SU(3) gauge theory with 12 flavours in a twisted box

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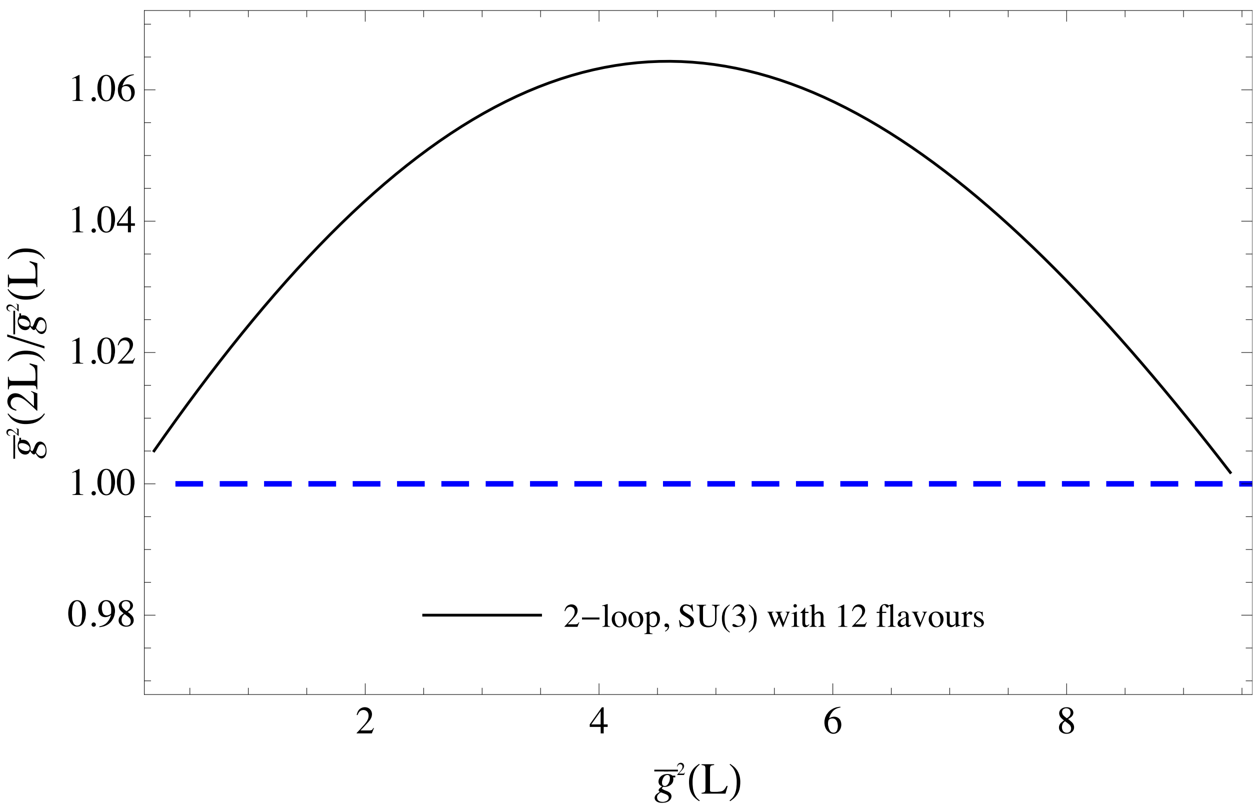
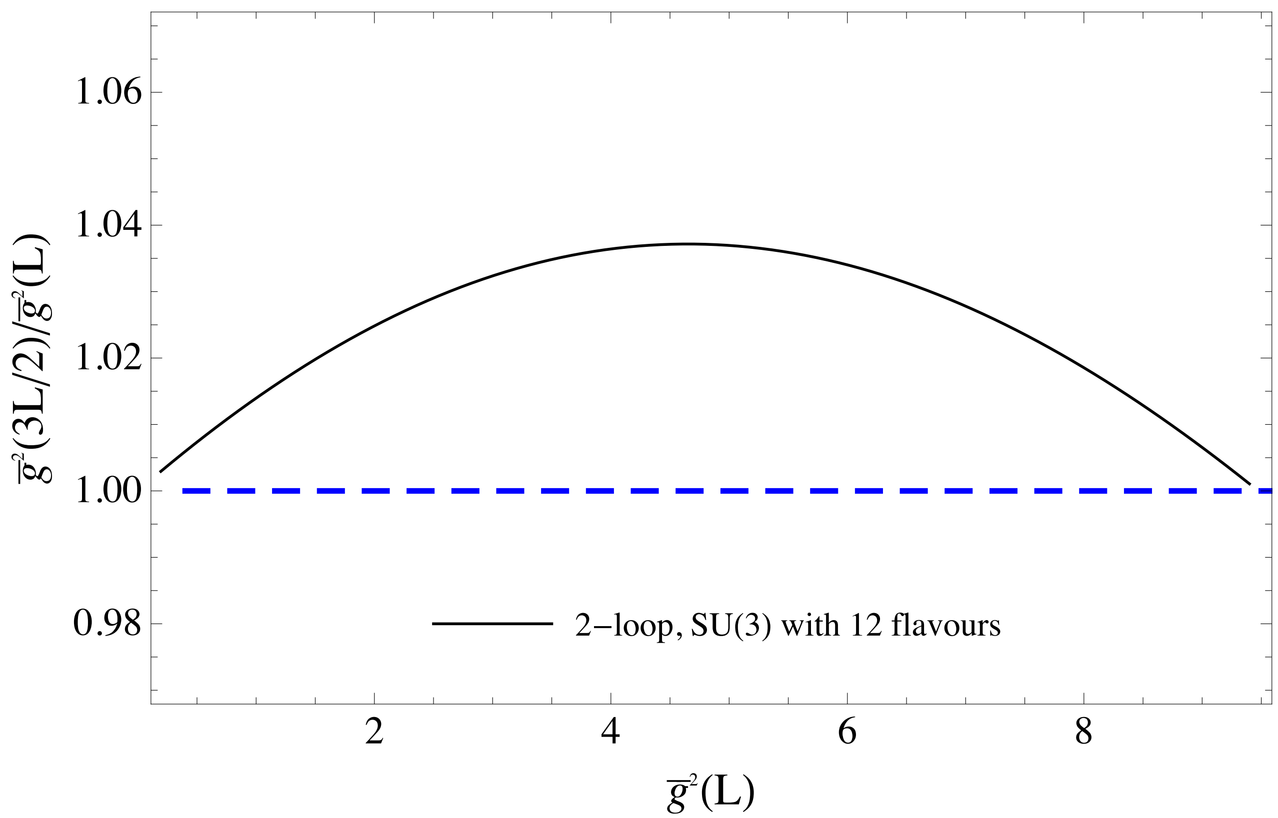
Controversy over the IR behaviour of SU(3) gauge theory with 12 flavour.

Strategy/challenges for the lattice search of IRFP

- Spectrum: Large finite-volume effects.
- Finite-size scaling *a'la* M. Fisher : universal curves.
- Running coupling: (slow) running within error.





