

# Some Progress on the Construction of Technicolor and Extended Technicolor Models

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SCGT12 Mini-Workshop, KMI, Nagoya University, March, 2012

# Outline

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# Motivations for Dynamical Electroweak Symmetry Breaking

Recall the motivations for considering dynamical electroweak symmetry breaking (EWSB). Standard Model (SM) Higgs mechanism for EWSB works but leaves some questions:

To get EWSB, one sets  $\mu^2 < 0$  in the scalar potential of the SM Lagrangian,  $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$ , yielding  $\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ . But why should  $\mu^2$  be negative rather than positive?

$\mu^2$  and hence  $m_H^2 = -2\mu^2 = 2\lambda v^2$  with  $v = (2/g)m_W = 246$  GeV are unstable to large radiative corrections from much higher energy scales - gauge hierarchy problem, fine-tuning needed to keep the scalar light.

What is the origin of the large range of SM Yukawa couplings, from  $O(1)$  for top quark to  $10^{-5}$  for electron mass (with additional inputs needed for neutrino masses)?

Furthermore, in two major previous cases where fundamental scalar fields were used in phenomenologically modelling SSB, the underlying physics involved bilinear fermion condensates:

**Superconductivity:** the Ginzburg-Landau free energy functional was a successful phenomenological description, using complex scalar field  $\phi$  with  $V = c_2|\phi|^2 + c_4|\phi|^4$ , with  $c_2 \propto (T - T_c)$ , so for  $T < T_c$ ,  $c_2 < 0$  and  $\langle \phi \rangle \neq 0$ . But the underlying origin of superconductivity is the dynamical formation of a condensate of Cooper pairs  $\langle ee \rangle$  in BCS theory.

$\sigma$  model for spontaneous chiral symmetry breaking ( $S\chi SB$ ) in hadronic physics, with  $V = (\mu^2/2)\vec{\phi}^2 + (\lambda/4)\vec{\phi}^4$ , where  $\vec{\phi} = (\sigma, \vec{\pi})$ . Here, one produces  $S\chi SB$  by the choice  $\mu^2 < 0$ , leading to  $\langle \sigma \rangle = f_\pi \neq 0$ . But the underlying origin of  $S\chi SB$  in QCD is the dynamical formation of a  $\langle \bar{q}q \rangle$  condensate.

These examples suggest the possibility that the underlying physics responsible for EWSB may also be a dynamically induced fermion condensate.

Indeed, there is one known source of dynamical EWSB via a fermion condensate: the  $\langle \bar{q}q \rangle$  condensate in QCD breaks electroweak symmetry.

Consider, e.g., QCD with  $N_f = 2$  massless quarks,  $u, d$ . This theory has a global  $SU(2)_L \times SU(2)_R$  chiral sym. quark condensate  $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R \rangle + \langle \bar{q}_R q_L \rangle$  transforms as an  $I_w = 1/2, |Y| = 1$  operator and breaks this symmetry to the diagonal, vectorial isospin  $SU(2)_V$ . Resultant Nambu-Goldstone bosons (NGB's) -  $\pi^\pm$  and  $\pi^0$  - are absorbed to become the longitudinal components of  $W^\pm$  and  $Z$ , giving them masses:

$$m_W^2 = \frac{g^2 f_\pi^2}{4}, \quad m_Z^2 = \frac{(g^2 + g'^2) f_\pi^2}{4}$$

With  $f_\pi \sim 93$  MeV, this yields  $m_W \simeq 30$  MeV,  $m_Z \simeq 33$  MeV, satisfying tree-level relation  $\rho = 1$ , where  $\rho = m_W^2 / [m_Z^2 \cos^2 \theta_W]$ . (A gedanken world where this is the only source of EWSB is discussed in Quigg and RS, Phys. Rev. D79, 096002 (2009)).

Scale here is too small by  $\sim 10^3$  to explain the observed  $W$  and  $Z$  masses, but suggests a more realistic model for dynamical EWSB.

# Basics of Technicolor

Technicolor (TC) is an asymptotically free vectorial gauge theory with gauge group that can be taken as  $SU(N_{TC})$  and a set of fermions  $\{F\}$  with zero Lagrangian masses, transforming according to some representation(s) of  $G$ . The TC interaction becomes strong at a scale  $\Lambda_{TC}$  of order the electroweak scale, confining and producing a chiral symmetry breaking technifermion condensate (Weinberg, Susskind, 1979); recent review: Sannino, Acta Phys. Polon., arXiv:0911.0931).

Assign technifermions so  $L$  ( $R$ ) components form  $SU(2)_L$  doublets (singlets). Minimal choice: “one-doublet” (1DTC) model with fund. rep. for technifermions uses

$$\begin{pmatrix} F_1^\tau \\ F_2^\tau \end{pmatrix}_L \quad F_{1R}^\tau, \quad F_{2R}^\tau$$

with TC indices  $\tau$  and  $Y = 0$  ( $Y = \pm 1$ ) for  $SU(2)_L$  doublet (singlets).

The  $SU(N_{TC})$  TC theory is asymp. free, so as energy scale decreases,  $\alpha_{TC}$  increases, eventually producing condensates  $\langle \bar{F}_1 F_1 \rangle$  and  $\langle \bar{F}_2 F_2 \rangle$  transforming as  $I_w = 1/2$ ,  $|Y| = 1$ , breaking EW symmetry at  $\Lambda_{TC}$ .

The  $W$  and  $Z$  pick up masses

$$m_W^2 \simeq \frac{g^2 F_{TC}^2 N_D}{4}, \quad m_Z^2 \simeq \frac{(g^2 + g'^2) F_{TC}^2 N_D}{4}$$

again satisfying  $\rho = 1$  because of the  $I_w$  and  $Y$  of  $\langle \bar{F}_i F_i \rangle$ ,  $i = 1, 2$ . Here  $F_{TC} \sim \Lambda_{TC}$  is the TC analogue to  $f_\pi \sim \Lambda_{QCD}$  and  $N_D =$  number of  $SU(2)_L$  technidoublets. For 1DTC,  $N_D = 1$ , so  $F_{TC} = 250$  GeV. One can add SM-singlet technifermions to get walking.

Another class of TC models that was studied in the past (but is now disfavored) used one SM family of technifermions (1FTC)

$$\begin{pmatrix} U^{a\tau} \\ D^{a\tau} \end{pmatrix}_L \quad U_R^{a\tau}, \quad D_R^{a\tau}$$

$$\begin{pmatrix} N^\tau \\ E^\tau \end{pmatrix}_L \quad N_R^\tau, \quad E_R^\tau$$

(where  $a, \tau$  are color, TC indices). For 1FTC,  $N_D = N_c + 1 = 4$ , so  $F_{TC} \simeq 125$  GeV.

Some appealing properties of TC:

- Given the asymp. freedom of TC theory, the condensate formation and hence EWSB are automatic, as in QCD, and do not require a specific parameter choice like  $\mu^2 < 0$  in SM.
- TC has no fundamental Higgs, so no hierarchy problem.
- Because  $\langle \bar{F}F \rangle = \langle \bar{F}_L F_R \rangle + \langle \bar{F}_R F_L \rangle$ , technicolor explains why the chiral part of  $G_{SM}$  is broken and the residual exact gauge symmetry,  $SU(3)_c \times U(1)_{em}$ , is vectorial (also explained in SM).

To give masses to quarks and leptons, embed TC in a larger, extended technicolor (ETC) gauge theory with ETC gauge bosons transforming SM fermions into technifermions and back (Dimopoulos and Susskind; Eichten and Lane, 1979-80).

ETC gauges SM fermion generation index and combines it with TC gauge indices in the full ETC symmetry group.

To satisfy constraints on flavor-changing neutral current (FCNC) processes, ETC gauge bosons must have large masses. These masses naturally arise from sequential breaking of a strongly coupled chiral ETC gauge symmetry.

