# SU(2) with six flavors
## A new kind of gauge theory

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for the LSD Collaboration

# Two-Color Gauge Theories

- Perturbatively, $SU(2)$ gauge theories behave like any other $SU(N_c)$ gauge theory.
- Non-perturbatively, $SU(2)$ could be quite different:
  - No complex representations (pseudo-real, real)
  - Enlarged global symmetry: $SU_L(N_f)\times SU_R(N_f)\rightarrow SU(2N_f)$.
  - Spontaneous symmetry breaking produces more NG bosons: $SU(2N_f)\rightarrow Sp(2N_f)$ gives $N_f(2N_f-1) - 1$ *vs.* $N_f^2 - 1$.
  - Can we establish the range of $N_f$ over which spontaneous symmetry breaking occurs, *i.e.* the conformal window?

# Two Colors and BSM Physics

- The special features of two-color gauge theories can lead to new models of BSM physics.
- The five NG bosons of the $N_f$=2 theory can yield a composite Higgs boson as pseudo-NG boson.
- Enlarged global symmetry suppresses charge radius and magnetic moment interactions in composite dark matter models.
- Enlarged NG boson sector could lead new kind of finite temperature phase transition for a confining gauge theory.
- If a confining, two-color gauge theory is realized in nature, what are the implications of this phase transition on cosmology?

# Two-Color Conformal Window

Perturbative Estimates

- Caswell-Banks-Zaks established that $SU(N_c)$ gauge theories with $N_f$ flavors of Dirac fermions in the fundamental representation have IR conformal fixed points if $N_f<11N_c/2$.

- This IR conformal behavior ends for $N_f < N_{f^*}(N_c)$ when the theory confines.

- Higher-loop calculations (Refs) can be used to test the reliability of perturbative estimates of $N_{f^*}$.

- A reasonable estimate is $N_{f^*} \lesssim 4\ N_c$.

# Two-Color Conformal Window

Ladder-Gap Equations

- The rainbow diagram approximation of the Schwinger-Dyson equation gives an estimate of the critical coupling $g_c^2$ of chiral symmetry breaking.

- For SU(2), $g_c^2 \approx 17.5$.

- Comparing this estimate to the IRFP coupling of the two-loop beta function gives an estimate of $N_{fc}$.

- For SU(2), $N_{fc} \approx 8$, consistent with pert. theory.

# Two-Color Conformal Window

Cardy's a-theorem

- Much has been made of late of the proposed proof of Cardy's a-theorem. Can it constrain $N_{fc}$?

- $a_{UV} = 62(N_c^2-1) + 11\, N_c\, N_f$

- For broken SU(2): $a_{IR} = N_f\,(2N_f-1) - 1$

- Given massless gauge dofs count 62 times massless scalars means the a-theorem, even if true, provides no useful constraint.

# Two-Color Conformal Window

ACS Thermal Inequality Conjecture

- Another way to count massless dofs is via the thermodynamic free energy: $f(T) = 90\,F(T) / \pi^2\,T^4$.
- In $T \rightarrow 0$ limit, massive contributions are suppressed.
- SU($N_c$): $f_{UV}(0) = 2\,(N_c^2 - 1) + 3.5\,N_c\,N_f$
- SU(2): $f_{IR}(0) = N_f\,(2N_f - 1) - 1$.
- ACS conjecture: $f_{UV}(0) \geq f_{IR}(0)$. If true, this leads to a significant bound for SU(2): $N_{fc} \lesssim 4.7$.
- Further, it is significantly different from perturbative estimates.

