Lattice QCD with 4 and 8 flavors as exploration for the walking behavior

Kei-ichi Nagai[a],
Yasumuchi Aoki[a], Tatsumi Aoyama[a], Masafumi Kurachi[a],
Toshihide Maskawa[a], Hiroshi Ohki[a], Akihiro Shibata[b],
Koichi Yamawaki[a] and Takeshi Yamazaki[a].
(LatKMI Collaboration)

[a]Kobayashi-Maskawa Institute (KMI), Nagoya University,
[b]KEK, Computing Research Center, Tsukuba,

March 19, 2012

Introduction

Strong coupling gauge theories

To Understand,

EW symmetry breaking by composite particles,
many flavor QCD system

- Nambu and Jona-Lasinio ('61),
- Maskawa and Nakajima ('74),
- Miransky and Yamawaki ('97),
- Caswell ('74),
- Banks and Zaks ('82),
- Conformal/Walking Technicolor, $\cdots$
- *etc.*

Introduction

♠ Perturbation theory → non-trivial IR Fixed Point (IRFP)

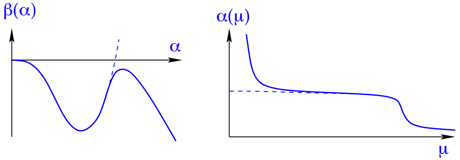
$$\beta(g) = -b_0 g^3 - b_1 g^5 + \cdots ,$$
$$b_0 = \frac{1}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right),\ b_1 = \frac{1}{(4\pi)^4}\left[\frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_c}\right)N_f\right].$$

- In $SU(3)$,
  - $N_f < 8.052\cdots$ → asymptotic free
  - $8.052\cdots < N_f < 16.5$ → non-trivial IR fixed point (conformal)
  - $16.5 \leq N_f$ → asymptotic non-free

♠ Schwinger-Dyson equation

- $N_f^{cr} = 11.9\cdots$

Why $N_f = 8$? (Walking/Conformal)



approximate IRFP

Walking Technicolor (K.Yamawaki *et.al.*)
$\rightarrow$ large anomalous mass dimension ($\gamma_m \simeq 1$) and $\langle\bar{\psi}\psi\rangle \neq 0$

Phenomenologically;
Farhi-Susskind model (one-family model):
3×2 in quark sector $+$ 2 in lepton $= 8$ flavors.

It's interesting to find gauge theories for the model building of the walking technicolor.

Beyond perturbation theory ⇒ Lattice Gauge Theory

Conformal window (IRFP) searches on the lattice:
$\Longrightarrow$ Application of lattice QCD technique

- Running coupling constant (lattice β-function)
- Spectroscopy (ChPT, Hyperscaling, ...)

A lot of lattice studies !

On the lattice:
χ-symmetry breaking / conformal ?
$\Rightarrow$ Walking (χSB and conformal-like with $\gamma_m \simeq 1$)

Our project

♠ We want to know the phase structure of many flavor gauge theories for the EW sym. breaking by the composite particle.
♠ We (LatKMI) investigate the spectrum for $N_f = 0$, 4, 8, 12, 16 systematically. $\rightarrow$ H. Ohki's talk about $N_f = 12$ and 16.
♠ Exploration of the walking behavior $\leftarrow$ flavor dependence.
SD-eq. , perturbation, phenomenological model
$\rightarrow$ near conformal for $N_f = 8 \sim 12$.

Then,
♠ In this talk, we investigate $N_f = 4$ as the typical χSB (hadronic phase) and $N_f = 8$ as the candidate for the walking technicolor model.

Lattice actions and the outline of simulations

♠ $S = S_G + \sum_{f=1}^{N_f} S_f^f$.

♠ S_G; Tree level Symanzik gauge action: $\beta = \frac{2N_c}{g^2}$ where g is the gauge coupling.

