Thermodynamic Study for (Pre-) Conformal and Dynamics in Many Flavor QCD

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Talk at KMI-SCGT, Nagoya Univ., March. 20, 2012

References

References

- K. Miura, M. P. Lombardo and E. Pallante, "Chiral phase transition at finite temperature and conformal dynamics in large N_f QCD," To appear in Phys. Lett. B, arXiv:1110.3152 [hep-lat].
- K. Miura, M. P. Lombardo and E. Pallante, "Thermodynamic Study for Conformal Phase in Large Nf Gauge Theory," PoS Lattice 2011, arXiv:1111.1098 [hep-lat].

Tools

- **MILC Code:** http://www.physics.utah.edu/˜detar/milc/milc_qcd.html
- **Argolithm:** Rational Hybrid Molecular-Dynamics with Omelyan-Integrator
- **Computers:** IBM-sp6 in CINECA, SP16000 in YITP, and Italian-Grid-Infrastructures

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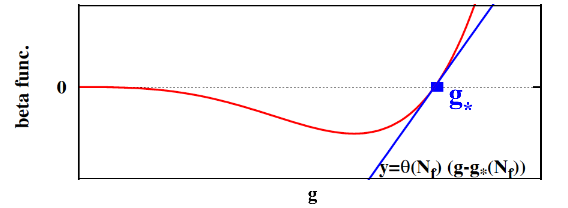
Scaling Property near IRFP



$$\frac{dg^2}{d\log(\mu/\Lambda_{\rm UV})} = \theta(N_f)\,(g^2 - g_*^2(N_f))\ , \qquad (1)$$

$$\mu_\chi \sim A(N_f)\left[g_\chi^2(N_f) - g_*^2(N_f)\right]^{1/\theta(N_f)}\ , \qquad (2)$$

$$N_f^* \ : \ \text{Lower Bound of CW.} \quad g_\chi^2(N_f^*) = g_*^2(N_f^*)\ , \qquad (3)$$

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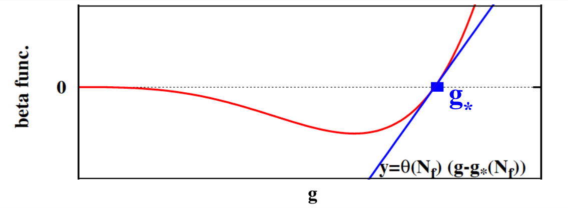

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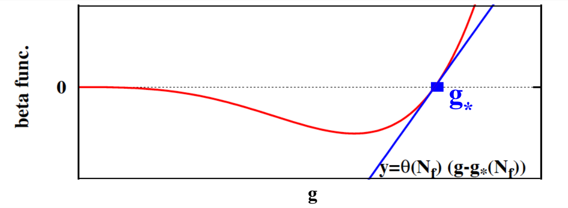

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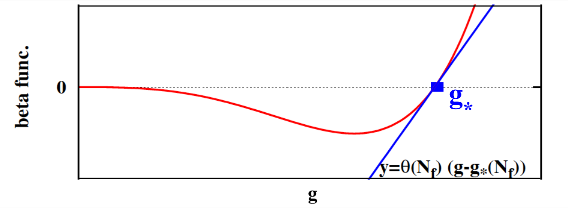

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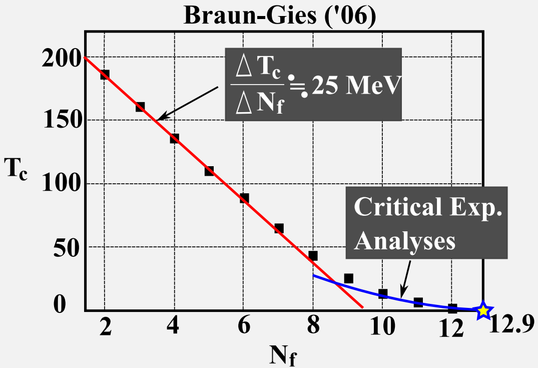
$T - N_f$ Phase Diagram: Functional Renormalization Group



$$T_c \propto \left| N_f - N_f^* \right|^{2.54} \quad (\text{Braun-Gies('06,'09,'11)}) \; , \tag{5}$$

$$c.f.\text{Miransky-Yamawaki Scaling ('97)}: \; T_c \sim \exp\left[-\frac{B(N_f^*)}{\sqrt{|N_f^* - N_f|}} \right] \; . \tag{6}$$

