Lattice Strong Dynamics for the LHC

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for the Lattice Strong Dynamics Collaboration

Conformality in Strong Coupling Gauge Theories at LHC and Lattice
Kobayashi–Maskawa Institute, Nagoya University, 20 March 2012

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PRD to appear (2012) [1201.3977]
1204.XXXX

Lattice Strong Dynamics Collaboration

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Performing non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider

Lattice Strong Dynamics projects

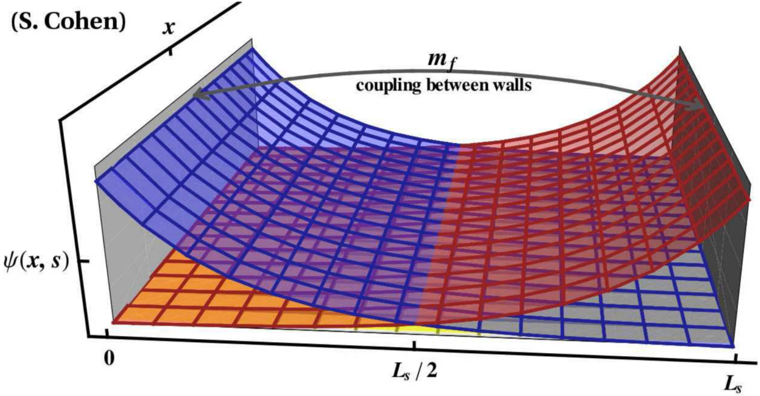
Strategy

- Use lattice QCD as baseline, focus on chirally broken systems
- Explore trends for increasing $N_f = 2 \longrightarrow 6 \longrightarrow 10$
- Attempt to match IR scale(s) for more direct comparison
- Use domain wall fermions for good chiral and flavor symmetries

Results

- Enhancement of chiral condensate for $N_f = 6$ — PRL 104:071601
- Suppression of S parameter for $N_f = 6$ — PRL 106:231601
- Relation between $\pi\pi$ and WW scattering — PRD to appear
  Scattering length decreases for $N_f = 6$
- Can't rule out IR conformality at $N_f = 10$ — **1204.XXXX**
- Running coupling of SU(2) with $N_f = 6$ — PoS Lattice 2011:093

Domain wall fermions



- Form a fifth dimension from L_s copies of the 4d gauge fields
- Exact chiral symmetry at finite lattice spacing in the limit $L_s \to \infty$
- At finite L_s, "residual mass" $m_{res} \ll m_f$; $m = m_f + m_{res}$
- $32^3 \times 64$ with $L_s = 16$: **significant computational expense**

  $m_{res} \approx 2.6 \times 10^{-5}$ [2f]; 82×10^{-5} [6f]; 170×10^{-5} [10f]

100s of millions of core-hours on clusters and supercomputers

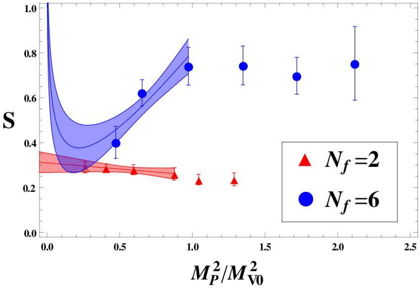
Livermore Nat'l Lab; USQCD (DOE); XSEDE (NSF); etc.

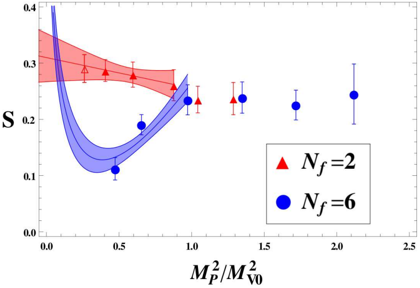
The S parameter

PRL 106:231601

Constraint from vacuum polarizations $\Pi^{\mu\nu}(Q)$ of EW gauge bosons

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$





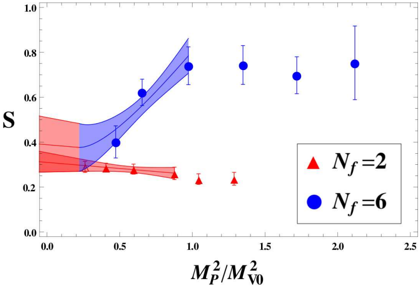
Maximum $N_D = N_f/2$

Minimum $N_D = 1$

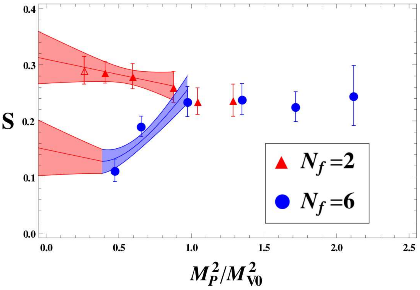
Connecting lattice results to phenomenology

- Lattice calculation involves $N_f^2 - 1$ degenerate pseudoscalars
- Only three would-be NGBs eaten in Higgs mechanism, $N_f^2 - 4$ must be massive PNGBs

Imagine freezing $N_f^2 - 4$ PNGB masses at the blue curve's minimum, and taking only three to zero mass