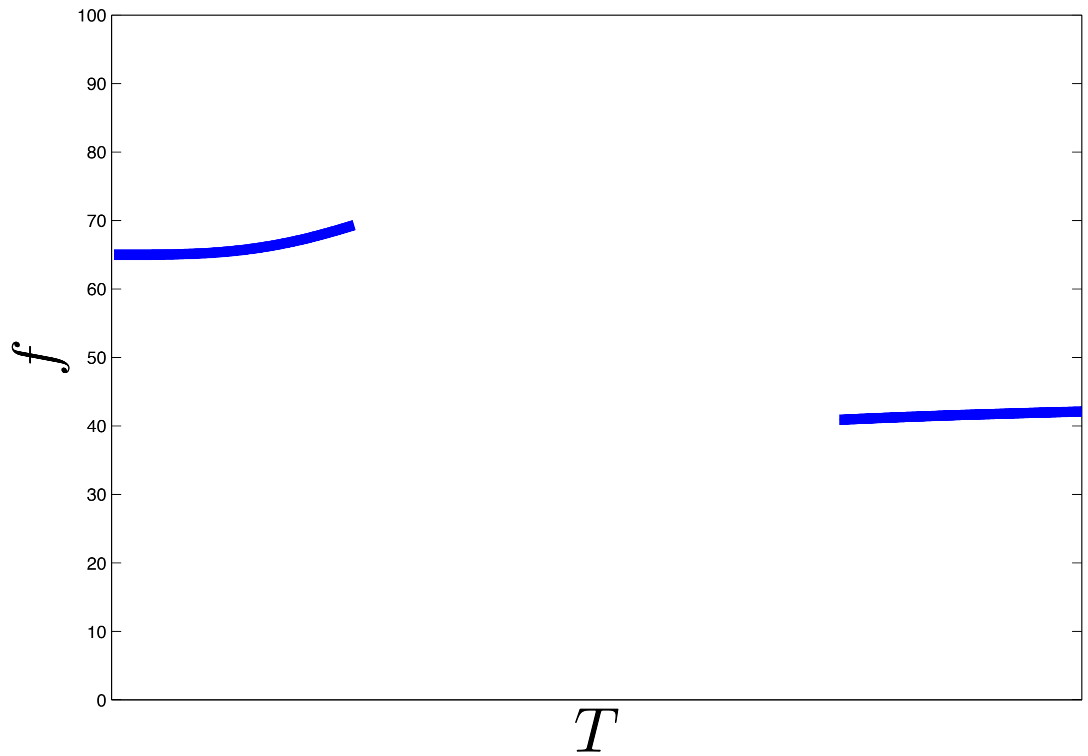
# Two-Color Conformal Window

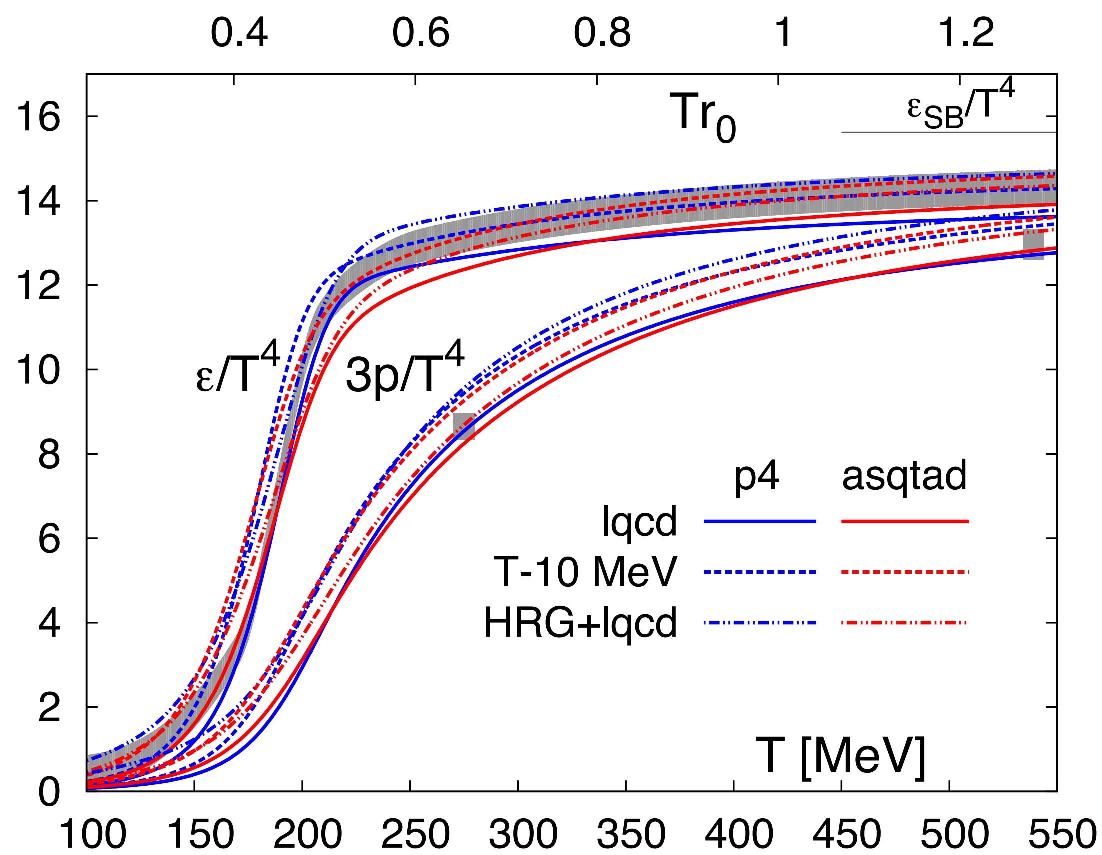
## Previous Lattice Results

- Numerous lattice results that demonstrate that the SU(2) $N_f=2$ theory is confining and chirally broken.
- Iwasaki et al (2004) infinite coupling confinement studies: $N_f=3$ inside the conformal window.
- Karavirta et al (2011) SF running coupling studies: $N_f=4$ outside conformal window.
- Other running coupling studies suggest $N_f=8$ (Ohki et al) and $N_f=10$ (Karavirta et al) are inside conformal window.
- $N_f=6$ is a difficult but very interesting case. Several early attempts were inconclusive (Bursa 2010, Karavirta 2011, Voronov 2011-2).
- There will be a presentation by N. Yamada about the calculation of the KEK group.

# SU(2) $N_f$=6 Thermodynamics

- In QCD, the equation of state outside the transition region is dominated by the Stefan-Boltzmann term.
- The ACS thermal inequality would mean that all confining asymptotically-free gauge theories have QCD-like thermodynamics.
- If SU(2) $N_f$=6 violates the ACS thermal inequality, the equation of state should be very different from QCD-like theories.