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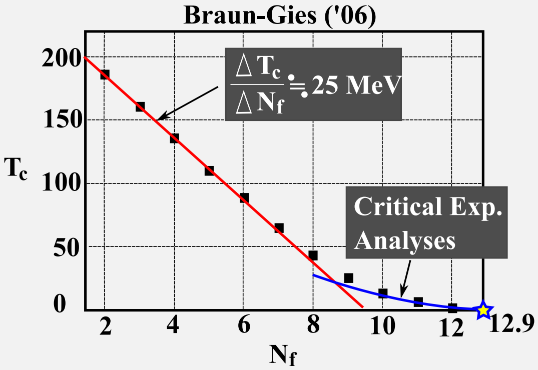

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Goal

Ultimate Goal

- We investigate finite T chiral phase transitions in $N_f = 0,\ 4,\ 6,\ 8,$ (and 12) cases, (and more in future).
- The lower edge of the conformal window is extracted from a vanishing $T_c(N_f)$.
- Miranski and/or BG Scaling are interesting physics, and useful to find and study the walking region.

Six Flavor QCD-Like? Theory

- Six flavor QCD is expected to be in the region of walking (pre-conformal) dynamics (Appelquist ('11)).
- The chiral and deconfinement dynamics at finite temperature has not been investigated, while they provide important data to complete the $T_c - N_f$ phase diagram.

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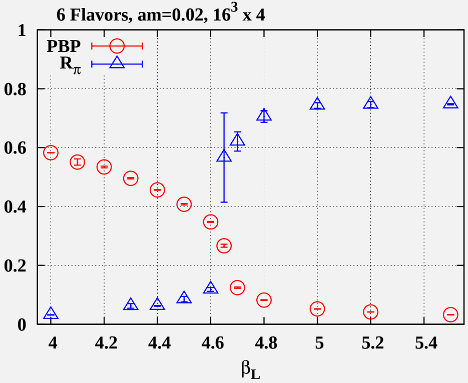
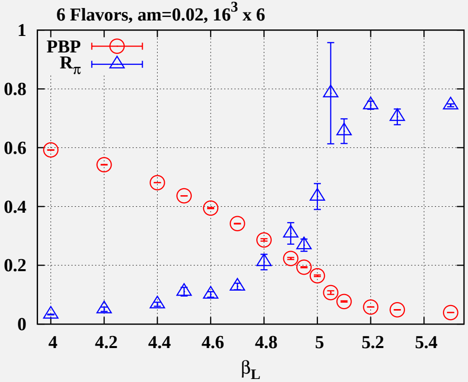
Critical Coupling Determination from R_π, $N_t = 4$



$$R_\pi \equiv \frac{\chi_\sigma}{\chi_\pi} \rightarrow \begin{cases} 0 & \text{(Broken)} \\ 1 & \text{(Restored)} \end{cases} . \tag{7}$$

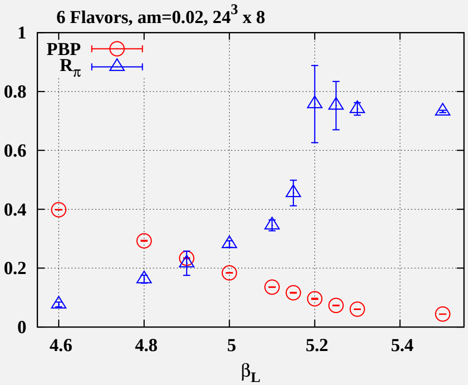
$$\beta_{\mathrm{L}}{}^{\mathrm{c}} = 4.65 \pm 0.05 \ .$$

Critical Coupling Determination from R_π, $N_t = 6$



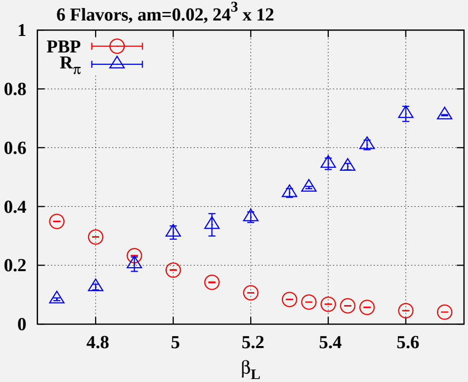
$$\beta_L{}^c = 5.05 \pm 0.05 \ .$$

Critical Coupling Determination from R_π, $N_t = 8$



$$\beta_L{}^c = 5.2 \pm 0.05 \ .$$

Critical Coupling Determination from R_π, $N_t = 12$



$$\beta_L{}^c = 5.45 \pm 0.15\ .$$

From $\beta_{\mathrm{L}}{}^{\mathrm{c}}$ To A Physical Quantity, T_c/Λ_{L}

Table: The summary table of β_c at $N_f = 6$ with $ma = 0.02$.

