Walking signals in Nf=8 QCD on the lattice

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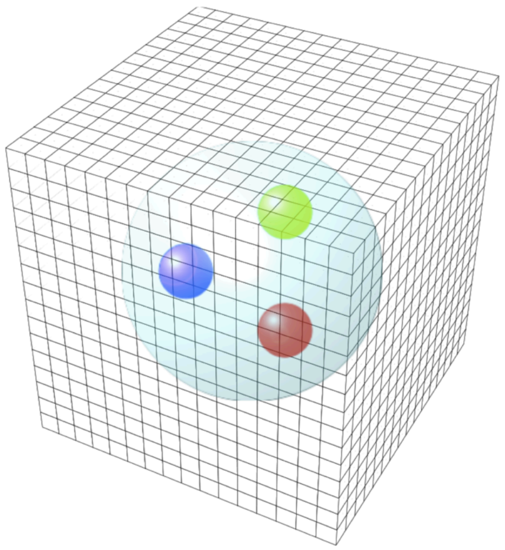
Plan of the Talk:



1. Introduction
2. Lattice study of Nf=8 QCD
   - Chiral Perturbation Theory (ChPT)
   - Finite Size Hyperscaling (FSHS)
3. Summary

♠ Nf=8 is a candidate of the walking behavior.

Walking signals in Nf=8 QCD on the lattice
Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kei-ichi Nagai, Hiroshi Ohki, Akihiro Shibata, Koichi Yamawaki, Takeshi Yamazaki,
Published in **Phys.Rev. D87 (2013) 094511**, e-Print: **arXiv:1302.6859 [hep-lat]**.

+ update

1. Introduction

- LQCD with many fermions
- BSM (quark mass, FCNC, ..., model building)
- ⇒ Candidate of the walking technicolor (WTC)

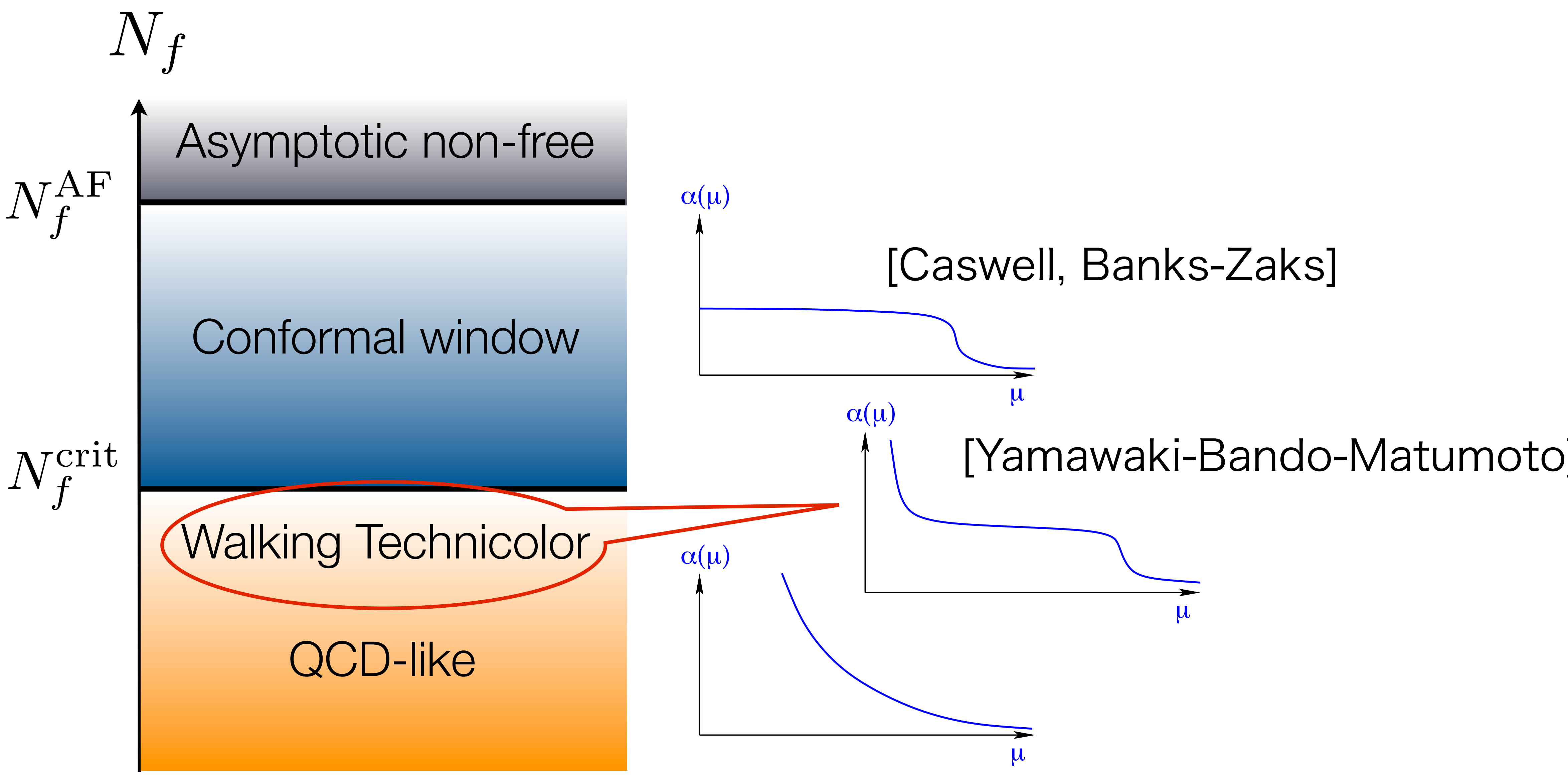
Requirements for the successful WTC theory

- spontaneous chiral symmetry breaking
- running coupling "walks" = slowly changing with μ: → nearly conformal
- large mass anomalous dimension: $\gamma_m \sim 1$
- light scalar 0^{++} (m_H = 126 GeV @ LHC !)
  - with input $F_\pi = 246/\sqrt{N}$ GeV (N: # weak doublet in techni-sector)
    - to reproduce $W^\pm$ mass
  - typical QCD like theory: $M_{Had} >> F_\pi$ (ex.: QCD: $m_\rho/f_\pi \sim 8$)
    - Naive TC: $M_{Had} > 1{,}000$ GeV
    - 0^{++} is a special case: pseudo Nambu-Goldstone boson of scale inv.

    ➡ is it really so ?

conformal window and walking coupling

- non-Abelian gauge theory with N_f massless fermions -



- Walking Technicolor could be realized just below the conformal window
- crucial information: $N_f^{\rm crit}$ & mass anomalous dimension around $N_f^{\rm crit}$

Many flavor QCD

⇒ Candidate of walking/conformal

Our investigation in Nf=12 (Ohki's talk)

→ consistent with the conformal with γ=0.4--0.5 .

not favored as WTC (model building)

Thus, we investigate Nf=8 QCD.

strong coupling dynamics and non-perturbative

Lattice simulation of Nf=8 QCD

One-family
Extended
TechniColor
model

$$
\begin{pmatrix} u \\ d \\ u \\ d \\ u \\ d \\ e \\ \nu_e \end{pmatrix}
\begin{pmatrix} c \\ s \\ c \\ s \\ c \\ s \\ \mu \\ \nu_\mu \end{pmatrix}
\begin{pmatrix} t \\ b \\ t \\ b \\ t \\ b \\ \tau \\ \nu_\tau \end{pmatrix}
\begin{pmatrix} U_1 \\ D_1 \\ U_1 \\ D_1 \\ U_1 \\ D_1 \\ E_1 \\ N_1 \end{pmatrix}
\cdots
\begin{pmatrix} U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ E_{N_{\rm TC}} \\ N_{N_{\rm TC}} \end{pmatrix}
$$

$$
SU(3+N_{\rm TC}) \xrightarrow{\Lambda_1} SU(2+N_{\rm TC}) \xrightarrow{\Lambda_2} SU(1+N_{\rm TC}) \xrightarrow{\Lambda_3} SU(N_{\rm TC})
$$

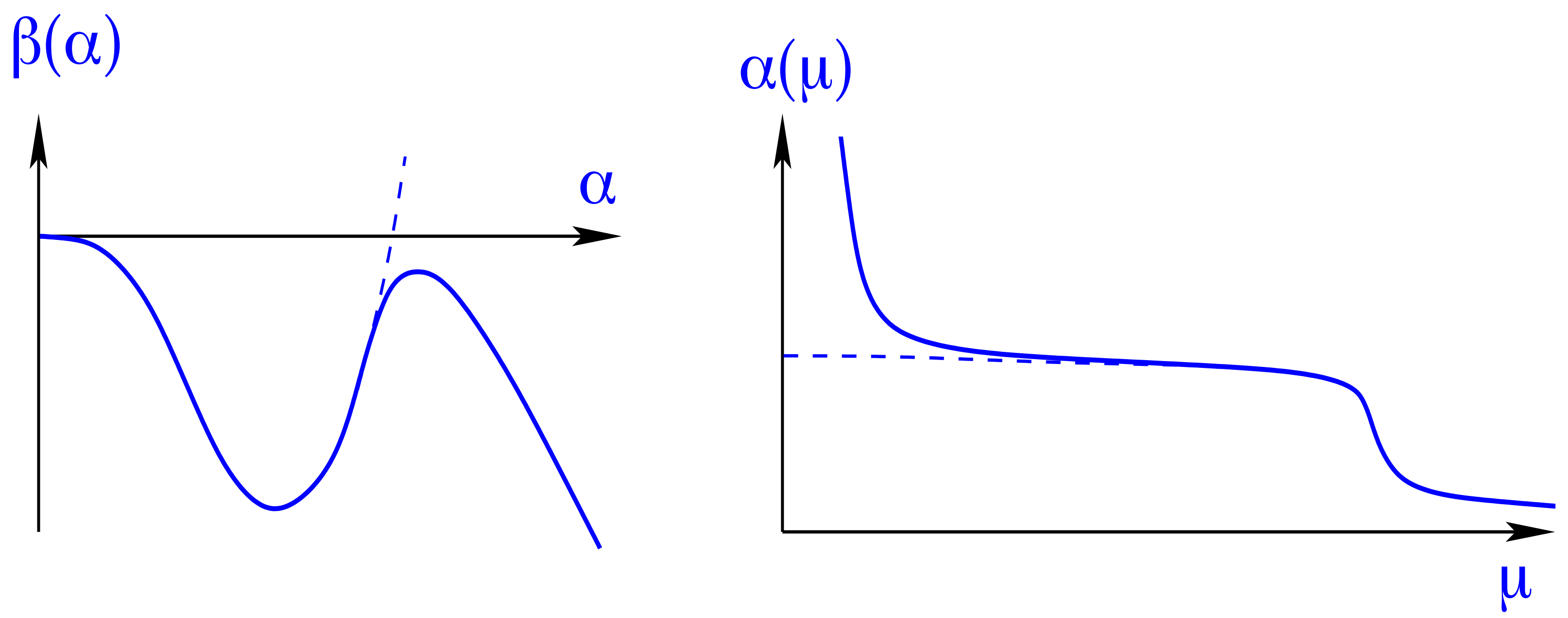
8-flavor $\mathrm{SU}(N_{\rm TC})$ technicolor

Why Nf=8 QCD?

We consider

$N_{\rm TC} = 3$

What is the signal of walking?



In Gauge Theories:

$\Rightarrow$ $\boxed{\text{Spectrum?}}$ Heuristically

SχSB and/or conformal ?

What is the signal of walking?

Scenario of Walking Dynamics ;

Schematic picture of the gauge coupling

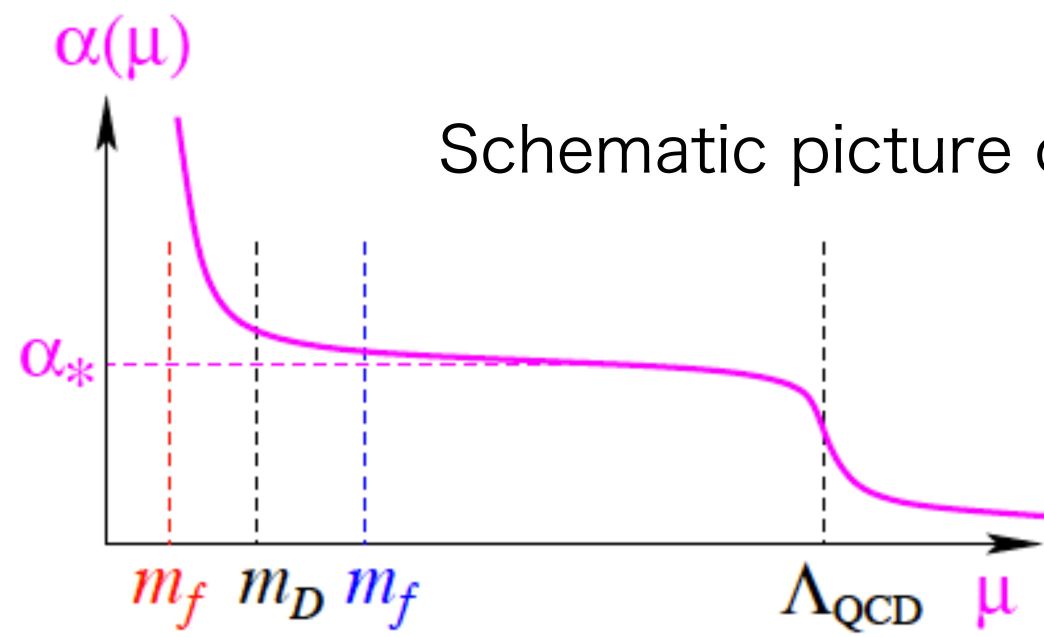


FIG. 1. Schematic two-loop/ladder picture of the gauge coupling of the massless large N_f QCD as a walking gauge theory in the SχSB phase near the conformal window. m_D is the dynamical mass of the fermion generated by the SχSB. The effects of the bare mass of the fermion m_f would be qualitatively different depending on the cases: Case 1: $m_f \ll m_D$ (red dotted line) well described by ChPT, and Case 2: $m_f \gg m_D$ (blue dotted line) well described by the hyper scaling.

