

# Dynamic AdS/QCD: walking and the conformal window

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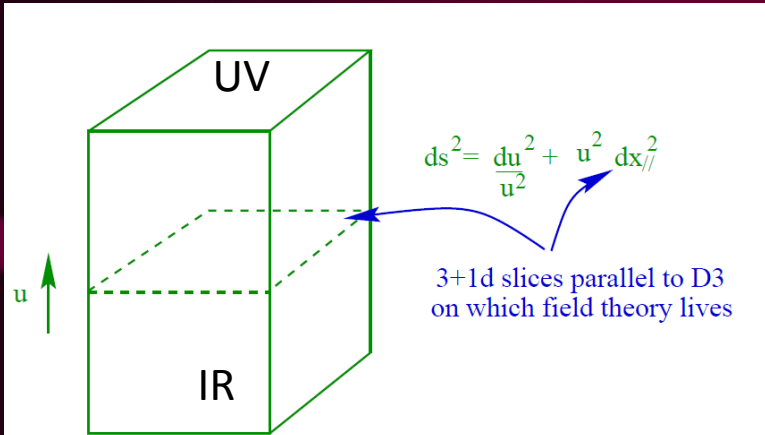
Keun-Young Kim, Kimmo Tuominen,  
Marc Scott



- AdS/QCD summary
- Conformal window dynamics
- Walking Technicolor
- Techni-dilatons

# Holography — a remarkable new way to think about RG flow

We treat RG scale as a direction of space-time....



Conformal theories eg

$$\int d^4x (\partial\phi)^2$$

have a dilatation symmetry

$$\phi \rightarrow e^\alpha \phi$$

$$x_{3+1} \rightarrow e^{-\alpha} x_{3+1}$$

Reproduced in AdS metric when

$$u \rightarrow e^\alpha u$$

O – J combinations are represented by fields in AdS with the same symmetries

$$S \sim \int d^4x du \sqrt{-g} (g^{MN} \partial_M \phi \partial_N \phi + M^2 \phi^2)$$

Consider spatially homogenous solutions where  $\phi = \phi(u)$

$$S \sim \int d^4x du u^3 (u^2 \partial_u \phi \partial_u \phi + M^2 \phi^2)$$

E-L equation:

$$\partial_u [u^5 \partial_u \phi] - u^3 M^2 \phi = 0$$

has solution

$$\phi = \frac{C}{u^\Delta} + \frac{C'}{u^{(4-\Delta)}}, \quad \Delta(\Delta - 4) = M^2$$

$\Delta$  of O – J fixes AdS field mass

Miracle: weak-strong coupling duality

# Traditional AdS/QCD

$$S = \int_{r_0}^{\infty} d^5x \sqrt{-g} T r \left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\}$$

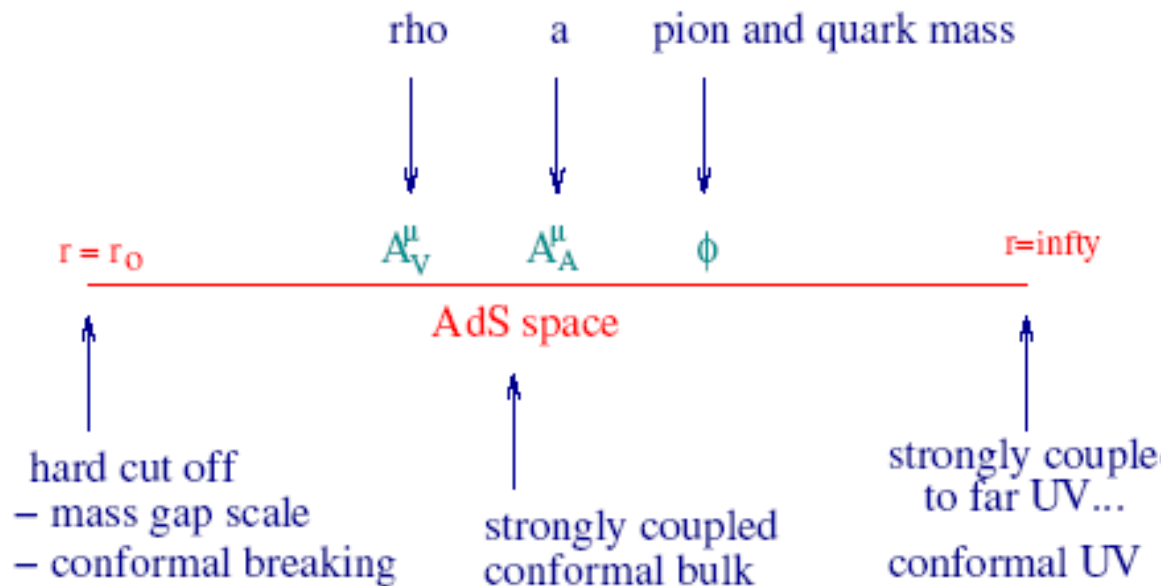
Parameter count

$r_0$

$c = qq$

$m$

$g_5$



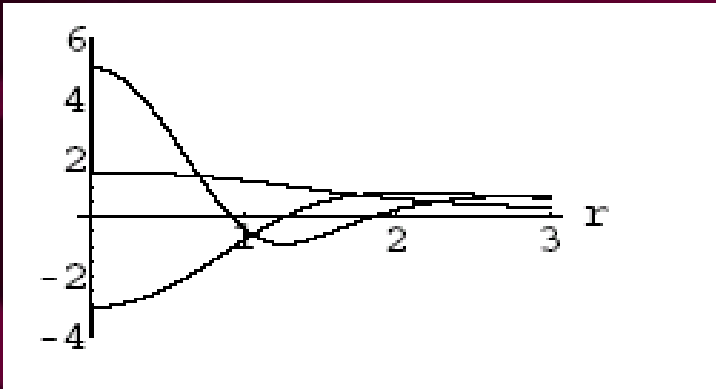
# Example Numerics - Meson Masses

The gauge field equation of motion

$$\left[ \partial_z \left( \frac{1}{z} \partial_z V_\mu^a(q, z) \right) + \frac{q^2}{z} V_\mu^a(q, z) \right]_\perp = 0.$$

$$V \sim V(z) e^{-iq \cdot x}, \quad q^2 = -M^2$$

Numerically shoot from UV... seek solutions with  $V'(\text{wall})=0$ ...



Sub  $V$ s back into the action and integrate over radial coordinate to determine decay constants....

Fix external currents normalization by matching to UV QCD  $\Pi_{vv}/\Pi_{aa}$

Observable	Measured (MeV)	AdS A (MeV)	AdS B (MeV)
$m_\pi$	$139.6 \pm 0.0004$	139.6*	141
$m_\rho$	$775.8 \pm 0.5$	775.8*	832
$m_{a_1}$	$1230 \pm 40$	1363	1220
$f_\pi$	$92.4 \pm 0.35$	92.4*	84.0
$F_\rho^{1/2}$	$345 \pm 8$	329	353
$F_{a_1}^{1/2}$	$433 \pm 13$	486	440

The basic ideas are remarkably good... but... to systematically move to QCD we would need to:

**IR improve** – include all operators that are non-zero in the vacuum and back react them on each other

**UV perfect** – match the running of operators to the true perturbative QCD values in the UV....

Of course as in any effective description of QCD this is overwhelming and you end up just re-parameterizing the data.... In QCD the **regime of validity** of the gravity description is **very tight**...

