The Muon $g-2$: present and future

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SCGT14Mini

Outline of Talk:

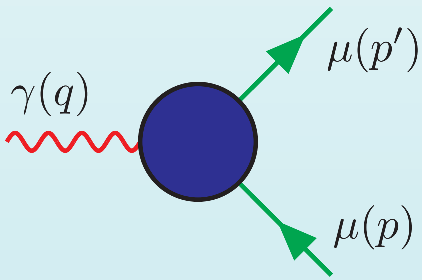
- Introduction
- Standard Model Prediction for a_μ
- The hadronic effects and precision limitations
- Effective field theory: the Resonance Lagrangian Approach
- The hadronic LbL: setup and problems
- Theory vs experiment: do we see New Physics?
- Future

Introduction

Particle with spin $\vec{s} \Rightarrow$ magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s}\ ; \quad g_\mu = 2\,(1 + a_\mu)$$

Dirac: $g_\mu = 2 \quad , \quad a_\mu = \frac{\alpha}{2\pi} + \cdots$ muon anomaly



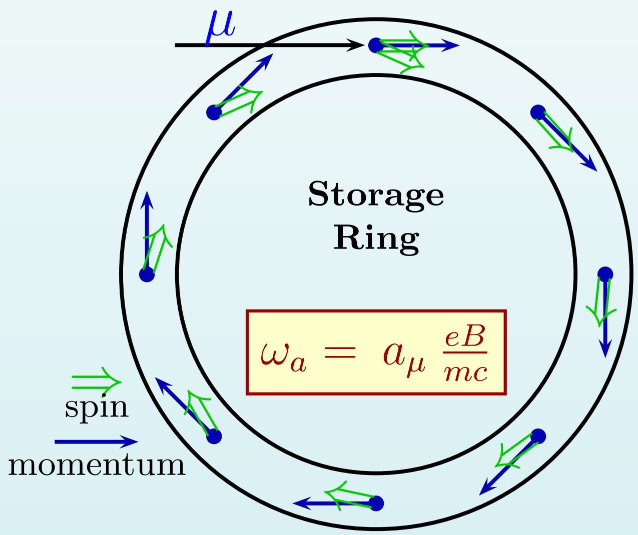
Electromagnetic Lepton Vertex

$$= (-\mathrm{i}e)\ \bar{u}(p')\left[\gamma^\mu F_1(q^2) + \mathrm{i}\,\frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2(q^2)\right] u(p)$$

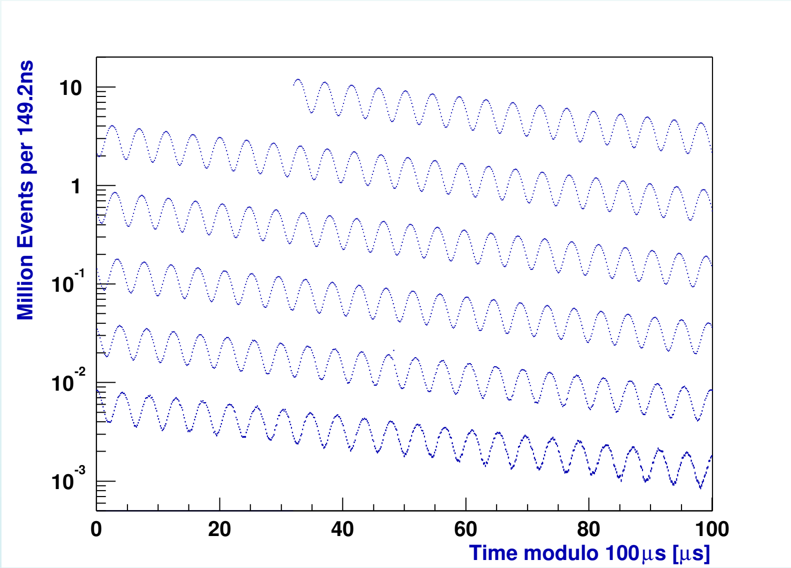
$$F_1(0) = 1\ ; \quad F_2(0) = a_\mu$$

a_μ responsible for the Larmor precession

Larmor precession $\vec{\omega}$ of beam of spin particles in a homogeneous magnetic field $\vec{B}$



actual precession × 2 $\sim 12'$/circle



Magic Energy: $\vec{\omega}$ is directly proportional to $\vec{B}$ at magic energy ~ 3.1 GeV

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]_{\text{at "magic } \gamma\text{"}}^{E\sim 3.1\,\text{GeV}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right]$$

CERN, BNL g-2 experiments

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2\,(1.00119 \pm 0.00005)$

In a uniform magnetic field, as in muon $g-2$ experimental setup:

$$a_\mu = \frac{\omega_a}{\gamma\, \omega_c} = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

- $\omega_p = (e/m_p c)\langle B\rangle$ free proton NMR frequency
- $\mathcal{R} = \omega_a/\omega_p = 0.003\,707\,2063(20)$ from E-821
- $\lambda = \omega_L/\omega_p = \mu_\mu/\mu_p =$ from hyperfine splitting of muonium

value used by E-821	$3.18334539(10)$	
new value	$3.183345107(84)$	CODATA 2011: [raXiv:1203.5425v1]

⇒change in a_μ: $+1.10 \times 10^{-10}$

$a_\mu^{\rm exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10}$ updated

Standard Model Prediction for a_μ

What is new?

- new CODATA values for lepton mass ratios m_μ/m_e, m_μ/m_τ
- spectacular progress by Aoyama, Hayakawa, Kinoshita and Nio on 5–loop QED calculation (as well as improved 4–loop results)
  - $O(\alpha^5)$ electron $g-2$, substantially more precise $\alpha(a_e)$
  - Complete $O(\alpha^5)$ muon $g-2$, settles better the QED part

❒ QED Contribution

The QED contribution to a_μ has been computed through 5 loops

Growing coefficients in the α/π expansion reflect the presence of large $\ln \frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops. Input:

$a_e^{\exp} = 0.001\,159\,652\,180\,73(28)$ Gabrielse et al. 2008

$\alpha^{-1}(a_e) = 137.0359991657(331)(68)(46)(24)[0.25\ \mathrm{ppb}]$ Aoyama et al 2012

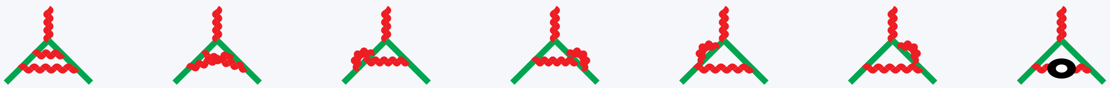
$$a_\mu^{\mathrm{QED}} = 116\,584\,718.851\ \underbrace{(0.029)}_{\alpha_{\mathrm{inp}}}\ \underbrace{(0.009)}_{m_e/m_\mu}\underbrace{(0.018)}_{\alpha^4}\underbrace{(0.007)}_{\alpha^5}[0.36]\times 10^{-11}$$

The current uncertainty is well below the $\pm 60\times 10^{-11}$ experimental error from E821

# n of loops	$C_i\ [(\alpha/\pi)^n]$	$a_\mu^{\mathrm{QED}}\times 10^{11}$
1	+0.5	116140973.289 (43)
2	+0.765 857 426(16)	413217.628 (9)
3	+24.050 509 88(32)	30141.9023 (4)
4	+130.8796(63)	381.008 (18)
5	+753.290(1.04)	5.094 (7)
tot		116584718.851 (0.036)

❶ **1 diagram** Schwinger 1948

❷ **7 diagrams** Peterman 1957, Sommerfield 1957

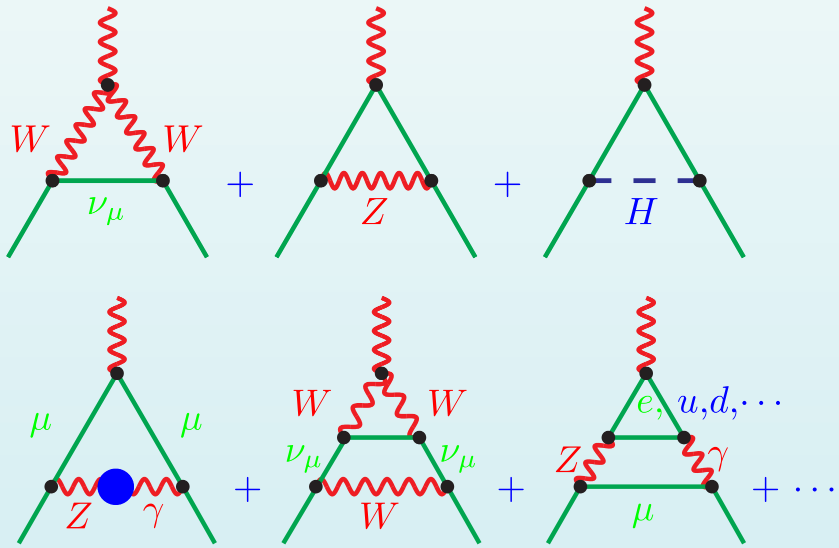


❸ **72 diagrams** Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996

❹ **871 diagrams** Kinoshita 1999, Kinoshita, Nio 2004, Ayoama et al. 2009/2012

❺ **estimates of leading terms** Karshenboim 93, Czarnecki, Marciano 00, Kinoshita, Nio 05
❏ **all 12672 diagrams (fully automated numerical)** Ayoama et al. 2012

Weak contributions



Brodsky, Sullivan 67, ...,
Bardeen, Gastmans, Lautrup 72
Higgs contribution tiny!

$a_\mu^{\mathrm{weak}(1)} = (194.82 \pm 0.02) \times 10^{-11}$

Kukhto et al 92
potentially large terms $\sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$
Peris, Perrottet, de Rafael 95
quark-lepton (triangle anomaly) cancellation
Czarnecki, Krause, Marciano 96

Heinemeyer, Stöckinger, Weiglein 04, Gribouk, Czarnecki 05 full 2–loop result

Most recent evaluations: improved hadronic part (beyond QPM)

$$a_\mu^{\mathrm{weak}} = (154.0 \pm 1.0[\mathrm{had}] \pm 0.3[\mathrm{m_H}, \mathrm{m_t}, 3-\mathrm{loop}]) \times 10^{-11}$$

new: m_H known!