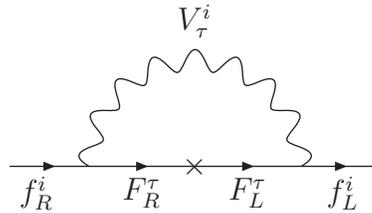
Diagrams for generating SM fermion masses involve virtual exchanges of ETC gauge bosons, so resultant fermion masses depend on inverse powers of these ETC gauge boson masses. To account for the hierarchy in the three generations of SM fermion masses, the ETC theory should break sequentially at three corresponding scales,  $\Lambda_1 > \Lambda_2 > \Lambda_3$ , e.g.,  $\Lambda_1 \simeq 10^3$  TeV,  $\Lambda_2 \simeq 50 - 100$  TeV,  $\Lambda_3 \simeq$  few TeV.

The ETC theory is constructed to be asymptotically free, so as energy decreases from a high scale, ETC coupling  $\alpha_{ETC}$  grows, eventually becomes large enough to form condensates that sequentially break the ETC symmetry to a residual exact subgroup, which is the TC gauge group; so  $G_{ETC} \supset G_{TC}$ .

An ETC theory is much more ambitious than the SM or MSSM because a successful ETC model would predict the entries in the SM fermion mass matrices and the resultant values of the quark and lepton masses and mixings. It would explain longstanding mysteries like the mass ratios  $m_e/m_\mu$ ,  $m_u/m_d$ ,  $m_d/m_s$ , etc. Not surprisingly, no fully realistic ETC model has yet been constructed, and TC/ETC models face many stringent constraints.

# Mass Generation Mechanism for Fermions

The ETC gauge bosons enable SM fermions, which are TC singlets, to transform into technifermions and back, communicating the EWSB in the TC sector to these SM fermions and producing masses for them. The figure shows a one-loop graph contributing to diagonal entries in mass matrix for SM fermion  $f^i$ , where  $i =$  generation index. Basic ETC vertex is  $f^i \rightarrow f^j + V_j^i$ , with  $V_j^i =$  ETC gauge boson,  $1 \leq i, j \leq 5$ ; here we distinguish first three ETC indices, which refer to SM fermion generations, from other ETC indices that are TC indices, by denoting the latter as  $\tau$  (with any color indices suppressed):



Rough estimate:

$$M_{ii}^{(f)} \simeq \frac{2\alpha_{ETC} C_f}{\pi} \int dk^2 \frac{k^2 \Sigma_{TC}(k)}{[k^2 + \Sigma_{TC}(k)^2][k^2 + M_i^2]}$$

where  $M_i \simeq (g_{ETC}/2)\Lambda_i \simeq \Lambda_i$  is the mass of the ETC gauge bosons that gain mass at scale  $\Lambda_i$ ,  $C_f =$  quadratic Casimir invariant. For Euclidean  $k \gg \Lambda_{TC}$ ,

$\Sigma_{TC}(k) \simeq \Sigma_{TC}(0)[\Sigma_{TC}(0)/k]^{2-\gamma}$ . In walking TC (WTC),  $\gamma$  may be  $\sim O(1)$ , so  $\Sigma_{TC}(k) \simeq \Sigma_{TC}(0)^2/k$ ; contrast with QCD, where  $\Sigma(k) \simeq \Sigma(0)^3/k^2$  for  $k \gg \Lambda_{QCD}$ . In general, the TC/ETC calculation of  $M_{ii}^{(f)}$  gives

$$M_{ii}^{(f)} \simeq \frac{\kappa C_f \eta \Lambda_{TC}^3}{\Lambda_i^2}$$

where  $\kappa \simeq O(10)$  is a numerical factor from the integral and  $\eta$  is a RG factor that enhances the mass. This is only a rough estimate, since ETC coupling is strong, so higher-order diagrams are also important.

The sequential breaking of the ETC symmetry at the highest scale,  $\Lambda_1$ , the intermediate scale,  $\Lambda_2$ , and the lowest scale,  $\Lambda_3$ , thus produces the generational hierarchy in the SM fermion masses. Since these ETC scales enter as inverse powers in the resultant SM fermion masses and since  $\Lambda_1$  is the largest ETC scale, it follows that first-generation fermion masses are the smallest, and since  $\Lambda_3$  is the smallest ETC scale, third-generation fermion masses are the largest.

There are mixings among the interaction eigenstates of the ETC gauge bosons to form mass eigenstates. Insertions of these on ETC gauge boson lines lead to CKM and lepton mixing ( Appelquist and RS, Phys. Lett. B 548, 204 (2002); Appelquist and RS, Phys. Rev. Lett. 90, 201801 (2003); Appelquist, Piai, RS, Phys. Rev. D 69, 015002 (2004); Christensen and RS, Phys. Rev. D 74, 015004 (2006)).

Since SM fermion masses arise dynamically, the running mass  $m_{f_i}(p)$  of a SM fermion of generation  $i$  is constant up to the ETC scale  $\Lambda_i$  and has the power-law decay (Christensen and RS, Phys. Rev. Lett. 94, 241801 (2005)):

$$m_{f_i}(p) \sim m_{f_i(0)} \frac{\Lambda_i^2}{p^2}$$

for Euclidean momenta  $p \gg \Lambda_i$  (neglect subdominant logarithmic factors).

Thus, e.g., the third-generation quark masses  $m_t(p)$  and  $m_b(p)$  decay like  $\Lambda_3^2/p^2$  for  $p \gg \Lambda_3$ , while the first-generation quark masses  $m_u(p)$  and  $m_d(p)$  are hard up to the much higher scale  $\Lambda_1$ , eventually decaying like  $\Lambda_1^2/p^2$  for  $p \gg \Lambda_1$ .

# UV to IR Evolution and Walking (Quasi-Conformal) TC

TC models that behaved simply as scaled-up versions of QCD were excluded by their inability to produce sufficiently large fermion masses without having ETC scales so low as to cause excessively large FCNC.

Modern TC theories are constructed to have a coupling  $g_{TC}$  that gets large, but runs slowly (“walks”) over an extended interval of energy (WTC) (Holdom, Yamawaki et al., Appelquist, Wijewardhana...).

This walking (quasi-conformal) behavior arises naturally from an approximate IR zero of the perturbative beta function:

$$\beta(\alpha_{TC}) = \frac{d\alpha_{TC}}{dt} = -\frac{\alpha_{TC}^2}{2\pi} \left( b_1 + \frac{b_2 \alpha_{TC}}{4\pi} + O(\alpha_{TC}^2) \right)$$

where  $t = \ln \mu$ , with  $b_1 > 0$  ( $N_f < N_{f,max}$ ) for asymptotic freedom. For sufficiently many technifermions,  $b_2 < 0$ , so  $\beta$  has a second zero, i.e., approx. IR fixed point (IRFP) of RG, at  $\alpha_{TC} = -4\pi b_1/b_2 \equiv \alpha_{IR}$ .

If  $N_f < N_{f,cr}$  (depending on technifermion rep. of  $G_{TC}, R$ ), as the theory evolves from the UV to IR,  $\alpha_{TC}$  gets large, but runs slowly because  $\beta$  approaches this zero at  $\alpha_{IR}$ . For TC, we want to choose  $N_f$  so that  $\alpha_{IR}$  is slightly greater than the minimal value  $\alpha_{cr}$  for technifermion condensation. Then the TC theory has quasi-conformal behavior, with a large  $\alpha_{TC}(\mu)$  over an extended interval of energies  $\mu$ .

