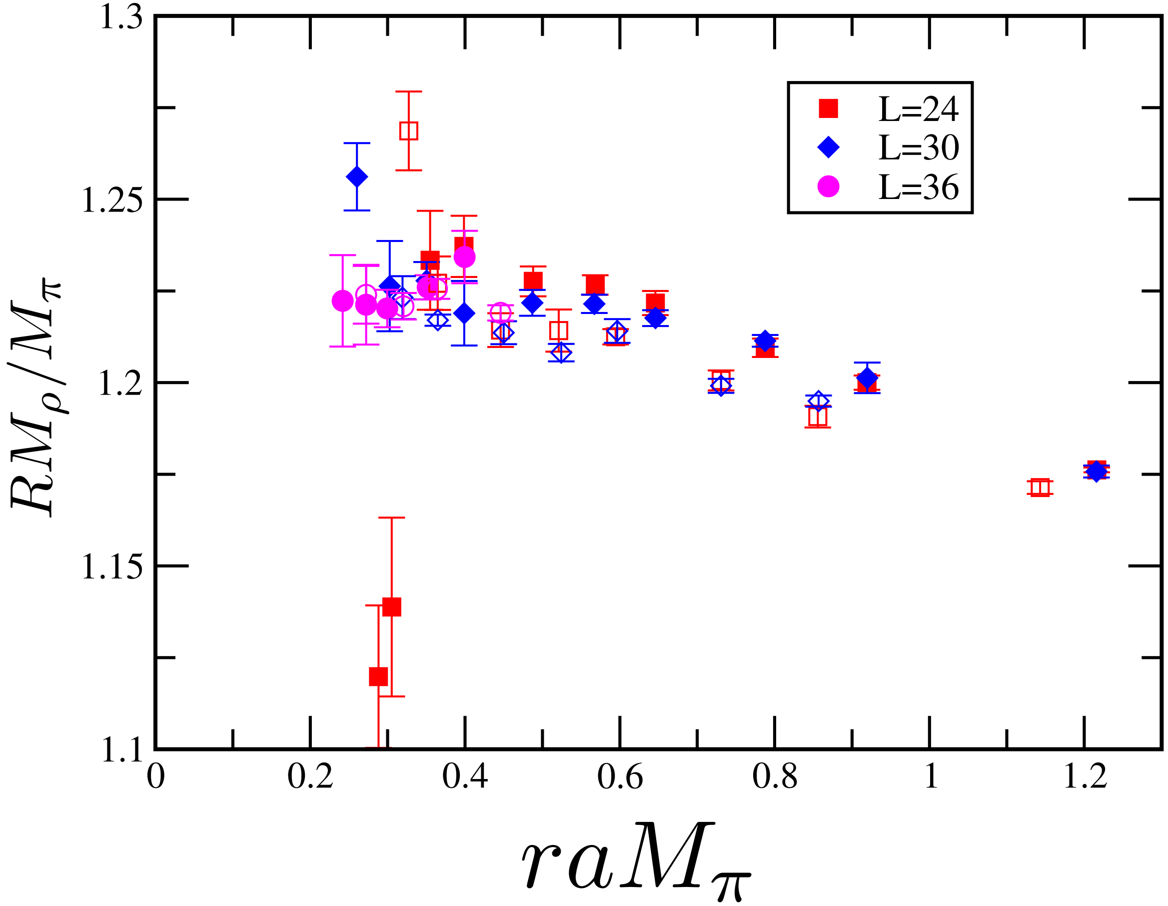


2. Further tuning for the remaining difference which may appear at the tail by introducing the horizontal factor r as r Mπ.