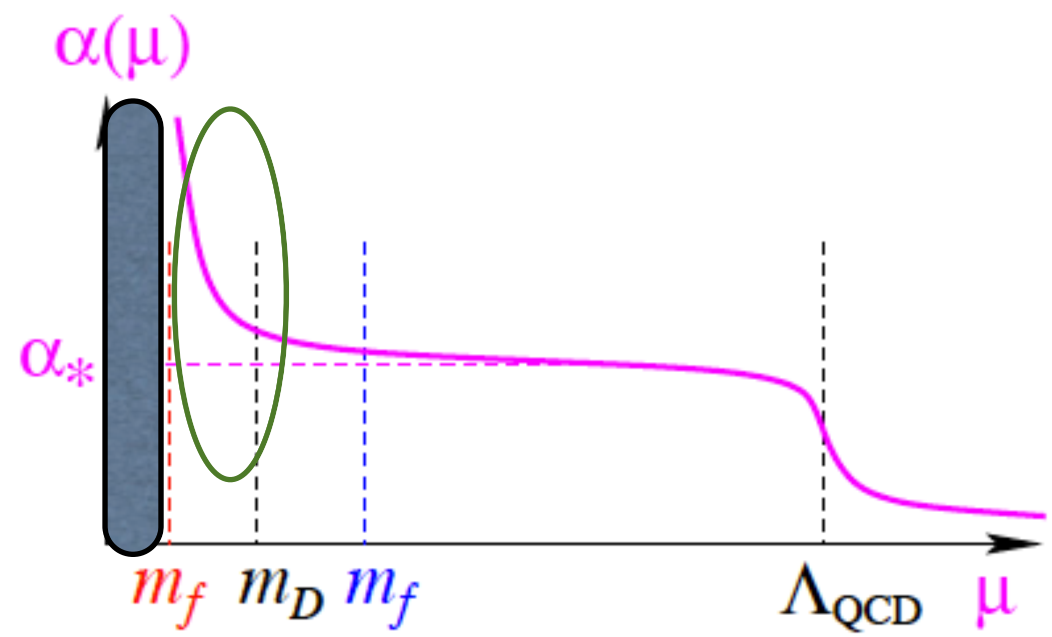
$\Rightarrow$ $\boxed{\text{Spectrum?}}$

SχSB and/or conformal ?

What is the signal of walking?

Scenario of Walking Dynamics ;

Case-1: probe mf << m_D → SχSB-like



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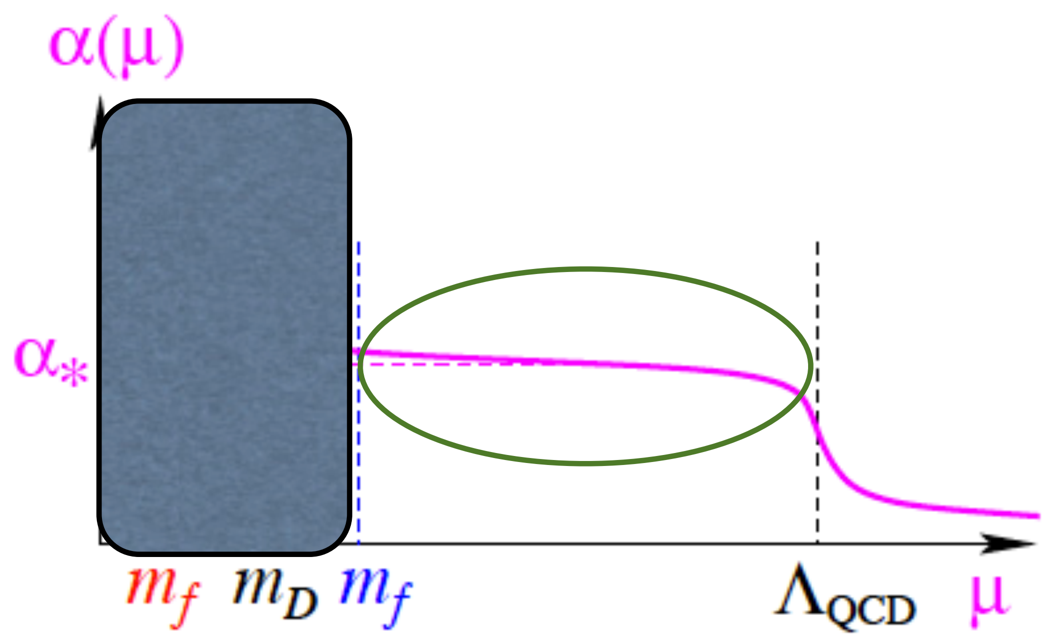
⇒ $\boxed{\text{Spectrum?}}$

SχSB and/or conformal ?

What is the signal of walking?

Scenario of Walking Dynamics ;

Case-2: probe mf >> m_D → conformal-like



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⇒ $\boxed{\text{Spectrum?}}$

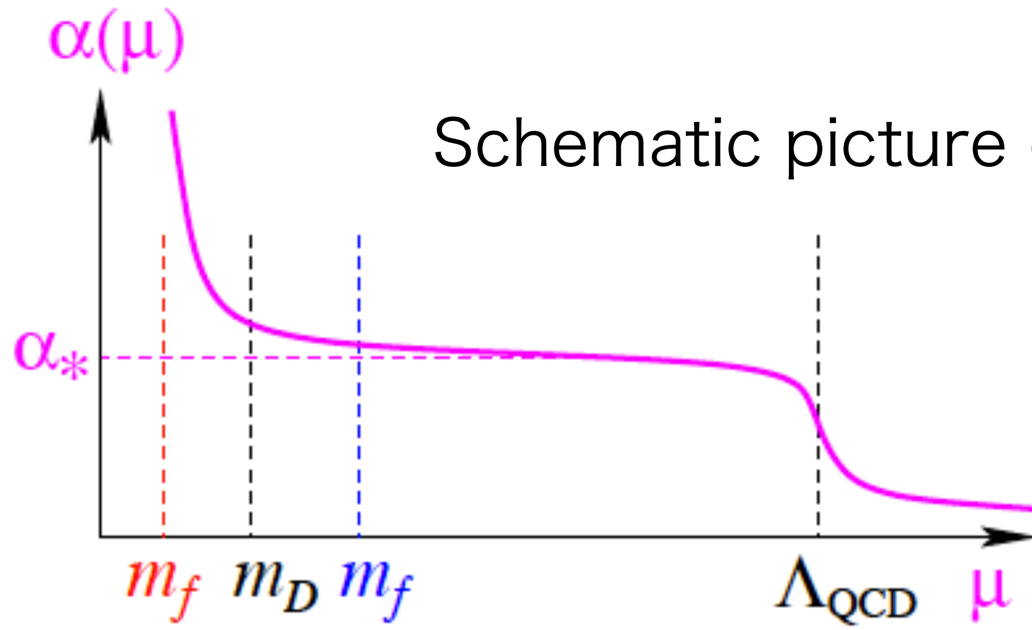
SχSB and/or conformal ?

What is the signal of walking?

Scenario of Walking Dynamics ;

Case-1: probe mf << m_D → SχSB-like
Case-2: probe mf >> m_D → conformal-like

Schematic picture of the gauge coupling



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⇒ $\boxed{\text{Spectrum?}}$

SχSB and/or conformal ?

2. Lattice Study of Nf=8 case

Simulation for Nf=8 (same setup with Nf=12)

lattice action (Hybrid Monte-Carlo simulation)

- Tree-level Symanzik gauge action
- Highly Improved Staggered Quarks = **HISQ**

(without tadpole improvement and mass correction in Naik term)

★ parameter set

- $\beta\,(\equiv 6/g^2) =$ **3.8,** V=L^{3}xT , T/L=4/3 fixed.

V	12^3x16	18^3x24	24^3x32	30^3x40	36^3x48	42^3x56
mf	0.01~0.16	0.04~0.1	0.02~0.1	0.02, 0.03,0.04~0.07	0.015,0.02	0.012

(updated) (new)

★ Gauge configurations for scalar measurement

★ Measurements (P+AP method ⇒ double size in T-dir.)

- Mπ, Fπ, Mρ, chiral condensate
- analysis for MπL > 6

data in the Nf=8 paper +   updated data

Preliminary

HISQ with Nf=8:

effective mass for the lowest mf(=0.015)
on the largest size(L=36)

$M_\pi^{PS}=M_\pi^{SC}$, $M_\rho^{PV}=M_\rho^{VT}$
→ good flavor symmetry

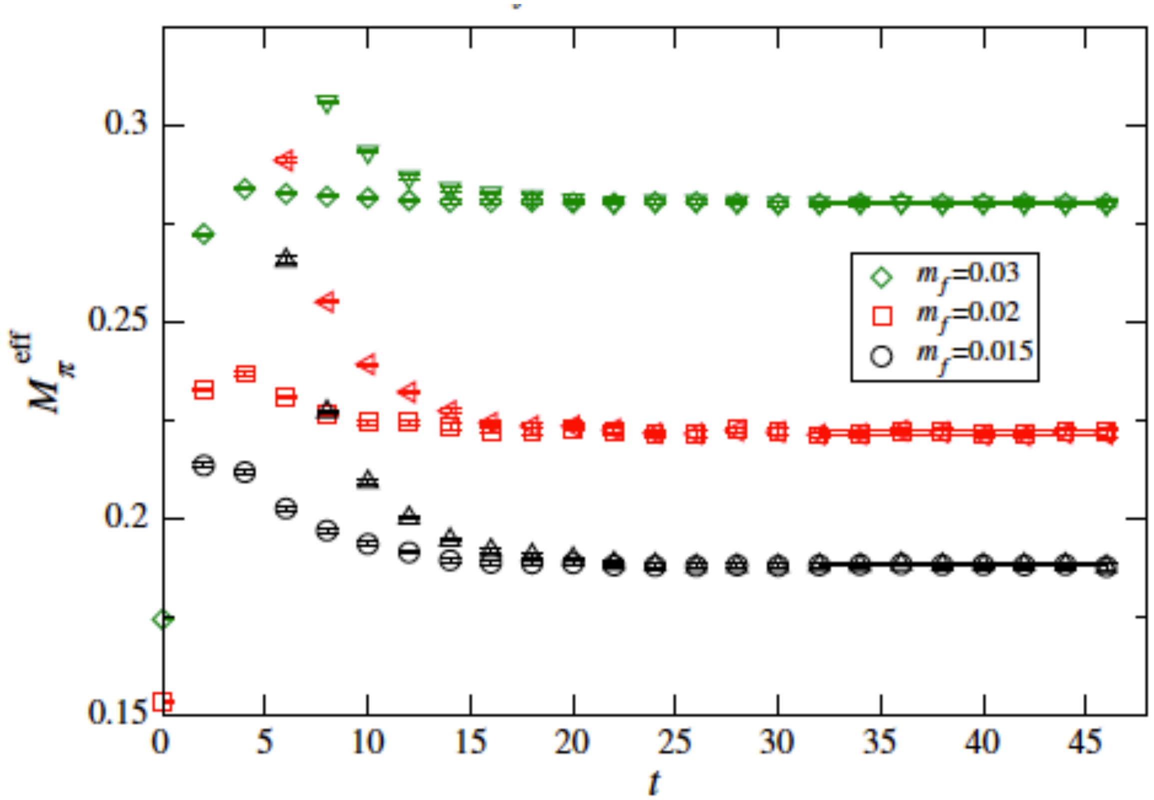


FIG. 2 (color online). Effective masses of PS meson, $M_\pi^{\rm eff}$, at $L = 36$. Triangles and other symbols denote results from point sink correlators with random wall source and corner wall source, respectively. Fit results with error band obtained from random wall source correlator are also plotted by solid lines.