As  $\alpha_{TC}(\mu)$  eventually exceeds  $\alpha_{cr}$  at  $\mu \sim \Lambda_{TC}$ , the technifermion condensate  $\langle \bar{F} F \rangle$  forms, the technifermions gain dynamical masses, and in the low-energy theory at smaller  $\mu$ , they are integrated out, so the TC beta function changes, and  $\alpha_{TC}$  evolves away from  $\alpha_{IR}$  which is thus an approximate IR fixed point.

Because WTC has approx. dilatational invariance, which is dynamically broken by the  $\langle \bar{F} F \rangle$  condensate, it has been suggested that this could lead to a light approx. Nambu-Goldstone boson (NGB), the techidilaton (Yamawaki..Goldberger, Grinstein, Skiba; Sannino...; Appelquist and Bai; Logan, Barger, Ellis...; see also Bardeen et al.; Holdom and Terning). This might be as light as 125 GeV.

The initial ATLAS and CMS indications, seen in several channels, of a possible state at 125 GeV, if confirmed with more data, might be a Higgs but instead might be a technidilaton, as discussed at this conf. Further experimental and theoretical studies are necessary to decide this.

For  $N_f > N_{f,cr}$ , the theory would evolve from the UV to the IR in a chirally symmetric manner, without ever producing  $\langle \bar{F}F \rangle$ , so the (initially massless) technifermions remain massless, and the IRFP is exact. This IR-conformal phase is of basic field-theoretic interest, although for TC, we should choose the technifermion content so that we are in the phase with  $S_\chi$ SB, as is necessary for EWSB.

In Walking TC, SM fermion masses are enhanced by the factor

$$\eta_i = \exp \left[ \int_{\Lambda_{TC}}^{\Lambda_i} \frac{d\mu}{\mu} \gamma(\alpha_{TC}(\mu)) \right]$$

where  $\gamma =$  anomalous dimension of  $\bar{F}F$  operator. If  $\gamma$  is approximately constant over this range of  $\mu$ , then  $\eta_i = (\Lambda_i/\Lambda_{TC})^\gamma$ , which can be substantially larger than 1. So one can increase ETC scales  $\Lambda_i$  for a fixed  $m_{f_i}$ , reducing FCNC effects.

One method ( $\beta$ DS) for studying quasi-conformal TC: use 2-loop  $\beta$  function to calculate running  $\alpha$ ; combine with sol. of Dyson-Schwinger (DS) eq. for technifermion propagator in the improved ladder (one-gluon exchange) approx. (Yamawaki, Miransky..., Appelquist et al., Lane..). DS eq. yields dynamical technifermion mass generation for  $\alpha > \alpha_{cr}$ , with  $\alpha_{cr} C_f \sim O(1)$ , where  $R$  is fermion rep.

As number of technifermions,  $N_f$ , increases,  $\alpha_{IR}$  decreases, and  $N_f \nearrow N_{f,cr}$  as  $\alpha_{IR} \searrow \alpha_{cr}$ . This yielded the  $\beta$ DS estimate  $N_{f,cr} \simeq 4N_{TC}$  for fund. rep.

The DS eq. captures some of the relevant physics but does not directly include effects of confinement or instantons, both of which are important for  $S\chi$ SB.

Because of confinement, the technifermions and gluons have maximum wavelengths  $\lambda \sim 1/\Lambda_{TC}$  and minimum momenta  $k_{min} \sim \Lambda_{TC}$ . The conventional DS eq. takes the lower end of the Euclidean loop integration to  $k = 0$ , but confinement raises this to  $k_{min}$ . This correction decreases the integration region, decreases tendency to  $S\chi$ SB (Brodsky and RS, Phys. Lett. B666, 95 (2008)). The correction for the neglect of instantons goes in the opposite direction; inclusion of instantons increases the tendency to  $S\chi$ SB.

Since these corrections are in opposite directions, the net shift in  $N_{f,cr}$  may not be too great, which helps to explain the rough agreement, to within the uncertainties of the  $\beta$ DS  $N_{f,cr}$  and general lattice data.

Other approaches to estimating  $N_{f,cr}$  are also of interest, e.g., analysis of gluon propagation (Oehme-Zimmermann, Frandsen et al..)

# Higher-loop corrections to UV $\rightarrow$ IR evolution of gauge theories

Because of the strong-coupling nature of the physics at an approximate IRFP of interest to TC theories, there are generically significant higher-order corrections to results obtained from the two-loop  $\beta$  function.

This motivates the calculation of the location of the IR zero in  $\beta$  and the value of  $\gamma = \gamma(\alpha)$  for an  $SU(N)$  gauge thy. evaluated at  $\alpha = \alpha_{IR}$  to higher-loop order. We have done this to 3-loop and 4-loop order (Ryttov and RS, PRD 83, 056011 (2011), arXiv:1011.4542; see also Pica and Sannino, PRD 83,035013 (2011), arXiv:1011.5917). This is of general field-theoretic interest, beyond the specific application to technicolor.

We have extended this analysis to an  $\mathcal{N} = 1$  supersymmetric  $SU(N)$  theory in Ryttov and RS, arXiv:1202.1297. First discuss non-supersymmetric theory.

Although the coefficients in the beta function at 3-loop and higher-loop order are scheme-dependent, the results give a measure of the accuracy of the 2-loop calculation of the IR zero, and similarly with the value of  $\gamma$  evaluated at this IR zero. For QCD, the value of such higher-loop calculations is shown by the increased accuracy in fits to data on  $\alpha_s(\mu)$  (Bethke).

We use the  $\overline{MS}$  scheme, for which the coefficients of  $\beta$  and  $\gamma$  have been calculated up to 4-loop order (highest-order from Vermaseren, Larin, and van Ritbergen). With  $\alpha \equiv \alpha_{TC}$  and  $a \equiv \alpha/(4\pi)$ , one has  $d\alpha/dt = -2\alpha \sum_{\ell=1}^{\infty} b_{\ell} a^{\ell}$ , where  $b_{\ell}$  is the  $\ell$ -loop coefficient. Recall

$$b_1 = \frac{1}{3}(11C_A - 4T_f N_f)$$

$$b_2 = \frac{1}{3} [34C_A^2 - 4(5C_A + 3C_f)T_f N_f]$$

$$b_3 = \frac{2857}{54} C_A^3 + T_f N_f \left[ 2C_f^2 - \frac{205}{9} C_A C_f - \frac{1415}{27} C_A^2 \right] \\ + (T_f N_f)^2 \left[ \frac{44}{9} C_f + \frac{158}{27} C_A \right]$$

and so forth for  $b_4$ , where  $\sum_a \sum_j \mathcal{D}_R(T_a)_{ij} \mathcal{D}_R(T_a)_{jk} = C_f \delta_{ik}$  and  $\sum_{j,k} \mathcal{D}_R(T_a)_{jk} \mathcal{D}_R(T_b)_{kj} = T_f \delta_{ab}$  so  $C_A \equiv C_{Adj} = N$  for  $SU(N)$ , and, e.g., for fund. rep.  $C_f = (N^2 - 1)/(2N)$ ,  $T_f(\text{fund.}) = 1/2$ , etc.

Similarly, for the anomalous dimension  $\gamma$  of  $\bar{F}F$ ,  $\gamma = \sum_{\ell=1}^{\infty} c_{\ell} a^{\ell}$  with

$$c_1 = 6C_f$$

$$c_2 = 2C_f \left[ \frac{3}{2}C_f + \frac{97}{6}C_A - \frac{10}{3}T_f N_f \right].$$

$$c_3 = 2C_f \left[ \frac{129}{2}C_f^2 - \frac{129}{4}C_f C_A + \frac{11413}{108}C_A^2 + C_f T_f N_f (-46 + 48\zeta(3)) \right. \\ \left. - C_A T_f N_f \left( \frac{556}{27} + 48\zeta(3) \right) - \frac{140}{27}(T_f N_f)^2 \right]$$

and so forth for  $c_4$ .