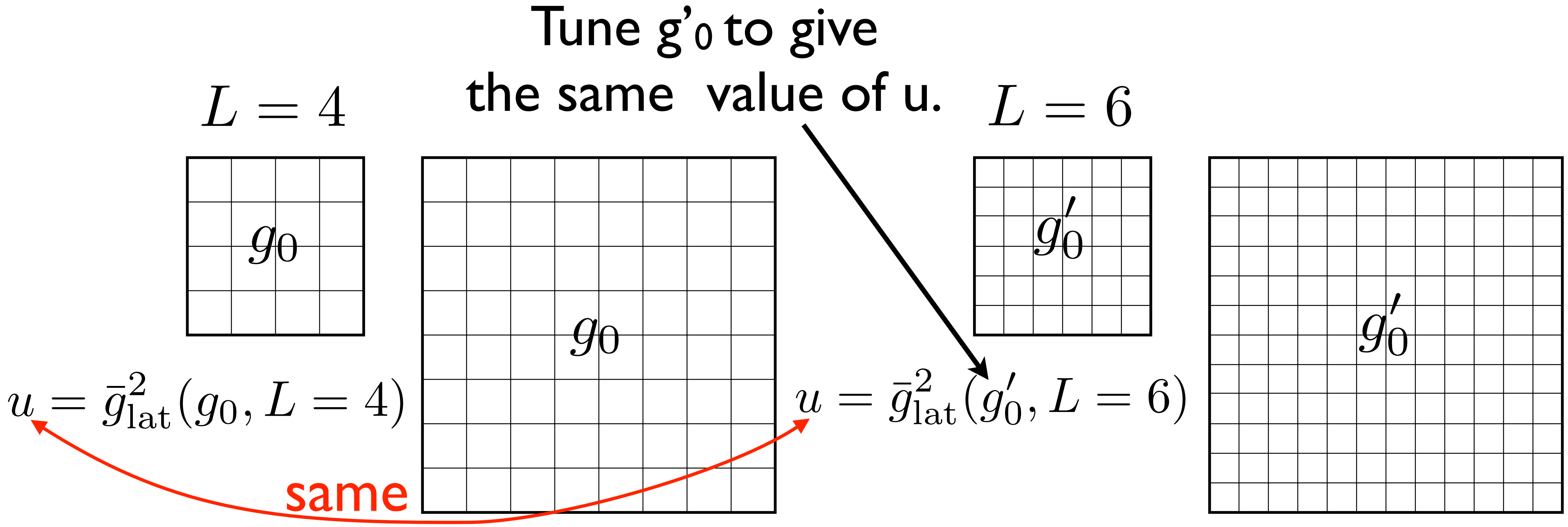
Outline

- Step scaling.
- Two schemes on the twisted box:
  - ☆ Twisted Polyakov Loop (TPL) scheme.
  - ☆ Wilson flow (WF) scheme.
- Numerical (preliminary) results.
- Outlook.

The step-scaling method

The idea

M. Luscher, P. Weisz, U. Wolff, 1991.

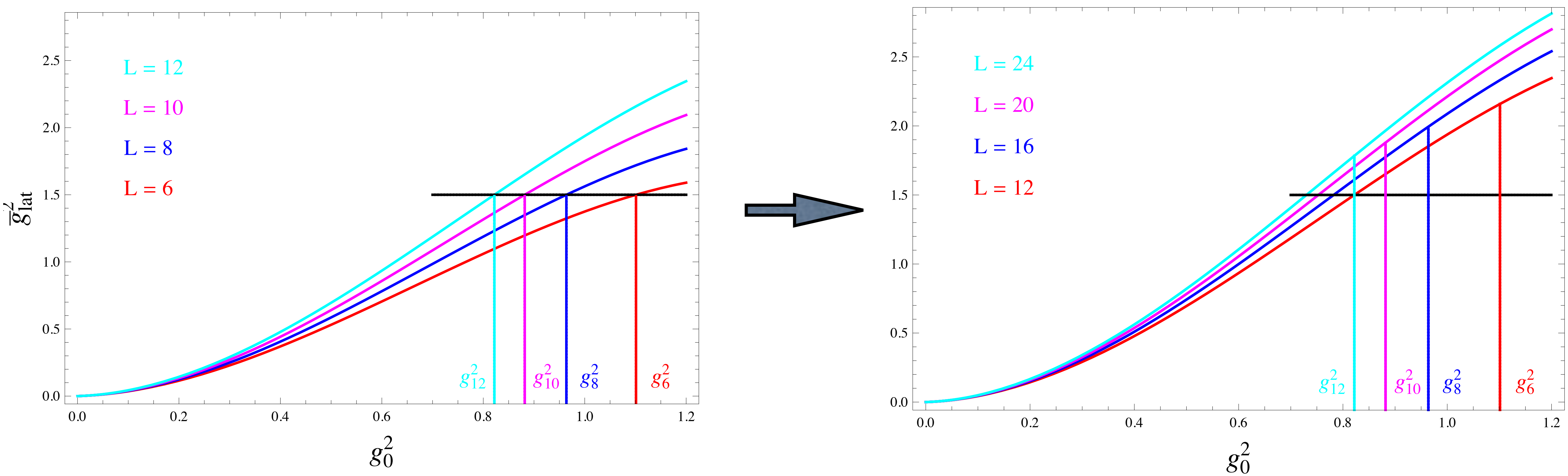


$\Sigma(u, L=4) = \bar{g}^2_{\rm lat}(g_0, L=8)$ $\quad$ $\Sigma(u, L=6) = \bar{g}^2_{\rm lat}(g_0', L=12)$

$$\sigma(u) = \lim_{a\to 0}\Sigma, \quad r_\sigma = \frac{\sigma(u)}{u} \xrightarrow{\text{fixed points}} 1.$$

The step-scaling method

The practice



- Massless unimproved staggered fermions with Wilson's plaquette gauge action.
- Compute $\bar{g}^2_{\rm lat}$ at many g_0^2 for each volume, and then interpolate volume by volume.
- Very challenging to pin down percentage-level effects in $r_\sigma = \dfrac{\sigma(u)}{u}$.

Twisted box

removing the zero modes

- **Gauge field:** G. 't Hooft, 1979

$$U_\mu(x+\hat{\nu}L) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger,\ \ \nu = 1,2,$$

where the twist matrices Ω_ν satisfy

$$\Omega_1\Omega_2 = \mathrm{e}^{2i\pi/3}\Omega_2\Omega_1,\ \ \Omega_\mu\Omega_\mu^\dagger = 1,\ \ (\Omega_\mu)^3 = 1,\ \ \mathrm{Tr}(\Omega_\mu) = 0.$$

- **Fermion:** If $\psi(x+\hat{\nu}L) = \Omega_\nu\psi(x)$

$$\Rightarrow\ \psi(x+\hat{\nu}L+\hat{\rho}L) = \Omega_\rho\Omega_\nu\psi(x) \neq \Omega_\nu\Omega_\rho\psi(x)$$

- **The fermion "smell" dof:** $N_s = N_c$ G. Parisi, 1983

$$\psi_\alpha^a(x+\hat{\nu}L) = \mathrm{e}^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(x)(\Omega_\nu)_{\beta\alpha}^\dagger.$$

TPL scheme

- Polyakov loops in the twisted directions:

$$P_1(y,z,t) = \mathrm{Tr}\langle \Pi_j U_1(j,y,z,t)\Omega_1 \mathrm{e}^{2iy\pi/3L}\rangle$$

with gauge and translation invariance.

