Conformal Window and MWT on the Lattice

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KMI, SCGT12Mini

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Particle Physics & Origin of Mass

University of Southern Denmark

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Outline

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- Conformal Window and β-function at 4-loops

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- Minimal Walking Technicolor on the Lattice

spectrum ◀

β and γ function ◀

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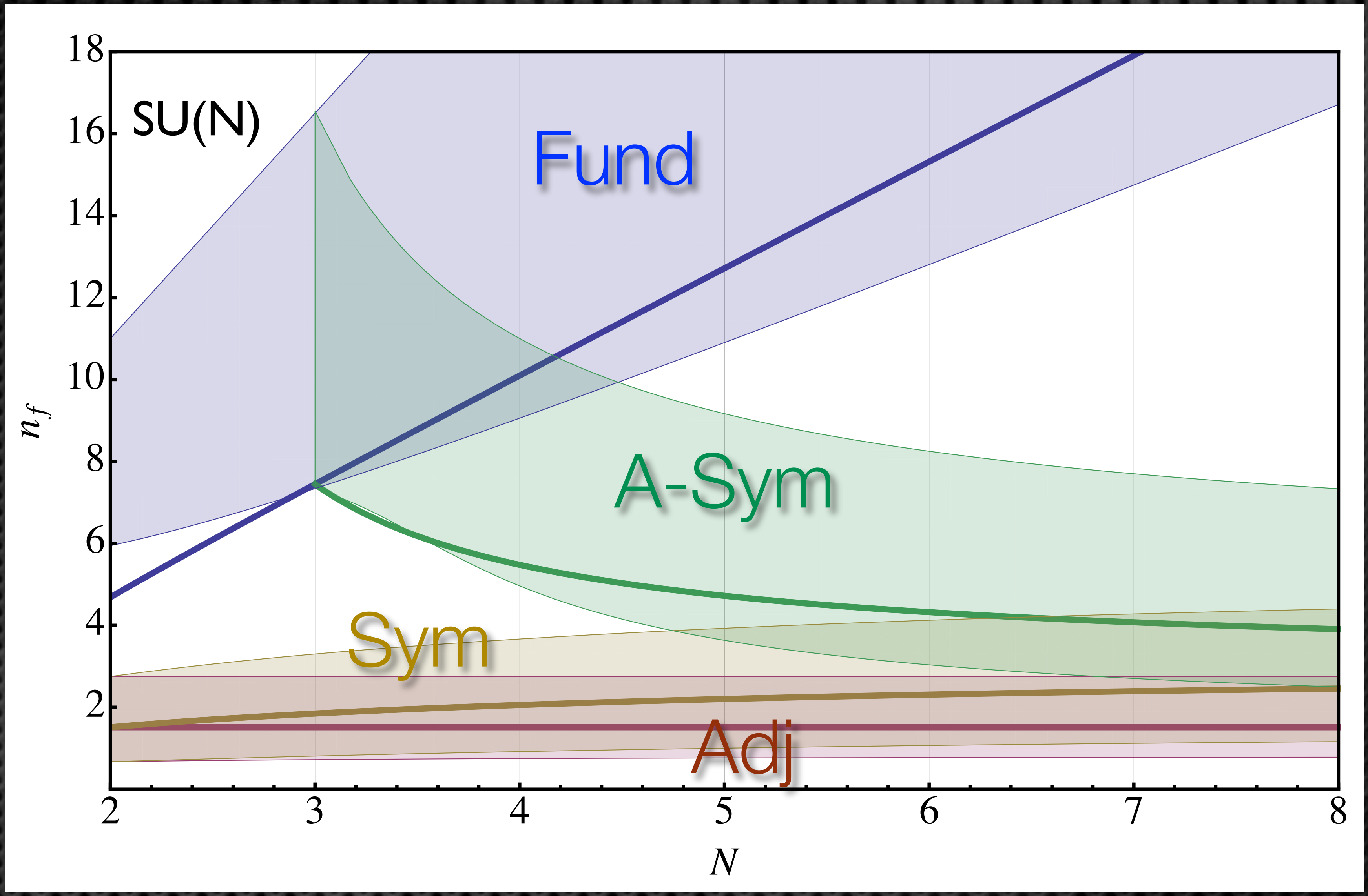
a few more words on the conformal window...

CP, F. Sannino, Phys.Rev. D83, 116001 (2011)
CP, F. Sannino, Phys.Rev. D83, 035013 (2011)

Conformal Window at 4-loops



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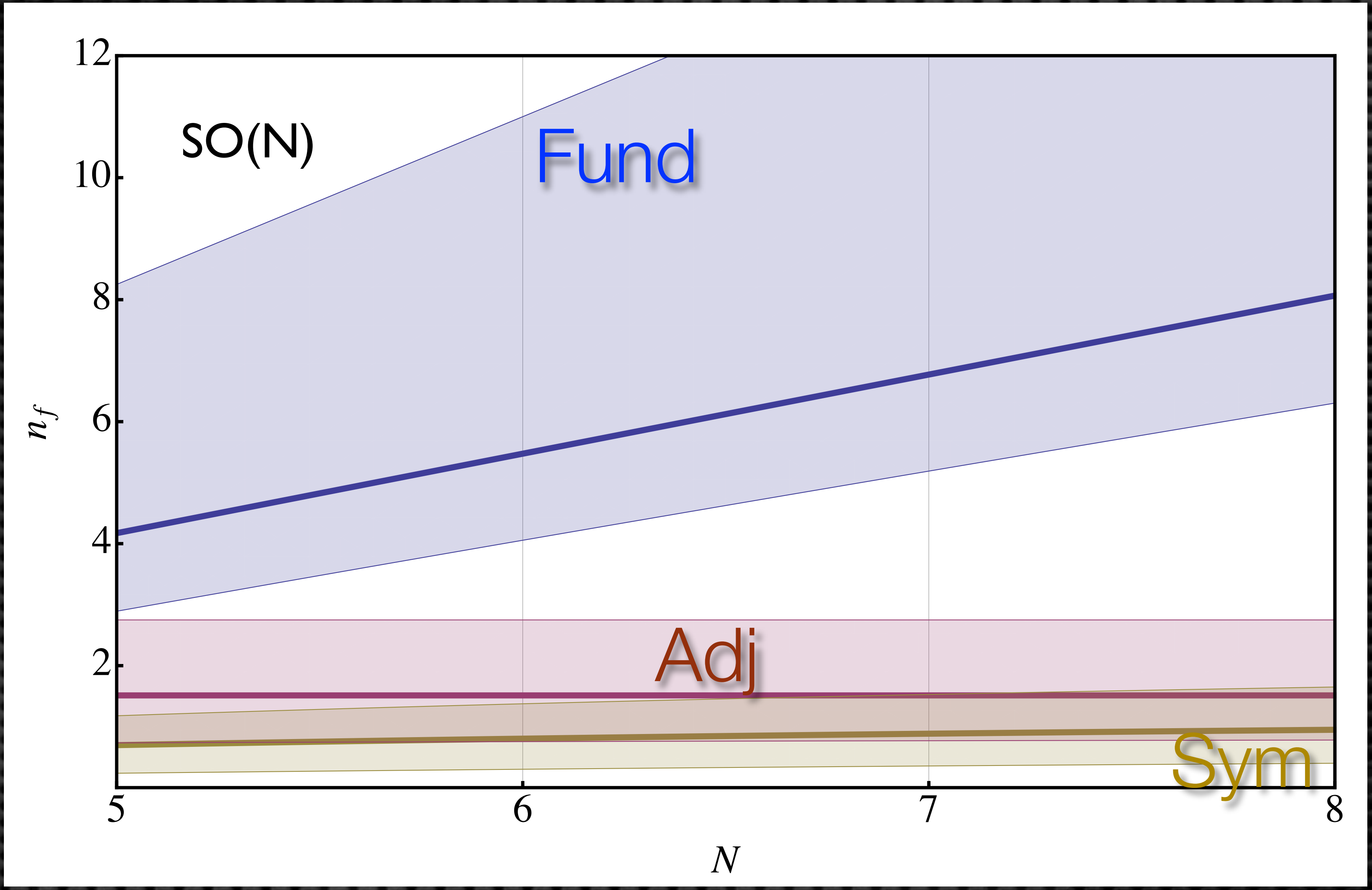
Conformal Window at 4-loops



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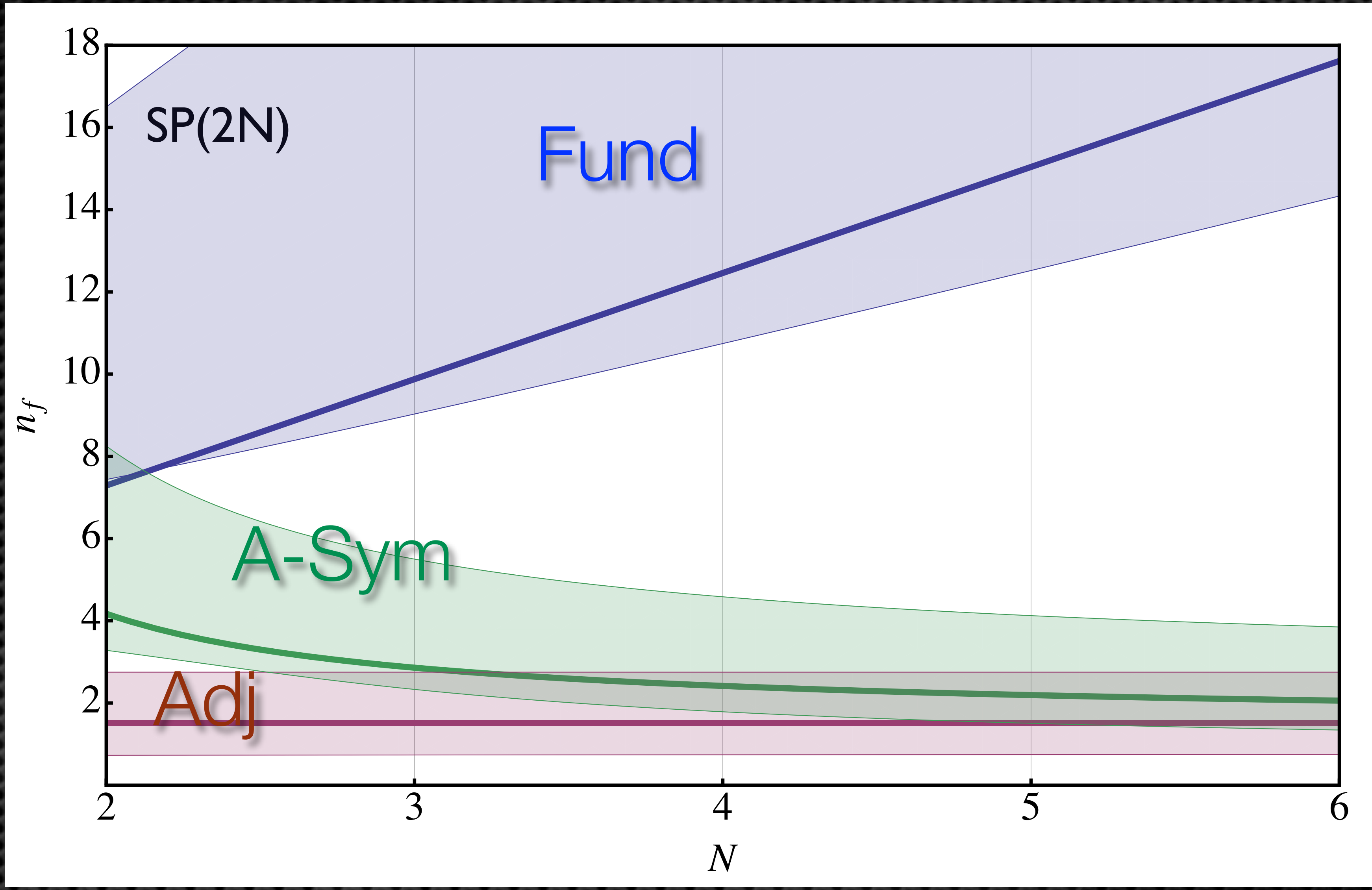