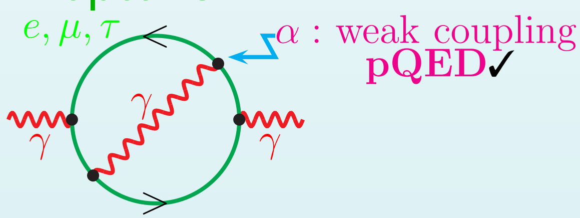
(Knecht, Peris, Perrottet, de Rafael 02, Czarnecki, Marciano, Vainshtein 02, FJ 12, Gnendiger, Stöckinger, Stöckinger-Kim 13)

Hadronic stuff: the limitation to theory

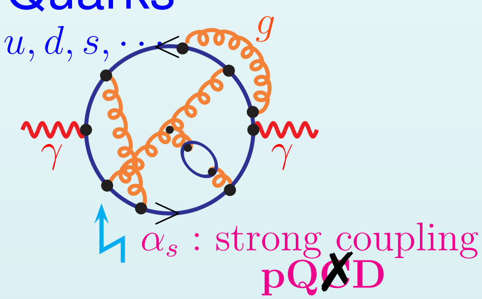
General problem in electroweak precision physics:

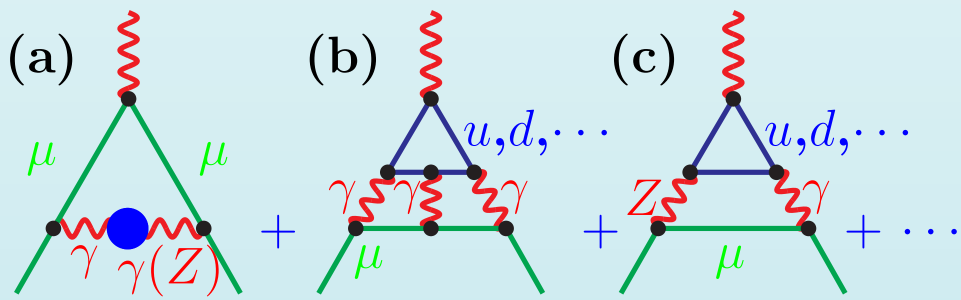
contributions from hadrons (quark loops) at low energy scales

Leptons



Quarks





(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$ — Light quark loops

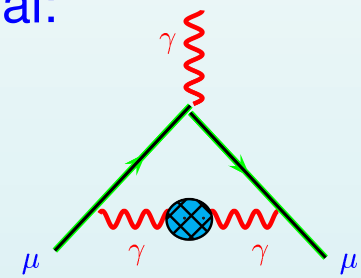
(b) Hadronic light-by-light scattering $O(\alpha^3)$ — $\downarrow$

(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$ — Hadronic “blobs”

❐ Evaluation of $a_\mu^{\rm had}$

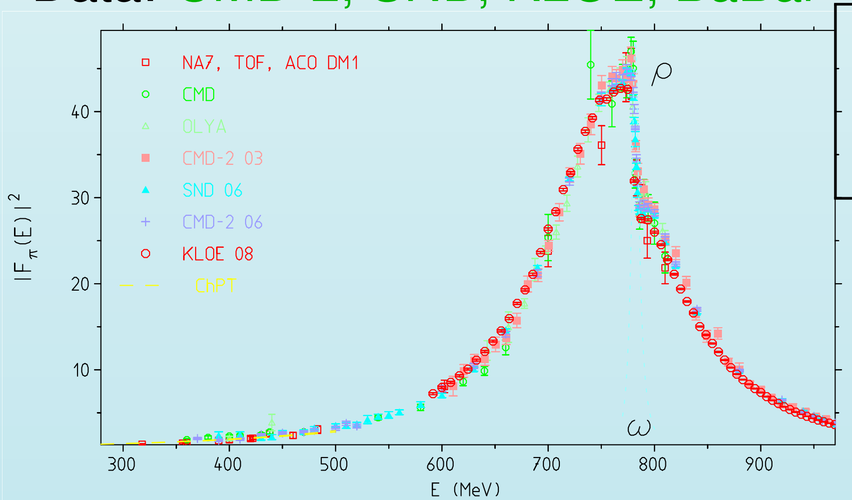
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} ds \, \frac{R_\gamma^{\rm data}(s)\; \hat{K}(s)}{s^2} + \int_{E^2_{\rm cut}}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\; \hat{K}(s)}{s^2} \right)$$



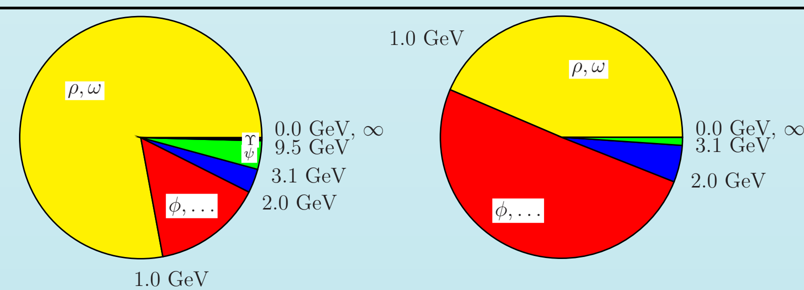
- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

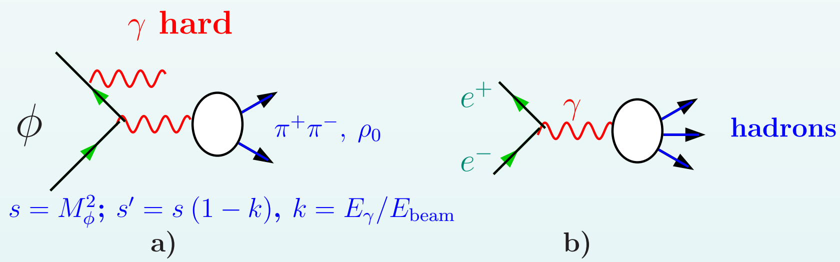
Data: CMD-2, SND, KLOE, BaBar



$$a_\mu^{\rm had(1)} = (690.7 \pm 4.7)[695.5 \pm 4.1]\; 10^{-10}$$

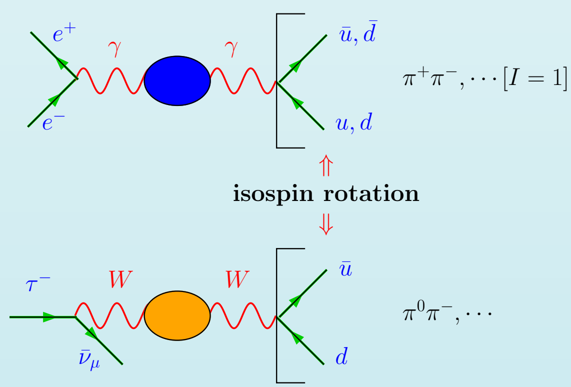
e^+e^-–data based [incl. BaBar MD09]





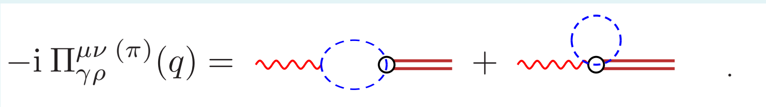
a) Radiative return,   b) Standard energy scan.

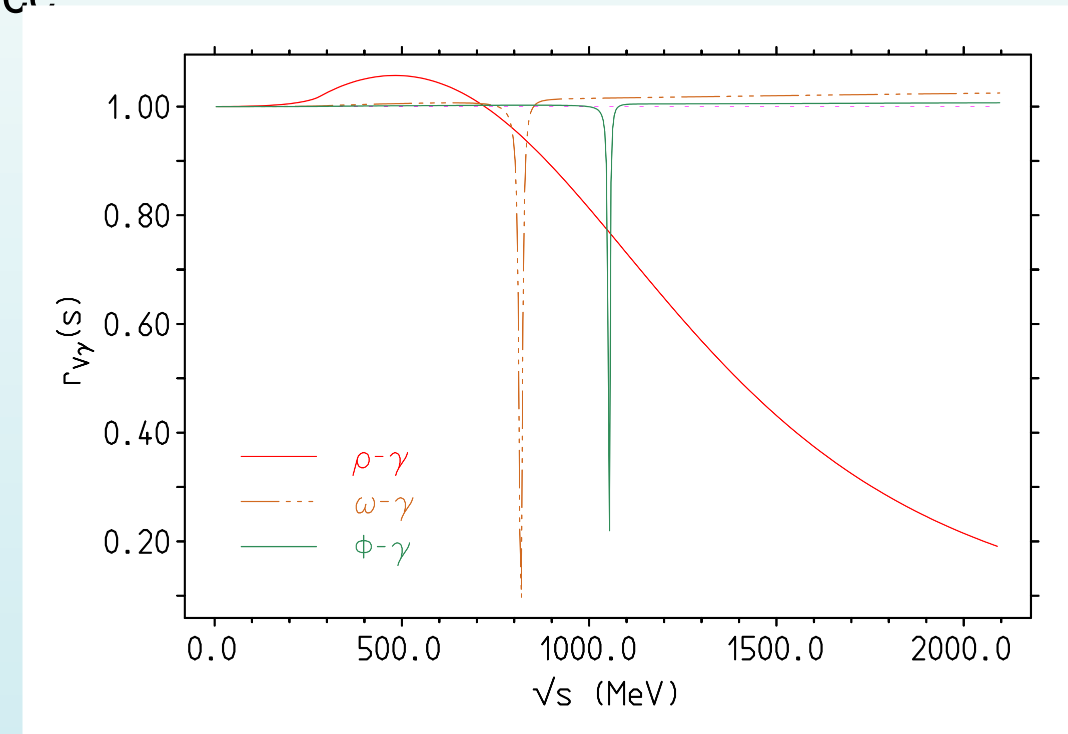
❖Good old idea: use isospin symmetry to include existing high quality τ–data (including isospin corrections)



Corrected data: large discrepancy [∼ 10%] persists! τ vs. e^+e^- problem! [manifest since 2002]