The two-loop zero of  $\beta$  away from the origin is given by  $a = -b_1/b_2$  and is physical for  $b_2 < 0$ .

At the three-loop level, we identify the physical IR zero of  $\beta$  as the relevant physical solution of the quadratic eq.  $b_1 + b_2 a + b_3 a^2 = 0$  (non-negative solution closest to origin).

Similarly, at the four-loop level, we pick out the relevant root from the cubic  $b_1 + b_2 a + b_3 a^2 + b_4 a^3 = 0$ . Behavior of the higher-order coefficients  $b_j$  as functions of  $N_f$  and rep.  $R$  is studied in paper.

We then evaluate the  $n$ -loop ( $\ell$ ) expression for  $\gamma$ , denoted  $\gamma_{n\ell}(\alpha_{IR,n\ell})$  at the  $n$ -loop zero of  $\beta$ ,  $\alpha_{IR,n\ell}$ .

Detailed analytic and numerical results are presented in our paper; here we give only some illustrative numerical results for low values of  $N \equiv N_{TC}$ .

For abstract field-theoretic purposes (not for TC application), we also give analytic results on the approach to the large- $N$  limit with fixed coupling  $\lambda = \alpha N$ . For the fund. rep. we combine this with the limit  $N_f \rightarrow \infty$  with  $r = N_f/N$  fixed.

We find that for given  $SU(N)$  and fermion content for which there is an IR zero of  $\beta$ , the 3-loop and 4-loop values of  $\alpha_{IR}$  are smaller than the 2-loop value.

Results for  $N_f$  technifermions in the fundamental rep. of  $SU(N)$  for  $N = 2, 3$ :

$N$	$N_f$	$\alpha_{IR,2\ell}$	$\alpha_{IR,3\ell}$	$\alpha_{IR,4\ell}$
2	7	2.83	1.05	1.21
2	8	1.26	0.688	0.760
2	9	0.595	0.418	0.444
2	10	0.231	0.196	0.200
3	10	2.21	0.764	0.815
3	11	1.23	0.578	0.626
3	12	0.754	0.435	0.470
3	13	0.468	0.317	0.337
3	14	0.278	0.215	0.224
3	15	0.143	0.123	0.126
3	16	0.0416	0.0397	0.0398

Similarly, we find that for given  $N$ ,  $R$ , and  $N_f$ , the value of  $\gamma$  calculated to 3-loop and 4-loop order and evaluated at the value of  $\alpha_{IR}$  calculated to the same order is somewhat smaller than the 2-loop value:

For  $N_f$  technifermions in  $R =$  fundamental rep. of  $SU(N)$  for  $N = 2, 3$ :

$N$	$N_f$	$\gamma_{2l}(\alpha_{IR,2l})$	$\gamma_{3l}(\alpha_{IR,3l})$	$\gamma_{4l}(\alpha_{IR,4l})$
2	7	(2.67)	0.457	0.0325
2	8	0.752	0.272	0.204
2	9	0.275	0.161	0.157
2	10	0.0910	0.0738	0.0748
3	10	(4.19)	0.647	0.156
3	11	1.61	0.439	0.250
3	12	0.773	0.312	0.253
3	13	0.404	0.220	0.210
3	14	0.212	0.146	0.147
3	15	0.0997	0.0826	0.0836
3	16	0.0272	0.0258	0.0259

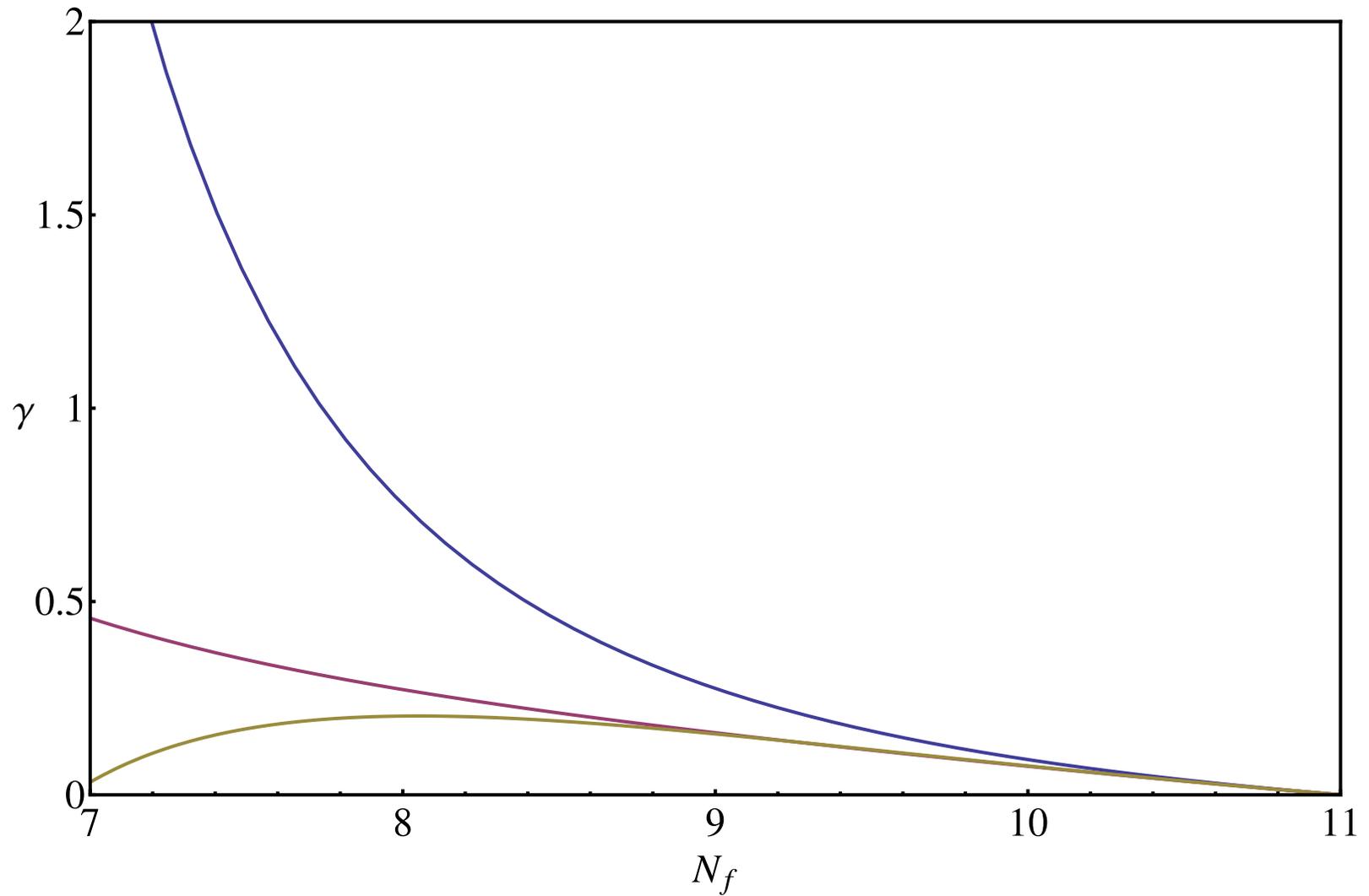


Figure 1: Anomalous dimension  $\gamma$  for SU(2) for  $N_f$  fermions in the fundamental representation; (i) blue: 2-loop; (ii) red: 3-loop; (iii) brown: 4-loop calculation ( $N_{f,max} = 11$ ).

