Large N ϕ^4 model in 3-dimensions and the conformal fixed point

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work in progress in collaboration with

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1. Introduction

Construction of non-trivial quantum field theories

1. Around asymptotic free fixed point

QCD: non-abelian gauge theory with a few fermions

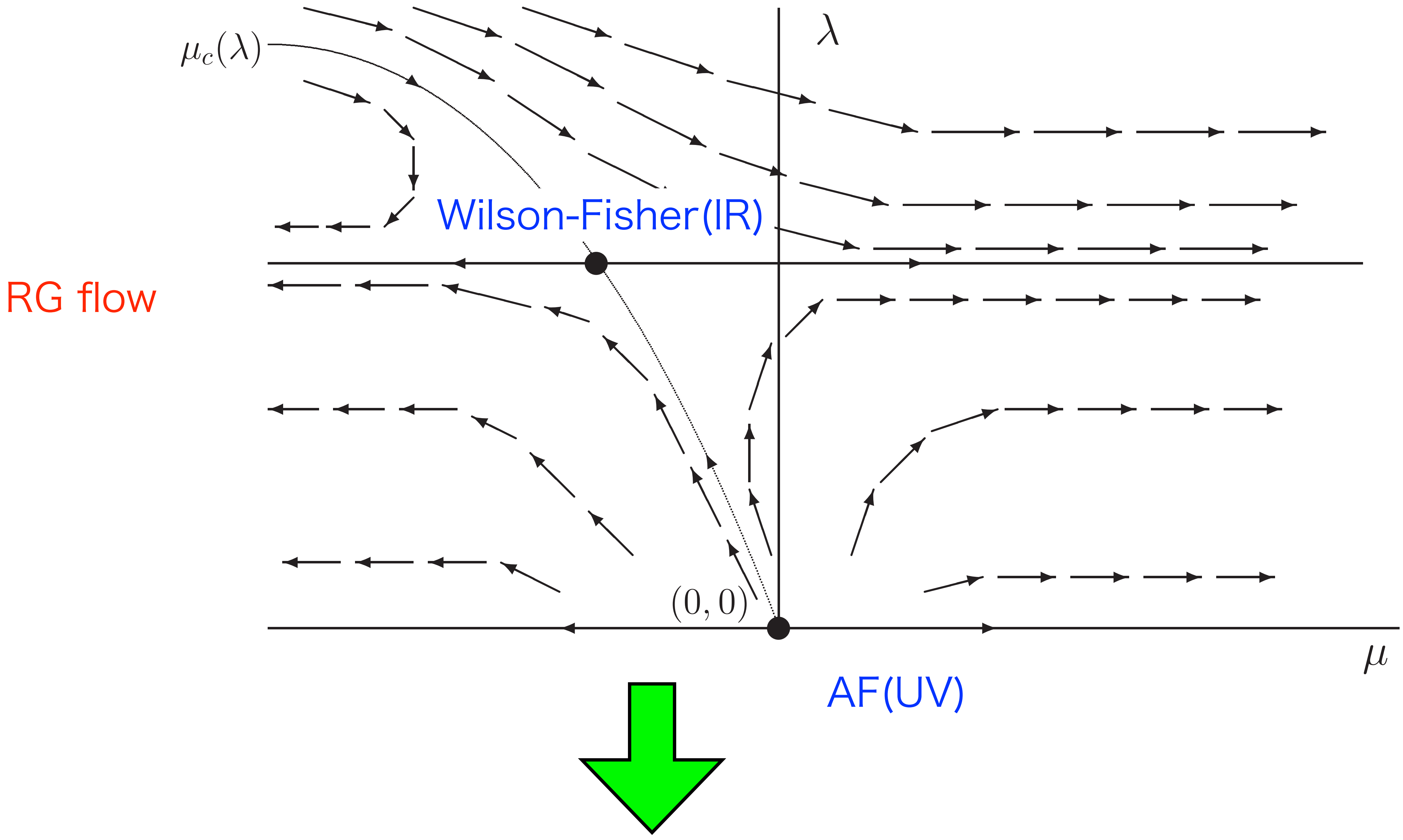
QCD-like theories with more fermions "walking" coupling ?