- The renormalised coupling constant:

$$g^2_{\mathrm{TP}}(L) = \frac{1}{k}\frac{\langle \sum y,z P_1(y,z,L/2)P_1^*(0,0,0)\rangle}{\langle \sum x,y P_3(x,y,L/2)P_3^*(0,0,0)\rangle},$$

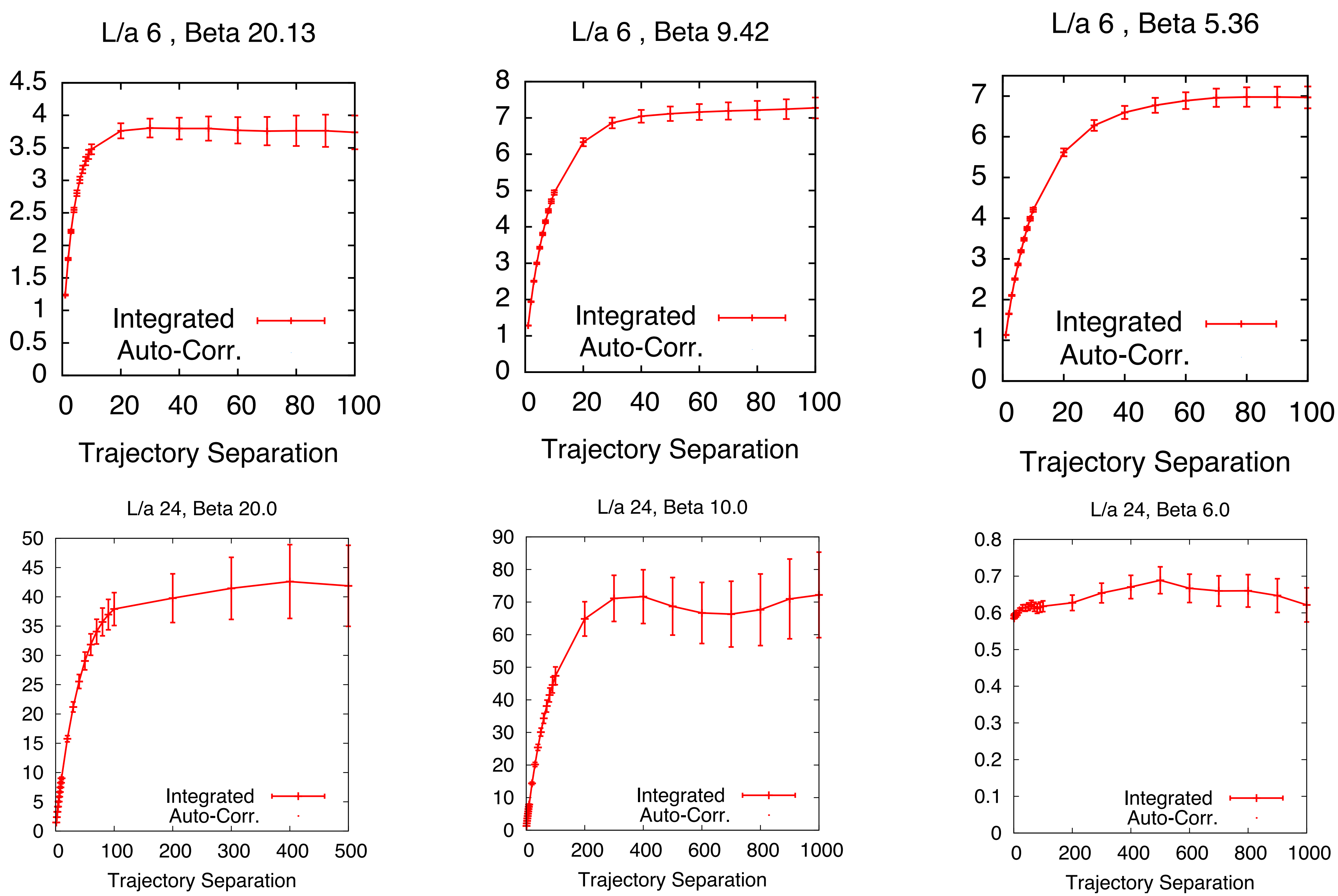
where $k = \frac{1}{24\pi^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{n^2+(1/3)^2} \sim 0.031847$

- Special feature:

At $L \to \infty$, $g^2_{\mathrm{TP}} \to \frac{1}{k} \sim 32$ if there is no IRFP.

Challenge in using the TPL scheme

Autocorrelation of the coupling

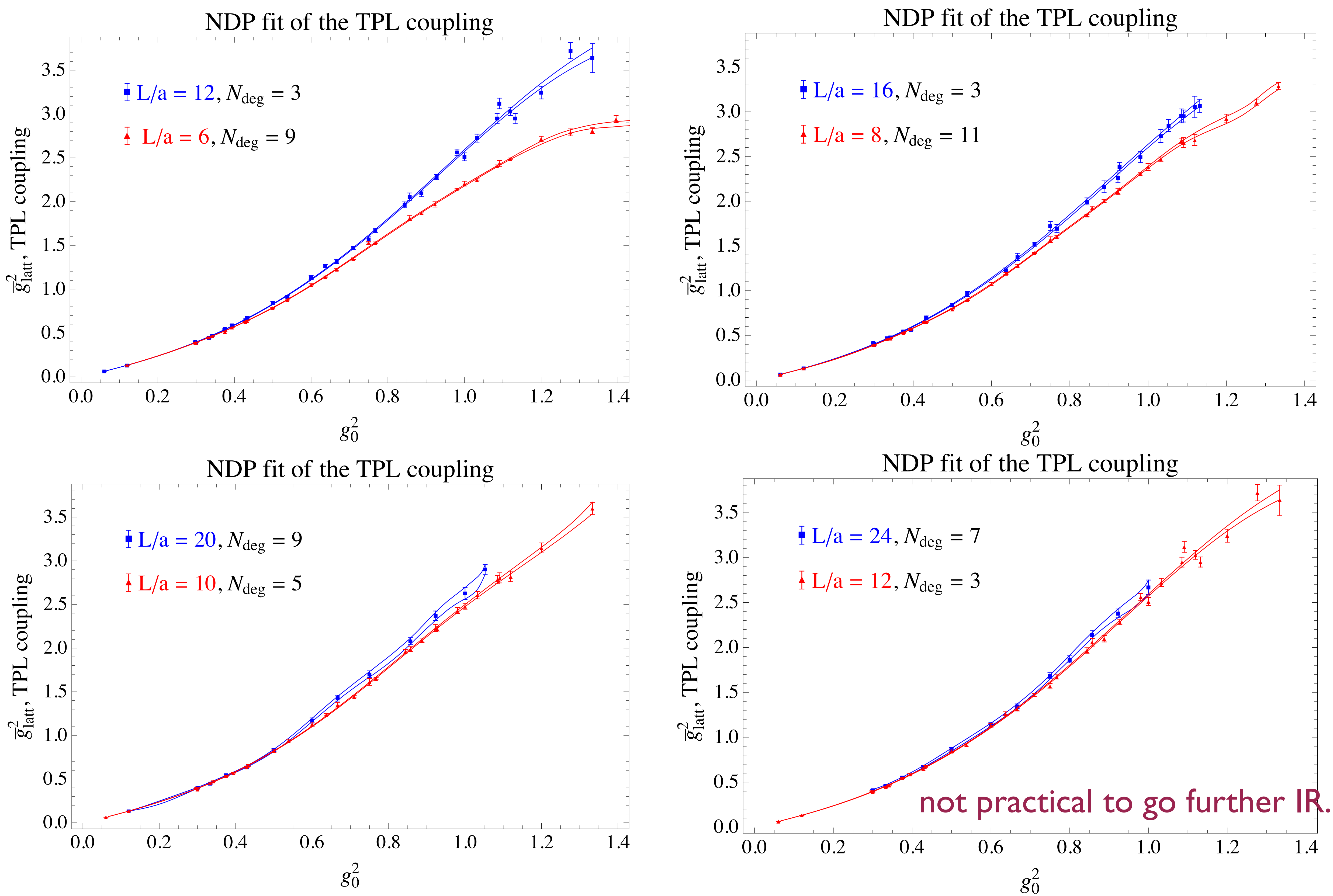


- Autocorrelation time grows with physical volume.