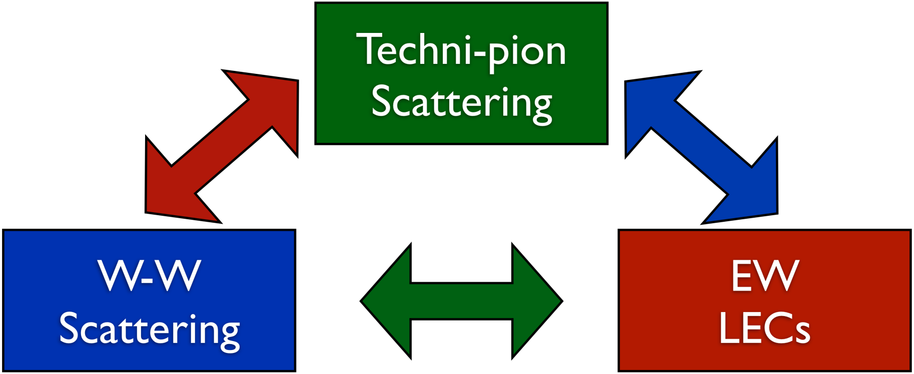
Maximum $N_D = N_f/2$



Minimum $N_D = 1$

WW scattering from the lattice: The Big Picture

WW scattering guaranteed to contain information about EWSB
Most direct probe (though **not** easiest) at LHC
On the lattice, restricted to **low-energy** scattering



(M. Buchoff)

WW scattering from the lattice

PRD to appear

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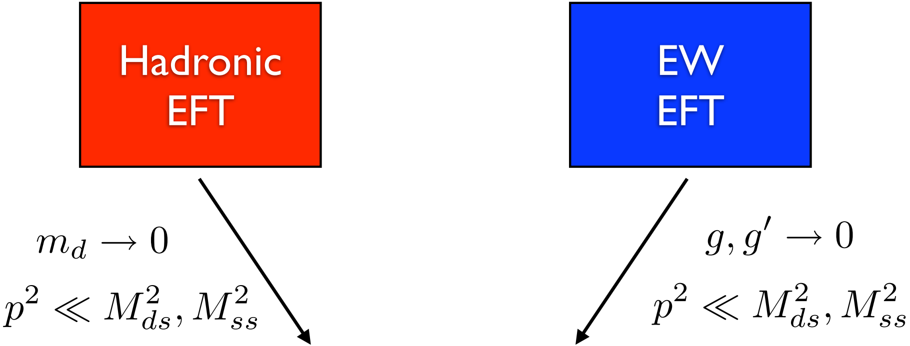
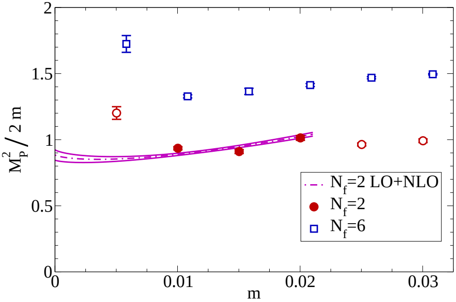
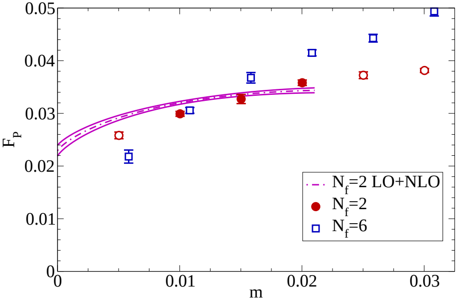
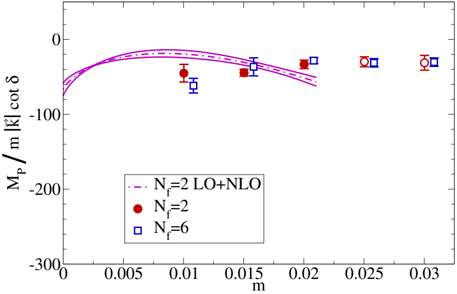


$$\frac{f^2}{4}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \alpha_5 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + \alpha_4 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2$$

Joint chiral fit to $M_P^2/2m$; F_P; $\langle\overline{\psi}\psi\rangle$; and $M_P/m|\vec{k}|\cot\delta$







$a_{PP} \approx 1/|\vec{k}|\cot\delta$ for $|\vec{k}|^2 \ll M_P^2$
$\langle\overline{\psi}\psi\rangle$ very linear, not shown
Only $N_f = 2$ fit feasible
Fit range restricted to
$0.01 \leq m_f \leq 0.02$
(solid points)

$\chi^2/dof = 83/6$

$N_f = 2$ NLO contribution to WW scattering

(As for S parameter)
One-loop SM subtraction removes would-be NGBs from spectrum
and introduces Higgs mass M_H

$$\alpha_4 + \alpha_5 = \left(3.34 \pm 0.17^{+0.08}_{-0.71}\right) \times 10^{-3} - \frac{1}{128\pi^2}\left[\log\left(\frac{M_H^2}{v^2} + \mathcal{O}(1)_{SM}\right)\right]$$

(dominant systematic error from chiral fit range)

Context for our $N_f = 2$ result

Unitarity bounds [hep-ph/0604255]:

$$\alpha_4 + \alpha_5 \geq 1.14 \times 10^{-3} \qquad \alpha_4 \geq 0.65 \times 10^{-3}$$