**AdS/QCD should be better in models with long running near strong conformality..**

# The Conformal Window

SU(N<sub>c</sub>) gauge theory with N<sub>f</sub> fundamental quarks

N<sub>f</sub>=11/2 N<sub>c</sub> \_\_\_\_\_ No AF

N<sub>f</sub> = ? N<sub>c</sub> \_\_\_\_\_ CFT

χ SB

m  $\bar{q}q$

$$\gamma_m^{(1)} = \mu \frac{d \ln m_q}{d\mu} = \frac{3(N_c^2 - 1)}{4N_c\pi} \alpha$$

If critical  $\gamma = 1 \dots$  N<sub>f</sub>/N<sub>c</sub> ~ 4

Yamawaki, Appelquist, Terning, Sannino, ...

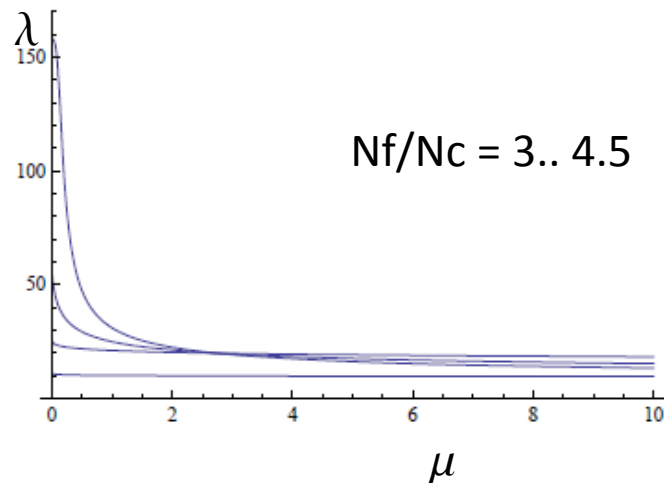
$$\beta(g) = -\frac{g^3}{(4\pi)^2} \left\{ \frac{11}{3}N_c - \frac{2}{3}N_f \right\} - \frac{g^5}{(4\pi)^4} \left\{ \frac{34}{3}N_c^2 - \frac{N_f}{N_c} \left[ \frac{13}{3}N_c^2 - 1 \right] \right\} + \dots$$

Using the 't Hooft coupling, and setting  $\frac{N_f}{N_c} \rightarrow x$  we obtain

$$\lambda \equiv g^2 N_c, \quad \dot{\lambda} = -b_0 \lambda^2 + b_1 \lambda^3 + \mathcal{O}(\lambda^4)$$

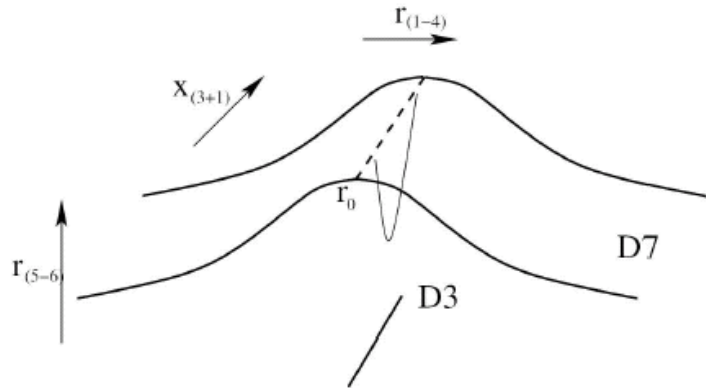
with

$$b_0 = \frac{2(11-2x)}{3(4\pi)^2}, \quad b_1 = -\frac{3(34-13x)}{2(11-2x)^2}$$





# Top-Down Models of Chiral Symmetry Breaking and Confinement



## Chiral Symmetry Breaking and Pions in Non-Supersymmetric Gauge/Gravity Duals

J. Babington <sup>a</sup>, J. Erdmenger <sup>a</sup>, N. Evans <sup>b</sup>, Z. Guralnik <sup>a</sup> and I. Kirsch <sup>a\*</sup>

## Towards a holographic dual of large- $N_c$ QCD

Martín Kruczenski, <sup>a</sup> David Mateos, <sup>b</sup> Robert C. Myers <sup>b,c</sup> and David J. Winters <sup>b,d</sup>

## Mesons in Gauge/Gravity Duals A Review

Johanna Erdmenger <sup>a</sup>, Nick Evans <sup>bc</sup>, Ingo Kirsch <sup>d</sup> and Ed Threlfall <sup>b\*</sup>

## Flavoured Large $N$ Gauge Theory in an External Magnetic Field

Veselin G. Filev<sup>\*</sup>, Clifford V. Johnson<sup>\*</sup>, R. C. Rashkov<sup>†1</sup> and K. S. Viswanathan<sup>†</sup>

## Towards a Holographic Model of the QCD Phase Diagram

Nick Evans,<sup>\*</sup> Astrid Gebauer,<sup>†</sup> and Maria Magou<sup>‡</sup>

*School of Physics and Astronomy, University of Southampton, Southampton, SO17 1RJ UK*

Keun-Young Kim<sup>§</sup>

## Low Energy Hadron Physics in Holographic QCD

Tadakatsu SAKAI<sup>1,\*</sup>) and Shigeki SUGIMOTO<sup>2,\*\*</sup>)

# Dynamic AdS/QCD

NB a model of quarks/mesons – we do not have a backreacted description of the glueballs! Kiritsis & Jarvinen try..

$$S = \int d^4x d\rho \text{Tr} \rho^3 \left[ \frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 + \frac{1}{2\kappa^2} (F_V^2 + F_A^2) \right]$$

$$X = L(\rho) e^{2i\pi^a T^a}.$$

$$ds^2 = \frac{d\rho^2}{(\rho^2 + |X|^2)} + (\rho^2 + |X|^2) dx^2,$$

X is now a dynamical field **whose solution will determine the condensate** as a function of m