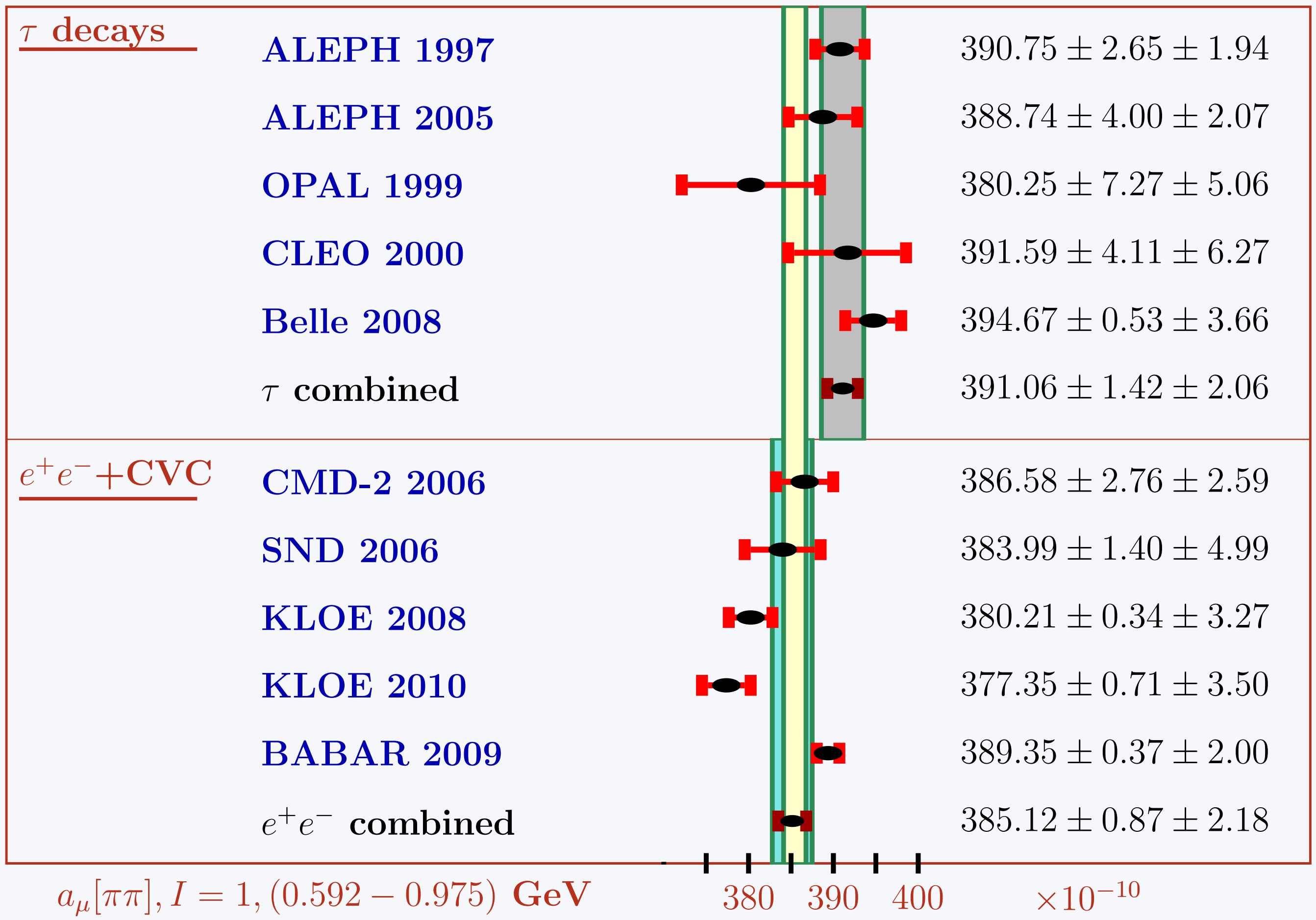
Recent: τ (charged channel) vs. e^+e^- (neutral channel) puzzle resolved
F.J.& R. Szafron, $\rho - \gamma$ interference
(absent in charged channel):



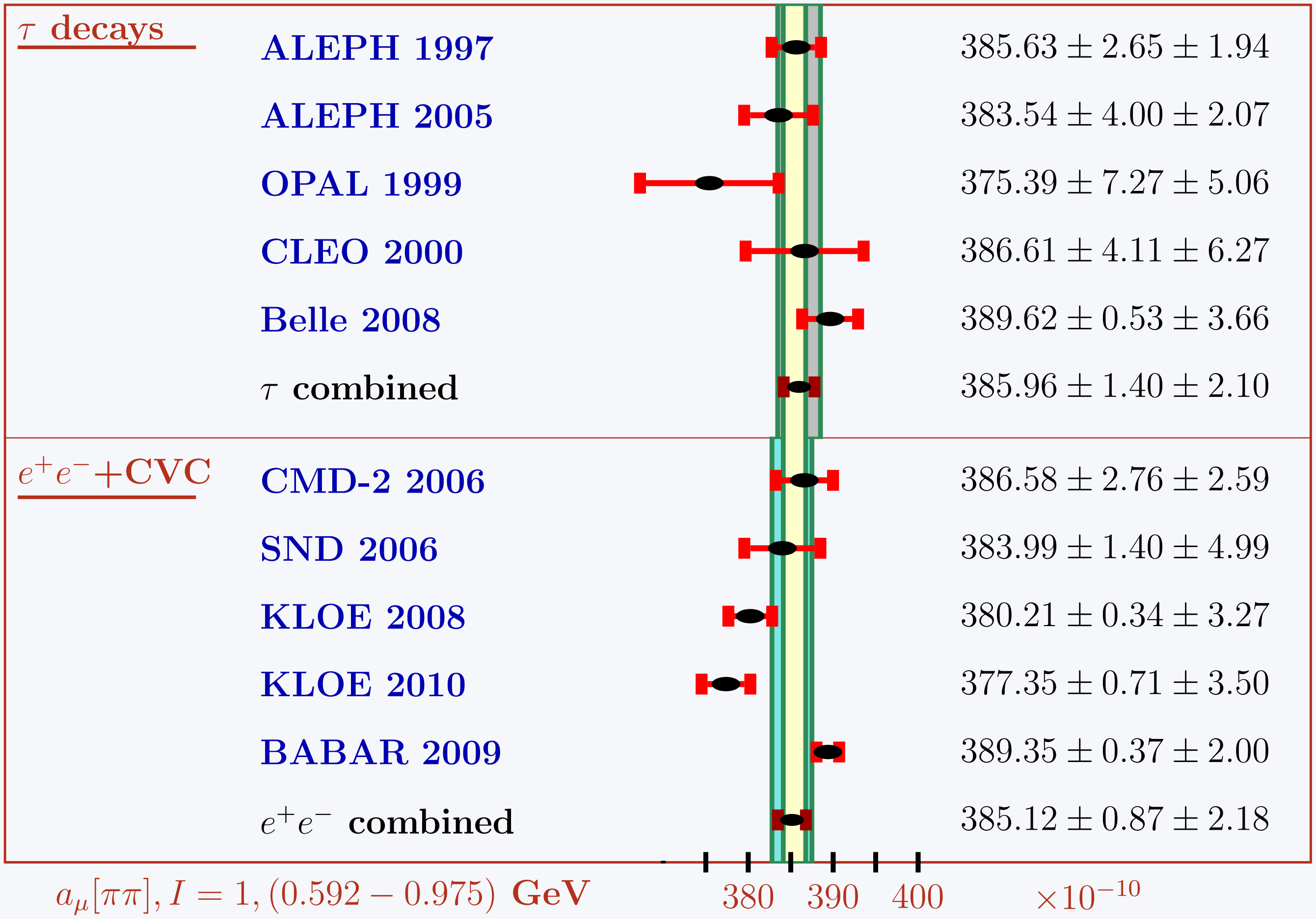


$$v_0(s) = r_{\rho\gamma}(s)\, R_{\mathrm{IB}}(s)\, v_-(s)$$

- τ require to be corrected for missing $\rho - \gamma$ mixing!
- results obtained from e^+e^- data is what goes into a_μ
- off-resonance tiny for ω, ϕ in $\pi\pi$ channel (scaled up $\Gamma_V/\Gamma(V \to \pi\pi)$)



I=1 part of $a_\mu^{\rm had}[\pi\pi]$



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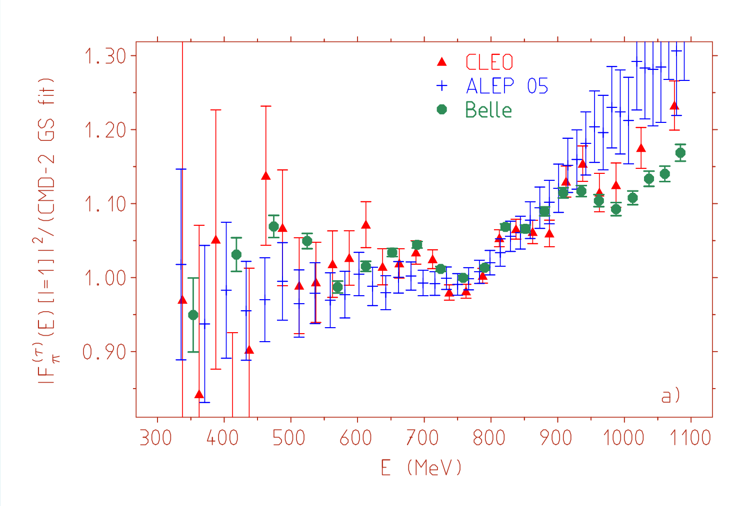
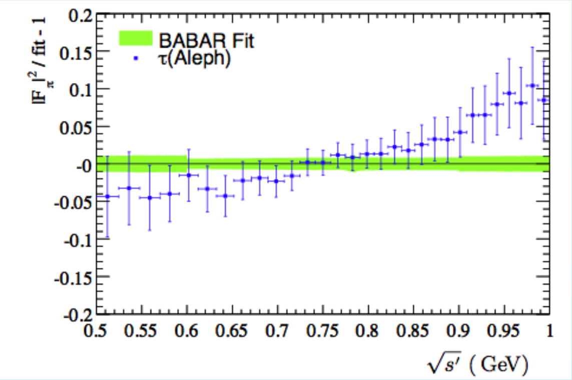
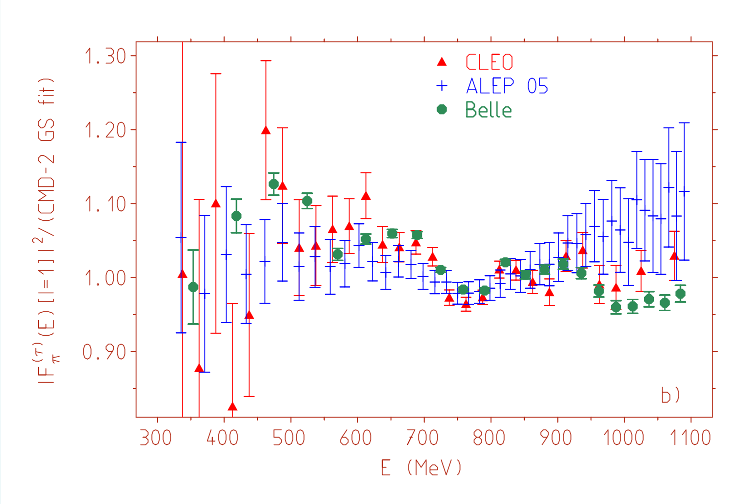