Expected LHC bounds [hep-ph/0606118]: (99% CL; 100/fb; 14 TeV)

$$-7.7 < \alpha_4 \times 10^3 < 15 \qquad -12 < \alpha_5 \times 10^3 < 10$$

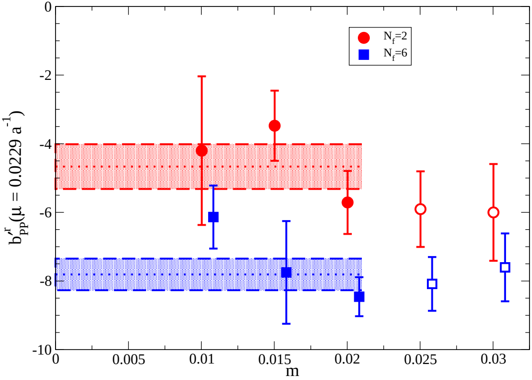
Scattering length decreases for $N_f = 6$

Reorganize a_{PP} chiral expansion in terms of measured M_P and F_P

New NLO low-energy constant is

$$b'_{PP} = -256\pi^2 \left[L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8\right]$$

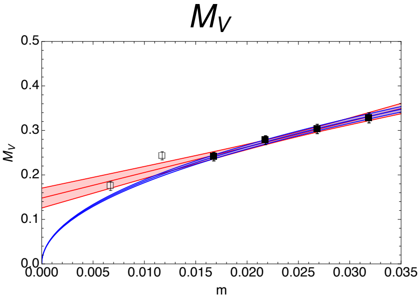
Can't extract $\alpha_4 + \alpha_5$, but can directly compare $N_f = 2$ and $N_f = 6$

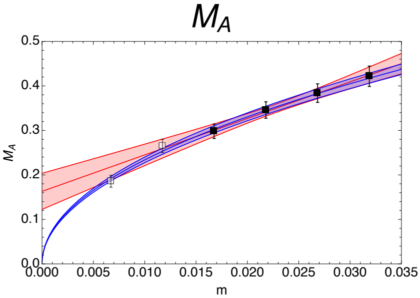


$b'_{PP} = -4.67 \pm 0.65^{+1.08}_{-0.05}$ (2f); $b'_{PP} = -7.81 \pm 0.46^{+1.23}_{-0.56}$ (6f)

Approaching IR-conformality with $N_f = 10$ **1204.XXXX**

Moving on to $N_f = 10$ has been a long-term project
Ordered- and disordered-start runs show long autocorrelations,
statistically significant differences in observables
Combination procedure produces fairly conservative error bars
Conclude $N_f = 10$ close to bottom of conformal window, $\gamma_m \approx 1$

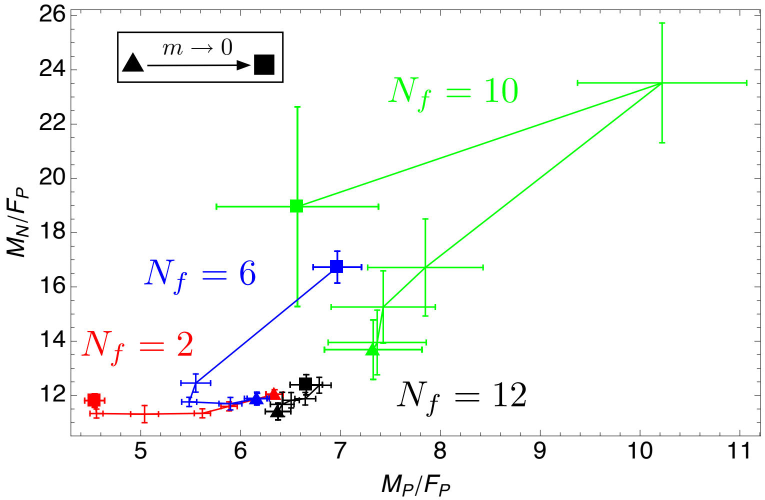




Compare $M = a + bm$ with $M = am^{1/(1+1)}$ ($\gamma_m = 1$)
Fit only to solid points ($m_f \geq 0.015$) due to finite-volume effects...

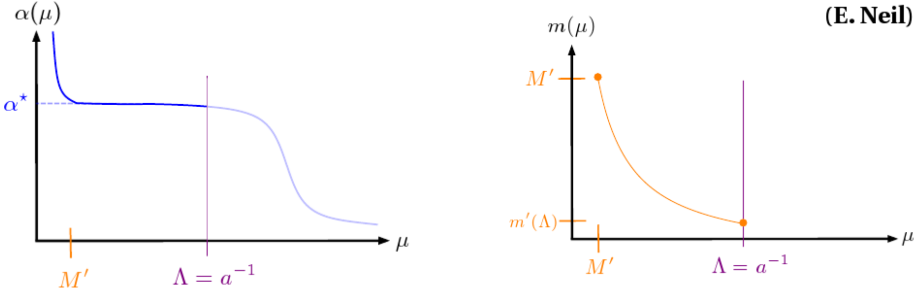
Finite-volume diagnostic: Edinburgh-style plot

Expect finite-volume effects to push points up and to the right



$N_f = 12$ data from Fodor *et al.*, PLB 703:348 (2011) [1104.3124]

Slow running is almost no running



- IR fixed point governs physics up to lattice cutoff $\Lambda = a^{-1}$
- Small fermion mass $m(\Lambda) = m$ at cutoff runs according to $\gamma_\star$
- Fermions screen out around $m(M) = M$, inducing confinement
  All masses and decay constants scale $\sim m^{1/(1+\gamma_\star)}$

