IR fixed points in gauge theories from lattice simulations

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Plan

- RG flows, fixed points, anomalous dimensions
- Anomalous dimension of four fermi operators
- Schrödinger functional step scaling functions
- Results
- work in collaboration with L Keegan, C Pena

RG flows

Consider a theory with a UV cutoff - integrate out UV modes:

$$\boxed{\mathcal{O}(\hat{g};\mu) = \mathcal{O}(\hat{g}';\mu')\,, \quad \mu' = \mu/b < \mu}$$

Dependence of the dimensionless couplings on the cut-off

The effect of integrating out the high-energy modes is compensated by the change in the couplings (and rescaling of the fields).

1.Scheme-dependence - caveats in RB's talk

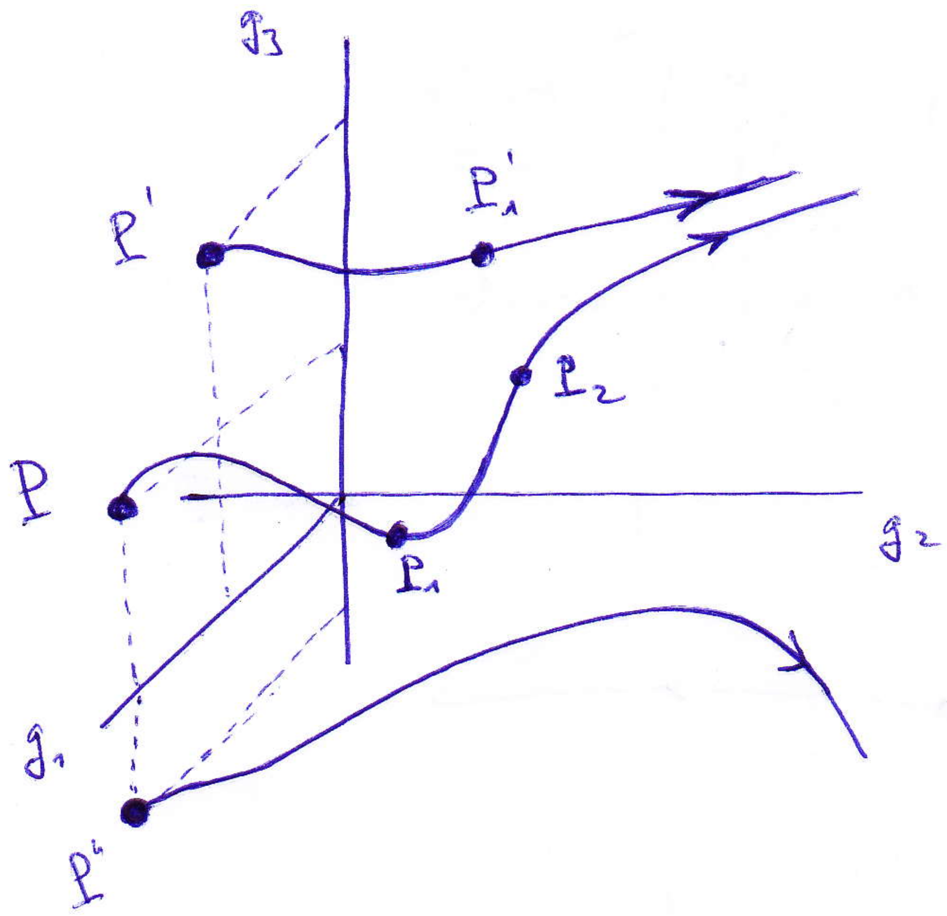
2. Integrating out the UV degrees of freedom generates all the interactions that are compatible with the symmetries of the system.

$$S[\phi;g,\mu] = \int d^D x \left[\frac{1}{2}\left(\partial_\mu\phi\right)^2 + \sum_k \mu^{d_i}\hat{g}_k O_k(x)\right]$$

$$O_k = \partial^{p_k}\phi^{n_k}\,, \quad D - d_k = n_k\frac{D-2}{2} + p_k$$

RG flows

An RG transformation can be seen as a **flow** in the space of couplings.

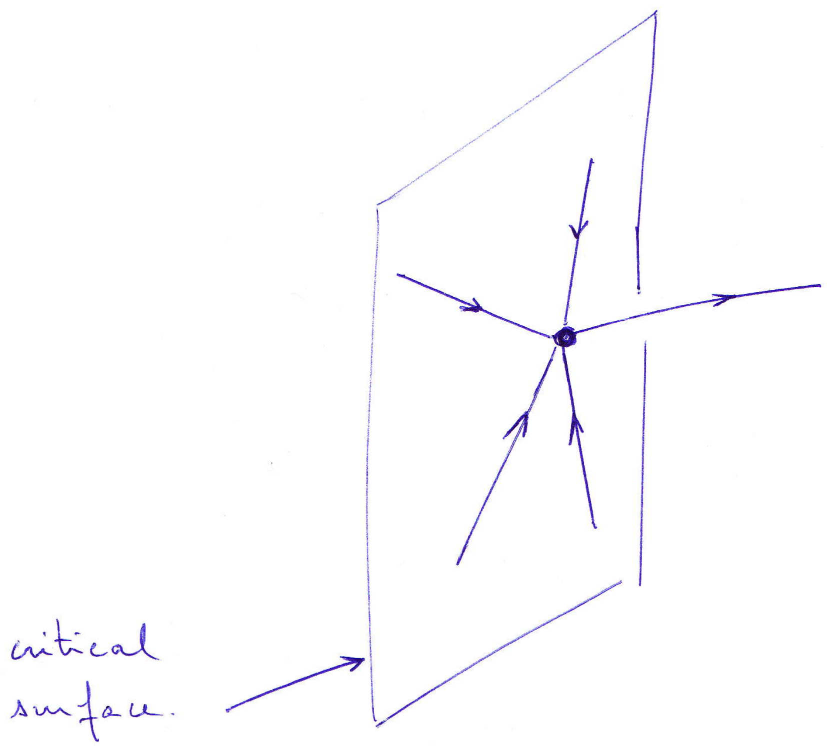


$$\mu \frac{d}{d\mu} \hat{g}_k = \hat{\beta}_k(\hat{g})$$

$$\hat{\beta}'_k(\hat{g}') = \frac{\partial \hat{g}'_k}{\partial \hat{g}_j} \hat{\beta}_j(\hat{g})$$

IR fixed points

Long-distance dynamics is described by $\lim_{\mu\to 0} \hat{g}_k(\mu)$



The couplings may have a finite limit in the IR, and flow towards fixed point values:

$$\boxed{\hat{\beta}_k(\hat{g}^*) = 0}$$

Scale invariant theory at the fixed point

Hierarchies and scaling dimensions

Linearized RG flow in a neighbourhood of a fixed point

$$\mu \frac{d}{d\mu} \hat{g}_k = -y_k \hat{g}_k \,, \qquad \hat{g}_k(\mu) = \left(\frac{\mu}{\Lambda_{\mathrm{UV}}} \right)^{-y_k} \hat{g}_0$$

Associated IR scale:

$$\Lambda_{\mathrm{IR}} \sim \hat{g}_0^{1/y_k} \Lambda_{\mathrm{UV}} \,, \qquad y_k \ll 1 \quad \Longrightarrow \quad \text{natural hierarchy}$$

Global-singlet relevant operators (GSRO) require fine-tuning.