2. Around conformal fixed point ?

QCD-like theories with many fermions theory is still AF in UV

A toy model which has both AF(UV) and conformal(IR) fixed points

φ^4 theory in 3-dimension



What are the continuum limits from these two fixed points ?

This talk

Consider φ^4 theory in 3-dim. in the large N limit. exactly solvable

David, Kassier, Neuberger, 1985. Zinn-Justin, Moshe-Moshe, 2003(Review)

Construct quantum field theories by taking "continuum limits".

classification of the continuum field theories

running coupling constant

scattering phase shift

1. Introduction
2. Model and Analysis
3. Continuum Limits
4. Scattering Phase Shift
5. Conclusion

2. Model and Analysis

Lagrangian $$L(\varphi) = \frac{1}{2}\{\partial_\mu \varphi(x)\}^2 + \frac{r}{2}\varphi^2(x) + \frac{u}{4!N}[\varphi^2(x)]^2$$ $$\varphi^2(x) = \sum_{i=1}^{N} \varphi^i(x)\varphi_i(x)$$

$$Z(H) = \int [d\varphi] \exp\left[-\int d^3x L(\varphi) + \int d^3x H(x)\varphi(x)\right]$$

large N limit

$$\varphi_H(x) = \frac{\delta}{\delta H(x)} \log Z(H)$$

$$H \to 0$$

Effective action (symmetric phase)

$$\Gamma(\varphi_H) = \frac{1}{2}\int d^3x\, \varphi_H(x)(-\nabla^2 + m_R^2)\varphi_H(x) + \frac{1}{4!N}\int d^3x\, d^3y\, \varphi_H^2(x) u_R(x-y)\varphi_H^2(y)$$

mass $$m_R = \sqrt{\tau + \left(\frac{u}{48\pi}\right)^2} - \frac{u}{48\pi}$$ $$\tau = r + \frac{u}{24\pi}\Lambda$$ cut-off

"coupling" $$u_R(p) = \frac{u}{1 + uB_\Lambda(p, m_R)}$$ $$B_\Lambda(p,m) = \frac{1}{24\pi|p|}\left[\tan^{-1}\frac{|p|}{2m} - \tan^{-1}\frac{|p|}{\Lambda}\right]$$

Renormalization

$u \to \Lambda_X$ mass scale in the continuum limit

special in 3-dimension

$$\alpha \equiv \frac{m_R}{\Lambda_X}$$ renormalized mass in units of the mass scale Λ_X

$$g_R \equiv \frac{u_R(0)}{m_R} = \frac{48\pi}{1 - \frac{m_R}{\Lambda} + 48\pi \frac{m_R}{u}} = \frac{48\pi}{1 + 48\pi \alpha}$$ fixed $0 \le g_R \le 48\pi$

Line of constant physics (LCP)

cut-off effect

dimensionless bare coupling $$u_0(\Lambda) \equiv \frac{u}{\Lambda} = \frac{\delta_X}{1 + \frac{\delta_X}{48\pi}}$$ $$\delta_X = \frac{\Lambda_X}{\Lambda}$$

dimensionless bare "mass" $$\tau_0(\Lambda) \equiv \frac{\tau}{\Lambda^2} = \delta_X^2 \left(\alpha^2 + \frac{\alpha}{48\pi + \delta_X} \right)$$

bare beta function

$$\beta_0(u_0) \equiv \Lambda \frac{d\, u_0(\Lambda)}{d\Lambda} = -u_0(\Lambda) \frac{(48\pi - u_0(\Lambda))}{48\pi}$$

$u_0 = 0$ AF(UV) $u_0 = 48\pi$ Wilson-Fisher(IR)

