# Strongly coupled gauge theories: In and out of the conformal window

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SCGT14Mini

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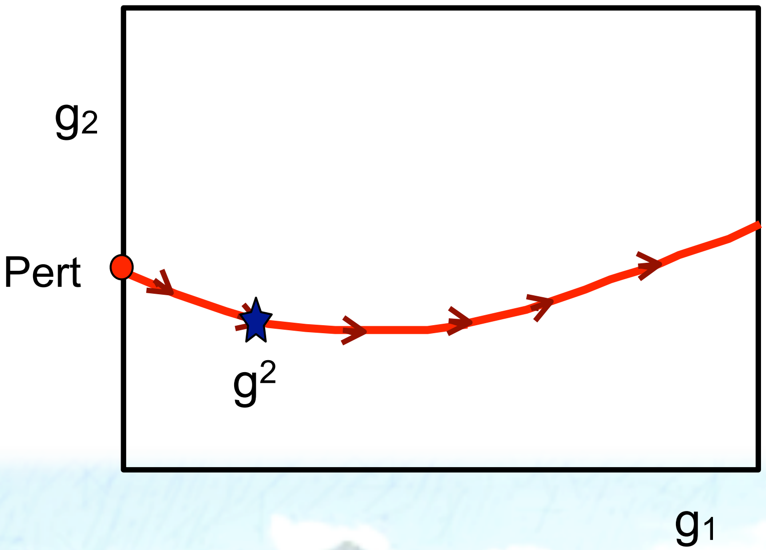
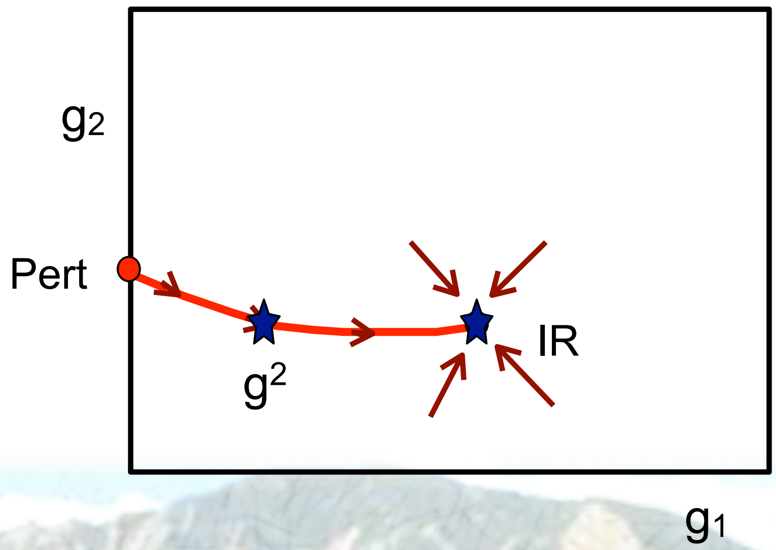
- Nearly conformal  models are very different from QCD
  yet difficult to distinguish from chirally broken systems

Compare the phase diagram on the m=0 chiral surface:

chirally broken




conformal




$g_2$ represents all irrelevant operators

# Universality

Systems

- with identical field content
- identical symmetries
- at criticality (basin of attraction of the FP)

are expected to show universal critical behavior.

Lattice symmetries:

- $SU(N_c)$ gauge preserved ✔
- $SU(N_f) \times SU(N_f)$ chiral symmetry is not:
  - staggered fermions : only $U(N_f/4) \times U(N_f/4)$ flavor symm.
  - Wilson fermions : no chiral symmetry
  - Domain Wall fermions : approximate chiral symm.

At the $g^2 = 0$ UVFP all formulations approach continuum fermions

At the $g^2 \neq 0$ conformal IRFP that is not the case

Universality should be investigated more carefully

(but only staggered fermions in this talk)

# Strongly coupled gauge-fermion systems

Nearly conformal models are very different from QCD, yet difficult to distinguish from chirally broken systems

→ numerical methods from QCD are not always effective

Combine

- standard QCD methods
- modified methods
- new approaches

# Strongly coupled gauge-fermion systems

## Modified methods

- Finite size scaling for $N_f$=12 (poster):
  - FSS is inconsistent if only the relevant exponent/ operator is considered
  - becomes consistent across different observables, gauge couplings, even actions if the leading irrelevant correction is included and predicts

$$\gamma^*_m = 0.235(15)$$

- Running gauge coupling with $N_f$=12 (poster):
  - Investigated both MCRG and gradient flow matching
  - After careful $(a/L)^2$ extrapolation gradient flow predicts an IRFP at $g^2_c = 6.21(25)$ (Scheme dependent!)

# Strongly coupled gauge-fermion systems

New approach:

– Running anomalous mass dimension from the spectral density of the Dirac operator ($N_f$=4, 8, 12, 16)

  - $N_f$=4 : chirally broken; test case
  - $N_f$=8 : near the conformal boundary at 2-loop;
  - $N_f$=12 : has been rather controversial
  - $N_f$= 16 : weakly coupled conformal

These methods probe the systems very differently:

Finite size scaling: L,m finite ;

Running coupling calculations: L finite, m=0;

Spectral density: $L \to \infty$, m=0

# Spectral density of the Dirac operator

Spectral density: $\rho(\lambda)$
Mode number : $\nu(\lambda)=V\int_{-\lambda}^{\lambda}\rho(\lambda')d\lambda'$

Chirally broken systems: $\rho(0)=\Sigma/\pi$ (Banks-Casher)
Conformal systems are chirally symmetric: $\rho(0)=0$

critical behavior suggests $\rho(\lambda)\propto\lambda^{\alpha}$ , $\lambda\approx 0$

.  $\nu(\lambda)\propto V\lambda^{\alpha+1}$

The **mode number** is RG invariant, (Giusti,Luscher)
unchanged under scale change s: $V\rightarrow s^4V$, $\lambda\rightarrow\lambda/s^{1+\gamma}$ , $\nu\rightarrow\nu$□ □□

→ $\alpha$ is related to the anomalous dimension (Zwicky,DelDebbio;Patella)

$$\frac{4}{1+\alpha}=y_m=1+\gamma_m$$

# Conformal system

Eigenvalue density $\rho(0)=0$ , scales as $\rho(\lambda) \propto \lambda^{\alpha(\lambda)}$

RG invariance implies $$\frac{4}{1+\alpha} = y_m = 1+\gamma_m$$

λ provides an energy scale

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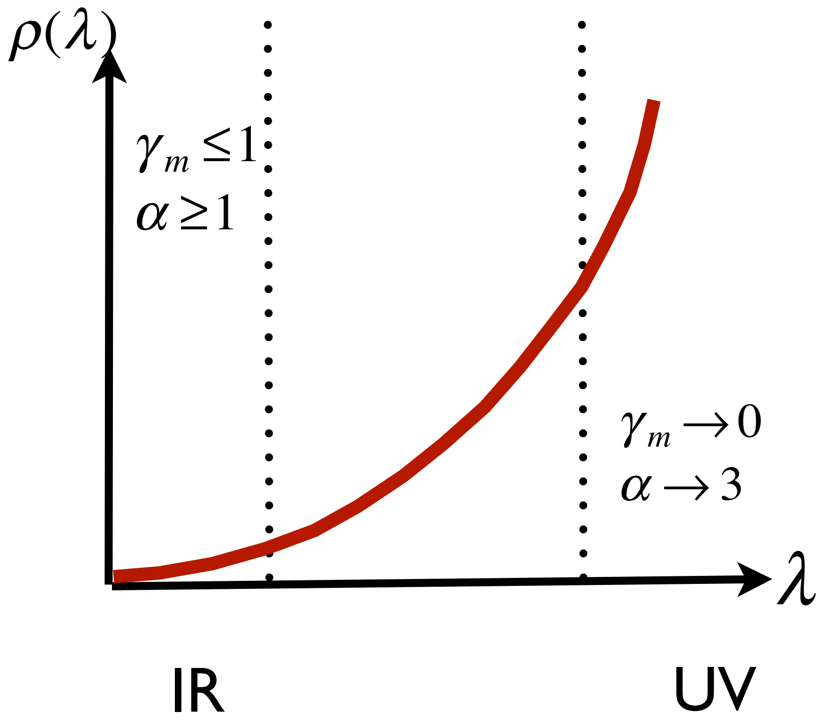


**IR** – small λ region:

$$\gamma_m(\lambda\to 0)=\gamma_m^*$$

predicts the universal anomalous dimension at the IRFP

**UV** – large λ =O(1) region:
if governed by the asymptotically free perturbative FP

$$\gamma_m(\lambda=\mathcal{O}(1))=\gamma_0 g^2+\ldots$$

**In between:**
scale dependent effective $\gamma_m$

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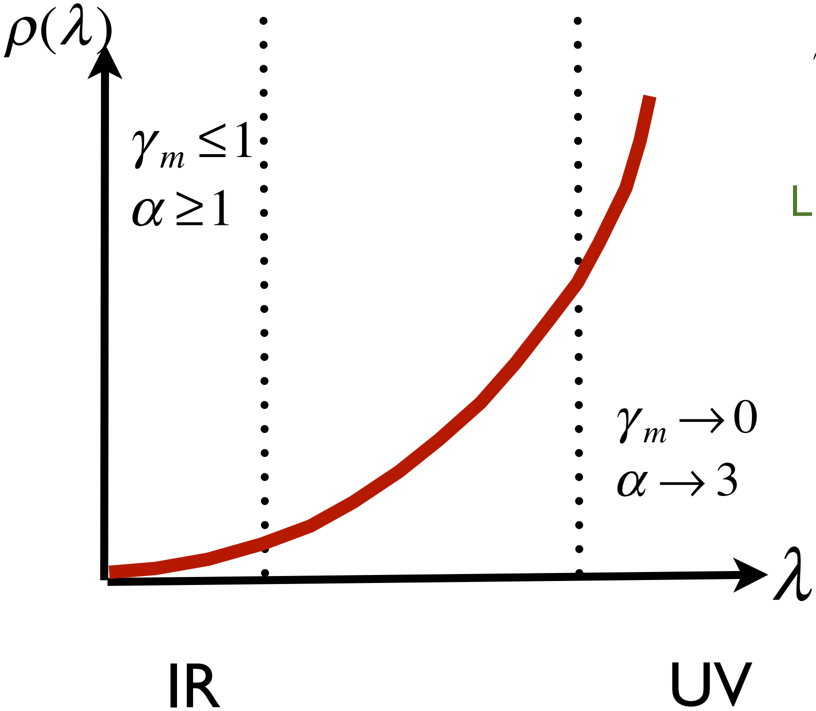