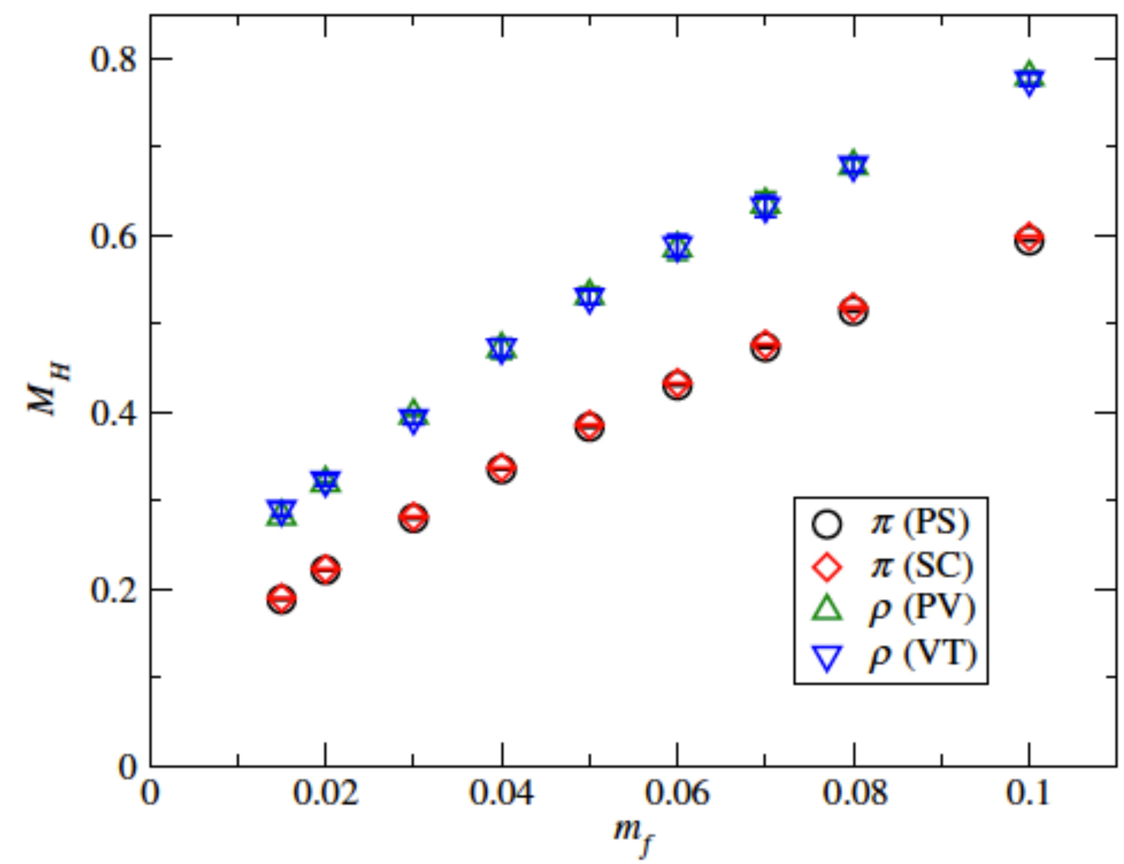
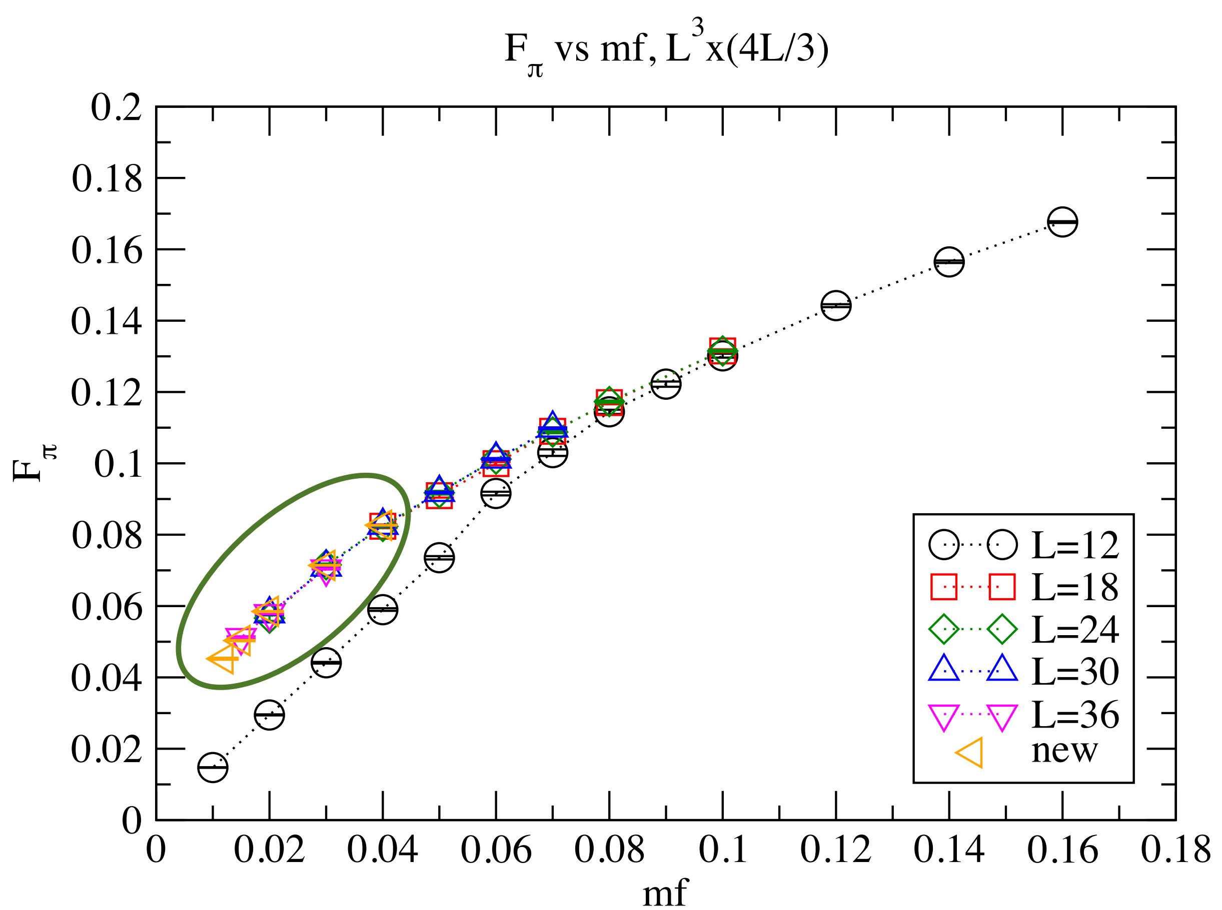
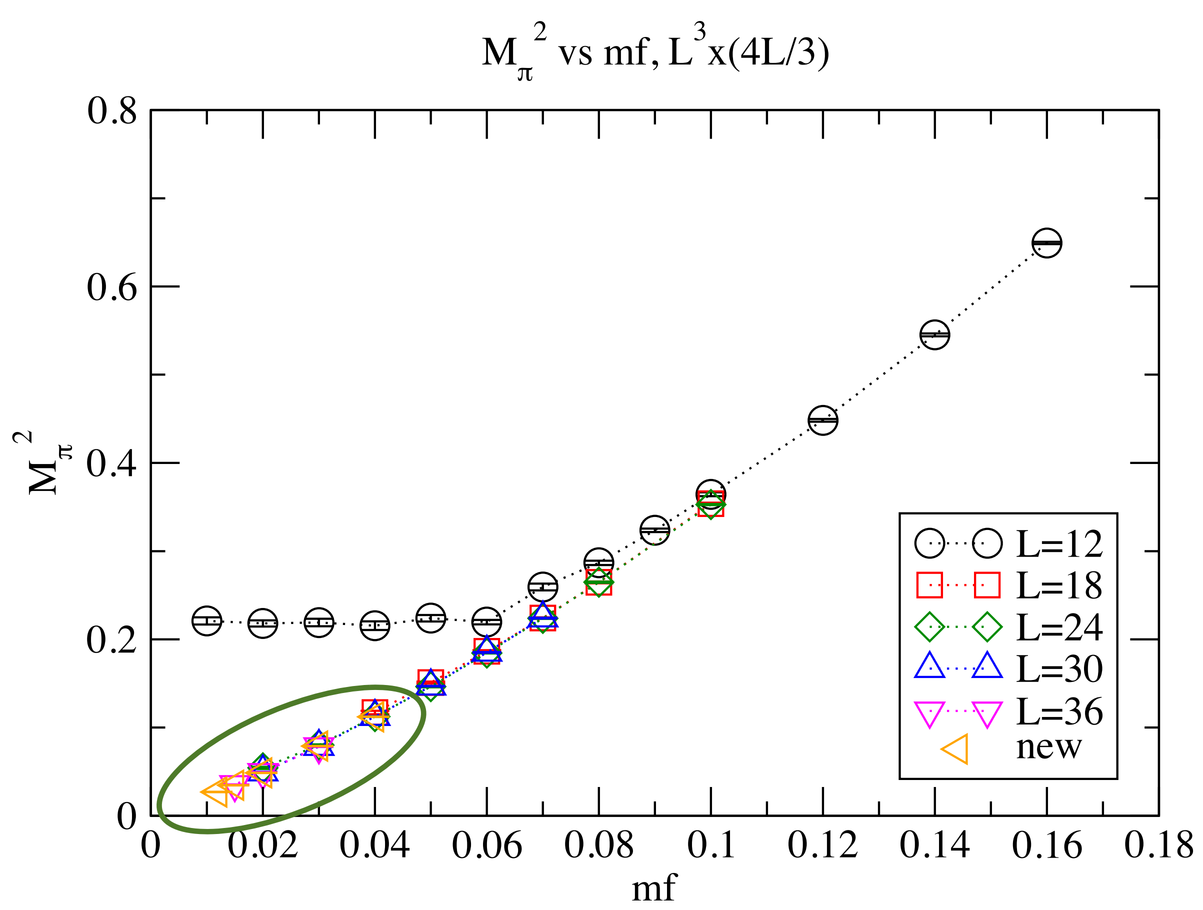


FIG. 4 (color online). Comparisons of M_π and $M_{\rm SC}$, and of $M_{\rho({\rm PV})}$ and $M_{\rho({\rm VT})}$ as a function of m_f with largest volume data at each m_f.