"Running coupling constant" at finite cut-off

$$g_R(\mu,\Lambda) \equiv \frac{u_R(\mu)}{\mu} = \frac{48\pi}{t_x(48\pi+\delta_X)+2\left[\tan^{-1}\left(\frac{t_x}{2\alpha}\right)-\left(\frac{t_x\delta_X}{2}\right)\right]}$$

$$t_x = \frac{\mu}{\Lambda_X}$$ "scale" in units of the mass scale Λ_X

3. Continuum limits

"Running coupling constant"in the continuum limit $\Lambda \longrightarrow \infty$

I. AF continuum limit

$$g_R(\mu) = \frac{1}{t_x + \frac{1}{24\pi}\tan^{-1}\left(\frac{t_x}{2\alpha}\right)}$$

$$\lim_{\mu\to 0} g_R(\mu) = \infty$$

$$\lim_{\mu\to\infty} g_R(\mu) = 0$$

QCD-like

I-a. + massless limit $\alpha \longrightarrow 0$

$$g_R(\mu) = \frac{48}{1+48t_x}$$

$$\lim_{\mu\to 0} g_R(\mu) = 48$$ IR conformal

$$\lim_{\mu\to\infty} g_R(\mu) = 0$$ UV AF

beta function

$$\beta_g(g_R) \equiv \mu \frac{d\, g_R(\mu)}{d\mu} = -\frac{g_R(\mu)}{48}\frac{(48 - g_R(\mu))}{48}$$

anomalous dimension

$$\gamma_g(g_R = 48) = 1$$

$$\gamma_g(g_R = 0) = -1$$

II. Wilson-Fisher continuum limit

$$\Lambda_X = O(\Lambda) \quad \Rightarrow \quad \delta_X \neq 0, \alpha = 0$$

$$g_R(\mu, \Lambda) \equiv \frac{u_R(\mu)}{\mu} = \frac{48\pi}{t_x(48\pi + \delta_X) + 2\left[\tan^{-1}\left(\frac{t_x}{2\alpha}\right) - \left(\frac{t_x \delta_X}{2}\right)\right]}$$