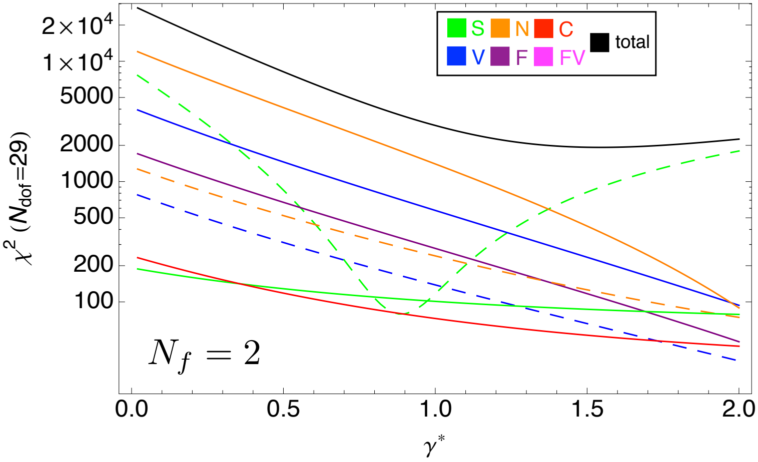
Appelquist *et al.*, PRD 84:054501 (2011) [1106.2148]

Del Debbio and Zwicky, PRD 82:014502 (2010) [1005:2371]

Miransky, PRD 59:105003 (1999) [hep-ph/9812350]

Conformal fit χ^2 vs. γ_m, $N_f = 2$

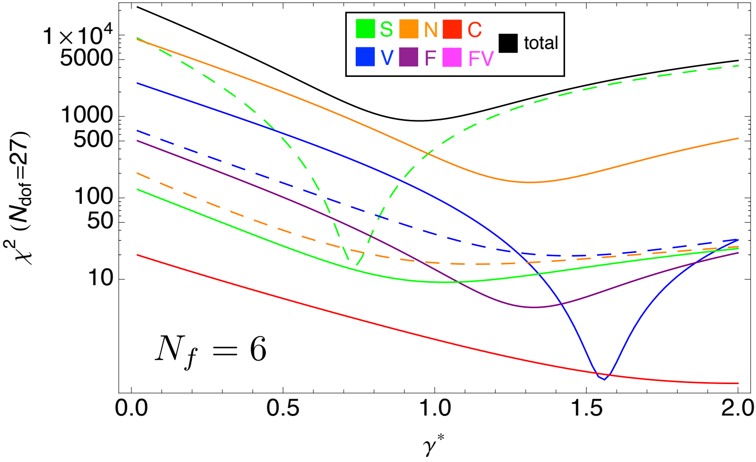
As N_f increases, minima develop and move to smaller γ_m



$N_f = 2$ is QCD; only M_P shows a minimum: $\gamma_m \approx 1 \Longrightarrow M_P \sim m^{1/2}$

Conformal fit χ^2 vs. γ_m, $N_f = 6$

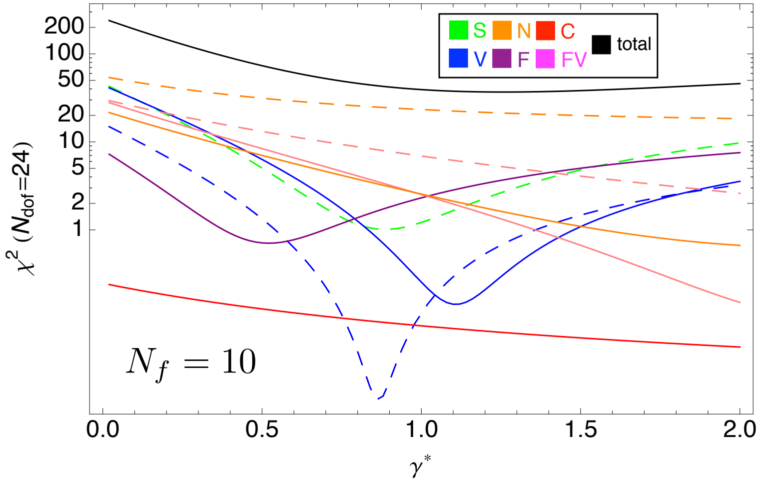
As N_f increases, minima develop and move to smaller γ_m



$N_f = 6$ is QCD-like; minima around $\gamma_m \approx 1.5$ are spurious

Conformal fit χ^2 vs. γ_m, $N_f = 10$

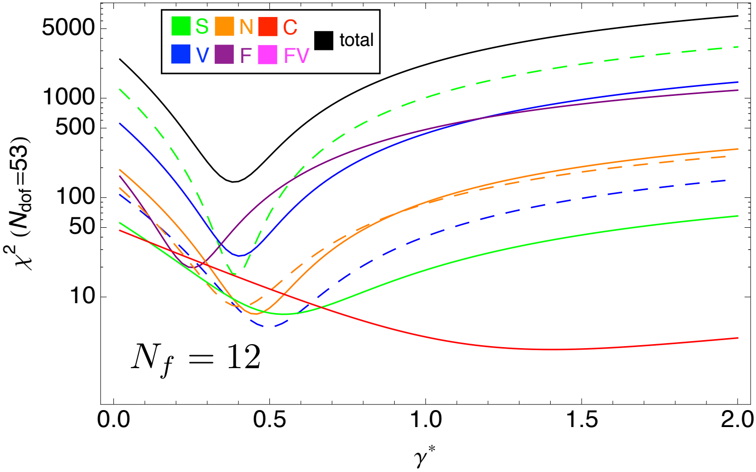
As N_f increases, minima develop and move to smaller γ_m