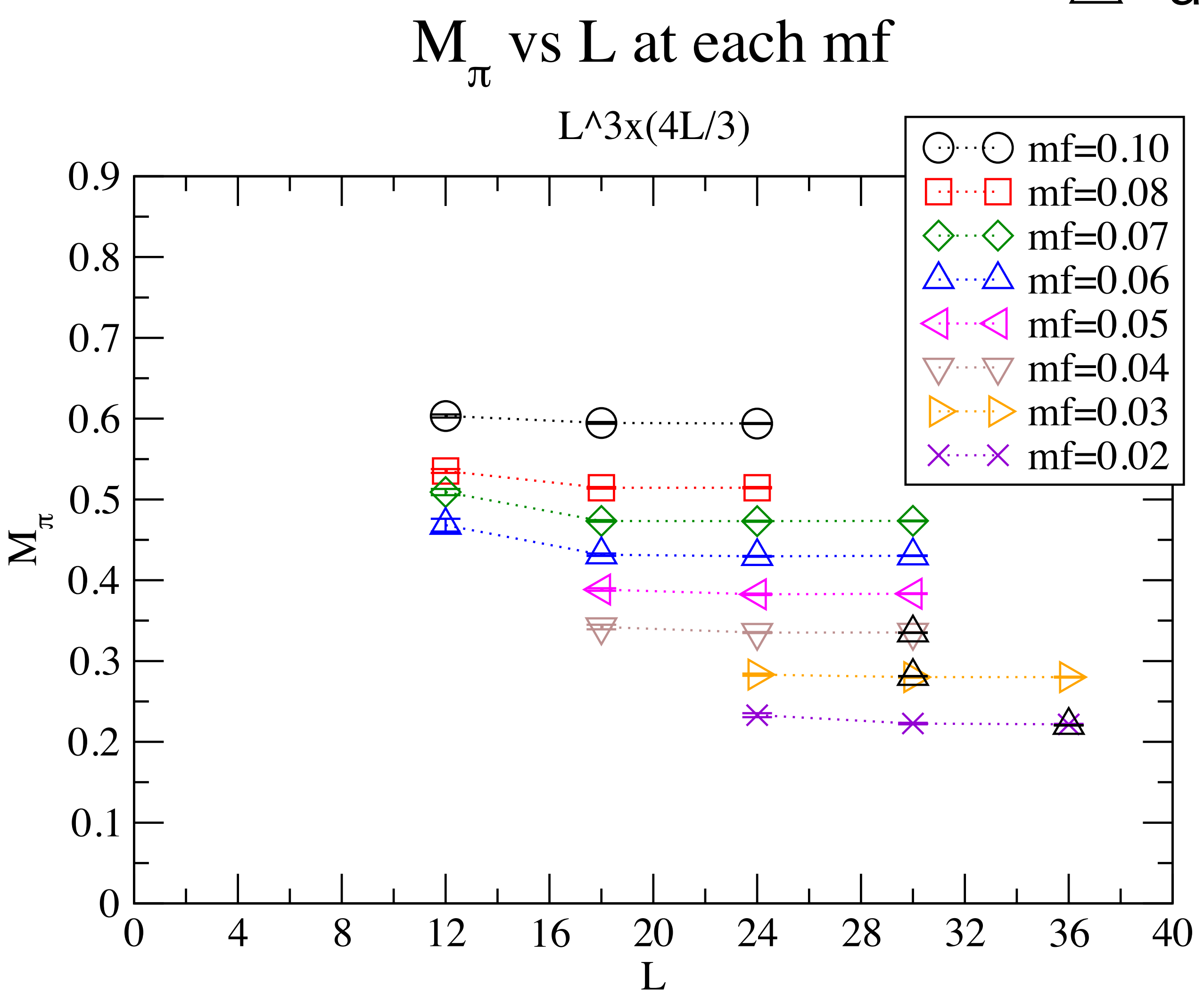
$M_\pi{}^2$, F_π vs. m_f on various L

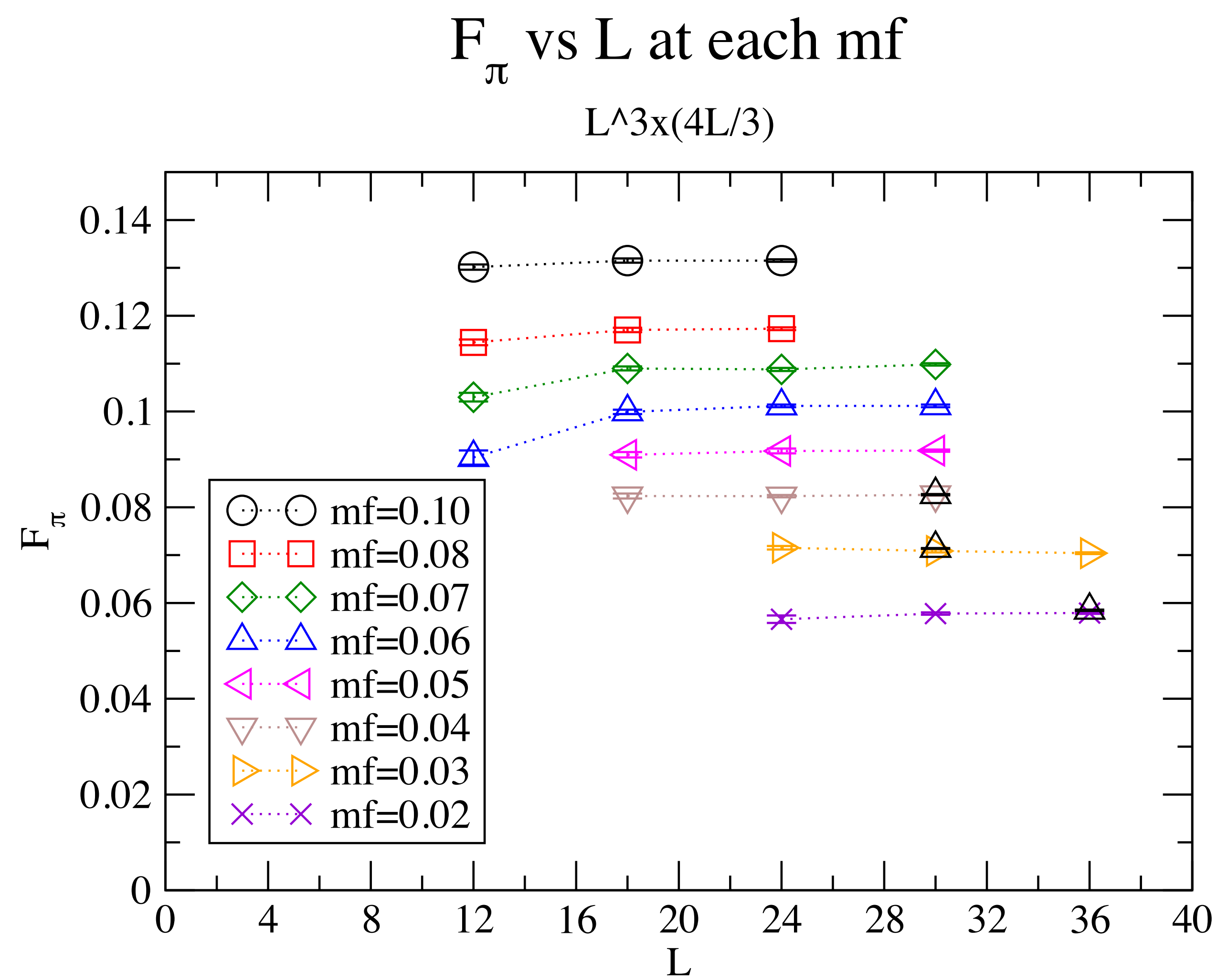




Size dependence of $M\pi$ and $F\pi$ at β=3.8

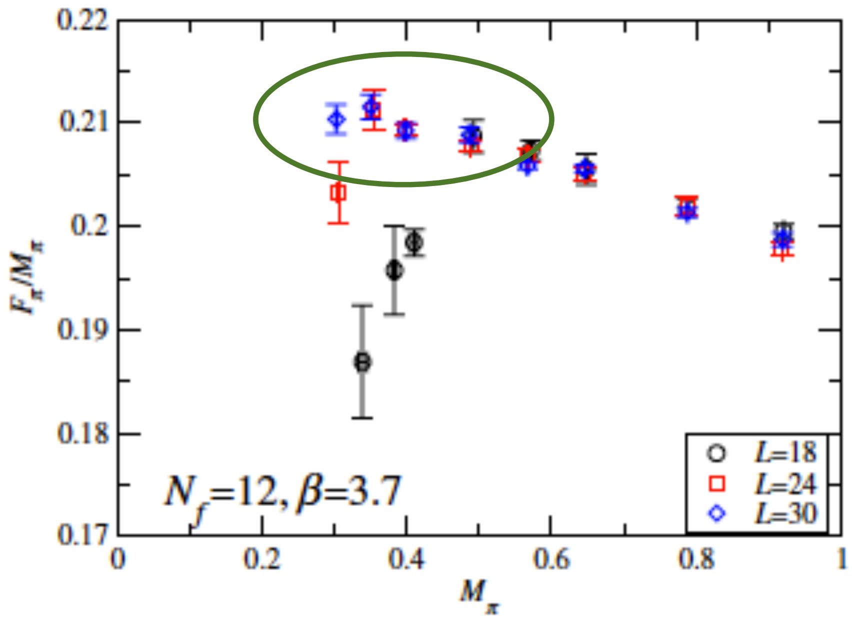
△=updated

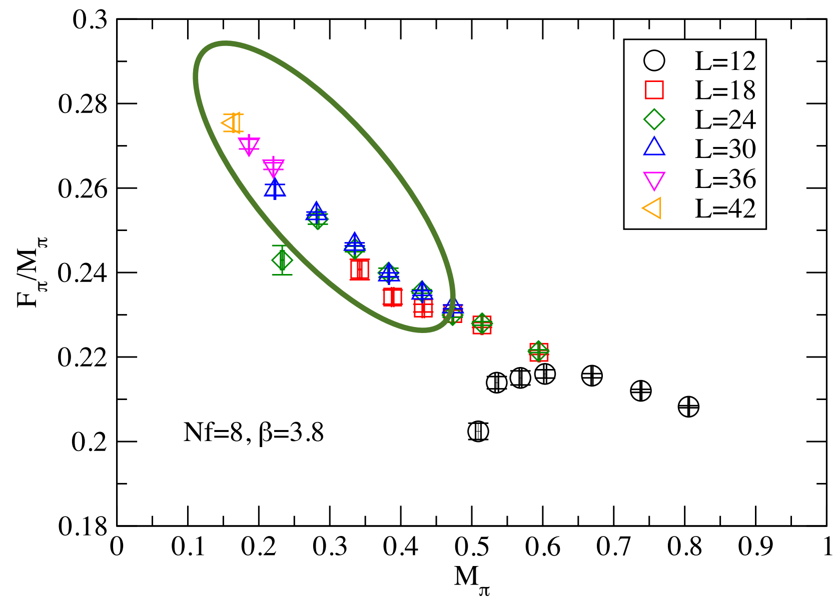


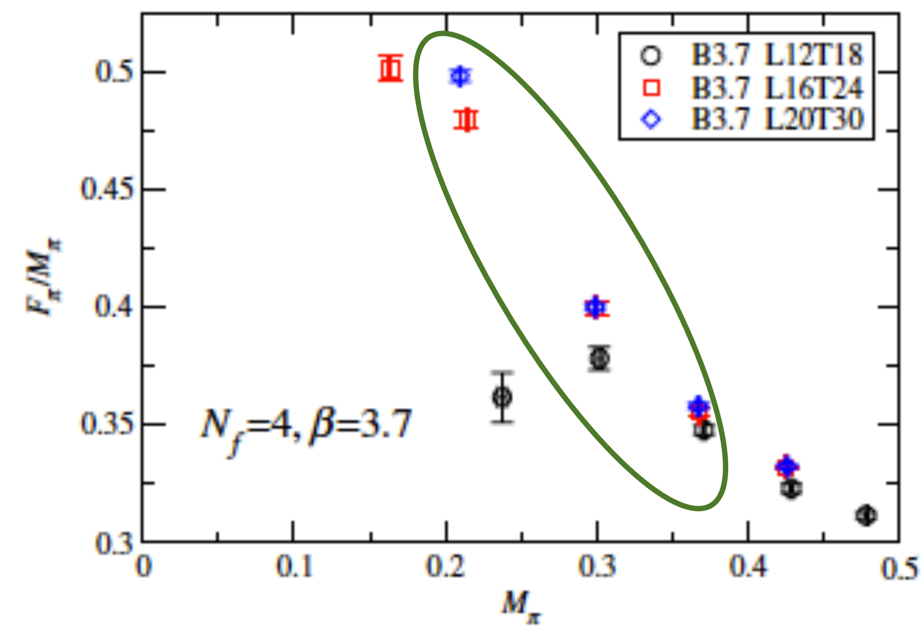


On the larger volume, there is not (or very tiny) size dependence.

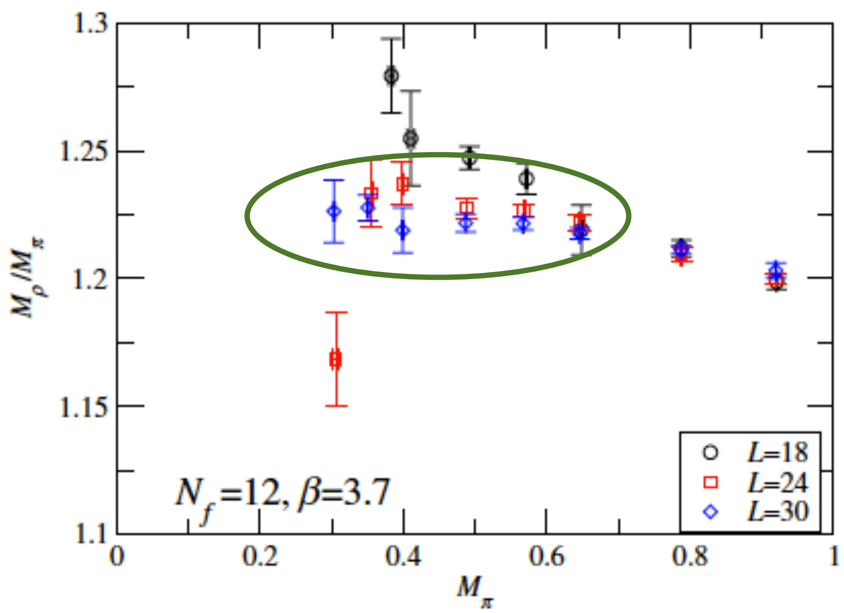
F_π/M_π for Nf=12, 8 and 4 (flat or divergent in χ-limit?)

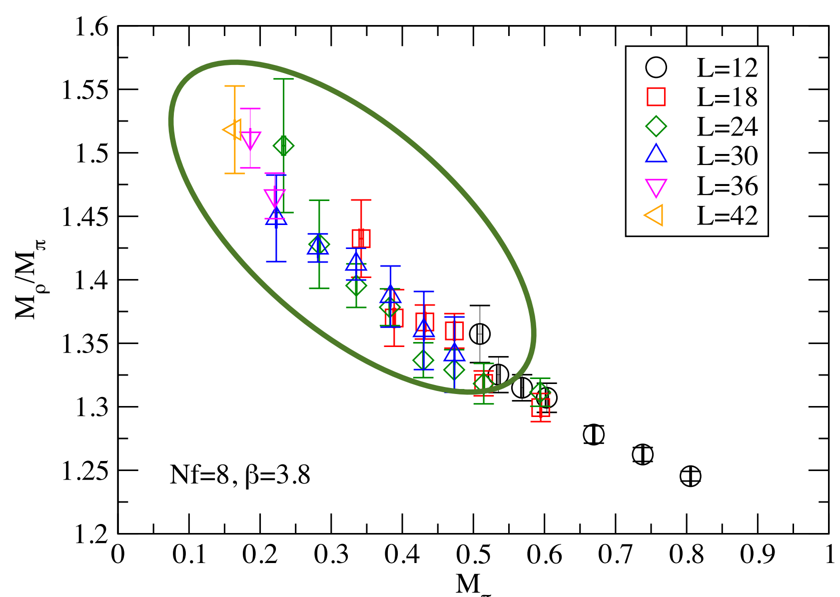


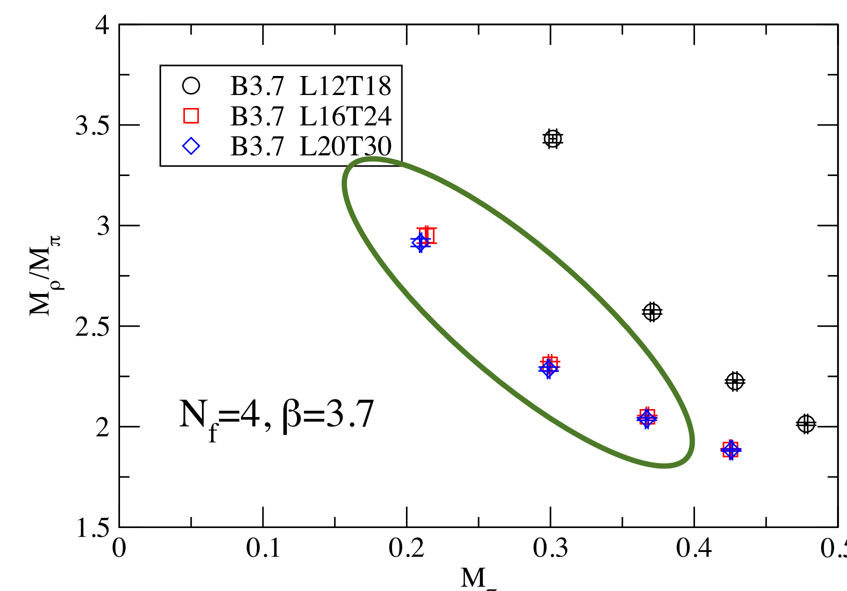




M_ρ/M_π for Nf=12, 8 and 4 (flat or divergent in χ-limit?)