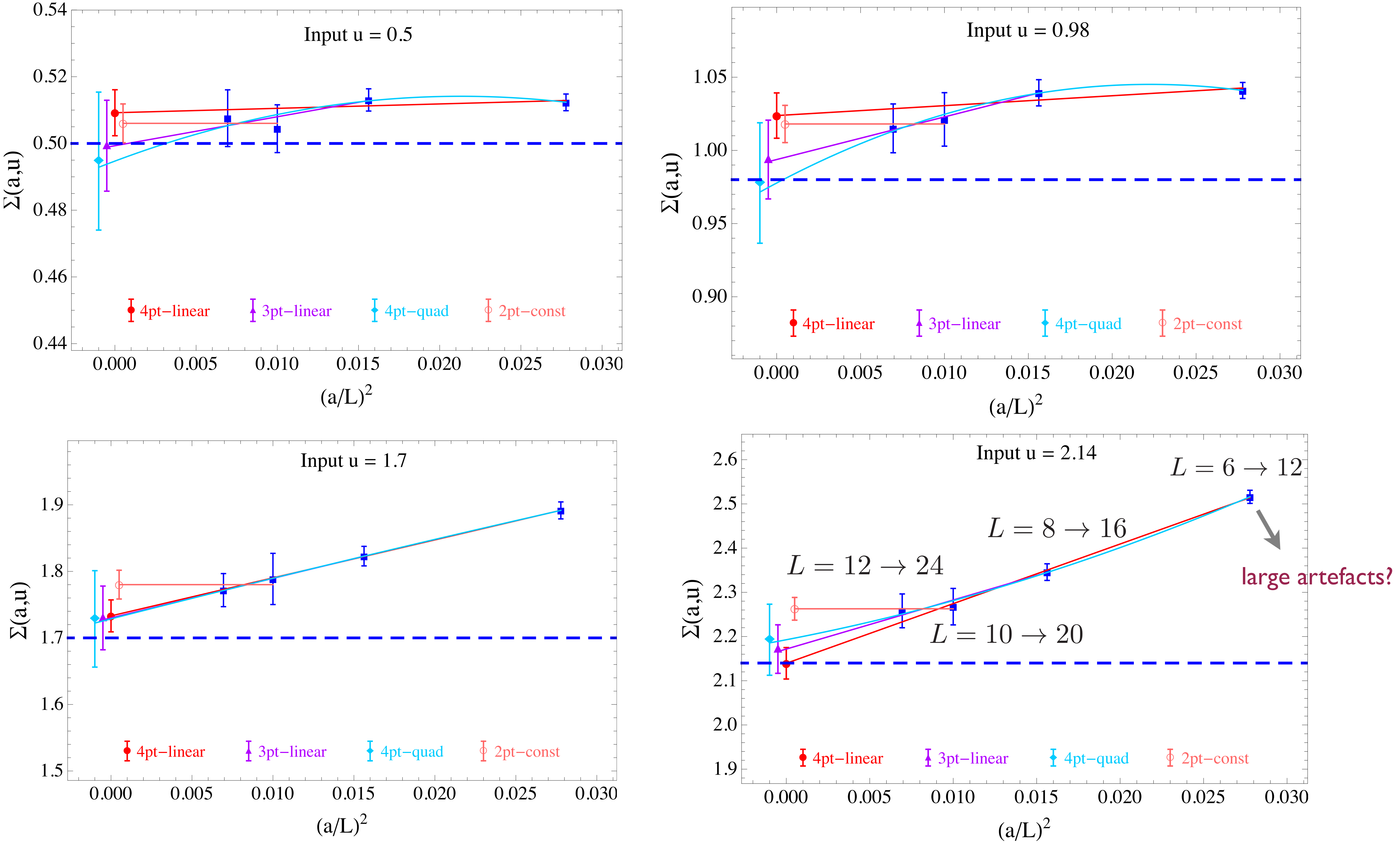
  ➡ Very challenging to have good statistics for large volumes at low beta.