Stable hierarchy generated by *weakly* relevant operators. [Strassler 03, Sannino 04, Luty&Okui 04]

YM theory at the GFP is a limiting case:

$$\Lambda_{\mathrm{IR}} \sim \Lambda_{\mathrm{UV}} \exp\{-\frac{1}{\beta_0 g^2}\}$$

Hierarchies and the flavor sector

In the SM + elementary Higgs:

GSRO

$$\boxed{\dim(H^\dagger H) \simeq 2}$$

$$\mathcal{L}_Y = Y^u\, H\, \bar{L} u_R + Y^d H^\dagger \bar{L} d_R$$

dimension = 1+3 = 4

In DEWSB: scalar is composite [Dimopoulos et al 79, Eichten et al 1980]

$$\mathcal{L}_Y = \frac{Y^q}{\Lambda_{\mathrm{UV}}^2}\, \bar{Q} Q\, \bar{q} q$$

dimension = 3+3 = 6

Tension with suppressing FCNC

$$\frac{f}{\Lambda_{\mathrm{UV}}^2}\, \bar{q} q \bar{q} q$$

dimension = 6

Fermionic operators

Alleviate the problem due to the **large** dimension of the composite scalar

Theory at the EW scale is **near** a non-trivial fixed point

Scaling dimension of the fermion bilinear is smaller

$$\dim\ H = \dim\ \bar{Q}Q = 3 - \gamma_m$$ [Holdom,Yamawaki, Appelquist, Eichten, Lane]

➡ Find numerical evidence for the existence of a fixed point - **scheme independent?**

➡ Characterize the fixed point by computing the **mass anomalous dimension**

➡ and the anomalous dimension of **4 fermi operators**

$$\dim(H)\ \text{small,}\ \ \mathbf{but}\ \ \dim(H^\dagger\, H) > 2\,\dim(H)$$

[Sannino 04, Luty 04, Rattazzi et al 08]

Anomalous dimension of the four-fermi operator

Under axial transformations in flavor space:

$$\delta^a \left[(\bar\psi\psi)(\bar\psi\psi)\right] = 4i(\bar\psi\psi)\,(\bar\psi\tau^a\gamma_5\psi)$$
$$\delta^b \left[(\bar\psi\psi)\,(\bar\psi\tau^a\gamma_5\psi)\right] = 2i(\bar\psi\tau^b\gamma_5\psi)\,(\bar\psi\tau^a\gamma_5\psi) + \ldots$$

Flavor non-singlet lies in the same multiplet

Construct a basis of operators that is closed under renormalization:

$$(\bar\psi_1\Gamma_1\psi_2)(\bar\psi_3\Gamma_2\psi_4)$$
$$(\bar\psi_1\Gamma_1 T^A\psi_2)(\bar\psi_3\Gamma_2 T^A\psi_4)$$

parity-even	parity-odd
$\gamma_\mu \otimes \gamma_\mu$	$\gamma_\mu \otimes \gamma_\mu\gamma_5$
$\gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5$	$\gamma_\mu\gamma_5 \otimes \gamma_\mu$
$\mathbf{1} \otimes \mathbf{1}$	$\mathbf{1} \otimes \gamma_5$
$\gamma_5 \otimes \gamma_5$	$\gamma_5 \otimes \mathbf{1}$
$\sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$	$\sigma_{\mu\nu} \otimes \tilde\sigma_{\mu\nu}$

Fierzing

Color trace identity:

$$(T^A)_{\alpha\beta}(T^A)_{\gamma\delta} = \frac{1}{2}\,\delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{1}{2N}\,\delta_{\alpha\beta}\delta_{\gamma\delta}$$

Fierz transformation:

$$(\Gamma_1^{(r)})_{ij}(\Gamma_2^{(r)})_{kl} = \sum_s f_{rs}(\Gamma_1^{(s)})_{il}(\Gamma_2^{(s)})_{kj}$$

$$\begin{aligned}(\bar\psi_1\Gamma_1^{(r)}T^A\psi_2)(\bar\psi_3\Gamma_2^{(r)}T^A\psi_4) &\mapsto \\ &- \frac{1}{2N}(\bar\psi_1\Gamma_1^{(r)}\psi_2)(\bar\psi_3\Gamma_2^{(r)}\psi_4) + \sum_s f_{rs}(\bar\psi_1\Gamma_1^{(s)}\psi_4)(\bar\psi_3\Gamma_2^{(s)}\psi_2)\end{aligned}$$

Parity-even sector & discrete symmetries

$$O^{\pm}_{\Gamma_1\Gamma_2} = \frac{1}{2}\,\left[(\bar\psi_1\Gamma_1\psi_2)(\bar\psi_3\Gamma_2\psi_4) \pm (\bar\psi_1\Gamma_1\psi_4)(\bar\psi_3\Gamma_2\psi_2)\right]$$

$$\begin{aligned} Q_1^{\pm} &= O^{\pm}_{VV+AA} \\ Q_2^{\pm} &= O^{\pm}_{VV-AA} \\ Q_3^{\pm} &= O^{\pm}_{SS-PP} \\ Q_4^{\pm} &= O^{\pm}_{SS+PP} \\ Q_5^{\pm} &= O^{\pm}_{TT} \end{aligned}$$

$$\Delta^{\pm} = \begin{pmatrix} 0 & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} \\ \Delta_{21} & 0 & 0 & \Delta_{24} & \Delta_{25} \\ \Delta_{31} & 0 & 0 & \Delta_{34} & \Delta_{35} \\ \Delta_{41} & \Delta_{42} & \Delta_{43} & 0 & 0 \\ \Delta_{51} & \Delta_{52} & \Delta_{53} & 0 & 0 \end{pmatrix}^{\pm}$$

$$Q^{\pm}_{i,R} = Z^{\pm}_{ij}\,\left[\delta_{jk} + \Delta^{\pm}_{jk}\right] Q^{\pm}_k$$

scale-independent

[Donini et al 99, Guagnelli et al 05]