**Two-Color Theory with Novel Infrared Behavior**

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# LSD Publication

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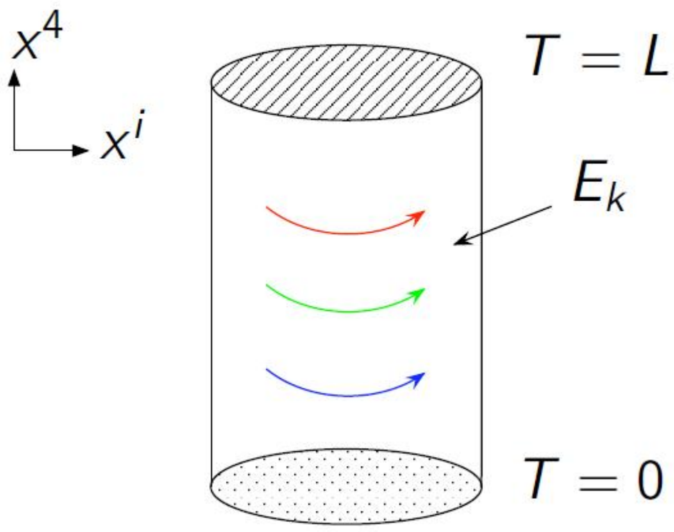
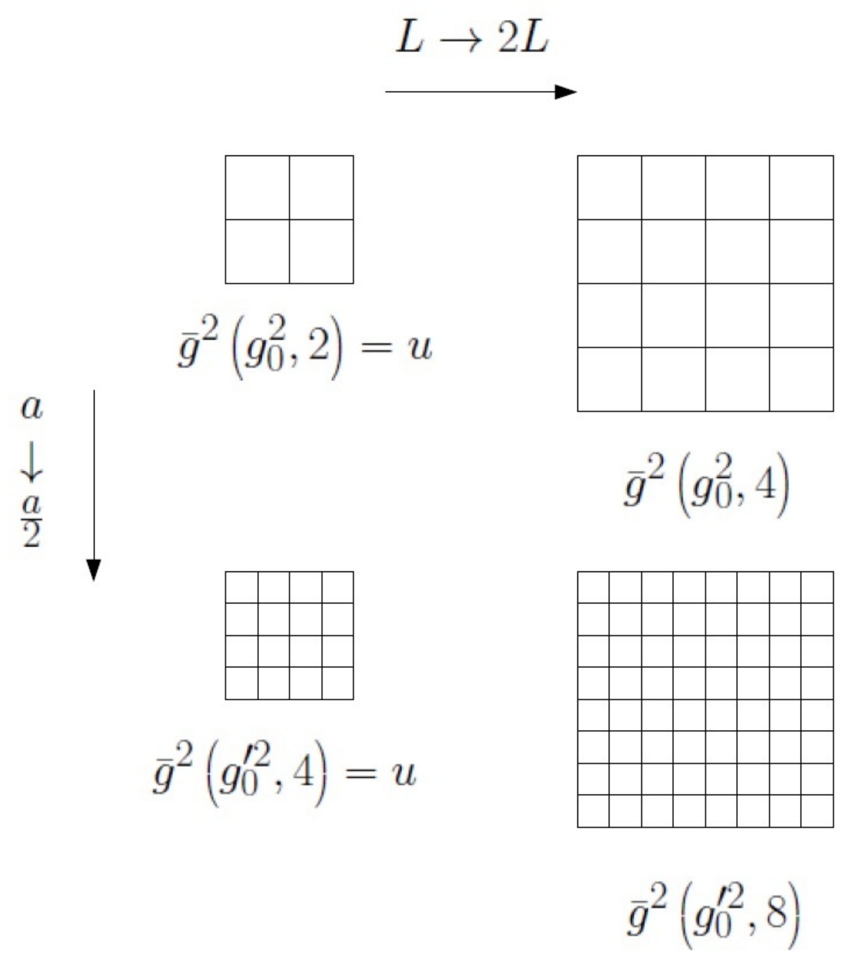
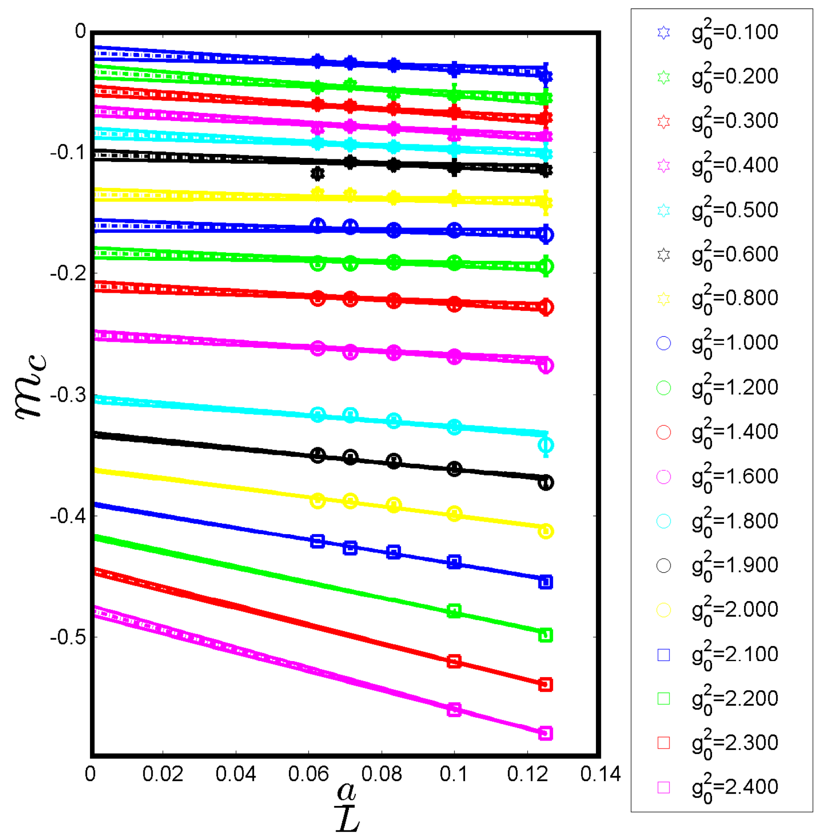
# SU(2) $N_f$=6 Calculational Details