Bare-coupling interpolation

TPL scheme



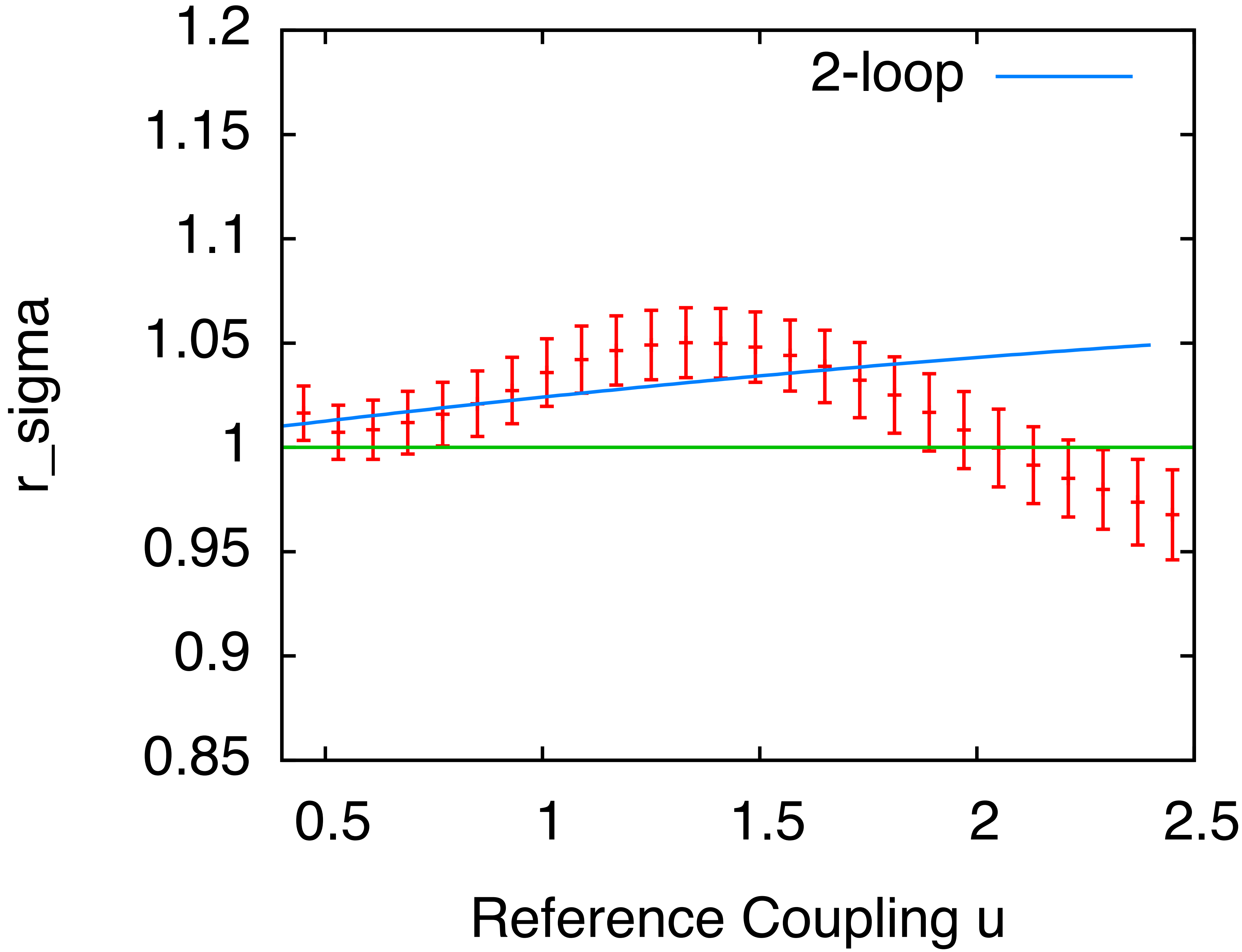
Continuum extrapolation
TPL scheme



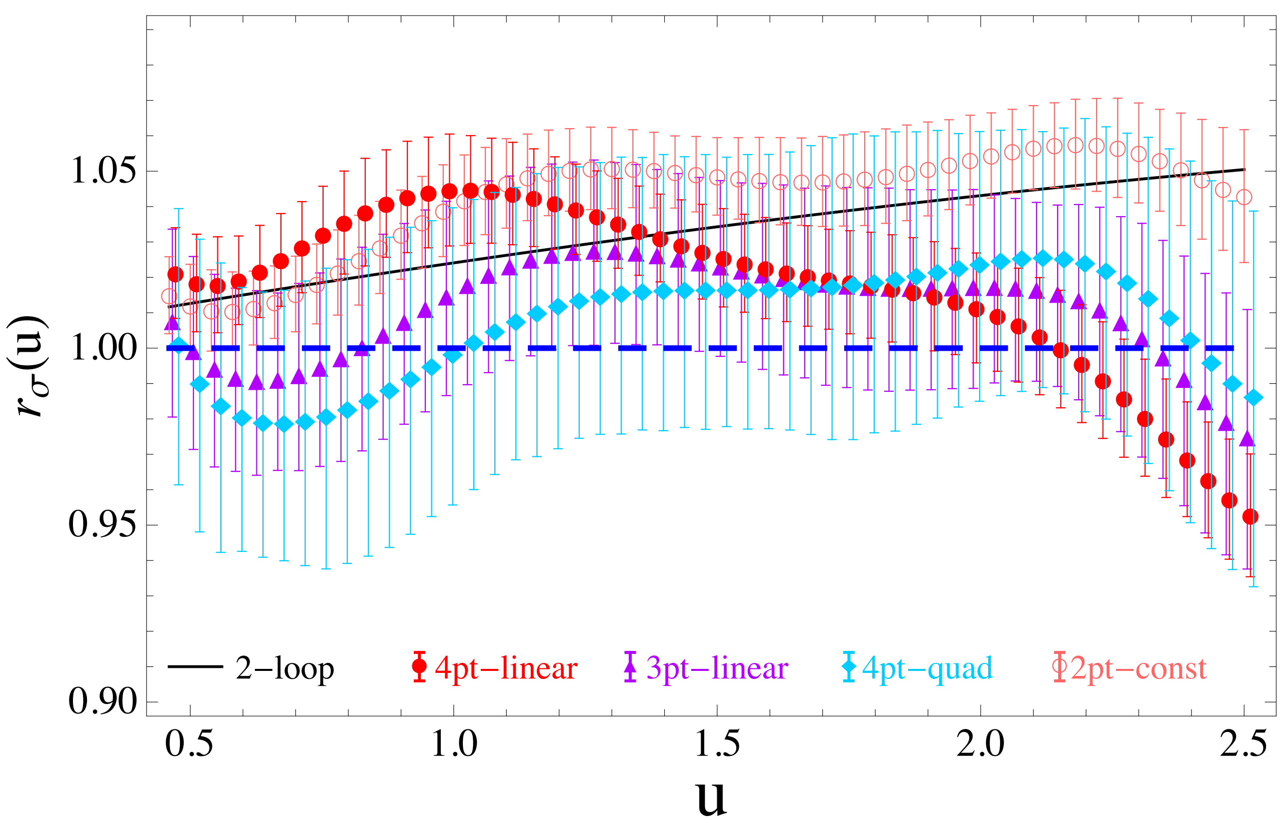
Result without the L/a=24 lattices

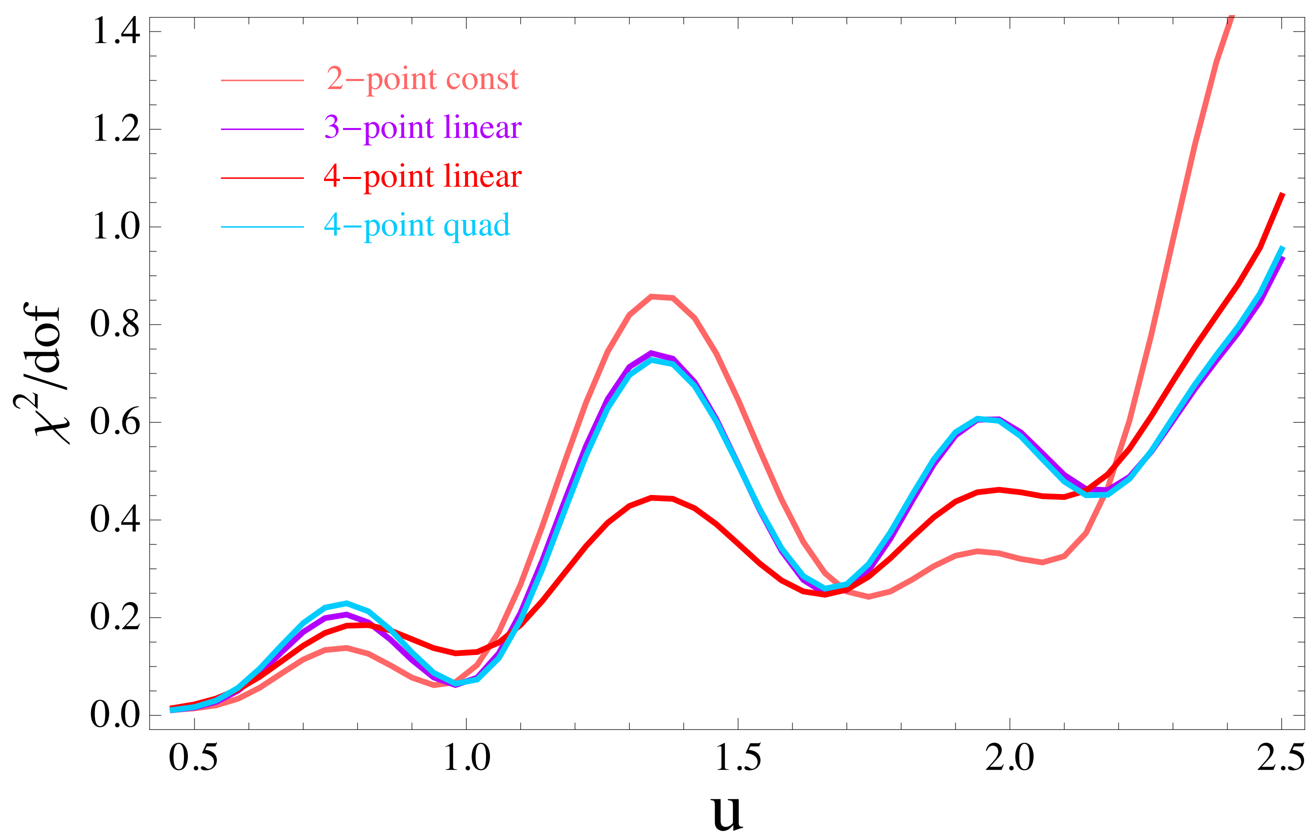
TPL scheme

C-JDL, K.Ogawa, H.Ohki, E.Shintani, JHEP 1208 (2012) 096



Result with the L/a=24 lattices
TPL scheme





Systematic error was severely underestimated without the L/a=24 data.

The Gradient Flow

- "Diffusion" of the gauge fields:

$$\dot{V}_t(x,\mu) = -g_0^2\,\{\partial_{x,\mu} S_{\rm w}(V_t)\}\,V_t(x,\mu),\;\; V_t(x,\mu)|_{t=0} = U(x,\mu).$$

- The radius of diffusion is $\sqrt{8t}$.

$$c_\tau = \frac{\sqrt{8t}}{L}$$

- Local operators are also diffused.