ChPT analysis

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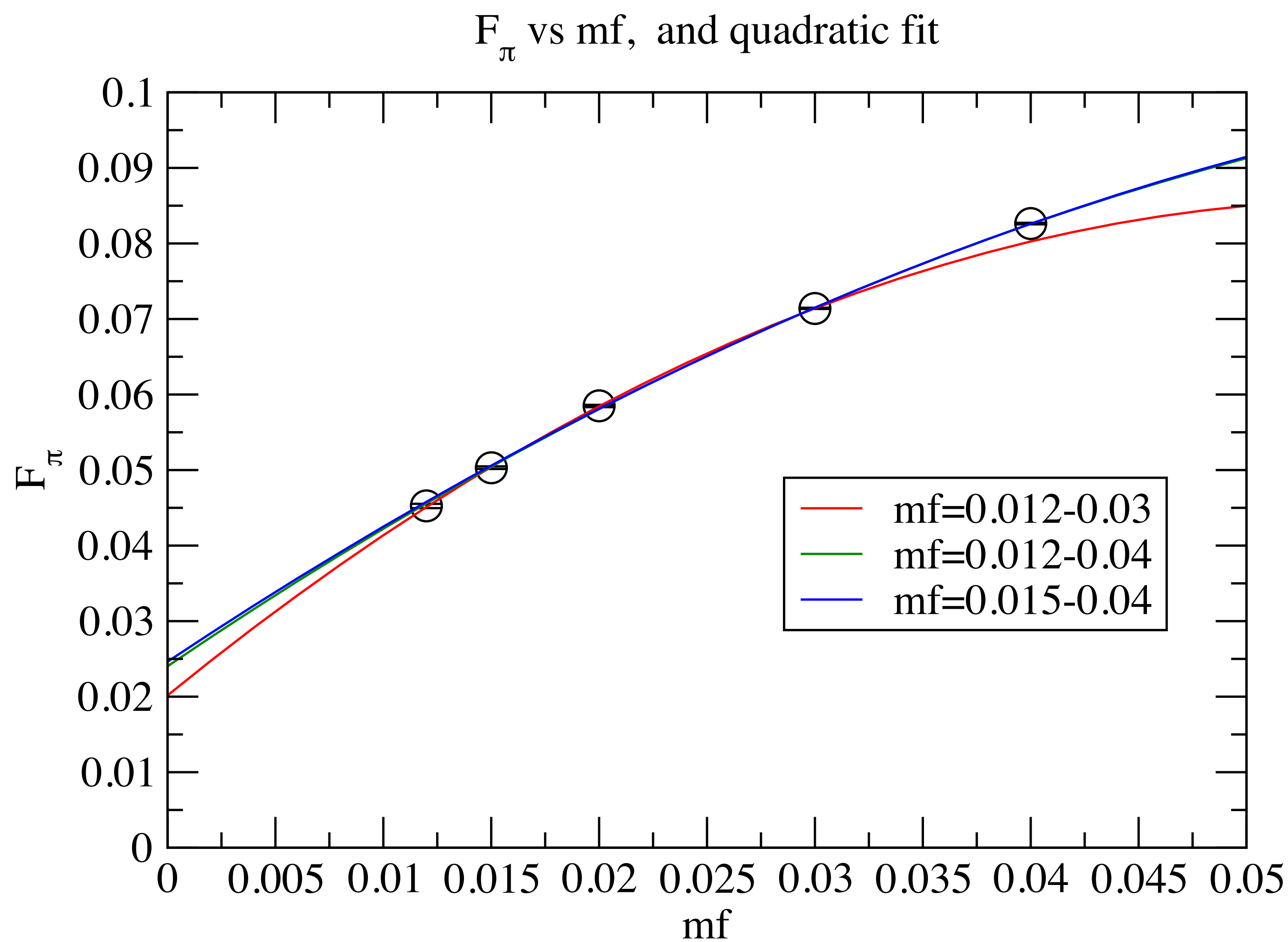
ChPT(SχSB) in $0.015 \leqq mf \leqq 0.04$

In the χ-limit; $F\pi=0.031(1)$, $M\rho/(F/\sqrt{2})=7.7(1.5)$

condensate~ 0.0005

Re-analysis with updated data ⇓

$F\pi$ vs mf with quadratic-func. fit : $y=C_0+C_1*mf+C_2*mf^2$



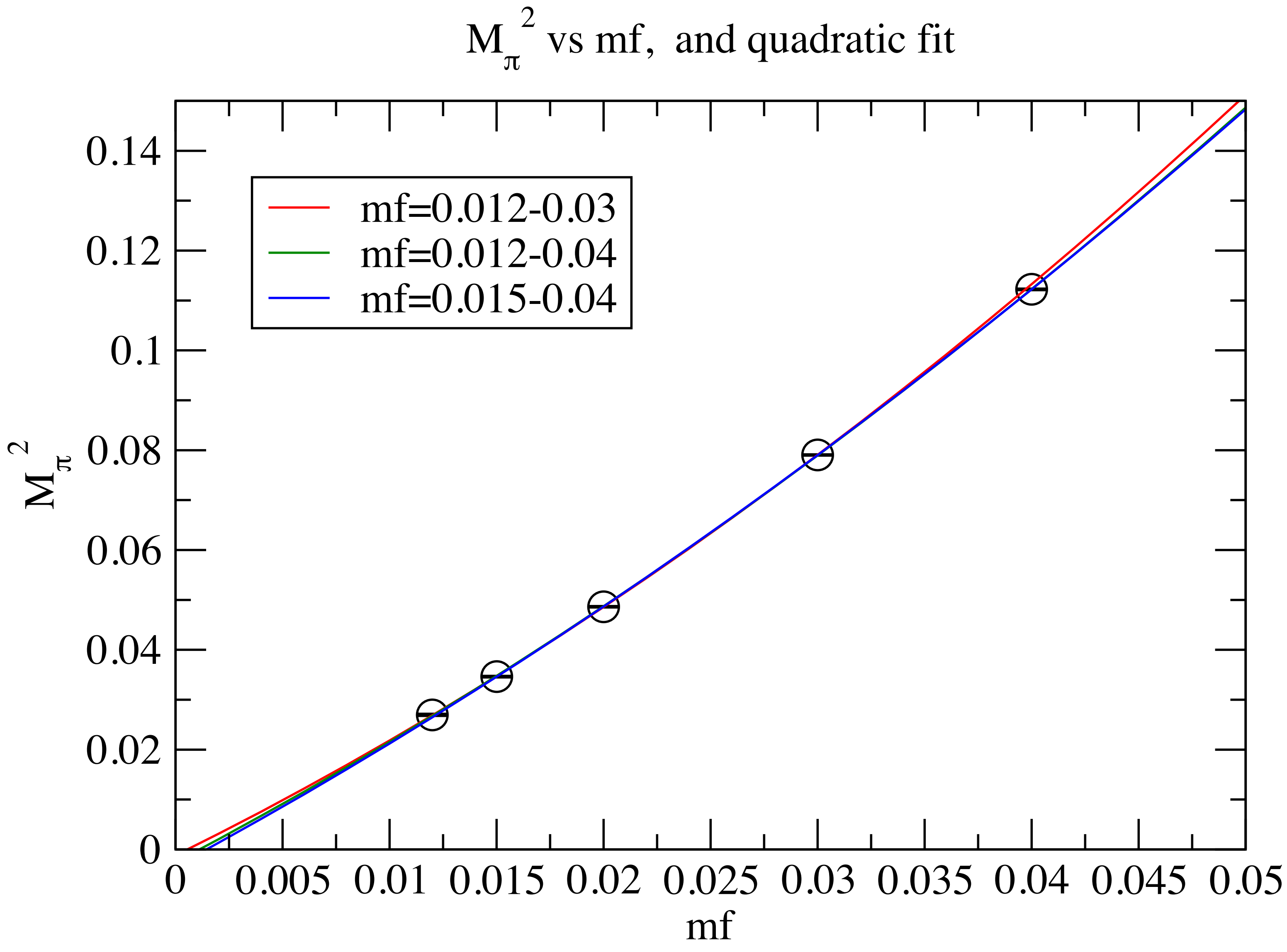
red : C_0=0.0202(13), $\chi 2$/dof=0.60 (dof=1)

green: C_0=0.0240(5), $\chi 2$/dof=5.9 (dof=2)

blue : C_0=0.0246(7), $\chi 2$/dof=9.4 (dof=1)

ChPT is good in $0.012 \leqq mf \leqq 0.03$.

$M_\pi{}^2$ vs mf with quadratic-func. fit



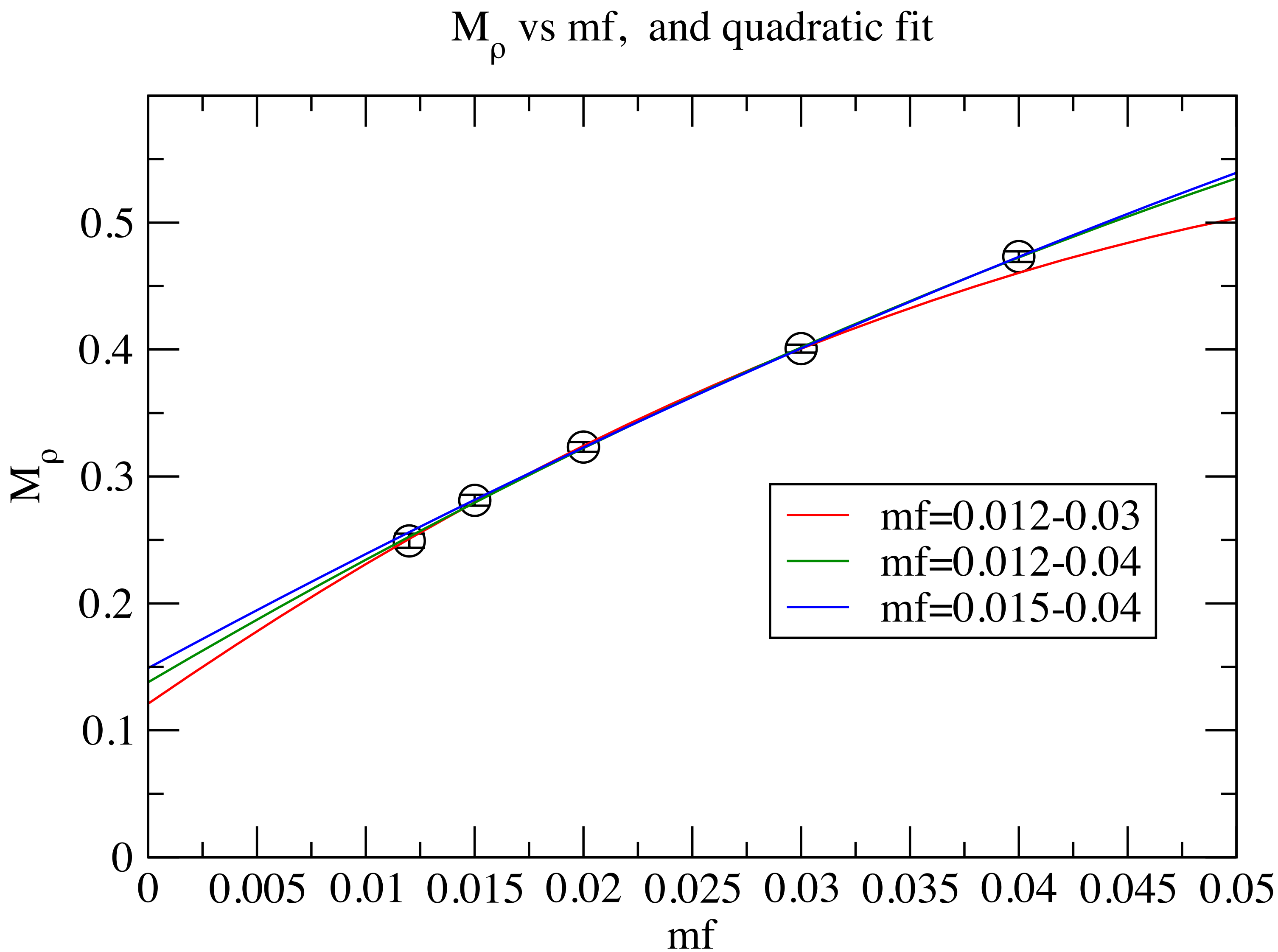
red : C_0=-0.0012(11), χ2/dof=0.79 (dof=1)

green: C_0=-0.0025(6), χ2/dof=1.4 (dof=2)

blue : C_0=-0.0034(8), χ2/dof=0.24 (dof=1)

ChPT is good in 0.012≦mf≦0.03.