$m_f \geq 0.015$; Relatively small χ^2 may be due to conservative error bars

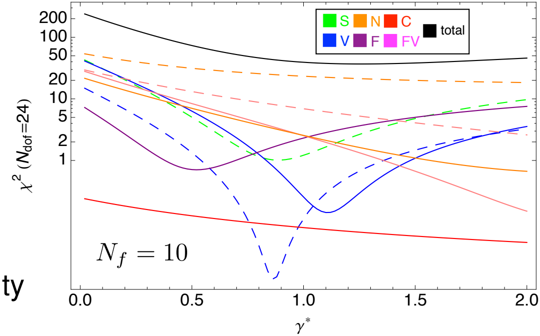
Conformal fit χ^2 vs. γ_m, $N_f = 12$ comparison

As N_f increases, minima develop and move to smaller γ_m



$N_f = 12$ data from Fodor *et al.*, PLB 703:348 (2011) [1104.3124]

$N_f = 10$ fit results



$N_f = 10$ spectrum appears consistent with IR-conformality

Global fit with $m_f \geq 0.015$:	$\gamma_m = 0.999(11)_{\text{stat}}$	$\chi^2/dof = 41/24$
Restricting to $m_f \geq 0.02$:	$\gamma_m = 0.986(14)_{\text{stat}}$	$\chi^2/dof = 27/15$

Compare quality of joint NLO chiral fits to M_P, F_P and $\langle\overline{\psi}\psi\rangle$

$m_f \geq 0.015$: $\chi^2/dof = 176/7$ $m_f \geq 0.02$: $\chi^2/dof = 85/4$

Due to large finite-volume effects, (NLO χPT needs $m \lesssim 0.005$)
cannot rule out spontaneous chiral symmetry breaking for $N_f = 10$

Conclusions and next steps

Results

- Enhancement of chiral condensate for $N_f = 6$ — PRL 104:071601
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- Relation between $\pi\pi$ and WW scattering — PRD to appear
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- Running coupling of SU(2) with $N_f = 6$ — PoS Lattice 2011:093

Future directions (selected highlights)

- Technibaryon form factors for dark matter — 120X.XXXX
- Runs on more volumes to quantify finite-volume effects
  and perform finite-volume scaling analyses
- Stout smearing to avoid strong-coupling lattice artifact transition
- Technipion form factors and D-wave scattering
  to sharpen and extend WW scattering results

Thank you!

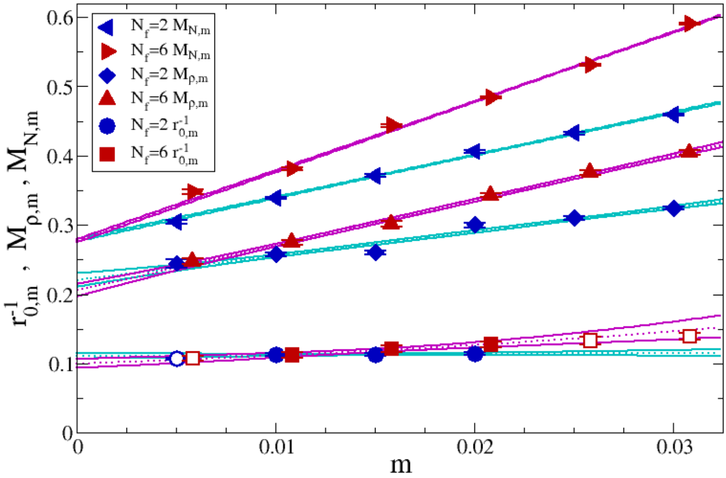
Thank you!

Collaborators

Tom Appelquist, Ron Babich, Rich Brower, Mike Buchoff, Michael Cheng, Mike Clark, Saul Cohen, George Fleming, Joe Kiskis, Meifeng Lin, Heechang Na, Ethan Neil, James Osborn, Claudio Rebbi, Chris Schroeder, Sergey Syritsyn, Pavlos Vranas, Gennady Voronov, Joe Wasem, Oliver Witzel

Funding and computing resources

Backup: matching IR scales in the chiral limit

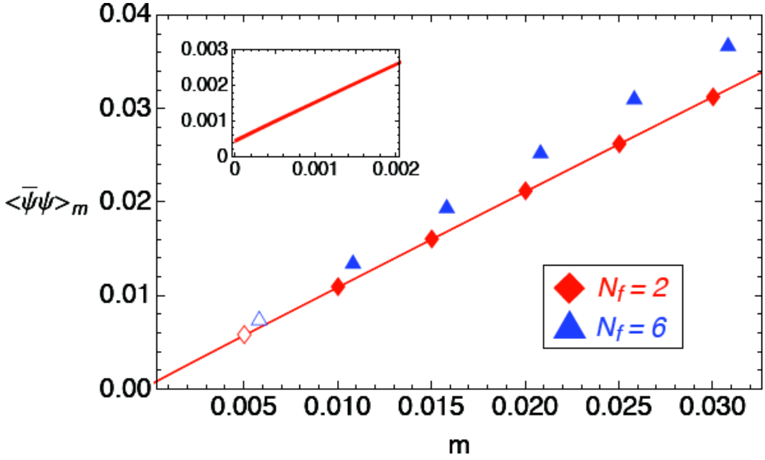


Vector mass, nucleon mass, and inverse Sommer scale

all match at 10% level between $N_f = 2$ and $N_f = 6$

$M_{V0} = 0.215(3)$ [2f]; 0.209(3) [6f]; 0.148(22) [10f, not shown]