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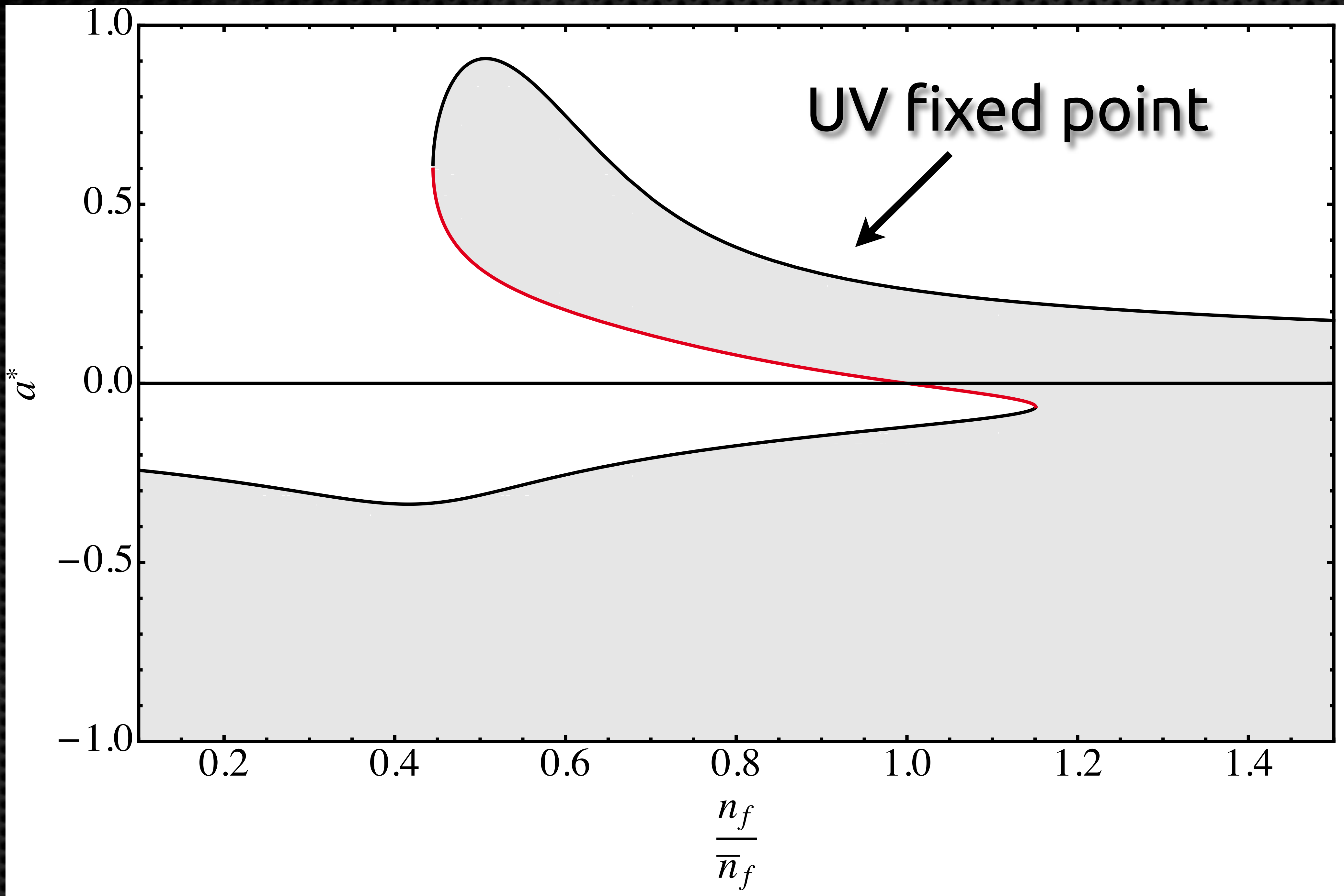
UV fixed point at large n_f



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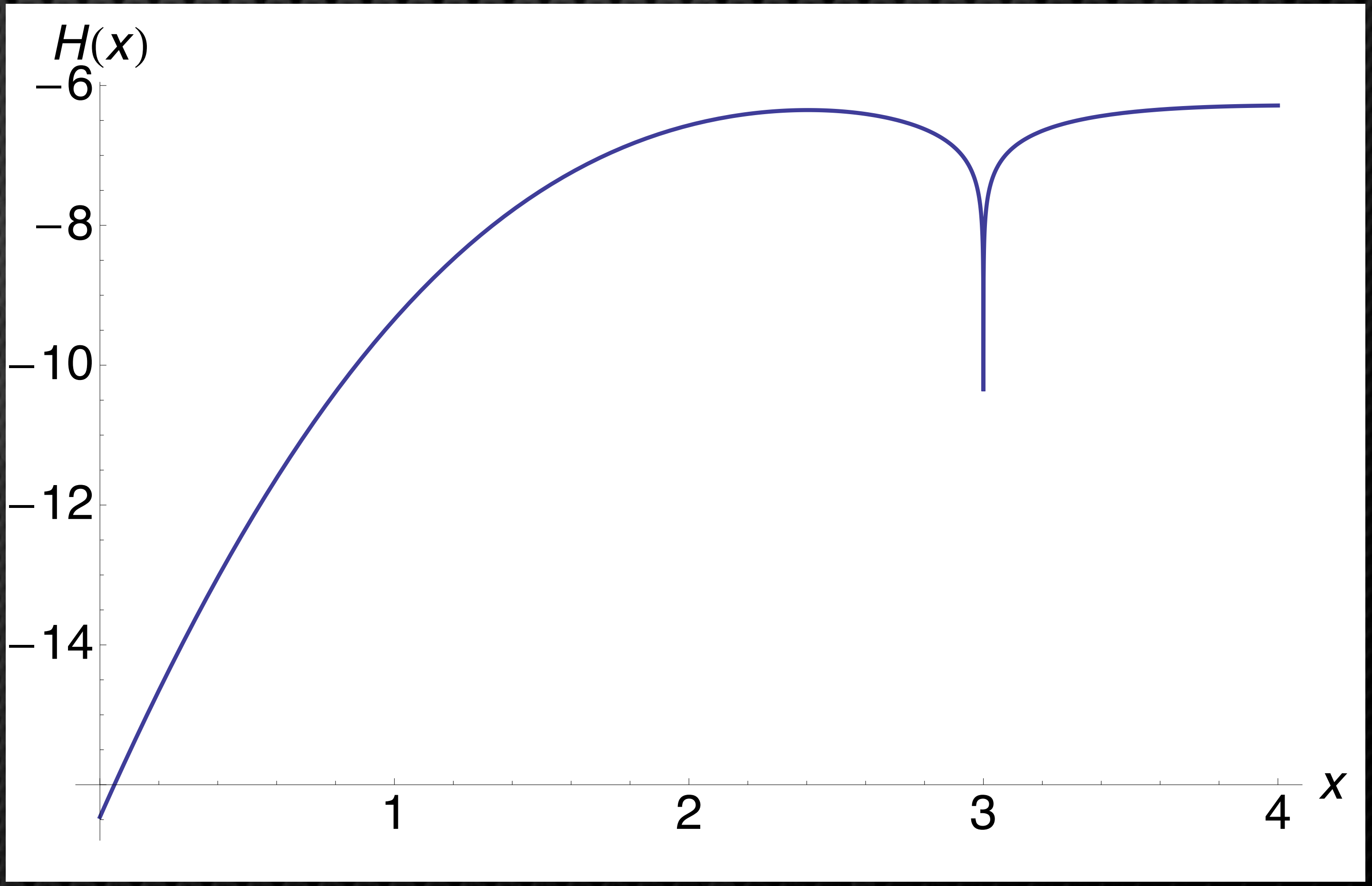
UV fixed point at large n_f

At leading order in n_f:

$$\frac{3}{4 n_f T_F} \frac{\beta(a)}{a^2} = 1 + \frac{H(a n_f T_F/\pi)}{n_f} + \mathcal{O}\left(n_f^{-2}\right) \Rightarrow \boxed{a_{UV} = \frac{3\pi}{T_F n_f}}$$

where H(x) is:



H(x) can be found, f.x., in this review:

B. Holdom, Phys. Lett. B694,74-79 (2010).

Minimal Walking Technicolor on the Lattice

L. Del Debbio, B. Lucini, A. Patella, CP, A.Rago, Phys. Rev. D82, 014509 (2010)
L. Del Debbio, B. Lucini, A. Patella, CP, A.Rago, Phys. Rev. D82, 014510 (2010)
E. Kerrane, et al. , Phys.Rev. D84, 034506 (2011)
L. Del Debbio, B. Lucini, A. Patella, CP, A. Rago, arXiv:1111.4672 [hep-lat]

Minimal Walking Technicolor

$SU(2)_{TC}$ + 2 Dirac Adjoint fermions

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- close/inside the conformal window by analytic estimates

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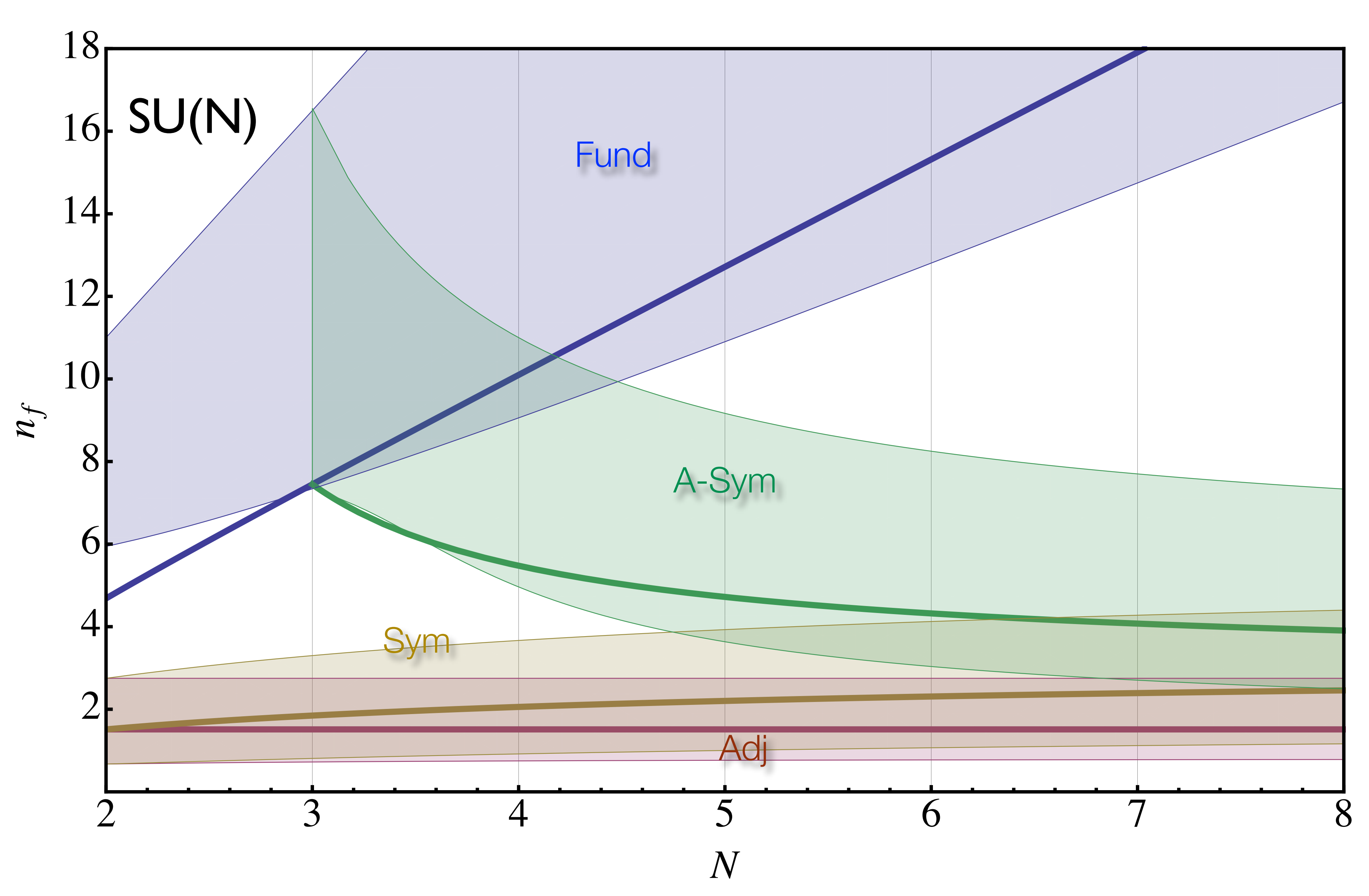
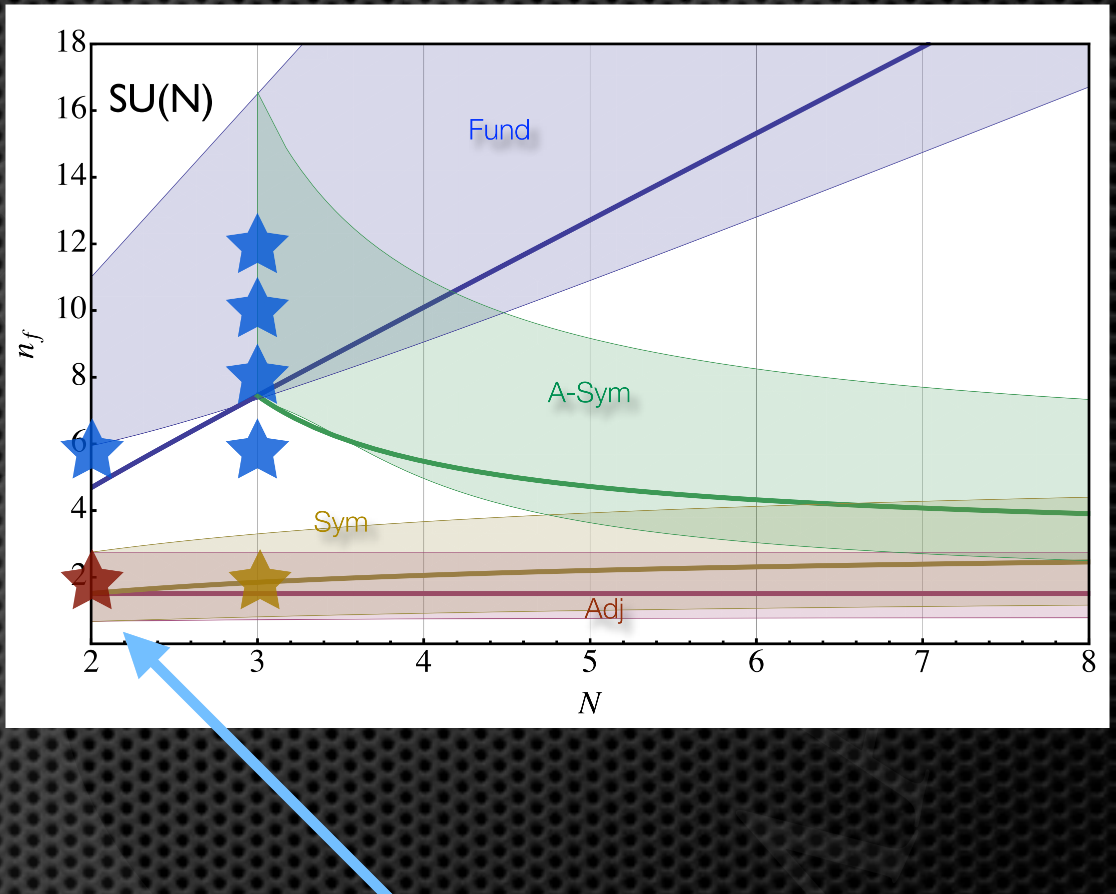
- glue-fermion composite states
- dark matter candidates

In or Out ?