♠ $S_f = S_{HISQ}$; HISQ action for the fermion sector : (E. Follana et al., PRD75(2007); A.Bazavov et al., PRD82(2010))

HISQ = Highly Improved Staggered Quarks

♠ First application of HISQ action to many flavor system.

♣ KMI computer system, "φ".

♣ We use the code based on MILC code version-7.

♣ Simulation → standard HMC for $N_f = 4n$.

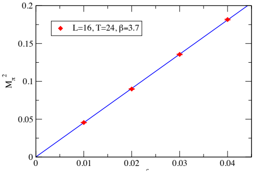
♣ Observables: M_π, M_ρ, f_π, $\langle\bar{\psi}\psi\rangle$

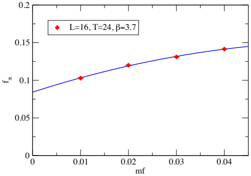
Search parameters

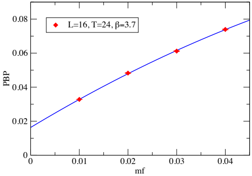
♣ $N_f = 4$;
$\beta = 3.5$, 3.6, 3.7, 3.8 on various lattice at various fermion masses.

♣ $N_f = 8$;
$\beta = 3.6$, 3.7, 3.8, 3.9 on on $12^3 \times 32$, $18^3 \times 24$, $24^3 \times 32$ and $30^3 \times 40$ at various fermion masses.
(and on $36^3 \times 48$ at $m_f = 0.015$ for $\beta = 3.8$)

$N_f = 4$







$M_\pi^2 \propto m_f$, $f_\pi \neq 0$ and $\langle \bar{\psi}\psi \rangle \neq 0$ at $m_f = 0$ (as $m_f \to 0$) in the quadratic fit. $\Longrightarrow$ χSB phase in $N_f = 4$

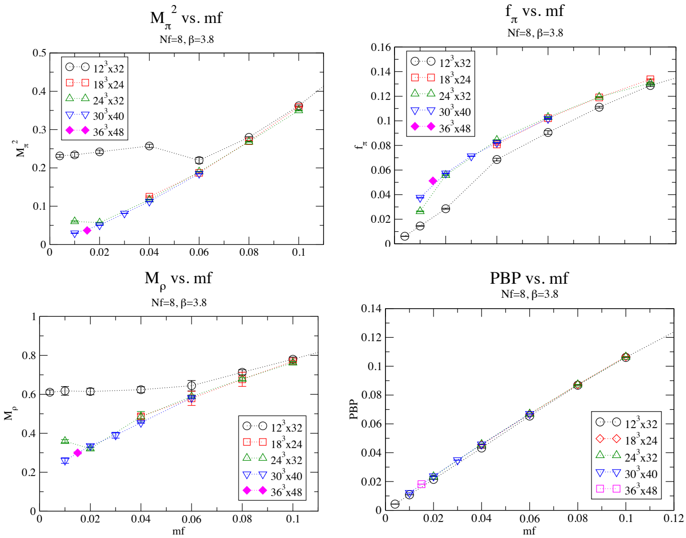
$N_f = 8$

In χSB phase (See $N_f = 4$ case in the previous page.);

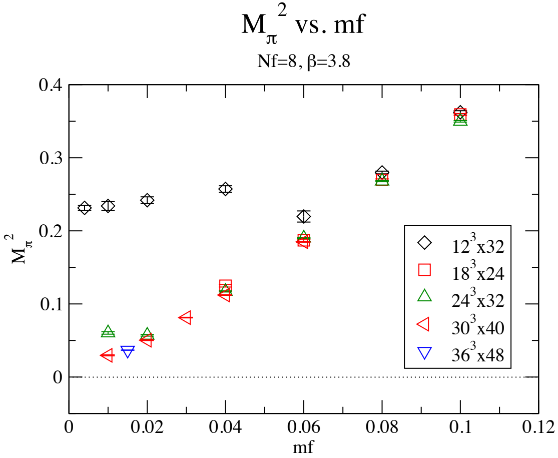
♠ χPT analysis

- $M_\pi^2 = c_1 m_f + c_2 m_f^2$??
- $f_\pi \neq 0$ at $m_f = 0$ (as $m_f \to 0$)??
- $\langle \bar{\psi}\psi \rangle \neq 0$ at $m_f = 0$ (as $m_f \to 0$)??
- $M_\rho \neq 0$ at $m_f = 0$ (as $m_f \to 0$)??