We use the top-down IR boundary condition on mass-shell:  $X'(\rho=X) = 0$

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate – no hard wall

The gauge DYNAMICS is input through  $\Delta m$

$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c \pi} \alpha$$

$$m^2 = \Delta(\Delta - 4)$$

The only free parameters are m and  $\Delta$



# Stability Bound

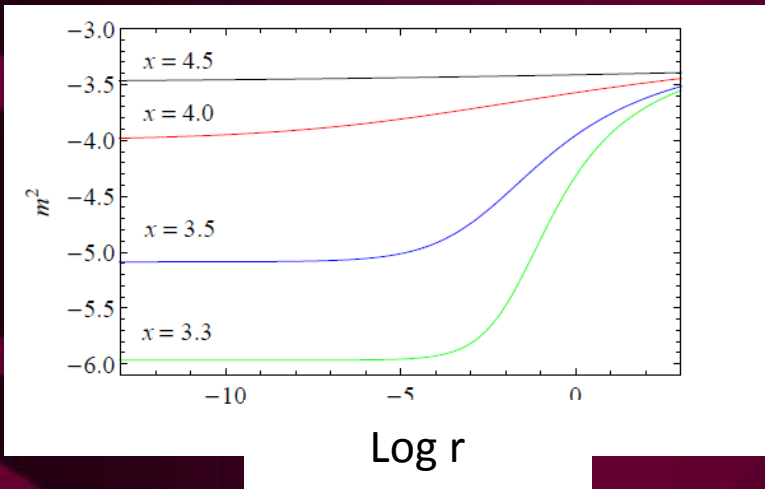
$N_f = 11/2 N_c$  ————— No AF  
CFT  
 $N_f = ? N_c$  —————

We model the  $\bar{q}q$  condensate by a scalar in AdS with a running mass

## Breitenlohmer-Freedman Bound

A scalar in AdS is stable until  
 $m^2 < -4$   
ie  $\Delta < 2$

$\chi$  SB



A hard prediction that matches gap equation-ology... using 2-loop  $\alpha$  plus 1-loop  $\gamma$  we predict

$$N_{f\_critical} = 4 N_c$$

ie 12 in QCD (same algebra as gap equations)

# Formation of the Chiral Condensate

We solve for the vacuum configuration of  $L$

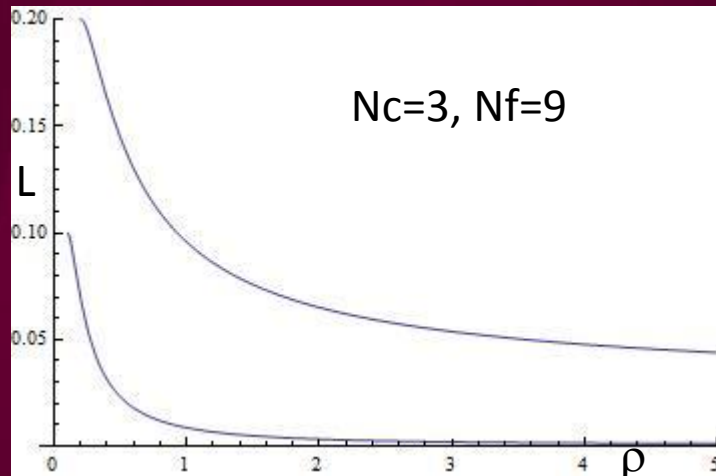
$$\partial_\rho[\rho^3 \partial_\rho L] - \rho \Delta m^2 L = 0.$$

We've dropped a term that depends on the derivative wrt  $L$  of  $\Delta m$  – AdS/CFT predicts its there... its not in perturbative QCD.. so we don't know how to fix its form... in anycase its tiny when  $\Delta m$  runs slowly...

Shoot out with

$$L'(\rho=L) = 0$$

If we included backreaction the decoupling of the flavours from the running would make the embedding go flat at lower  $\rho$ ...



Read off  $m$  and  $qq$  in the UV...

# In The CW

The massless quark theory is conformal... so consider the theory with a quark mass...

At a fixed point  $\gamma$  is constant and  $0 < \gamma < 1$

$$L = \frac{m}{\rho^\gamma} + \frac{\bar{q}q}{\rho^{2-\gamma}}, \quad |X|=L$$

Imposing the IR boundary condition gives analytically

$$\bar{q}q = \frac{\gamma}{2\gamma - 2} \left( \frac{2\gamma - 2}{\gamma - 2} \right)^{\frac{3-\gamma}{1+\gamma}} m^{\frac{3-\gamma}{1+\gamma}}$$

This is the hyperscaling relation of Del Debbio & Zwicky – ie dimensional analysis as the fixed point....