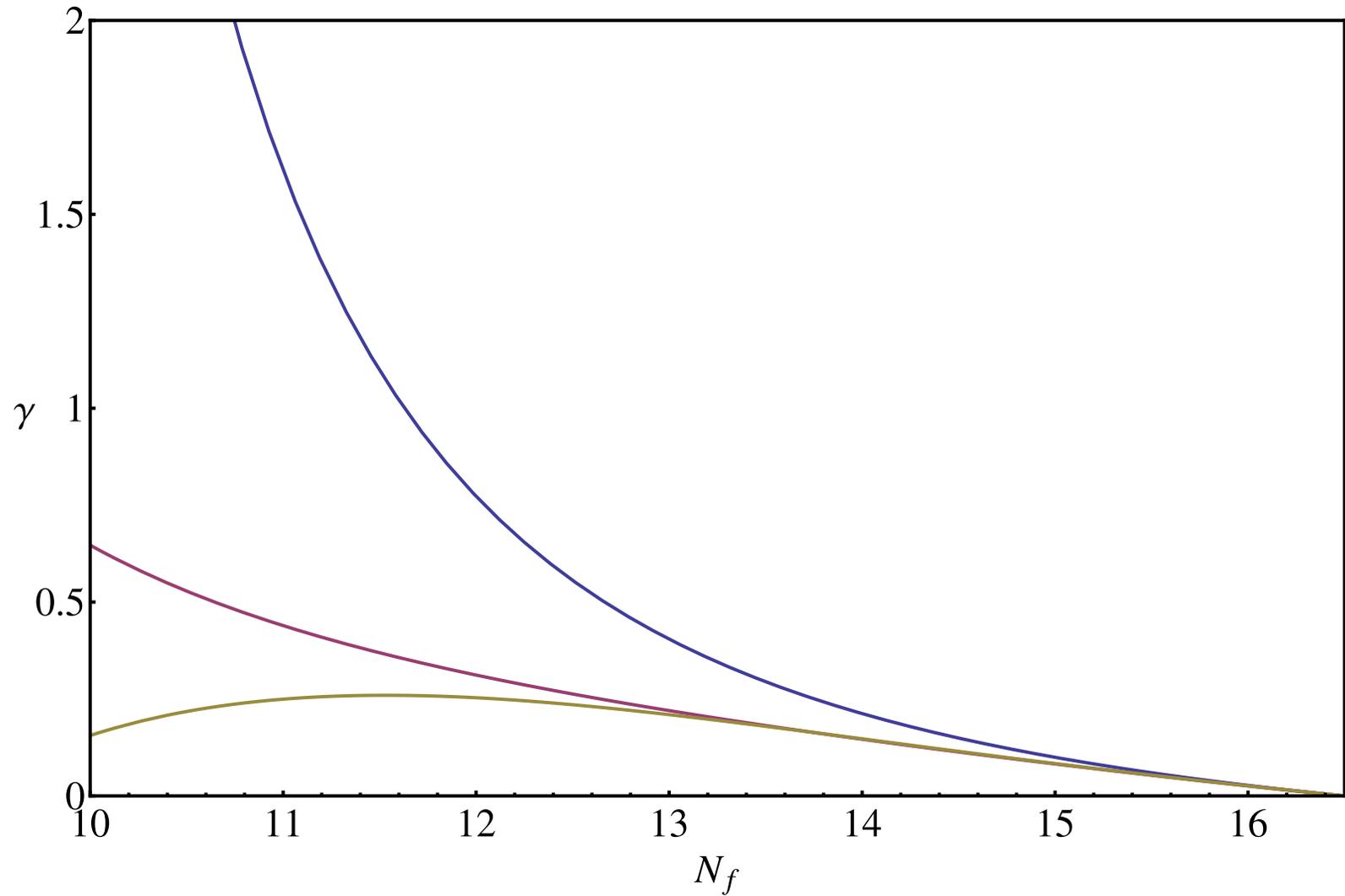


Figure 2: Anomalous dimension  $\gamma$  for SU(3) for  $\mathbf{N}_f$  fermions in the fundamental representation; (i) blue: 2-loop; (ii) red: 3-loop; (iii) brown: 4-loop calculation ( $\mathbf{N}_{f,max} = \mathbf{16.5}$ ).

A necessary condition for a perturbative calculation to be reliable: higher-order contributions do not modify the result very much. Our results show that for a given  $N$  and  $N_f$ , there is a substantial decrease in  $\alpha_{IR}$  and  $\gamma$  when one goes from 2-loop to 3-loop order, but for a reasonable range of  $N_f$ , the 3-loop and 4-loop results are close to each other all the way down to the  $\beta$ DS estimate of  $N_{f,cr}$ .

Thus, these higher-loop calculations of  $\alpha_{IR}$  and  $\gamma$  allow one to probe the theory more reliably down to smaller values of  $N_f$  nearer to estimated  $N_{f,cr}$ . With the increase in  $\alpha_{IR}$  as  $N_f$  decreases, perturbative calcs. of  $\alpha_{IR}$  and  $\gamma$  eventually get less reliable. (Values of  $\gamma$  in parentheses are unphysically large.)

In phase with confinement and  $S\chi SB$ ,  $\alpha_{IR}$  is only an approximate IRFP and  $\gamma$  is only an effective quantity describing the theory at scales  $\mu$  where  $\alpha$  is near to  $\alpha_{IR}$ . In the conformal phase, an IRFP is exact (although our perturbative calculation of it is only approximate), and  $\gamma$  describes the scaling of the bilinear  $\bar{F}F$  at this IRFP.

Lattice gauge simulations provide a fully nonperturbative determination of  $N_{f,cr}$  and  $\gamma$ , motivating intensive lattice studies. Although it is difficult to determine the precise value of  $N_{f,cr}$  for a given  $SU(N)$  and rep.  $R$ , these studies have shown definite walking behavior for  $N_f$  near to the  $\beta$ DS estimate of  $N_{f,cr}$ .

Some examples of comparison with lattice measurements:

For SU(3) with  $N_f = 12$ , from the table above,

$$\gamma_{IR,2\ell} = 0.77, \quad \gamma_{IR,3\ell} = 0.31, \quad \gamma_{IR,4\ell} = 0.25$$

Lattice results:

$\gamma = 0.414 \pm 0.016$  (Appelquist, Fleming, Lin, Neil, Schaich, PRD 84, 054501 (2011), arXiv:1106.2148, analyzing data of Kuti et al., PLB 703, 348 (2011), arXiv:1104.3124, inferring consistency with conformality)

$\gamma \sim 0.35$  (DeGrand, arXiv:1109.1237, also analyzing Kuti et al. data ).

So here the 2-loop value is slightly larger than, and the 3-loop and 4-loop values closer to, these lattice measurements.

Thus, our higher-loop calculations improve the agreement with these lattice computations. Lattice measurements are making good progress, with many talks here.

We have also carried out these higher-loop calculations for fermions in larger representations. For fermions in the adjoint representation,  $N_f \leq 2$  to maintain asymptotic freedom. For  $N_f = 2$  we find

$N$	$\alpha_{IR,2l,adj}$	$\alpha_{IR,3l,adj}$	$\alpha_{IR,4l,adj}$
2	0.628	0.459	0.493
3	0.419	0.306	0.323

$N$	$\gamma_{2l,adj}(\alpha_{IR,2l,adj})$	$\gamma_{3l,adj}(\alpha_{IR,3l,adj})$	$\gamma_{4l,adj}(\alpha_{IR,4l,adj})$
2	0.820	0.543	0.571
3	0.820	0.543	0.561

For SU(2) with  $N_f = 2$  fermions in the adjoint rep., lattice results include (caution: various groups quote uncertainties differently):

$$\gamma = 0.49 \pm 0.13 \quad (\text{Catterall, Del Debbio et al., arXiv:1010.5909, PoS(Lat2010) 057})$$

$$\gamma = 0.31 \pm 0.06 \quad (\text{DeGrand, Shamir, Svetitsky, PRD 83, 074507 (2011)})$$

$$\gamma = 0.17 \pm 0.05 \quad (\text{Appelquist et al., PRD 84, 054501 (2011), arXiv:1106.2148})$$

$$-0.6 < \gamma < 0.6 \quad (\text{Catterall, Del Debbio, et al., arXiv:1108.3794})$$

It is of interest to carry out a similar analysis in an asymptotically free  $\mathcal{N} = 1$  supersymmetric gauge theory with vectorial chiral superfield content  $\Phi, \tilde{\Phi}$  in the  $R, \bar{R}$  reps. for various  $R$ , since here  $N_{f,cr}$  is known (Seiberg for  $F = R$ ; Rytov and Sannino for higher  $R$ ).