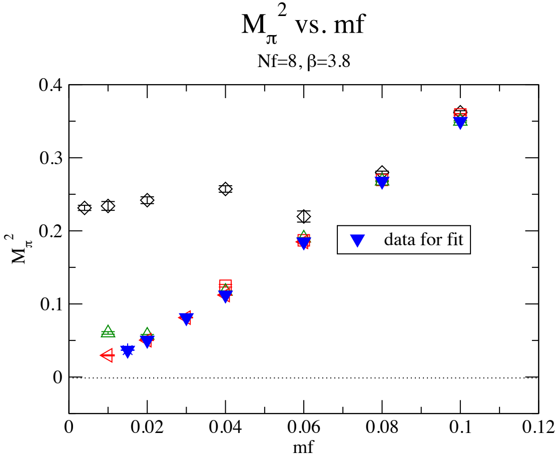
$N_f = 8, \beta = 3.8$


M_π^2 vs. mf
Nf=8, β=3.8
M_π^2
12^3x32
18^3x24
24^3x32
30^3x40
36^3x48
f_π vs. mf
Nf=8, β=3.8
f_π
M_ρ vs. mf
Nf=8, β=3.8
M_ρ
mf
PBP vs. mf
Nf=8, β=3.8
PBP
mf

$N_f = 8, \beta = 3.8$

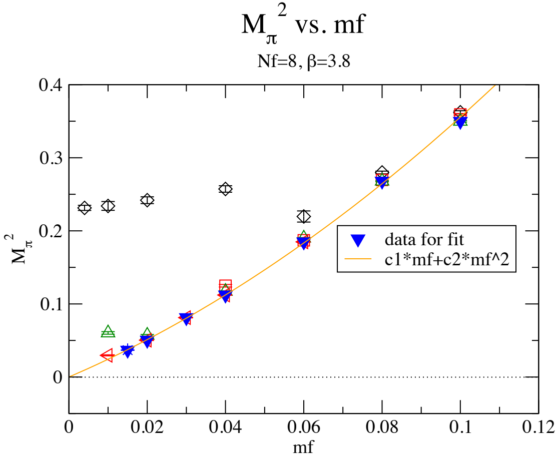

M_π^2 vs. mf
Nf=8, β=3.8
M_π^2
mf
0.4
0.3
0.2
0.1
0
0.02
0.04
0.06
0.08
0.1
0.12
12^3x32
18^3x24
24^3x32
30^3x40
36^3x48

$N_f = 8, \beta = 3.8$



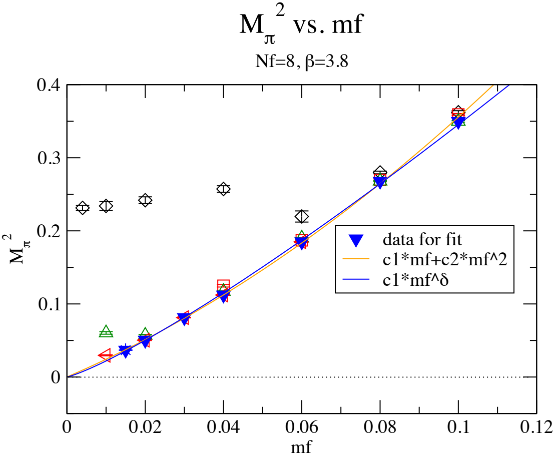
M_π^2 vs. mf
Nf=8, β=3.8
0.4
0.3
0.2
0.1
0
M_π^2
0
0.02
0.04
0.06
0.08
0.1
0.12
mf
data for fit