# In The UV

In the UV we impose the perturbative log running of  $\Delta m$

$$L = \frac{m}{(\ln \rho)^k} + \frac{\bar{q}q}{\rho^2} (\ln \rho)^k,$$

$$|X|=L$$

$$k = 3C_2/(4\pi b_0)$$

Imposing the IR boundary condition (for large  $m$ ) gives analytically

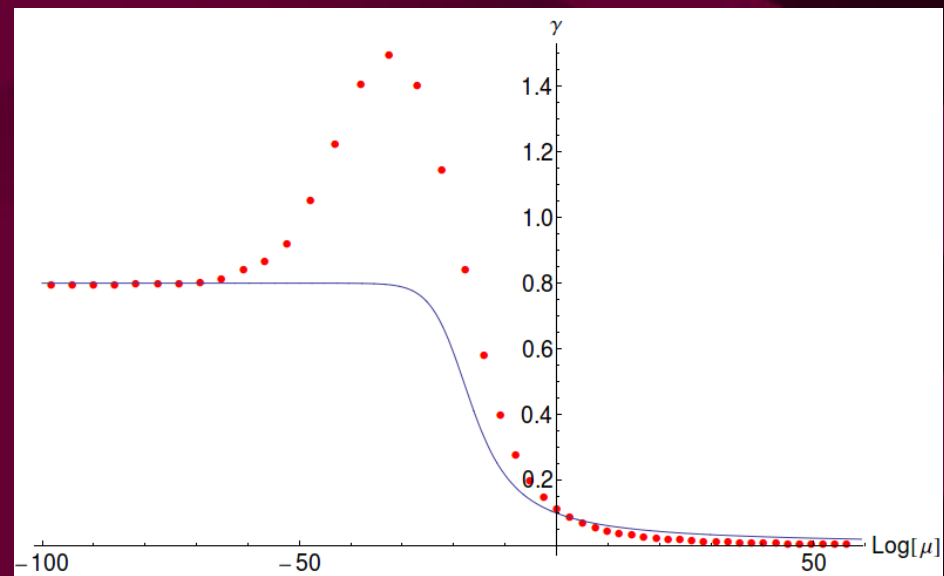
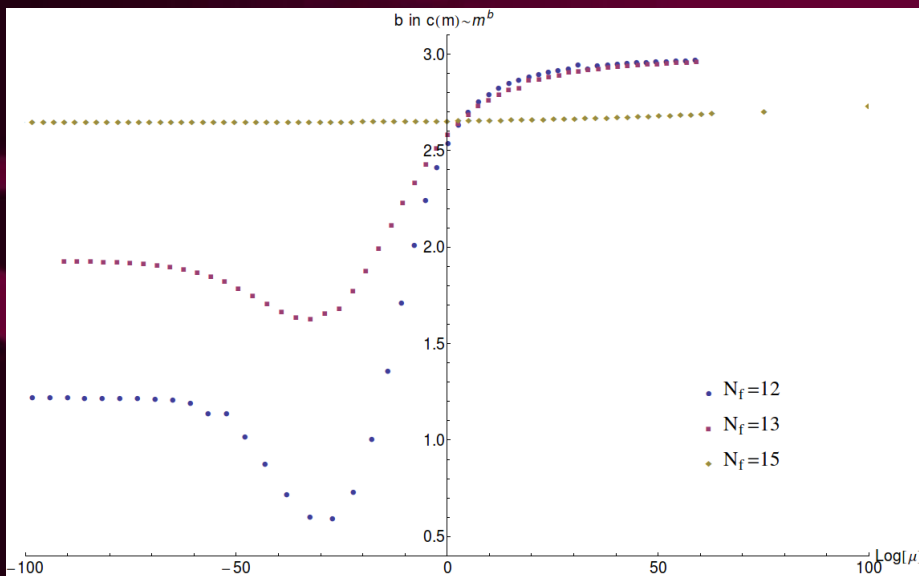
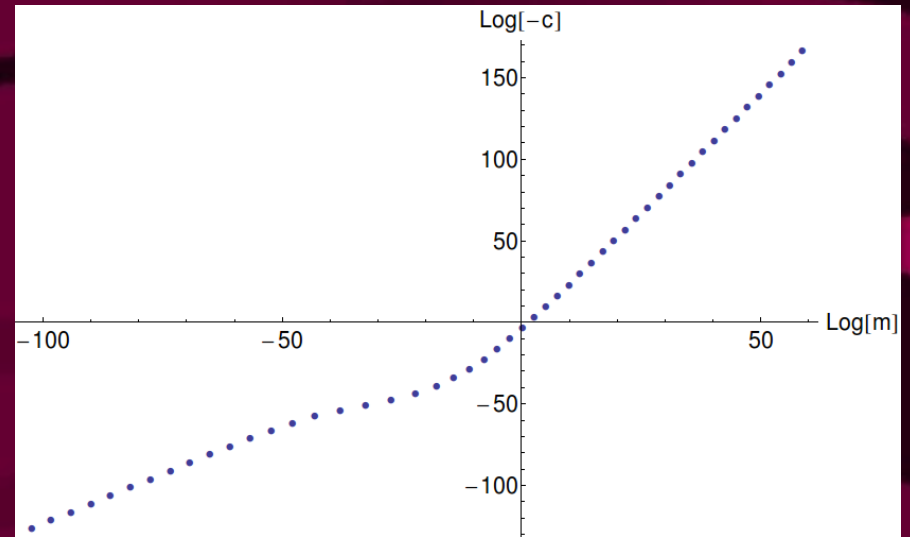
$$\bar{q}q = \frac{4}{\left(\ln \frac{L_0}{\Lambda}\right)^{4k}} m^3$$

This is dimensional analysis  $\bar{q}q \sim m^3$  upto log running....

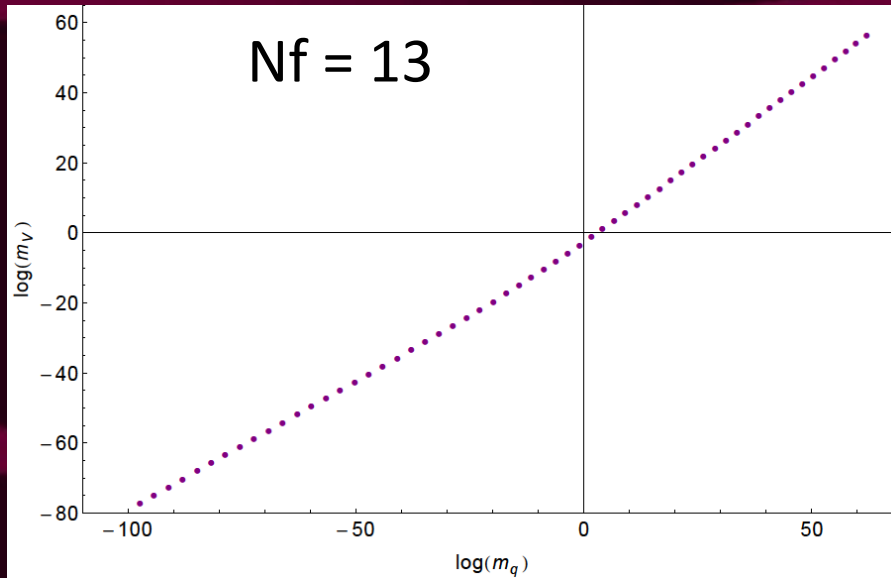
$qq(at\ m)$  vs  $m$  for the  
 full running theory  $N_c=3$   
 $N_f=12$

$$qq \sim m^b$$

$$qq \sim \Lambda^a m^b$$



# The $\rho$ mass

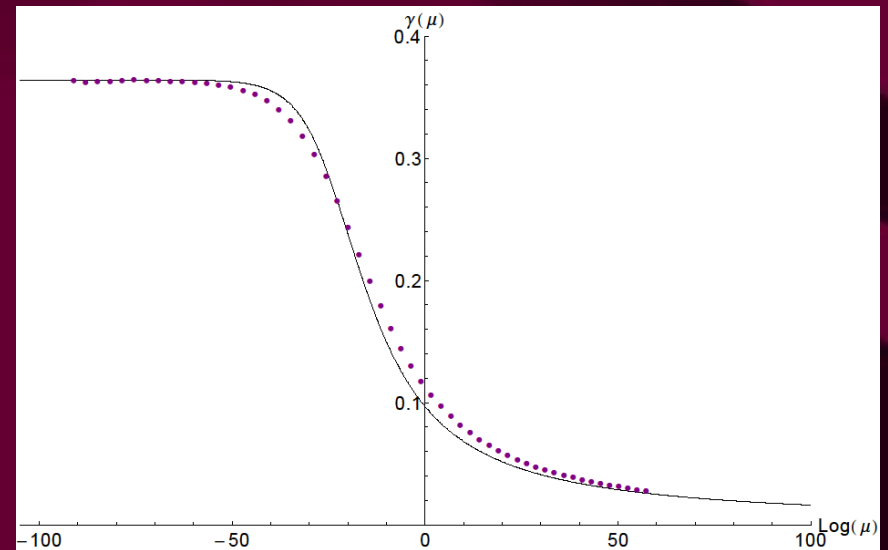


A plot of the  $\rho$  mass against the quark mass with the full two loop running included...