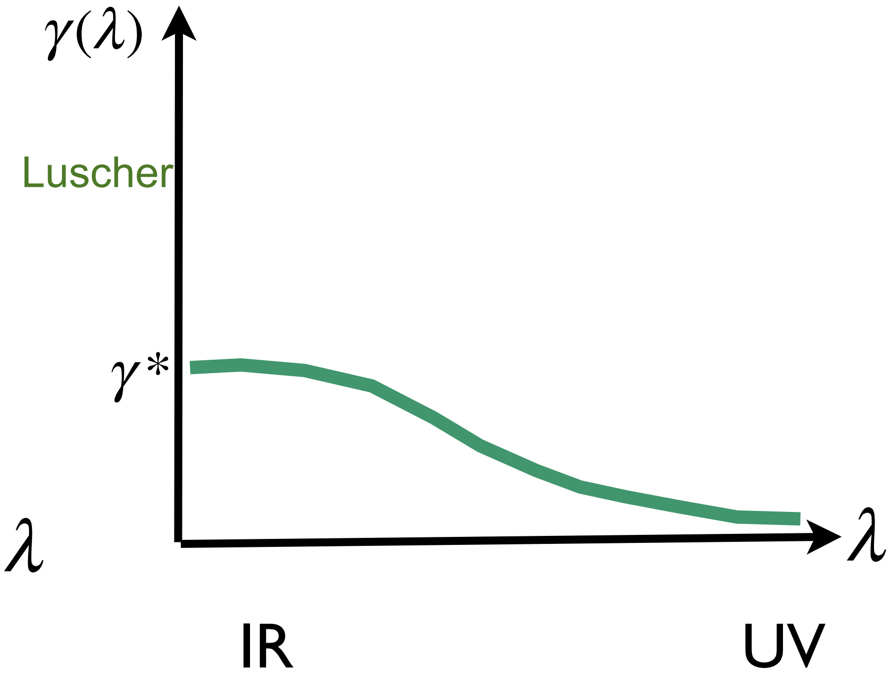

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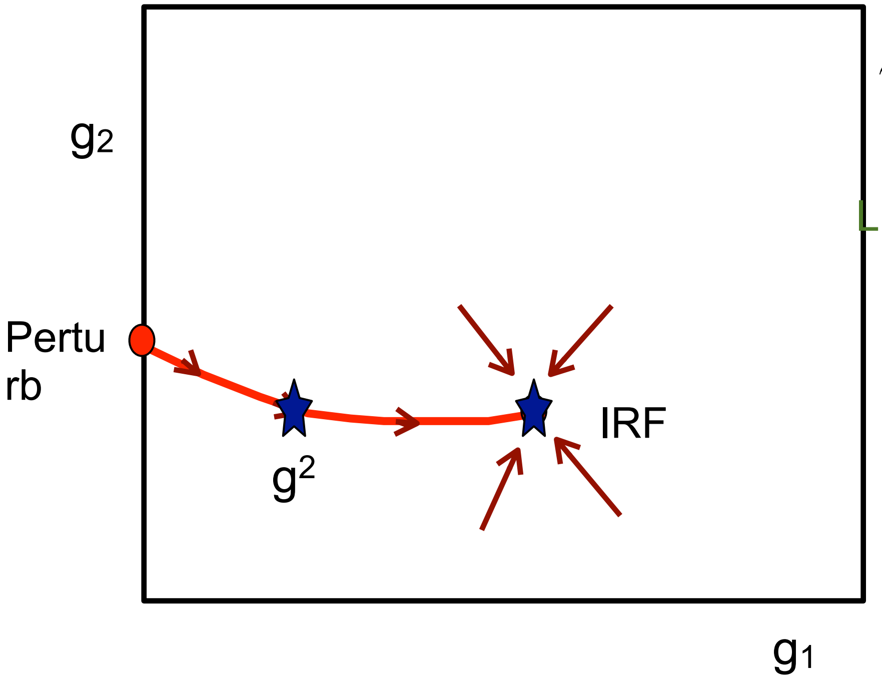