$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit)

Best “proof”:





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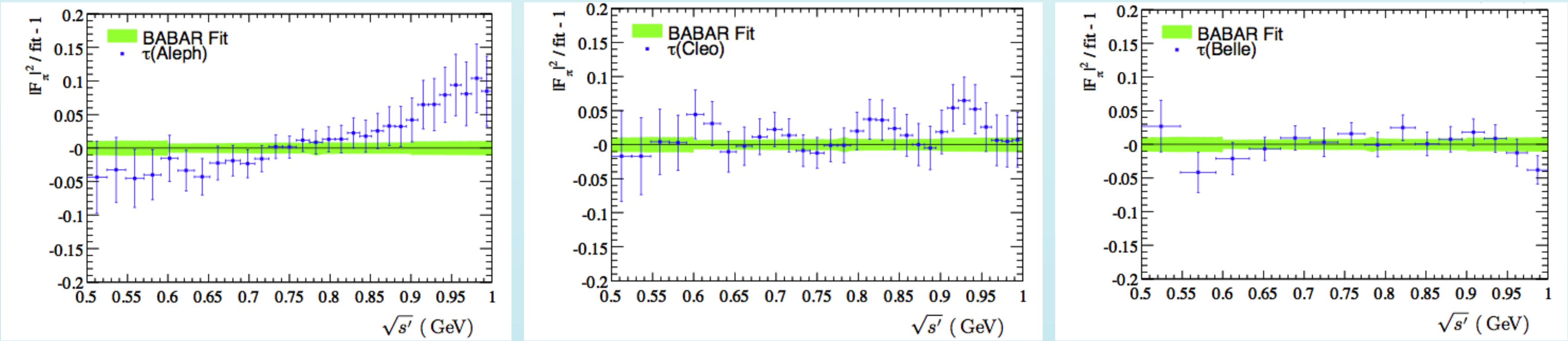
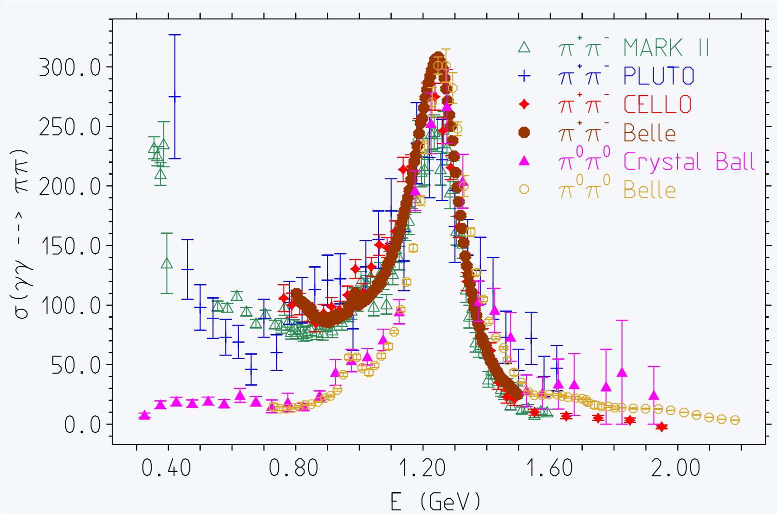


Is our model viable?



How photons couple to pions? Use $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ as a probe: what we see: 1) below about 1 GeV photons couple to pions as point-like objects (i.e. to the charged ones overwhelmingly), 2) at higher energies the photons see the quarks exclusively and form the prominent tensor resonance $f_2(1270)$. Plotted $2\,\sigma(\pi^0\pi^0)$ vs. $\sigma(\pi^+\pi^-)$

Strong tensor meson resonance in $\pi\pi$ channel $f_2(1270)$ with photons directly probe the quarks! Contribution to $a_\mu^{\mathrm{had\,LbL}}$?

Effective field theory: the Resonance Lagrangian Approach

HVP dominated by spin 1 resonance physics! need theory of $\rho, \omega, \phi, \cdots$

❒ Principles to be included: Chiral Structure of QCD, VMD & electromagnetic gauge invariance.

❖General framework: resonance Lagrangian extension of chiral perturbation theory (CHPT), i.e. implement VMD model with Chiral structure of QCD. Specific version Hidden Local Symmetry (HLS) effective Lagrangian Bando, Kugo, Yamawaki. First applied to HLbL of muon $g-2$ Hayakawa, Kinoshita, Sanda.

Global Fit strategy:
Data below $E_0 = 1.05$ GeV (just above the ϕ) constrain effective Lagrangian couplings, using 45 different data sets (6 annihilation channels and 10 partial width decays).

❒ Effective theory predicts cross sections:

$$\pi^+\pi^-,\ \pi^0\gamma,\ \eta\gamma,\ \eta'\gamma,\ \pi^0\pi^+\pi^-,\ K^+K^-,\ K^0\bar{K}^0 \quad \text{(83.4\%)},$$

- Missing part:

$$4\pi, 5\pi, 6\pi, \eta\pi\pi, \omega\pi \text{ and regime } E > E_0$$

evaluated using data directly and pQCD for perturbative region and tail

- Including **self-energy effects** is mandatory ($\gamma\rho$-mixing, $\rho\omega$-mixing ..., decays with proper phase space, energy dependent width etc)

- Method works in reducing uncertainties by using **indirect constraints**

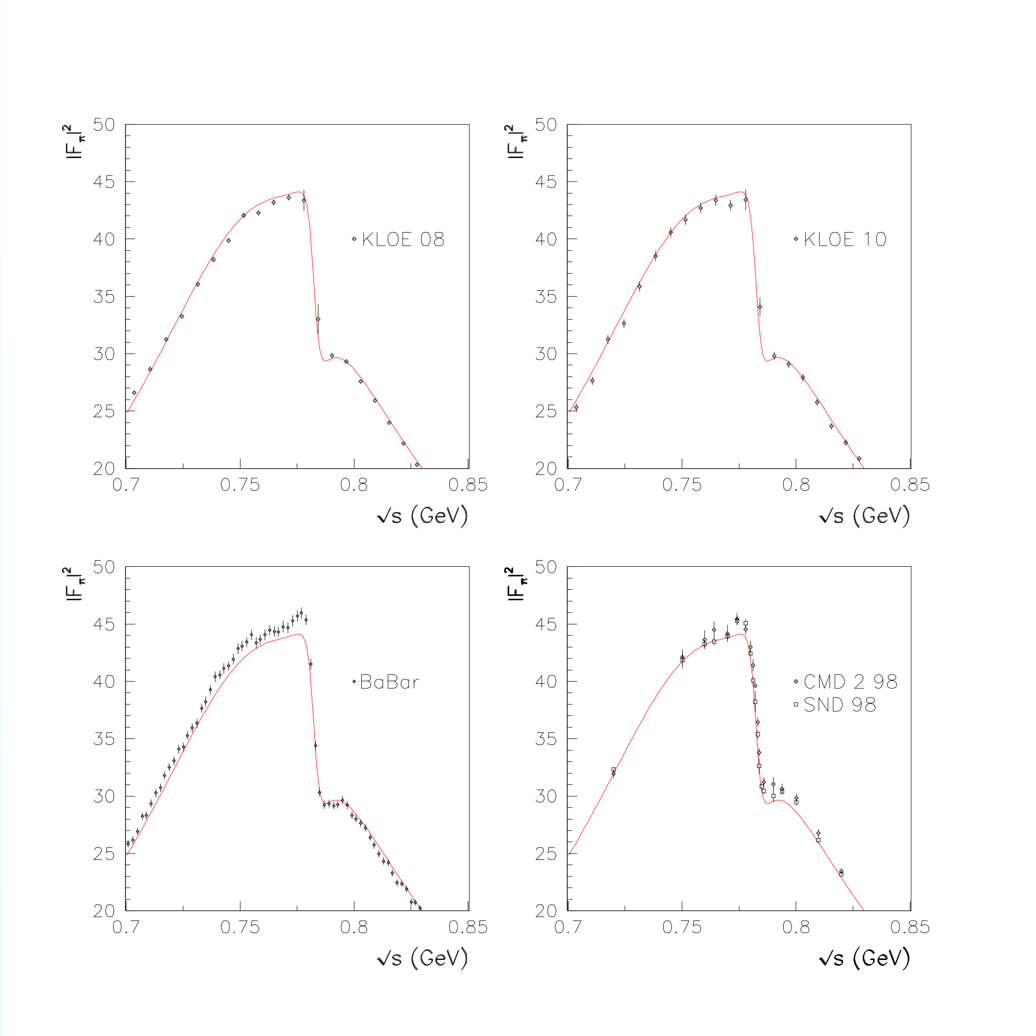
- Able to reveal inconsistencies in data, e.g. KLOE vs BaBar

Main goal:

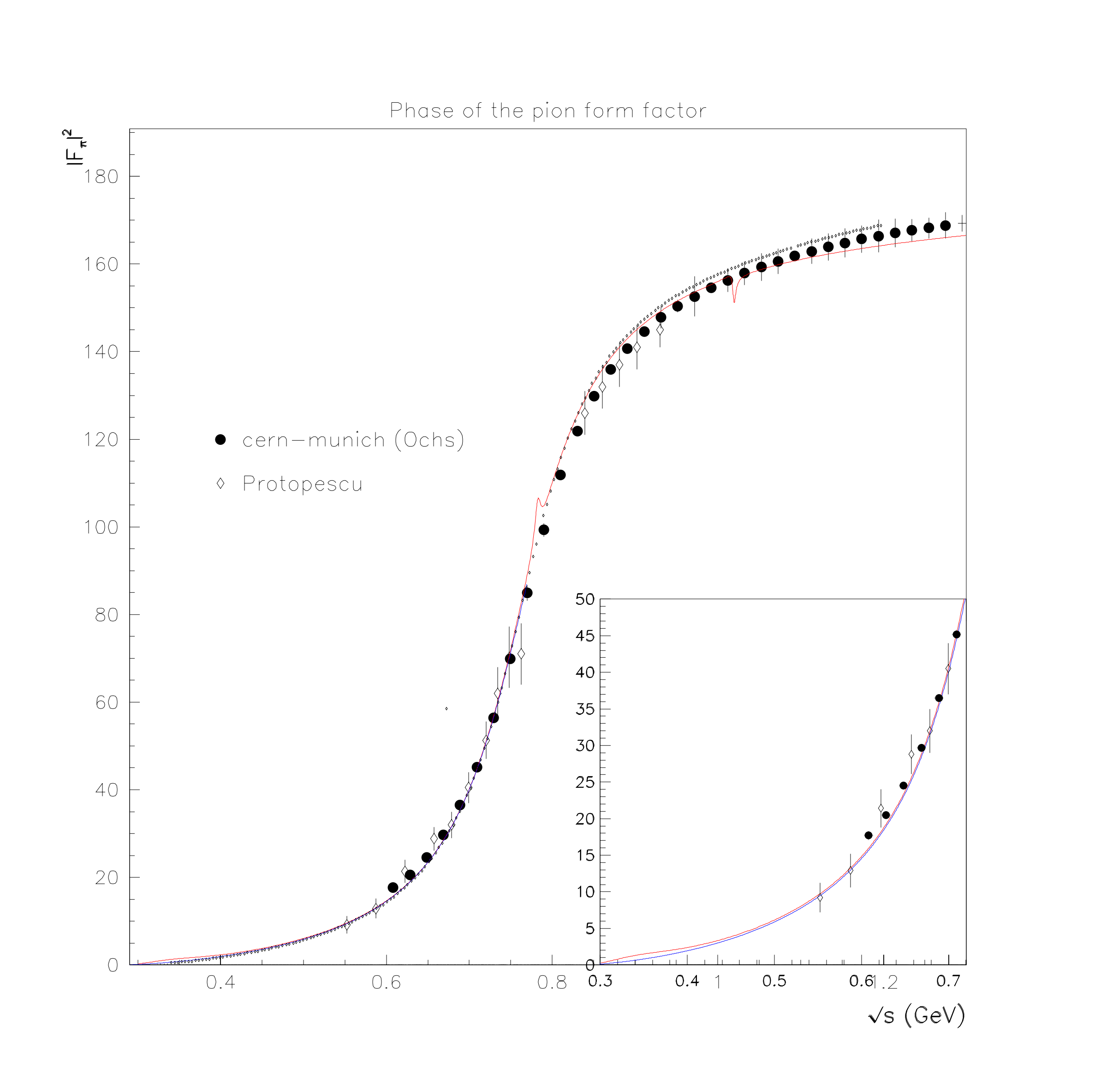
- **Single out** representative effective resonance Lagrangian by global fit is expected to help in improving EFT calculations of hadronic light-by-light scattering (such concept so far missing)

- could help improving uncertainty on hadronic VP (besides e^+e^- and τ decay data other experimental information

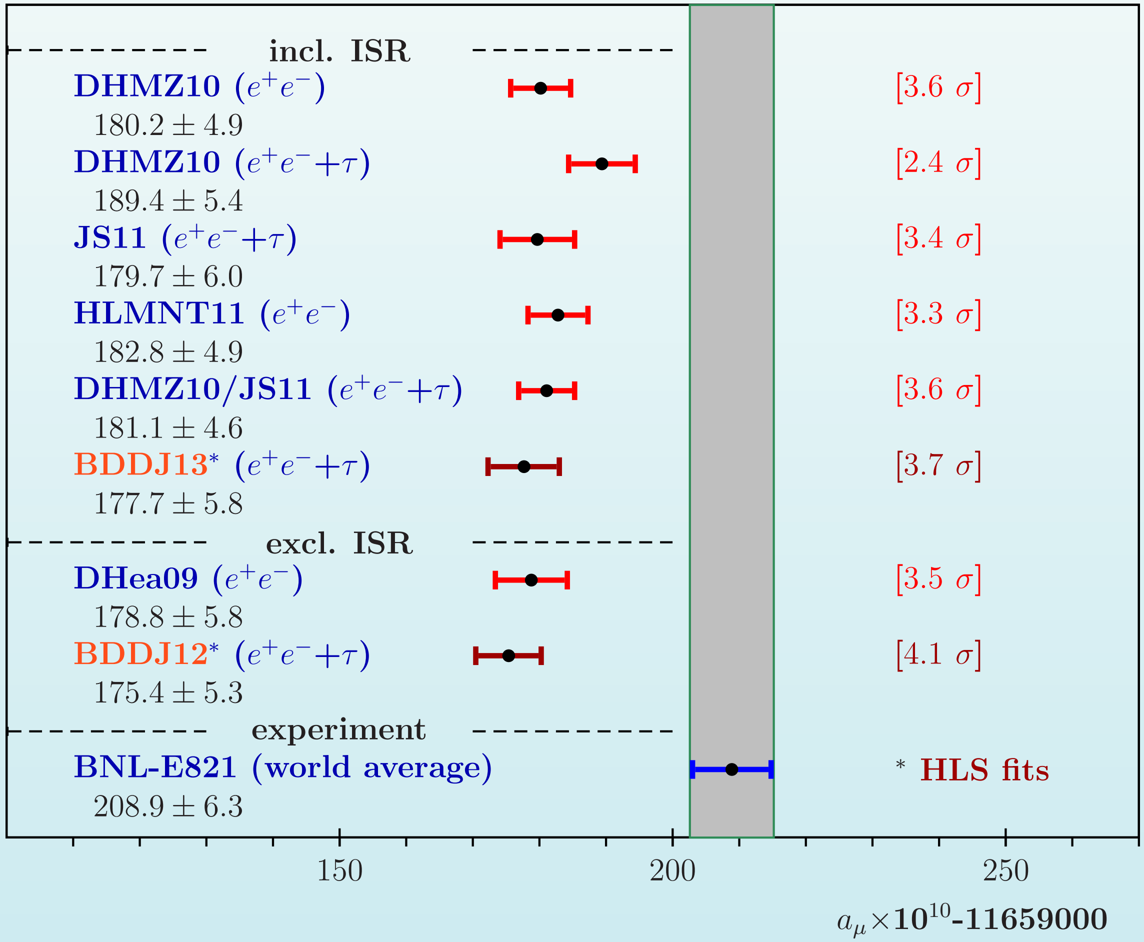
❑ Fit of τ +PDG vs $\pi^+\pi^-$–data



Benayoun et al 2012/13



The $\pi\pi$ scattering phase of our HLS prediction

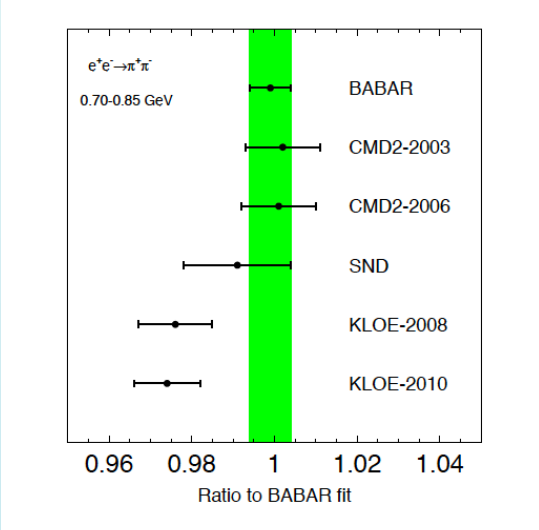


Comparison with other Results. Note: results depend on which value is taken for HLbL. JS11 and BDDJ13 includes $116(39) \times 10^{-11}$ [JN], DHea09, DHMZ10, HLMNT11 and BDDJ12 use $105(26) \times 10^{-11}$ [PdRV].

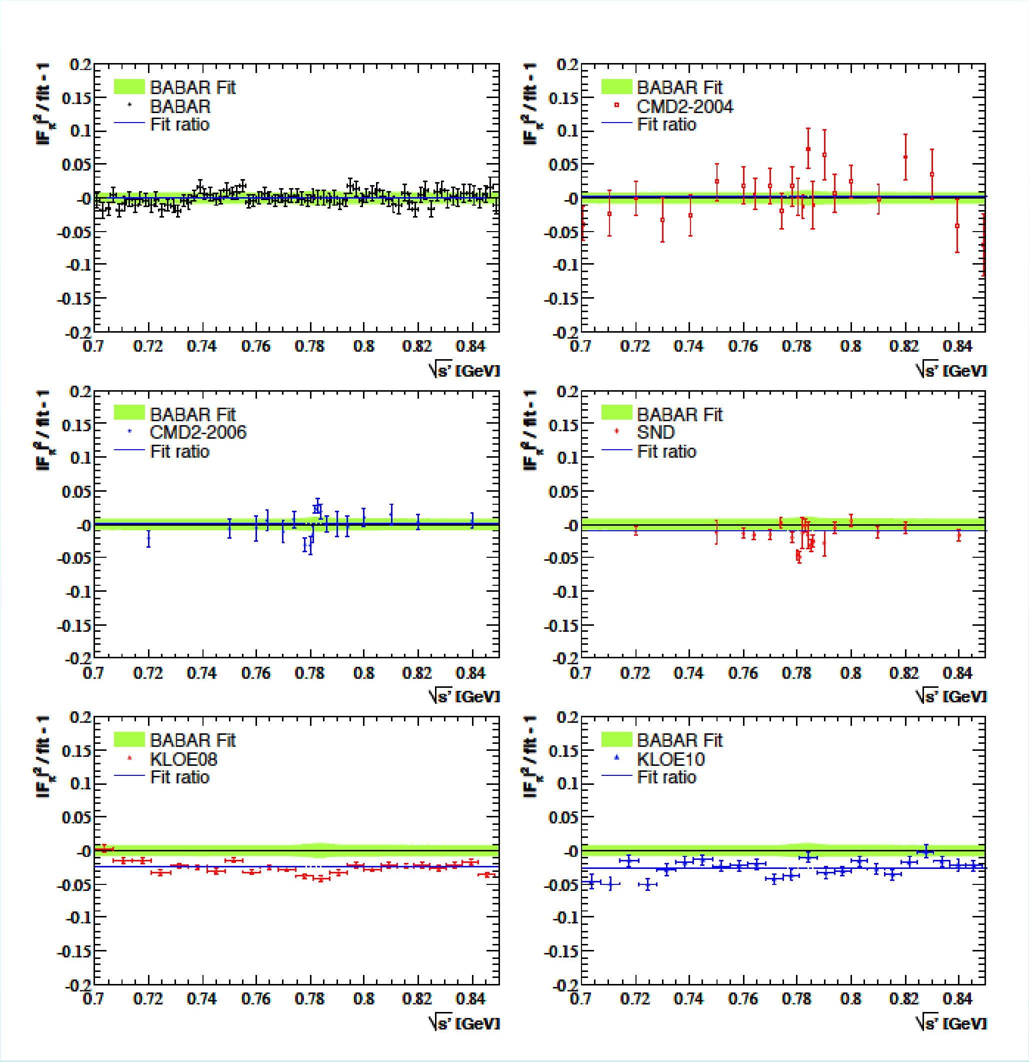
Main issues

- region 1.2 to 2 GeV bad data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) Who will do it? BES III radiative return!