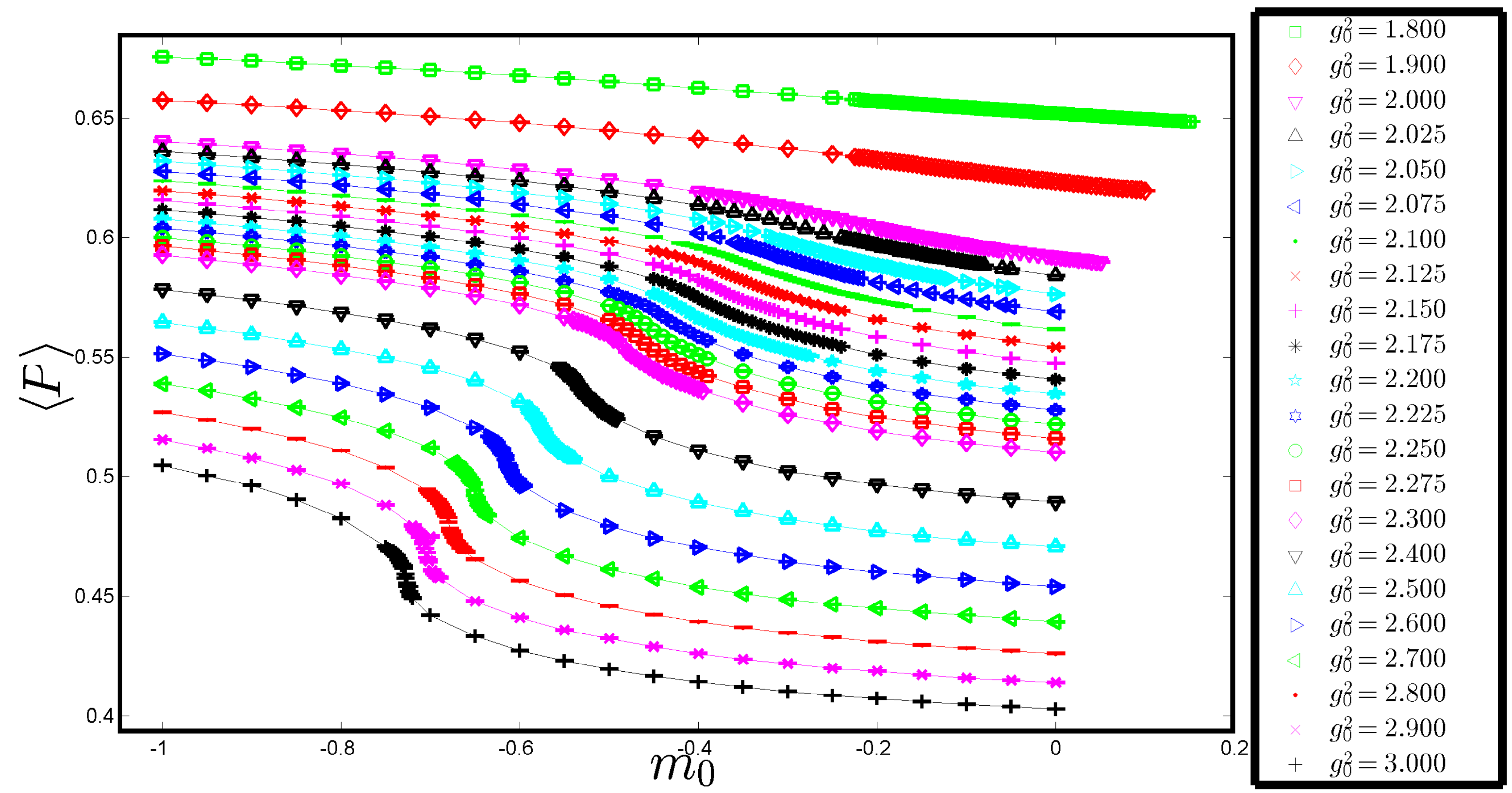
- We use the standard Schrödinger functional running coupling formulation.
- We use step-scaling to compute the lattice step scaling function:
  $\Sigma(u,s,a/L) \equiv g^2(g_0^2,sL/a)$ if $u=g^2(g_0^2,L/a)$.
- We compute the continuum step scaling function by taking the limit:
  $\sigma(u,s) = \Sigma(u,s,a/L)$ as $a/L \to 0$.
- The quantity $[\sigma(u,s)-u]/u$ is analogous to the continuum beta function.
- We use the Wilson fermion action with one level of stout smearing, tuned to massless point.