$$g_R(\mu) = \frac{48\pi}{2\tan^{-1}\left(\frac{\mu}{2m_R}\right)}$$

$$\lim_{\mu\to 0} g_R(\mu) = \infty$$

$$\lim_{\mu\to\infty} g_R(\mu) = 48$$

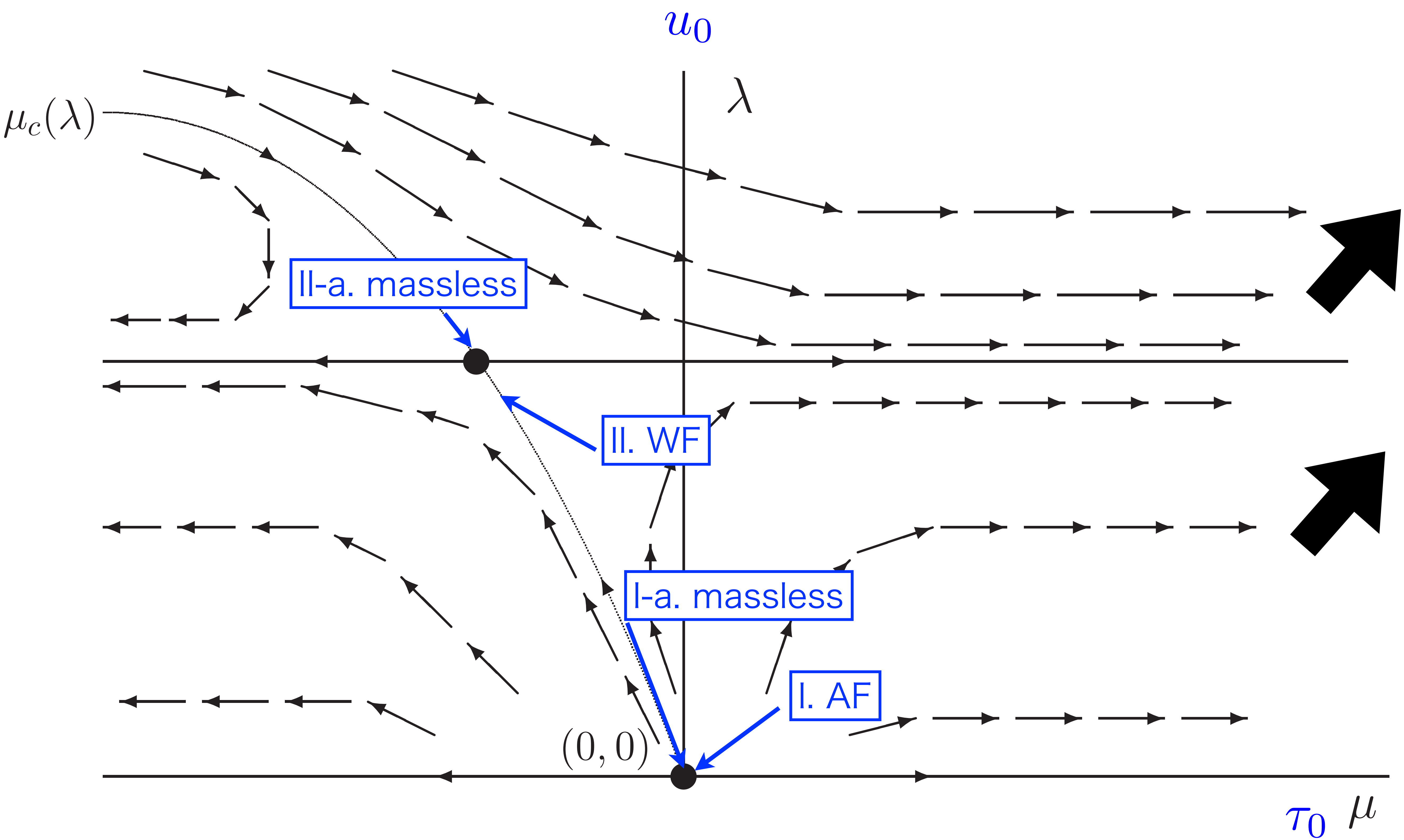
UV conformal

II-a. + massless limit

$$m_R \to 0$$

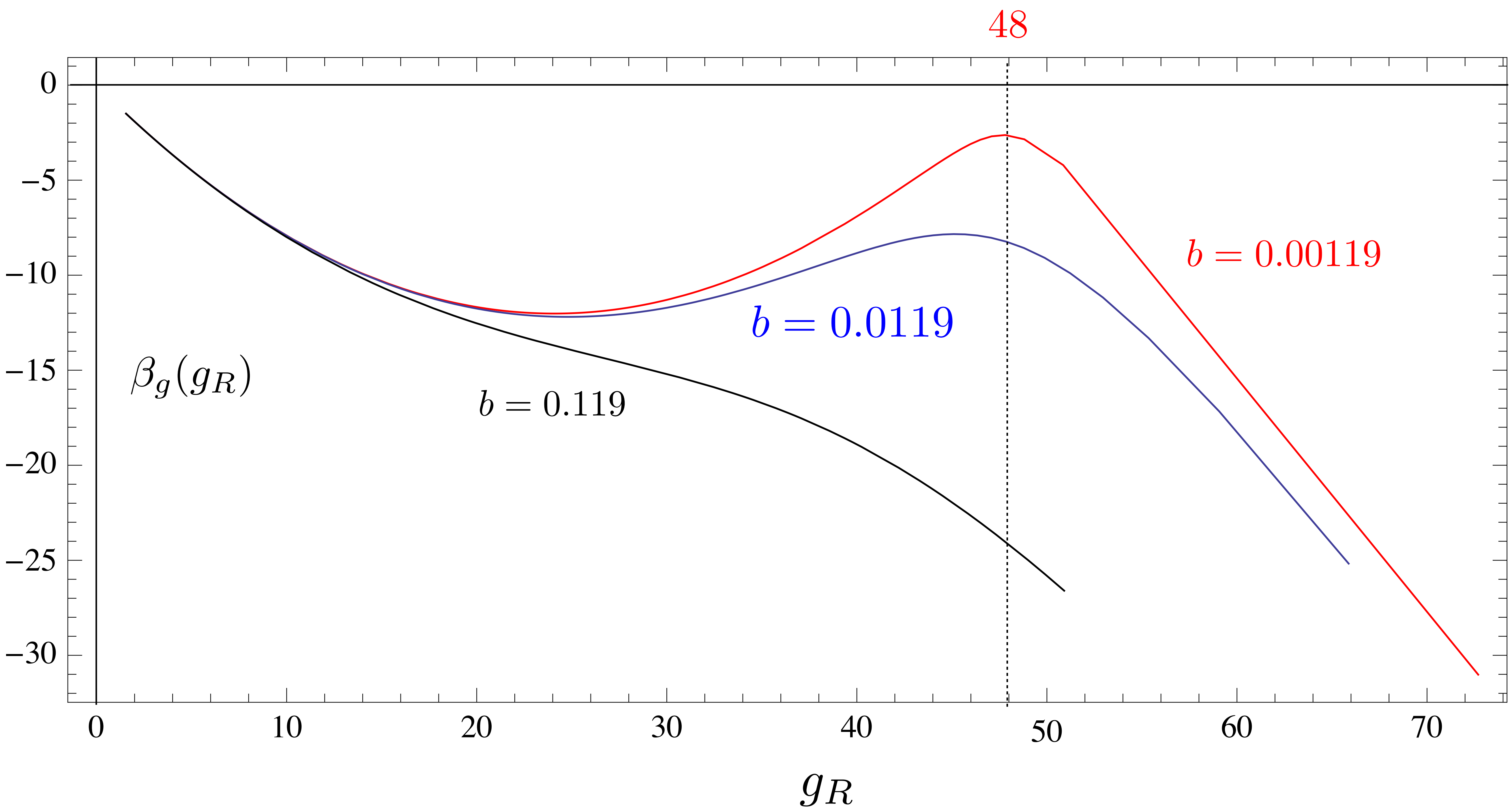
$$g_R(\mu) = 48$$