- discrepancy BaBar vs KLOE $\pi\pi$ data. Who can clarify it?



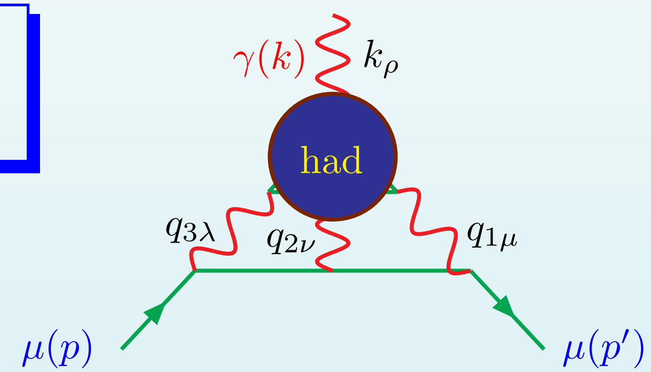
Davier&Malaescu 2013



Davier&Malaescu 2013

The hadronic LbL: setup and problems

Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$



Key object full rank-four hadronic vacuum polarization tensor

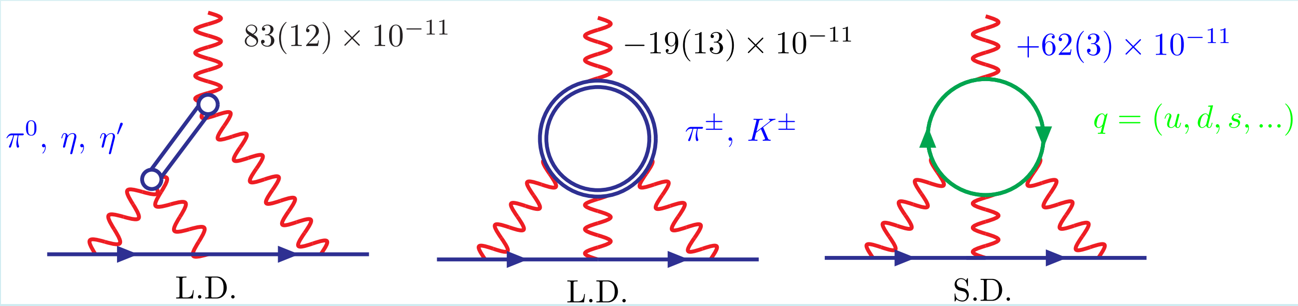
$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int d^4x_1\, d^4x_2\, d^4x_3\, e^{i\,(q_1x_1+q_2x_2+q_3x_3)} \times\langle\, 0\,|\,T\{j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0)\}\,|\,0\,\rangle\;.$$

- non-perturbative physics
- general covariant decomposition involves 138 Lorentz structures of which
- 32 can contribute to $g-2$
- fortunately, dominated by the pseudoscalar exchanges $\pi^0,\eta,\eta',...$ described by the effective Wess-Zumino Lagrangian

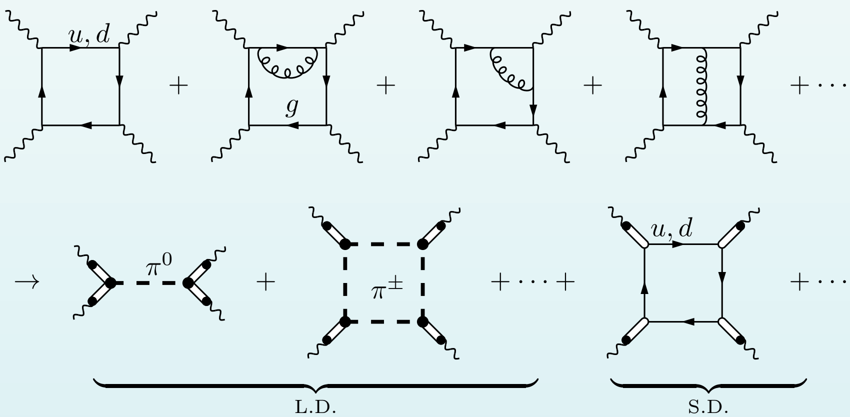
- generally, pQCD useful to evaluate the short distance (S.D.) tail

- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar Goldstone bosons as well as the vector mesons which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large N_c inspired ansätze, and others

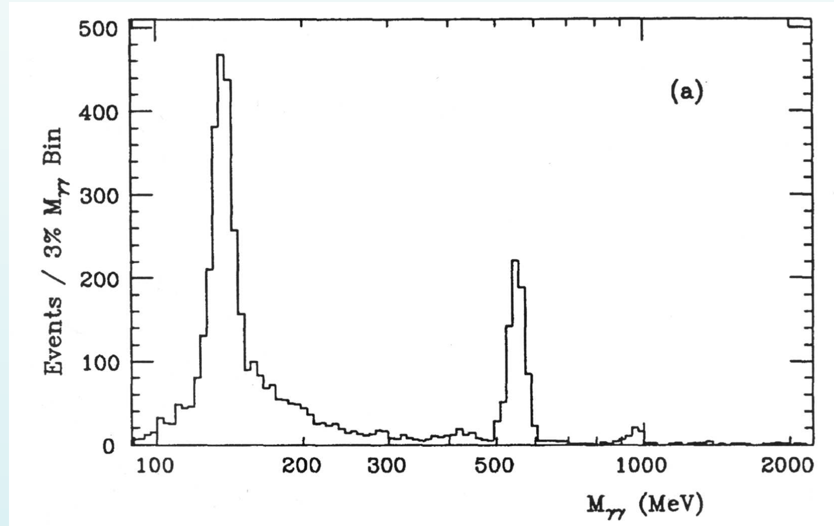
Need appropriate low energy effective theory ⇒amount to calculate the following type diagrams



LD contribution requires low energy effective hadronic models: simplest case $\pi^0\gamma\gamma$ vertex



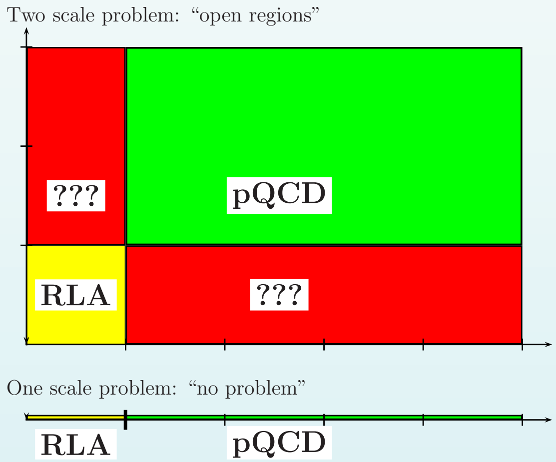
Crystal Ball 1988



Data show almost background free spikes of the PS mesons! Substantial background form quark loop is absent. Clear message from data: fully non-perturbative, evidence for PS dominance. However, no information about axial mesons (Landau-Yang theorem). Illustrates how data can tell us where we are.

Low energy expansion in terms of hadronic components: theoretical models vs experimental data ➠ KLOE, KEDR, BES, BaBar, Belle, ?

Basic problem: (s, s_1, s_2)–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane



??? – Data + Dispersion Relation, OPE,
– QCD factorization,
– Brodsky-Lepage approach
– Models constrained by data

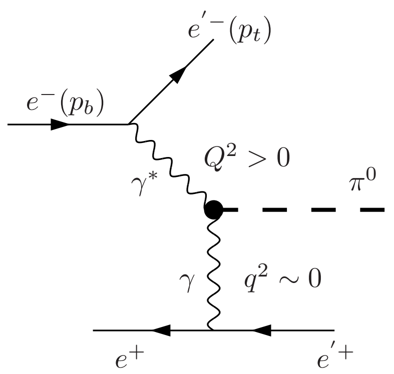
Novel approach: refer to quark–hadron duality of large-N_c QCD, hadron spectrum known, infinite series of narrow spin 1 resonances 't Hooft 79 ⇒no matching problem (resonance representation has to match quark level representation) De Rafael 94, Knecht, Nyffeler 02

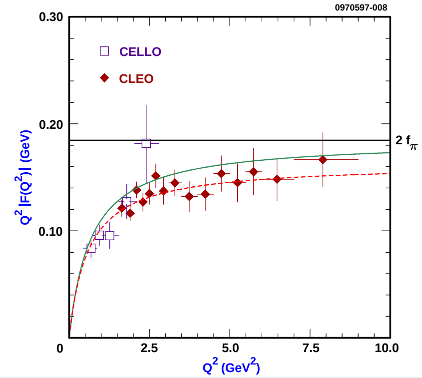
Constraints for on-shell pions (pion pole approximation)

❖ General form–factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$ is largely unknown

- The constant $e^2\,\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2,0,0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025$ GeV^{-1} well determined by $\pi^0 \to \gamma\gamma$ decay rate (from Wess-Zumino Lagrangian); experimental improvement needed!

- Information on $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ from $e^+e^- \to e^+e^-\pi^0$ experiments





CELLO and CLEO measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ at high space–like Q^2. Outdated by BaBar? Belle conforms with theory expectations!

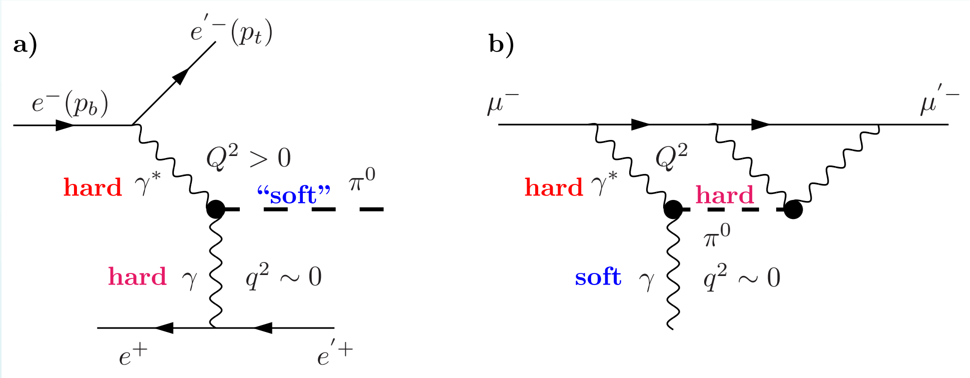
Brodsky–Lepage interpolating formula gives an acceptable fit.