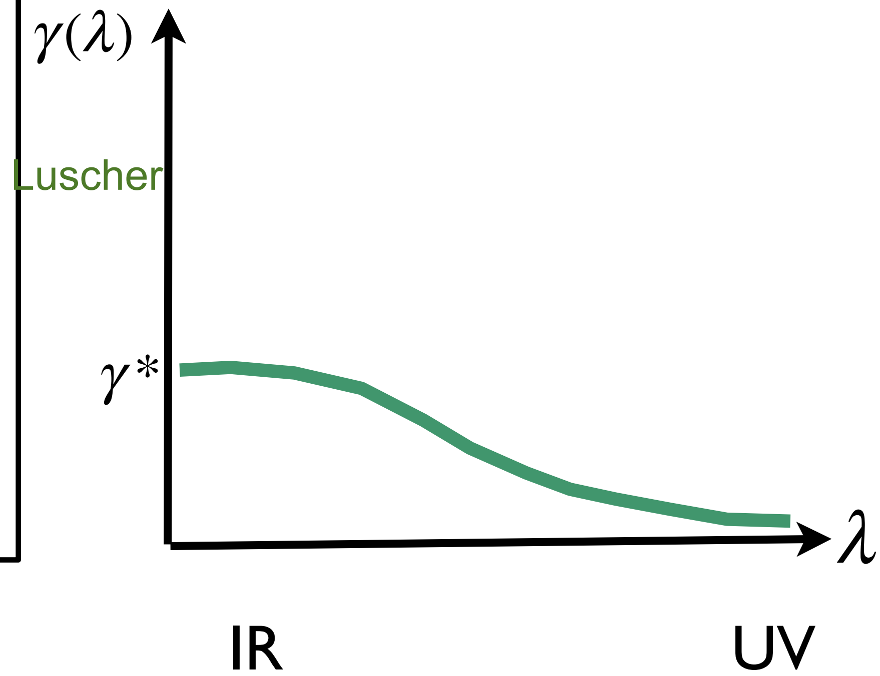

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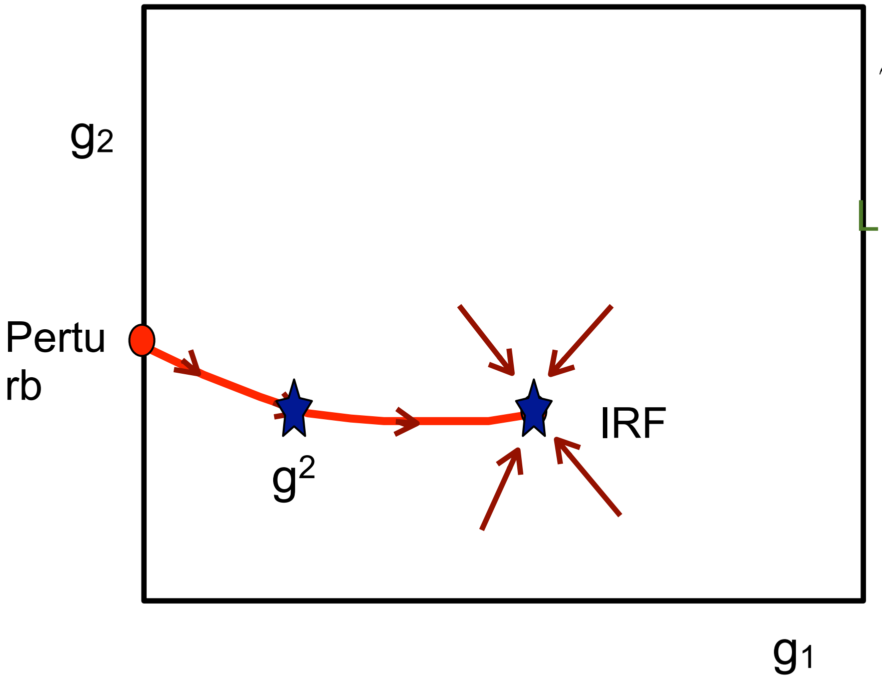
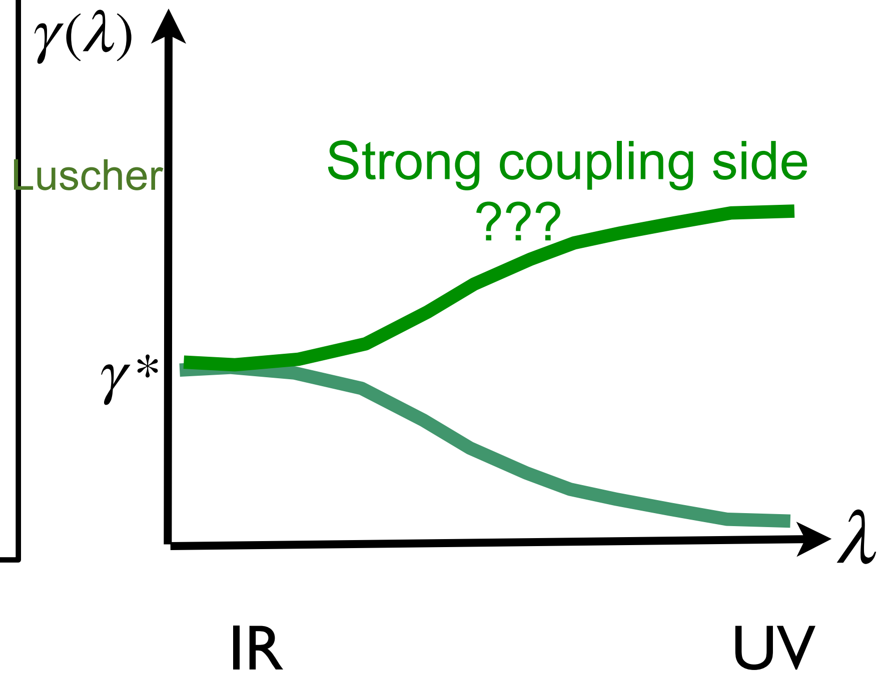
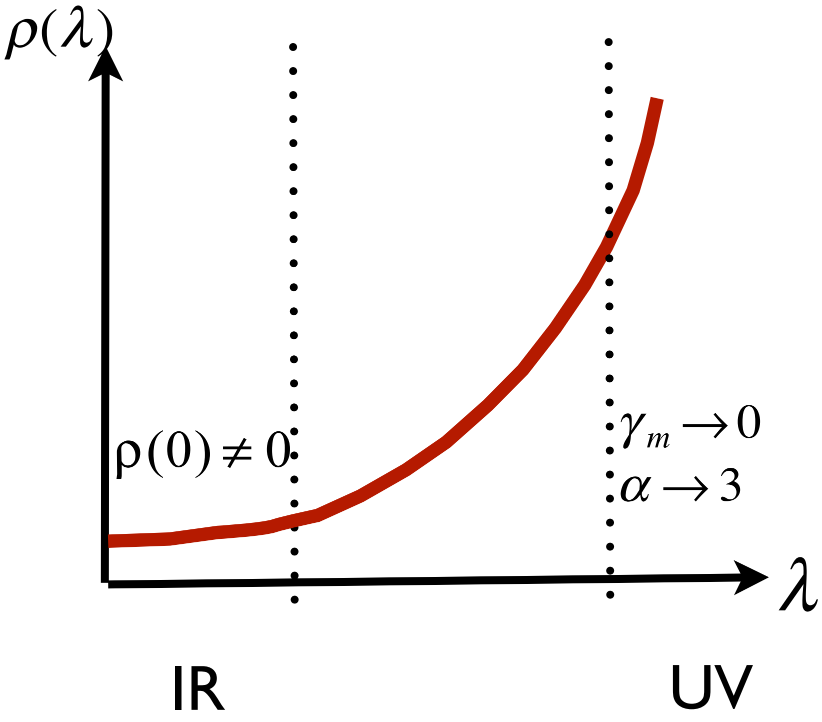


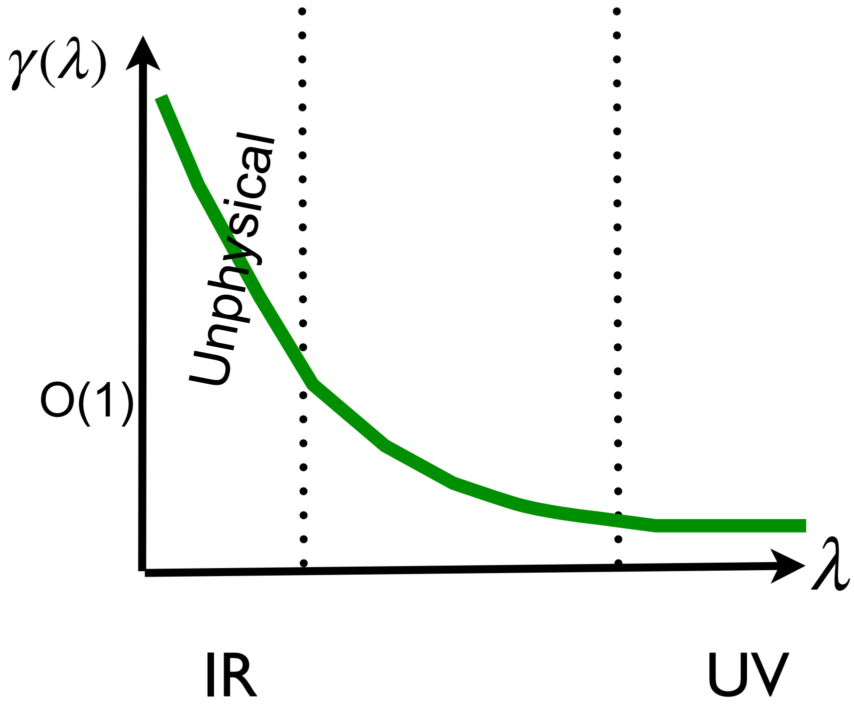



# Chirally broken system

Chirally broken systems show only the asymptotically free region

$$\frac{4}{1+\alpha} = y_m = 1 + \gamma_m$$



# Dirac operator eigenvalue spectrum and spectral density

- calculate $\nu(\lambda)$ stochastically
- fit $\nu(\lambda) \propto V \lambda^{\alpha(\lambda)+1}$
  $\log \nu = (\alpha(\lambda)+1)\log(\lambda) + c$
- extract the scale dependent $\gamma_m(\lambda)$

This should be done in the

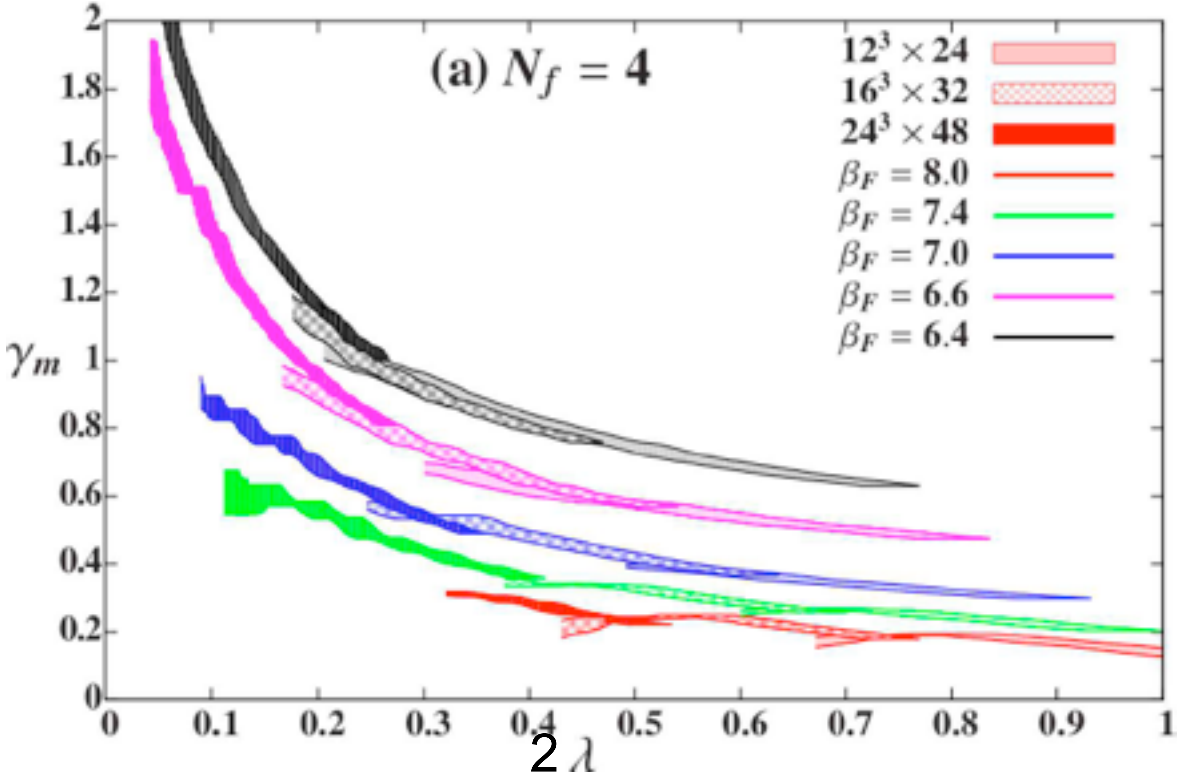
chiral m=0 and infinite volume $L \to \infty$ limit :

finite mass, volume introduces only small $\lambda$ transient effects

Important: fit $\nu(\lambda)$ , not $\rho(\lambda)$; The two are not the same!