Backup: Chiral condensate with chiral fit



Joint NNLOχPT fit to $N_f = 2$ F_P, M_P^2, $\langle\overline{\psi}\psi\rangle$
Linear term clearly dominant

Backup: Calculating S on the lattice

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right)\Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

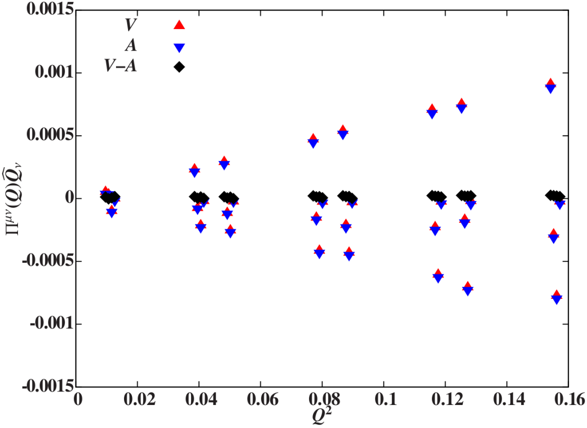
- Renormalization constant $Z = Z_A = Z_V$ for chiral fermions
  Non-perturbatively, $Z = 0.85$ (2f); 0.73 (6f); 0.71 (10f)
- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel...

Backup: Lattice currents and Ward identities

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

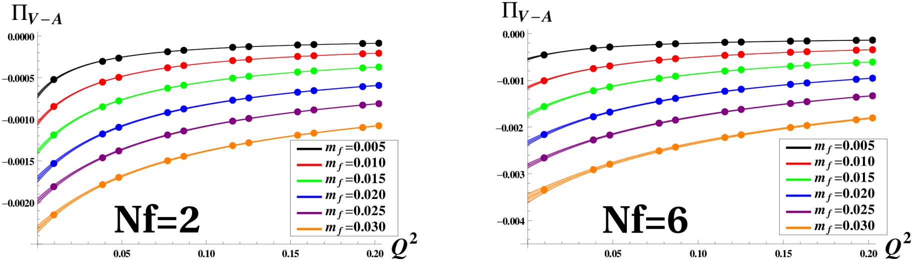
Ward identity violations of mixed correlators cancel in $V\!-\!A$ difference

Save an order of magnitude in computing costs



Backup: rational function fits to $\Pi_{V-A}(Q^2)$

$$S = 4\pi N_D \lim_{Q^2\to 0} \frac{d}{dQ^2}\Pi_{V-A}(Q^2) - \Delta S_{SM}$$



Very smooth data ⇒ fit to "Padé-(1, 2)" functional form

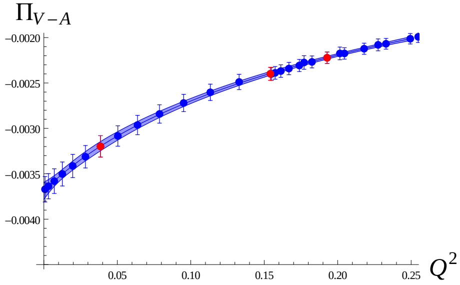
$$\Pi_{V-A}(Q^2) = \frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} = \frac{\sum_{m=0}^{1} a_m Q^{2m}}{\sum_{n=0}^{2} b_n Q^{2n}}$$

(similar to single-pole-dominance approximation)

Results stable and $\chi^2/dof \ll 1$ as Q^2 fit range varies

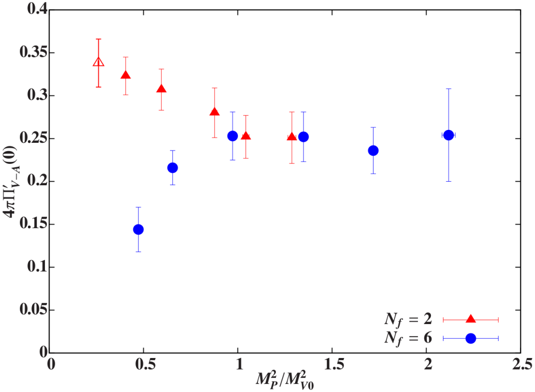
Backup: Twisted boundary conditions for $\Pi_{V-A}(Q^2)$

- Introduce external abelian field
  (equivalent to adding phase at lattice boundaries)
- Allows access to arbitrary Q^2, not just lattice modes $2\pi n/L$



- Correlations $\Longrightarrow$ TwBCs do not improve Padé fit results for slope
- May make it easier to apply chiral perturbation theory
- May help IR-conformal analysis with $m \to 0$ at small $Q^2 > 0$

Backup: Fit results for $\Pi'_{V-A}(0)$



Vertical axis: $4\pi\Pi'_{V-A}(0)$

where

$$\Pi'(0) = \lim_{Q^2 \to 0} \frac{d}{dQ^2}\Pi(Q^2)$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