completely conformal


u_0
λ
$\mu_c(\lambda)$
II-a. massless
II. WF
I-a. massless
I. AF
$(0,0)$
τ_0
μ

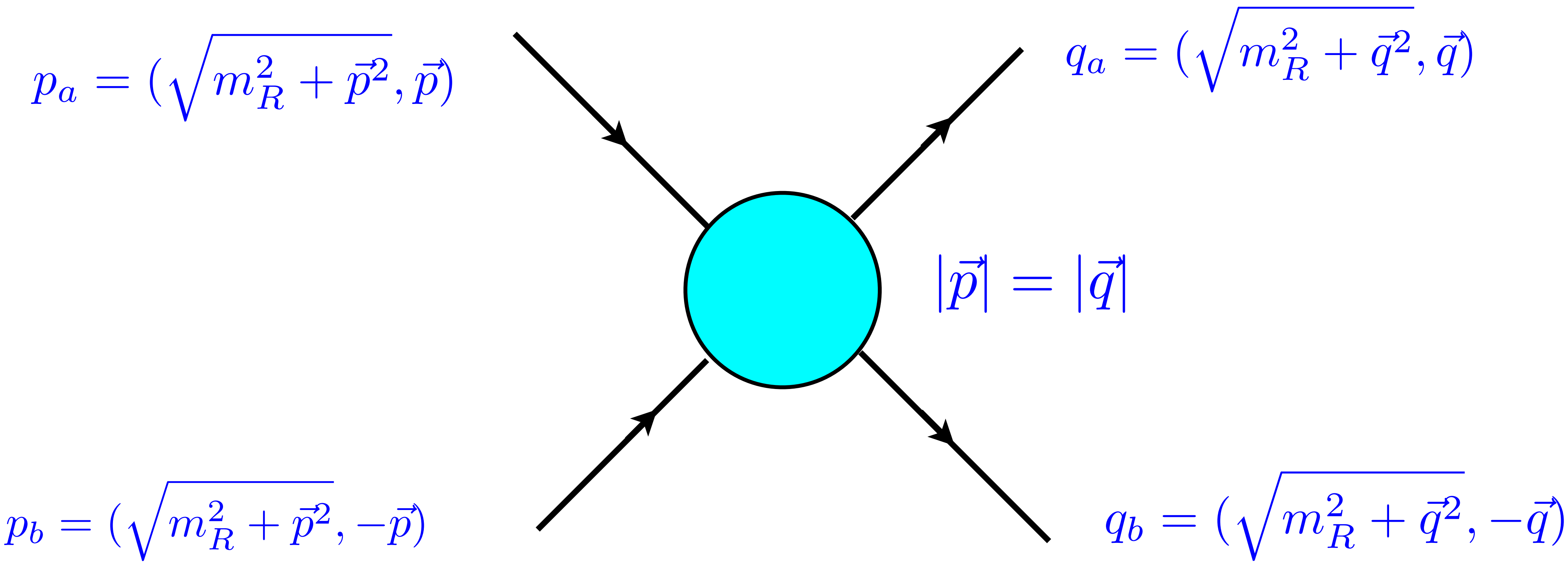
"Walking" ?