In or Out ?

What can we look at on the Lattice?

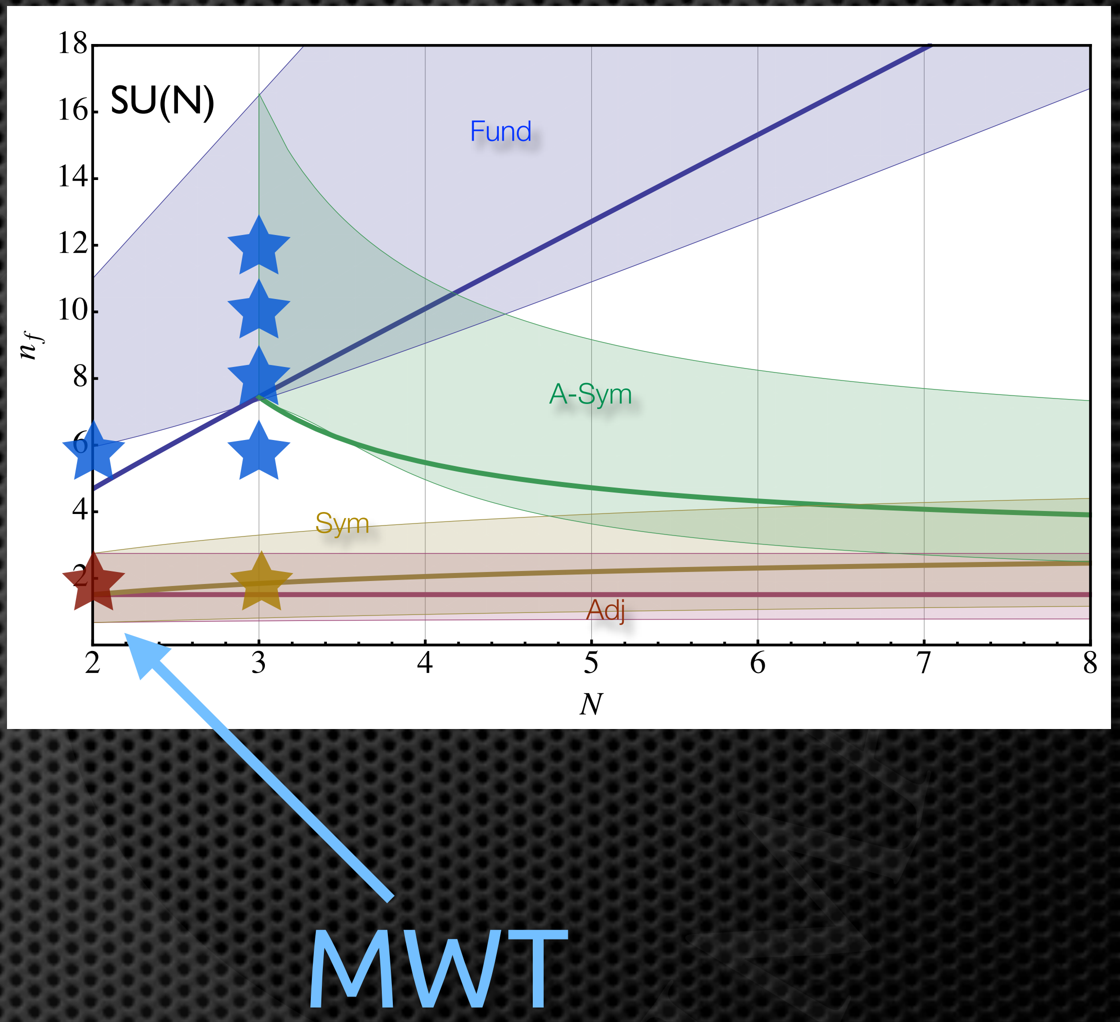


MWT

In or Out ?

What can we look at on the Lattice?

- study the spectrum of the theory
  - meson/baryon spectrum
  - gluonic spectrum
  - scaling laws



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What can we look at on the Lattice?

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  - Schrödinger functional scheme
  - Wilson / twisted Polyakov loop scheme
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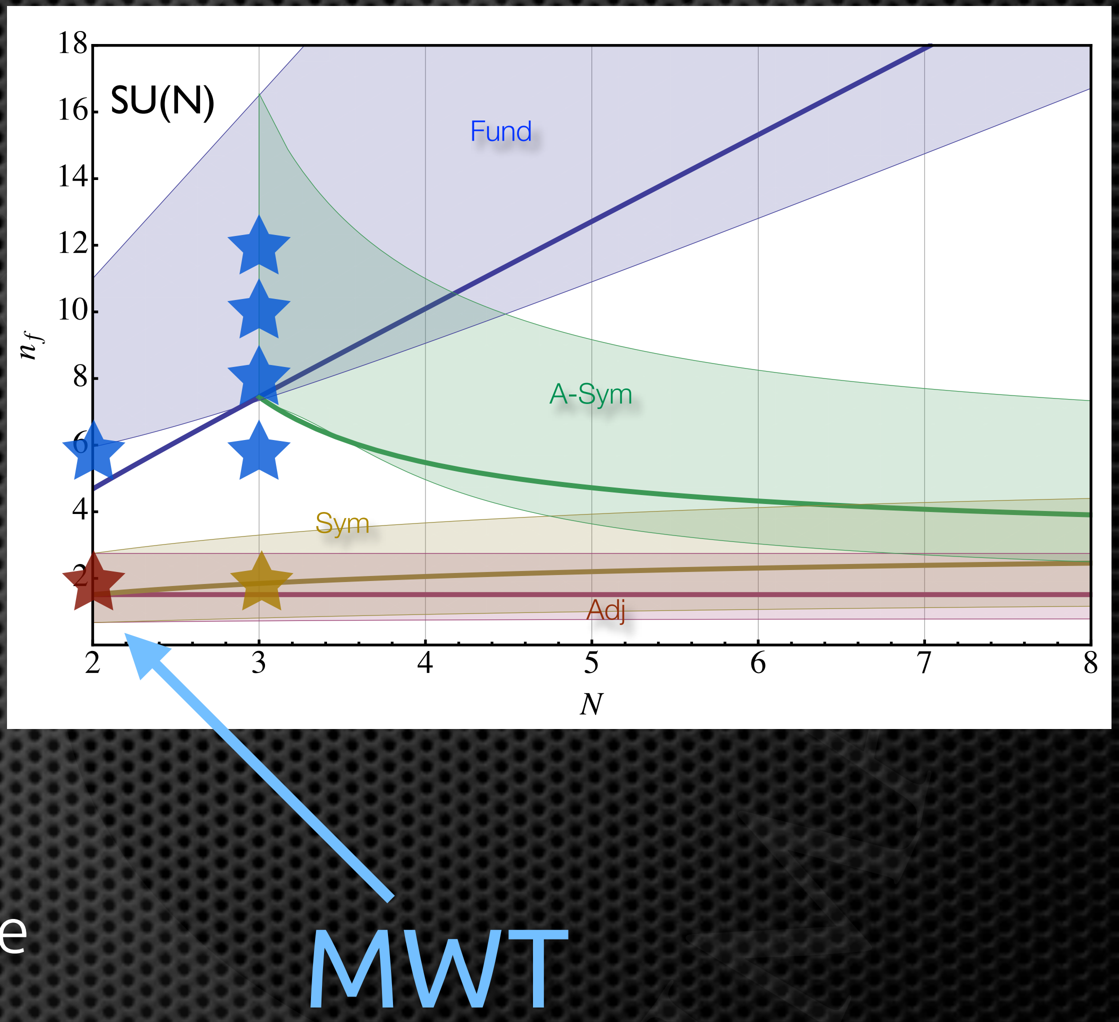

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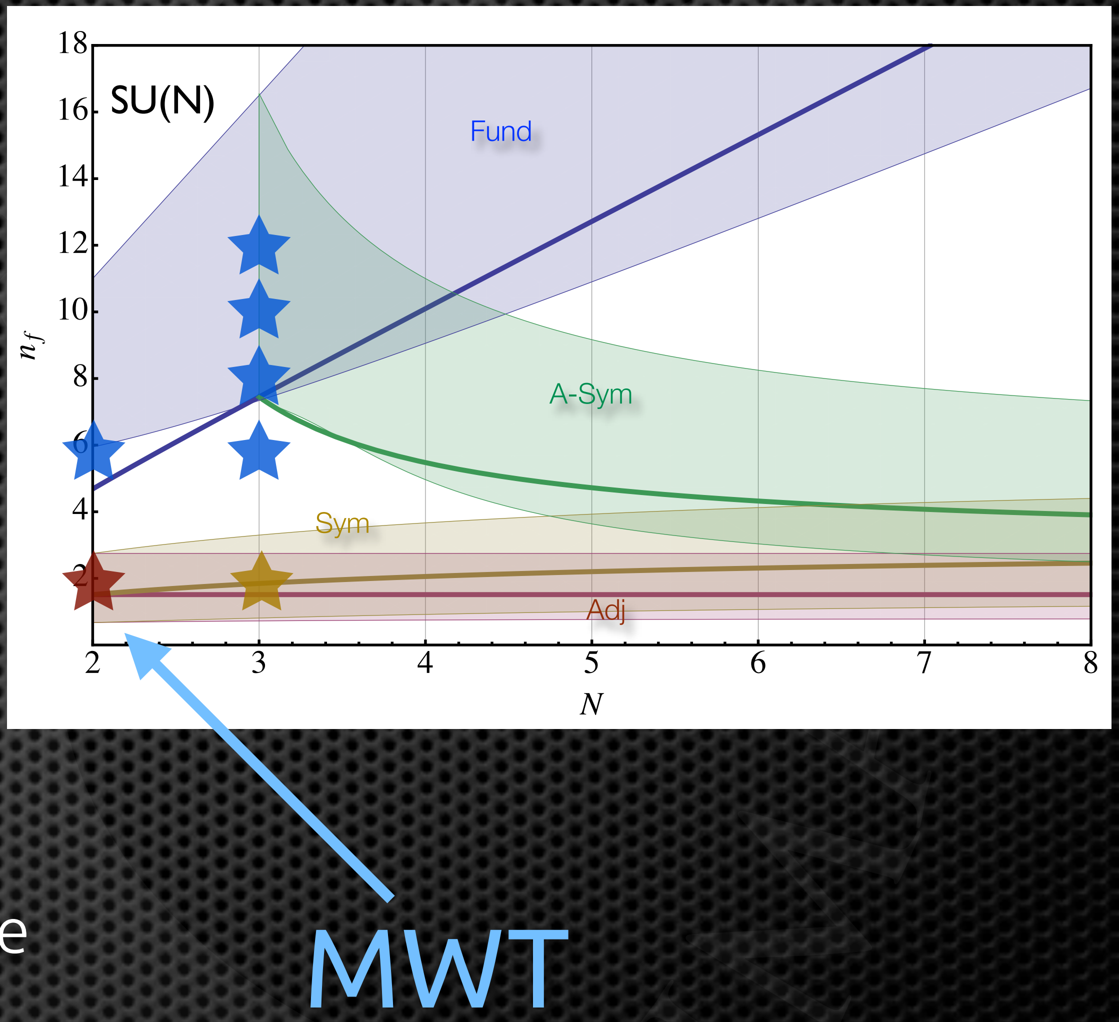

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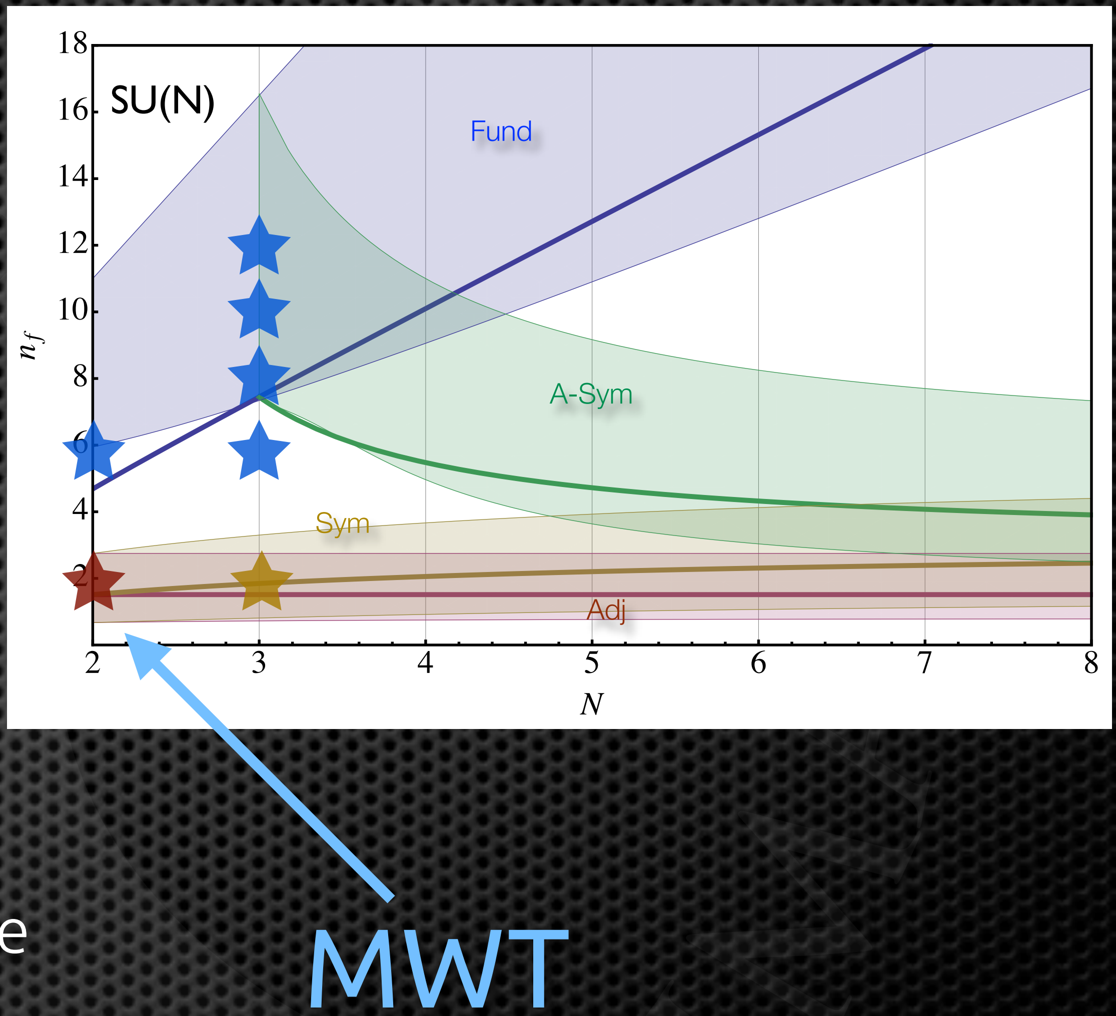