$$aM_\pi \longrightarrow raM_\pi$$ for β=4

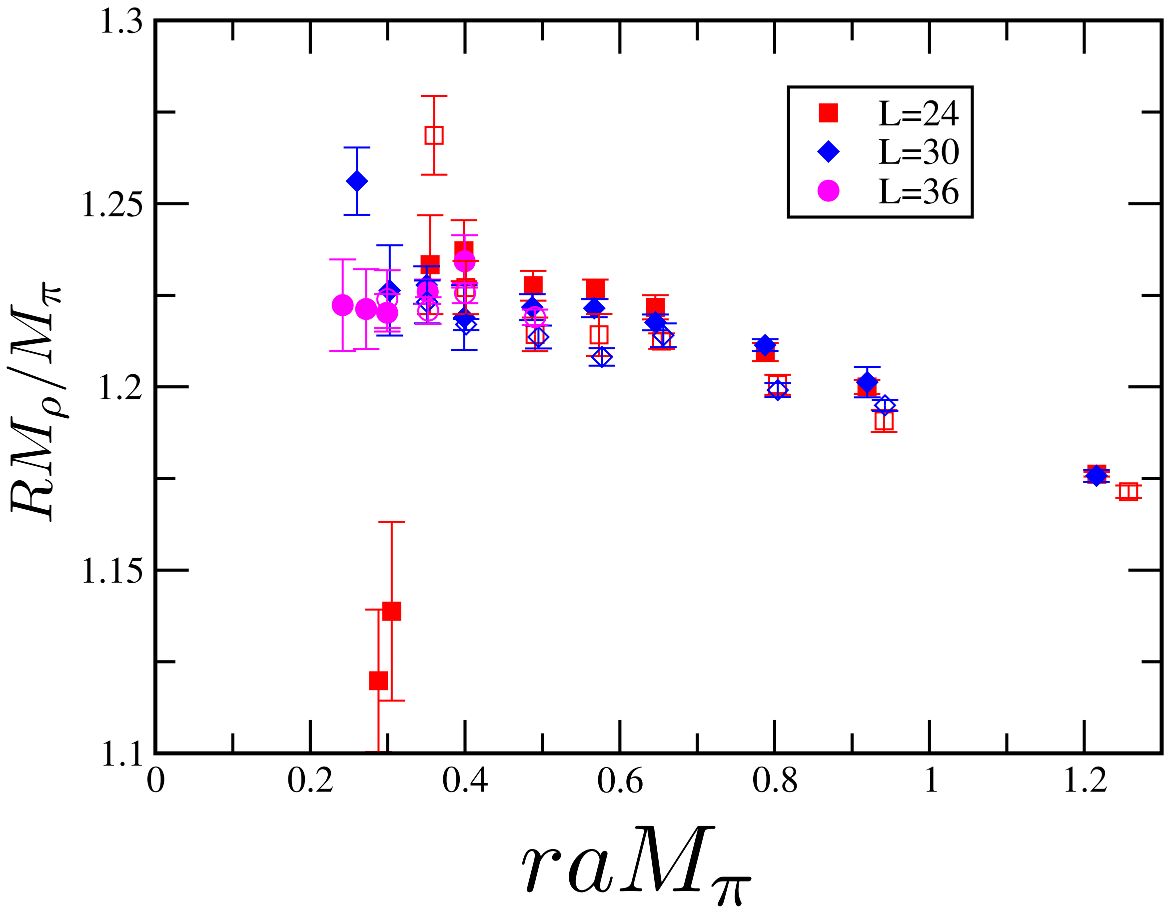
■How to match the scale

R=1.03, r=1.00



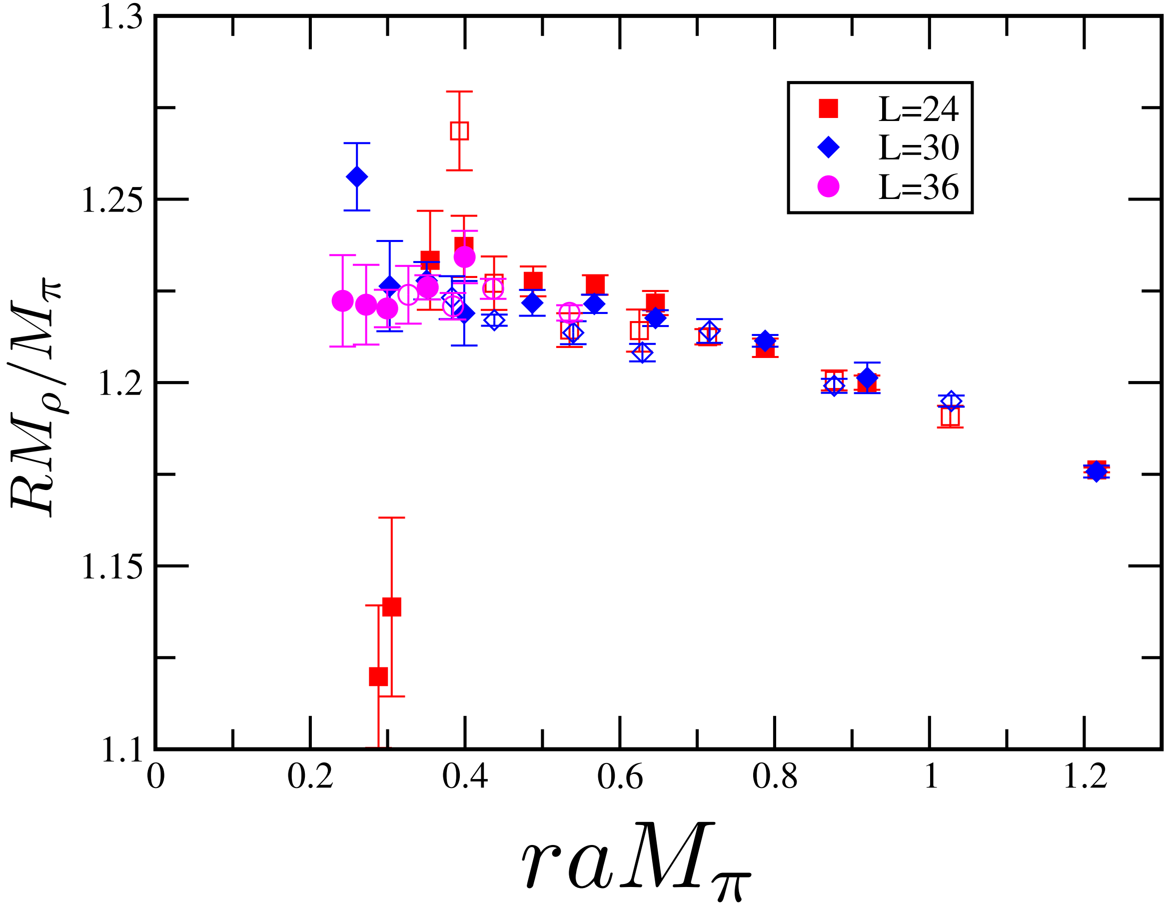
■How to match the scale

R=1.03, r=1.10



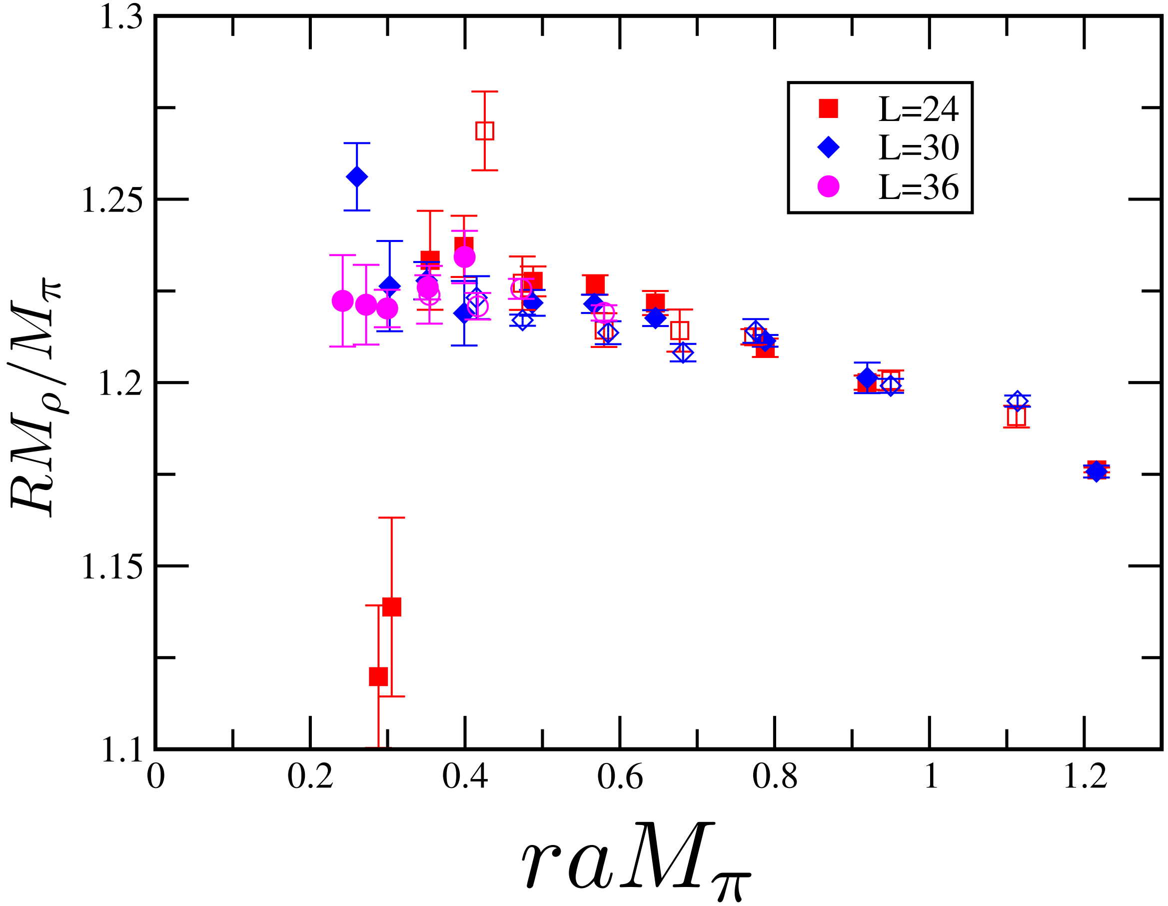