We have done this for an  $SU(N)$  gauge theory in Rytov and RS, arXiv:1202.1297. We find that, as in the non-susy theory, for a given  $N, N_f$ , and rep.  $R$ , the IR zero of  $\beta$  decreases when one goes from the 2-loop to the 3-loop level; e.g., for  $N = 2, 3$ :

$N$	$N_f$	$\alpha_{IR,2\ell}$	$\alpha_{IR,3\ell}$
2	4	6.28	2.65
2	5	1.14	0.898
3	5	18.85	3.05
3	6	2.69	1.40
3	7	0.992	0.734
3	8	0.343	0.308

(where the entries with excessively large values of  $\alpha$  are not reliable).

In the susy case, there is a bound  $\gamma \leq 1$  from unitarity for a theory in the IR conformal phase. Insofar as perturbative calculations are reliable, they indicate that  $\gamma$  increases (from 0) as  $N_f$  decreases from  $N_{f,max}$ . So we get a perturbative estimate for  $N_{f,cr}$  by setting the perturbatively calculated  $\gamma = 1$  and solving for  $N_f$ .

For example, for  $R = F$ , fundamental rep.,

$$N_{f,max} = 3N, \quad N_{f,cr} = \frac{N_{f,max}}{2} = \frac{3N}{2}$$

Our perturbative estimates are approx. 1.3 to 1.4 times larger than exact result. Similar results for higher-dim. reps.

So this comparison suggests that, in this susy case at least, perturbative results slightly overestimate the value of  $N_{f,cr}$  compared with the exact results, i.e., slightly underestimate the size of the IR-conformal phase.

We are continuing our studies of higher-loop and nonperturbative effects on  $\alpha_{IR}$  and  $\gamma$ .

## Some Constraints on TC/ETC Models

Early studies of ETC considered the TC theory as an effective low-energy theory and added various plausible four-fermion operators linking SM fermions and technifermions.

Part of our work has focused on constructing reasonably UV-complete ETC models that predict the forms and coefficients of the four-fermion operators in the effective low-energy technicolor theory.

Typically, ETC is arranged to be an asymptotically free chiral gauge theory, and includes a set of SM-singlet, ETC-nonsinglet fermions chosen so that as the scale decreases from the deep UV, the ETC gauge coupling becomes large enough to produce condensates of these SM-singlet fermions, which break the ETC gauge symmetry.

Since this involves strongly coupled gauge interactions, it is not precisely calculable, but the pattern of condensate formation can be plausibly determined by the most attractive channel (MAC) criterion. Some studies include Appelquist and Terning, PRD 50, 2116 (1994); Appelquist and RS, PLB 548, 204 (2002); PRL 90, 201801 (2003); Appelquist, Piai, RS, PRD 69, 015002 (2004); Christensen and RS, PRD 74, 015004 (2006); Rytov and RS PRD 81, 115013 (2010); Rytov and RS, PRD 84, 056009 (2011).

To account for the three generations of SM fermion masses, there is a sequential breaking of the ETC gauge symmetry, at the three scales  $\Lambda_i$ ,  $i = 1, 2, 3$ . Although the full ETC theory is chiral, we focus here on ETC models with vectorial couplings to quarks and charged leptons, denoted VSM ETC models.

At the highest scale,  $\Lambda_1$ ,  $G_{ETC}$  breaks to  $H_{ETC}$ , and the gauge bosons in the coset  $G_{ETC}/H_{ETC}$  gain masses  $\sim g_{ETC}\Lambda_1 \sim \Lambda_1$ , and so forth for the breakings at the two lower scales  $\Lambda_2$  and  $\Lambda_3$ .

Studies of reasonably UV-complete models showed how not just diagonal, but also off-diagonal, elements of SM fermion mass matrices could be produced, via nondiagonal propagator corrections to ETC gauge bosons,  $V_\tau^i \rightarrow V_\tau^j$ , where  $i, j$  are generation indices and  $\tau$  is a TC index (Appelquist, Piai, RS, PRD 69, 015002 (2004)).

A feature that was found in these studies of reasonably UV-complete ETC models was the presence of approximate residual generational symmetries that naturally suppress these ETC gauge boson propagator corrections and hence also off-diagonal elements of SM fermion mass matrices.

Further, a possible mechanism to account for the very small neutrino masses was presented. This made use of suppressed Dirac and Majorana neutrino masses leading to a low-scale seesaw (Appelquist and RS, PLB 548, 204 (2002); PRL 90, 201801 (2003)).

TC/ETC theories are constrained by FCNC processes. These can be suppressed by making the ETC breaking scales  $\Lambda_i$  sufficiently large, but this is restricted by the requirement that one not cause excessive suppression of SM fermion masses.

One insight from studies of reasonably UV-complete ETC models was that the approximate residual generational symmetries suppress the FCNC effects.

For example, consider  $K^0 - \bar{K}^0$  mixing and resultant  $K_L - K_S$  mass difference  $\Delta m_{K_L K_S}$ . SM contribution consistent with experimental value  $\Delta m_{K_L K_S}/m_K \simeq 0.7 \times 10^{-14}$ .

Simple effective Lagrangian used in early studies without a UV-complete ETC theory:  $\mathcal{L}_{eff} = c[s\gamma_\mu d]^2$  with coefficient  $c \sim 1/\Lambda_{ETC}^2$ , usually with just a single generic ETC scale.

Now in terms of ETC eigenstates, an  $s\bar{d}$  in a  $\bar{K}^0$  produces a  $V_1^2$  ETC gauge boson, but this cannot directly yield a  $d\bar{s}$  in the final-state  $K^0$ ; the latter is produced by a  $V_2^1$ . So this requires either the ETC gauge boson mixing  $V_1^2 \rightarrow V_2^1$  or the related mixing of ETC quark eigenstates to produce mass eigenstates.

The ETC gauge boson propagator insertion  $\frac{1}{2}\Pi_1^2$  required for this breaks the generational symmetries associated with the  $i = 1$  and  $i = 2$  generations, and hence

$$|{}^1_2\Pi_1^2| \lesssim \Lambda_2^2$$

Therefore, the contribution to  $\bar{K}^0 \rightarrow K^0$  transition from  $V_1^2 \rightarrow V_2^1$ :

$$|c| \lesssim \frac{1}{\Lambda_1^2} {}^1_2\Pi_1^2 \frac{1}{\Lambda_1^2} \sim \frac{\Lambda_2^2}{\Lambda_1^2} \frac{1}{\Lambda_1^2} \ll \frac{1}{\Lambda_1^2}$$

With above values for  $\Lambda_1$  and  $\Lambda_2$ , the suppression factor is  $(\Lambda_2/\Lambda_1)^2 \simeq 10^{-2}$ . So rather than the naive result  $\Delta m_{K_L K_S}/m_K \sim \Lambda_{QCD}^2/\Lambda_1^2$ , this yields the considerably smaller result

$$\frac{\Delta m_{K_L K_S}}{m_K} \sim \frac{\Lambda_2^2 \Lambda_{QCD}^2}{\Lambda_1^4} \sim 10^{-15}$$

which agrees with experimental limits on new-physics contributions.

