Study of the hyperscaling relation with the Schwinger-Dyson equation

Masafumi Kurachi
KMI, Nagoya University

SCGT12 mini, March 18-20, 2012
KMI, Nagoya University

LatKMI Collaboration

Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa,
K. Nagai, H. Ohki, A. Shibata, K. Yamawaki, and T. Yamazaki

Reference:
arXiv: 1201.4157 [hep-lat]
(to be published in PRD)

Contents

Motivation

Technicolor model is an attractive candidate for the origin of the Electroweak symmetry breaking, though one based on naive scale up of QCD is phenomenologically disfavored...

→ **Walking Technicolor**

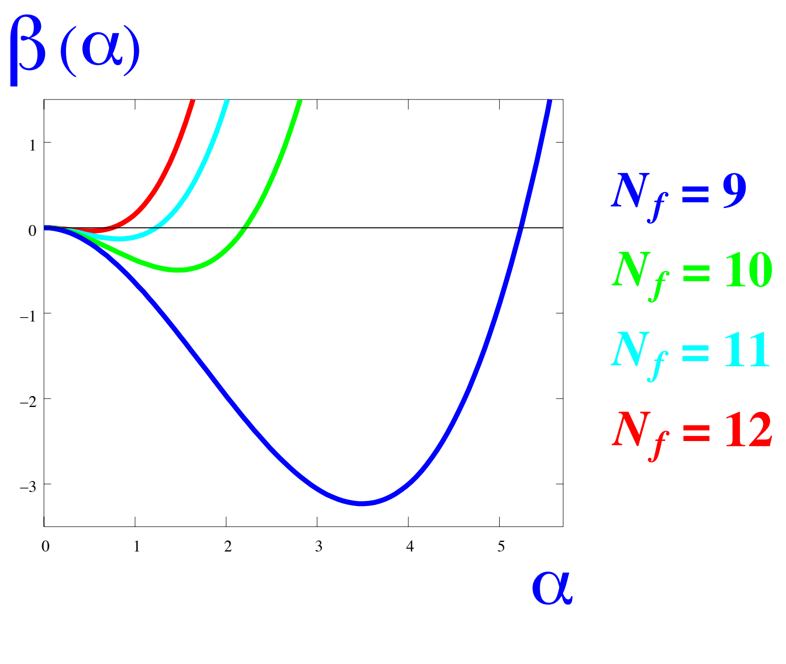
Yamawaki-Bando-Matumoto (1986)

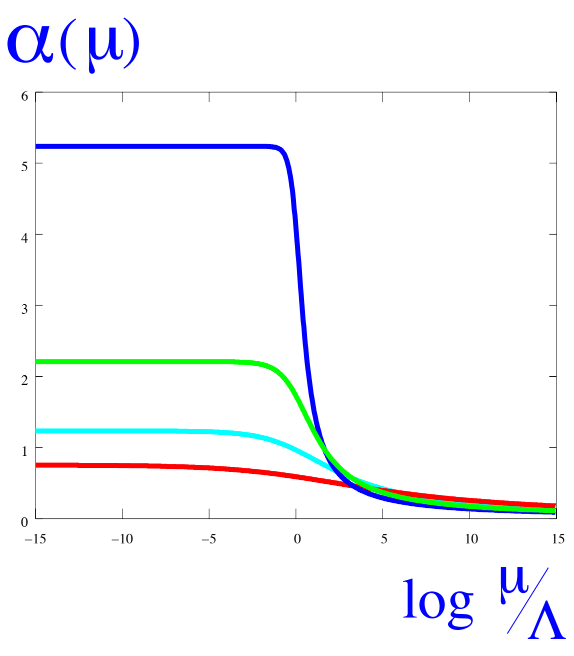
A theory which has an (approximate) infrared fixed point with large mass anomalous dimension is preferable

Large flavor QCD

RGE $\mu \frac{d}{d\mu}\alpha(\mu) = \beta(\alpha) = -b\,\alpha^2(\mu) - c\,\alpha^3(\mu)$

$(N_c = 3)$	$N_f < 8.05$	$8.05 < N_f < 16.5$	$16.5 < N_f$
$b = \frac{1}{6\pi}(33 - 2N_f)$	+	+	−
$c = \frac{1}{12\pi^2}(153 - 19N_f)$	+	−	−





← Indication from the 2-loop analysis

⋮

Fully non-perturbative (lattice) study is desirable

Several ways of checking infrared conformality

- measure the running coupling
- study the spectrum
- ....

↑ We discuss this here

Hyperscaling relation (review)

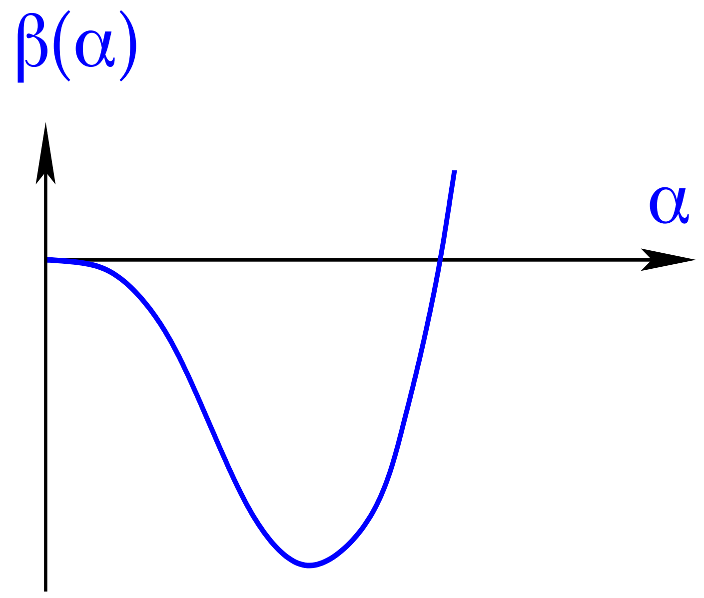
Ref : Miransky, PRD59 105003, 1999

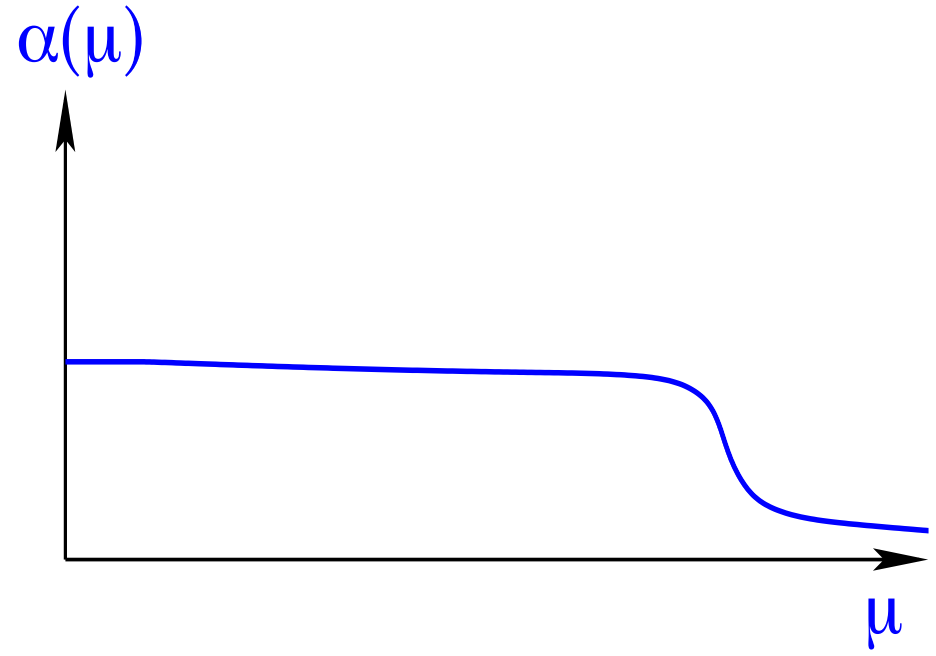
Del Debbio, Zwicky, PRD82 014502, 2010

Hyperscaling relation (review)