■How to match the scale

R=1.03, r=1.20



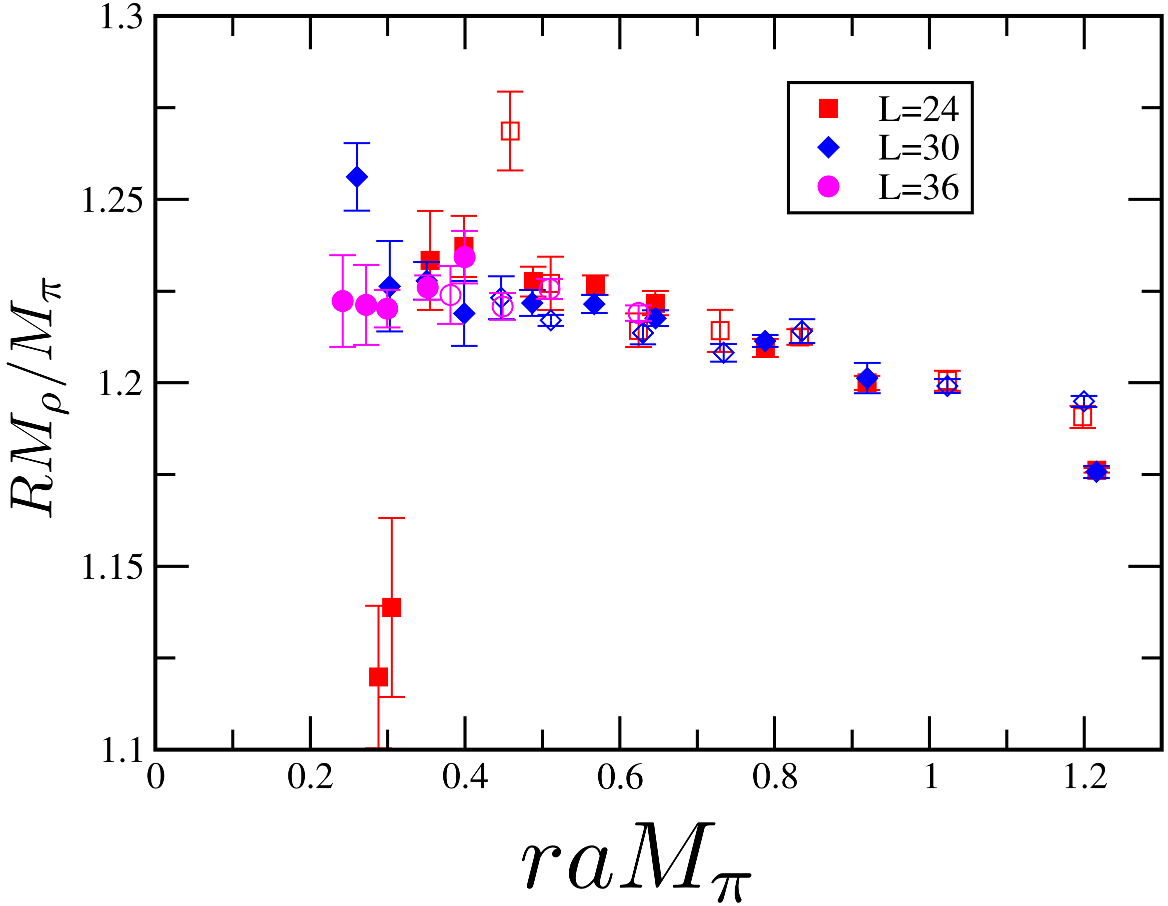
■How to match the scale

R=1.03, r=1.30



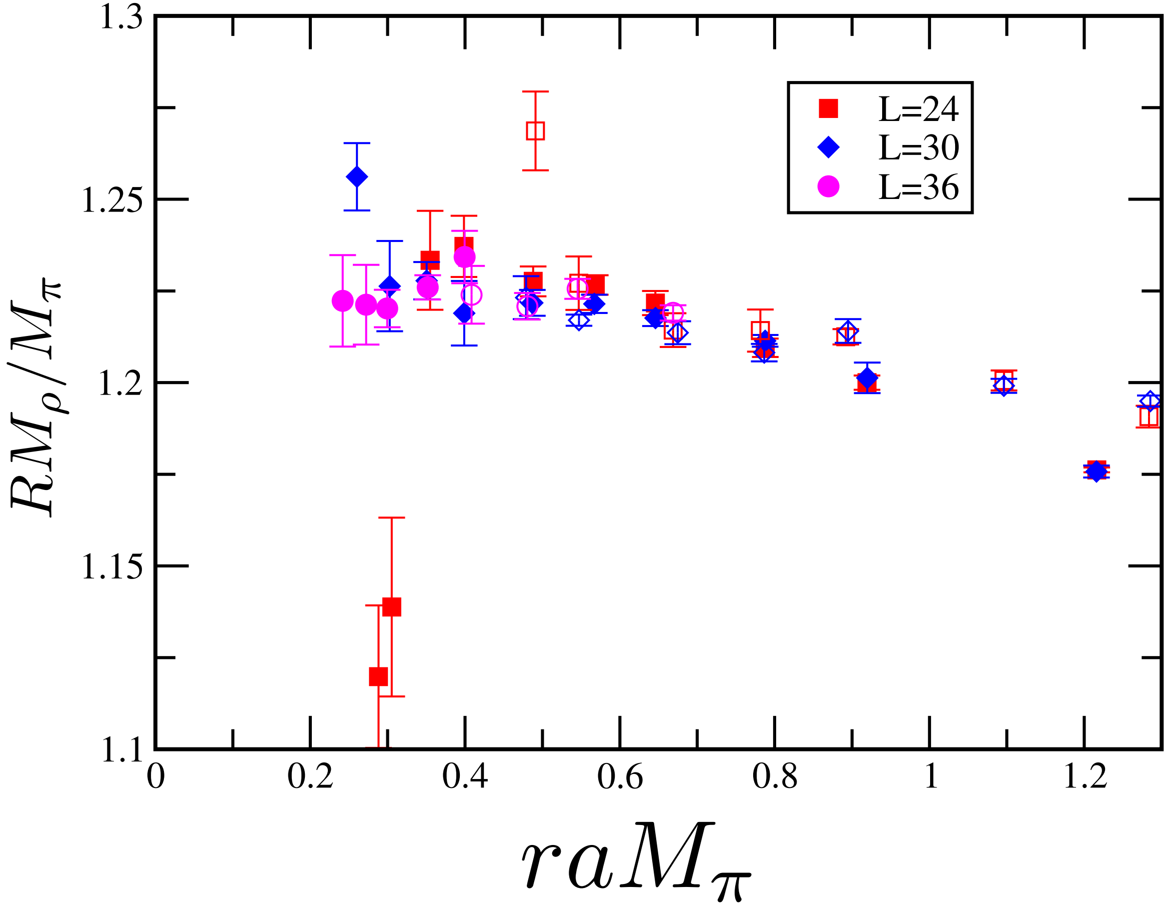
■How to match the scale

R=1.03, r=1.40



■How to match the scale