$$g_R(\mu) = \frac{1}{t_x + \frac{1}{24\pi}\tan^{-1}\left(\frac{t_x}{2\alpha}\right)} \qquad t_x = \frac{\mu}{\Lambda_X} \qquad b = 96\alpha \qquad \alpha = \frac{m_R}{\Lambda_X}$$



4. Scattering Phase Shift

Since the running coupling constant and beta function are not observables, we investigate properties of a physical observable, scattering phase shift.



Scattering amplitude

$$T(p_a, p_b | q_a, q_b) = \sum_{I=0}^{2} Q^I T^I(\vec{p}, \vec{q})$$

Q^I : iso-spin projection

In the large N limit, we have

$$T^0(\vec{p},\vec{q}) = -\frac{u}{3}\frac{1}{1+uB_\Lambda((i-\epsilon)(p_a+p_b),m_R)} \qquad \epsilon \to 0 \qquad p_a+p_b=(W,\vec{0})$$

$$T^1(\vec{p},\vec{q}) = T^2(\vec{p},\vec{q}) = 0$$

Unitarity

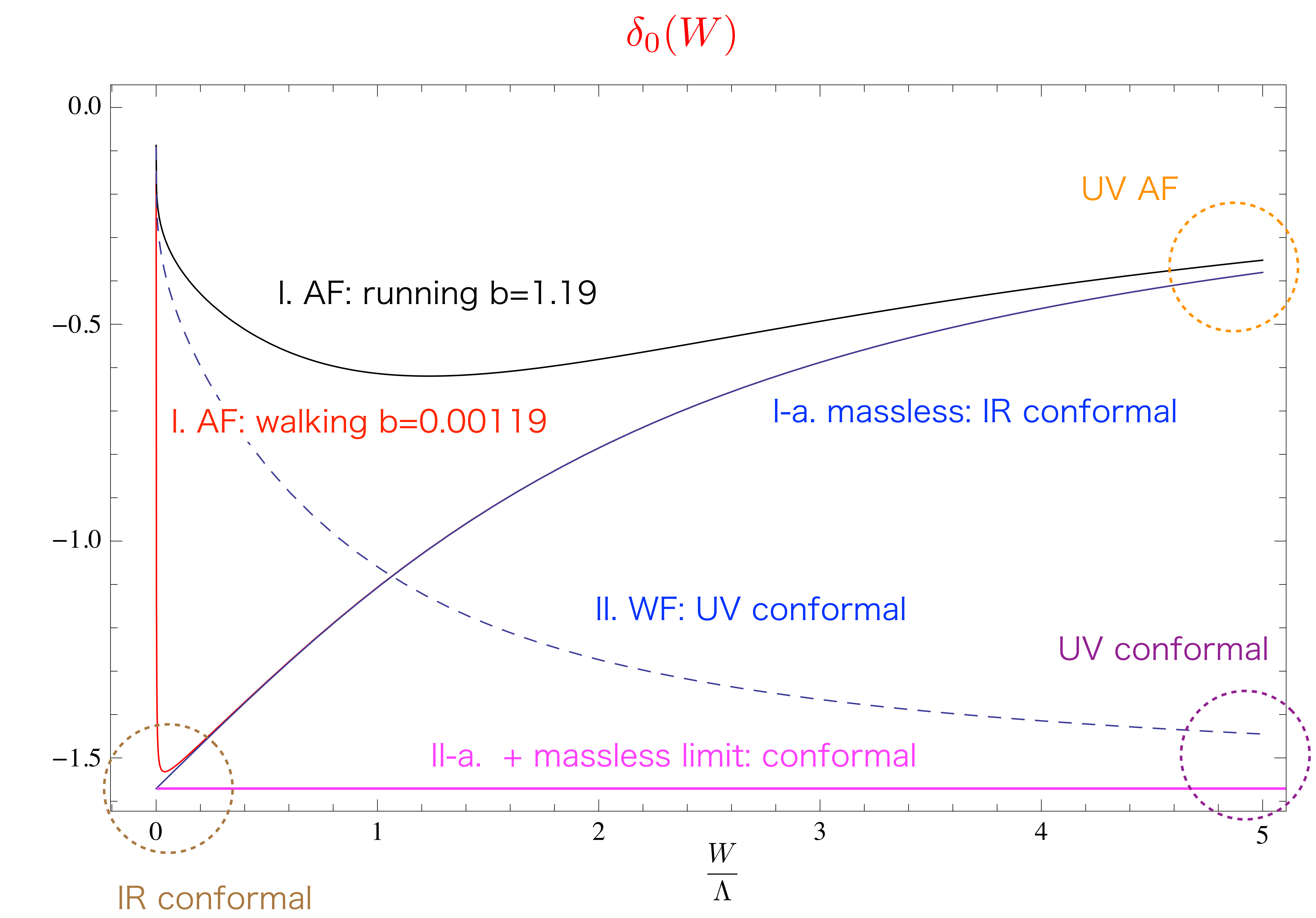
$$T^0(\vec{p},\vec{q}) = 16We^{i\delta_0(W)}\sin\delta_0(W)$$

$\delta_0(W)$: $I=0$ scattering phase shift

$$\cot\delta_0(W) = -\frac{48W}{u} - \frac{2}{\pi}\left[\coth^{-1}\left(\frac{W}{2m_R}\right) - \tanh^{-1}\left(\frac{W}{2\Lambda}\right)\right]$$

$\Lambda \to \infty$

$$\delta_0(W) = -\cot^{-1}\left[\frac{48W}{\Lambda_X} + \frac{2}{\pi}\coth^{-1}\left(\frac{W}{2m_R}\right)\right]$$



$\delta_0(W)$
0.0
−0.5
−1.0
−1.5
0
1
2
3
4
5
$\frac{W}{\Lambda}$
UV AF
I. AF: running b=1.19
I. AF: walking b=0.00119
I-a. massless: IR conformal
II. WF: UV conformal
UV conformal
II-a. + massless limit: conformal
IR conformal

5. Conclusion

- Large N ϕ^4 model in 3-dimensions has various continuum limits
  - I. continuum limit around AF fixed point: QCD-like theory
    - I-a. + massless limit: + IR conformal
  - II. continuum limit around Wilson-Fisher point: UV conformal
    - II-a. + massless limit: completely conformal
- “Walking” in case I. if the mass is very small.
- scattering phase shift reflects above difference of the continuum limits.

Works in progress

Broken phase

Effect of ϕ^6 terms -> just shift the ϕ^4 coupling

$$u \to u + \frac{6 g_6 \Lambda}{\pi}$$

Effect of finite cut-off -> not so large

Effect of finite volume

Cut-off effect

$$g_R(\mu,\Lambda) \equiv \frac{u_R(\mu)}{\mu} = \frac{48\pi}{t_x(48\pi+\delta_X) + 2\left[\tan^{-1}\left(\frac{t_x}{2\alpha}\right) - \left(\frac{t_x\delta_X}{2}\right)\right]}$$

$b = 0.00119$