$M\rho$ vs mf with quadratic-func. fit



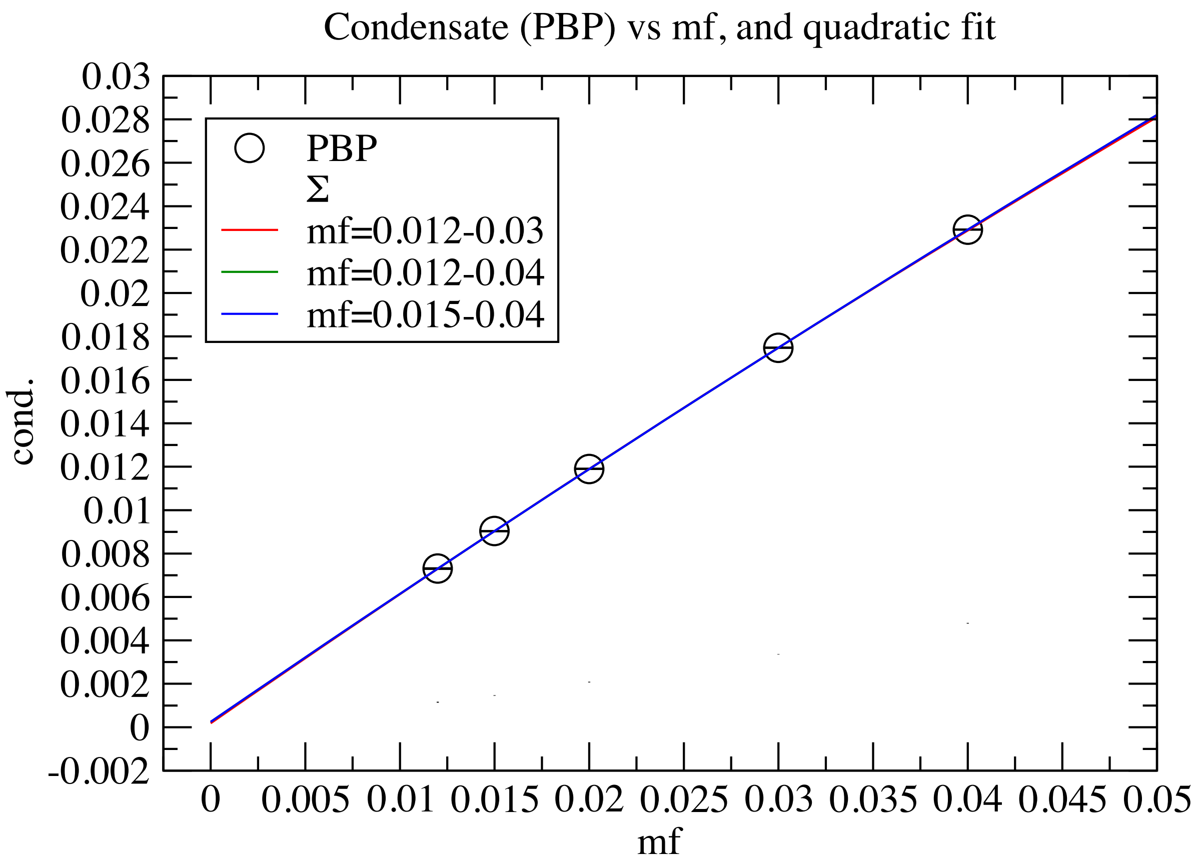
red : C_0=0.121(29), $\chi 2$/dof=0.27 (dof=1)

green: C_0=0.138(15), $\chi 2$/dof=0.36 (dof=2)

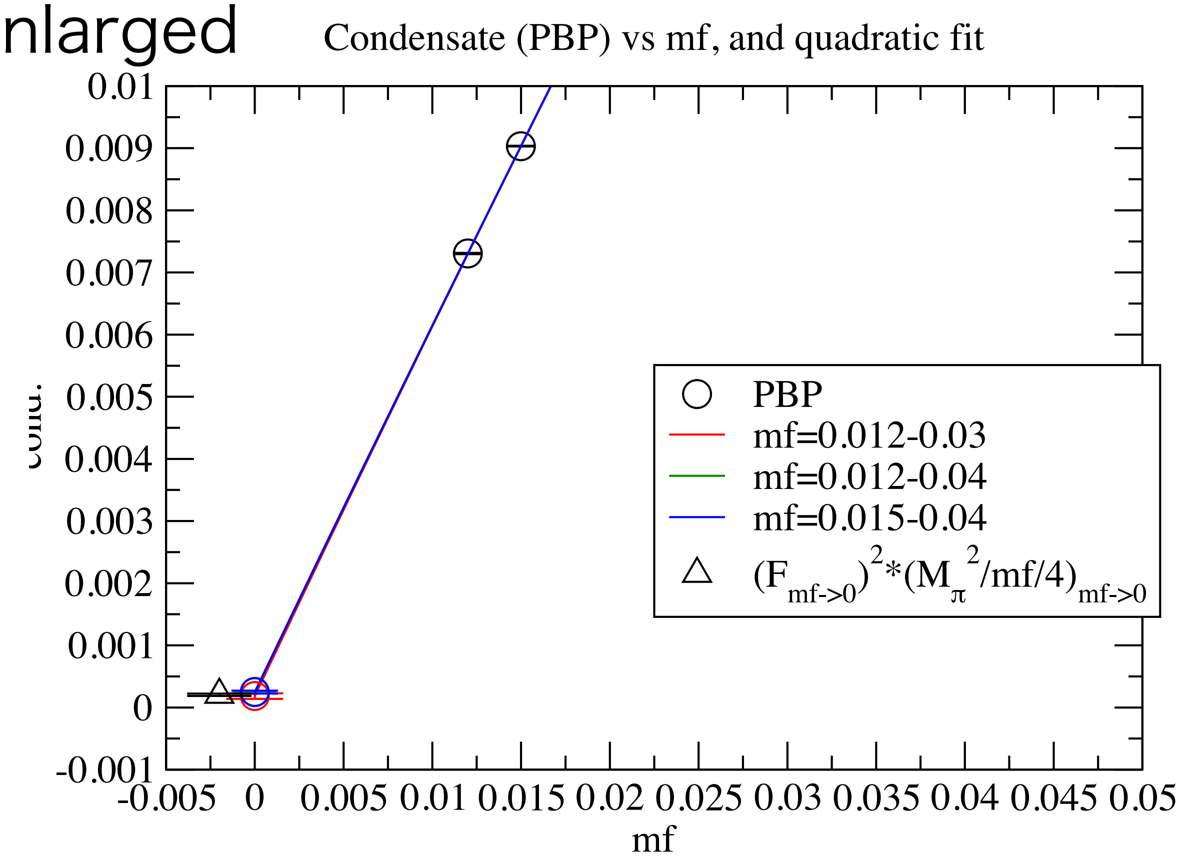
blue : C_0=0.149(20), $\chi 2$/dof=0.025 (dof=1)

Condensate (PBP and Σ) with quadratic-func. fit

PBP=Tr[Prop(x,x)/4]



enlarged



PBP

red : C_0=0.00018(5), $\chi 2$/dof=0.89

green: C_0=0.00024(2), $\chi 2$/dof=1.61

blue : C_0=0.00025(2), $\chi 2$/dof=3.18

$\Sigma = F_\pi M_\pi{}^2/mf/4$

$(F_\pi)^2{}_{mf\to0} * (M_\pi{}^2/mf/4)_{mf\to0} = 0.00021(2)$

consistent value

Summary-1,

old data

- The quadratic fit
- Nf=8 is consistent with ChPT(SχSB) in the small mf region (0.015≦mf≦0.04).
- In the χ-limit. F=0.031(1), $M_\rho/(F/\sqrt{2})$=7.7(1.5) condensate〜0.0005.
- The expansion parameter χ=O(1) of ChPT in the smallest mf (self-consistent), in contrast to Nf=12.
- ⇒ in the intermediate mf region?

$$\boxed{\chi = N_f \left(\frac{M_\pi}{4\pi F}\right)^2}$$

Summary-1,

+updated

- The quadratic fit
- Nf=8 is consistent with ChPT(SχSB) in the small mf region (0.012≦mf≦0.03).
- In the χ-limit. F=0.0202(13), $M_\rho/(F/\sqrt{2})$=8.5(2.1) condensate〜0.0002.
- The expansion parameter χ=O(1) of ChPT in the smallest mf (self-consistent), in contrast to Nf=12.
- ⇒ in the intermediate mf region?

$$\boxed{\chi = N_f \left(\frac{M_\pi}{4\pi F}\right)^2}$$

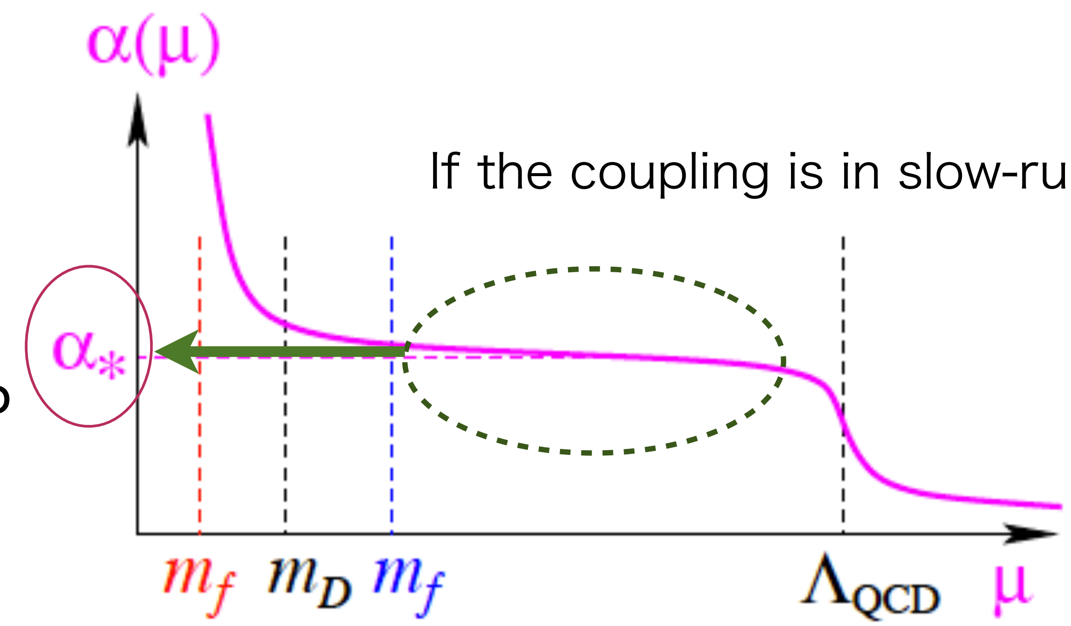
Finite size Hyperscaling analysis in a intermediate mf region

$$\boxed{LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)}$$

(DeGrand, Del Debbio et al.)

$x \equiv L\, m^{1/1+\gamma}$ (: universal γ)

information
at (approximate) IRFP



c.f. Finite Size Scaling (FSS) of 2nd order phase transition

$$\xi_L(T) = L f_\xi\left(\frac{L}{\xi_\infty}\right). \qquad \xi_\infty \propto \left|\frac{T_c - T}{T_c}\right|^{-\nu}$$

Finite size Hyperscaling analysis in a intermediate mf region

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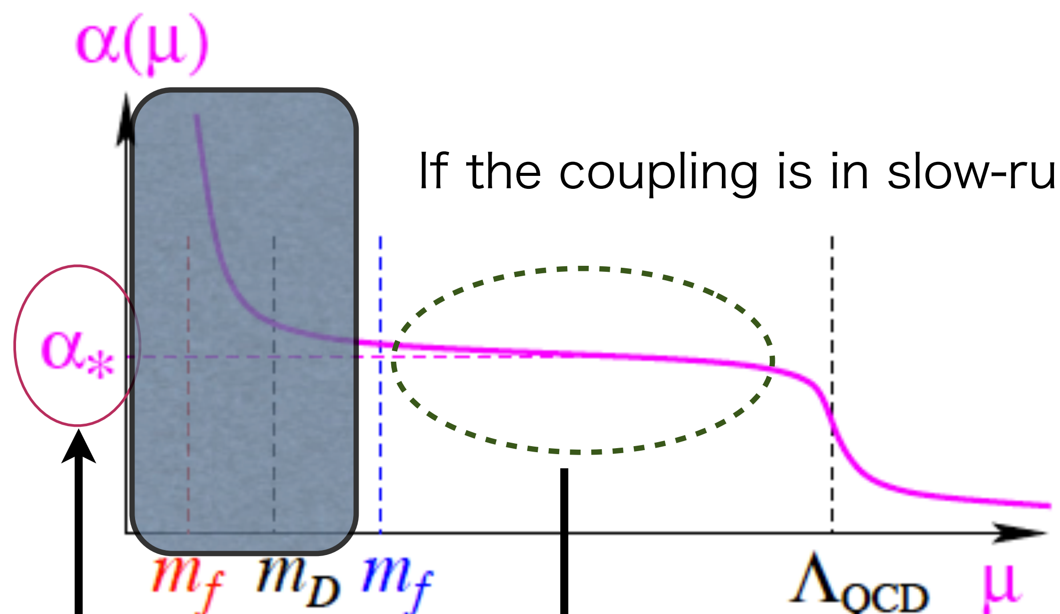


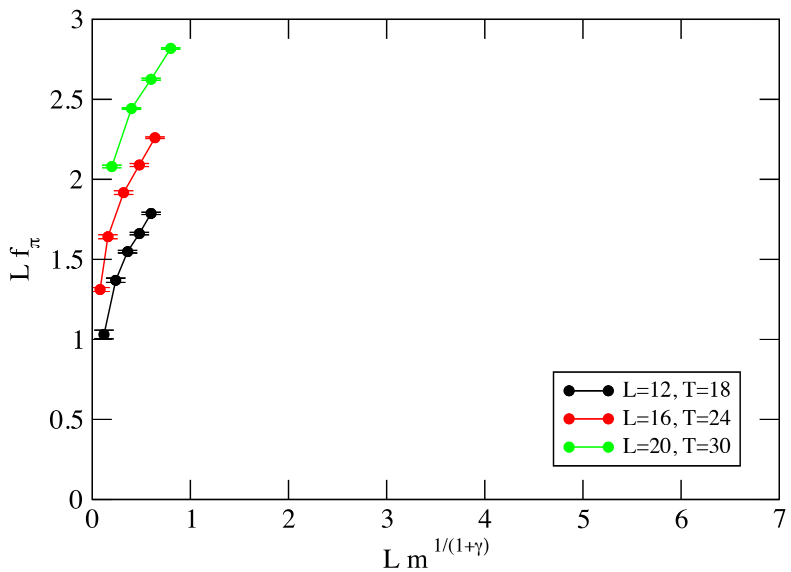
Dig it out!

from the data in this region

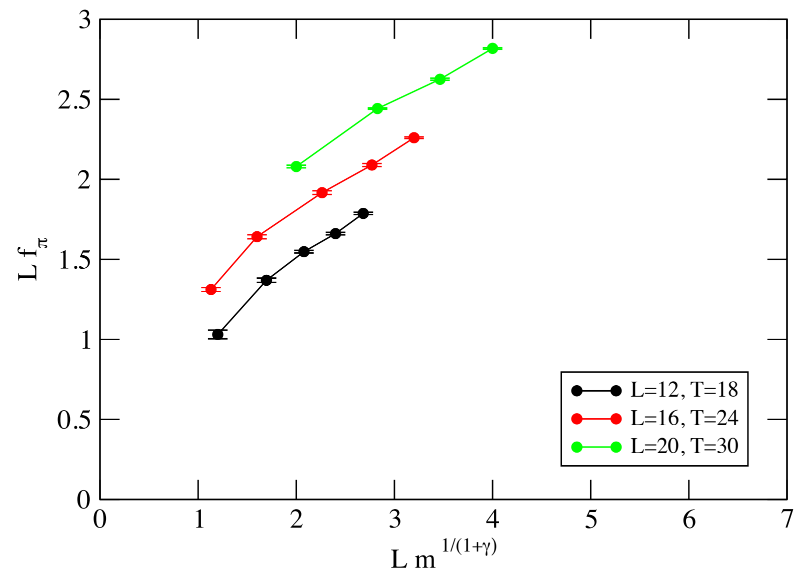
The hyperscaling with mass corrections is needed,
from the lesson of Schwinger-Dyson analysis (our paper, PRD(2012)).