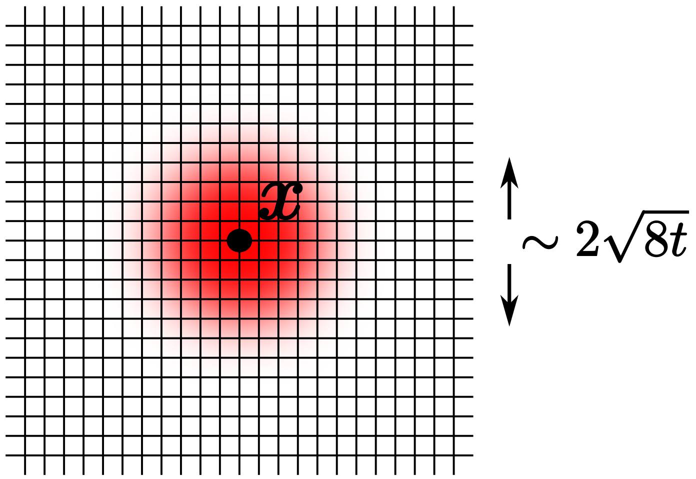


Figure taken from M.Luscher, Lattice 2013

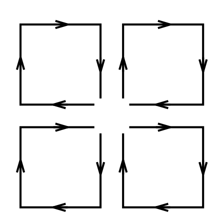
The Gradient Flow scheme

- The quantity, $\langle E(t)\rangle = \frac{1}{4}\langle G_{\mu\nu}(t)G_{\mu\nu}(t)\rangle$, is finite when expressed in terms of renormalised coupling at positive flow time.

- In a colour-twisted box, can define,

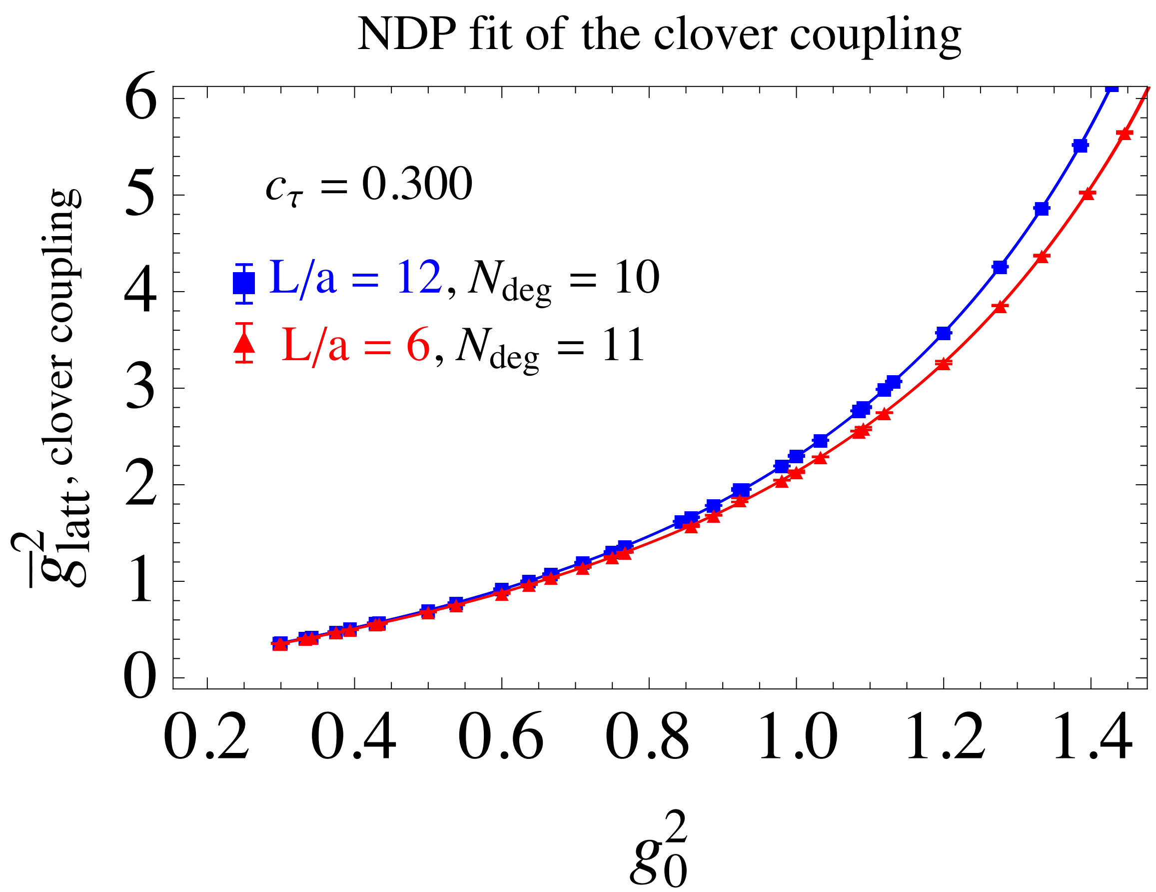
$$\overline{g}^2_{\mathrm{GF}}(L) = \mathcal{N}^{-1}t^2\langle E(t)\rangle = \overline{g}^2_{\mathrm{MS}} + \mathcal{O}(\overline{g}^4_{\mathrm{MS}}),$$

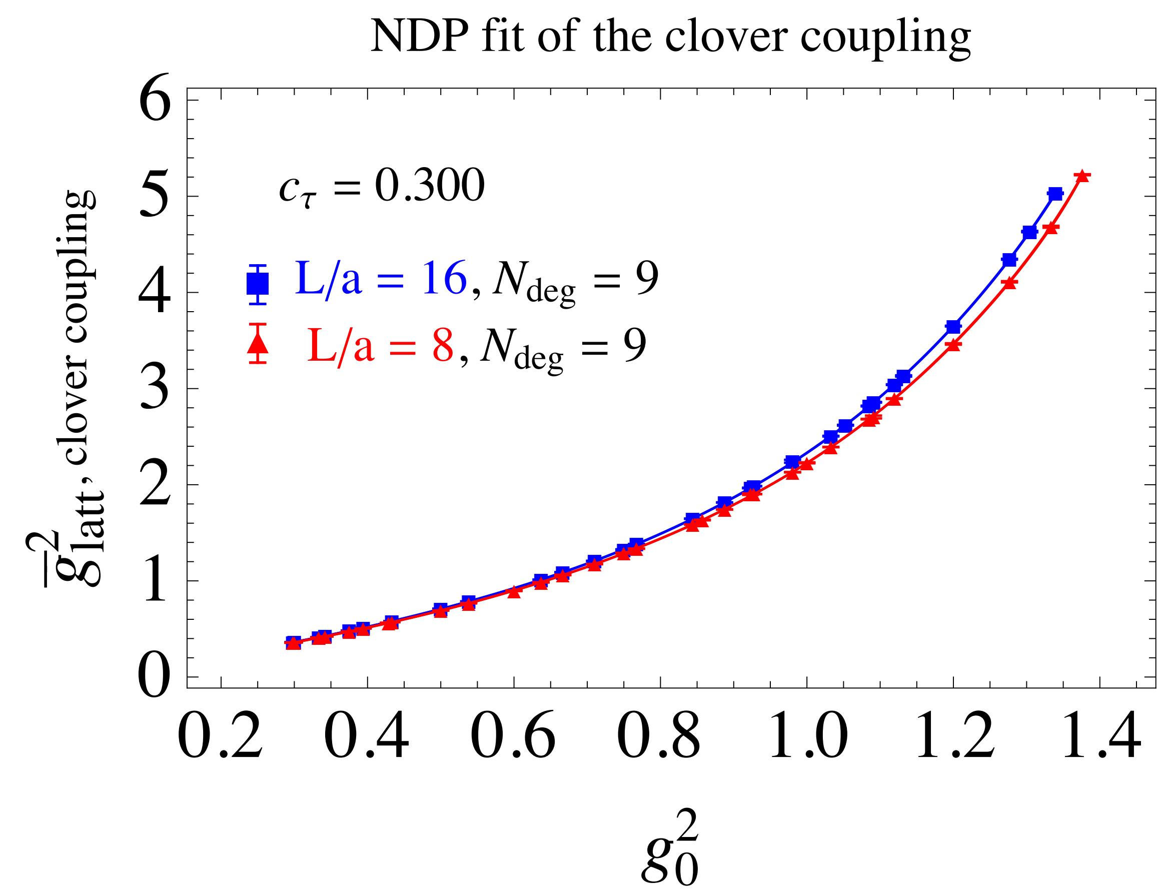
  where $\mathcal{N}$ can be computed in perturbation theory.