# Results: $N_f = 4$

Broken chiral symmetry in IR, asymptotic freedom in UV



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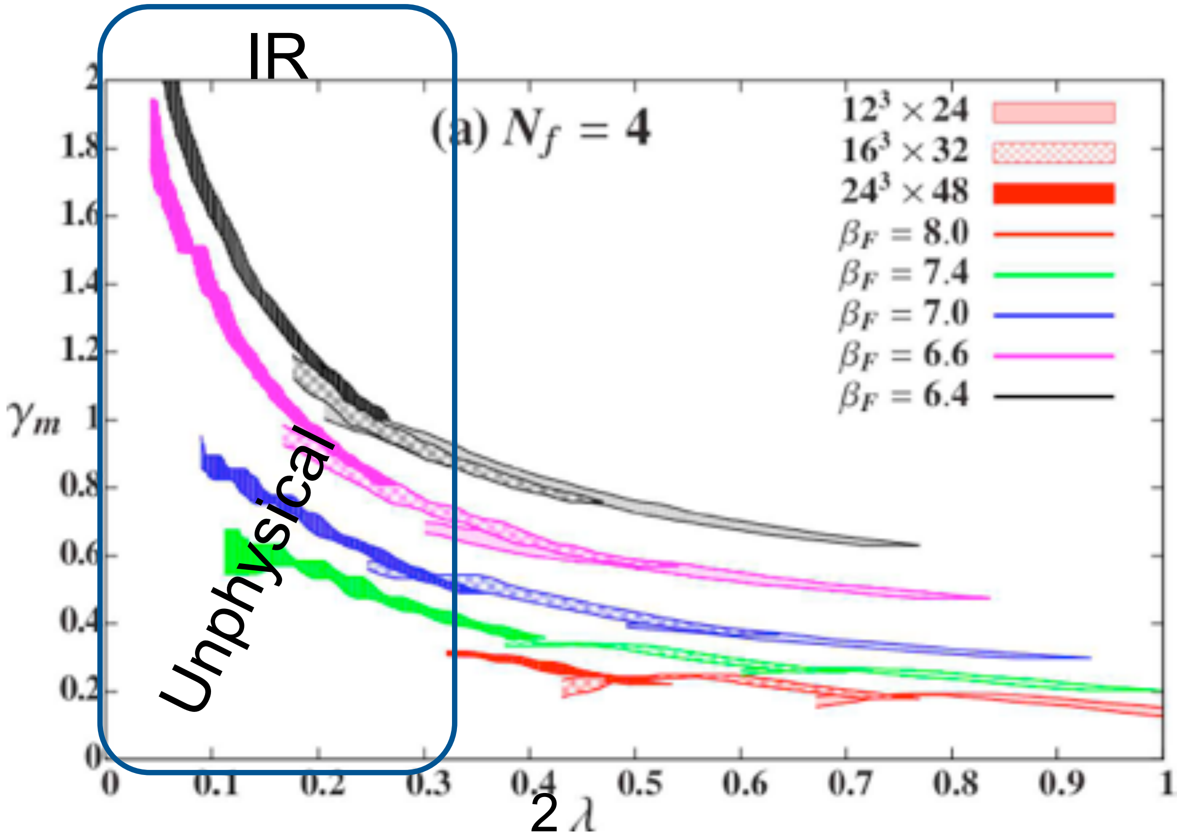

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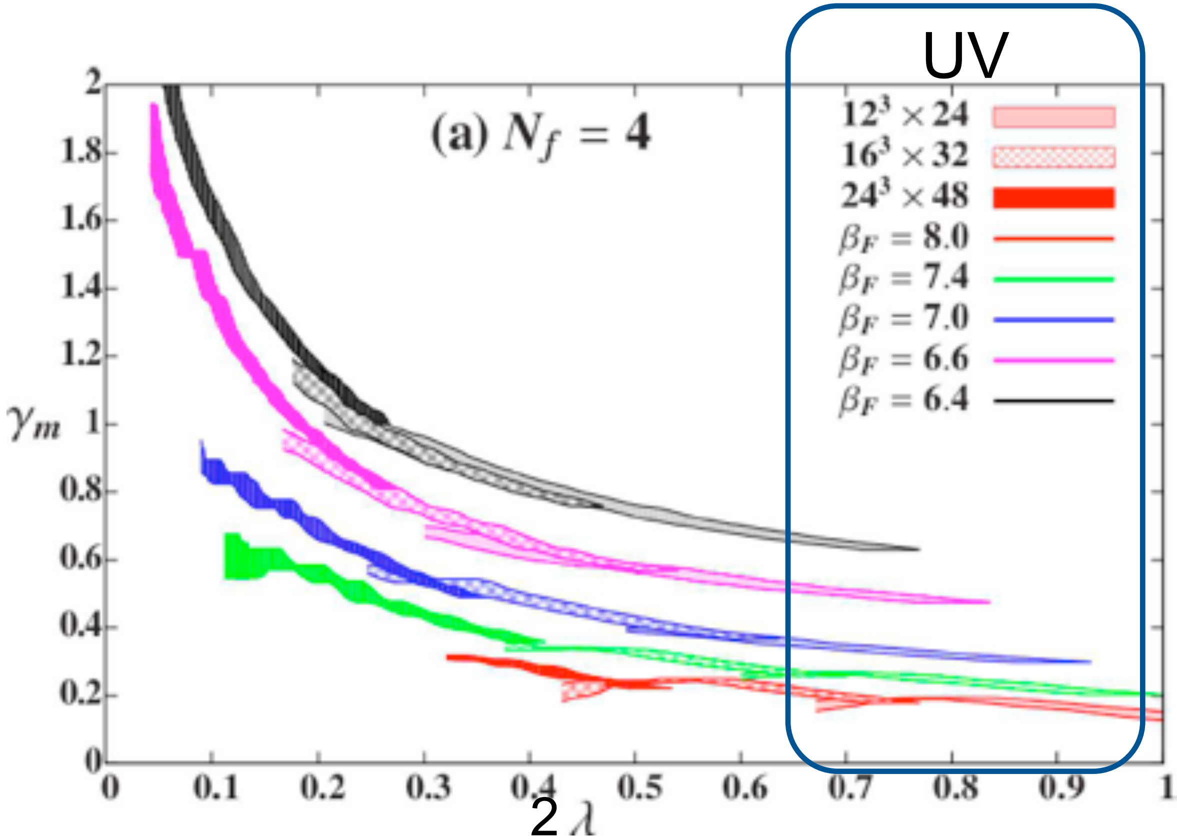

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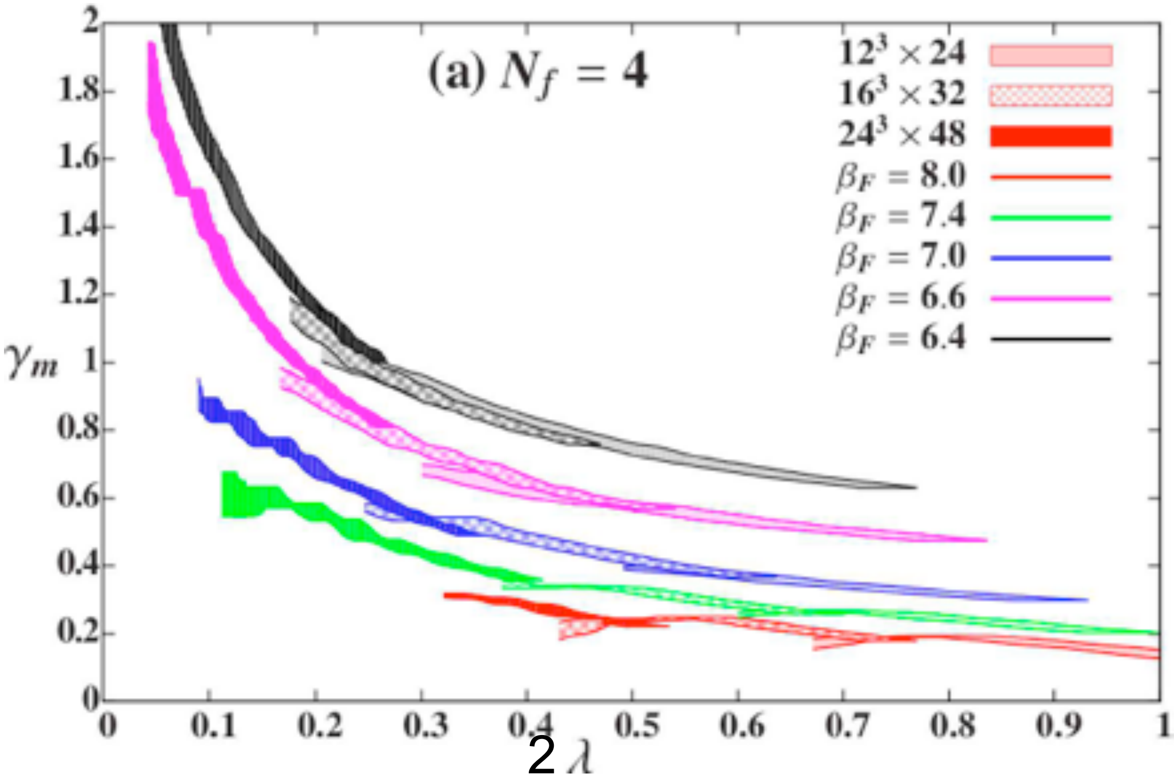



Can the different couplings be rescaled?