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Spectrum

Deforming an IR conformal theory

Consider a 2-point function at zero-momentum:

$$C(t,g,m,\mu) = \int d^3x \, \langle \Phi_R(t,\mathbf{x}) \Phi_R(0) \rangle (g,m,\mu)$$

we have

$$\left\{ t\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - [1+\gamma(g)]\, m\frac{\partial}{\partial m} + 2\left[d_\Phi - \gamma_\Phi(g)\right] \right\} C(t,g,m,\mu) = 0$$

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the solution is:

$$C(t,g,m,\mu) = b^{2(d_\Phi - \gamma_\Phi)} C(bt, g_*, b^{-(1+\gamma)} m, \mu) =$$

$$= \mu^{2d_\Phi} \left(\frac{m}{\mu}\right)^{2\frac{d_\Phi - \gamma_\Phi}{1+\gamma}} F\left(t m^{\frac{1}{1+\gamma}}, \mu\right)$$

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With an explicit mass *m* a mass gap is expected to be generated:

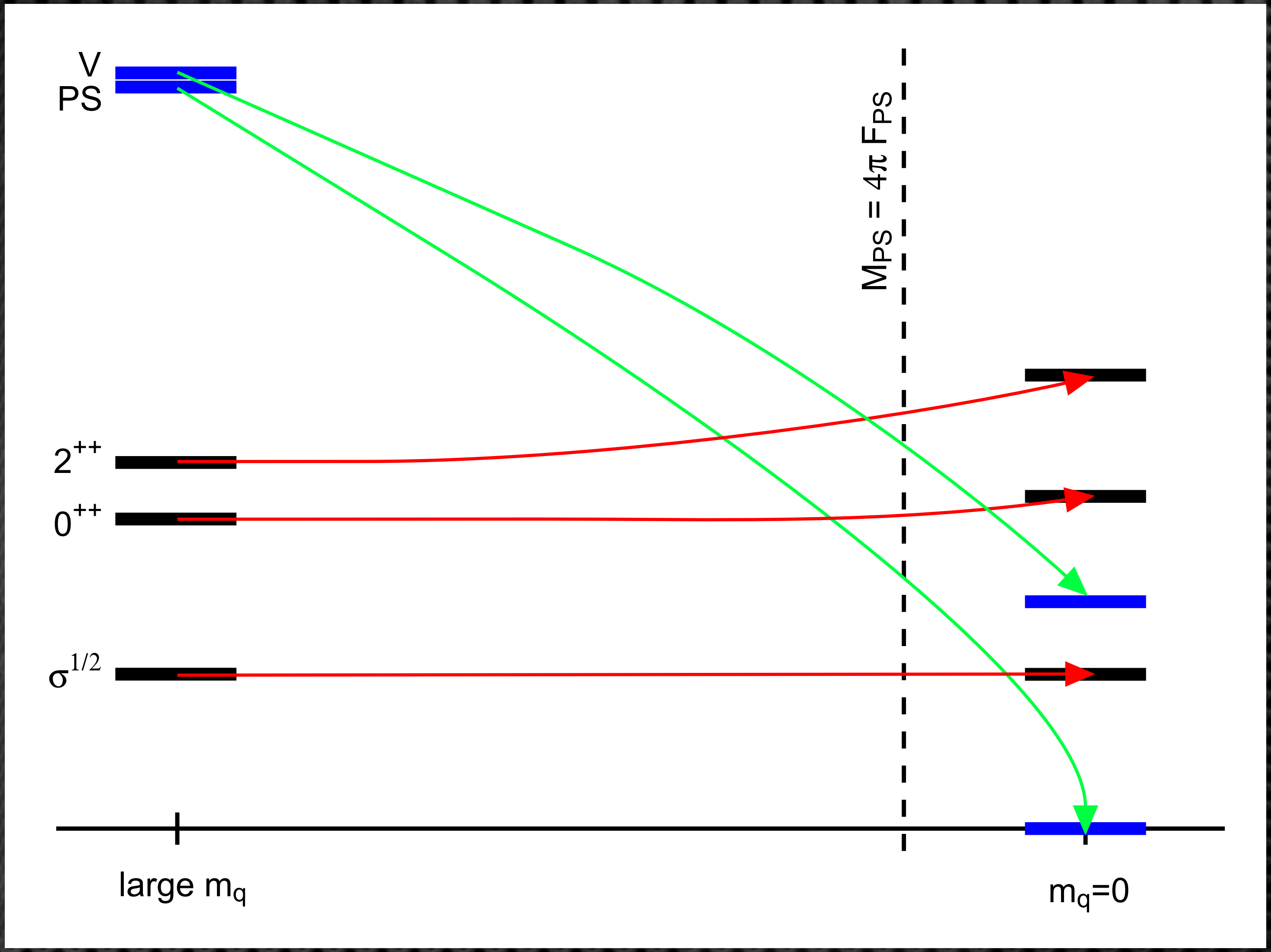
$$C(t, g, m, \mu) \sim A \exp(-M_\Phi t)$$

so that

$$M_\Phi = a_\Phi \mu \left(\frac{m}{\mu}\right)^{\frac{1}{1+\gamma}} \qquad m \to 0$$

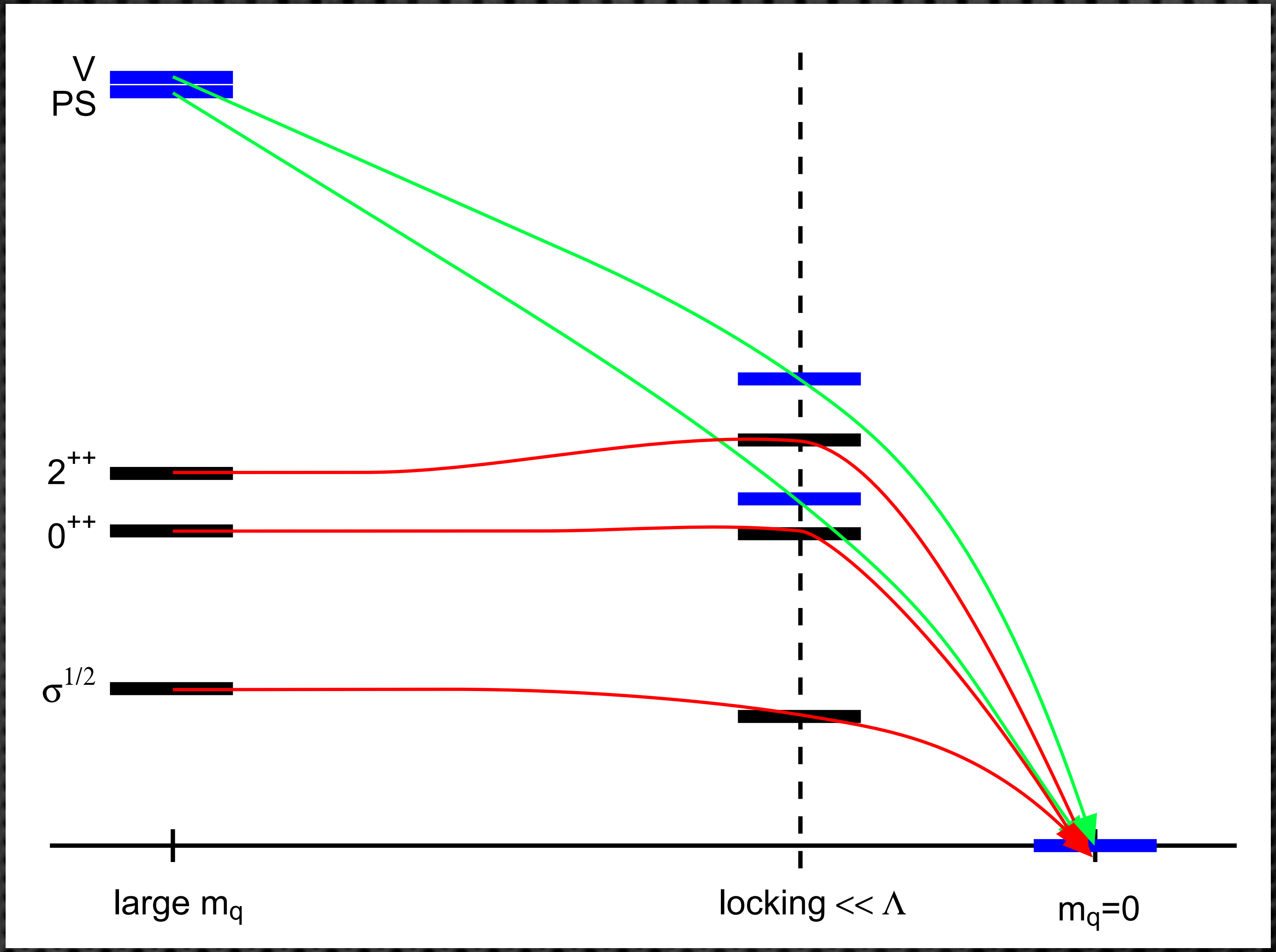
Qualitative behavior

with chiral SB:



Qualitative behavior

IR conformal:



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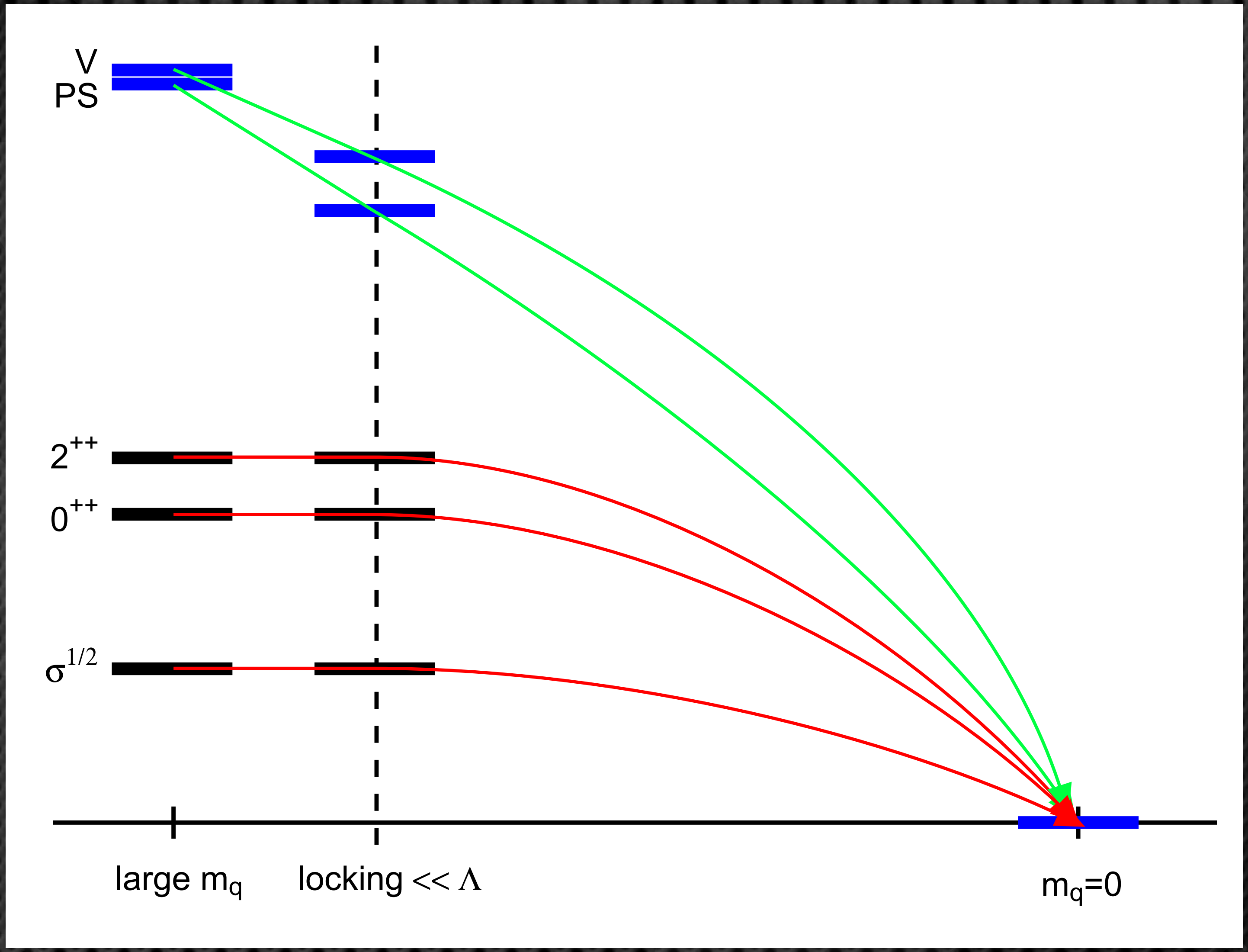
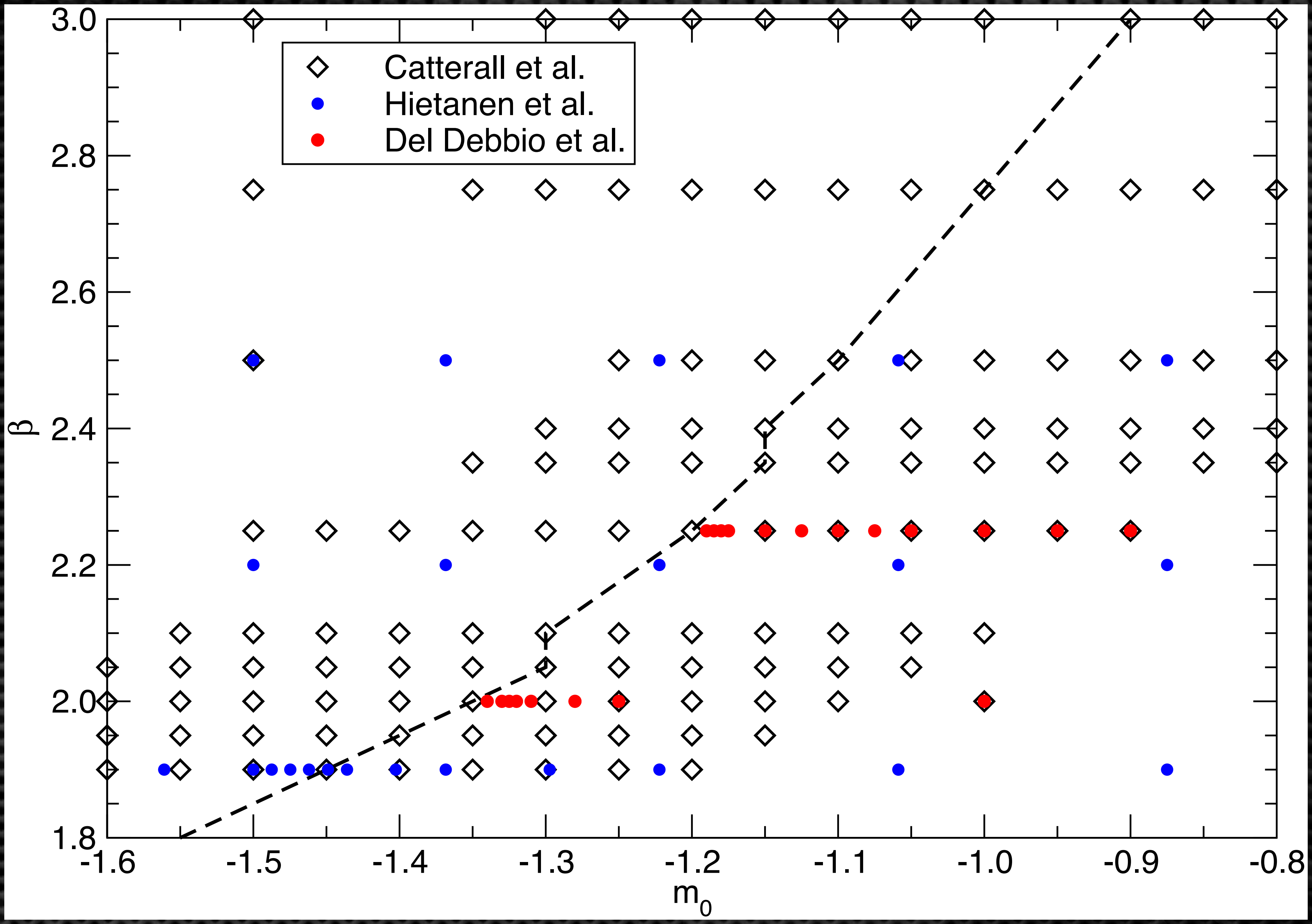


Lattice phase structure



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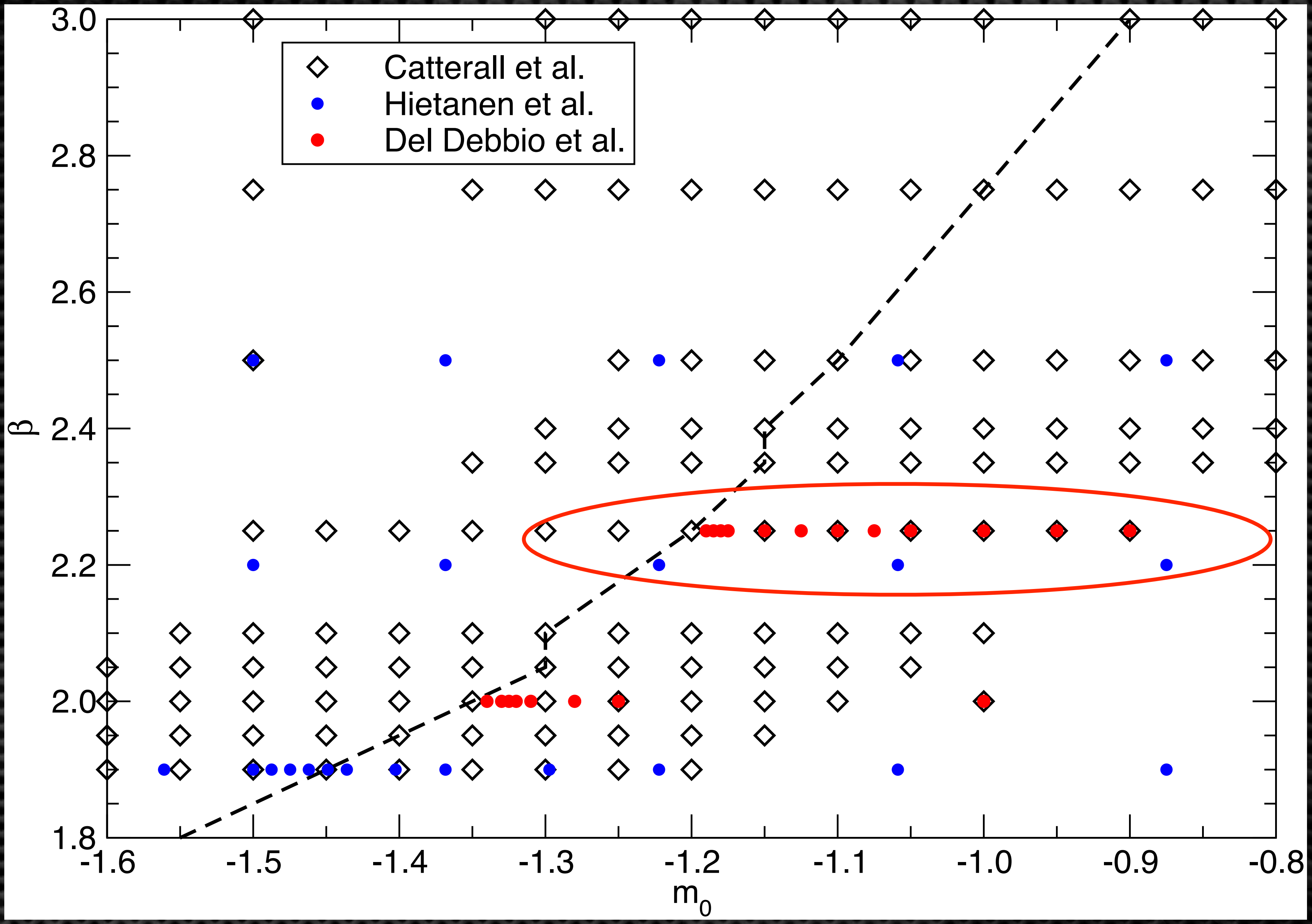

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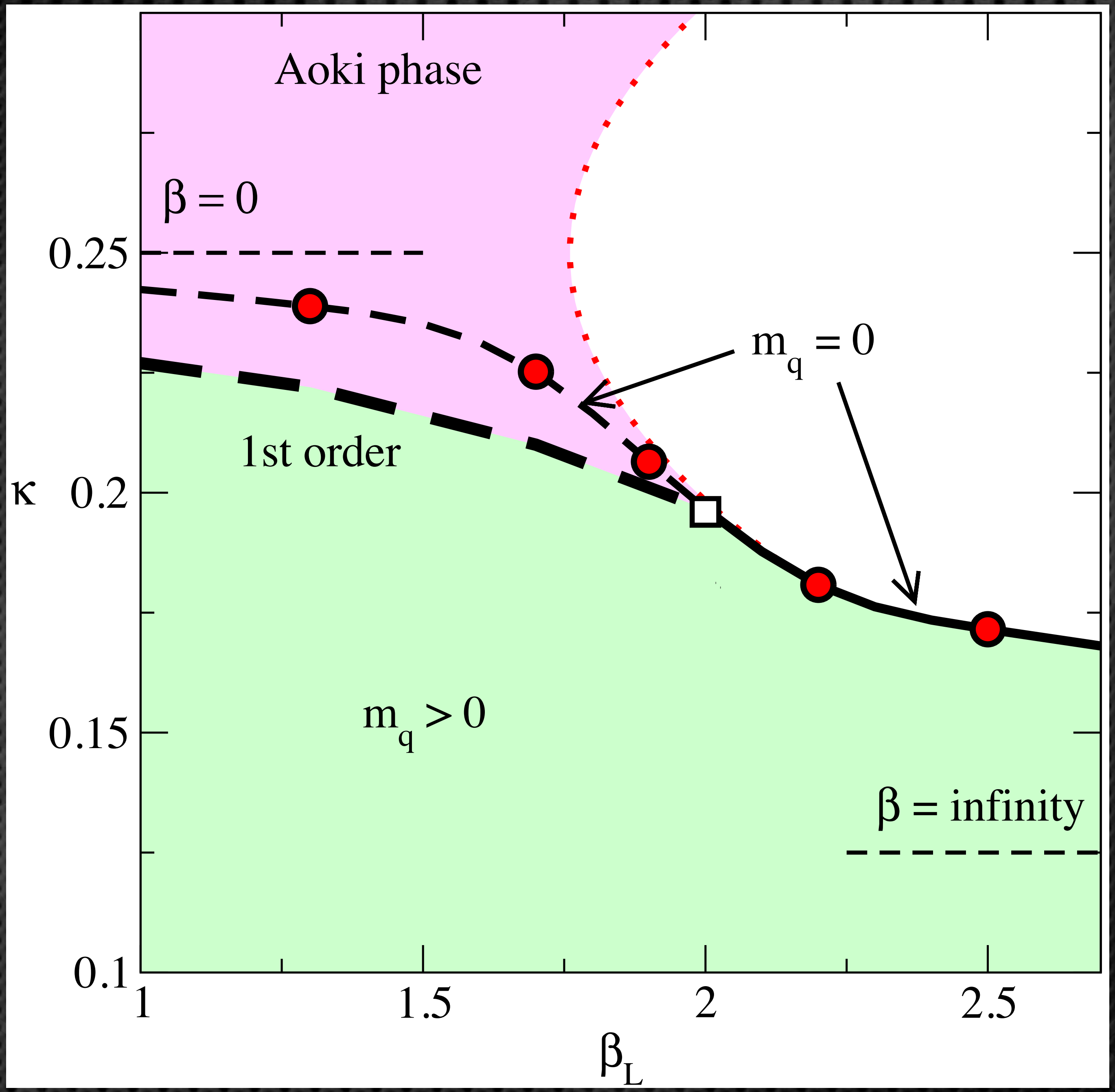
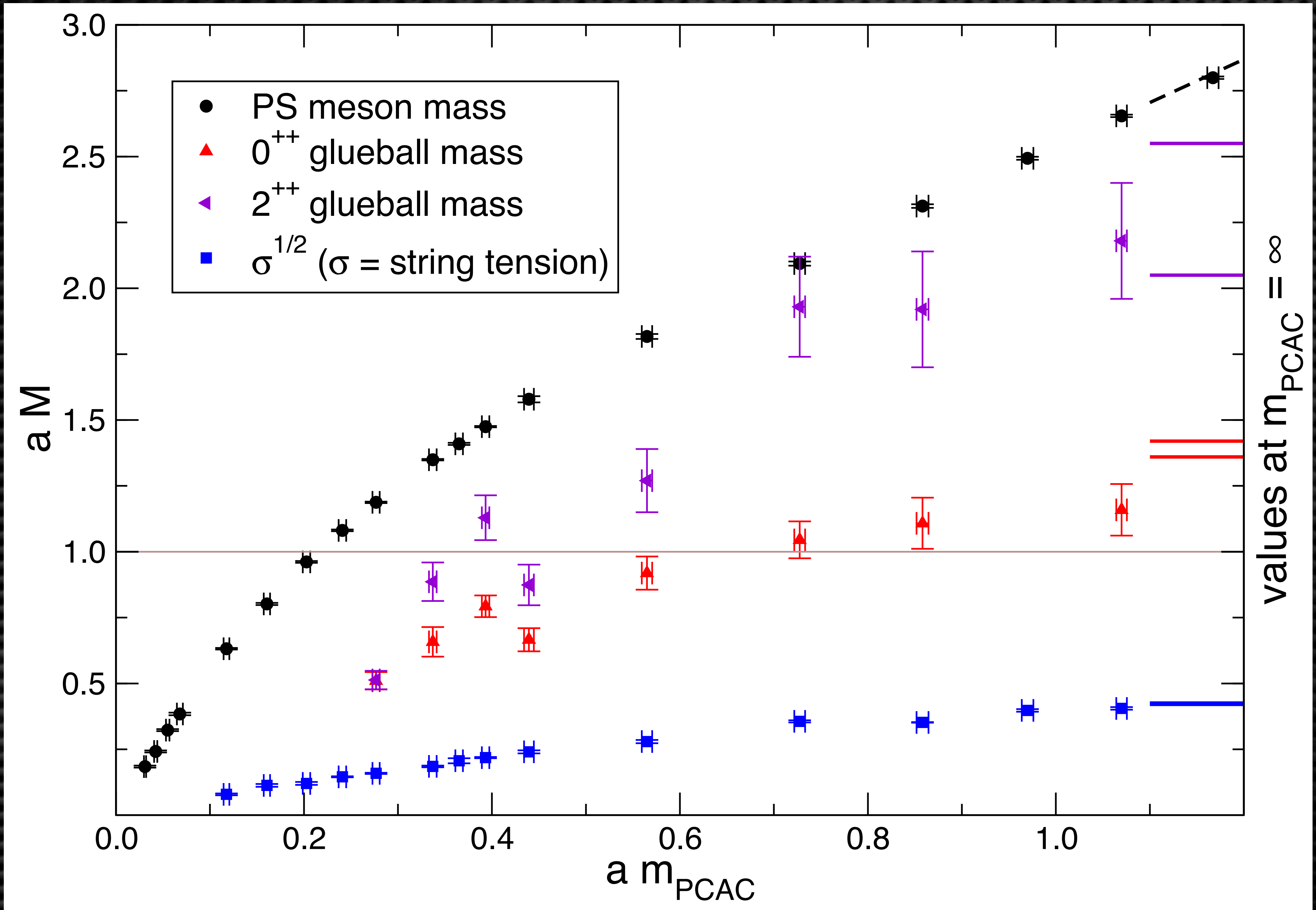