Similar comments applies to ETC contributions to a number of other FCNC processes. Some studies of FCNC constraints that take account of these approximate generational symmetries include Appelquist, Piai, RS, PLB 593, 175 (2004); PLB 595, 442 (2004); Appelquist, Christensen, Piai, RS, PRD 70, 093010 (2004). Recent bound  $Br(B_s \rightarrow \mu^+ \mu^-) < 4.5 \times 10^{-9}$  from LHCb is a new constraint. Other phenom. aspects include muon  $g - 2$  constraint, dark matter candidates, etc.

It remains a challenge to construct a TC/ETC model that does everything that is demanded of it, including sufficient suppression of FCNC effects and accounting for realistic quark, charged lepton, and neutrino masses and quark and lepton mixing.

A particular challenge is to get splitting of  $m_t$  and  $m_b$  without excessive contributions to  $\rho$ . One cannot do this via the dynamically generated technifermion masses  $\Sigma_{TC,U} > \Sigma_{TC,D}$  because  $\Sigma_{TC,U} \simeq \Sigma_{TC,D}$ , and anyway, this would violate custodial symmetry too much.

One approach: topcolor assisted TC, i.e. TC2 (Hill, Chivukula + Simmons talks here; Eichten, Lane..). TC2 models get splitting between  $m_t$  and  $m_b$  by using separate, asymp. free SU(3) gauge interactions acting on the third generation of quarks and on the first two generations, denoted SU(3)<sub>1</sub> and SU(3)<sub>2</sub>, respectively. The SU(3)<sub>1</sub> is arranged to become sufficiently strong, at a scale  $\Lambda_t \simeq O(1)$  TeV, to produce a  $\langle \bar{t}t \rangle$  condensate and hence a dynamically generated  $m_t$ .

As a third-generation symmetry, the  $SU(3)_1$  treats the  $t$  and  $b$  quarks in the same way. To avoid producing  $\langle \bar{b}b \rangle \simeq \langle \bar{t}t \rangle$ , and hence  $m_b \simeq m_t$ , one uses an additional set of hypercharge-type interactions,  $U(1)_1 \otimes U(1)_2$ . The  $U(1)_1$  is attractive in the  $\bar{t}t$  channel and repulsive in the  $\bar{b}b$  channel.

The  $SU(3)_1 \otimes SU(3)_2$  and  $U(1)_1 \otimes U(1)_2$  symmetries are assumed to break to their respective diagonal subgroups, color  $SU(3)_c$  and hypercharge  $U(1)_Y$ . A UV completion in which one can show that these breakings plausibly occur is Ryttov and RS, Phys. Rev. D82, 055012 (2010).

The scale  $\Lambda_t$  is fixed in TC2 models by  $m_t$ , and the scale at which  $SU(3)_1 \otimes SU(3)_2$  breaks to  $SU(3)_c$  cannot be larger than this, or else the  $SU(3)_1$  interaction would break before it could produce the desired  $\langle \bar{t}t \rangle$  condensate. This yields an upper bound on the masses of the eight vector bosons in the coset  $SU(3)_1 \otimes SU(3)_2/SU(3)_c$  of order TeV.

The current ATLAS/CMS lower bounds of  $\sim 2.5$  TeV on coloron/axigluons can cause some tension with certain TC2 models.

Constraints from precision electroweak data:  $\Delta\rho = \alpha_{em}(m_Z)T$  and  $S$ , where

$$\frac{\alpha_{em}S}{\sin^2(2\theta_W)} = \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}$$

$S$  is sensitive to heavy fermion loop contributions to  $Z$  propagator.

From experimental data, SM fits obtain allowed regions in  $S$  and  $T$ , depending on an assumed value of  $m_H$  mass; in general,  $S \lesssim 0.2$  (90 % CL).

Naive perturbative estimate (which is not applicable, since TC is nonperturbative at scale  $m_Z$ ):

$$(\Delta S)_{TC,pert.} \simeq \frac{\dim(R_{TC}) N_D}{6\pi}$$

where  $\dim(R_{TC})$  is the dimension of the TC fermion rep., e.g.,  $\dim(R_{TC}) = N_{TC}$  for fundamental. If TC were QCD-like, nonperturbative effects would yield  $(\Delta S)_{TC} \simeq 2(\Delta S)_{TC,pert.}$  (Peskin-Takeuchi, 1990), which, together with too-small SM fermion masses, showed that TC could not be a scaled-up QCD-like theory.

A viable TC model must have a reduction in  $(\Delta S)_{TC}$  wrt. its QCD-like value. This motivates building TC models with the minimal content of  $SU(2)_L$ -nonsinglet technifermions.

Studies of Dyson-Schwinger equations have shown that  $(\Delta S)_{TC}$  (per EW doublet) is somewhat reduced in walking TC as compared with its QCD-like value: Harada, Kurachi and Yamawaki, Prog. Theor. Phys. 115, 765 (2006); Kurachi and RS, Phys. Rev. D 74, 056003 (2006); see also Sannino, Phys. Rev. D82, 081701 (2010).

It is also useful to calculate  $S$  taking into account not just TC, but also ETC effects (Kurachi, RS, Yamawaki, Phys. Rev. D 76, 035003 (2007)). We found that ETC effects do not increase  $S$  significantly wrt. the value for the pure TC theory.

Lattice simulations (Appelquist et al., (LSD Collab.), with  $SU(3)$  with  $N_f = 6$ , fund. rep., have also found a reduction in  $(\Delta S)_{TC}$  (per EW doublet) wrt. its QCD-like value (PRL 106, 231601 (2011)).

In general, the constraint from the  $S$  parameter remains an important one for TC/ETC theories. The  $T$  constraint is easier to satisfy, since  $\Sigma_{TC,U} \simeq \Sigma_{TC,D}$ .

# Collider signals for TC/ETC theories and constraints from early LHC data

Although TC/ETC theories have been constrained indirectly from flavor physics, precision electroweak quantities, and Tevatron searches, key tests are now forthcoming with LHC data.

Some collider signals for TC depend on the type of model. A general signature that applies to all technicolor models results from the property that the technihadrons include a technivector mesons, in particular, techni- $\rho$ , denoted  $\rho_{TC}$ . In QCD the  $\rho$  couples strongly to  $\pi\pi$  and decays to  $\pi\pi$  with a large width, so also in technicolor.

In TC, the technipions are absorbed to become the longitudinal components of the  $W^\pm$  and  $Z$ . Hence at sufficiently high energy the scattering of longitudinally polarized  $W$  and  $Z$ 's will be enhanced by resonant  $s$ -channel contributions:

$$W_L^+ W_L^- \rightarrow \rho_{TC}^0 \rightarrow W_L^+ W_L^-$$

$$W_L^+ Z_L \rightarrow \rho_{TC}^+ \rightarrow W_L^+ Z_L$$

The  $\rho_{TC}$  mass,  $m_{\rho_{TC}}$  mass can be roughly estimated from

$$\frac{m_{\rho_{TC}}}{m_{\rho}} \simeq \frac{\Lambda_{TC}}{\Lambda_{QCD}} \simeq \frac{f_{TC}}{f_{\pi}} \left( \frac{N_c}{N_{TC}} \right)^{1/2}$$

where  $f_{TC} \simeq 250$  GeV for a one-doublet TC theory. With  $f_{\pi} = 93$  MeV and  $m_{\rho} = 775$  MeV, this yields

$$m_{\rho_{TC}} \simeq (2.0 \text{ TeV}) \left( \frac{N_c}{N_{TC}} \right)^{1/2}$$

Studies of meson masses in WTC (Kurachi and RS, JHEP 12, 034 (2006)) obtained an approx. 30 % increase in  $m_{\rho_{TC}}/m_{\rho}$  relative to this QCD-like estimate, suggesting that

$$m_{\rho_{TC}} \simeq (2.6 \text{ TeV}) \left( \frac{N_c}{N_{TC}} \right)^{1/2}$$

By analogy with  $\rho \rightarrow \pi\pi$  in QCD, the  $\rho_{TC}$  would decay as  $\rho_{TC}^0 \rightarrow W^+W^-$  and  $\rho_{TC}^{\pm} \rightarrow W^{\pm}Z$ . For the width of such a technihadron, a rough estimate is

$$\frac{\Gamma_{\rho_{TC}}}{\Gamma_{\rho}} \sim \frac{\Lambda_{TC}}{\Lambda_{QCD}}$$

so, with  $\Gamma_\rho \simeq 150$  MeV, one has  $\Gamma_{\rho_{TC}} \sim 250$  GeV. Similar for other technihadrons.