Lattice spacing from Wilson flow:

$$a_{6.4} \;/\; a_{7.4} = 2.84(3)$$

$$a_{6.6} \;/\; a_{7.4} = 2.20(5)$$

$$a_{7.0} \;/\; a_{7.4} = 1.45(3)$$

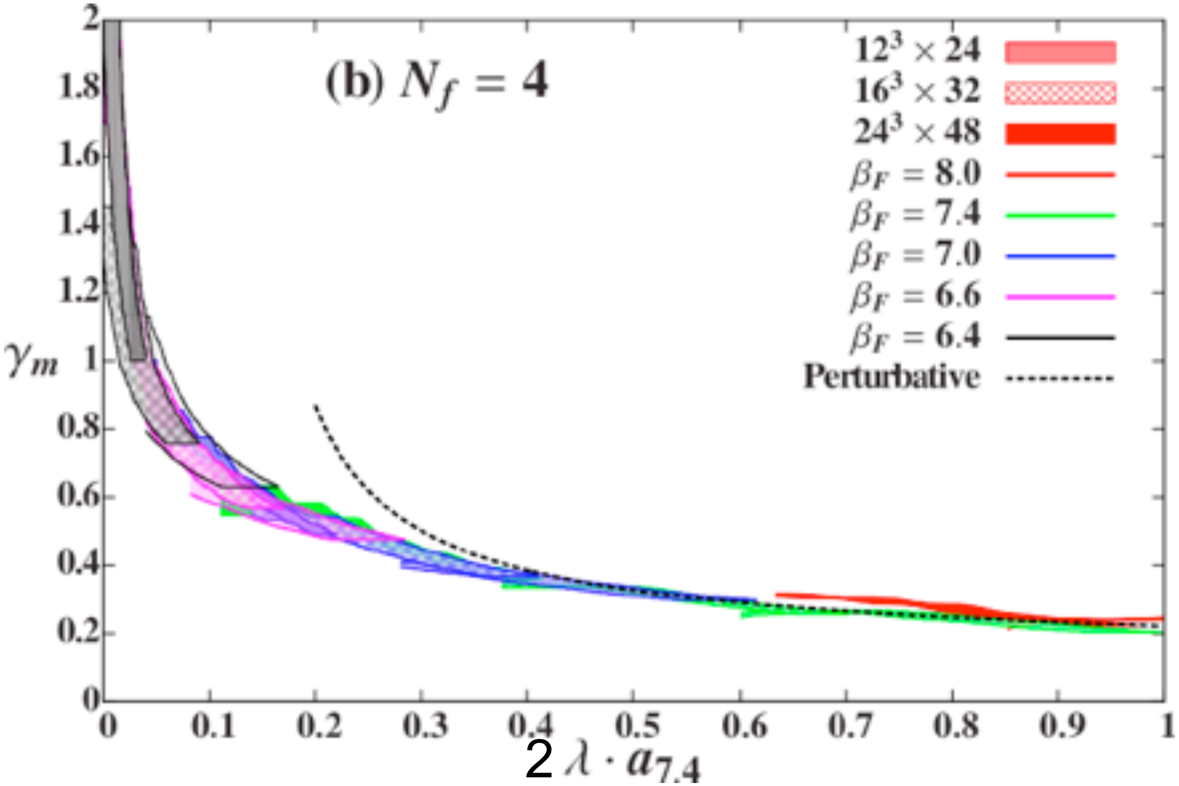
$$a_{8.0} \;/\; a_{7.4} = 0.60(4)$$

# Rescaling: $N_f$ =4

The dimension of λ is carried by the lattice spacing: $\lambda_{lat} = \lambda_p a$

Rescale to a common physical scale:

$$\lambda_\beta \to \lambda_\beta \left( \frac{a_{7.4}}{a_\beta} \right)^{1+\gamma_m(\lambda_\beta)}$$

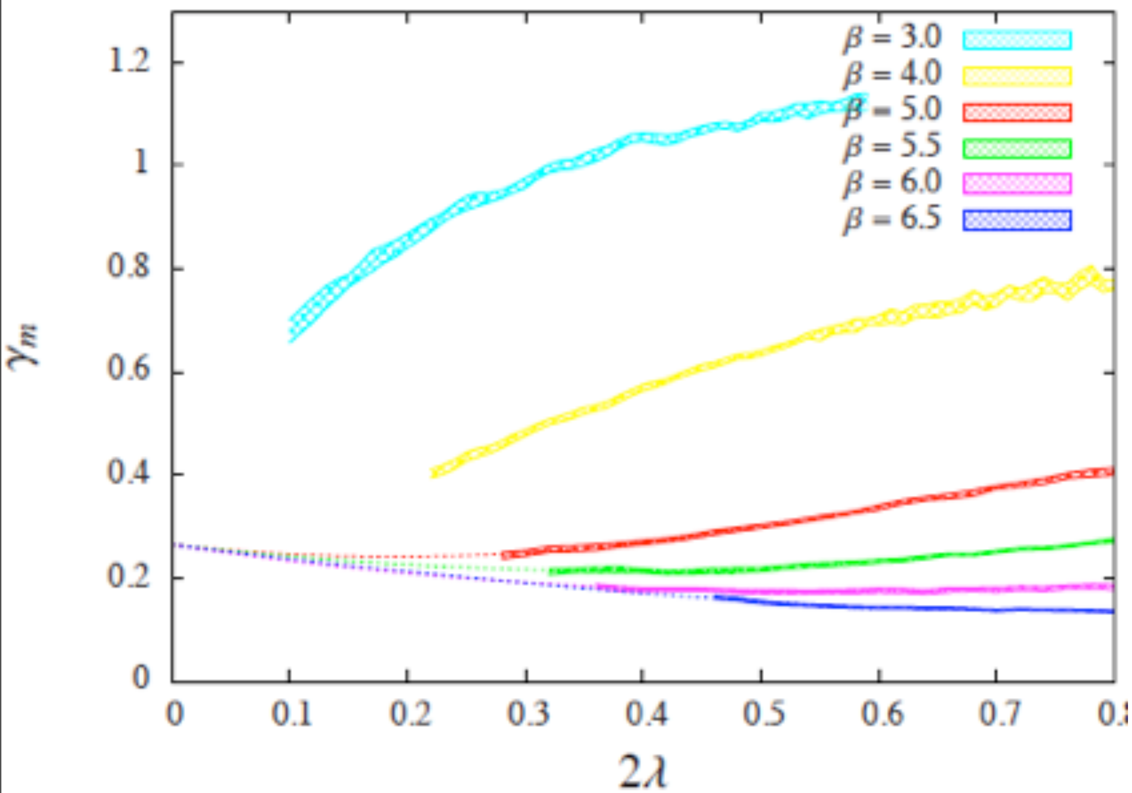



Universal curve covering almost 2 orders of magnitude in energy!

Perturbative: functional form from 1-loop PT, relative scale is fitted

Most of these data were obtained on deconfined (small) volumes with m=0!

# Spectral density results: $N_f = 12$




All simulations are in the m=0 chiral limit,
$32^4$ and $36^4$ volumes to take $V \to \infty$

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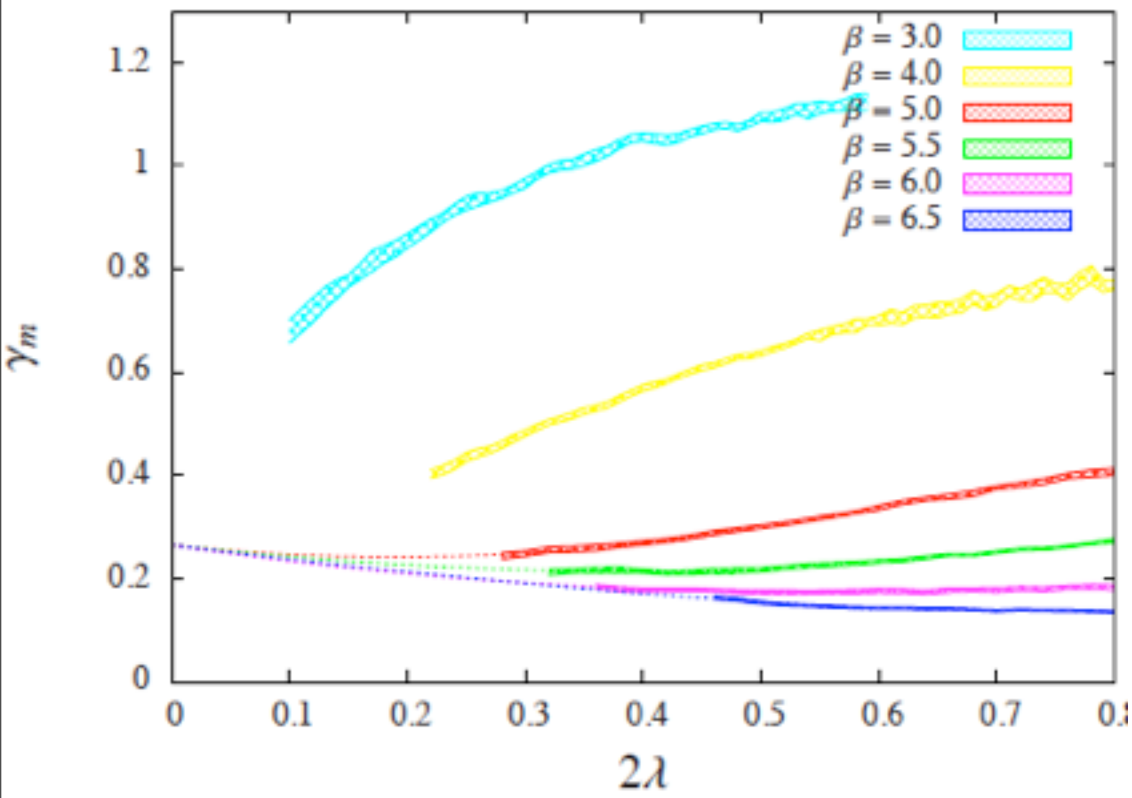


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UV:

- There is no sign of asymptotic freedom behavior for β<6.0,

$\gamma_m$ grows towards UV

- Not in the basin of attraction of the UVFP

IR:

- Extrapolation to λ=0 (quadratic with common intercept for 4 couplings) predicts $\gamma^*_m = 0.26(3)$
- Consistent with conformal behavior