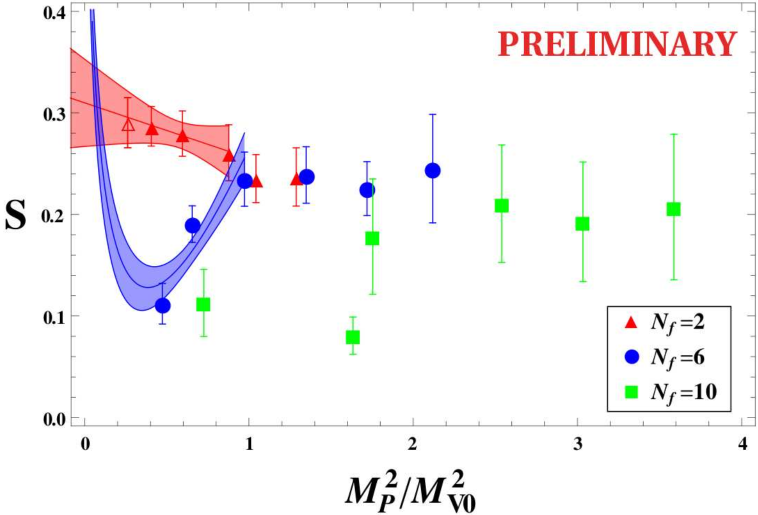
Horizontal axis: M_P^2/M_{V0}^2 gives a more physical comparison than m

$M_{V0} \equiv \lim_{m \to 0} M_V$ is matched between $N_f = 2$ and $N_f = 6$

Expect agreement in the quenched limit $M_P^2 \to \infty$

Backup: 10f results for S parameter

NB: assumes $M_{V0} > 0$



10f finite-volume effects set in for $M_P^2 \approx 1.6 M_{V0}^2$

Expect (and observe) naïve scaling for $M_P^2 > M_{V0}^2$

Backup: Spurious $S \to 0$ from finite-volume effects

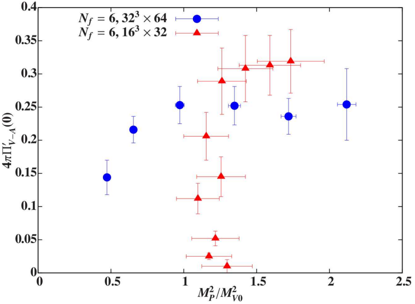
If m too small compared to L, system deconfines

$\Longrightarrow$ chiral symmetry restored, parity doubling

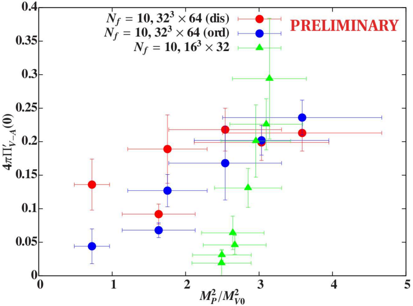
$$4\pi\Pi'_{V-A}(0) = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left[R_V(s) - R_A(s)\right] \longrightarrow 0$$

Also clearly distorts spectrum

$N_f = 6$



$N_f = 10$

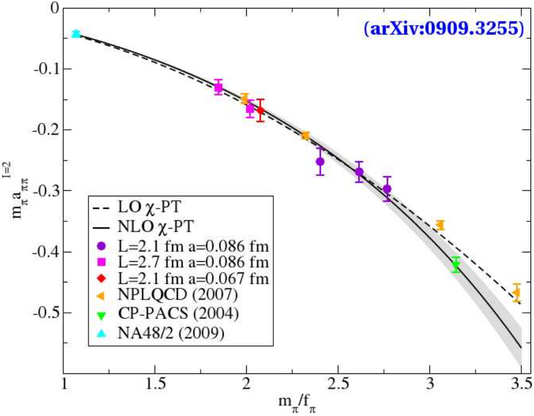


Backup: Reorganized expansion in QCD

Replace low energy constants B and F by measured M_P and F_P

Expansion parameter is M_P^2/F_P^2, leading order is $M_P a_{PP} = -\frac{M_P^2}{16\pi F_P^2}$

(Weinberg, 1966)

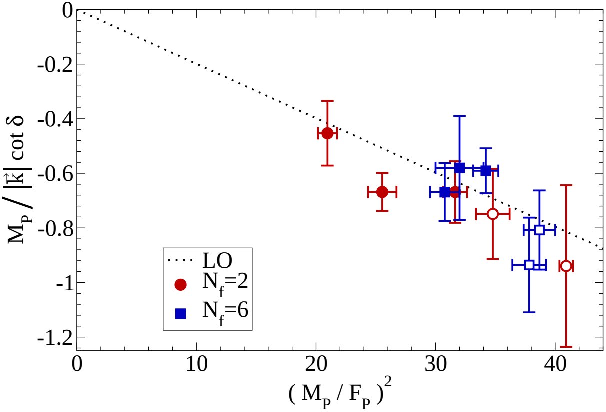


Puzzling persistence of leading-order relation

well beyond expected radius of convergence

Backup: Our results in reorganized expansion

Leading-order relation is straight line for $M_P/(|\vec{k}|\cot\delta)$ vs. M_P^2/F_P^2

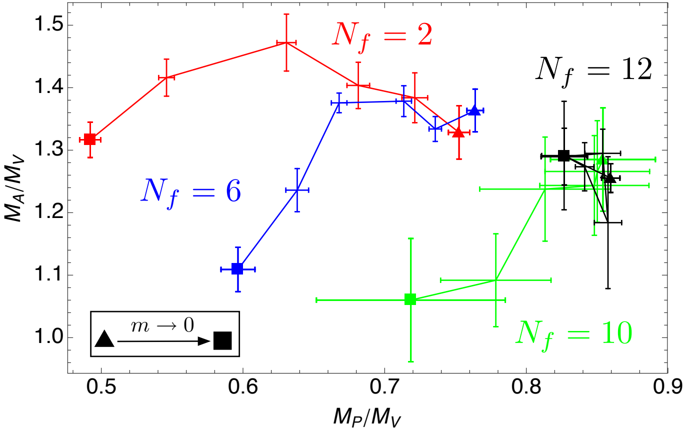


Leading order continues describing data far better than expected

Small upward shift (somewhat less-repulsive scattering)