We can extract  $\gamma$  as a function of RG scale from

$$M \sim m^{1/1+\gamma}$$

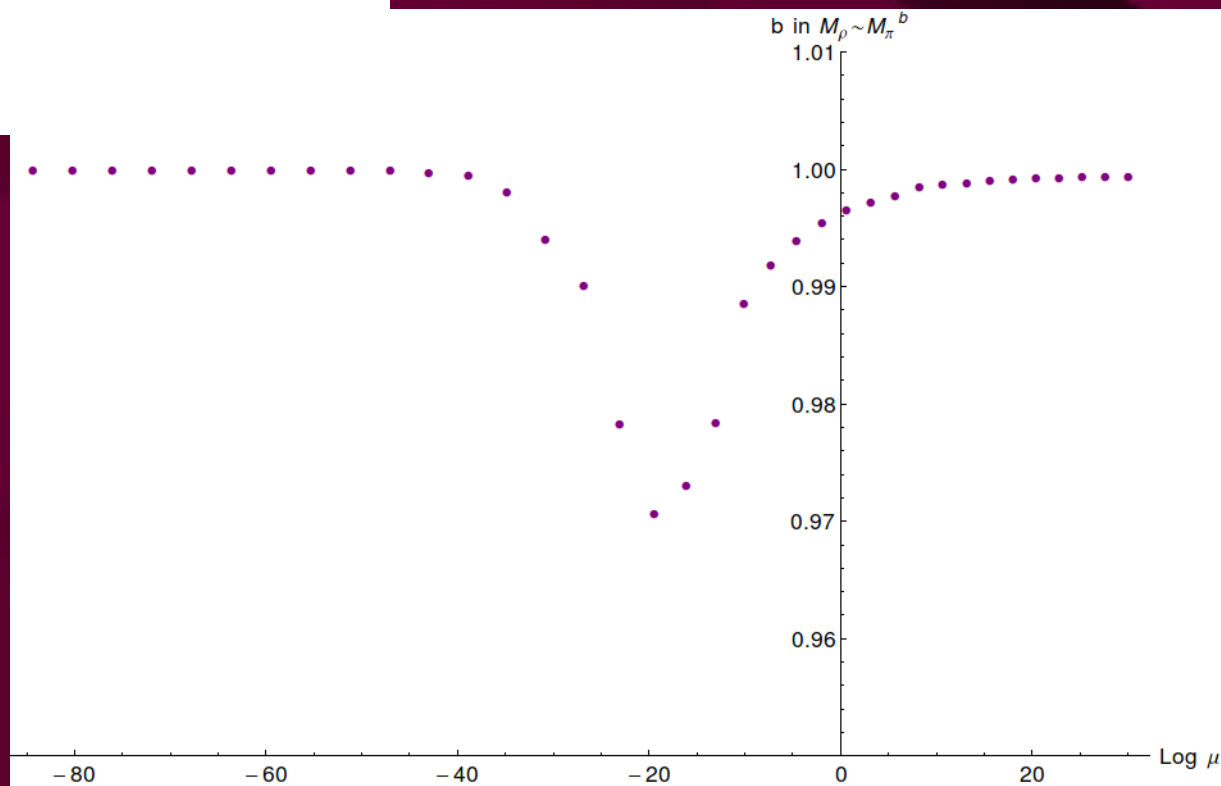
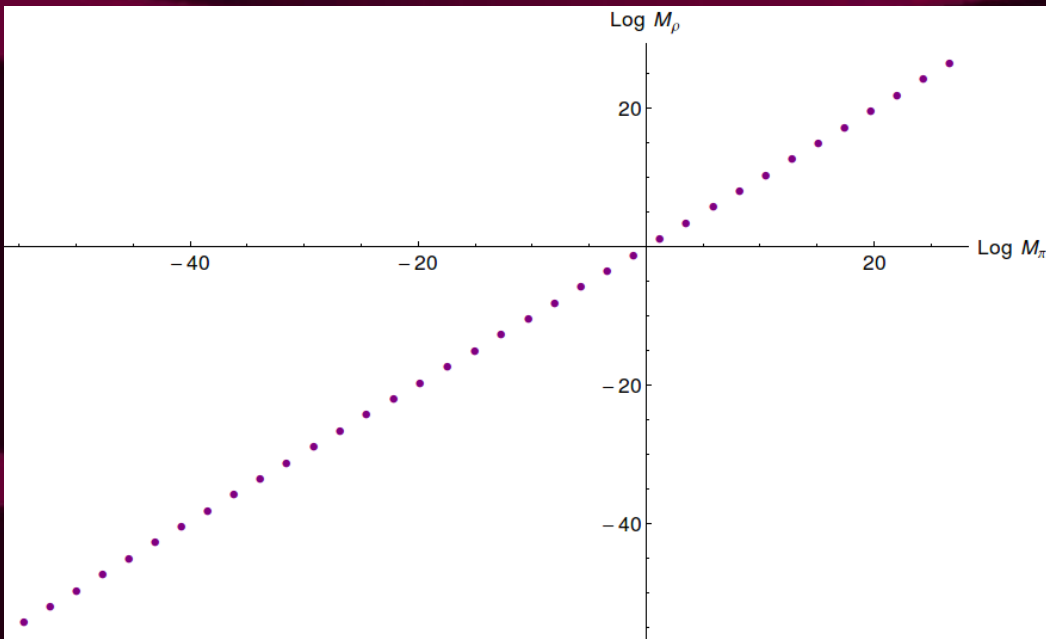
And compare to underlying running... small effects from  $\Lambda$





# $M_\rho$ vs $M_\pi$

$N_c=3$   $N_f=12$



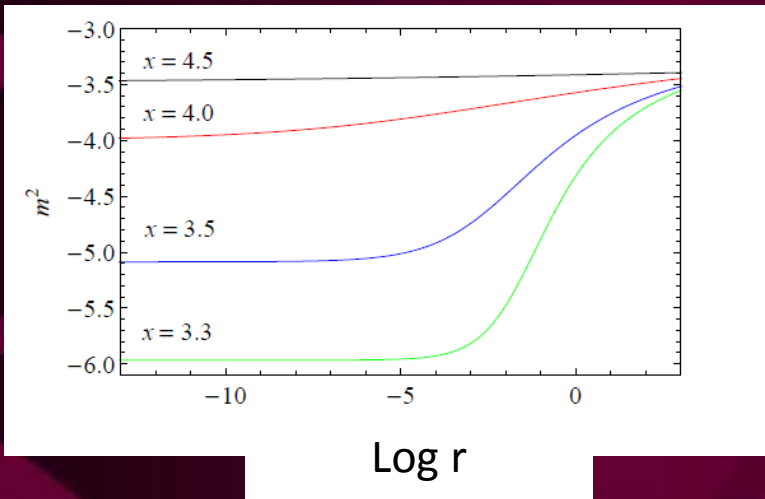
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$$N_{f\_critical} = 4 N_c$$

ie 12 in QCD (same algebra as gap equations)

# BKT transition

arXiv:0905.4752[hep-th]; arXiv:1002.3159 [hep-th]

A transition due to a violation of the BF bound in the deep IR is of holographic BKT type...

$$L = L_0 + \delta(\rho)e^{ikx}$$

$$k^2 = -M^2$$

The Schroedinger equation for the mesonic fluctuations at  $m^2 = -4$  has an infinite number of unstable modes...