$N_f = 8, \beta = 3.8$



M_π^2 vs. mf
Nf=8, β=3.8
0.4
0.3
0.2
0.1
0
M_π^2
0
0.02
0.04
0.06
0.08
0.1
0.12
mf
data for fit
c1*mf+c2*mf^2

$N_f = 8,\ \beta = 3.8$

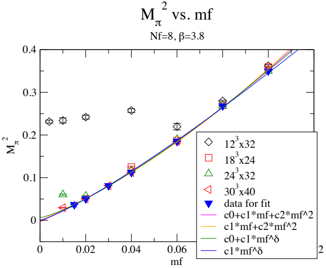

M_π^2 vs. mf
Nf=8, β=3.8
data for fit
c1*mf+c2*mf^2
c1*mf^δ
M_π^2
mf
0.4
0.3
0.2
0.1
0
0
0.02
0.04
0.06
0.08
0.1
0.12

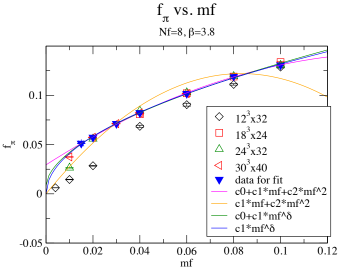
Fit result of M_π^2 in $N_f = 8$ at $\beta = 3.8$

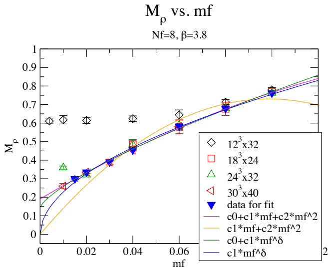
$$M_\pi^2 = 5.43(4) m_f^{1.197(3)}, \quad \chi^2/dof = 34.0\,,$$
$$M_\pi^2 = 2.31(2) m_f + 12.5(1) m_f^2, \quad \chi^2/dof = 17.9\,.$$

In χ^2/dof monitoring,
the polynomial fit is better than the power fit.

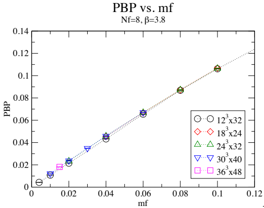
$N_f = 8, \beta = 3.8$


M_π^2 vs. mf
Nf=8, β=3.8
M_π^2
mf
12^3x32
18^3x24
24^3x32
30^3x40
data for fit
c0+c1*mf+c2*mf^2
c1*mf+c2*mf^2
c0+c1*mf^δ
c1*mf^δ

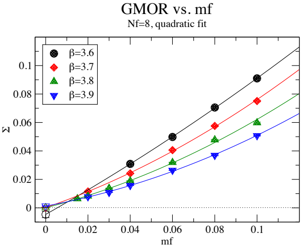

f_π vs. mf
Nf=8, β=3.8
f_π
mf
12^3x32
18^3x24
24^3x32
30^3x40
data for fit
c0+c1*mf+c2*mf^2
c1*mf+c2*mf^2
c0+c1*mf^δ
c1*mf^δ

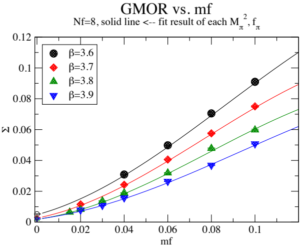

M_ρ vs. mf
Nf=8, β=3.8
M_ρ
mf
12^3x32
18^3x24
24^3x32
30^3x40
data for fit
c0+c1*mf+c2*mf^2
c1*mf+c2*mf^2
c0+c1*mf^δ
c1*mf^δ

Condensate: PBP=$Tr[S(x,x)]$ and GMOR relation; $\Sigma = \frac{f_\pi^2 m_\pi^2}{m_f}$



$PBP \sim m_f$, in contrast to $N_f = 4$ case.