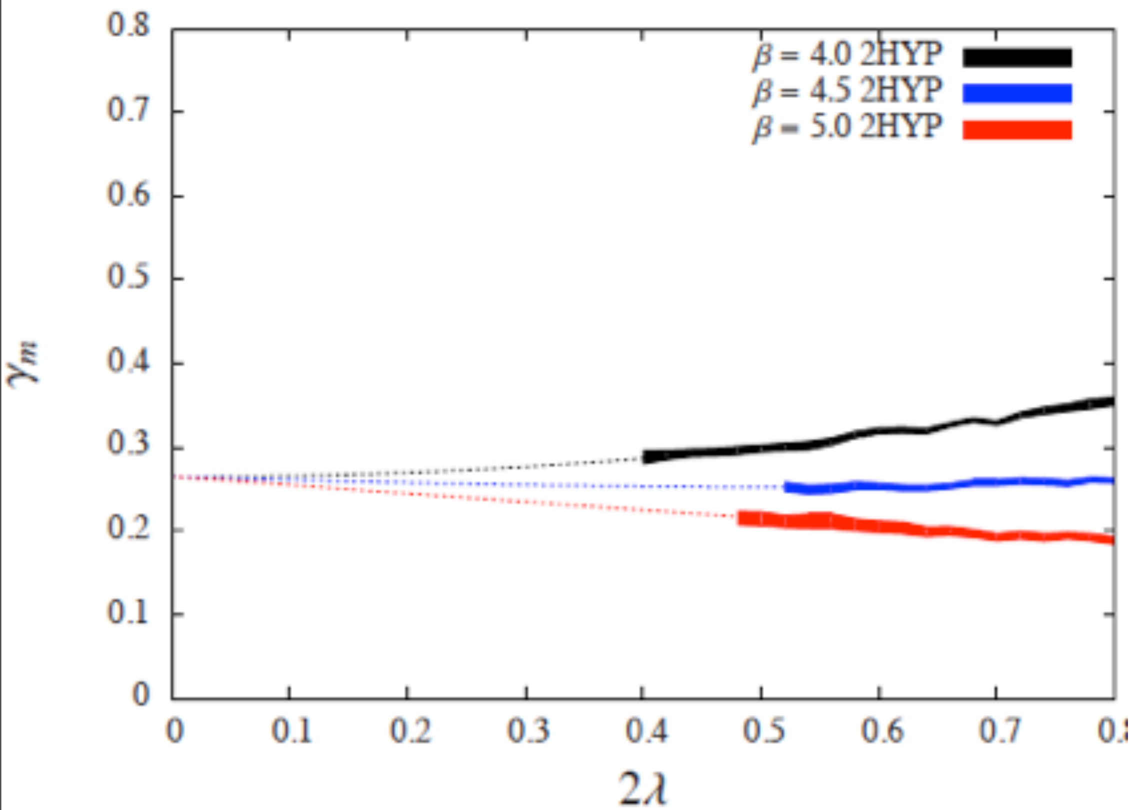
# Universality

## Is this result universal?

- We use nHYP staggered action. What is the prediction from other actions ?
  (In the FSS analysis we showed that stout and HISQ staggered actions from LH and LatKMI collaborations are consistent with nHYP)

- Look at 2x nHYP smeared and naive staggered actions for comparison

# Spectral density results: nHYP and 2nHYP, $N_f$ =12




Combine nHYP and 2nHYP staggered actions

- λ=0 extrapolation predicts $\gamma^*_m = 0.27(3)$
- 2nHYP has smaller UV effects

Spectral density results are consistent with FSS ( $\gamma^*_m = 0.235(15)$ )

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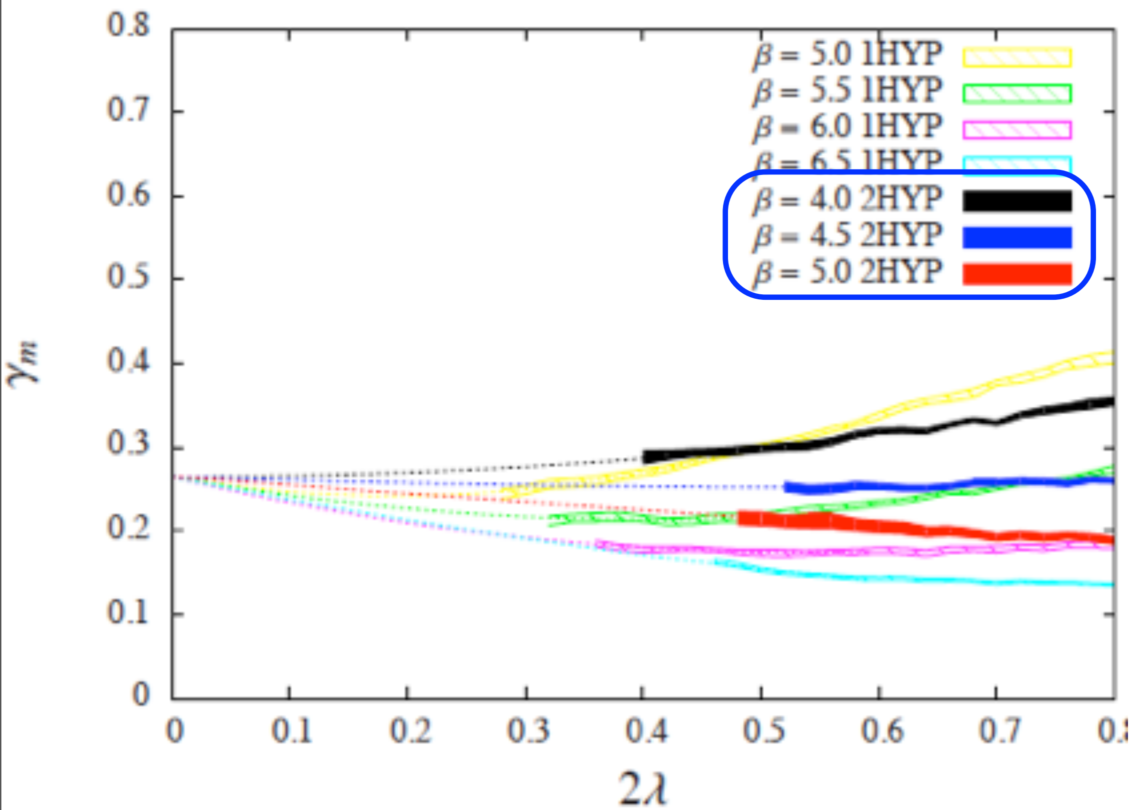


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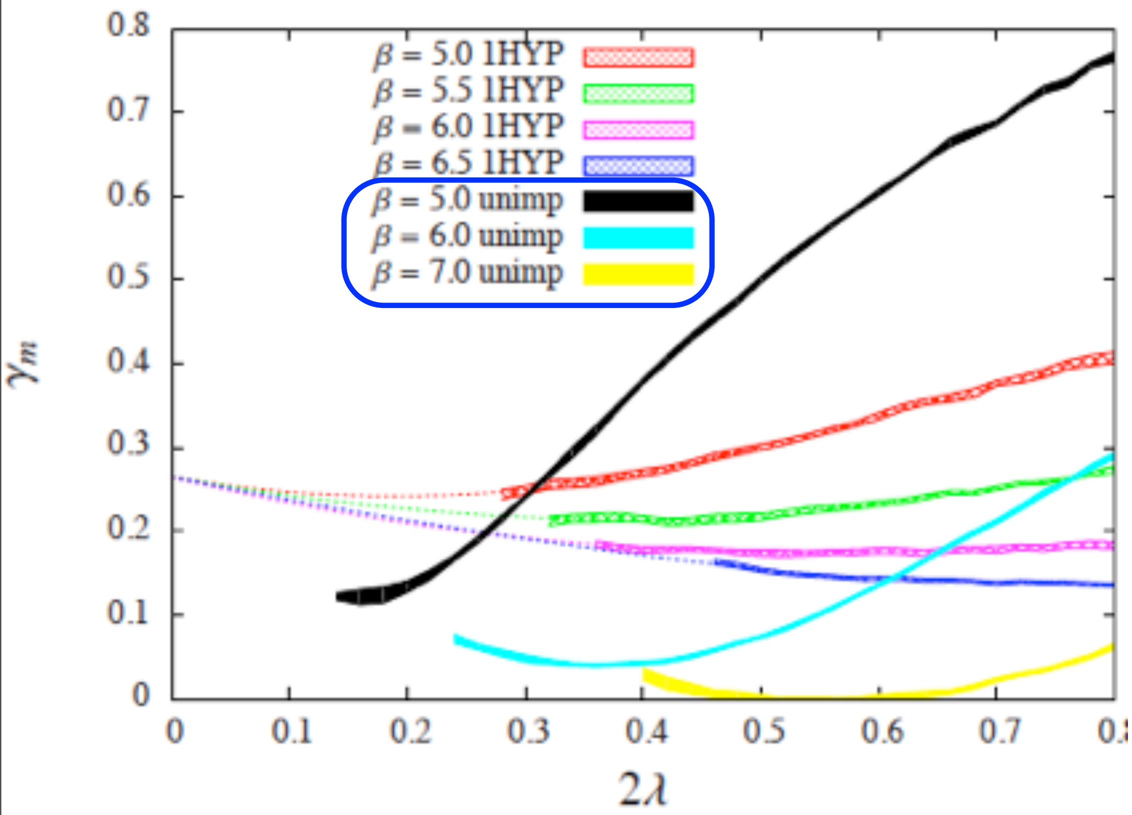