Comparison with Nf=4
: hyperscaling of Fπ (in SχSB)

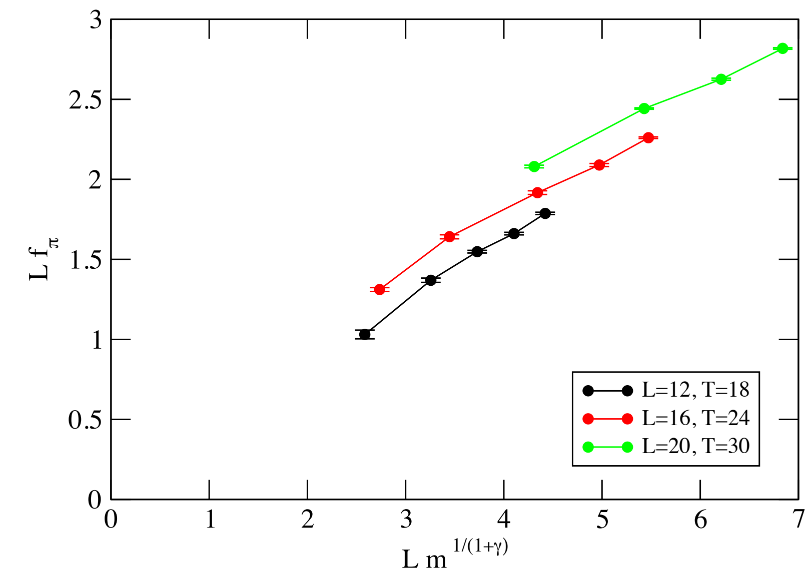
β=3.7, γ = 0.0



β=3.7, γ = 1.0



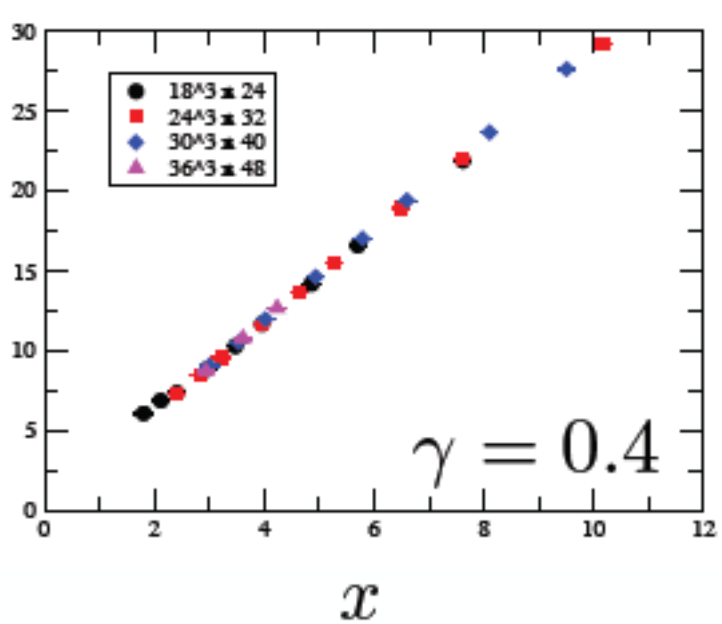
β=3.7, γ = 2.0



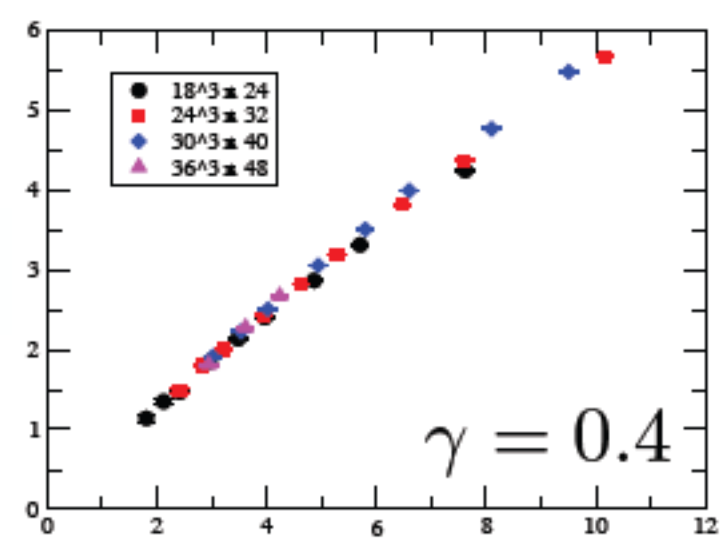
F$_\pi$: no scaling for $0 < \gamma < 2$

Comparison with Nf=12

LM_π



LF_π



Finite size Hyperscaling analysis

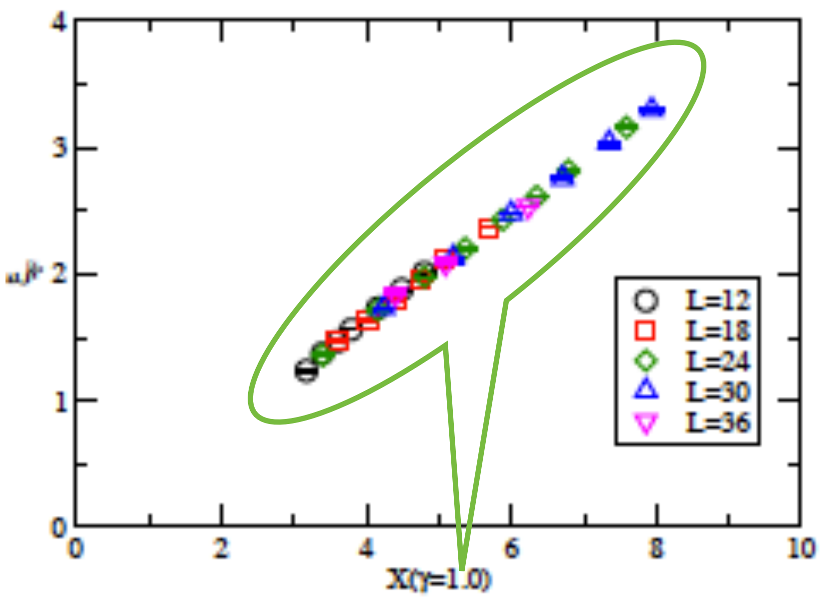
Comformal → Finite size Hyperscaling behavior with universal γ

(Critical exponent obtained from the finite volume setup)

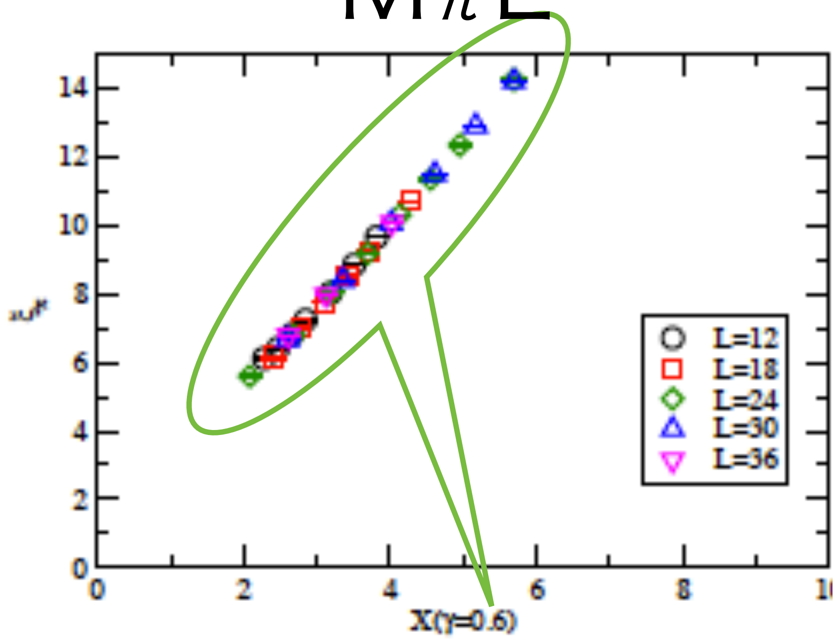
$$LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)$$

$$x \equiv L\, m^{1/1+\gamma}$$

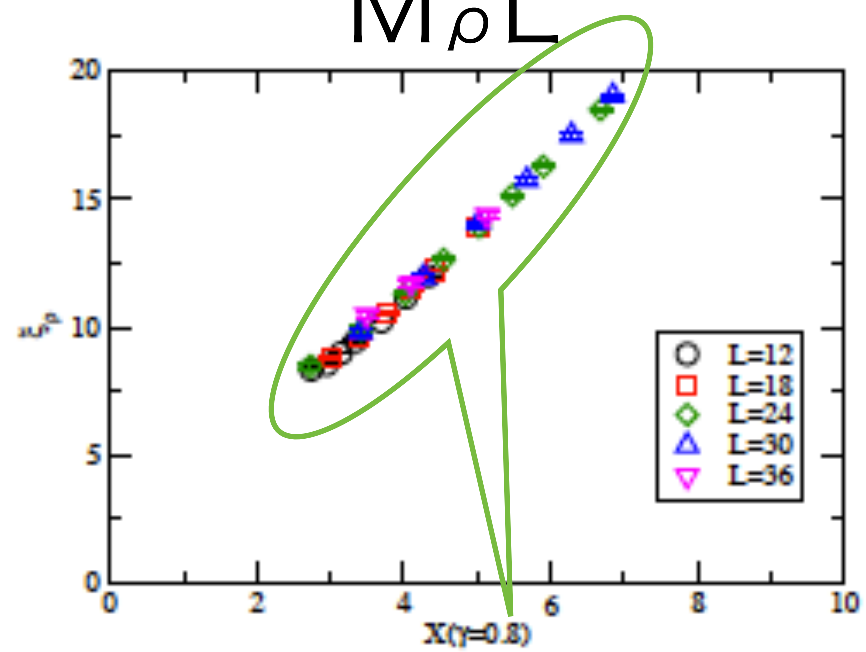
$F_\pi L$



$M_\pi L$



$M_\rho L$



in the middle region of mf≧0.05 and ξ_π ($=M_\pi L$) ≧8

$\gamma(F\pi)\sim1.0, \gamma(M\pi)\sim0.6, \gamma(M\rho)\sim0.8$

different from Nf=4 and12

Simultaneous fit of Hyperscaling with mass corrections

$$\xi_H = C_0^H + C_1^H X + C_2^H L m_f^{\alpha}.$$

in the middle region of mf≧0.05 and ξ_π (=M_πL) ≧8

(Schwinger-Dyson eq. analysis with large mass)

ξ_H(=M_HL) vs mf (not X): α=1 fixed (example)

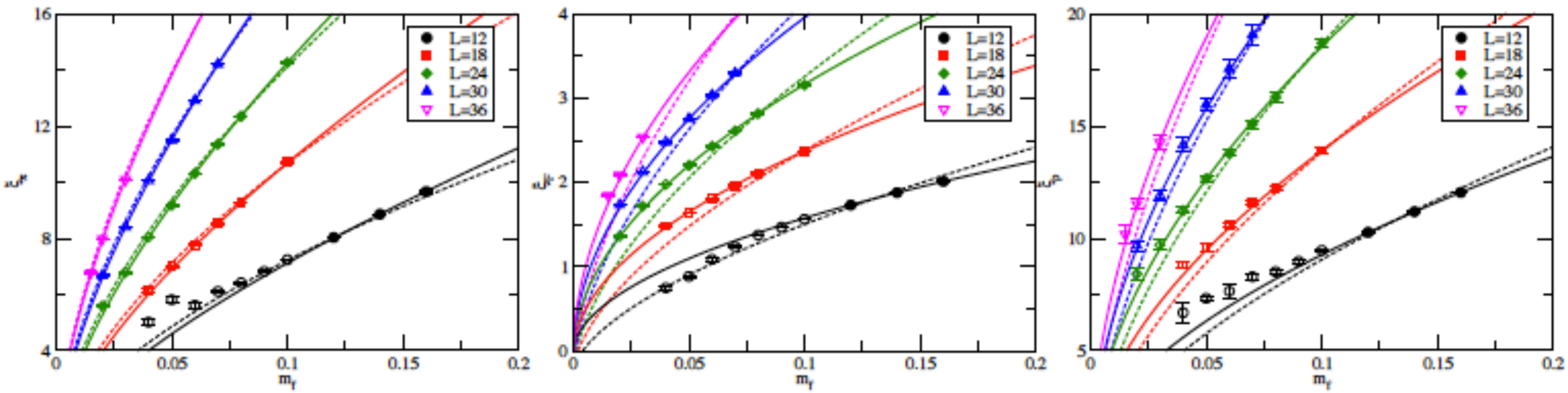


Fig. 5. Simultaneous FSHS fit in ξ_π(left), ξ_F(center) and ξ_ρ(right) with $\alpha = 1$. The filled symbols are included in the fit, but the open symbols are omitted. The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$. The solid curve is the fit result. For a comparison, the simultaneous fit result without correction terms is also plotted by the dashed curve, whose $\chi^2/dof = 83$.