Left: direct fit by quadratic, Right: Solid line $\leftarrow$ quadratic fit of each M_π^2 and f_π

The chiral condensate is not inconsistent with zero.

Short summary-1

$N_f = 8$ is consistent with ChPT behavior.

The chiral condensate in $N_f = 8$ is different from that in $N_f = 4$.
The (remnant of) conformal property??
$\rightarrow$ Hyperscaling analysis

Finite-size Hyperscaling analysis in $N_f = 8$

The mass deformed hyperscaling;

$$M_H \propto m_f^{\frac{1}{1+\gamma}} . \quad (1)$$

Finite-size Hyperscaling analysis;

$$M_H = \frac{1}{L}\mathcal{F}(X) \quad \text{where} \quad X = Lm_f^{\frac{1}{1+\gamma}} . \quad (2)$$

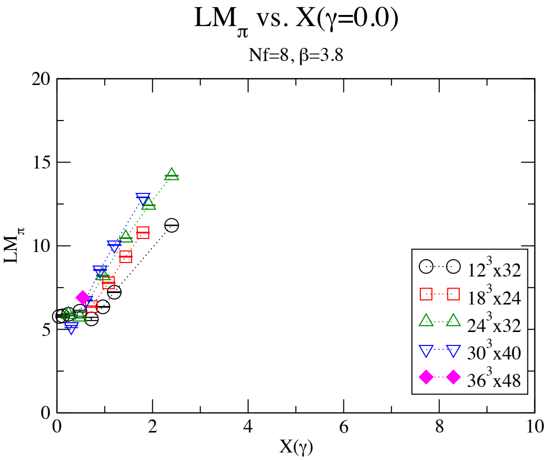
on $L^3 \times T$ at a fixed ratio, $\frac{L}{T}$.
Eq. (2) in the infinite volume limit $\longrightarrow$ Eq. (1)

$$\mathcal{F}(X) = C_0 + C_1 Lm_f^{\frac{1}{1+\gamma}} . \quad (3)$$

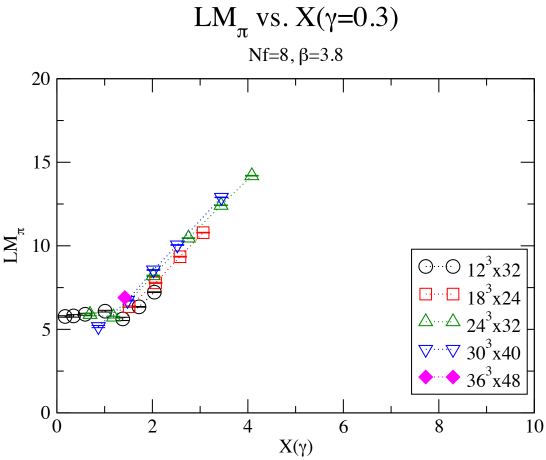
Thus, the finite-size hyperscaling

$$LM_H = C_0 + C_1 X, \quad \text{where} \quad X = Lm_f^{\frac{1}{1+\gamma}} . \quad (4)$$

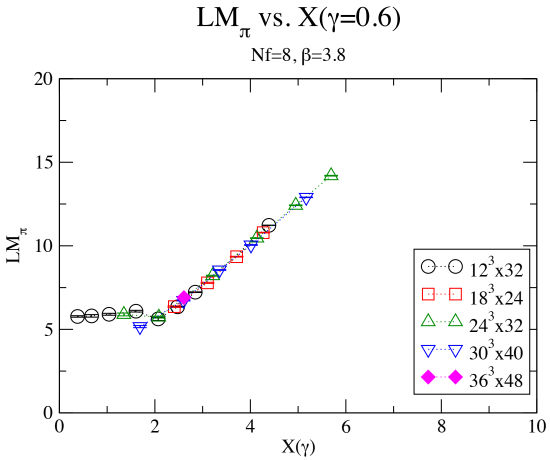
$N_f = 8,\ \beta = 3.8$


LMπ vs. X(γ=0.0)
Nf=8, β=3.8
LMπ
X(γ)
0
5
10
15
20
0
2
4
6
8
10
12³x32
18³x24
24³x32
30³x40
36³x48