Parity-odd sector & discrete symmetries

$$O^{\pm}_{\Gamma_1\Gamma_2} = \frac{1}{2}\ \left[(\bar{\psi}_1\Gamma_1\psi_2)(\bar{\psi}_3\Gamma_2\psi_4) \pm (\bar{\psi}_1\Gamma_1\psi_4)(\bar{\psi}_3\Gamma_2\psi_2)\right]$$

$$\begin{aligned}
\mathcal{Q}_1^{\pm} &= O^{\pm}_{VA+AV}\\
\mathcal{Q}_2^{\pm} &= O^{\pm}_{VA-AV}\\
\mathcal{Q}_3^{\pm} &= -O^{\pm}_{SP-PS}\\
\mathcal{Q}_4^{\pm} &= O^{\pm}_{SP+PS}\\
\mathcal{Q}_5^{\pm} &= O^{\pm}_{T\tilde{T}}
\end{aligned}$$

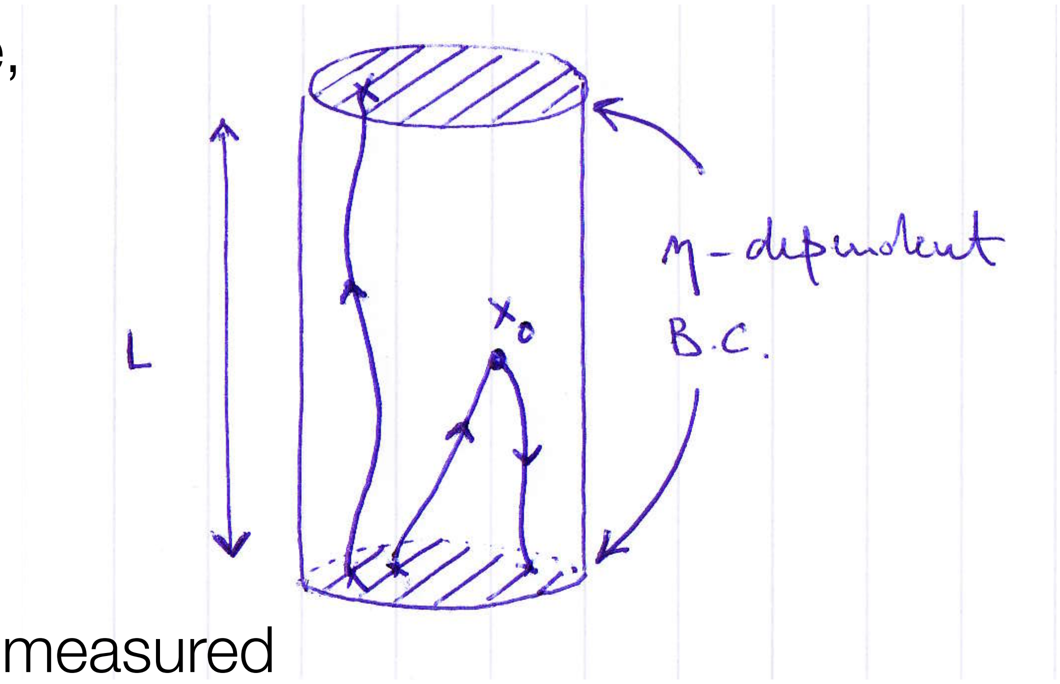
$$\begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}^{\pm}_R = \begin{pmatrix} \mathcal{Z}_{11} & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Z}_{22} & \mathcal{Z}_{23} & 0 & 0 \\ 0 & \mathcal{Z}_{32} & \mathcal{Z}_{33} & 0 & 0 \\ 0 & 0 & 0 & \mathcal{Z}_{44} & \mathcal{Z}_{45} \\ 0 & 0 & 0 & \mathcal{Z}_{54} & \mathcal{Z}_{55} \end{pmatrix}^{\pm} \begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}^{\pm}$$

Schrödinger functional

Finite-volume renormalization scheme:

- size of the system defines the renormalization scale,
- Dirichlet boundary conditions

$$\boxed{\overline{g}^2(L) = k \left\langle \frac{\partial S}{\partial \eta} \right\rangle^{-1}}$$



The renormalized charge is an *observable,* i.e. it can be measured by numerical simulations.

The definition above extends outside the perturbative regime and yields a *nonperturbative* coupling.

The coupling depends on one scale only, the finite size of the system.

[M Luscher et al]

SF - running of the coupling

The running of the coupling as the scale is varied by a factor s is encoded in the *step scaling* function:

$$\Sigma(u, s, a/L) = \overline{g}^2(g_0, sL/a)\big|_{\overline{g}^2(g_0, L/a)=u}$$

Lattice step scaling is affected by lattice artefacts, i.e. depends on the details of the UV regulator. We can compute the *continuous* step scaling:

$$\sigma(u, s) = \lim_{a/L \to 0} \Sigma(u, s, a/L)$$

L/a: *resolution* of the simulation

$$-2 \log s = \int_u^{\sigma(u,s)} \frac{dx}{\sqrt{x}\beta(\sqrt{x})}$$

At the fixed point: $\sigma(u, s) = u \quad \Longleftrightarrow \quad \sigma(u, s)/u = 1$