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \simeq \frac{1}{4\pi^2 f_\pi}\frac{1}{1+(Q^2/8\pi^2 f_\pi^2)} \sim \frac{2f_\pi}{Q^2}$$

Inspired by pion pole dominance idea this FF has been used mostly (HKS,BPP,KN) in the past, but has been criticized recently (MV and FJ07).

❒ Melnikov, Vainshtein: in chiral limit vertex with external photon must be non-dressed! i.e. use $\mathcal{F}_{\pi^0\gamma^*\gamma}(0,0,0)$, which avoids eventual kinematic inconsistency, thus no VMD damping ⇒result increases by 30% !

❒ In $g-2$ external photon at zero momentum ⇒ only $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ not $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ is consistent with kinematics. Unfortunately, this off–shell form factor is not known and in fact not measurable and CELLO/CLEO constraint does not apply!. Obsolete far off-shell pion (in space-like region).



Measured is $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space–like Q^2, needed at external vertex is $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$ or $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q^2, q^2, 0)$ if integral to be evaluated in Minkowsky space.

❒ I still claim using $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(0, 0, 0)$ in this case is not a reliable approximation!

Need realistic "model" for off–shell form–factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q^2, q^2, 0)$ via DR from data!

Note: $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$ is a one-scale problem. Self-energy type of problem $\Rightarrow$ can get it via dispersion relation from appropriate data

Is it really to be identified with $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(0, 0, 0)$?

Can we check such questions experimentally or in lattice QCD?

Evaluation of $a_\mu^{\rm LbL}$ in the large-N_c framework

- Knecht & Nyffeler and Melnikov & Vainshtein were using pion-pole approximation together with large-N_c $\pi^0\gamma\gamma$–form-factor

- FJ & A. Nyffeler: relax from pole approximation, using KN off-shell LDM+V form-factor

$$\begin{aligned}
\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(p_\pi^2,q_1^2,q_2^2) &= \frac{F_\pi}{3}\frac{\mathcal{P}(q_1^2,q_2^2,p_\pi^2)}{\mathcal{Q}(q_1^2,q_2^2)} \\
\mathcal{P}(q_1^2,q_2^2,p_\pi^2) &= h_7+h_6\,p_\pi^2+h_5\,(q_2^2+q_1^2)+h_4\,p_\pi^4+h_3\,(q_2^2+q_1^2)\,p_\pi^2 \\
&\quad +h_2\,q_1^2\,q_2^2+h_1\,(q_2^2+q_1^2)^2+q_1^2\,q_2^2\,(p_\pi^2+q_2^2+q_1^2)) \\
\mathcal{Q}(q_1^2,q_2^2) &= (q_1^2-M_1^2)\,(q_1^2-M_2^2)\,(q_2^2-M_1^2)\,(q_2^2-M_2^2)
\end{aligned}$$

all constants are constraint by SD expansion (OPE). Again, need data to fix parameters!

My estimations

Leading LbL contribution from PS mesons:

$$a_\mu[\pi^0, \eta, \eta'] \sim (93.91 = [63.14 + 14.87 + 15.90] \pm 12.40) \times 10^{-11}$$

Expected contribution from axial mesons:

$$a_\mu[a_1, f_1', f_1] \sim (28.13 = [7.02 + 19.38 + 1.74] \pm 5.63) \times 10^{-11}$$

Expected contribution from $q\bar{q}$ scalars:

$$a_\mu[a_0, f_0', f_0] \sim (-5.98 = [-0.17 - 2.96 - 2.85] \pm 1.20) \times 10^{-11}$$

depending slightly on assuming nonet symmetry, ideal mixing

V. Pauk, M. Vanderhaeghen: meson pole conributions

$$a_\mu[a_0, f_0', f_0] \sim (-3.1 = [-0.63 - 1.84 - 0.61] \pm 0.8) \times 10^{-11}$$

$$a_\mu[f_1', f_1] \sim (6.4 = [5.0 + 1.4] \pm 2.0) \times 10^{-11}$$

$$a_\mu[f_2', f_2, a_2', a_2] \sim (1.1 = [0.79 + 0.07 + 0.22 + 0.02] \pm 0.1) \times 10^{-11}$$

LbL: Present

JN09 based on Nyffeler 09: the only result relaxing from pole approximation

$$a_\mu^{\mathrm{LbL;had}} = (116 \pm 39) \times 10^{-11}$$

Summary of results

Contribution	HKS	BPP	KN	MV	PdRV	N/JN
π^0, η, η'	82.7±6.4	85±13	83±12	114±10	114±13	99±16
π, K loops	−4.5±8.1	−19±13	–	0±10	−19±19	−19±13
axial vectors	1.7±1.7	2.5±1.0	–	22± 5	15±10	22± 5
scalars	–	−6.8±2.0	–	–	−7± 7	−7± 2
quark loops	9.7±11.1	21± 3	–	–	2.3	21± 3
total	89.6±15.4	83±32	80±40	136±25	105±26	116±39

Is this the final answer? How to improve? A limitation to more precise $g-2$ tests?

Looking for new ideas to get ride of model dependence

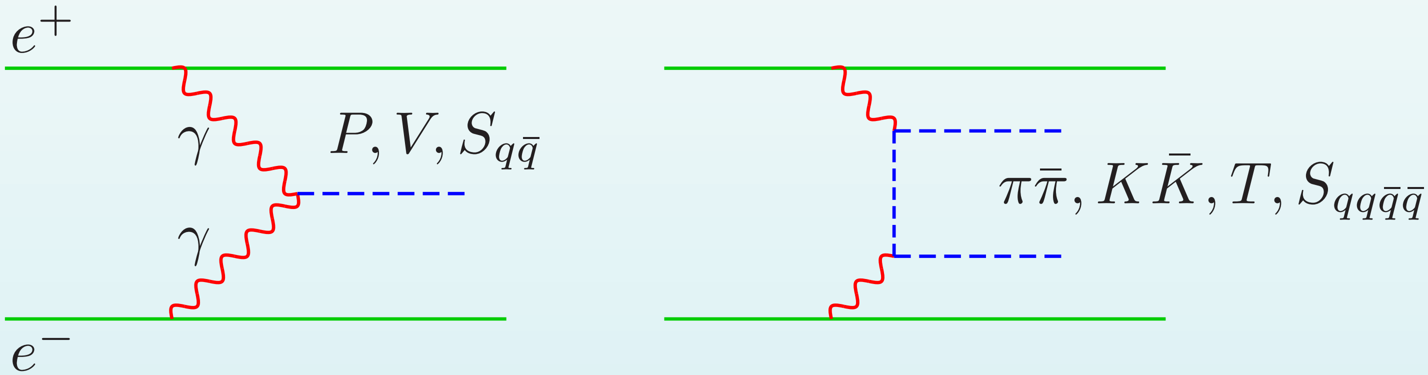
- Need better constrained effective resonance Lagrangian (e.g. HSL and ENJL models vs. RLA of Ecker et al). "Global effort" needed!
  recent: HLS global fit available Benayoun et al 2010/12

- Lattice QCD will provide an answer [take time ("yellow" region only?)]!

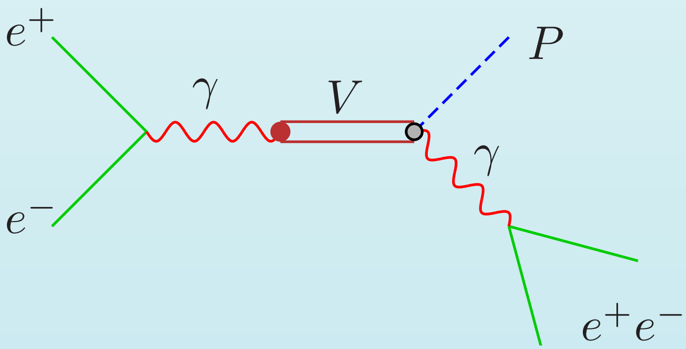
- Amplitudes in terms of Dispersion Relations (Cutkosky-rules technique) exploiting data! Which data needed?

- Using DESER-GILBERT-SUDARSHAN representation for vertex functions (analog to Kallen-Lehmann representation for two-point function) may be of help.

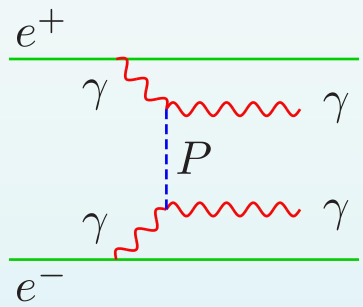
❒ Try exploiting possible new experimental constraints from $\gamma\gamma \to$ hadrons



mostly single-tag events: KLOE, KEDR (taggers), BaBar, Belle, BES III (high luminosity)



Dalitz-decays: $\rho, \omega, \phi \to \pi^0(\eta)e^+e^-$ Novosibirsk, NA60,JLab, Mainz, Bonn, Jülich, BES



would be interesting, but is buried in the background

❒ all in conjunction with DR Vanderhaeghen et al 2012/14

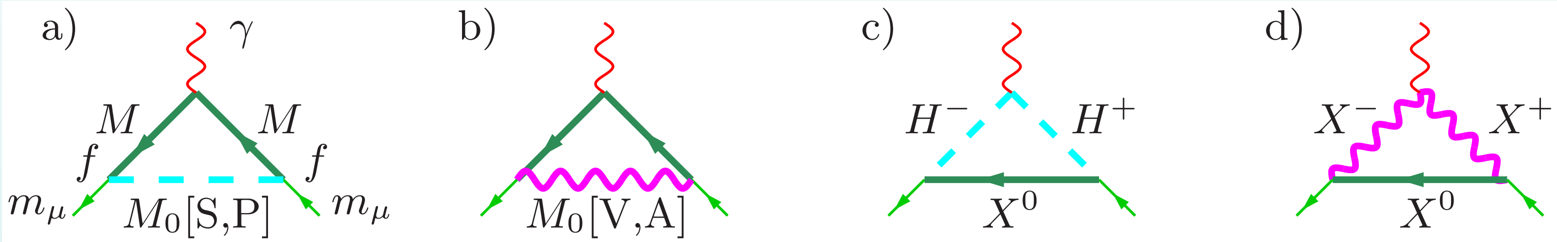
Theory vs experiment: do we see New Physics?