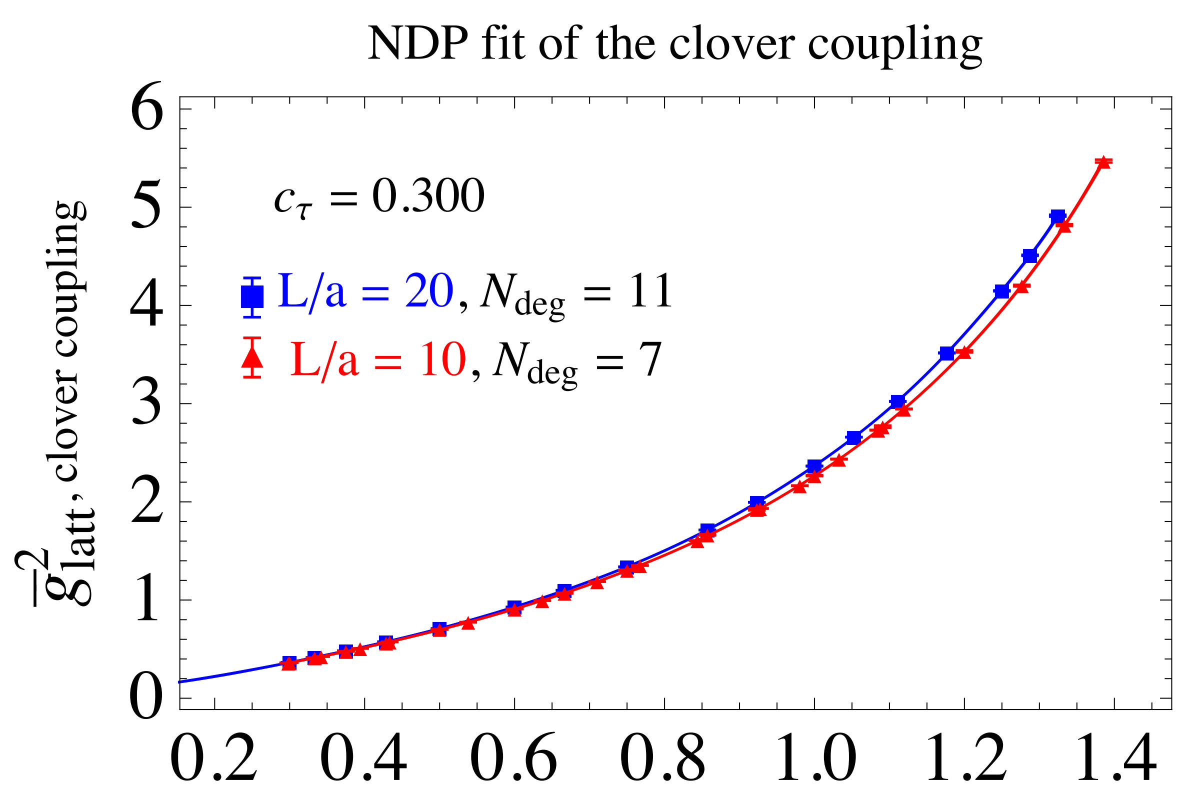
- Use the clover operator, , to extract $\langle E(t)\rangle$.

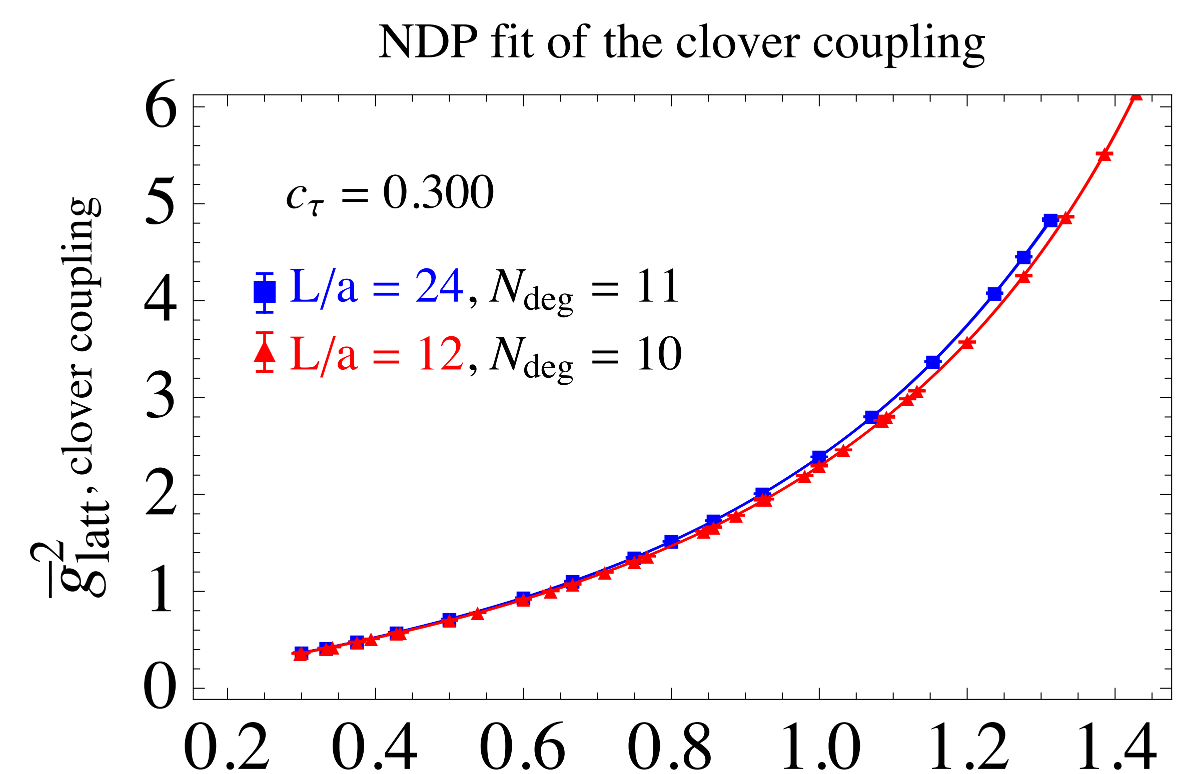
- Autocorrelation time ~25 HMC trajectories for all simulations.