$N_f \backslash N_t$	4	6	8	12
6	4.65 ± 0.05	5.05 ± 0.05	5.2 ± 0.05	5.45 ± 0.15

Comment

- We have used a single lattice mass $ma = 0.02$.
- Thermal scaling property indicates that $N_f = 6$ looks out of conformal window (*c.f.* Bulk Transition at $N_f = 12$ (Deuzaman et al, ('10))).

$$\Lambda_L a(\beta_c) = \left(\frac{\beta}{2N_c b_0}\right)^{(b1/(2b_0^2))} \exp\left[-\frac{\beta_c}{4N_c b_0}\right], \quad \text{(2-Loop)}, \tag{8}$$

$$\frac{1}{N_\tau} = \frac{T_c}{\Lambda_{\mathrm{L}}} \cdot \Lambda_{\mathrm{L}} a(\beta_c). \tag{9}$$

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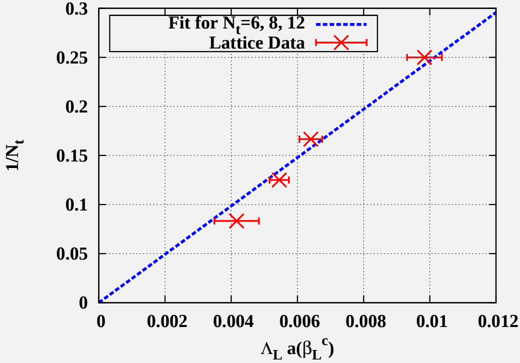
Asymptotic Scaling and Its Violation



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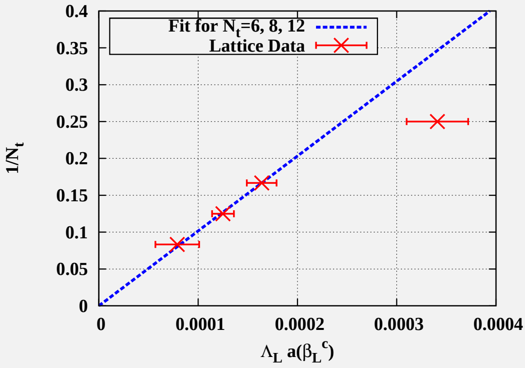

$$\frac{1}{N_\tau} = \frac{T_c}{\Lambda_{\rm L}} \cdot \Lambda_{\rm L} a(\beta_c) , \quad \beta = 10/g_0^2 \to 6/g^2 . \qquad (11)$$

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Collection of β_c, for several N_f

Table: The summary table of β_c. The values are obtained by using the same action except the number of flavors. Blue: Deuzeman-Lombardo-Pallante ('08).

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0	-	7.88 ± 0.05	-	-
4	-	5.89 ± 0.03	-	
6	4.675 ± 0.025	5.025 ± 0.025	5.225 ± 0.025	5.45 ± 0.05
8	-	4.1125 ± 0.0125	-	4.34 ± 0.04

Comment

- We have used a single lattice mass $ma = 0.02$.
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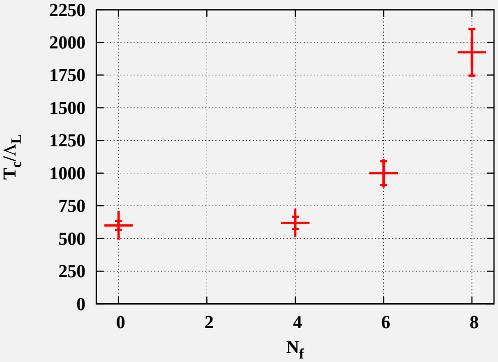

Enhancement of $T_c/\Lambda_{\rm L}$

- Vanishing IR-Scale $\Lambda_{\rm L}$ with closer to the conformal window.
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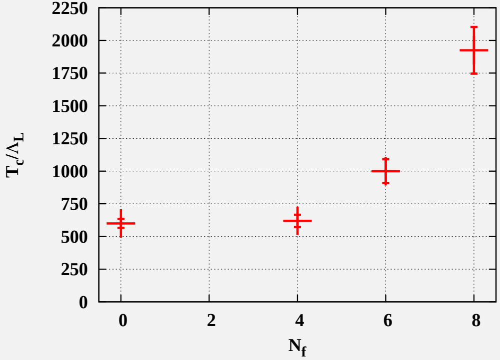

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Reference-Scale M: Renomalization Scale