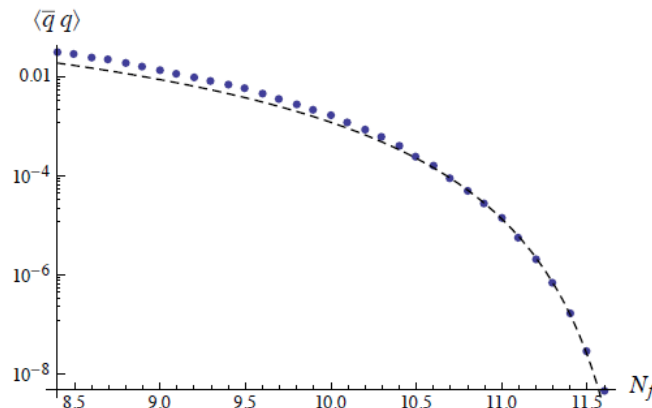
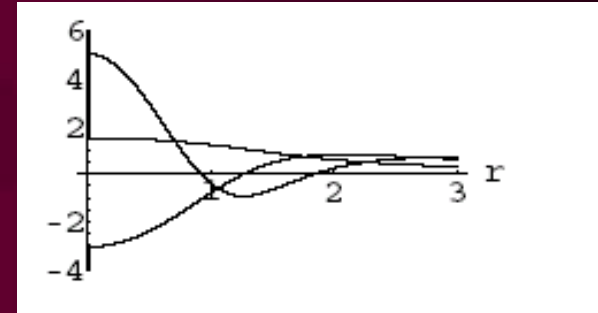
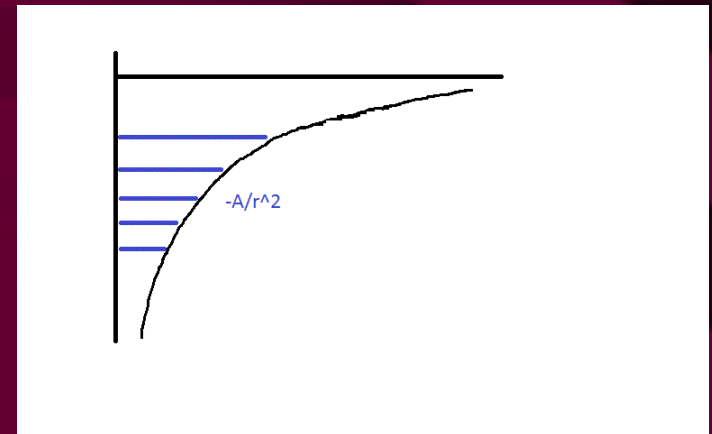


FIG. 4: The dots show numerical results for the quark condensate as a function of  $N_f$ . The dashed line is the BKT fit  $a \exp(-3b/(N_f^c - N_f)^{1/2})$  with parameters  $a = 63.090$  and  $b = 5.111$ .



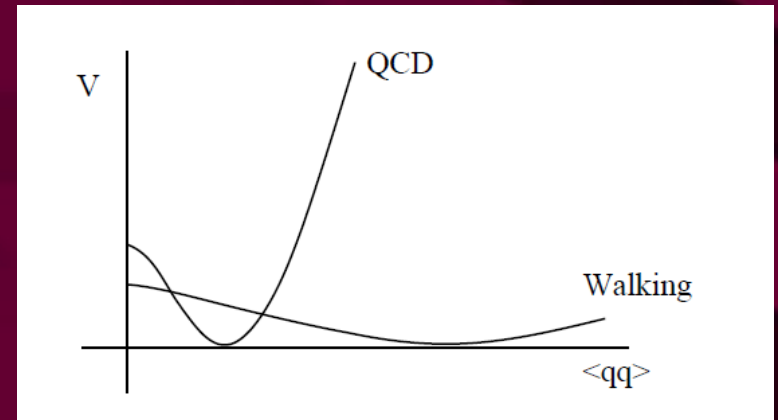
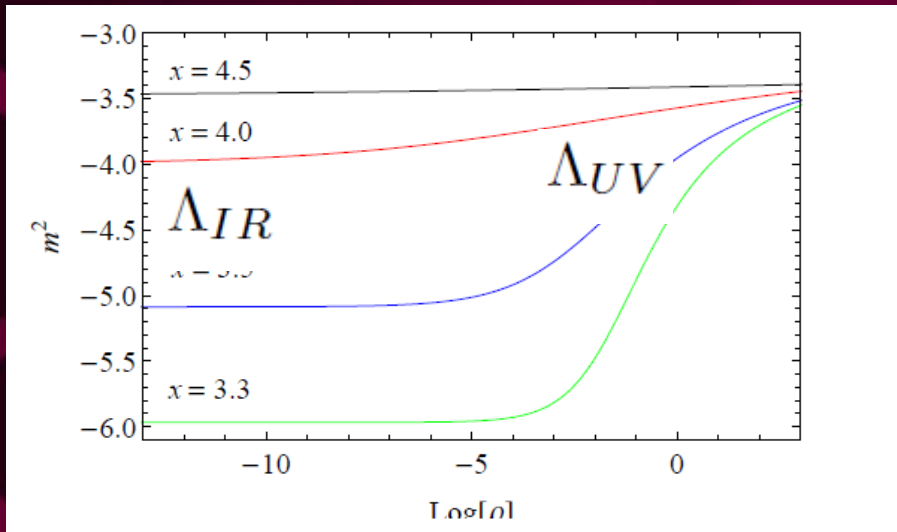
(Miransky scaling)

# Walking Dynamics

Just above the CW regime theories have an enhanced UV quark condensate

$$\langle \bar{q}q \rangle_{UV} \sim \Lambda_{UV} \langle \bar{q}q \rangle_{IR} \sim \Lambda_{UV} \Lambda_{IR}^2$$

$$f_\pi \sim \Lambda_{IR}$$



- Is the sigma particle light – a techni-dilaton?
- Do these make good inflation theories?