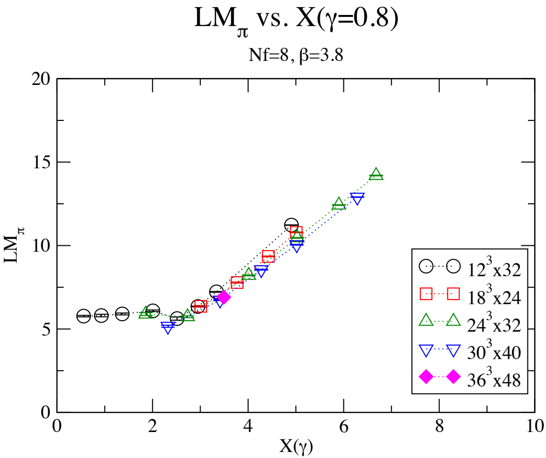
$N_f = 8, \beta = 3.8$


LMπ vs. X(γ=0.3)
Nf=8, β=3.8
LMπ
X(γ)
0
5
10
15
20
0
2
4
6
8
10
12^3x32
18^3x24
24^3x32
30^3x40
36^3x48

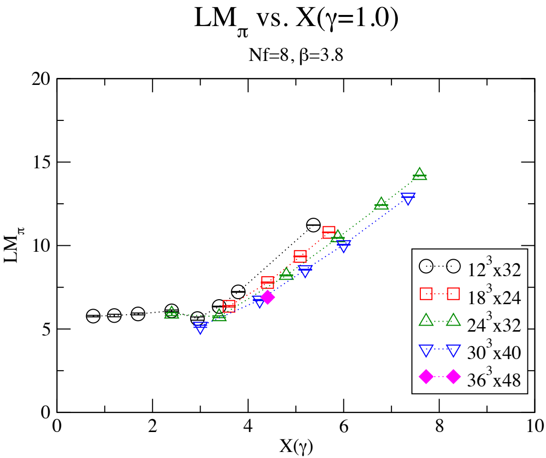
$N_f = 8,\ \beta = 3.8$


LMπ vs. X(γ=0.6)
Nf=8, β=3.8
LMπ
X(γ)
0
5
10
15
20
0
2
4
6
8
10
12³x32
18³x24
24³x32
30³x40
36³x48

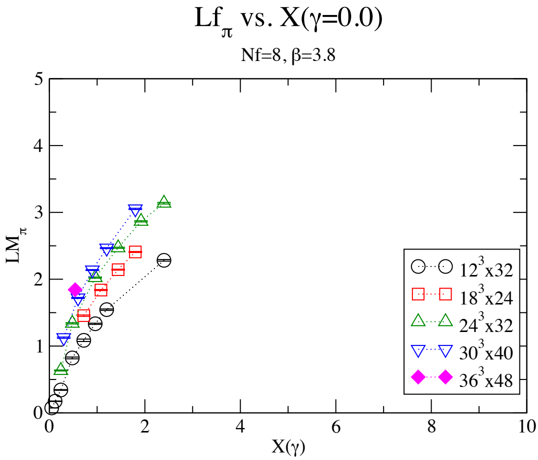
$N_f = 8,\ \beta = 3.8$


LMπ vs. X(γ=0.8)
Nf=8, β=3.8
20
15
10
5
0
LMπ
0
2
4
6
8
10
X(γ)
12³x32
18³x24
24³x32
30³x40
36³x48