A. J. Hietanen, K. Rummukainen, K. Tuominen. JHEP 0905 (2009) 025

Spectrum Hierarchy



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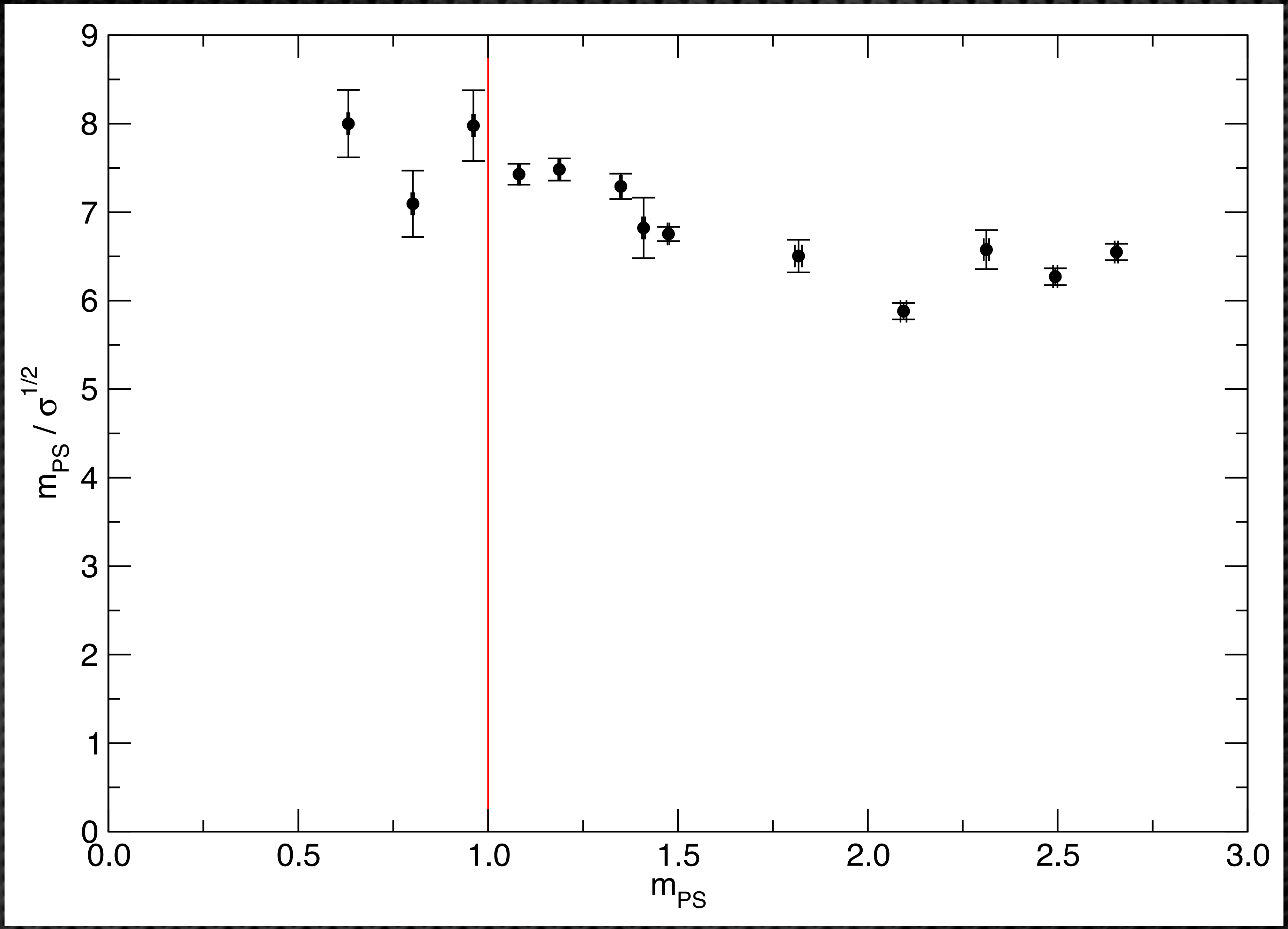
String Tension vs m_{PS}



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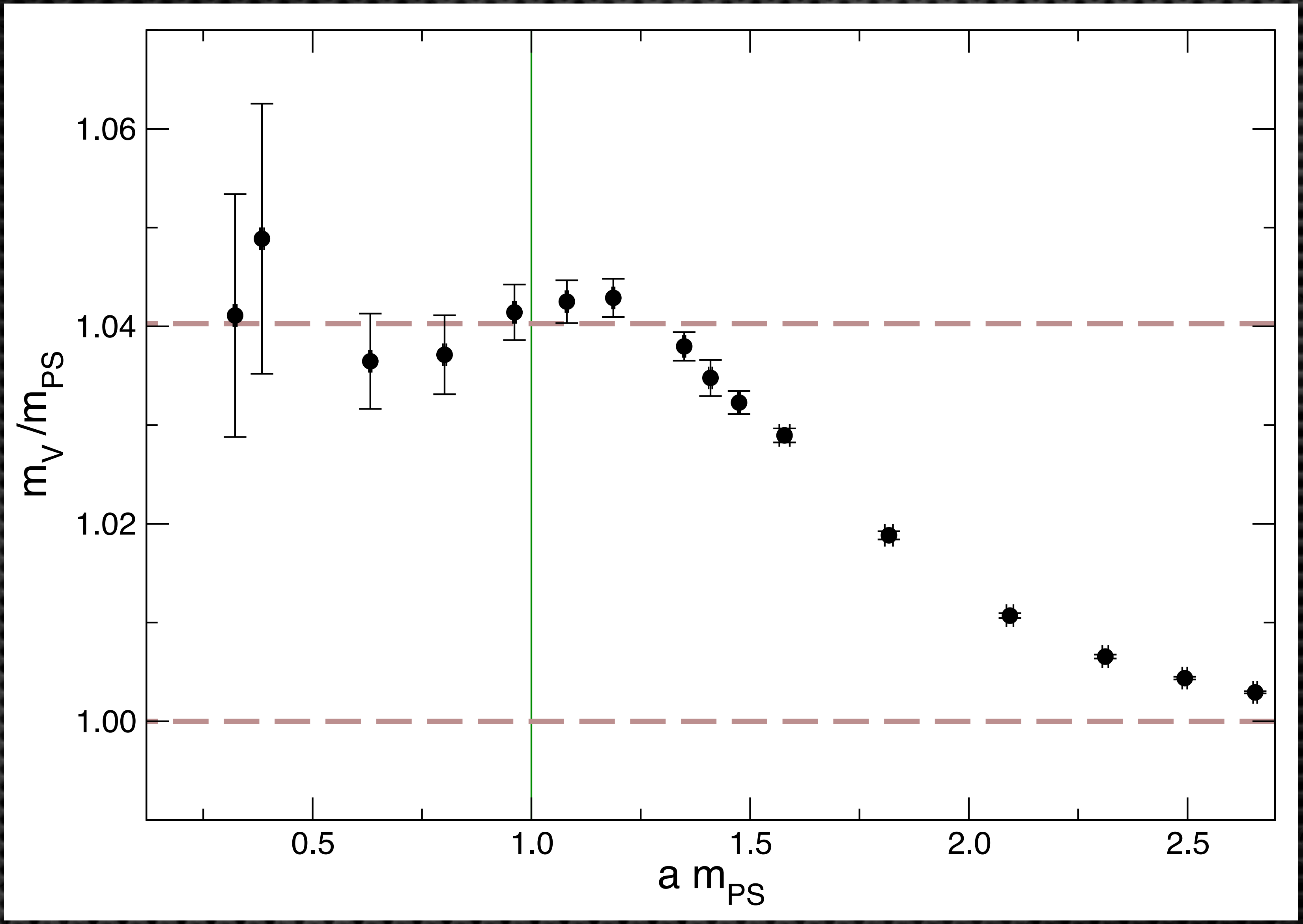
Vector vs Pseudoscalar



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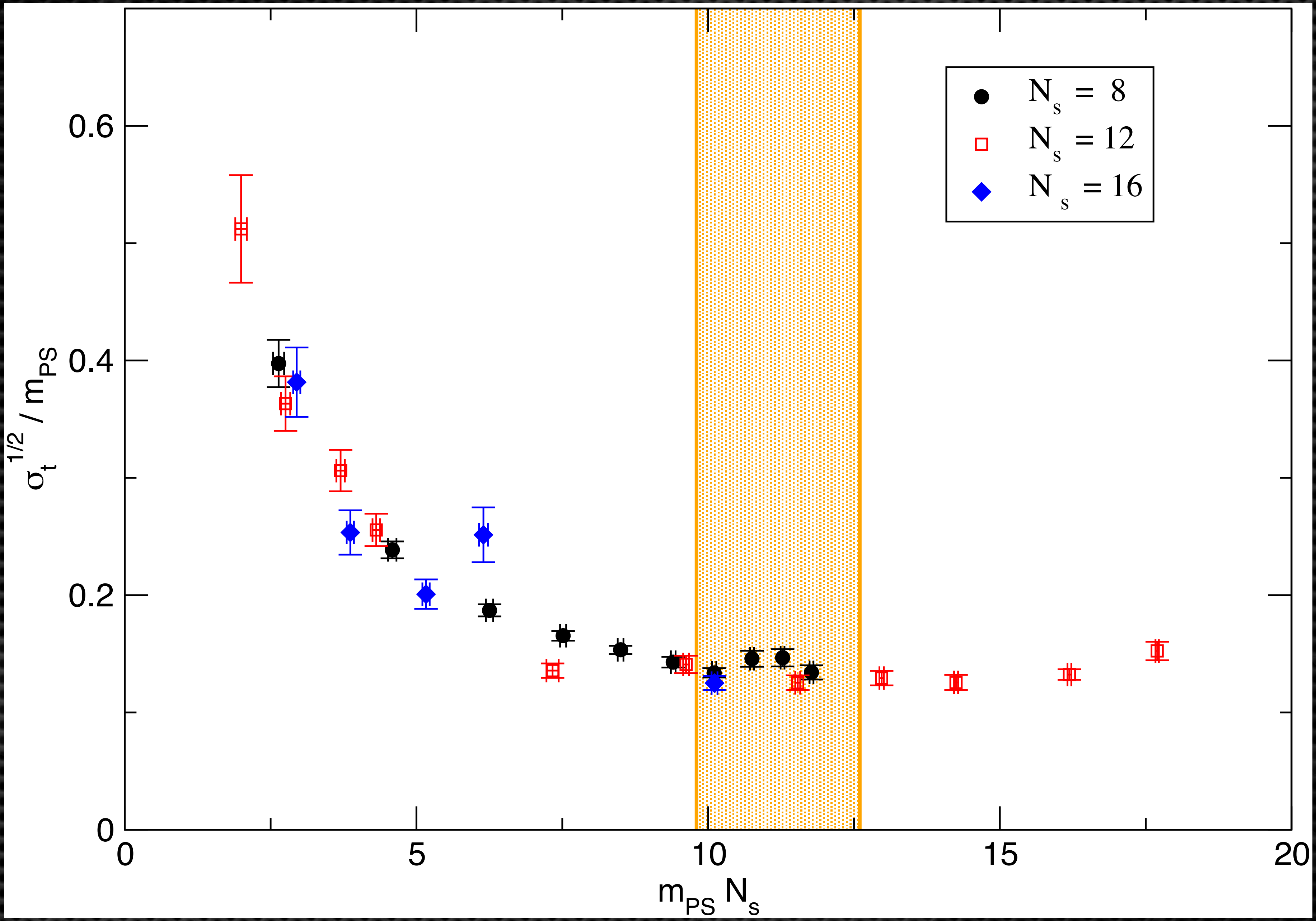
Finite Size Scaling