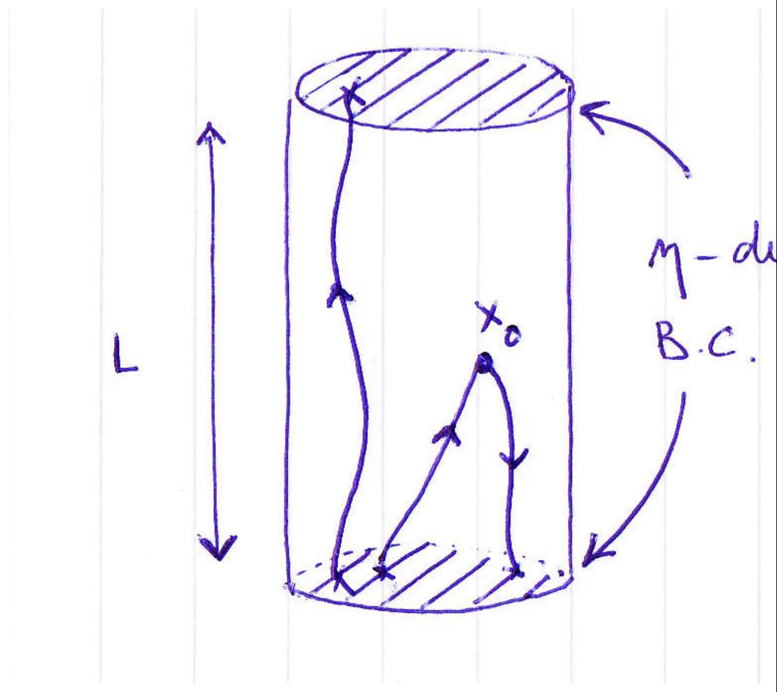
SF - running of the mass

The renormalized mass is defined as:

$$\bar{m}(\mu) = \frac{Z_A}{Z_P(\mu)} m$$

In order to study its running we need to compute nonperturbatively:

$$\boxed{Z_P(L) = \sqrt{3 f_1} / f_P(L/2)}$$



$$f_1 = -1/12L^6 \int d^3u\, d^3v\, d^3y\, d^3z\, \langle \overline{\zeta}'(u) \gamma_5 \tau^a \zeta'(v) \overline{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle \,,$$

$$f_P(x_0) = -1/12 \int d^3y\, d^3z\, \langle \overline{\psi}(x_0) \gamma_5 \tau^a \psi(x_0) \overline{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle \,.$$

SF - running of the mass

Step scaling functions for the mass:

$$\Sigma_P(u,s,a/L) = \left.\frac{Z_P(g_0, sL/a)}{Z_P(g_0, L/a)}\right|_{\overline{g}^2(L)=u}$$

$$\sigma_P(u,s) = \lim_{a\to 0} \Sigma_P(u,s,a/L)$$

Relation to the anomalous dimension:

$$\sigma_P(u) = \left(\frac{u}{\sigma(u)}\right)^{(d_0/(2\beta_0))} \exp\left[\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dx \left(\frac{\gamma(x)}{\beta(x)} - \frac{d_0}{\beta_0 x}\right)\right]$$

SF - running of the mass

In the neighbourhood of the fixed point:

$$\int_{\overline{m}(\mu)}^{\overline{m}(\mu/s)} \frac{dm}{m} = -\gamma_* \int_{\mu}^{\mu/s} \frac{dq}{q}$$

$$\log|\sigma_P(s,u)| = -\gamma_* \log s$$

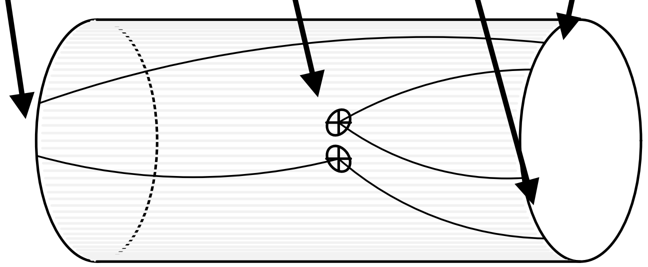
Hence we can define an estimator for the anomalous dimension:

$$\boxed{\hat{\gamma}(u) = -\frac{\log|\sigma_P(u,s)|}{\log|s|}}$$

Scheme-independent in a neighbourhood of a fixed point.

SF - four-fermi anomalous dimension

$$F^{\pm}_{i;A,B,C} = \frac{1}{L^3}\,\langle \mathcal{O}'_{53}[\Gamma_C]\,\mathcal{Q}^{\pm}_i\,\mathcal{O}_{21}[\Gamma_A]\,\mathcal{O}_{45}[\Gamma_B]\rangle$$



$$\mathcal{O}'_{f_1 f_2}[\Gamma] = a^6 \sum_{\mathbf{y},\mathbf{z}} \bar{\zeta}'_{f_1}(\mathbf{y})\Gamma\zeta'_{f_2}(\mathbf{z})$$

$$\mathcal{O}_{f_1 f_2}[\Gamma] = a^6 \sum_{\mathbf{y},\mathbf{z}} \bar{\zeta}_{f_1}(\mathbf{y})\Gamma\zeta_{f_2}(\mathbf{z})$$

SF - four-fermi anomalous dimension

$$f_1 = -\frac{1}{2L^6}\,\langle \mathcal{O}'_{12}[\gamma_5]\,\mathcal{O}_{21}[\gamma_5]\rangle$$

$$k_1 = -\frac{1}{6L^6}\,\sum_{k=1,2,3}\langle \mathcal{O}'_{12}[\gamma_k]\,\mathcal{O}_{21}[\gamma_k]\rangle$$

$$\boxed{h^{\pm}_{i;A,B,C}(x_0) = \frac{F^{\pm}_{i;A,B,C}(x_0)}{f_1^{\eta}k_1^{3/2-\eta}}}$$

$$h_1^{\pm} = h_{1;\gamma_5\gamma_5\gamma_5}$$

$$h_2^{\pm} = \frac{1}{6}\sum_{j,k,l}\epsilon_{jkl}\,h_{1;\gamma_j\gamma_k\gamma_l}$$

$$h_3^{\pm} = \frac{1}{3}\sum_{k} h_{1;\gamma_5\gamma_k\gamma_k}$$

$$h_4^{\pm} = \frac{1}{3}\sum_{k} h_{1;\gamma_k\gamma_5\gamma_k}$$

$$h_5^{\pm} = \frac{1}{3}\sum_{k} h_{1;\gamma_k\gamma_k\gamma_5}$$

$$Z_1^{\pm}(g_0, a\mu)\,h^{\pm}_{1;A,B,C}(L/2) = h^{\pm}_{1;A,B,C}(L/2)\Big|_{g_0=0}$$

SF - four-fermi anomalous dimension

Step-scaling functions:

$$\Sigma^{\pm}(s;u,L/a) = Z^{\pm}(g_0,sL/a)\, Z^{\pm}(g_0,L/a)^{-1}\big|_{\bar{g}(L)^2=u}$$