Ref : Miransky, PRD59 105003, 1999
Del Debbio, Zwicky, PRD82 014502, 2010

- Consider a theory which has an IRFP

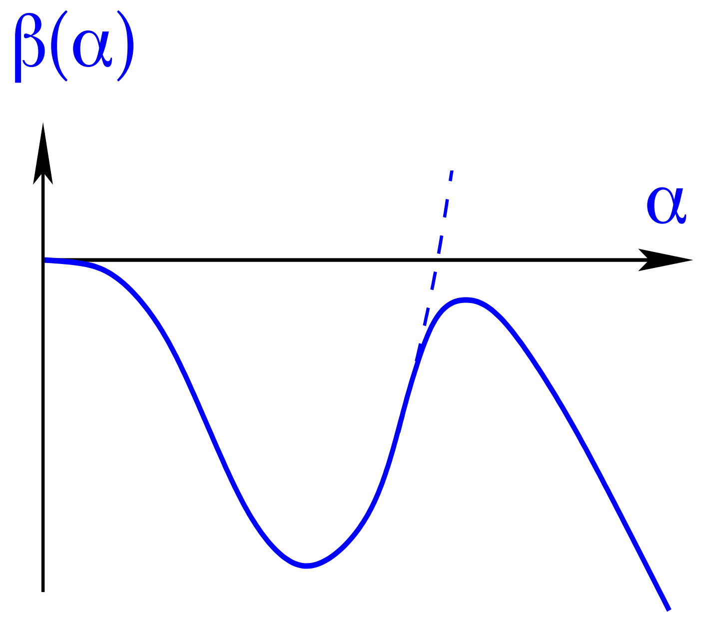


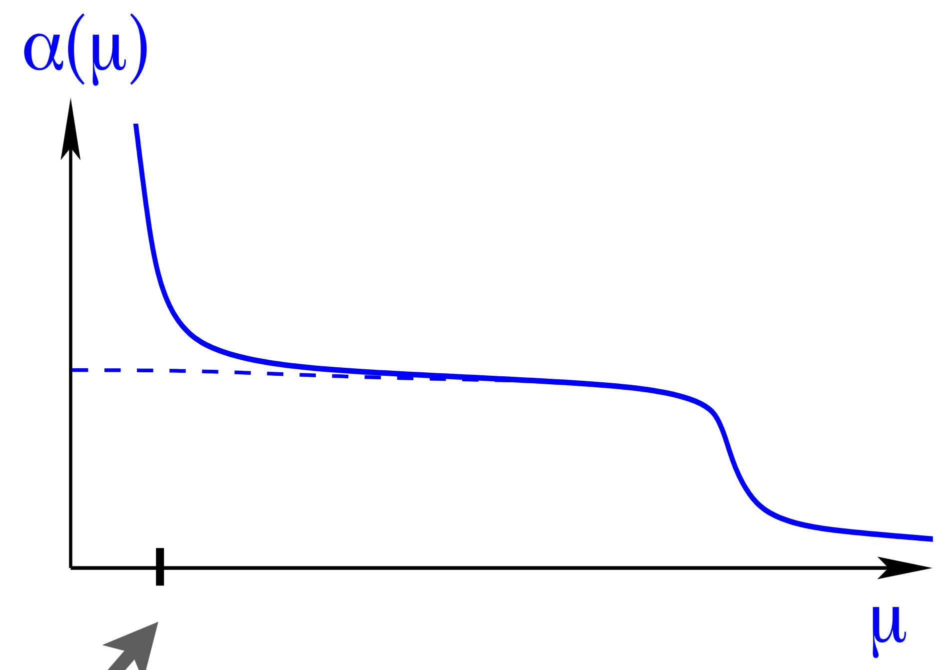


Hyperscaling relation (review)

Ref : Miransky, PRD59 105003, 1999
Del Debbio, Zwicky, PRD82 014502, 2010

- Consider a theory which has an IRFP
- Deform it by introducing small fermion bare mass m



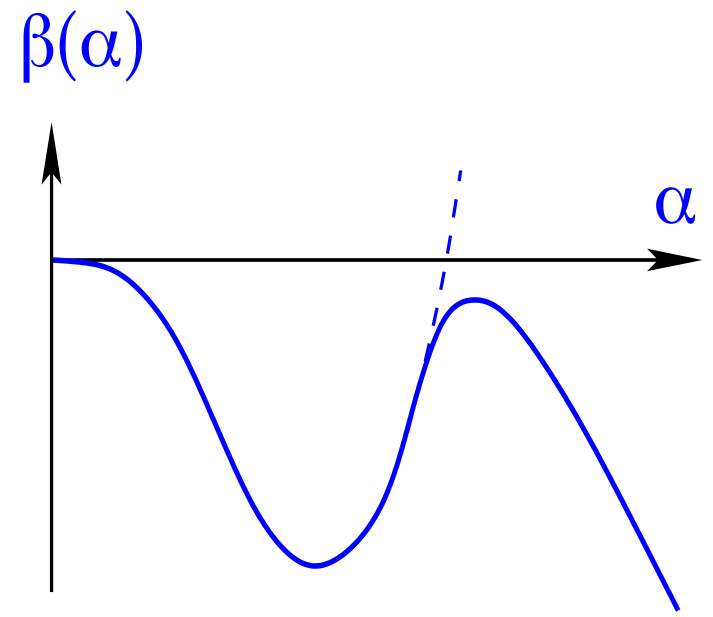


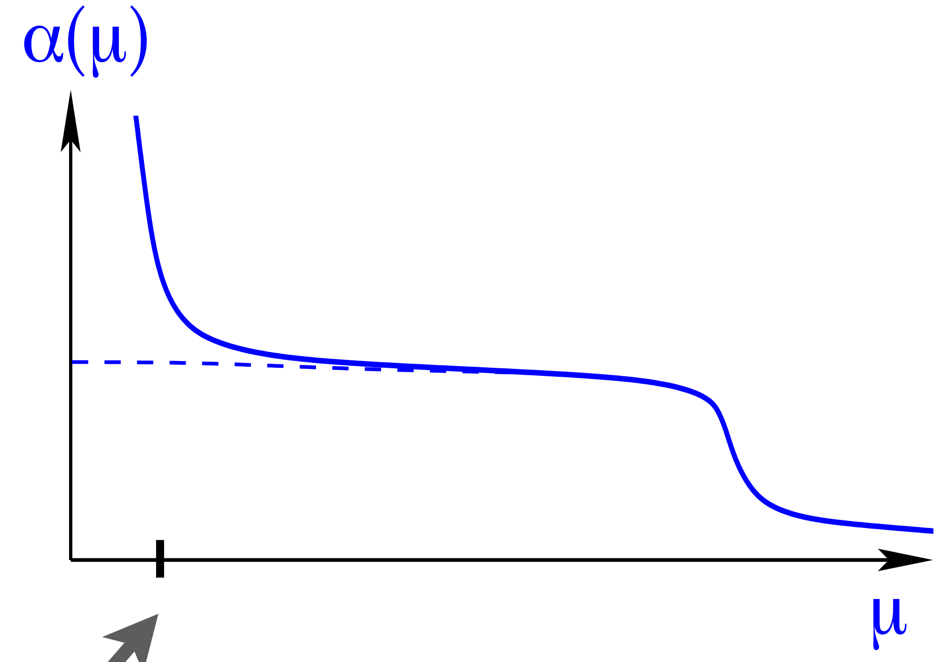
Hadrons emerge at this scale

Hyperscaling relation (review)