R=1.03, r=1.50



■The scale matching

The value of r~ 1.2- 1.3 shows a consistency between β=3.7 and 4.0 for a quantity of the ratio Mρ/Mπ

$$a_1(\beta = 3.7) \qquad a_2(\beta = 4.0)$$

$$\Rightarrow \frac{a_1}{a_2} = r \sim 1.2 - 1.3$$

This result is consistent with being in the asymptotically free region for our β's.

■Comparison between different beta's using $\xi = LM_\pi$

We assume that the two scales $a_1(\beta = 3.7)$ and $a_2(\beta = 4.0)$

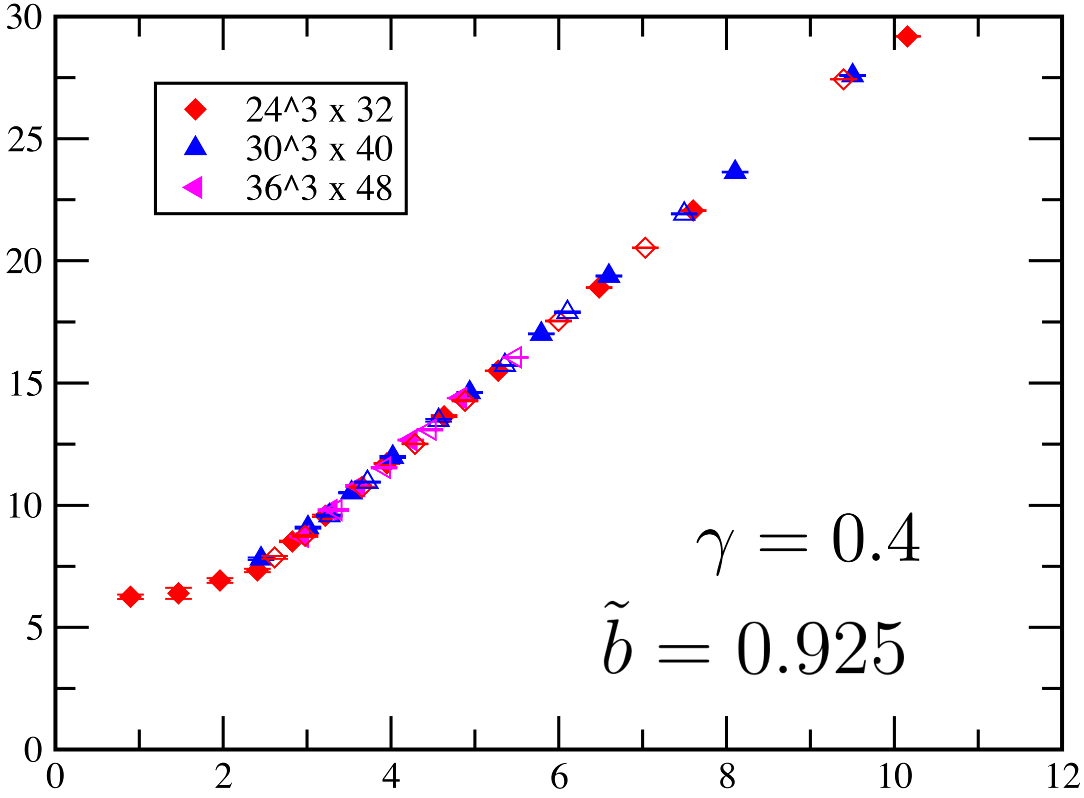
have the following relation $a_1 = ba_2$ where b is a factor.

$$\Rightarrow \quad \xi = f(x_1) = f(\tilde{b}x_2),$$

where $x_i = L/a_i(a_i m_f)^{1/(1+\gamma)}$ for $i = 1, 2$ and $\tilde{b} = b^{-\gamma/(1+\gamma)}$