Integrating 2-loop beta function, $dg/dM = -g^3(b_0 + b_1 g^2)$, from $M(\beta_{\mathrm{ref}})$ to $a^{-1}(\beta_c)$, We obtain

$$M(\beta_{\mathrm{ref}}) \times a(\beta_c) = \left(\frac{b_0^2}{b_1} \frac{\beta_c + 2N_c b_1/b_0}{\beta_{\mathrm{ref}} + 2N_c b_1/b_0} \right)^{b1/(2b_0^2)} \exp\left[-\frac{\beta_c - \beta_{\mathrm{ref}}}{4N_c b_0} \right] . \quad (13)$$

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- $M(\beta_{\mathrm{L}}^{\mathrm{ref}})$: Renormalization scale in 2-loop perturbations.
- c.f: Braun-Gies ('06), $\alpha_s(M = 1.7\mathrm{GeV}) = 0.322$.

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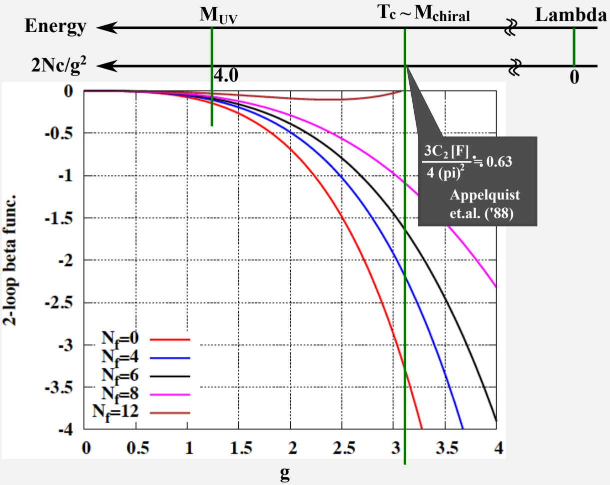
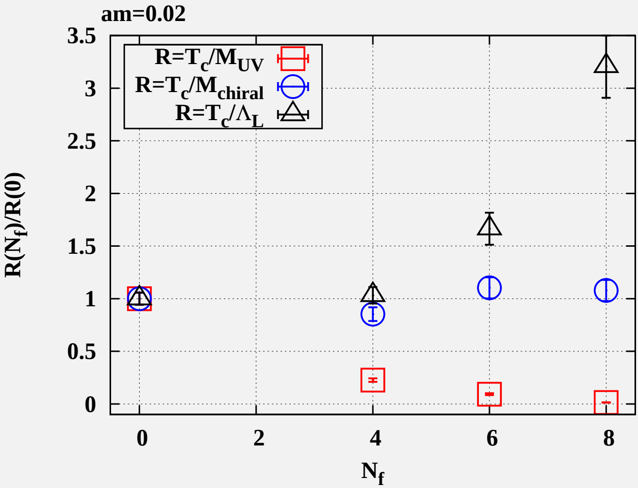
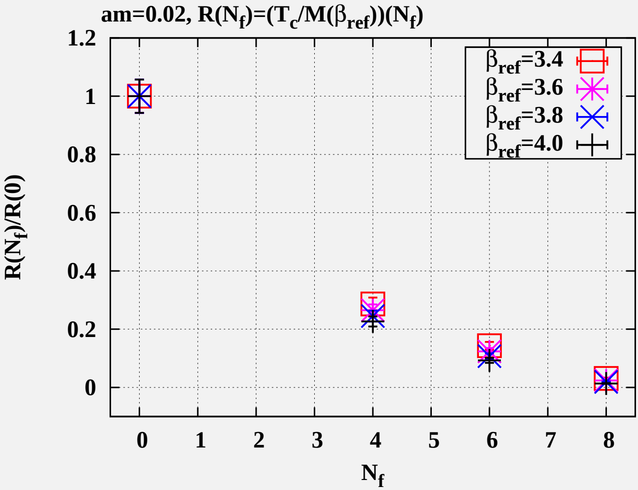
Reference-Scale M II