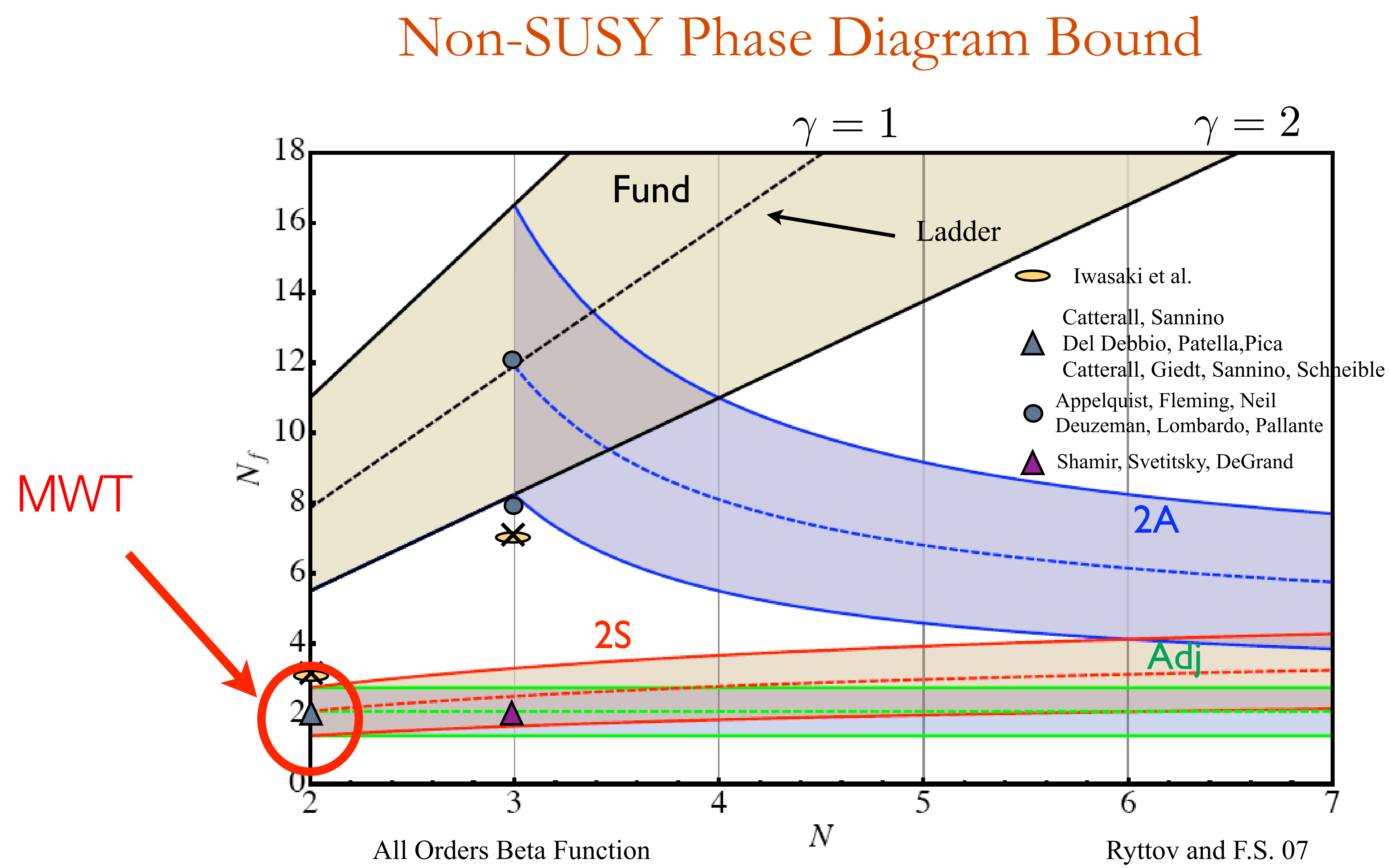
$$\sigma^{\pm}(s;u) = \lim_{a\to 0} \Sigma^{\pm}(s;u,L/a) = \mathrm{T}\,\exp\left\{\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dg\,\frac{\gamma^{\pm}(g)}{\beta(g)}\right\}$$

In a neighbourhood of a fixed point:

$$\boxed{\gamma^{\pm}(u) = \frac{\log\sigma^{\pm}(s;u)}{\log s}}$$

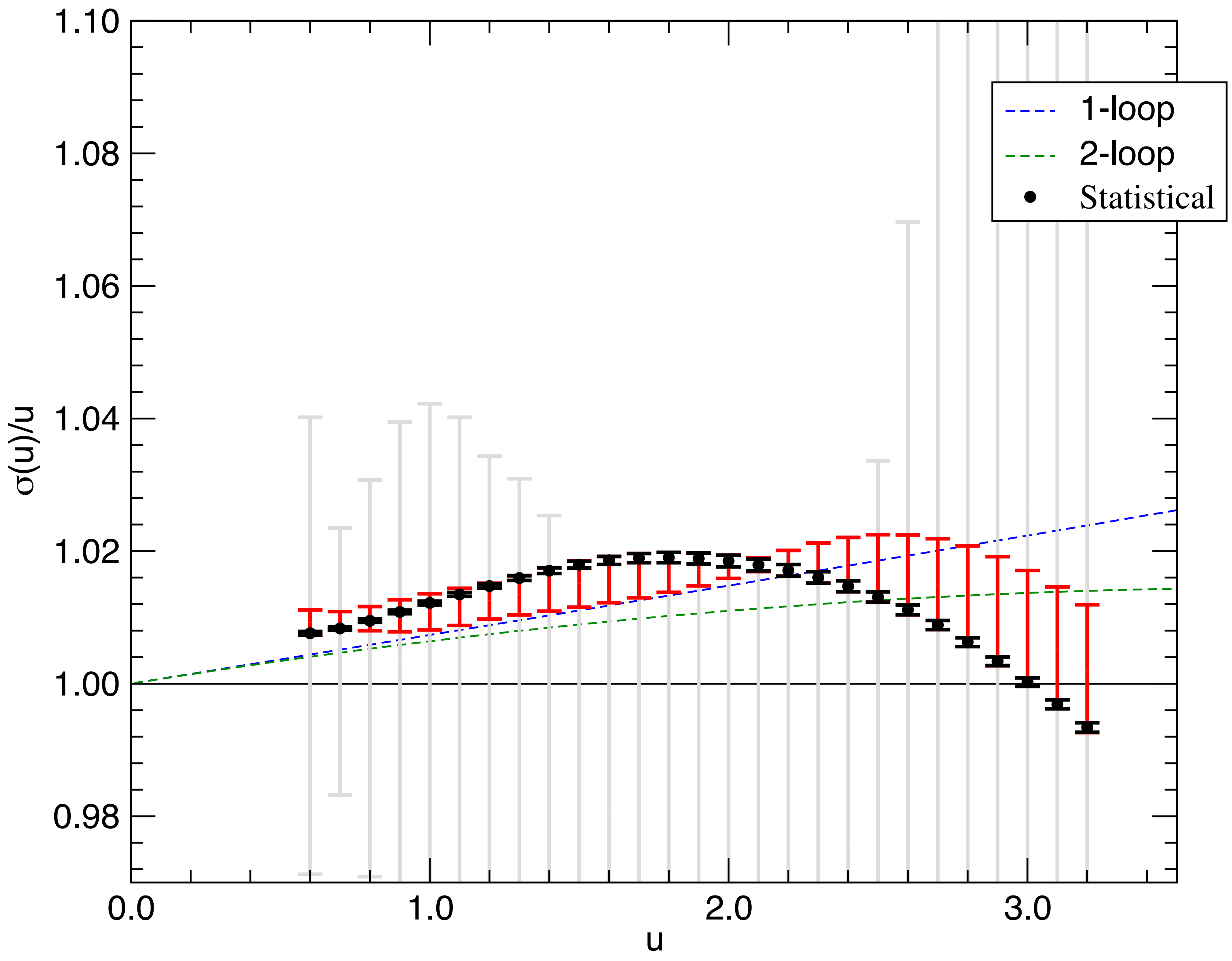
Phase diagram of SU(N) gauge theories

Use lattice tools to **search** for IRFPs in 4D SU(N) gauge theories



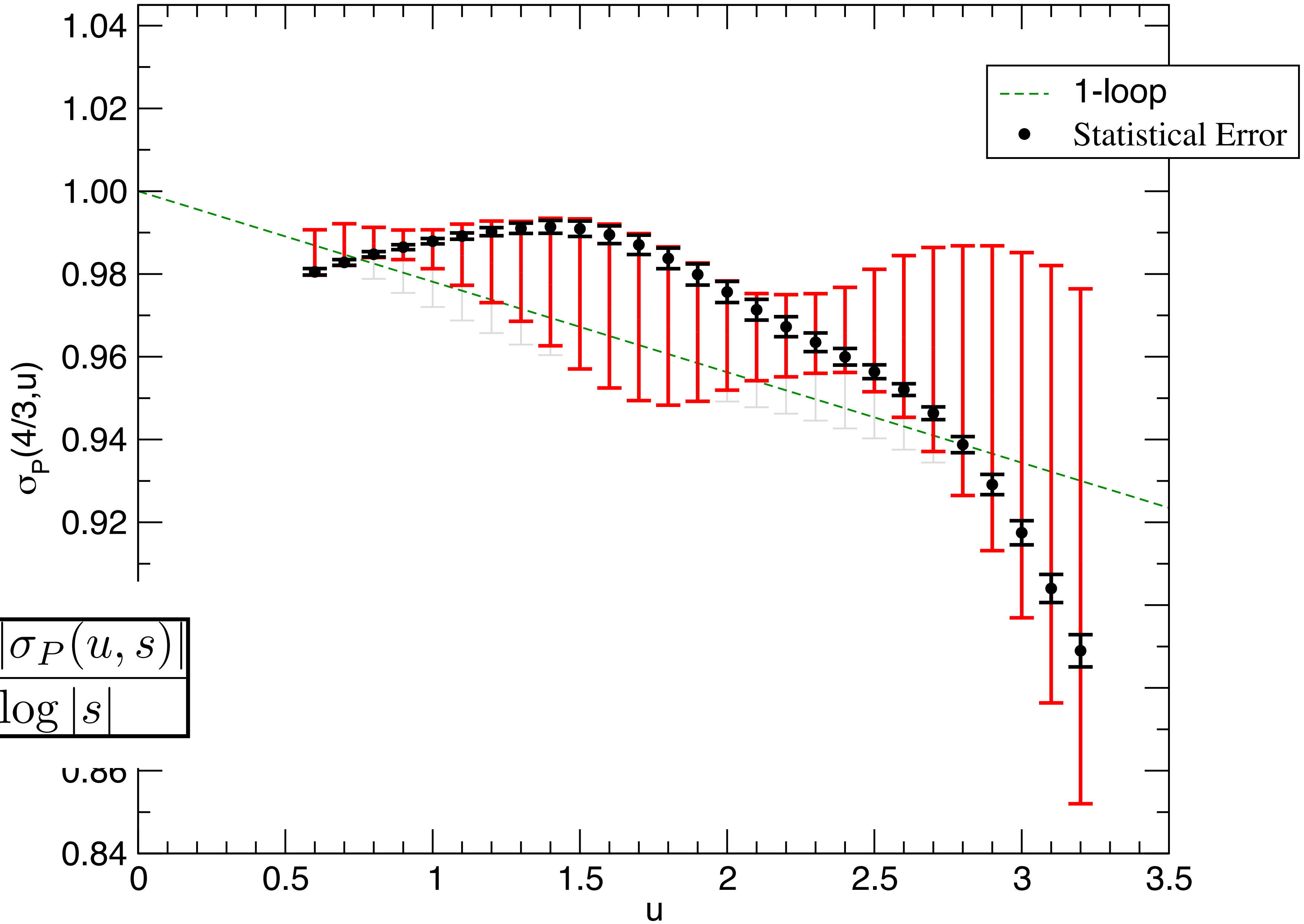
[Yamawaki, Appelquist, Miransky, Schrock, Nunez, Piai, Hong, Braun, Gies]

Running coupling



[Bursa et al 09]