■Comparison between different beta

$$\xi = LM_\pi$$



$$x = x_1 = \tilde{b} x_2$$

Fit results for combined data of beta=3.7 and 4.0 $\xi = LM_\pi$

$$\tilde{b} \sim 0.92,$$

$$\gamma \sim 0.43.$$

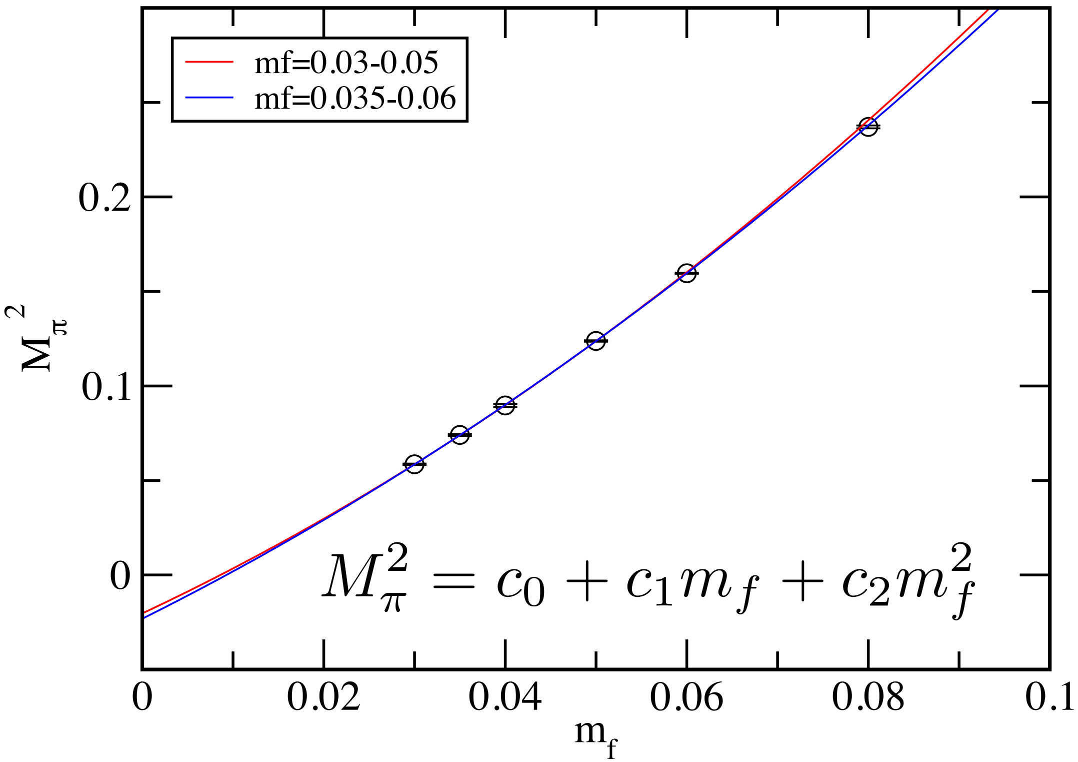
$$b = \tilde{b}^{-(1+\gamma)/\gamma} \sim 1.3$$

This results suggest that the data for both beta are consistent with the finite volume scaling and asymptotically free.

ChPT analysis

■Fit result on π mass (β=3.7 to see near the chiral limit)

We analyze the largest volume data only.



The fit results

fit range	c0	χ2/dof	dof
[0.03-0.05]	-0.02(1)	0.16	1
	[0]	2.4	2
[0.035-0.06]	-0.023(7)	0.16	1
	[0]	5.6	2