Contribution	Value	Error	Reference
QED incl. 4-loops+5-loops	11 658 471.8851	0.036	Remiddi, Kinoshita ...
Leading hadronic vac. pol.	688.60	4.24	HLS driven
Subleading hadronic vac. pol.	-9.832	0.082	2012 update
Hadronic light–by–light	11.6	3.9	evaluation (J&N 09)
Weak incl. 2-loops	15.40	0.10	CMV06/FJ12/BSS13
Theory	11 659 177.65	5.76	–
Experiment	11 659 209.1	6.3	BNL Updated
Exp.- The. 3.7 standard deviations	31.25	8.54	–

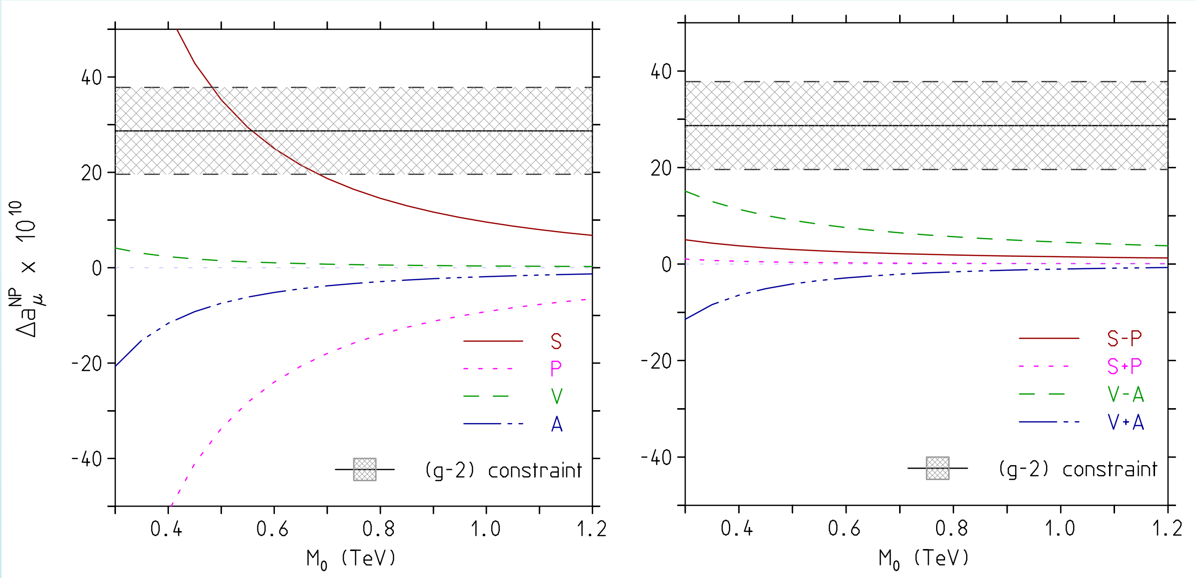
Standard model theory and experiment comparison [in units 10^{-10}]. What represents the 3 σ deviation: ❒ new physics? ❒ a statistical fluctuation? ❒ underestimating uncertainties (experimental, theoretical)?

❖do experiments measure what theoreticians calculate?

Most natural New Physics contributions: (examples)



neutral boson exchange: a) scalar or pseudoscalar and c) vector or axialvector, flavor changing or not, new charged bosons: b) scalars or pseudoscalars, d) vector or axialvector



Left: $m_\mu = M \ll M_0$ Right: $m_\mu \ll M_0 = M$

In general:

$$\Delta a_\mu^{\mathrm{NP}} = \alpha^{\mathrm{NP}} \frac{m_\mu^2}{M_{\mathrm{NP}}^2}$$

NP searches (LEP, Tevatron, LHC): typically $M_{\mathrm{NP}} >> M_W$, then $\Delta a_\mu^{\mathrm{exp-the}} = \Delta a_\mu^{\mathrm{NP}}$ requires $\alpha^{\mathrm{NP}} \sim 1$ spoiling perturbative arguments. Exception: 2HDM, SUSY $\tan\beta$ enhanced coupling! Note: NP sensitivity enhanced for muon by $\sim 40\,000$ relative to electron, while a_e is only 2250 times more precise than a_μ.

Problem: LEP, Tevatron and LHC direct bounds on masses of possible new states

[typically $M_X > 800$ GeV]

Future

The big challenge: two complementary experiments: Fermilab with ultra hot muons and J-PARC with ultra cold muons (very different radiation) to come

Provided deviation is real $3\sigma \to 9\sigma$ possible? Provided theory and needed cross section data improves the same as the muon $g-2$ experiments!

Key: need substantial progress in non-perturbative QCD

For muon $g-2$:

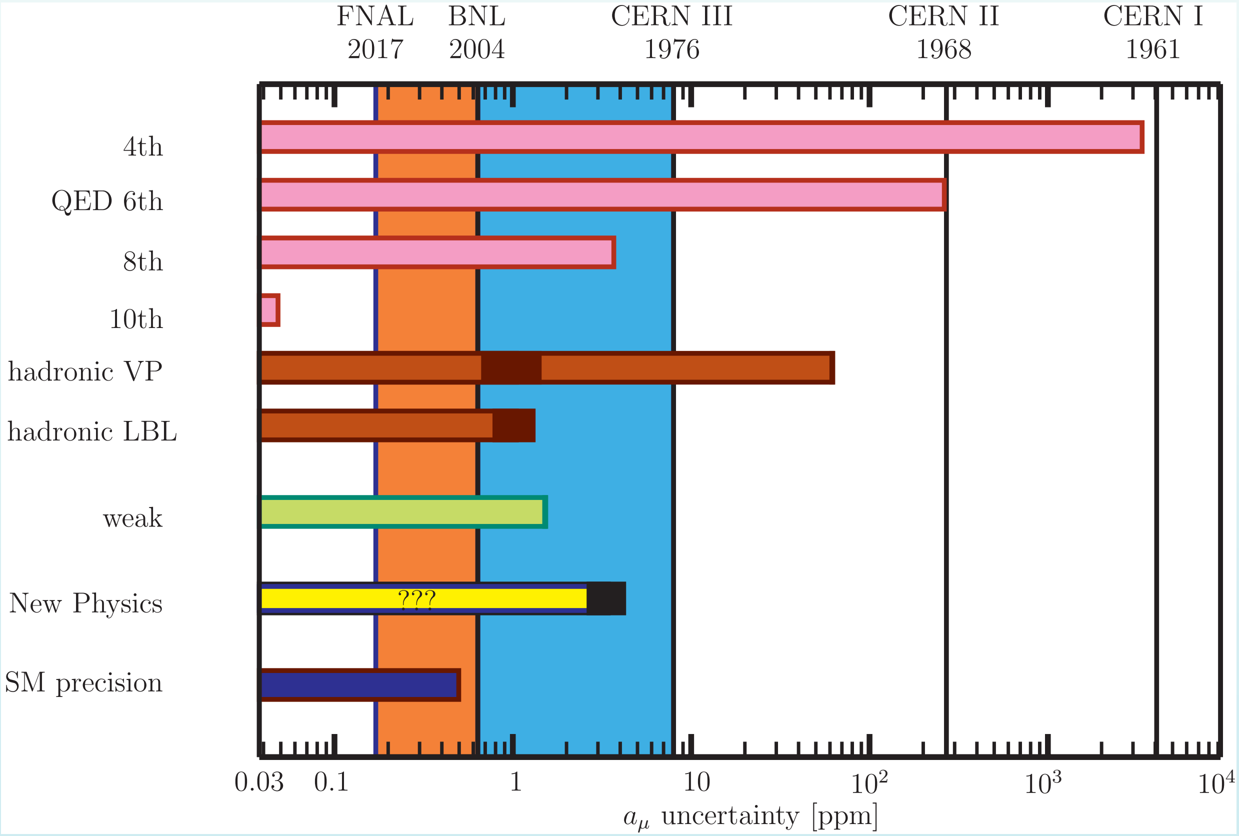
❖main obstacle: hadronic light-by-light [data, lattice QCD, RLA]

❖progress in evaluating HVP: more data (BaBar, Belle, VEPP 2000, BESIII,...), lattice QCD in reach (recent progress Jansen et al, Wittig et al, Blum et al)

in both cases lattice QCD will be the answer one day,

❖also low energy effective RL and DR approach need be further developed.

And here we are:



Sensitivity of $g-2$ experiments to various contributions. The increase in precision with the BNL $g-2$ experiment is shown as a cyan vertical band. New Physics is illustrated by the deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$

The challenge:

$a_\mu^{\text{had,VP}}[LO]$	$(6923 \pm 42) \times 10^{-11}$	+58.82 ±0.36 ppm
$a_\mu^{\text{had,VP}}[NLO]$	$(-98 \pm 1) \times 10^{-11}$	
a_μ^{EW}	$(154 \pm 1) \times 10^{-11}$	
$a_\mu^{\text{had,LbL}}$	$[(105 \div 115) \pm (26 \div 40)] \times 10^{-11}$	+0.90 ±0.22 ppm
$\delta a_\mu^{\exp}$ present	63×10^{-11}	±0.54 ppm
$\delta a_\mu^{\exp}$ future	16×10^{-11}	±0.14 ppm

Next generation experiments require a factor 4 reduction of the uncertainty
optimistically feasible is factor 2 we hope

What do we see in the muon $g-2$??? You may find what it is!

- Still a question: do we calculate what experiments measure?

Resent: Arbuzov and Kopylowa 2013: effect of real radiation on a_μ:

$$\Delta a_f^{(1,\kappa)} = \frac{\alpha}{2\pi}\left(1 + \delta d_f^{(\kappa)}\right)$$

$$\delta a_f^{(\kappa)} = \left(\frac{1}{4} + \frac{1}{2}\,\ln|\kappa|\right)\kappa + O(\kappa^2)$$

as it should smooth as $\kappa \to 0$ (“offshellness” of the muon)
Assume

$$\Delta a_\mu^{\mathrm{exp-SM}} \sim 3 \times 10^{-9} \simeq \frac{\alpha}{2\pi}\,\delta a_\mu^{(\kappa)}$$

$\Rightarrow \quad \kappa \simeq -3.5 \times 10^{-7}\ ; \ \ \kappa m_\mu \sim 35$ eV , remember $p_\mu \simeq 9.1$ GeV

- Do we need new ideas to pin down more precisely the hadronic effects?

The muon $g-2$ story continues!

Thank you for your attention!

Thanks to KIM for the invitation and the kind hospitality!

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