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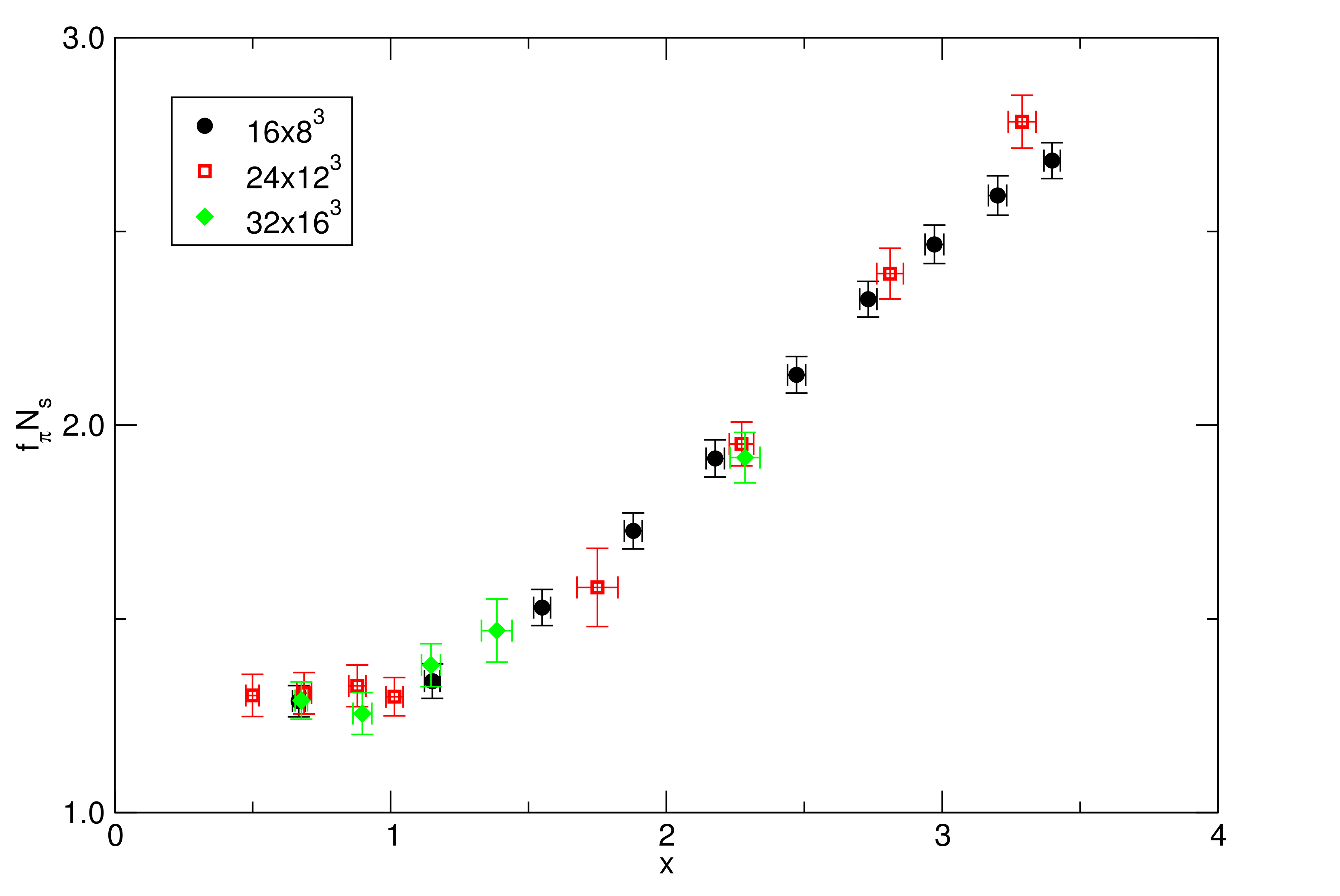

$$LF_{PS} = f(x) \quad \text{with} \quad x \equiv Lm^{1/(1+\gamma_*)}$$

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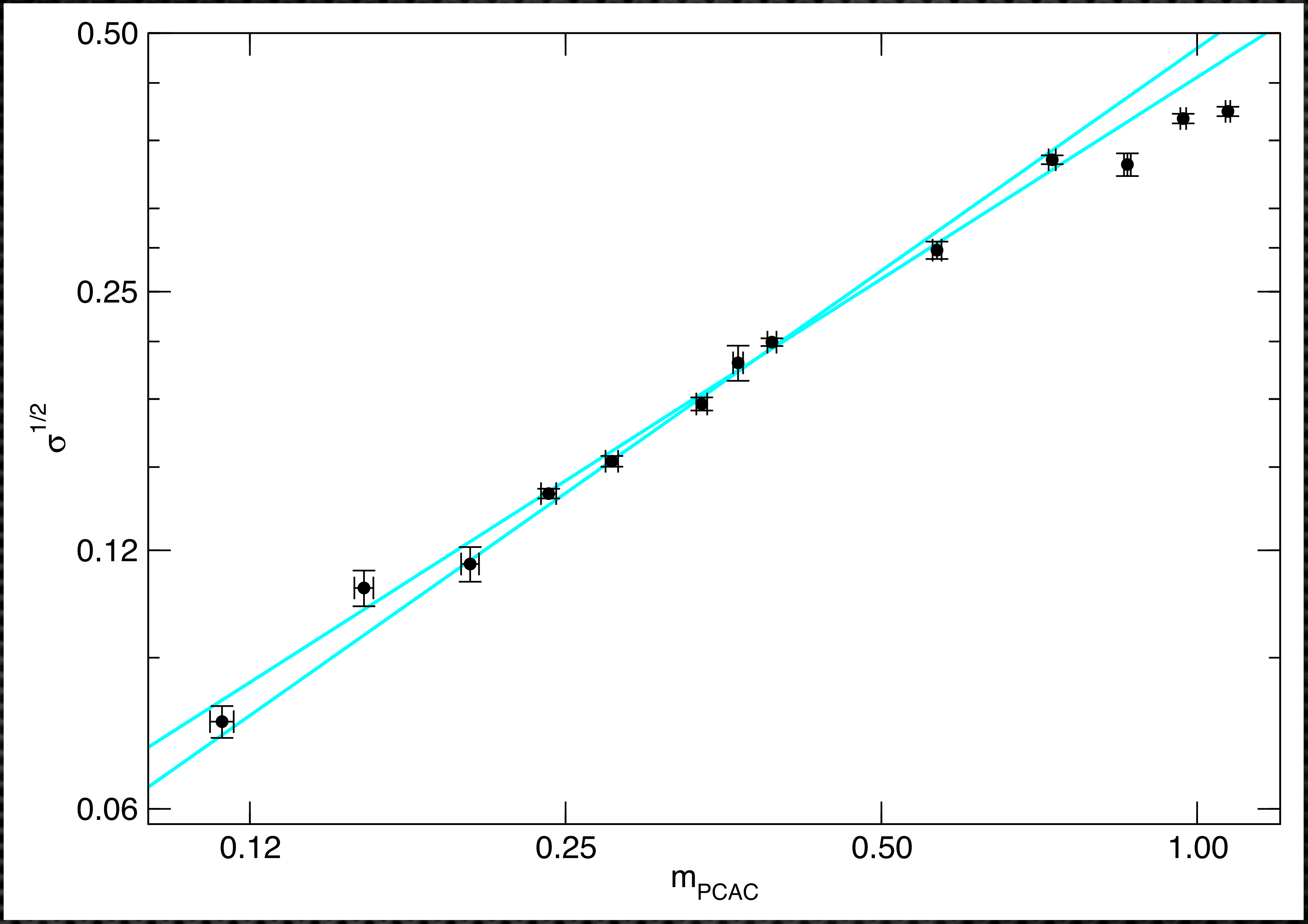
Scaling of the string tension



$$a\sqrt{\sigma} = A_{\sigma}(am_{PCAC})^{1/(1+\gamma_*)} \qquad \gamma \simeq 0.16 - 0.28$$

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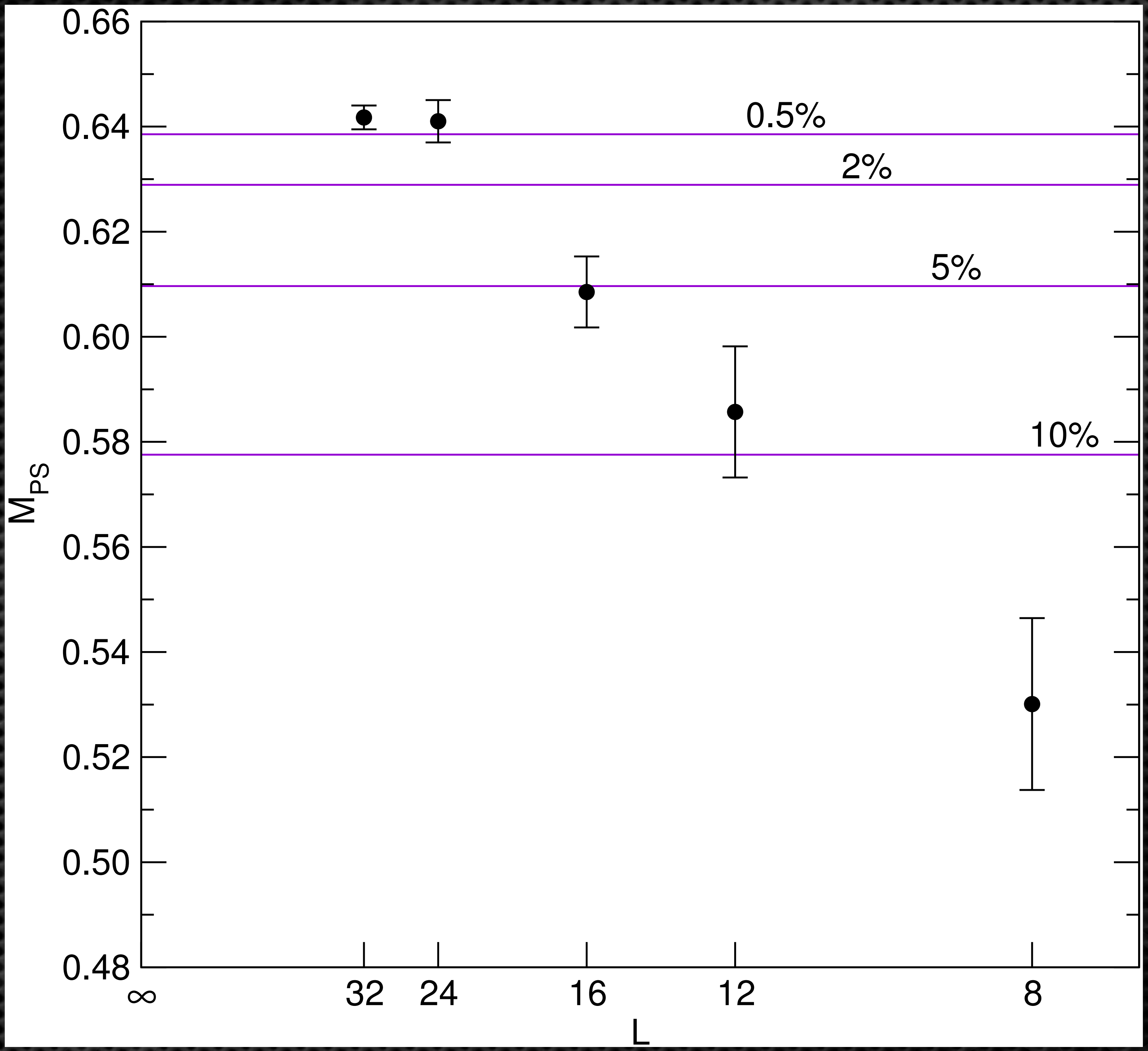
Finite Size Effects



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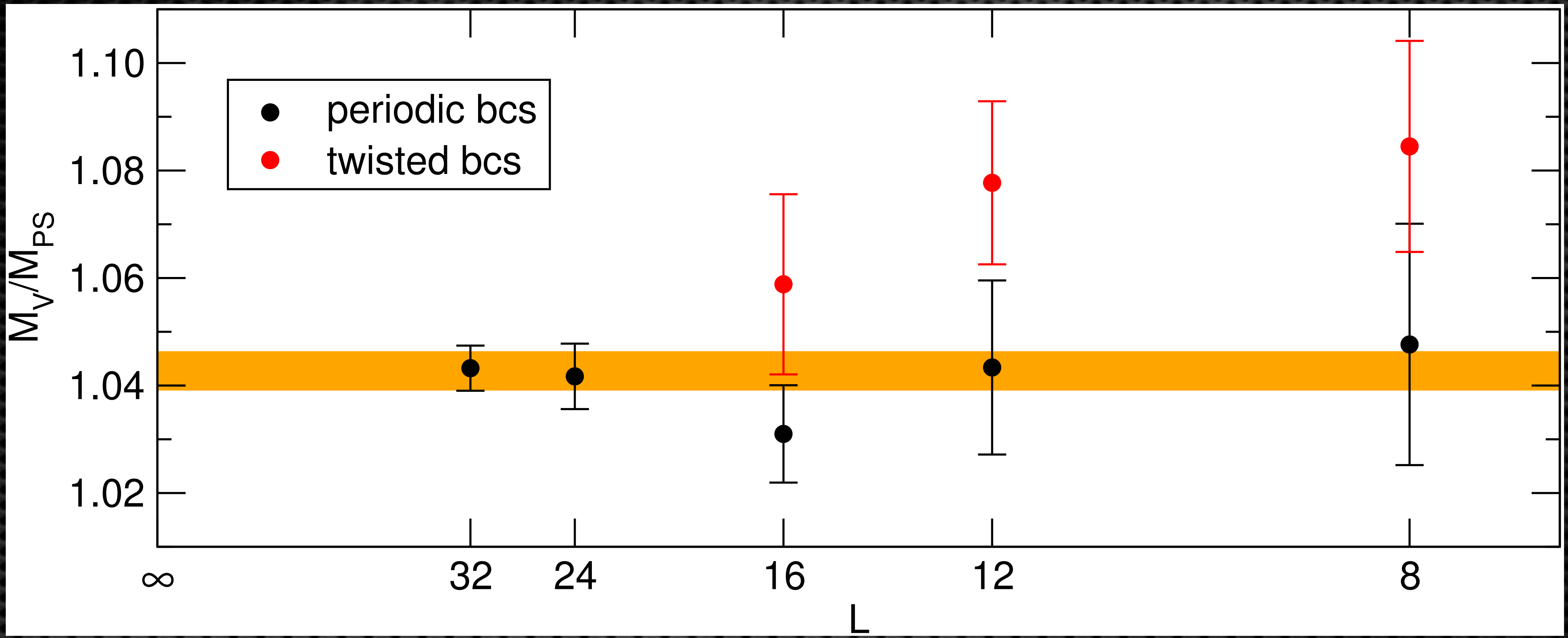