Running of the mass

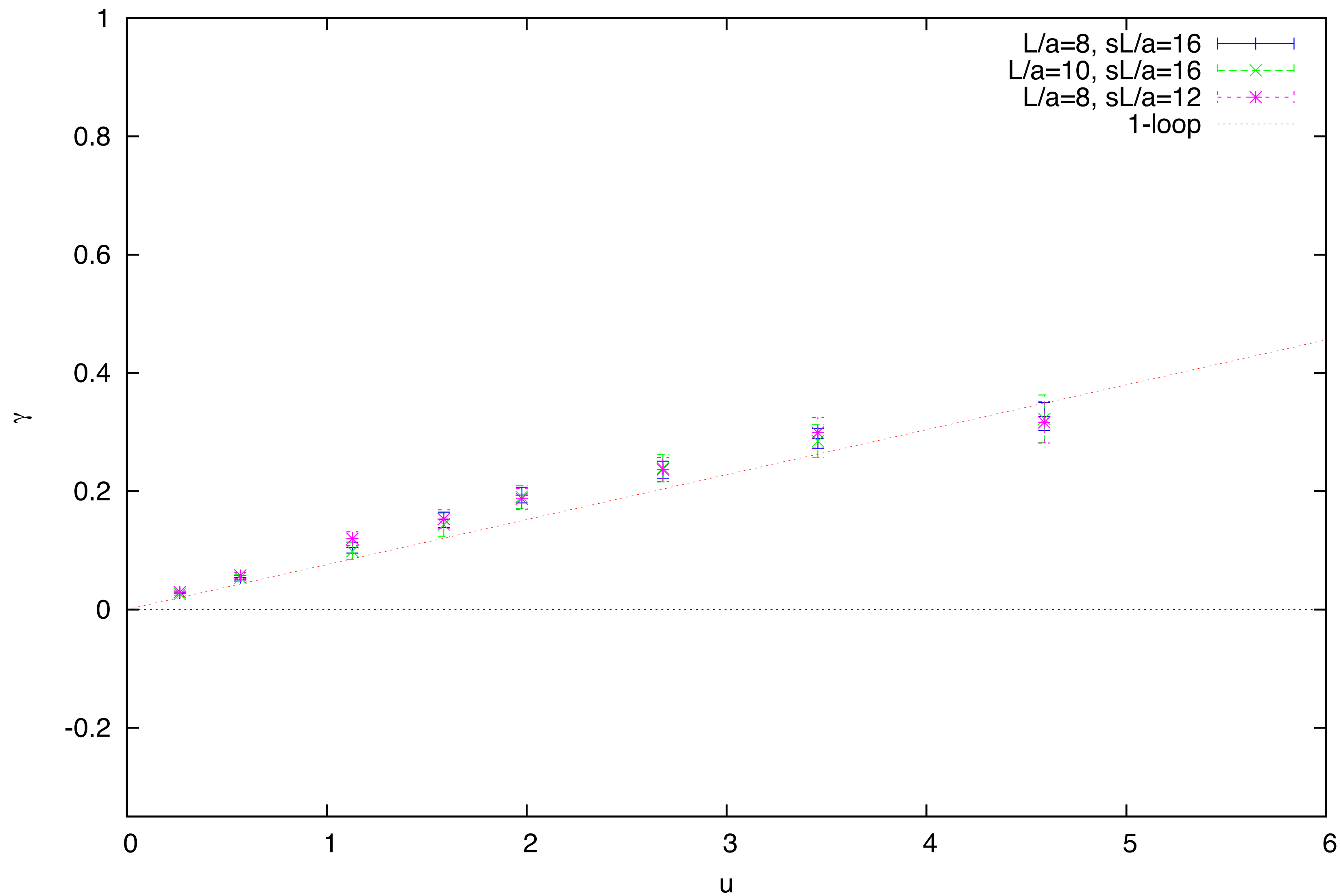


$$\hat{\gamma}(u) = -\frac{\log|\sigma_P(u,s)|}{\log|s|}$$

$\gamma < 0.6$

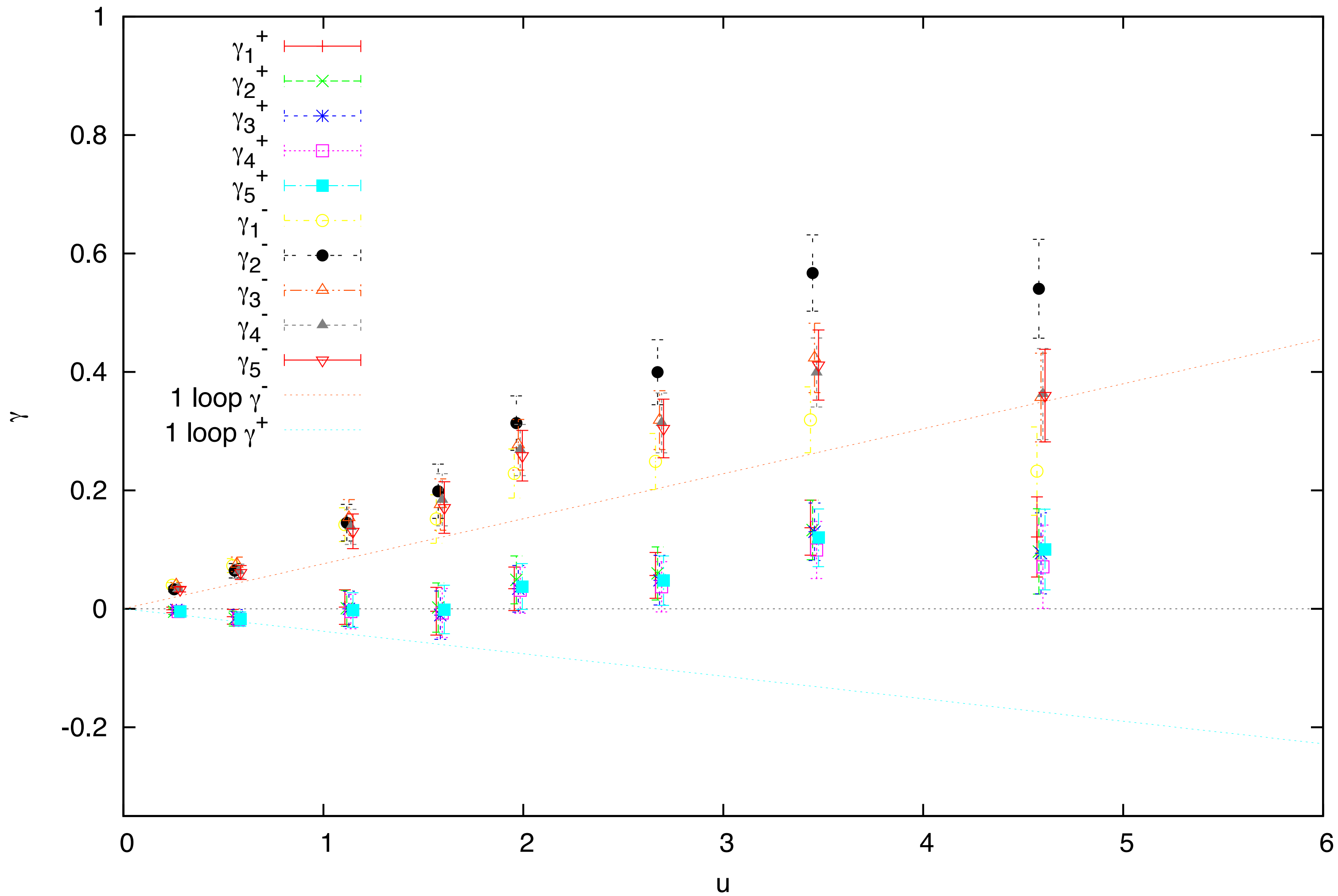
[Bursa et al 09]