The unimproved action has very strong lattice artifacts

- λ=0 extrapolation is difficult even on $48^4$

but it could be consistent

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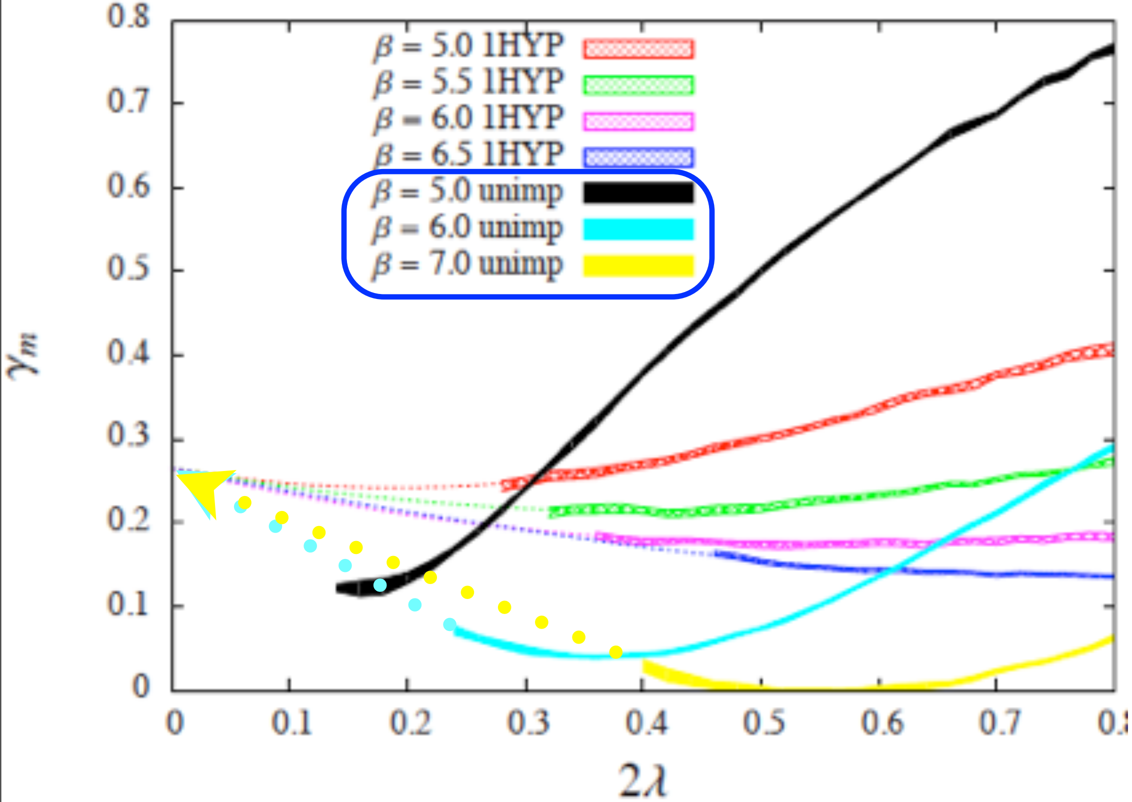


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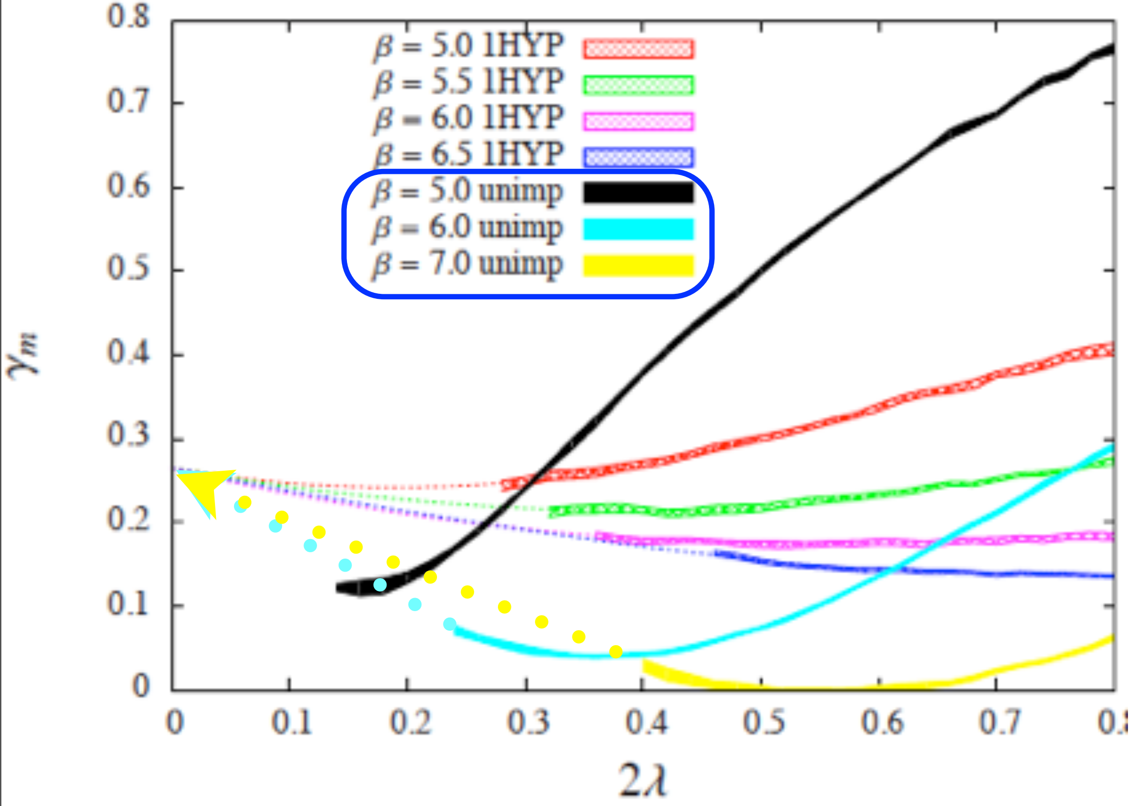


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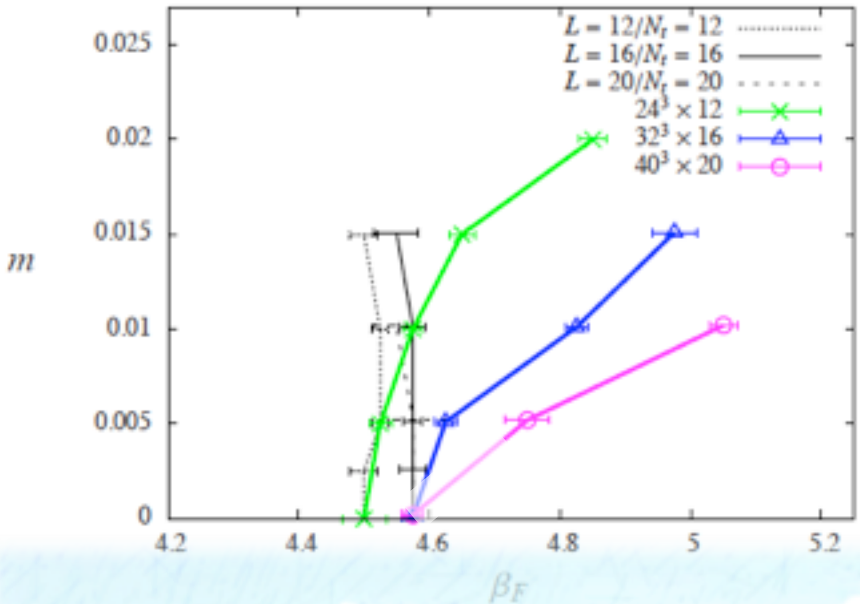
Beware of unimproved actions - they can bite

# $N_f$=8 flavors

Expected to be chirally broken

- 2-loop PT : close to conformal
- Numerical studies: (newer)
  - Boulder : finite temperature phase diagram cannot distinguish between conformal & chiral broken
  - LatKMI : walking with mixed ChPT/hyperscaling
  - USQCD : cannot distinguish between conformal & chiral broken

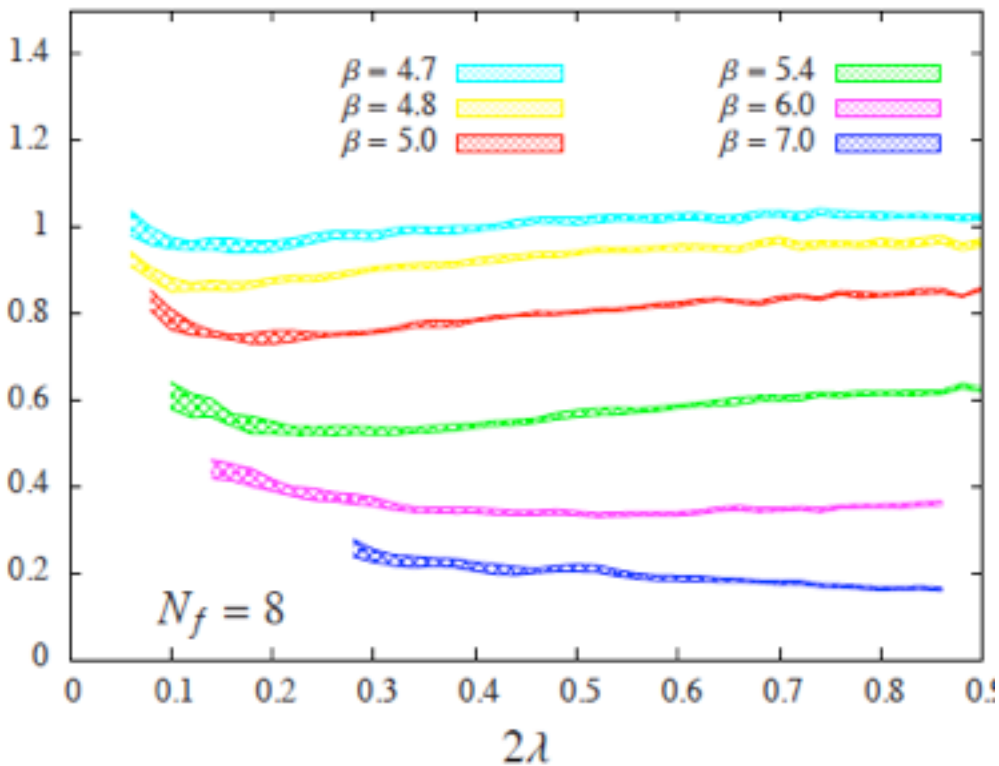



Phase diagram with nHYP action:
black lines: bulk $S^4b \rightarrow$ deconfined
colored lines: confined $\rightarrow$ deconfined

No clear confining phase in the chiral limit up to $N_T$=20 !