Ref : Miransky, PRD59 105003, 1999
Del Debbio, Zwicky, PRD82 014502, 2010

- Consider a theory which has an IRFP
- Deform it by introducing small fermion bare mass m





Hadrons emerge at this scale

Relations between low-energy physical quantities (for example, a hadron mass M_H) and the bare fermion mass m can be derived

$$\boxed{M_H \simeq (\mathrm{const.})\ \mu\ \hat{m}^{1/1+\gamma_*}}$$

mass anomalous dimension at the fixed point

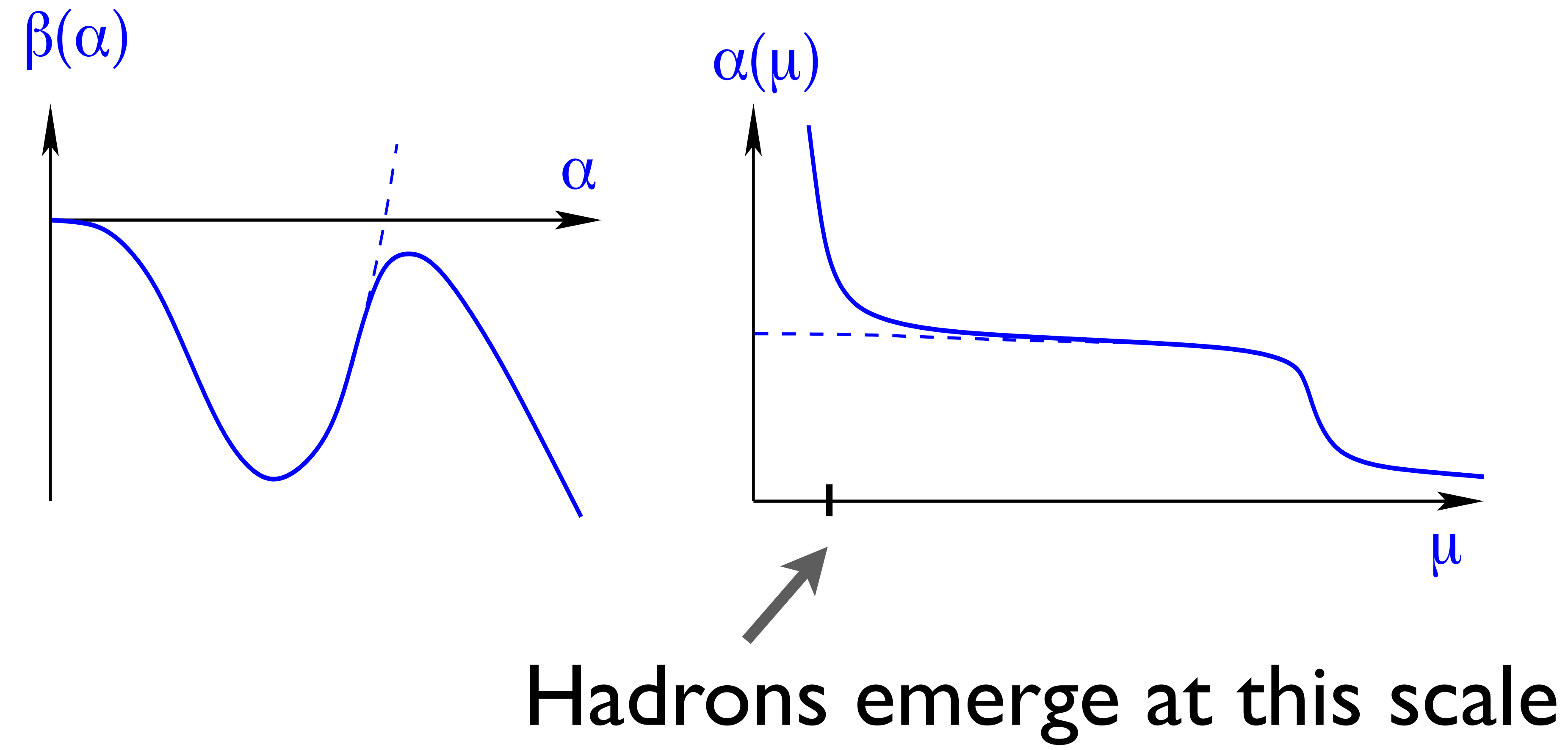
(normalized) bare quark mass: $\hat{m}(\mu) = m(\mu)/\mu$

a scale (in the case of lattice, one can consider $\mu = a^{-1}$)

Hyperscaling relation (review)

Ref : Miransky, PRD59 105003, 1999
Del Debbio, Zwicky, PRD82 014502, 2010

- Consider a theory which has an IRFP
- Deform it by introducing small fermion bare mass m



Relations between low-energy physical quantities (for example, a hadron mass M_H) and the bare fermion mass m can be derived

When one considers a theory in a finite volume $\sim L^4$

$$\boxed{M_H = L^{-1} f\,(x) \quad \text{where } x = \hat{L}\ \hat{m}^{1/(1+\gamma_*)}}$$

$$\hat{L} \equiv L\mu$$

Hyperscaling relation (review)

Results of lattice simulation with various values of input $(\hat{L}, \hat{m})$
should satisfy the hyperscaling relation (with an appropriate value of γ_*)
if the theory has an IRFP

(Wouldn't it be interesting to see that **all the data** you have
with **different values of** $(\hat{L}, \hat{m})$ align in a single curve?)

Many lattice groups use this method to judge whether a theory
is conformal, and if it is, to estimate the value of γ_*

Hyperscaling relation (review)

Results of lattice simulation with various values of input $(\hat{L}, \hat{m})$
should satisfy the hyperscaling relation (with an appropriate value of γ_*)
if the theory has an IRFP

Couple of questions arise here:

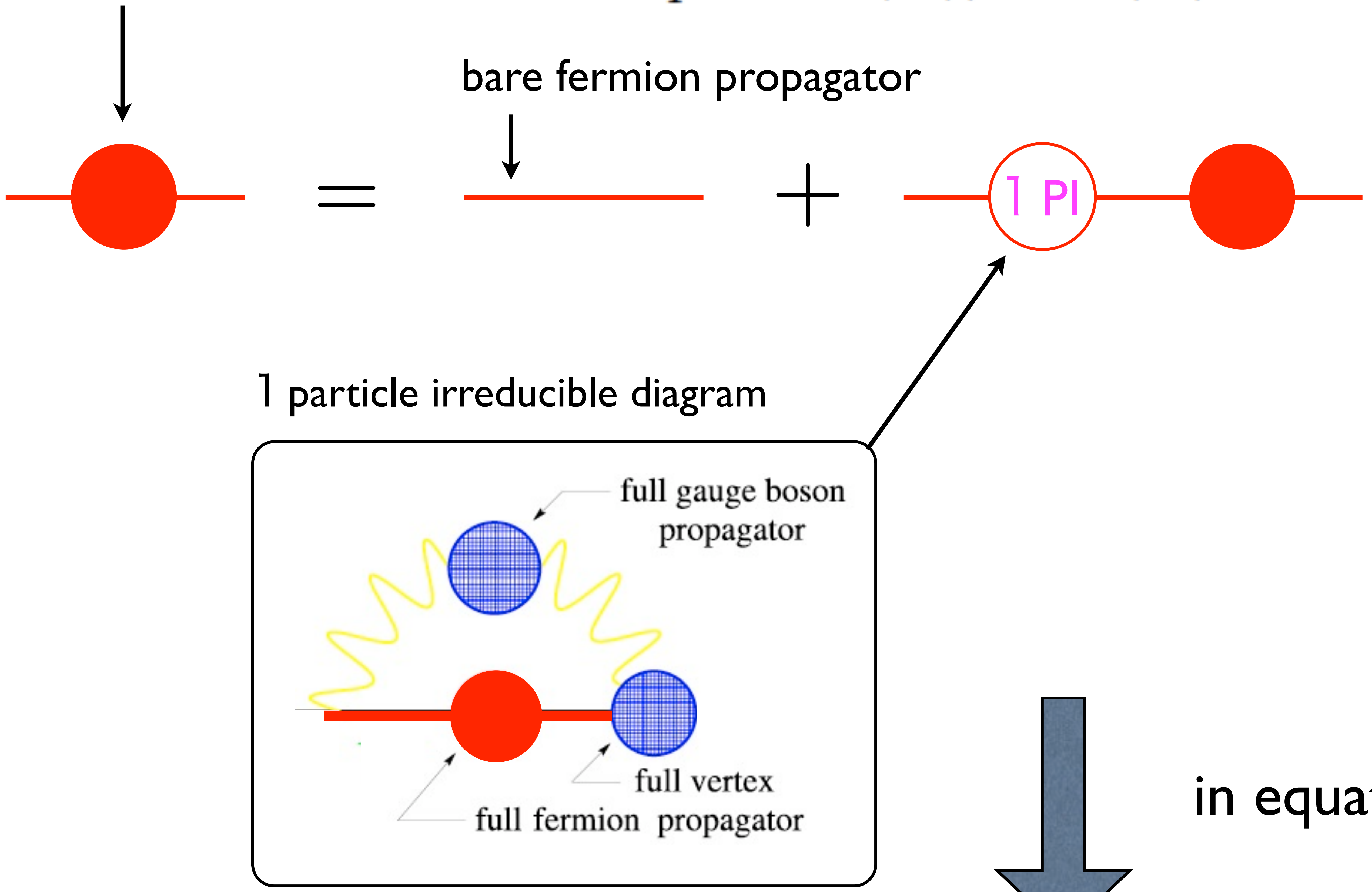
- How small m has be to observe the scaling?
  (What is the form of correction when it's not small enough?)
- When the original theory does not have an IRFP,
  how and how much the scaling relation is violated?

Schwinger-Dyson equation is a useful tool for such studies

- we know the phase structure, and a value of γ_* for a given theory
- analytic understanding can be obtained (to a certain extent)
- numerical calculations can be easily done in a wide range of parameter space (on your Mac (or PC))

Schwinger-Dyson equation (review)

Self-consistent equation for the
full fermion propagator $iS_F^{-1} \equiv A(p^2)\not{p} - B(p^2)$



in equation...

Schwinger-Dyson equation (review)

Self-consistent equation for the
full fermion propagator $iS_F^{-1} \equiv A(p^2)\not{p} - B(p^2)$

$$iS_F^{-1}(p) = \not{p} - m$$
$$+ C_2 \int \frac{d^4k}{i(2\pi)^4}\, \bar{g}^2(p,k)\, \frac{1}{(p-k)^2} \left(g_{\mu\nu} - \frac{(p-k)_\mu (p-k)_\nu}{(p-k)^2} \right) \gamma^\mu\, iS_F(k)\, \gamma^\nu$$

We adopted...
Landau gauge
Improved ladder approximation

$$C_2 = \frac{N_C^2 - 1}{2N_C}$$

$\bar{g}(p,k)$: running coupling

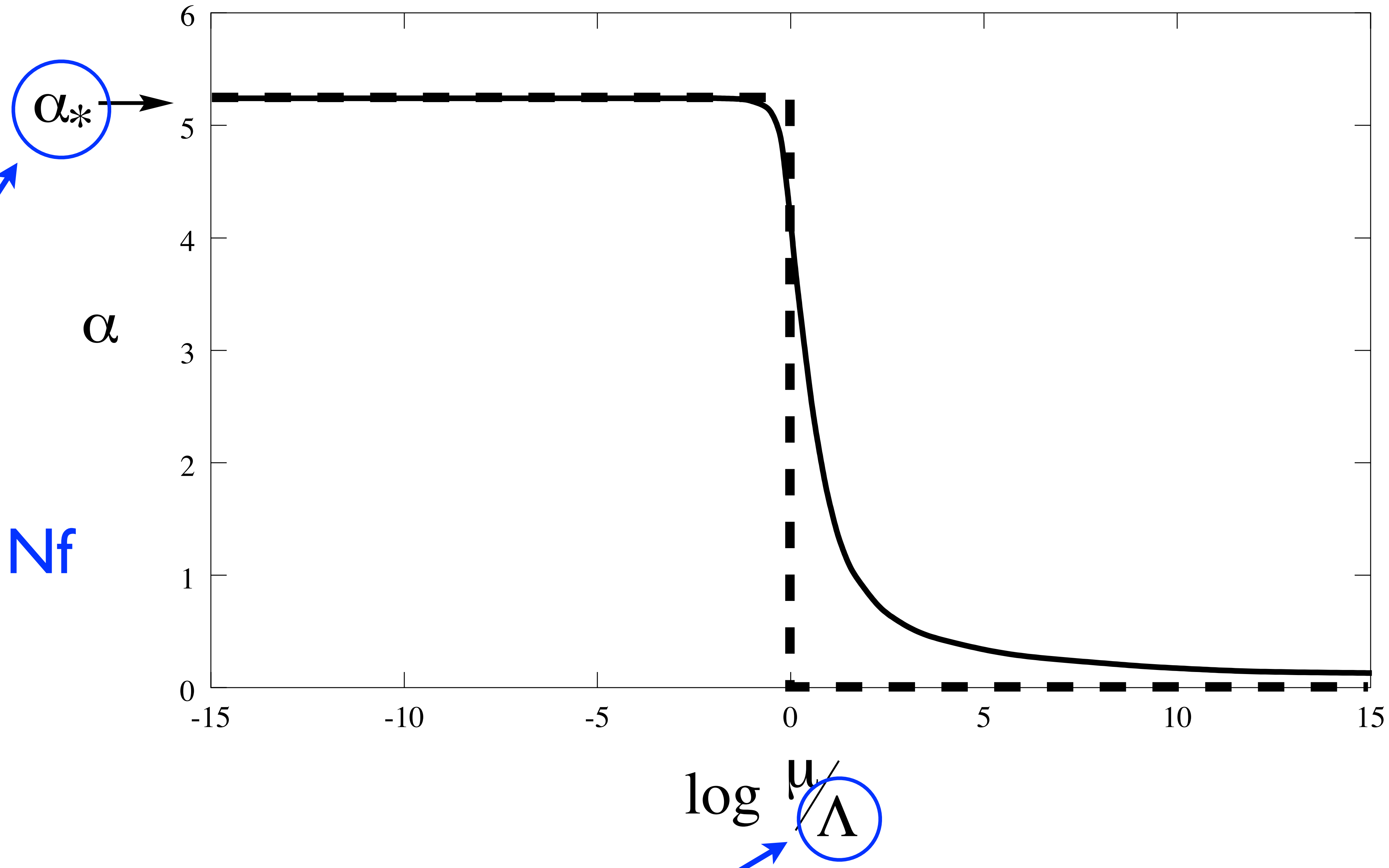
coupled equation for
$A(p^2),\ B(p^2)$
mass function
$\Sigma(p^2) \equiv B(p^2)/A(p^2)$