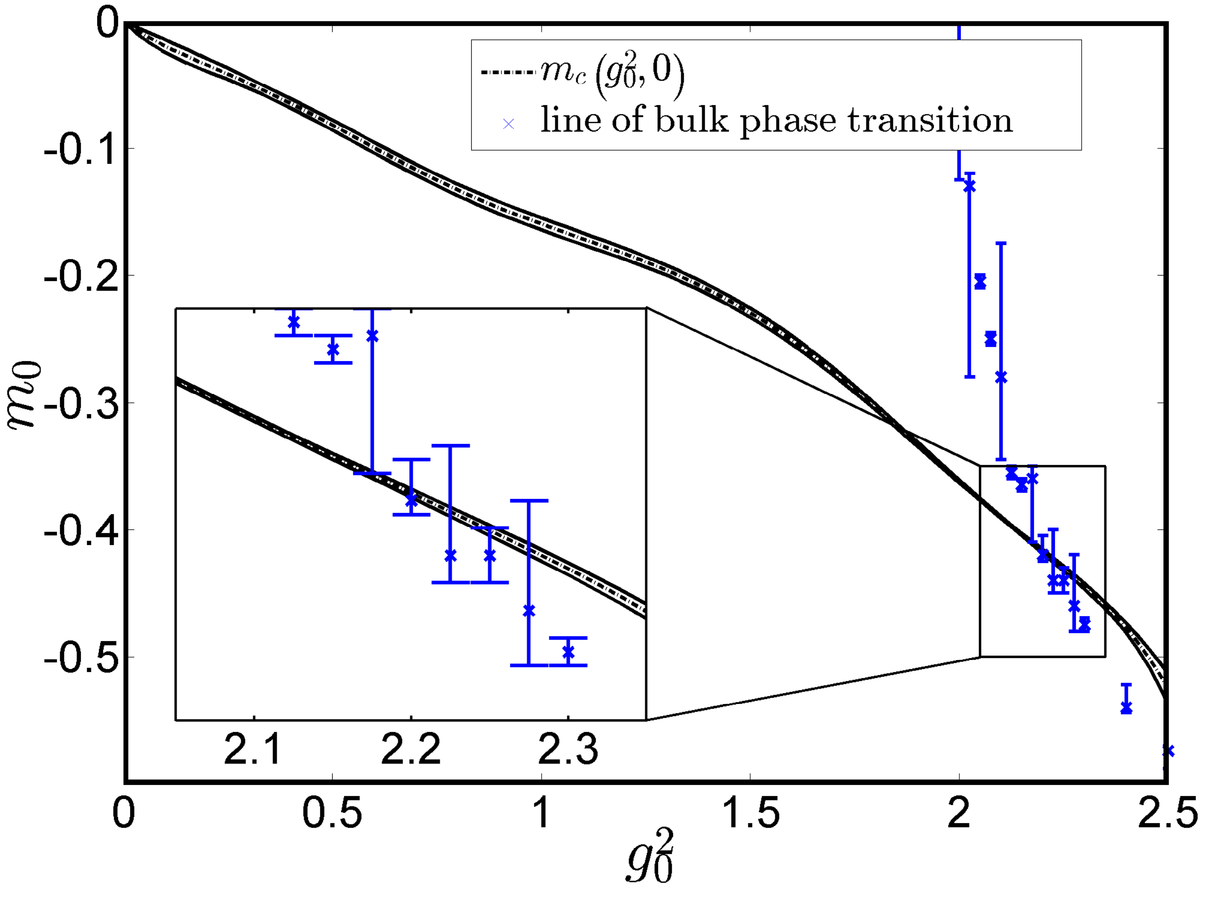



# Stout Wilson Parameter Space



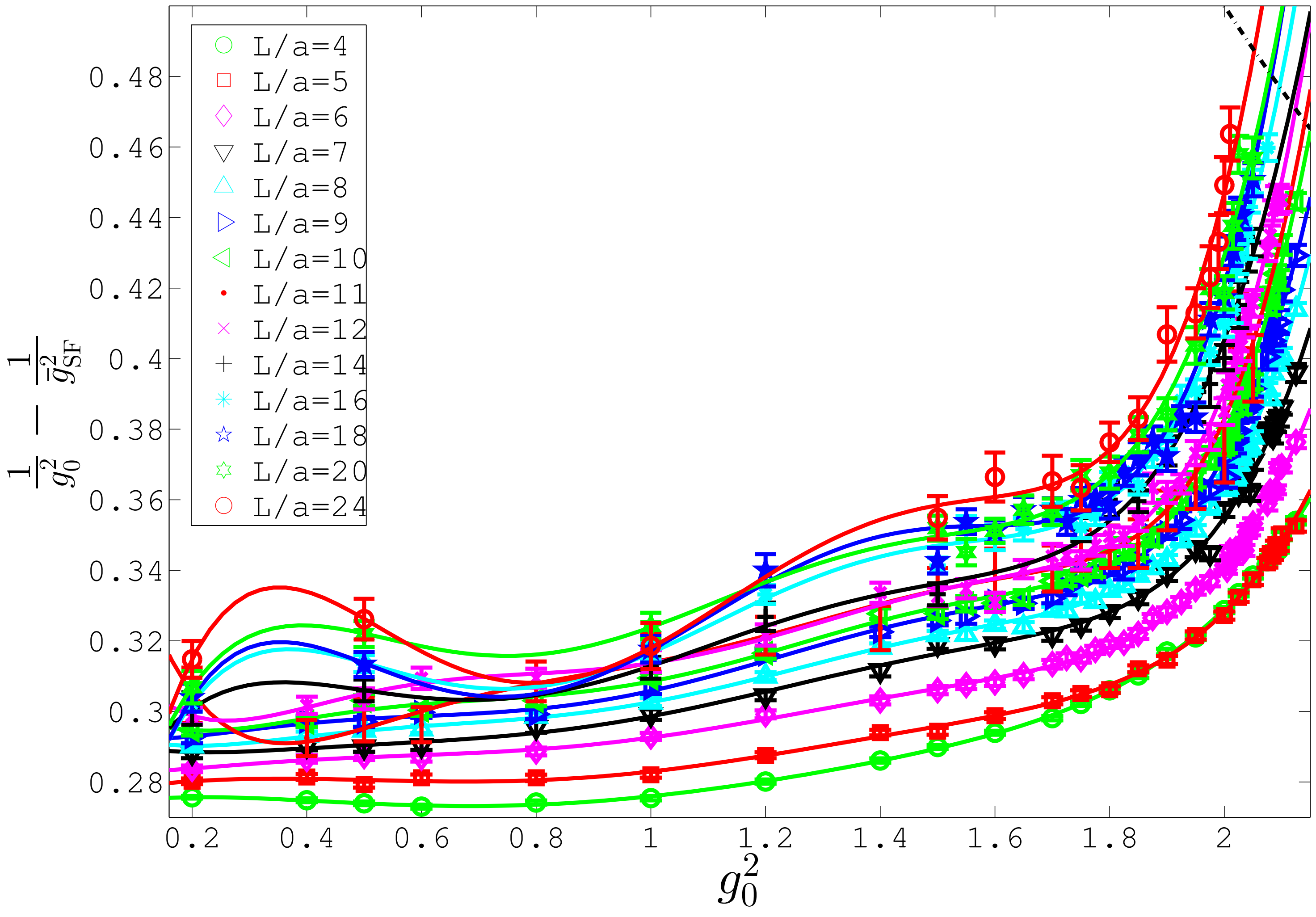




- We determined the massless point vs. coupling in infinite volume limit.
- We also located bulk phase transition/crossover line.
- Transition crosses massless curve around $g_0^2 = 2.2$.