LHC can search for this resonant behavior, but this will require substantially more integrated luminosity than the present  $\int \mathcal{L} dt = 5 \text{ fb}^{-1}$  per experiment. Many studies of this over the years agree that a clear observation of this resonant behavior may require  $\int \mathcal{L} dt \sim 50 - 100 \text{ fb}^{-1}$  at  $\sqrt{s} = 14$  TeV.

For an example of how current LHC data are useful in constraining technicolor models, consider the one-family TC model. This is already in some tension with precision electroweak constraints, since it yields  $(\Delta S)_{TC,pert.} = N_{TC} N_D / (6\pi)$ . Now  $N_D = N_c + 1 = 4$  in this one-family model, so even if one takes the minimum value,  $N_{TC} = 2$ , this is  $(\Delta S)_{TC,pert.} = 4 / (3\pi) = 0.4$ .

The one-family TC model makes two predictions for techni-hadrons that are tested with current LHC data.

The first is a large number of pseudo-NGB's (PNGB's). If one neglects ETC effects and uses the fact that the SM gauge interactions are weak at the EW scale, then a generic 1FTC model has an  $SU(8)_L \times SU(8)_R$  global chiral symmetry (where  $8 = N_w(N_c + 1)$ ). The technifermion condensates break this to  $SU(8)_V$ , yielding 63 (P)NGB's, of which 3 NGB's are eaten. The PNGB's gain masses from color and ETC

interactions that break the above global chiral symmetry, but some of them include color-nonsinglet states and could have masses of order several 100 GeV. There is no evidence for these at the LHC.

In particular, the one-family TC model predicts color-octet  $\bar{Q}_a (T_\alpha)_b^a Q^b$  pseudoscalar and vector states, where  $Q$  are techniquarks and  $T_\alpha, \alpha = 1, \dots, 8$  are  $SU(3)_c$  generators. The mass of the color-octet  $\rho_{TC}^{(8)}$  can be estimated as above, with  $f_{TC} \simeq 125$  GeV, yielding  $m_{\rho_{TC}^{(8)}} \simeq 1.3 \sqrt{N_c/N_{TC}}$  TeV. Walking might raise this mass slightly, as noted above.

CMS and ATLAS have set lower bounds on color-octet resonances of 2.5 TeV. Although the mass estimates for TC theories have significant uncertainties owing to the strongly coupled nature of the TC physics, this causes tension with the one-family TC model.

Minimal technicolor models with only color-singlet technifermions are consistent with these LHC data. They have no color-nonsinglet technihadrons, and they have only one  $SU(2)_L$  doublet of left-handed technifermions (with corresponding  $SU(2)_L$ -singlet right-handed technifermions), so the three SM-nonsinglet NGB's are all eaten by the  $W^\pm$  and  $Z$  and there are no residual SM-nonsinglet (P)NGB's. With their minimal SM-nonsinglet technifermion content, they may also yield an acceptably small  $S$ .

## Some Further Model-Building Results

LHC results thus motivate further study of TC (1DTC) models with minimal technifermion content, consisting of one color-singlet  $SU(2)_L$  doublet with corresponding right-handed components. This is also desirable to minimize TC contributions to  $S$ .

A recent study of a model with this minimal content and technifermions in the fundamental rep. of the TC group is Rytov and RS, PRD 84, 056009 (2011) with

$$F_L^\tau = \begin{pmatrix} F_1^\tau \\ F_2^\tau \end{pmatrix}_L \text{ with hypercharge } Y_{F_L}$$
$$F_{1R}^\tau, \quad F_{2R}^\tau, \quad Y_{f_{iR}}, \quad i = 1, 2$$

Electric charge is vectorial, so  $Y_{F_{1R}} = Y_{F_L} + 1$  and  $Y_{F_{2R}} = Y_{F_L} - 1$ .

In order to get approx. IR zero of  $\beta$  and walking behavior, we add  $N_{f,cr} - 2$  additional SM-singlet technifermions. These make no contribution to  $S$  or  $T$ .

If one took the simplest choice,  $G_{TC} = \text{SU}(2)$  and  $Y_{F_L} = 0$ , the EW theory would be free of gauge anomalies, but the most attractive channel would lead to the Majorana condensates ( $\alpha, \beta$  are  $\text{SU}(2)_L$  indices)

$$\langle \epsilon_{\alpha\beta} \epsilon_{\tau\tau'} F_L^{\alpha\tau}{}^T C F_L^{\beta\tau'} \rangle, \quad \langle \epsilon_{\tau\tau'} F_{1R}^{\tau}{}^T C F_{2R}^{\tau'} \rangle,$$

which would not break EW symmetry.

So consider the next step up,  $G_{TC} = \text{SU}(3)$ . By itself, this theory would have an odd number of  $\text{SU}(2)_L$  doublets, global  $\text{SU}(2)$  anomaly. Thus, add another  $\text{SU}(2)_L$  doublet (which is singlet under color and TC) to avoid this:

$$\psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L, \quad \psi_{1R}, \quad \psi_{2R}$$

One requires  $N_{TC} Y_{F_L} + Y_{\psi_L} = 0$ ; a reasonable choice is  $Y_{F_L} = 1/3$  and  $Y_{\psi} = -1$ . ETC gives heavy masses to these leptons.

One can get  $S_{TC,pert.} \lesssim 0.2$  here.

A general feature is that the embedding of TC in ETC is more complicated in TC models with color-singlet technifermions, because now the ETC gauge bosons carry color and charge, in contrast, in 1FTC models, where the ETC gauge bosons are SM-singlets.

Further, ETC symmetry breaking is more complicated than in 1FTC models.

Also studies of models with color-singlet technifermions in higher-dim. reps. of TC group (Sannino, Dietrich, Ryttov, Tuominen...)

Another question concerns the extent to which one can embed TC, ETC in a theory having higher gauge unification, using dynamical symmetry breaking. This would be desirable in order to explain features not explained by the standard model:

- unification of quarks and leptons
- charge quantization

We have shown how, in principle, this is possible, using an extended strong-EW gauge group  $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$  (Appelquist and RS, Phys. Rev. Lett. 90, 201801 (2003)). We have also studied prospects for possible higher unification of both TC and SM symmetries and explored contrasts between Higgs-type GUT breaking and dynamical GUT breaking; e.g., in Chen, Rytov, and RS, Phys. Rev. D82, 116006 (2010).

# Conclusions

Dynamical EWSB via technicolor is an interesting and well-motivated possibility which will be decisively tested by the ATLAS and CMS experiments at the LHC. This approach has shown

- how a new gauge interaction that becomes strongly coupled on the TeV scale naturally produces EWSB,  $W$  and  $Z$  masses.
- how an associated large but slowly running gauge coupling can result from an approximate IR fixed point, enhancing fermion mass generation; higher-loop calculations give further insight into these quasi-conformal theories.
- how fermion masses and generations could arise, by sequential breaking of ETC symmetry.
- Dynamical EWSB has distinctive experimental signatures that can be probed at the LHC, including resonant scattering of longitudinally polarized  $W$  and  $Z$ , and also a possible light technidilaton.
- Future LHC data should answer the question of the origin of EWSB.