Schwinger-Dyson equation (review)

setup for the current study

2-loop running coupling of the SU(Nc) gauge theory

Running coupling is approximated by the step function



fixed by Nc, Nf

Energy scale where the coupling begins to run

Schwinger-Dyson equation (review)

After several minor approximations... integral equation can be rewritten in a form of linear differential equation with UV and IR boundary conditions:

$$(x\Sigma(x))'' + \alpha_* \frac{3C_2}{4\pi} \frac{\Sigma(x)}{x + m_P^2} = 0$$

$$\mathrm{IRBC}: \quad \lim_{x \to 0} x^2 \Sigma(x)' = 0$$

$$\mathrm{UVBC}: \quad (x\Sigma(x))' \big|_{x=\Lambda^2} = m$$

where

$$x \equiv p_E^2$$

$$m_P \equiv \Sigma(x = m_P^2)$$

Schwinger-Dyson equation (review)

After several minor approximations... integral equation can be rewritten in a form of linear differential equation with UV and IR boundary conditions:

$$(x\Sigma(x))'' + \alpha_* \frac{3C_2}{4\pi} \frac{\Sigma(x)}{x + m_P^2} = 0$$

$$\mathrm{IRBC}: \quad \lim_{x \to 0} x^2 \Sigma(x)' = 0$$

$$\mathrm{UVBC}: \quad (x\Sigma(x))'\big|_{x=\Lambda^2} = m$$

where

$$x \equiv p_E^2$$

$$m_P \equiv \Sigma(x = m_P^2)$$

m_P is sometimes called the "pole mass", though it's not. Anyway, we identify this quantity as a typical scale of low-energy physical quantity.

Schwinger-Dyson equation (review)

After several minor approximations... integral equation can be rewritten in a form of linear differential equation with UV and IR boundary conditions:

$$(x\Sigma(x))'' + \alpha_* \frac{3C_2}{4\pi} \frac{\Sigma(x)}{x + m_P^2} = 0$$

$$\mathrm{IRBC}: \quad \lim_{x \to 0} x^2 \Sigma(x)' = 0$$

$$\mathrm{UVBC}: \quad (x\Sigma(x))'\big|_{x=\Lambda^2} = m$$

where

$$x \equiv p_E^2$$

$$m_P \equiv \Sigma(x = m_P^2)$$

m_P is sometimes called the "pole mass", though it's not. Anyway, we identify this quantity as a typical scale of low-energy physical quantity.

Note: $m_P \propto M_H$ is observed from the analysis based on BS equation

Schwinger-Dyson equation (review)

$$(x\Sigma(x))'' + \alpha_* \frac{3C_2}{4\pi} \frac{\Sigma(x)}{x + m_P^2} = 0$$

$$\mathrm{IRBC:} \quad \lim_{x \to 0} x^2 \Sigma(x)' = 0$$

$$\mathrm{UVBC:} \quad (x\Sigma(x))' \big|_{x=\Lambda^2} = m$$

By checking whether the solution exists or not for $\Sigma \neq 0, \ m = 0$, critical coupling is obtained as

$$\alpha_{\mathrm{cr}} = \frac{\pi}{3C_2}$$

e.g.) SU(3) fundamental:

$$\alpha_{\mathrm{cr}} = \pi/4 \quad \Leftrightarrow \quad N_f^{\mathrm{cr}} \simeq 11.9$$

for $N_f \geq 12$ an IRFP exists in the chiral limit

Schwinger-Dyson equation (review)

$$(x\Sigma(x))'' + \alpha_* \frac{3C_2}{4\pi} \frac{\Sigma(x)}{x + m_P^2} = 0$$

$$\text{IRBC}: \quad \lim_{x\to 0} x^2 \Sigma(x)' = 0$$

$$\text{UVBC}: \quad (x\Sigma(x))'\big|_{x=\Lambda^2} = m$$

By checking whether the solution exists or not for $\Sigma \neq 0, \;\; m = 0$, critical coupling is obtained as

$$\alpha_{\rm cr} = \frac{\pi}{3C_2}$$

e.g.) SU(3) fundamental:

$$\alpha_{\rm cr} = \pi/4 \quad \Leftrightarrow \quad N_f^{\rm cr} \simeq 11.9$$

for $N_f \geq 12$ an IRFP exists in the chiral limit

Since we are here interested in hyperscaling of IR conformal theories and correction by bare fermion mass, we consider

$$\alpha_* < \alpha_{\rm cr}, \quad m \neq 0$$

Schwinger-Dyson equation (review)

The solution of differential equation which satisfies the IRBC looks like

$$\Sigma(x) = \xi\, m_P\, F\left(\frac{1+\omega}{2}, \frac{1-\omega}{2}, 2, -\frac{x}{m_P^2}\right), \quad \text{where} \quad \boxed{\omega \equiv \sqrt{1-\frac{\alpha_*}{\alpha_{\rm cr}}}}$$

hypergeometric function

a constant which is determined by $m_P \equiv \Sigma(x = m_P^2)$

Schwinger-Dyson equation (review)

The solution of differential equation which satisfies the IRBC looks like

$$\Sigma(x) = \xi\, m_P\, F\left(\frac{1+\omega}{2}, \frac{1-\omega}{2}, 2, -\frac{x}{m_P^2}\right) , \quad \text{where} \quad \boxed{\omega \equiv \sqrt{1-\frac{\alpha_*}{\alpha_{\rm cr}}}}$$

In the limit of $x \gg m_P^2$

$$\Sigma(x) \simeq \xi\, m_P \left[\frac{\Gamma(\omega)}{\Gamma(\frac{\omega+1}{2})\,\Gamma(\frac{\omega+3}{2})} \left(\frac{x}{m_P^2}\right)^{\frac{\omega-1}{2}} + (\omega \leftrightarrow -\omega) \right]$$

Schwinger-Dyson equation (review)

The solution of differential equation which satisfies the IRBC looks like

$$\Sigma(x) = \xi\, m_P\, F\left(\frac{1+\omega}{2}, \frac{1-\omega}{2}, 2, -\frac{x}{m_P^2}\right) , \quad \text{where} \quad \boxed{\omega \equiv \sqrt{1-\frac{\alpha_*}{\alpha_{\rm cr}}}}$$

In the limit of $x \gg m_P^2$

$$\Sigma(x) \simeq \xi\, m_P \left[\frac{\Gamma(\omega)}{\Gamma(\frac{\omega+1}{2})\,\Gamma(\frac{\omega+3}{2})} \left(\frac{x}{m_P^2}\right)^{\frac{\omega-1}{2}} + \quad (\omega \leftrightarrow -\omega) \right]$$