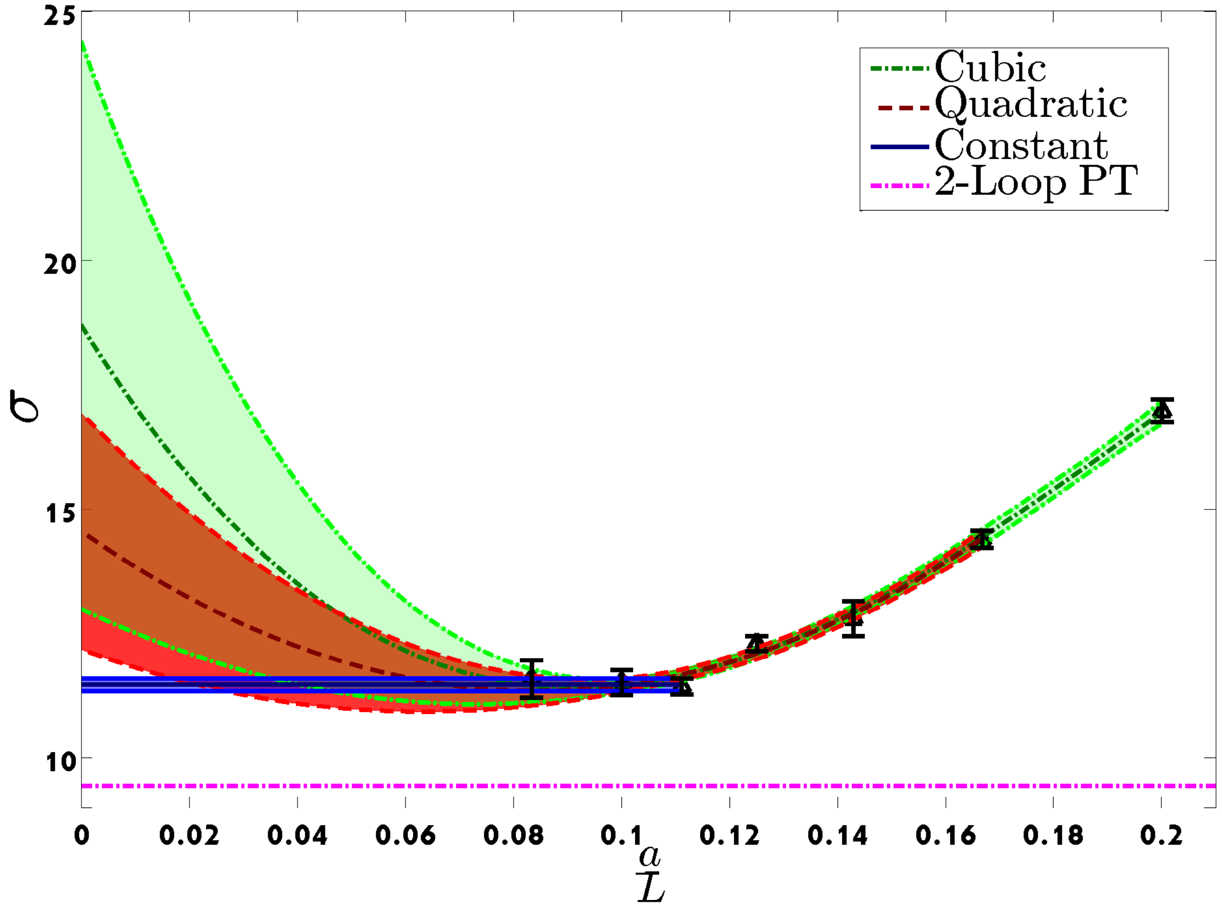
# Interpolating the data

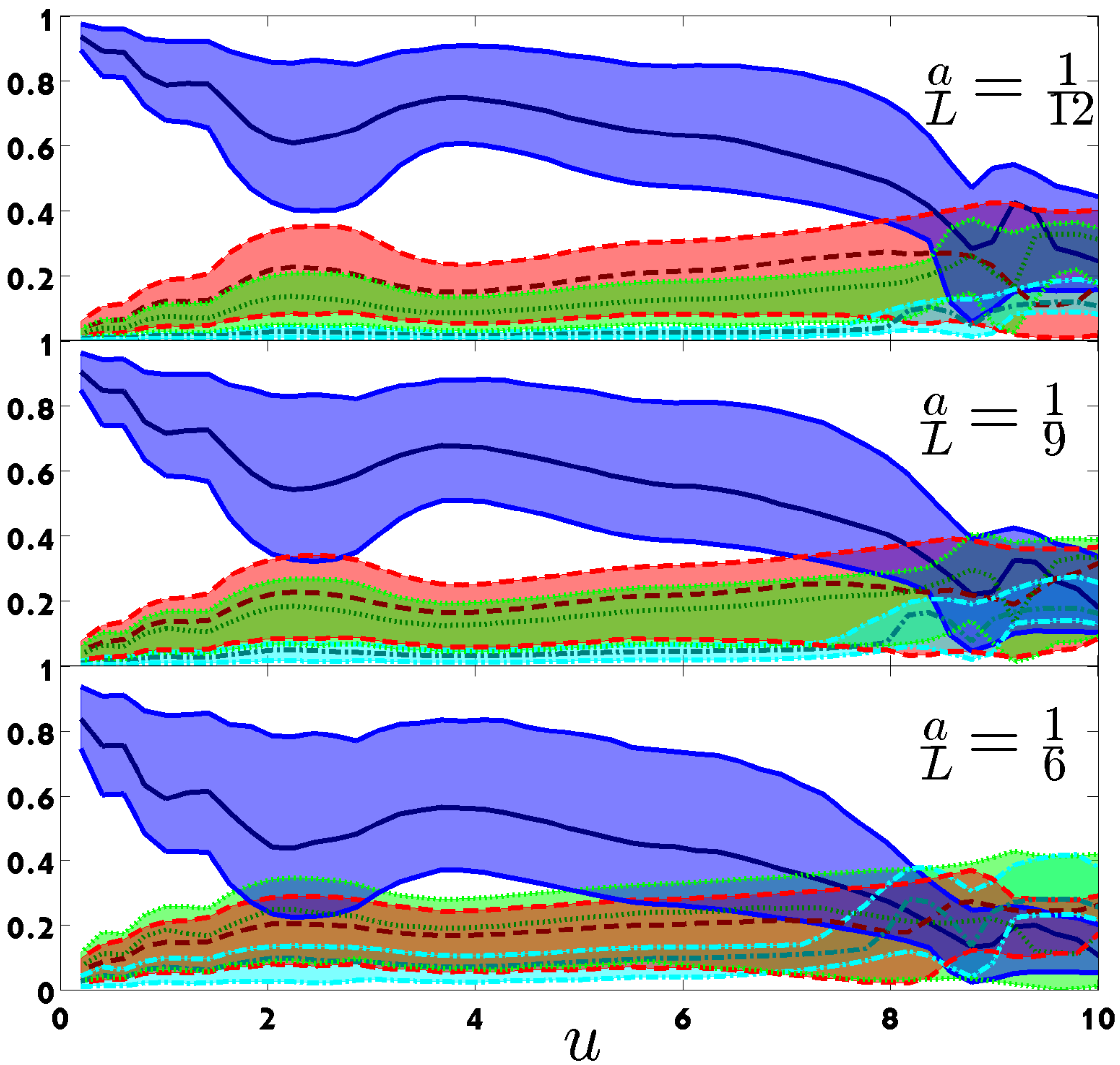
- Gennady generated a huge amount of data using many different computers over two years.
- For a slowly running theory, it is impossible to do step scaling tuning the lattice spacing by had at each and every step.
- We compute the SF coupling over a range of $g_0^2 < 2.2$ and $4 \leq L/a \leq 24$.
- We fit $(g_0^2)^{-1}$ - $(g_{SF}^2)^{-1}$ to polynomial in $g_0^2$ for each L/a.
- The functional form is inspired by perturbation theory but the coefficients are not constrained to p.t. values.
- We don't worry about wiggles at very weak coupling. They don't affect the result, as I will explain.