Bare-coupling interpolation
Wilson flow scheme

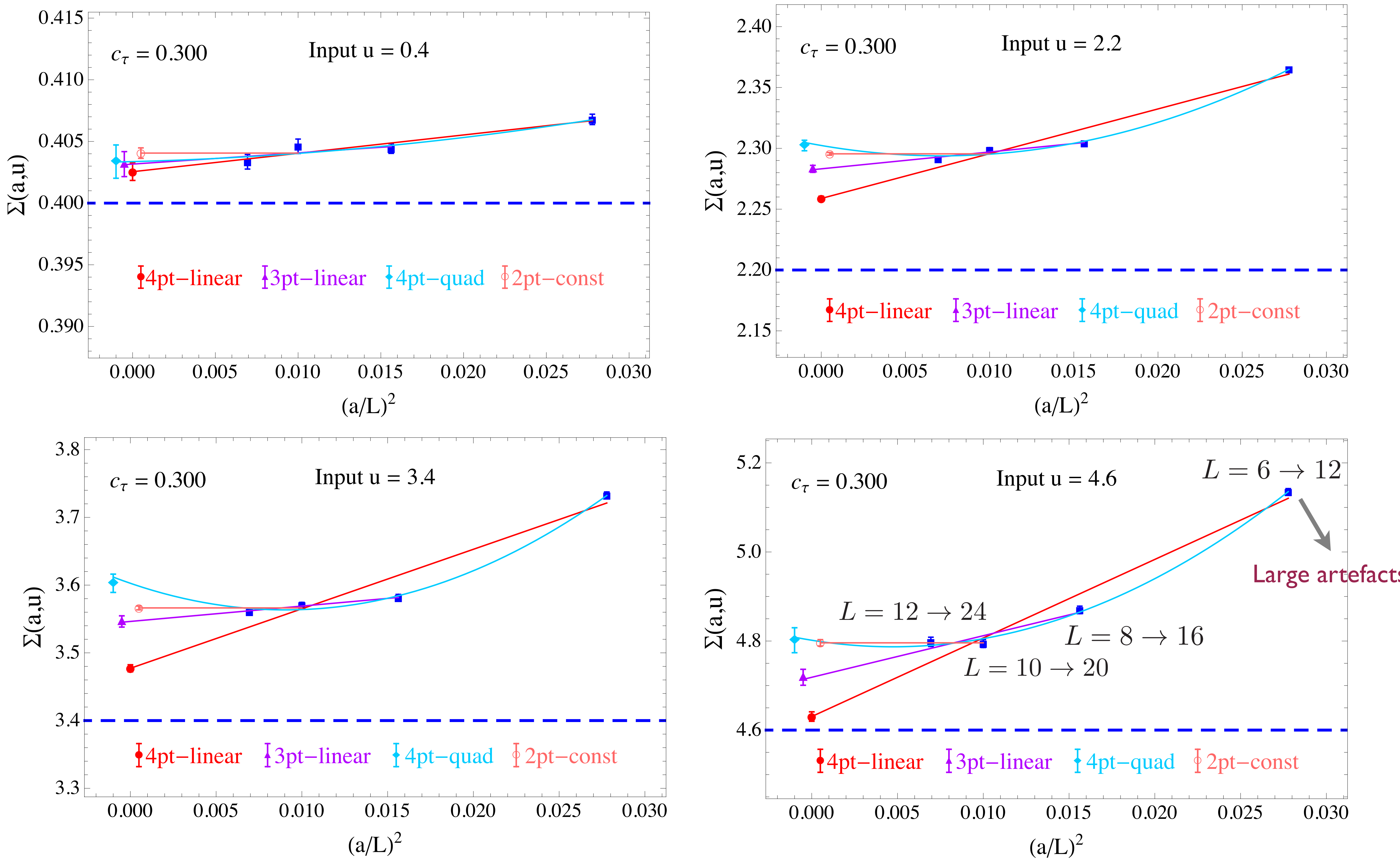




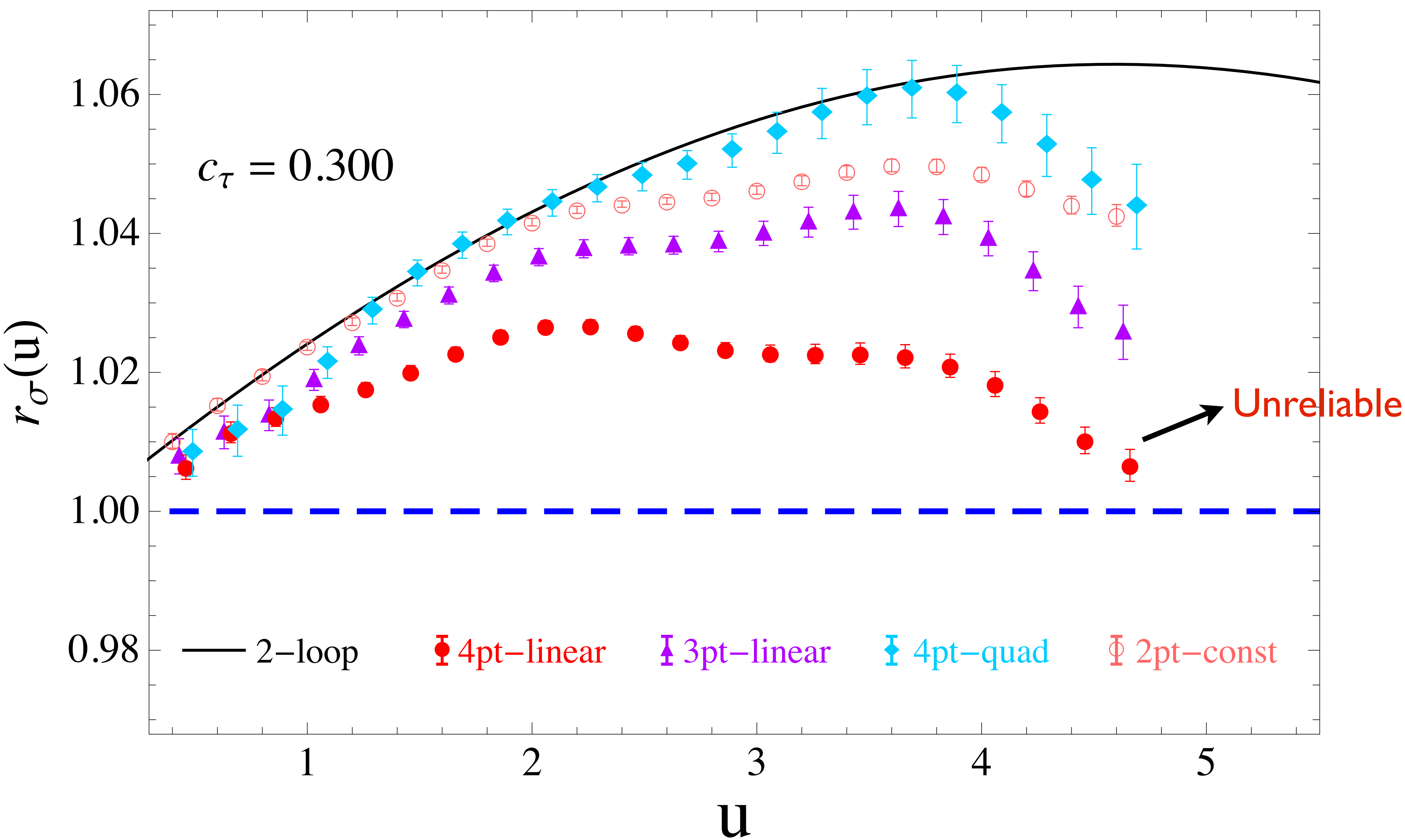




Continuum extrapolation
Wilson flow scheme



Preliminary result
Wilson flow scheme



Remarks and outlook

- Calculation in the TPL scheme shows no definite conclusion for IR comformality hitherto.
- On the other hand, the Gradient Flow scheme offers a very nice/promising tool.
- We are currently generating data to go further IR in the Gradient Flow scheme.