Running of the mass - revisited



Four-fermi anomalous dimension - VA+AV

Multiplicative renormalization in this channel

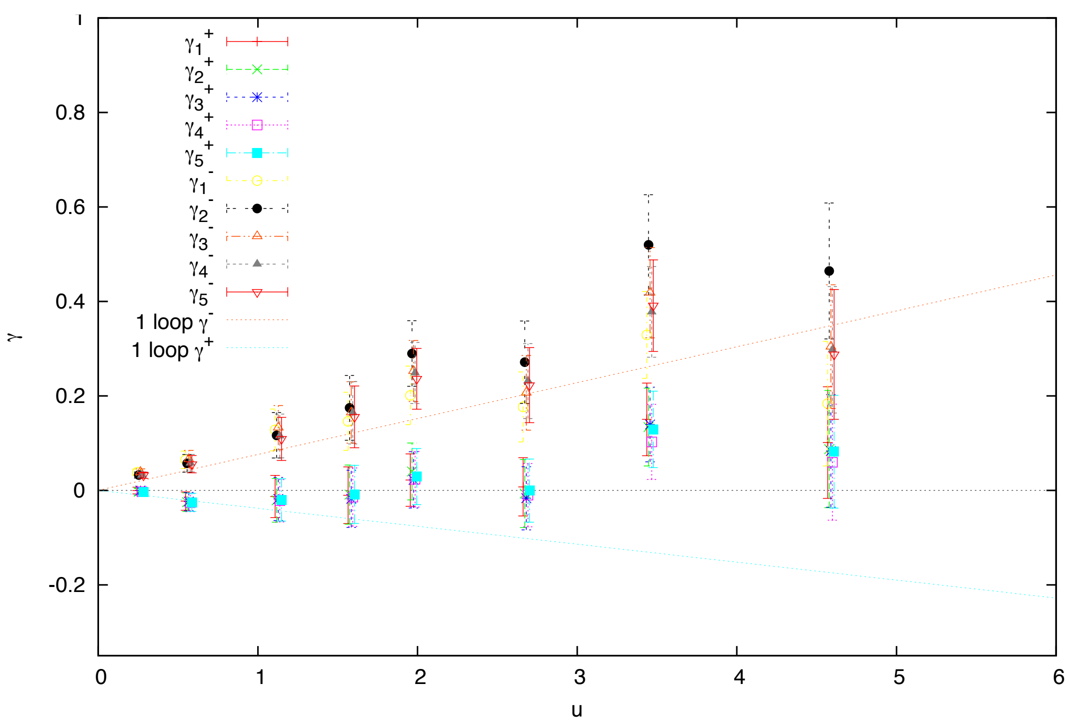


$$L/a = 8 \to L/a = 16$$

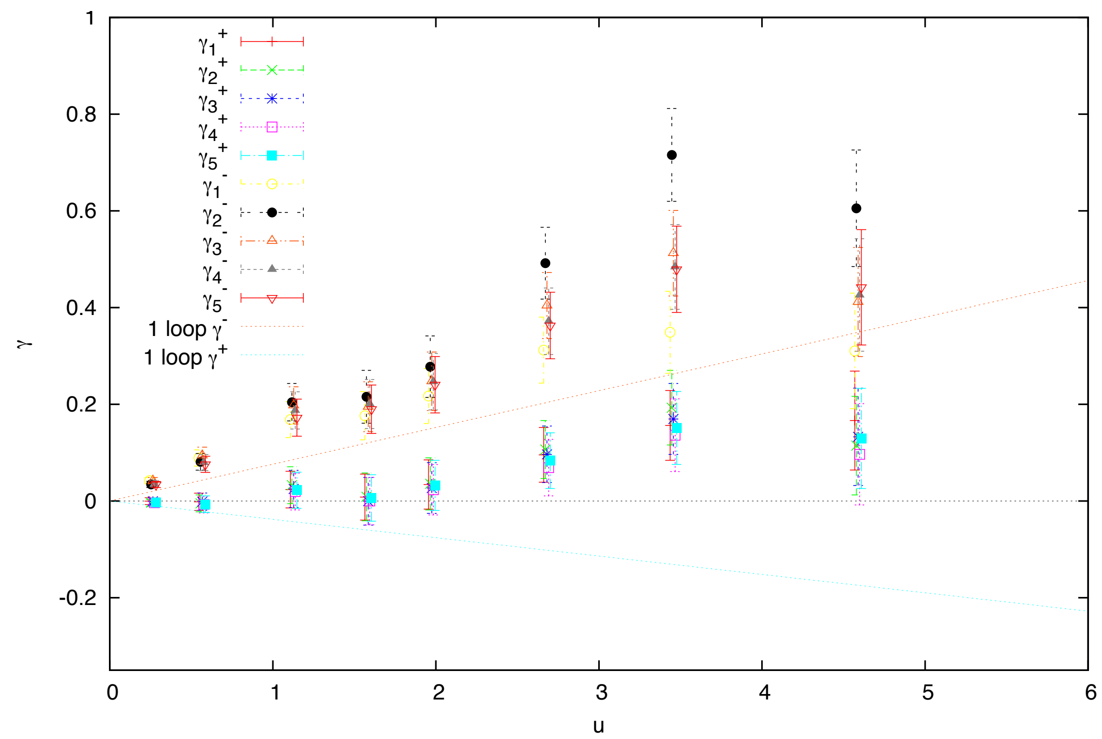
Scheme-independence: system in a neighbourhood of a fixed point?

Four-fermi anomalous dimension

Checks with different scaling steps



$$L/a = 10 \to L/a = 16$$



$$L/a = 8 \to L/a = 12$$

Results are consistent

Outlook

Goal: **robust evidence** for an IR fixed point in SU(2) adj

Determination of the critical exponents at the IRFP using SF

mass anomalous dimension & 4fermi anomalous dimensions are feasible

new couplings: gradient flow, chirally rotated [Ramos 13, Sint 10]

Comparison with other methods - scaling of spectral quantities

must find compatible results

Energy-momentum tensor and IRFP