In this case, the mass correction works well with γ=0.874(25), χ^2/dof=0.75, dof=32.

- In various trials of this analysis: γ=0.78—0.93 ～1

Summary-2, Finite-size Hyperscaling analysis

- In the region of mf≧0.05, hyperscaling is seen. (different from Nf=4)
- non-universal γ for each observable in the naive finite-size hyperscaling (different from Nf=12)
- Lesson from the Schwinger-Dyson analysis with the (large) mass.
- Simultaneous fit of hyperscaling with mass correction gives the universal γ=0.78--0.93 ～1. [requirement for the successful walking technicolor.]
- Nf=8 has the “remnant” of the conformality in the middle range of mf.

Summary

◆ SU(3) gauge theories with 4, 12, 16 and 8 HISQ quarks.

Nf=4 → Kurachi's poster

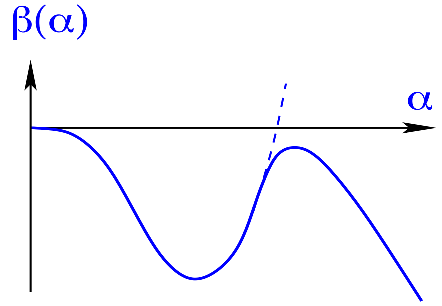
Nf=12 → Ohki's talk

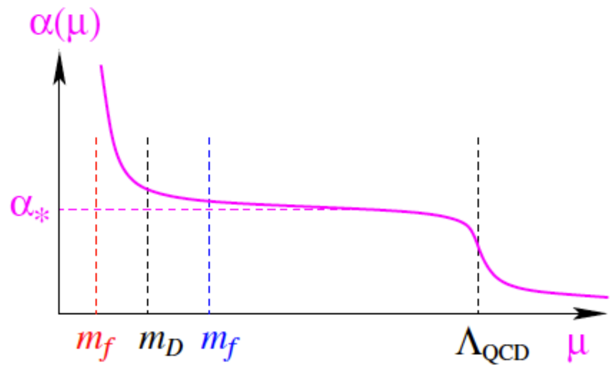
Nf=16 → Yamazaki's poster

Preliminary (data updated: 2013→2014)

◆ Nf=8; possibility of SχSB in the small mass region of our simulation and the remnant of the conformality in the middle region of mf with γ~1. (In contrast to Nf=4 and 12 cases.)

Nf=8 → Candidate of Walking dynamics





Phenomenologically preferable?

In Progress:

- Simulation on larger volumes at lighter masses
- Finite Size Effect (one-volume: mf=0.015 on L=36, mf=0.012 on L=42)
- Finite Size Scaling
- Lattice spacing dependence (Enhancement) ←many β
- Spectroscopy ($M_{glueball}$, $M_{\text{"scalar"}}$, M_{baryon}, M_{meson}, $F_{\rho/\sigma}$, S-param., string tension, etc.)

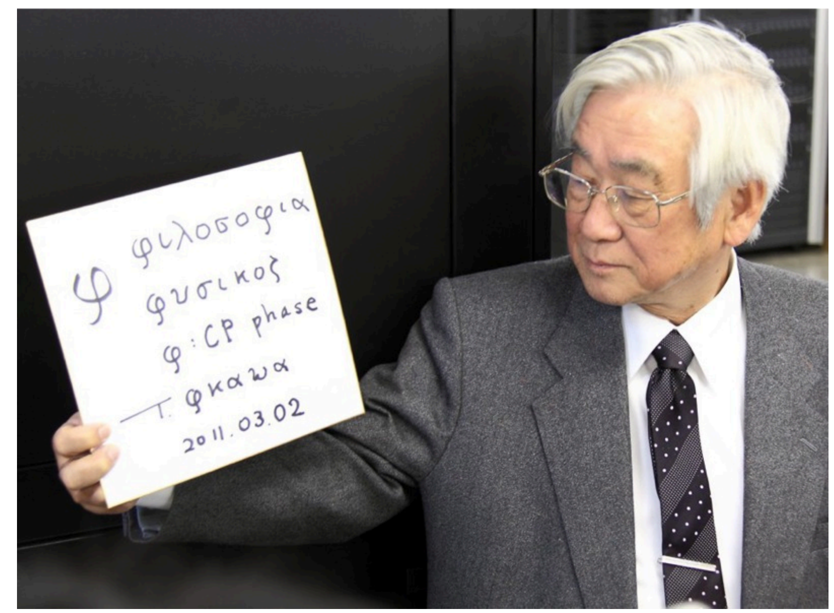
  $M_{glueball}$, string tension (→ Rinaldi's poster for Nf=12)
- $M_{\text{"flavor-singlet light scalar"}}$ → Yamazaki's talk

Thank you

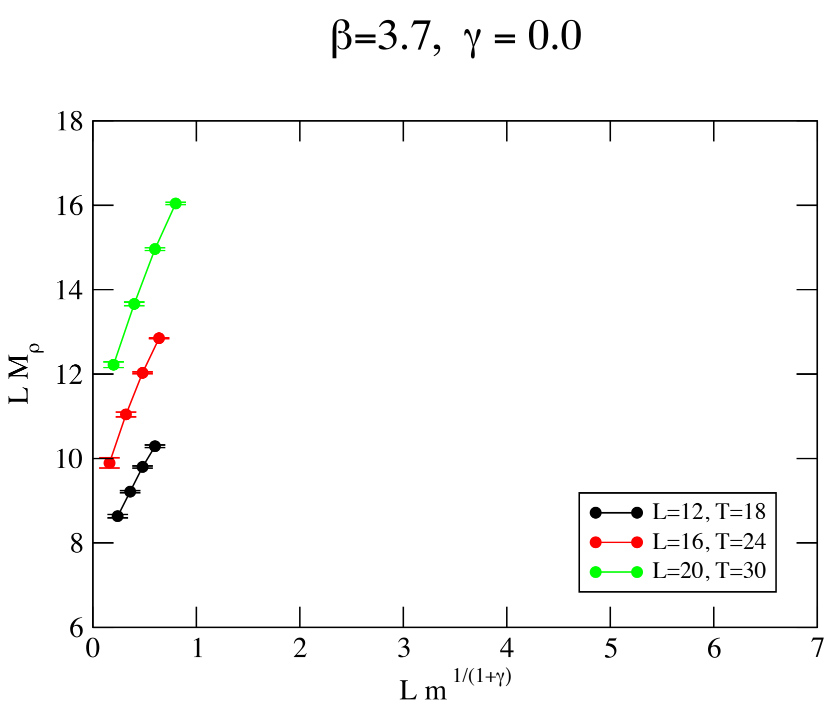
Backup

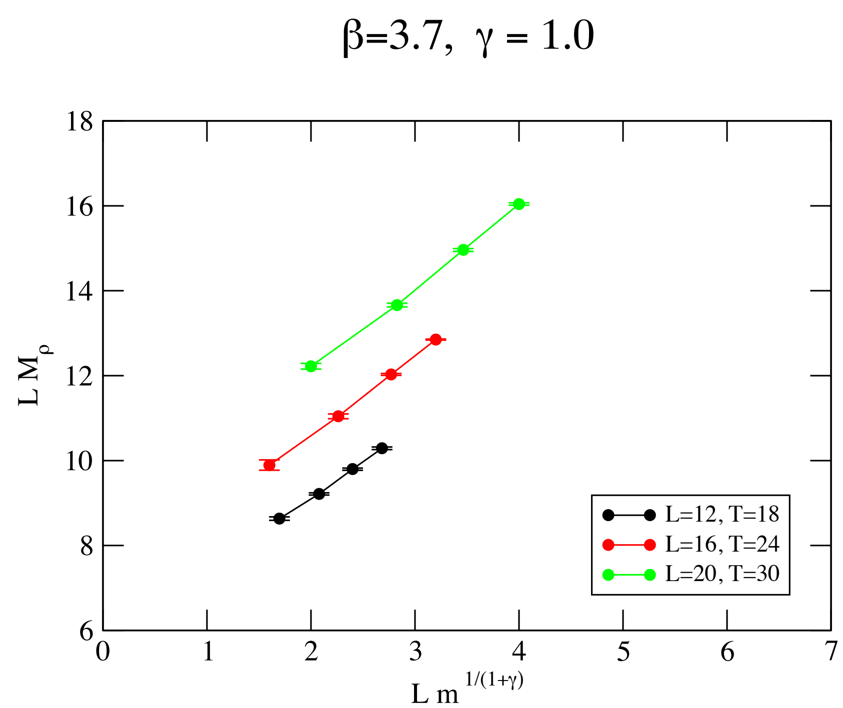
KMI computer, φ

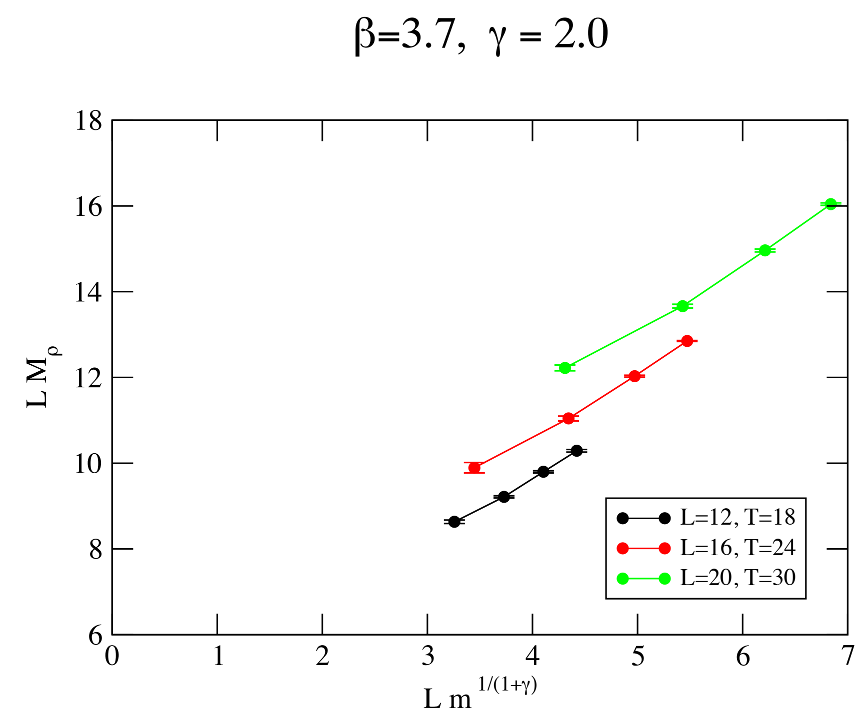
- non GPU nodes
  - 148 nodes
  - 2x Xenon 3.3 GHz
  - 24 TFlops (peak)
- GPU nodes
  - 23 nodes
  - 3x Tesla M2050
  - 39 TFlops (peak)



Comparison with Nf=4
: hyperscaling of $M_\rho L$ (in SχSB)







M_ρ : no scaling for $0 < \gamma < 2$

Simultaneous fit of hyperscaling with mass corrections

$\xi_H = C_0^H + C_1^H X + C_2^H L m_f^\alpha$. (same method with Nf=12)

Hyperscaling? in the middle region of mf (mf≧0.05 and ξ_π (=MπL) ≧8)

The mass corrections might be needed, as done in Nf=12,
from the lesson in SD analysis.

TABLE XI. Simultaneous FSHS fit with a correction term, $\xi = C_0^H + C_1^H X + C_2^H L m_f^\alpha$ using several choices of α. The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$.

$\alpha = 0.889(55)$	C_0^H	C_1^H	C_2^H
ξ_π	$-0.005(25)$	1.338(96)	1.494(37)
ξ_F	$-0.0275(98)$	0.4435(36)	—
ξ_ρ	0.53(16)	2.476(39)	—
	$\gamma = 0.9130(76)$, $\chi^2/\mathrm{dof} = 1.73$, dof = 33		

$\alpha = 1$ fixed	C_0^H	C_1^H	C_2^H
ξ_π	$-0.014(24)$	1.61(10)	1.31(15)
ξ_F	$-0.012(10)$	0.484(30)	$-0.068(44)$
ξ_ρ	0.01(19)	2.60(17)	0.25(24)
	$\gamma = 0.874(25)$, $\chi^2/\mathrm{dof} = 0.75$, dof = 32		

$\alpha = \frac{3-2\gamma}{1+\gamma}$ fixed	C_0^H	C_1^H	C_2^H
ξ_π	0.020(24)	1.52(39)	1.17(35)
ξ_F	$-0.011(10)$	0.572(34)	$-0.158(52)$
ξ_ρ	0.03(19)	2.91(30)	$-0.15(36)$
	$\gamma = 0.775(56)$, $\chi^2/\mathrm{dof} = 0.93$, dof = 32		

⇒ good χ^2/dof, but unclear which α is better.

Tools for analysis

(1) Chiral Perturbation Theory (ChPT)

Spontaneous chiral symmetry breaking (S χ SB)

$$M_\pi^2 = C_1^\pi m_f + C_2^\pi m_f^2 + \cdots, \quad F_\pi = F + C_1^F m_f + C_2^F m_f^2 + \cdots,$$

(2) (Finite size) Hyperscaling

(critical phenomena in conformal transition)

$$\boxed{LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)}$$ (DeGrand, Del Debbio et al.)

$$x \equiv L\, m^{1/1+\gamma}$$ (: universal γ)

Infinite volume limit:

$$M_H \propto mf^{1/(1+\gamma)}$$

$$F_\pi \propto mf^{1/(1+\gamma)}$$

c.f. Finite Size Scaling (FSS) of 2nd order phase transition

$$\xi_L(T) = L f_\xi\left(\frac{L}{\xi_\infty}\right). \qquad \xi_\infty \propto \left|\frac{T_c - T}{T_c}\right|^{-\nu}$$