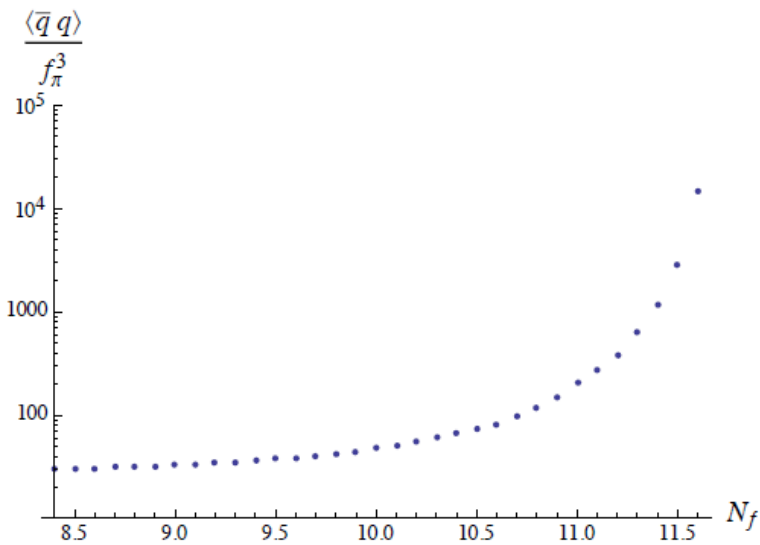


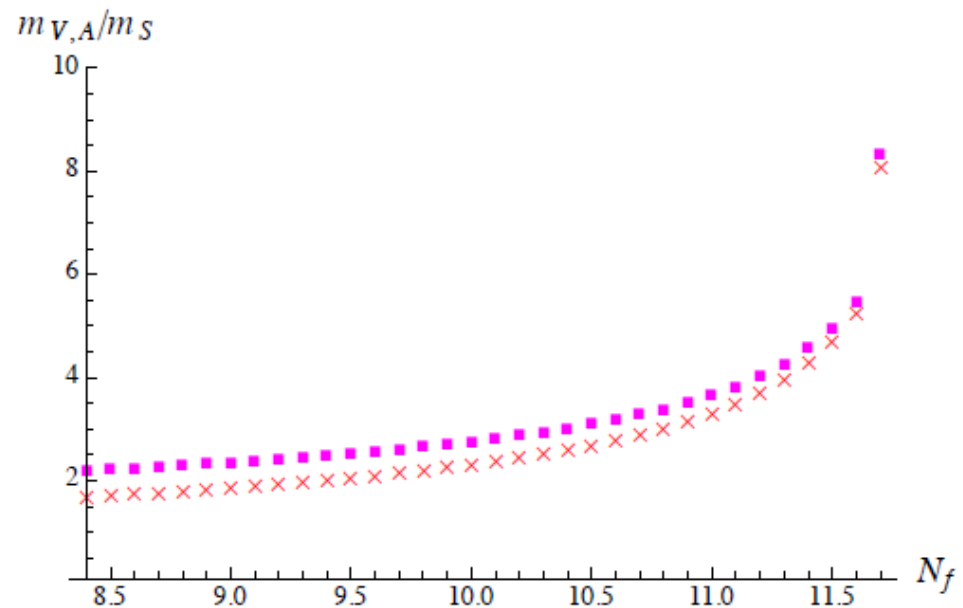
FIG. 5: The quark condensate normalized by  $f_\pi^3$  vs  $N_f$ .

Enhancement reproduced....

Cf FCNC problem in TC

Suppression reproduced...

Cf a light higgs in TC...

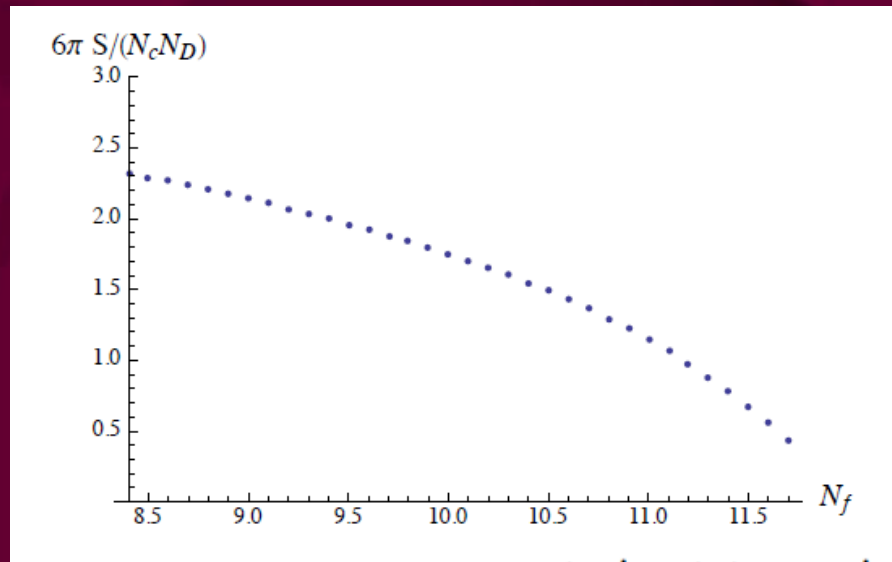
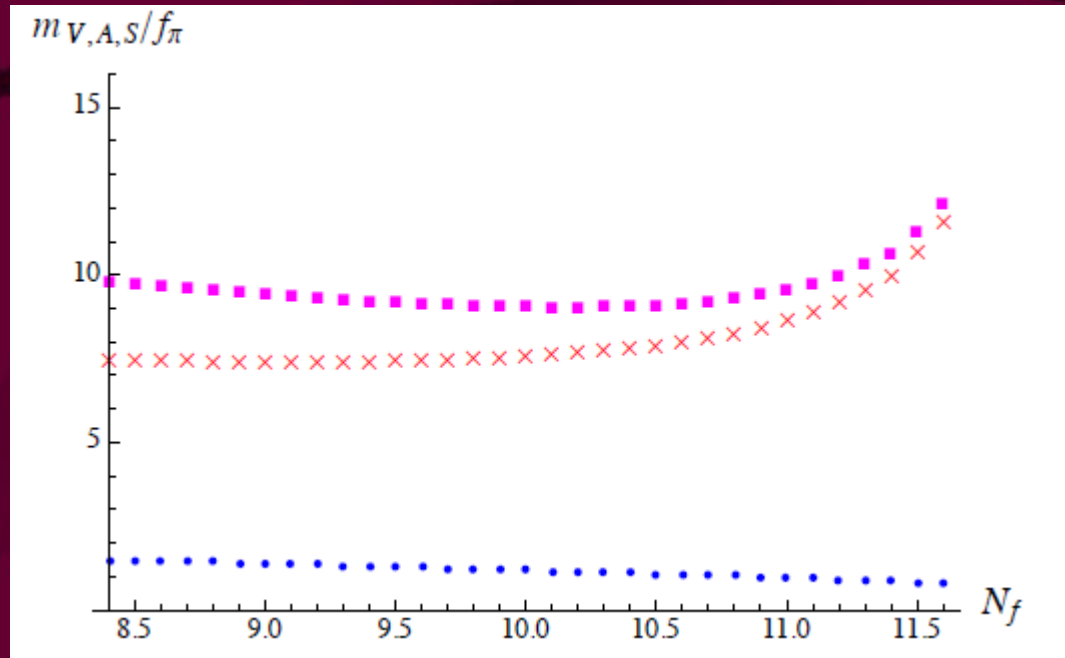


The BKT transition is continuous... so we expect vector-axial symmetry to be continuously restored as we approach the transition... so we impose...

$$\kappa^2 = 3.34(N_f - N_f^c).$$

Which imposes  $\rho$ -a degeneracy at the transition...