β and γ function

Schrödinger Functional

We consider the system in the presence of a background field parametrized by η:

$$Z[\eta] = e^{-\Gamma[\eta]} = \int_\eta DU D\psi D\bar{\psi} e^{-S[U,\psi,\bar{\psi}]}$$

The renormalized coupling is defined by:

$$\frac{k}{\bar{g}^2(L)} = \left.\frac{\partial\Gamma}{\partial\eta}\right|_{\eta=0}, \quad \bar{g}^2 = g_0^2 + \mathcal{O}(g_0^4)$$

We define the lattice step scaling function:

$$\Sigma(u,s,a/L) = \left.\bar{g}^2(g_0, sL/a)\right|_{\bar{g}^2(g_0,L/a)=u}$$

The continuum limit of gives the beta function:

$$\sigma(u,s) = \lim_{a/L\to 0} \Sigma(u,s,a/L) \qquad -2\log s = \int_u^{\sigma(u,s)} \frac{dx}{\sqrt{x}\beta(\sqrt{x})}$$

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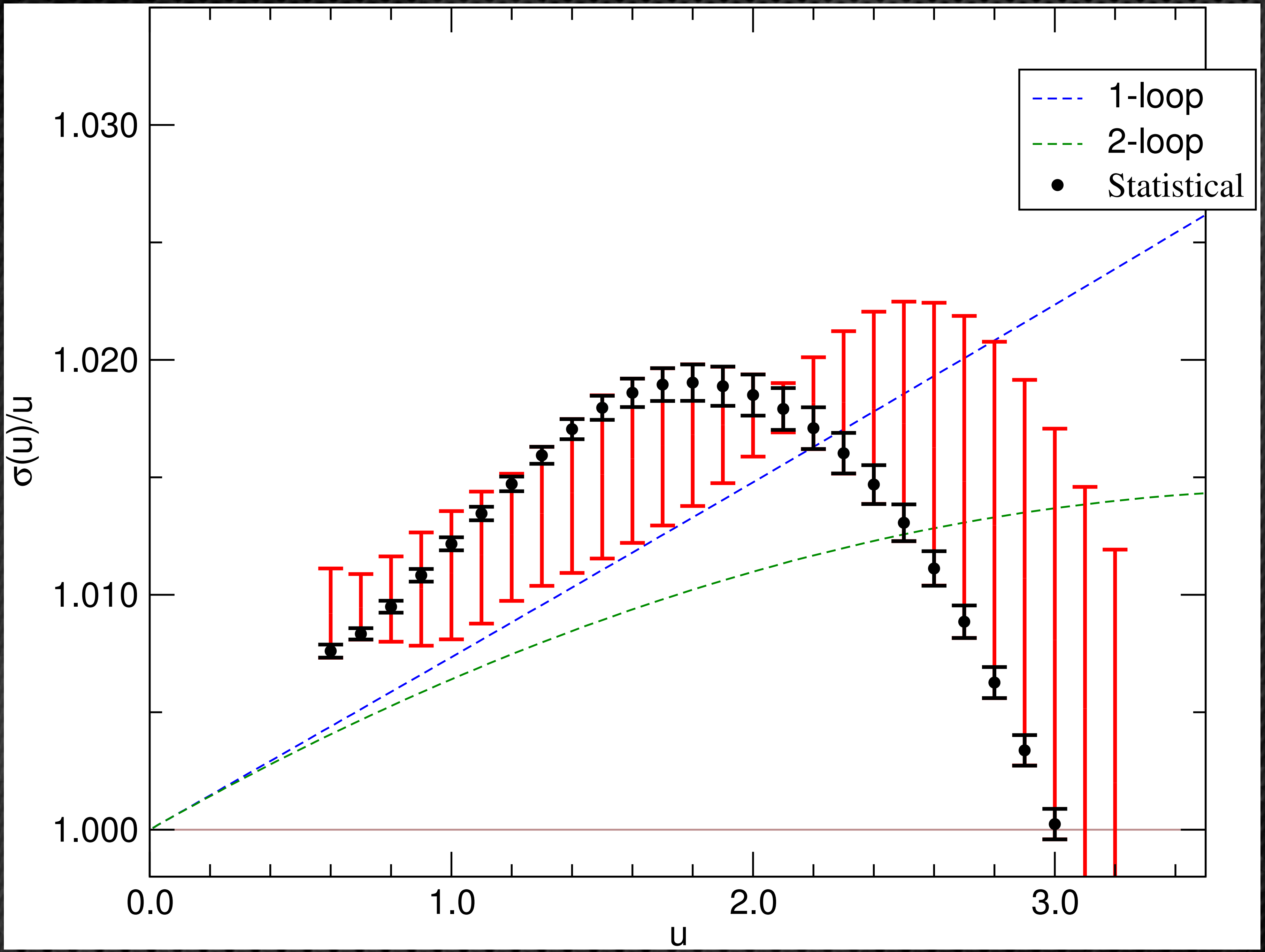
N.B.

$$\beta(u) = 0 \quad \Rightarrow \quad \sigma(u,s) = u$$

Running of the coupling



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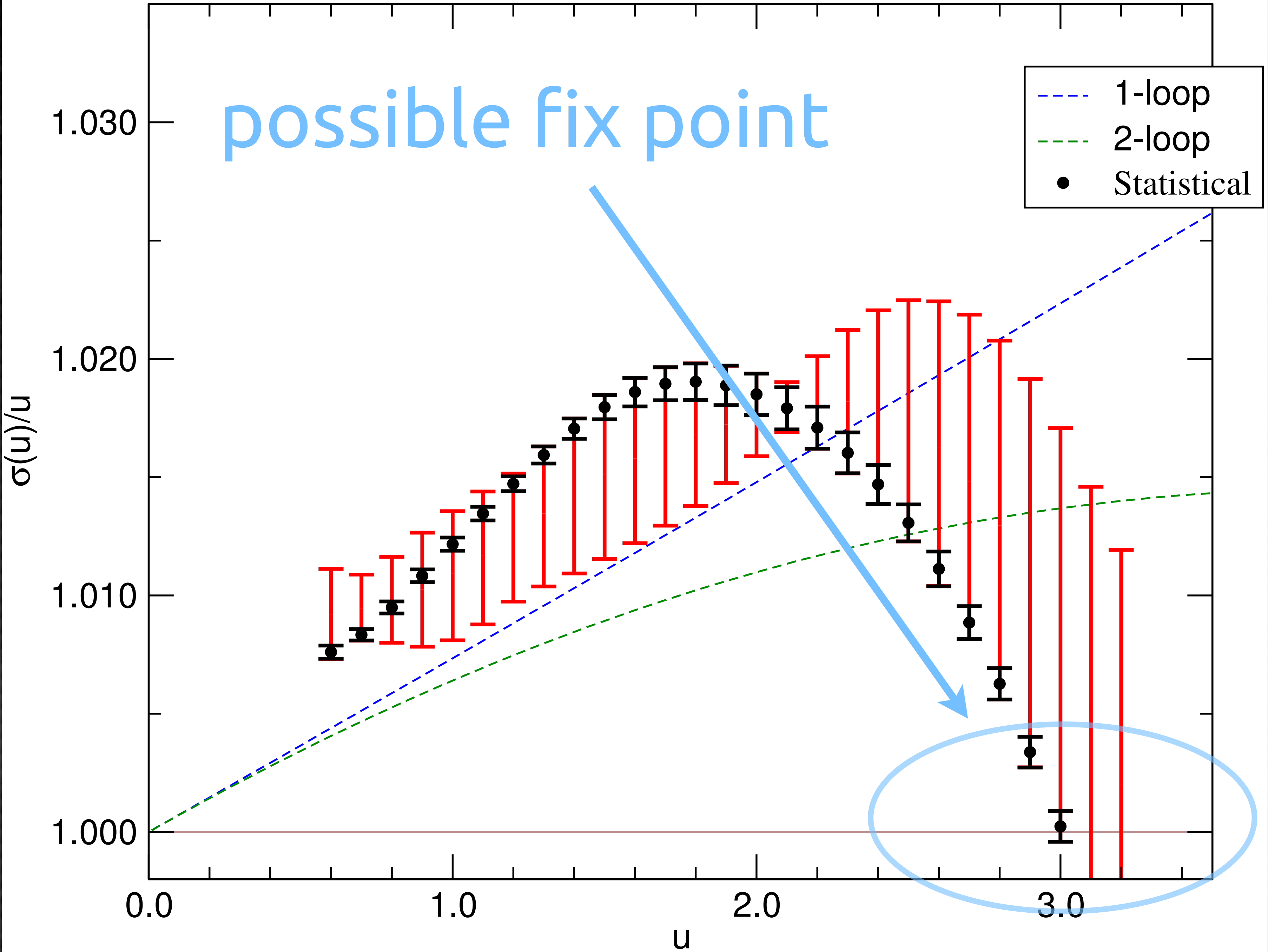
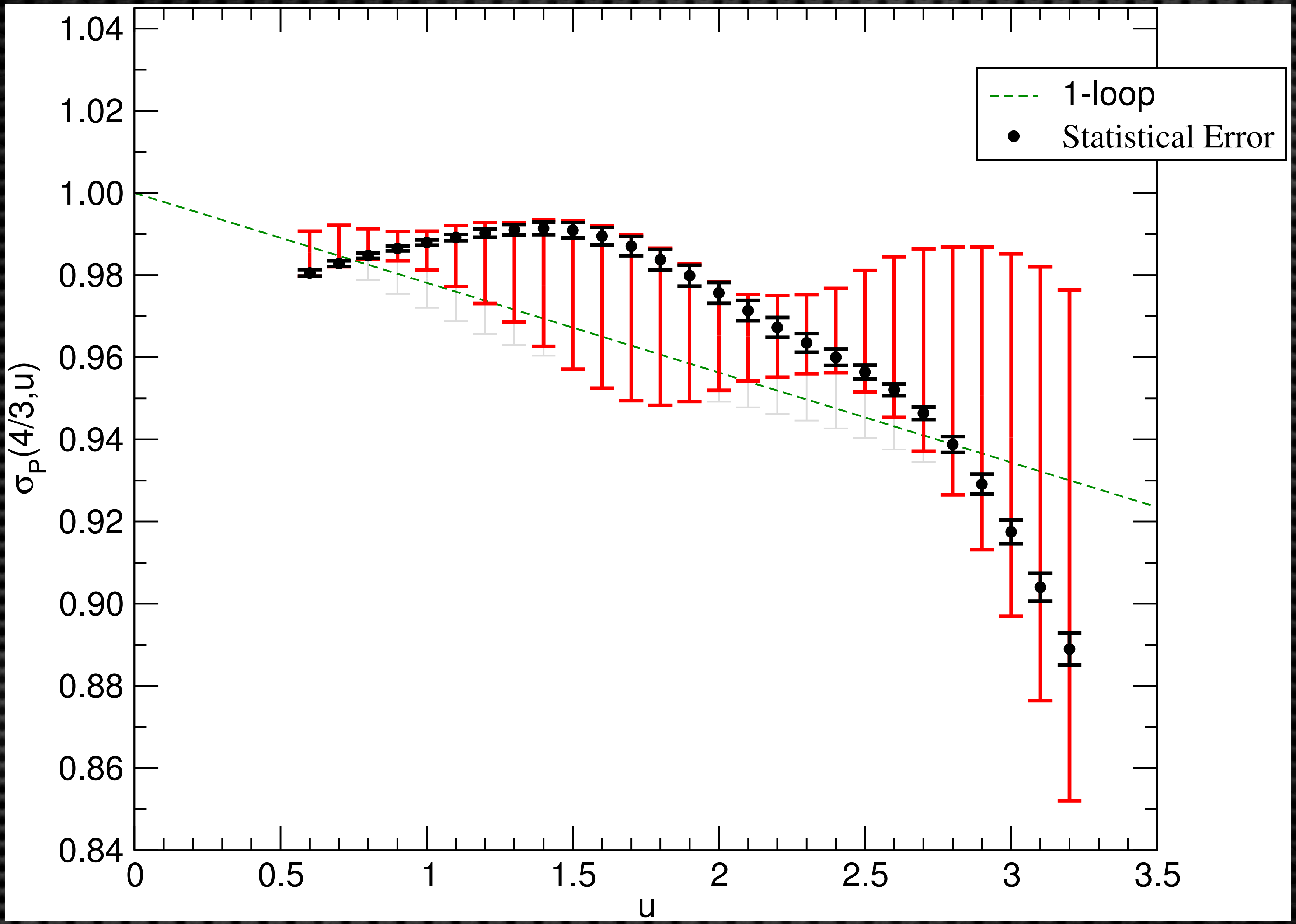


Running of the mass



Consistent with: $0.05 < \gamma_* < 0.56$

Conclusions

- MWT inside the CW

- spectrum scaling consistently with IR conformality
- SF analysis also consistent with an IR fixed point
- small $\gamma \leq 0.6$

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Thank you!