Inserting this into UVBC,
we obtain the ralation between m and m_P

$$m = \xi\, m_P \left[\frac{\Gamma(\omega)}{\Gamma(\frac{\omega+1}{2})^2} \left(\frac{\Lambda^2}{m_P^2}\right)^{\frac{\omega-1}{2}} + \quad (\omega \leftrightarrow -\omega) \right]$$

Schwinger-Dyson equation (review)

Rewriting ω in terms of the mass anomalous dimension,

$$\gamma_* = 1 - \omega \left(= 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{\rm cr}}} \right),$$

we obtain

$$m/\Lambda = \xi \left[\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*} + \frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*} \right]$$

This is a **textbook fact**,

(Refs: V.A. Miransky, "Dynamical Symmetry Breaking in Quantum Field Theory")

however, the importance of the second term in the context of hyperscaling relation hasn't been discussed

Now, let us discuss implications of this

Scaling violation

Rewriting ω in terms of the mass anomalous dimension,

$$\gamma_* = 1 - \omega \left(= 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{\rm cr}}} \right),$$

we obtain

$$m/\Lambda = \xi \left[\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*} + \frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*} \right]$$

Hyperscaling and deviation from it

Scaling violation

Rewriting ω in terms of the mass anomalous dimension,

$$\gamma_* = 1 - \omega \left(= 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{\rm cr}}} \right),$$

we obtain

$$\boxed{m/\Lambda = \xi \left[\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*}} + \underline{\frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*}} \right]$$

Hyperscaling and deviation from it

In what situation the solution approximates well the hyperscaling relation?

Scaling violation

Rewriting ω in terms of the mass anomalous dimension,

$$\gamma_* = 1 - \omega \left(= 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{cr}}} \right),$$

Note

α_*	0	$\longleftrightarrow$	α_{cr}
γ_*	0	$\longleftrightarrow$	1

we obtain

$$m/\Lambda = \xi \left[\boxed{\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*}} + \underline{\frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*}} \right]$$

Hyperscaling and deviation from it

In what situation the solution approximates well the hyperscaling relation?

Scaling violation

Rewriting ω in terms of the mass anomalous dimension,

$$\gamma_* = 1 - \omega \left(= 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{cr}}} \right),$$

Note

α_*	$0 \longleftrightarrow \alpha_{cr}$
γ_*	$0 \longleftrightarrow 1$

we obtain

$$\boxed{m/\Lambda = \xi \left[\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*}} + \underline{\frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*}} \right]$$

Hyperscaling and deviation from it

In what situation the solution approximates well the hyperscaling relation?

when $\frac{m_P}{\Lambda}$, or equivalently, $\frac{m}{\Lambda}$ is small since $1+\gamma_* < 3-\gamma_*$

but, it's difficult direction for lattice simulations

Scaling violation

Rewriting ω in terms of the mass anomalous dimension,

$$\gamma_* = 1 - \omega \left(= 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{cr}}} \right),$$

Note

α_*	$0 \longleftrightarrow \alpha_{cr}$
γ_*	$0 \longleftrightarrow 1$

we obtain

$$m/\Lambda = \xi \left[\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*} + \frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*} \right]$$

Hyperscaling and deviation from it

In what situation the solution approximates well the hyperscaling relation?

when γ_* is small since power suppression becomes strong, as well as the coefficient of the second term becomes small

but, our (phenomenological) motivation was large γ_* ...

Scaling violation

Rewriting ω in terms of the mass anomalous dimension,

$$\gamma_* = 1 - \omega \left(= 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{\rm cr}}} \right),$$

Note

$$\begin{array}{c|ccc} \alpha_* & 0 & \longleftrightarrow & \alpha_{\rm cr} \\ \hline \gamma_* & 0 & \longleftrightarrow & 1 \end{array}$$

we obtain

$$m/\Lambda = \xi \left[\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*} + \frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*} \right]$$

Hyperscaling and deviation from it

The message is: we should keep the existence of the correction term in mind when we analyze data in practical situations

Scaling violation

Rewriting ω in terms of the mass anomalous dimension,

$$\gamma_* = 1 - \omega \left(= 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{\rm cr}}} \right),$$

Note

$$\begin{array}{c|ccc} \alpha_* & 0 & \longleftrightarrow & \alpha_{\rm cr} \\ \hline \gamma_* & 0 & \longleftrightarrow & 1 \end{array}$$

we obtain

$$\boxed{m/\Lambda = \xi \left[\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*}} + \underline{\frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*}} \right]$$

Hyperscaling and deviation from it

The message is: we should keep the existence of the correction term in mind when we analyze data in practical situations

What happens if we forget it ?

Scaling violation

$$\boxed{m/\Lambda \;=\; \xi \left[\; \frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*}} + \underline{\frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*}} \;\right]$$

Hyperscaling correction

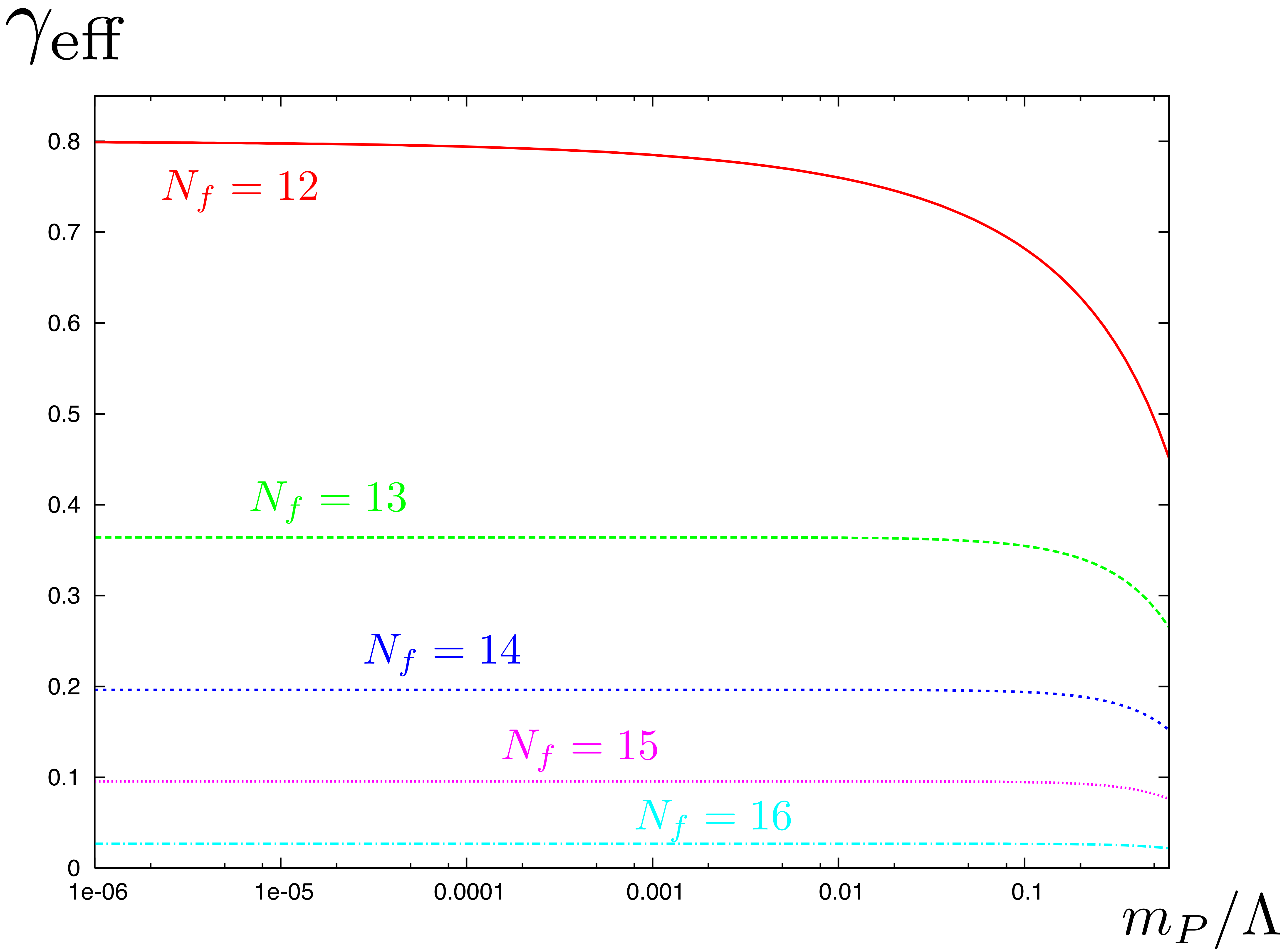
Consider $m_P \dfrac{\partial}{\partial m_P} (\log m) - 1 \;\equiv\; \gamma_{\mathrm{eff}}$