# Extrapolating the step scaling function

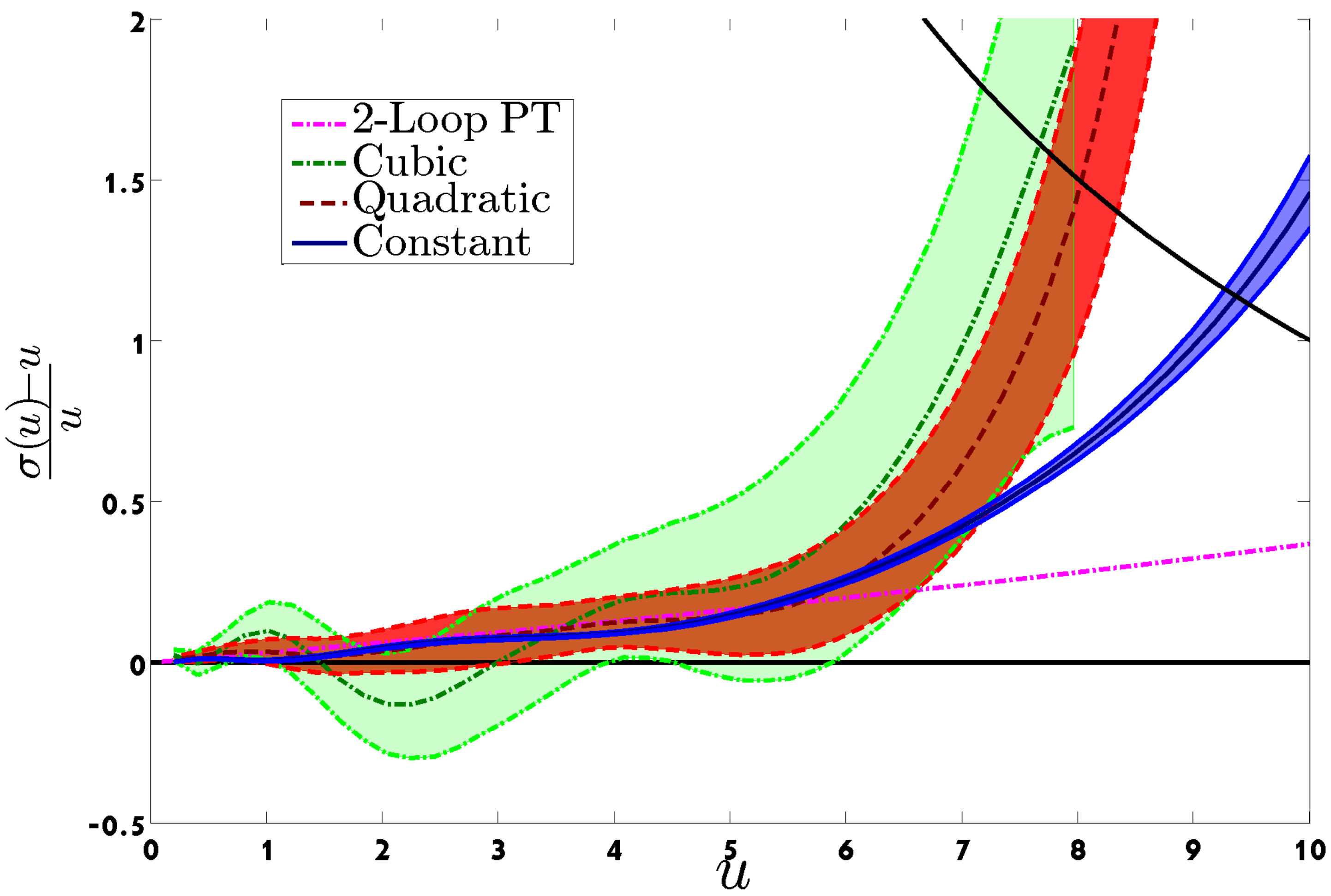
- Extrapolate Σ(u,s,a/L) to polynomial in a/L to extract σ(u,s).
- At weak coupling (u<6), a constant extrapolation is fine. At stronger coupling (u>6), a higher order continuum extrapolation is required.
- The quadratic term is as important at the linear term unless L/a is very large. Perhaps linear would be OK with 16→32 and larger volumes.



# Discrete beta function

- In the discrete beta function, we don't see any evidence for a fixed point.
- We don't expect that a fixed point will appear as the beta function dipping down to zero. It should cross zero and run backward all the way to strong coupling.
- You might recall for SU(3), $N_f$=12 the Yale group (pre-LSD) saw clear evidence for backward running and for SU(3), $N_f$=8 there was no such evidence.



# Comments

- I look forward to hearing about the latest KEK results for SU(2) $N_f$=6 from Yamada on Friday.

- I want to strongly emphasize that very slowly running theories are very hard to study on the lattice so it may take some time to get consistent results from all groups.

- Confining two-color theories always have composite Higgs candidates as pseudo-NG bosons.

- Studying the thermodynamics of the SU(2) $N_f$=6 theory could be very interesting.

- In the future, **lattice radial quantization** might be a better way to study (nearly-)conformal theories. See my poster.