$$LM_\pi = 8.71\ (m_f = 0.030)$$

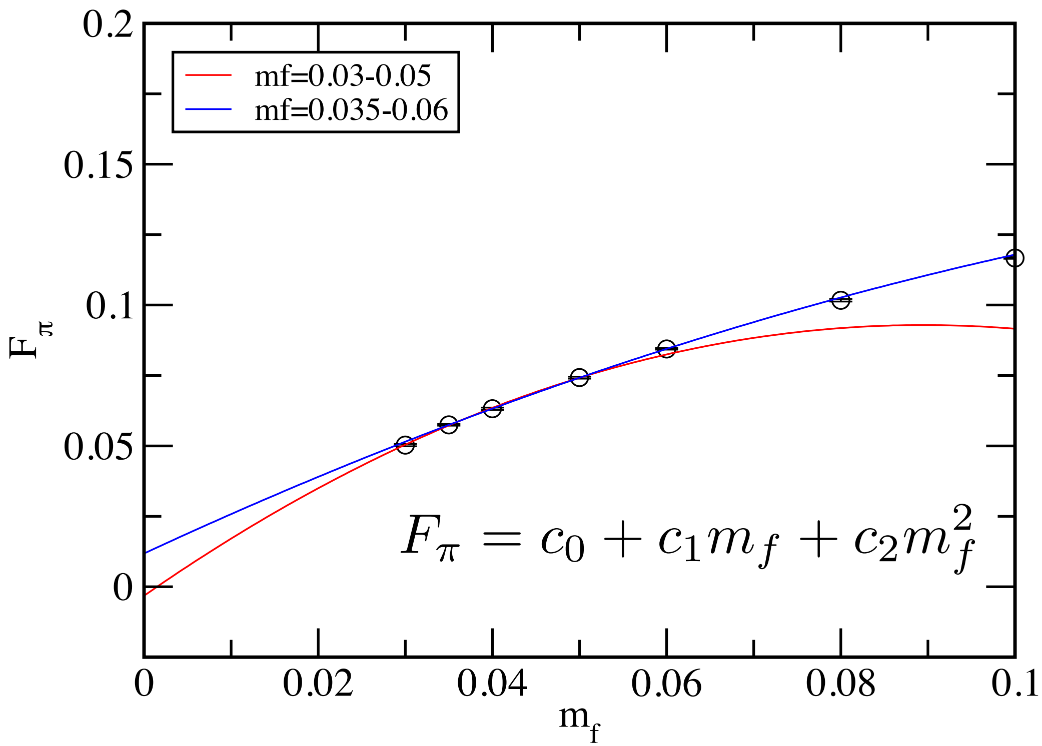
$$LM_\pi = 9.79\ (m_f = 0.035)$$

Fit results for Mπ

- **Polynomial fit is reasonable for small fermion mass range.**
  For the smallest mass range, Mπ goes to zero or negative.

preliminary

■Fit result on Fπ (β=3.7)



The fit results

fit range	c0	χ2/dof	dof
[0.03-0.05]	-0.003(7)	1.1	1
[0.035-0.06]	0.012(5)	0.01	1

Fit results for Fπ

- **Polynomial fit is reasonable for small fermion mass range.**
  For the smallest mass range, Mπ goes to zero or negative.
  Fπ in the chiral limit is tiny non-zero or consistent with zero.

preliminary

Note on ChPT fit in many flavor QCD

- **Natural chiral expansion parameter is** $$\boxed{\chi = N_f \left(\frac{M_\pi}{4\pi F} \right)^2}$$

[M. Soldate and R. Sundrum, Nucl.Phys.B340,1 (1990)],
[R. S. Chivukula, M. J. Dugan and M. Golden, Phys. Rev. D47,2930 (1993)]

The parameter χ should be less than 1 to be consistent with ChPT expansion.
χ ~ 3.5 at the lightest mass point and χ >30 using F in the chiral limit.

->It is difficult to tell real chiral behavior. e.g. Fπ in the chiral limit, if it exists.

Summary

- **Large Nf SU(3) gauge theory is being investigated in LatKMI project.**
- **We focus on the Nf=12 case.**

[LatKMI, PRD 2012 and some update].

- **Finite size hyper scaling is observed for the π (“NG-boson”) mass, decay constant and rho meson mass.**
- **Nf=12 is consistent with conformal gauge theory.**
- **The resulting universal γ ~0.4-0.5 (without correction), 0.2-0.4(with correction), (not favored as Walking Technicolor)**
- ChPT expansion is not valid, expansion parameter is much larger than 1. (Not yet exclude chiral broken scenario (very small Fπ))

How about other # of fermions??
-> e.g. 8 flavor case, talk by K.-i. Nagai (next)

END
Thank you