# Anomalous dimension, nHYP, $N_f$ =8

Expected to be chirally broken - looks like walking!




- No asymptotic free scaling at stronger coupling but
- Walking across orders of magnitude of energy

- At stronger coupling the $S^4b$ phase develops

- Would 2HYP allow stronger coupling/ explicit chiral breaking?

All simulations are in the m=0 chiral limit, $24^3$x48 and $32^3$x64 volumes

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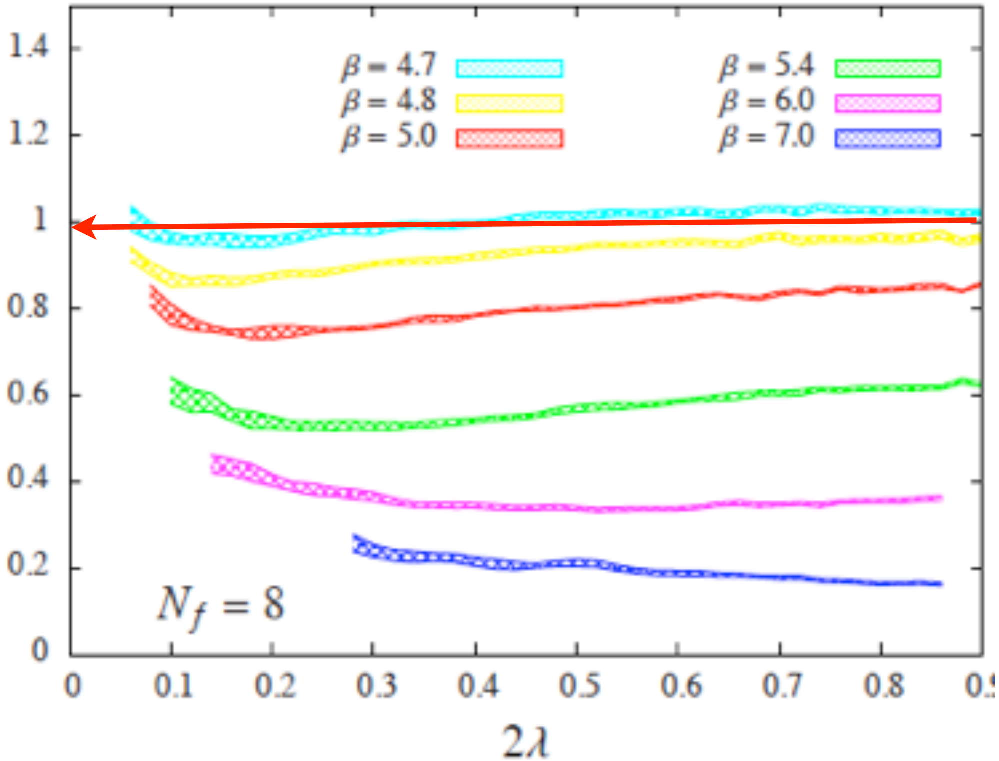


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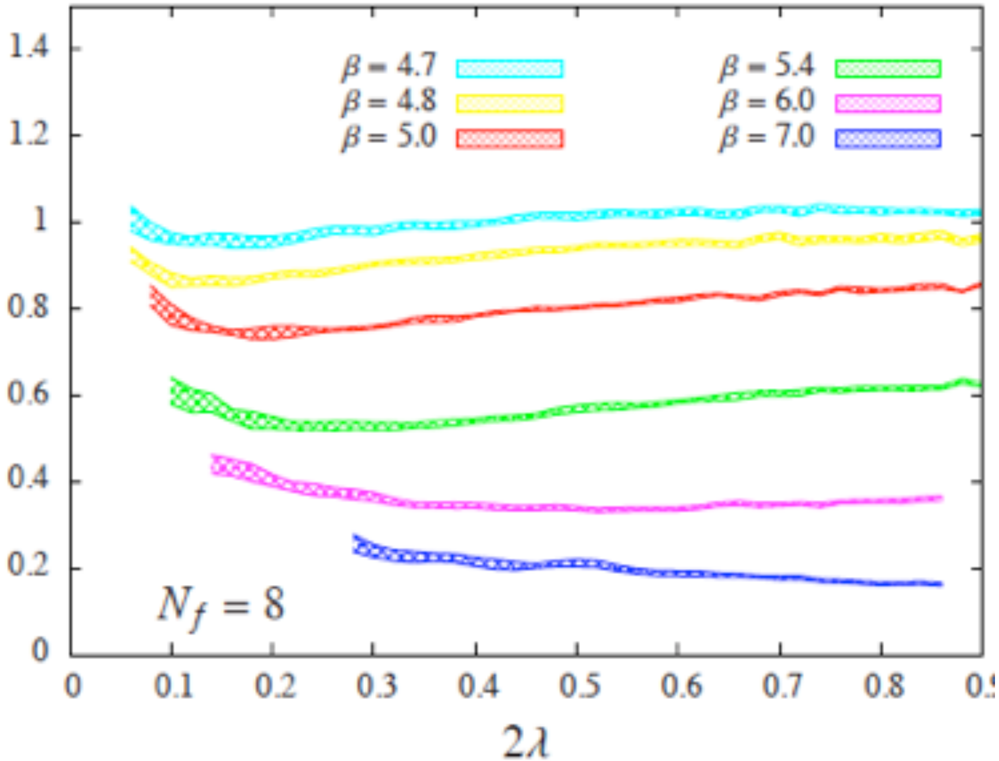


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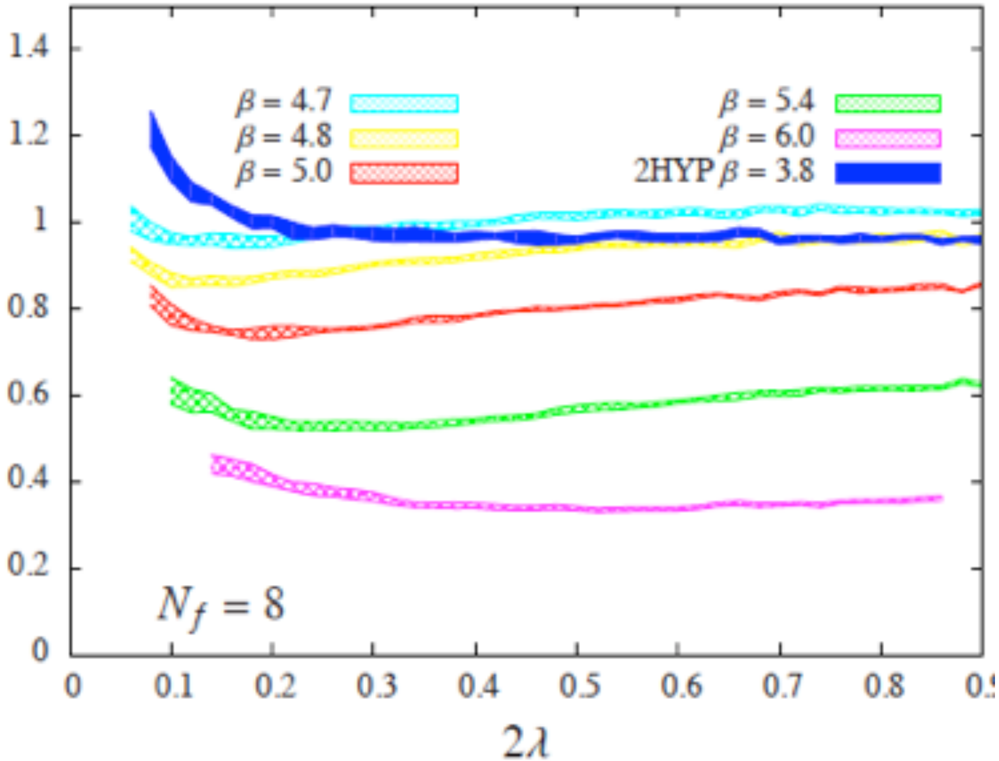
# Anomalous dimension, 2nHYP vs nHYP, $N_f$ =8

2nHYP breaks chiral symmetry in the chiral limit on $32^4$ before $S^4$b phase




- 2nHYP at β=3.8, m=0.0025

-Nearly constant $\gamma_m \approx 1$ until it breaks chiral symmetry

-2HYP shows less UV and allows stronger coupling

$N_f$=8 shows walking and chiral symmetry breaking
What else to ask for ? ($0^{++}$ mass)

# Dirac operator eigenvalue spectrum and spectral density

Unique & promising method !

Can distinguish strong and weak coupling region of conformal / chirally broken systems

It is important to look at

- the scale dependence of $\gamma_{\rm eff}$
- several gauge couplings, even actions

Predictions:

$N_f$=4 : scaling & anomalous dimension

$N_f$=12 : looks conformal with $\gamma^* = 0.26(3)$

$N_f$=8 : could be walking with large anomalous dimension!