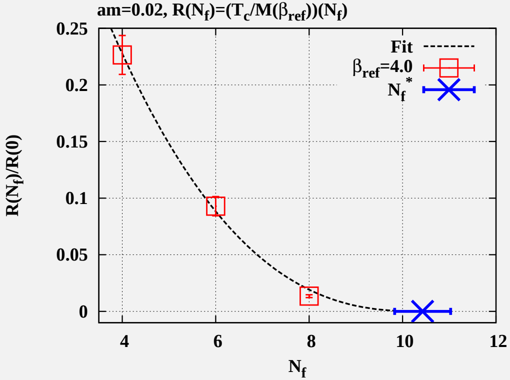
T_c/M as a function of N_f



T_c/M as a function of N_f: UV M



Critical Flavor Number N_f^*



$$
\begin{aligned}
T_c(N_f) &= K|N_f - N_f^*|^{2.54}\ , \quad \text{(Braun-Geis ('11))} && (14)\\
N_f^* &= 10.43 \pm 0.6\ , \quad \text{(Presnt Work with } \beta_{\rm ref}{=}4.0) && (15)
\end{aligned}
$$

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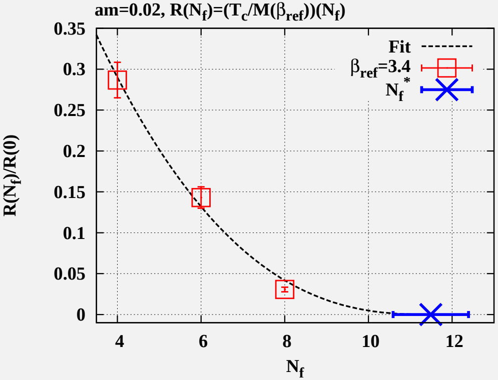
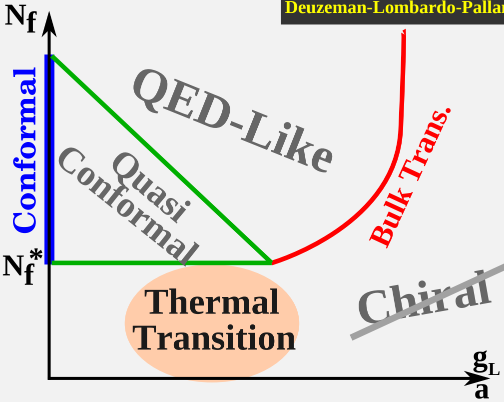


$$T_c(N_f) = K|N_f - N_f^*|^{2.54} , \quad \text{(Braun-Geis ('11))} \quad (16)$$

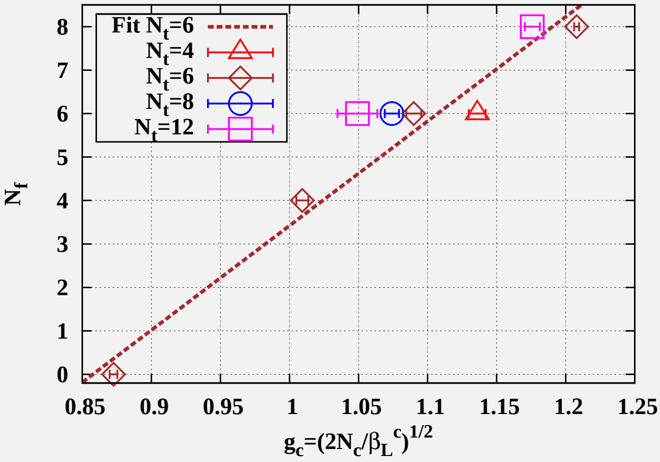
$$N_f^* = 11.49 \pm 0.9 , \quad \text{(Presnt Work with } \beta_{\mathrm{ref}}=3.4) \quad (17)$$

Miransky-Yamawaki Phase Diagram: Naive Speculation

Banks-Zaks ('82)
Miransky-Yamawaki ('97)
Deuzeman-Lombardo-Pallante ('10)



Thermal Transition Lines in Miransky-Yamwaki Phase Diagram



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- To clarify the $T_c - N_f$ and the MY phase diagram is important to investigate the (pre-) conformal nature of a strongly flavored gauge theory.
- We have investigated a finite T chiral phase transition (crossover) in the QCD-Like theory with six flavors, and observed thermal scaling property of the critical coupling.
- The ratio T_c/Λ_{L} starts increasing from $N_f = 6$, which implies a change of dynamics.
- The ratio T_c/M_{UV} with a UV renormalization scale M_{UV} turns to be a decreasing function of N_f. This property would be consistent with FRG investigations with the use of a UV reference scale.

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- The ratio T_c/M_{UV} with a UV renormalization scale M_{UV} turns to be a decreasing function of N_f. This property would be consistent with FRG investigations with the use of a UV reference scale.

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Future Works

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- To set a scale a^{-1} from zero T simulatoin and complete $T - N_f$ and MY Phase Diagram: Work in Progress!
- Critical behavior near the IR-Fixed Pt.
- The color SU($N_c = 2$) with 8 flavors at finite T.