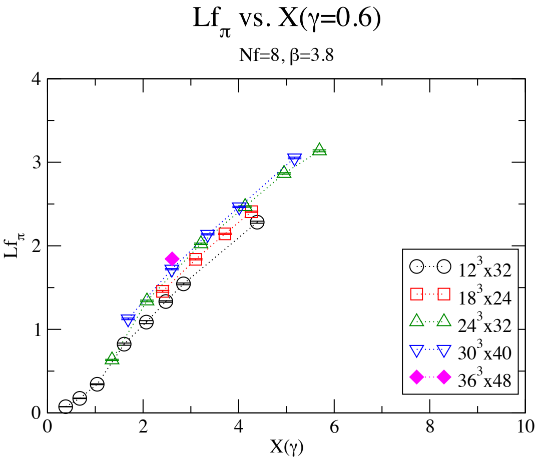
$N_f = 8, \beta = 3.8$


LMπ vs. X(γ=1.0)
Nf=8, β=3.8
LMπ
X(γ)
0
5
10
15
20
0
2
4
6
8
10
12³x32
18³x24
24³x32
30³x40
36³x48

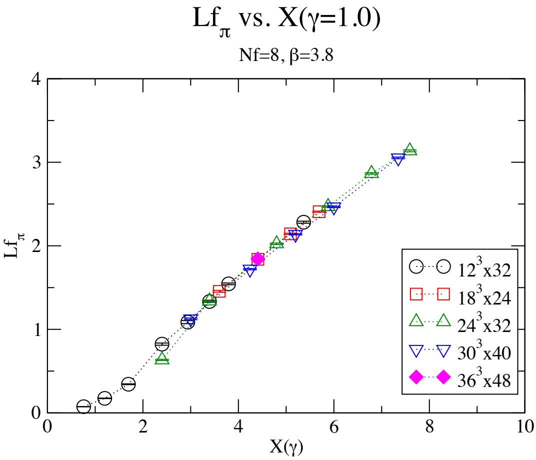
$N_f = 8,\ \beta = 3.8$


Lf_π vs. X(γ=0.0)
Nf=8, β=3.8
LM_π
X(γ)
0
1
2
3
4
5
0
2
4
6
8
10
12³x32
18³x24
24³x32
30³x40
36³x48

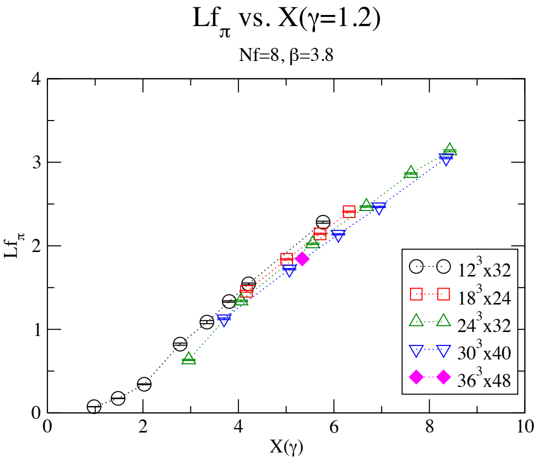
$N_f = 8,\ \beta = 3.8$


Lf_π vs. X(γ=0.6)
Nf=8, β=3.8
Lf_π
X(γ)
0
1
2
3
4
0
2
4
6
8
10
12^3x32
18^3x24
24^3x32
30^3x40
36^3x48

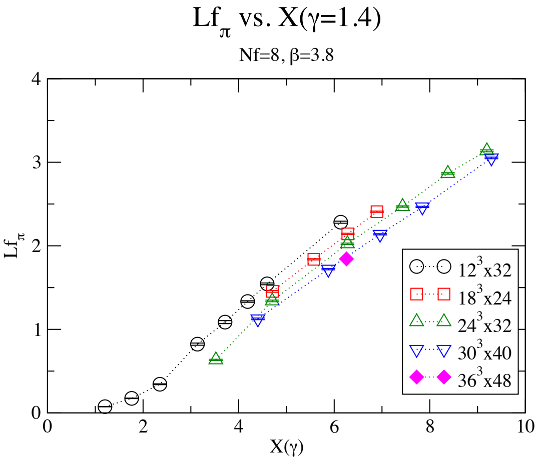
$N_f = 8$, $\beta = 3.8$


Lf_π vs. X(γ=1.0)
Nf=8, β=3.8
Lf_π
X(γ)
0
1
2
3
4
0
2
4
6
8
10
12^3x32
18^3x24
24^3x32
30^3x40
36^3x48