visible for $N_f = 6$ compared to $N_f = 2$

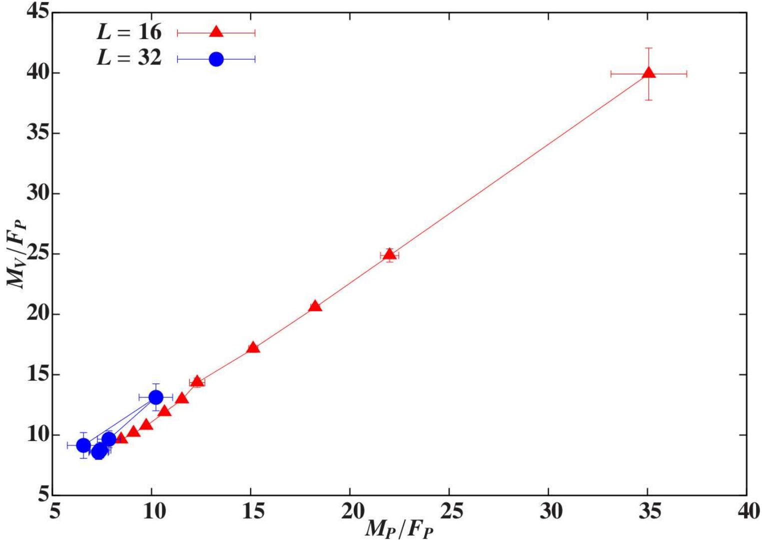
Backup: Edinburgh-style plot for M_A/M_V vs. M_P/M_V



Edinburgh-style plot illustrates (spurious?) parity doubling,
M_P/M_V changing less as N_f increases

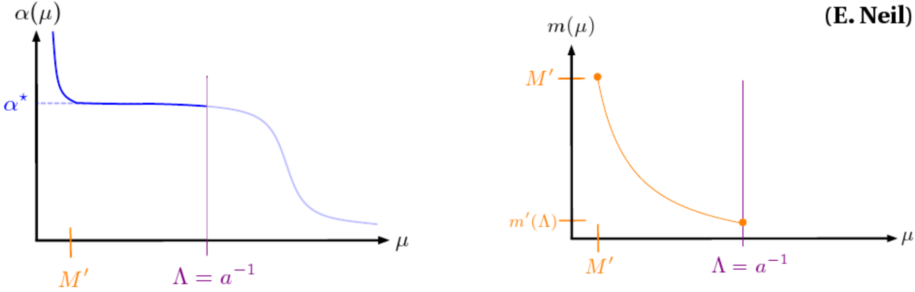
$N_f = 12$ data from Fodor *et al.*, PLB 703:348 (2011) [1104.3124]

Backup: 10f finite-volume effects on $16^3 \times 32$ volumes



Use M_V instead of M_N since latter not (yet) measured on $16^3 \times 32$

Mass-deformed IR-conformal spectrum analysis



- Leading order: $M_X = C_X m^{1/(1+\gamma_\star)}$
- Higher order: $M_X = C_X m^{1/(1+\gamma_\star)} + D_X m$
- Finite volume: $M_X = C_X M \left[1 + \frac{z_X}{ML}\right] + D_X m$
- $\langle\overline{\psi}\psi\rangle = A_C m + B_C m^{[(3-\gamma_\star)/(1+\gamma_\star)]} + C_C m^{[3/(1+\gamma_\star)]} + D_C m^3$

For now, we neglect higher-order and finite-volume corrections

A slowly-running theory will look IR-conformal for *m* too large

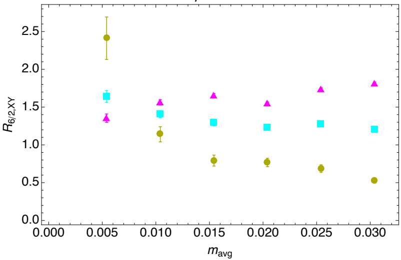
Backup: Condensate enhancement ratios

Three dimensionless ratios all approach $\langle\overline{\psi}\psi\rangle / F_P^3$ in the chiral limit:

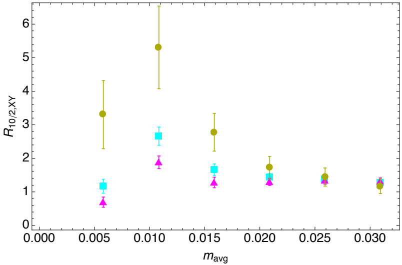
$$X^{(FM)} = \frac{M_P^2}{2mF_P} \qquad X^{(CM)} = \frac{\left(M_P^2/2m\right)^{3/2}}{\langle\overline{\psi}\psi\rangle^{1/2}} \qquad X^{(FM)} = \frac{\langle\overline{\psi}\psi\rangle}{F_P^3}$$

Condensate enhancement from "ratios of ratios": $R_{N_1/N_2}^{(AB)} = \frac{X_{N_f=N_1}^{AB}}{X_{N_f=N_2}^{AB}}$

$N_f = 6/N_f = 2$



$N_f = 10/N_f = 2$



Ordering CM < FM < CF consistent with IR conformality for $\gamma_m \approx 1$

Also consistent with large finite-volume effects