If there is no correction term, it gives γ_*

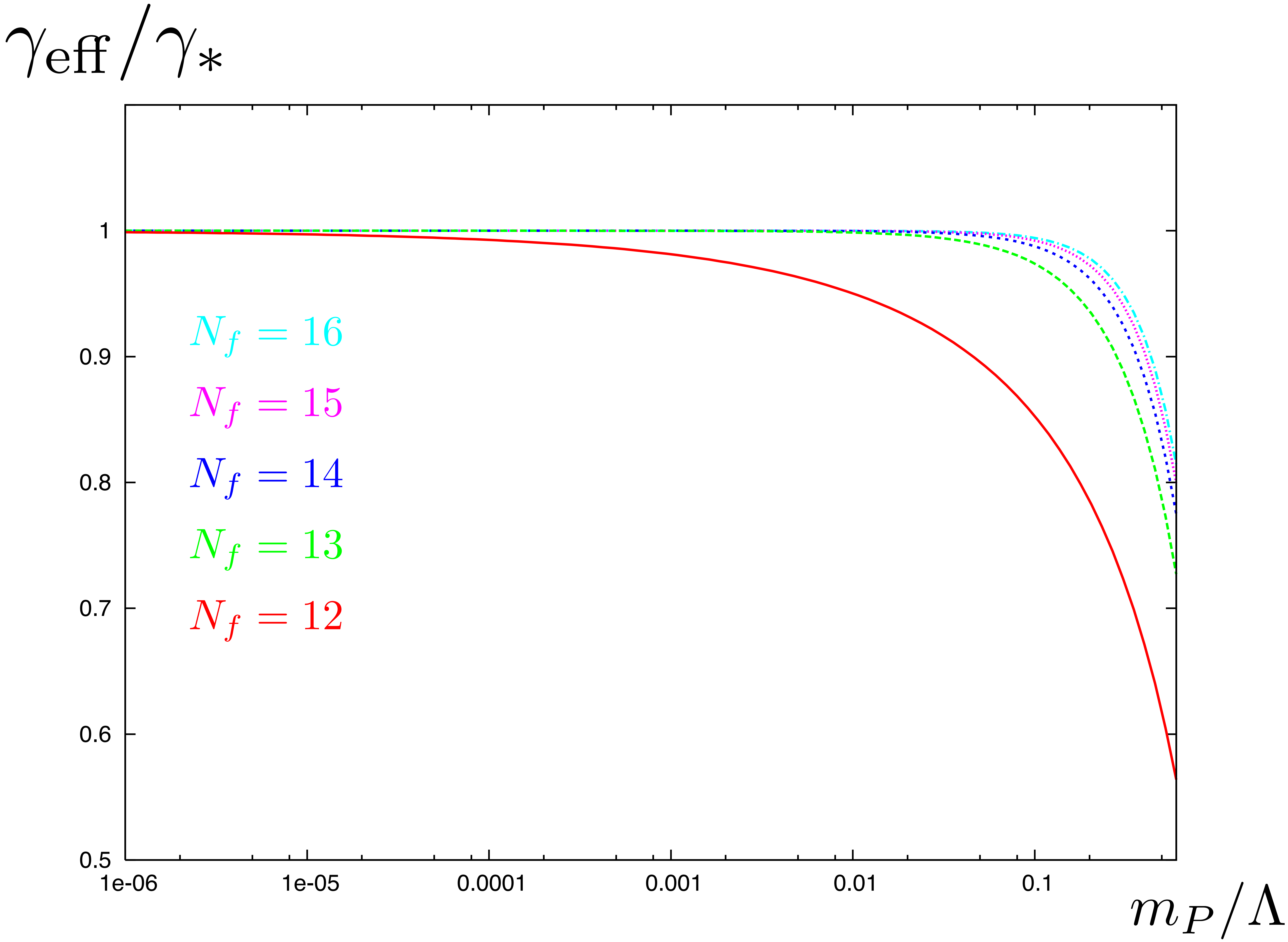
The question is: how much it deviates from γ_*

when the correction term exists

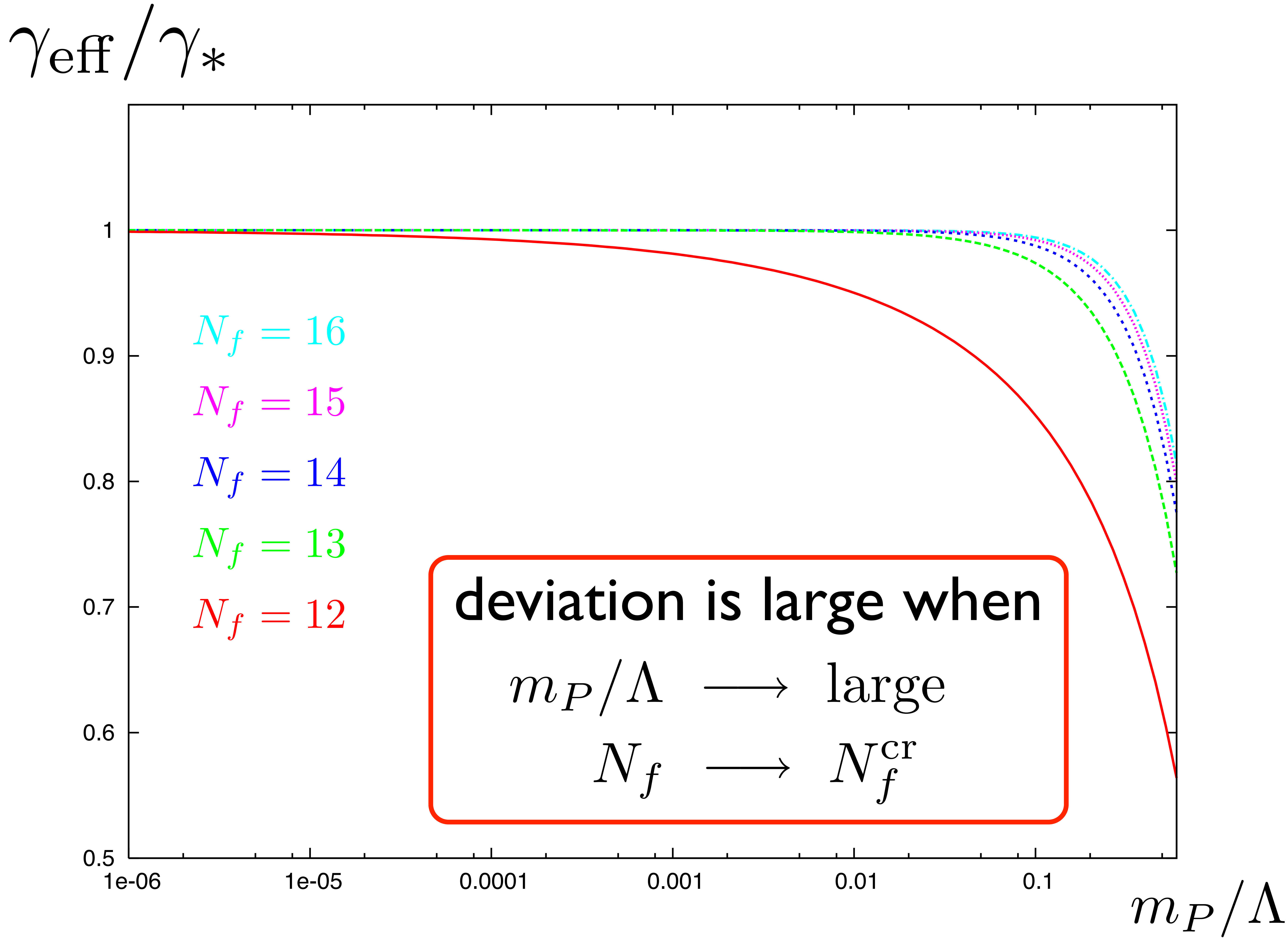
Effective anomalous dimension SU(3) fundamental