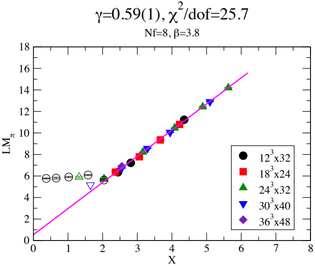
$N_f = 8, \beta = 3.8$

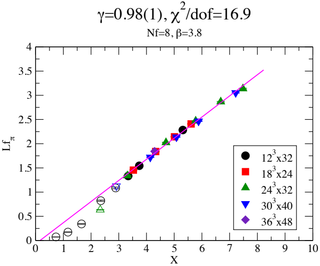

Lf_π vs. X(γ=1.2)
Nf=8, β=3.8
Lf_π
X(γ)
0
1
2
3
4
0
2
4
6
8
10
12^3x32
18^3x24
24^3x32
30^3x40
36^3x48

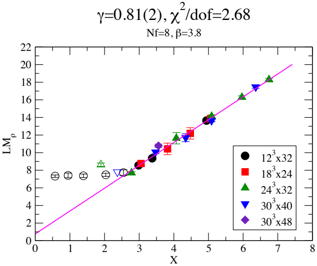
$N_f = 8,\ \beta = 3.8$


Lfπ vs. X(γ=1.4)
Nf=8, β=3.8
Lfπ
X(γ)
0
1
2
3
4
0
2
4
6
8
10
12³x32
18³x24
24³x32
30³x40
36³x48

$N_f = 8, \beta = 3.8$


γ=0.59(1), χ²/dof=25.7
Nf=8, β=3.8
LMπ
X
12³x32
18³x24
24³x32
30³x40
36³x48


γ=0.98(1), χ²/dof=16.9
Nf=8, β=3.8
Lfπ
X
12³x32
18³x24
24³x32
30³x40
36³x48


γ=0.81(2), χ²/dof=2.68
Nf=8, β=3.8
LMρ
X
12³x32
18³x24
24³x32
30³x40
30³x48

Short summary-2

	$\beta = 3.6$	$\beta = 3.7$	$\beta = 3.8$	$\beta = 3.9$
γ in M_π	0.64(1)	0.60(1)	0.59(1)	0.57(1)
γ in f_π	0.98(2)	0.99(1)	0.98(1)	0.94(1)
γ in M_ρ	1.02(2)	0.91(3)	0.81(2)	0.85(3)

Table: the statistical error only

For $N_f = 8$, $0.5 < \gamma \lesssim 1.0$.
However, $\gamma(M_\pi) \neq \gamma(f_\pi)$
$\rightarrow$ What's the meaning?? Walking??

Summary

♠ In LatKMI collaboration, the investigation of the many flavor QCD ($N_f = 0$, 4, 8, 12 and 16) on KMI computer system "φ", for IRFP search and the exploration of the walking behavior.
♠ Tree level Symanzik gauge action $+$ HISQ staggered fermion for many flavor system. (first attempt)
♠ $N_f = 4$ case shows the property of the χSB (hadronic phase).
♠ $N_f = 8$ is consistent with ChPT.
$+$ the remnant of the conformal property.
$\Longrightarrow$ Walking??
♠ Signal of the walking/conformal, far away from the IRFP??
Ex) mass and finite volume effect in SD $\rightarrow$ M. Kurachi's talk
♠ Effective theories/models (in vitro), corresponding to the role of ChPT??
♠ To make "the particle data book" (singlet-scalar, glueball, condensate, string tension, η', *etc.*) in $N_f = 8$, and compare it with $N_f = 4$.