Cf Kiritsis-Jarvinen

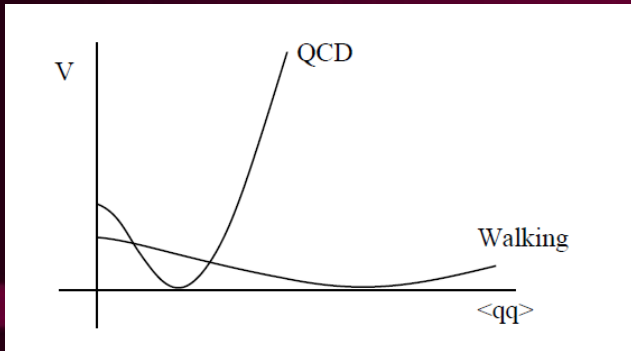


$$S = 4\pi (\Pi'_{AA}(0) - \Pi'_{VV}(0)) ,$$



# Inflation

arXiv:1009.5678 [hep-th]; arXiv:1208.3060 [hep-ph]



$$\frac{\bar{q}q}{V_0^{3/4}} \rightarrow \infty$$

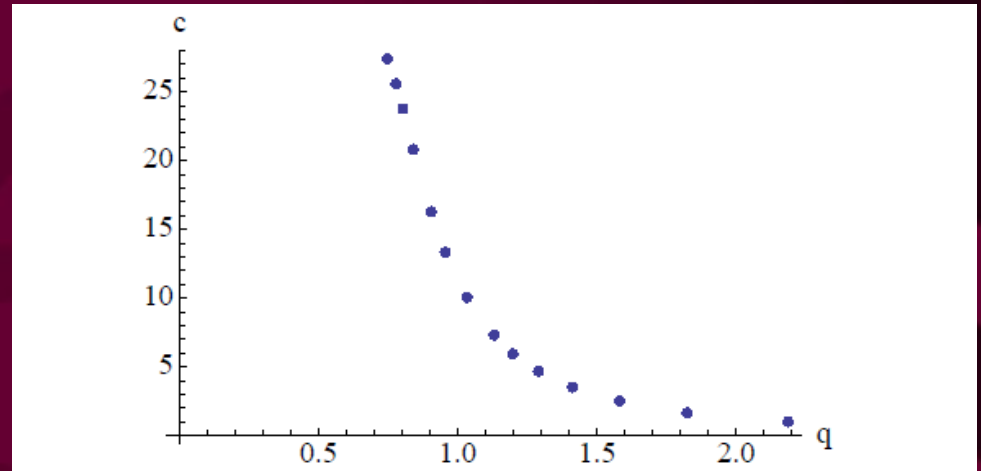
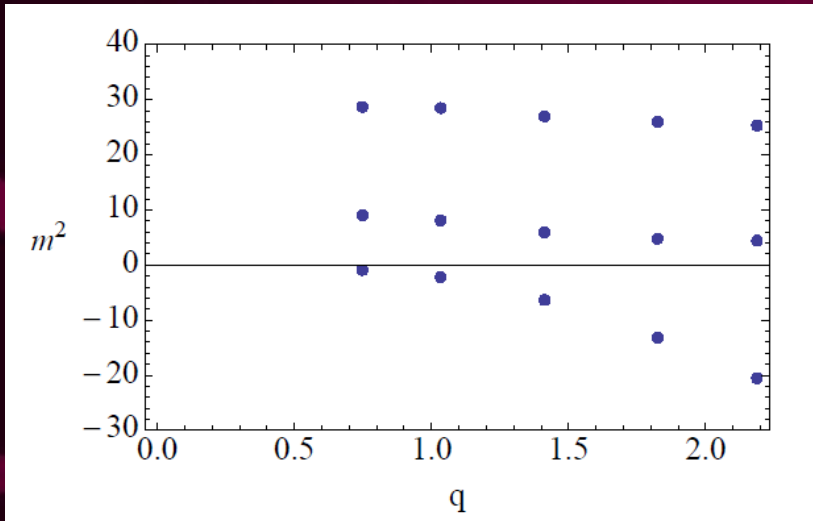
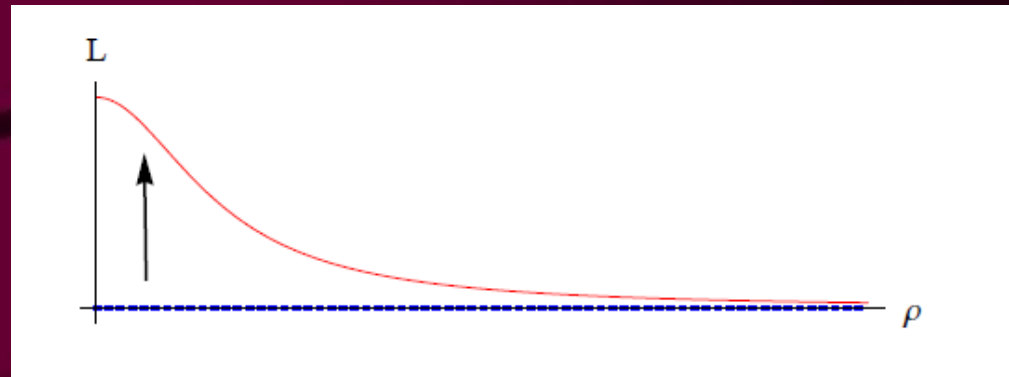


FIG. 3: The quark condensate  $\langle \bar{q}q \rangle$  against  $A$  in the model of (7) and  $q$  in (11) in each case for fixed vacuum energy  $\Delta V$ .

$$V \simeq \frac{\Delta V}{v^4} (\phi^2 - v^2)^2$$

The roll from false vacuum to true...



The meson masses of the unstable vacuum.. There is a single inflaton... whose mass falls to zero as one approaches the transition...

You can also time the roll in an explicit time dependent computation and show it increases in length as one approaches the CW transition...

If you're willing to fine tune to the transition you can get an arbitrarily flat potential/ long roll time...

# Conclusions

- Dynamic AdS/QCD is a very simple model that reproduces the known lore about the CW and walking dynamics. Useful toy for lattice practitioners?
- Holography predicts the conformal window becomes unstable when  $\gamma=1$
- Reproduces hyper-scalings in UV/IR (and transition)
- Holography predicts the chiral transition is of BKT/Miransky nature
- If the gradient of  $\gamma$  falls to zero at and around  $\gamma=1$  then a techni-dilaton is observed
- Tuning to the exit point gives a very flat effective potential for  $\bar{q}q$  suitable for naïve slow roll inflation

# Jarvinen & Kiritsis Holographic Model

arXiv:1112.1261 [hep-ph]

- 5d supergravity  $ds^2 = e^A dr^2 + dx_4^2$
- $\lambda$  scalar to represent running coupling
- $V(\lambda, A)$  – impose preferred IR and UV behaviour
- $\phi$  scalar to represent  $\bar{q}q$  condensate
- $V(\phi, \lambda, A)$  – to determine if  $\bar{q}q$  condenses

$$N_f/N_c \sim 4$$