Effective anomalous dimension SU(3) fundamental



Effective anomalous dimension SU(3) fundamental



Finite-volume scaling

When the correction term is important, what happens if we try to judge conformality, and extract the value of mass anomalous dimension from finite-volume hyperscaling relation?

$$\boxed{\begin{array}{l} M_H = L^{-1} f\left(x\right) \\ \text{where} \quad x = \hat{L}\ \hat{m}^{1/(1+\gamma_*)} \end{array}}$$

For the purpose of studying this, we formulate the Schwinger-Dyson equation in a finite size spacetime, and generate data for various values of $(\hat{L}, \hat{m})$

SD equation

Self-consistent equation for the full fermion propagator $iS_F^{-1} \equiv A(p^2)\not{p} - B(p^2)$

$$
\begin{aligned}
A(p^2) &= 1 + C_2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{g}^2(p,k)}{k^2 A(k^2)^2 + B(k^2)^2} \\
&\quad \cdot \left[A(k^2) \frac{(p \cdot k)}{p^2 (p-k)^2} + 2A(k^2) \frac{\{p \cdot (p-k)\}\{k \cdot (p-k)\}}{p^2 (p-k)^4} \right], \\
B(p^2) &= m_0 + 3 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{g}^2(p,k)}{k^2 A(k^2)^2 + B(k^2)^2} \frac{B(k^2)}{(p-k)^2}.
\end{aligned}
$$

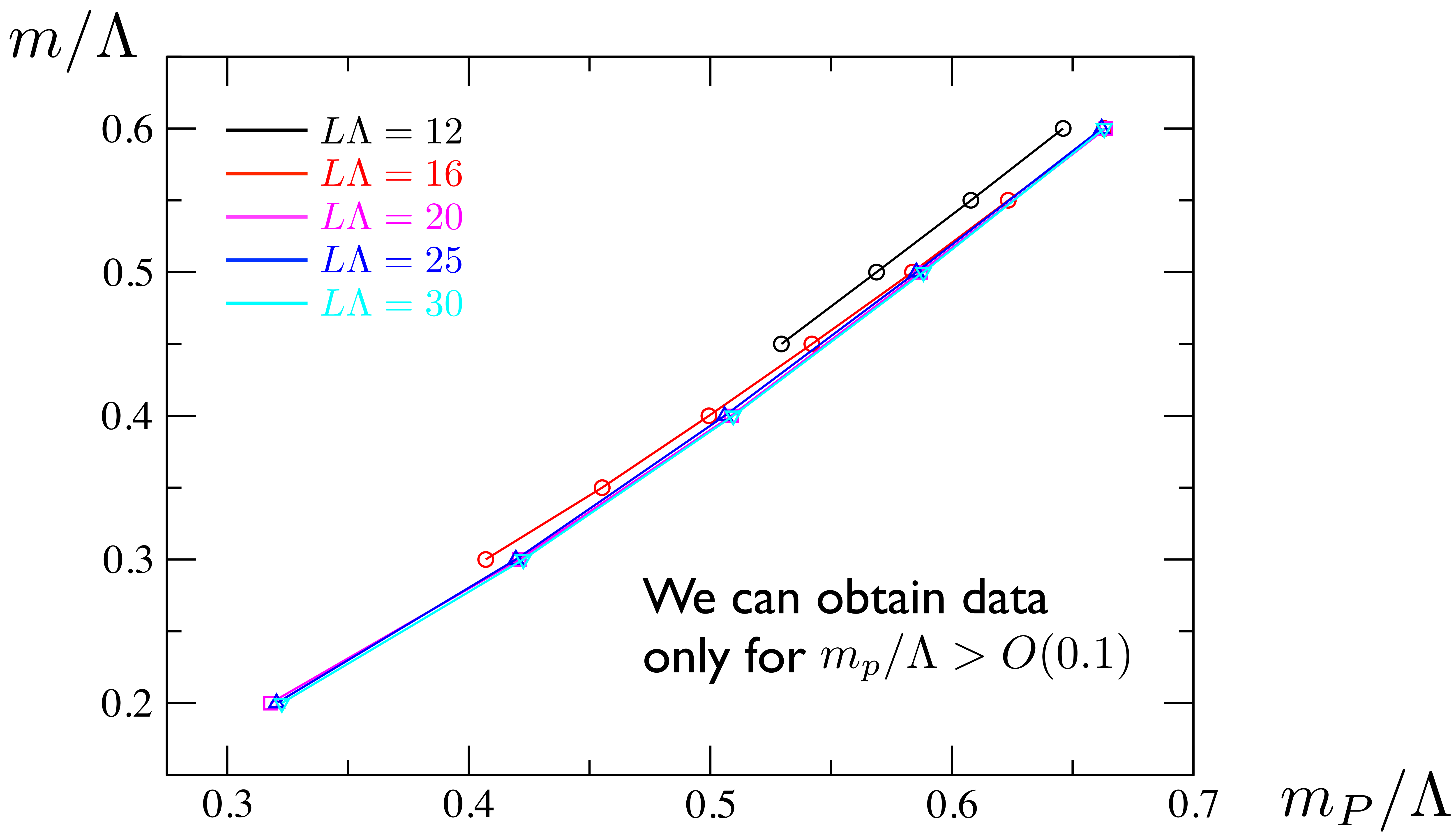
In the case of finite volume, we replace continuous momentum by the discrete one:

$$
p \longrightarrow p_n = \frac{2\pi n}{L}, \qquad \int_{-\infty}^{\infty} dp\, f(p) \longrightarrow \frac{2\pi}{L} \sum_{n=-\infty}^{\infty} f(p_n)
$$

Then, do the iteration

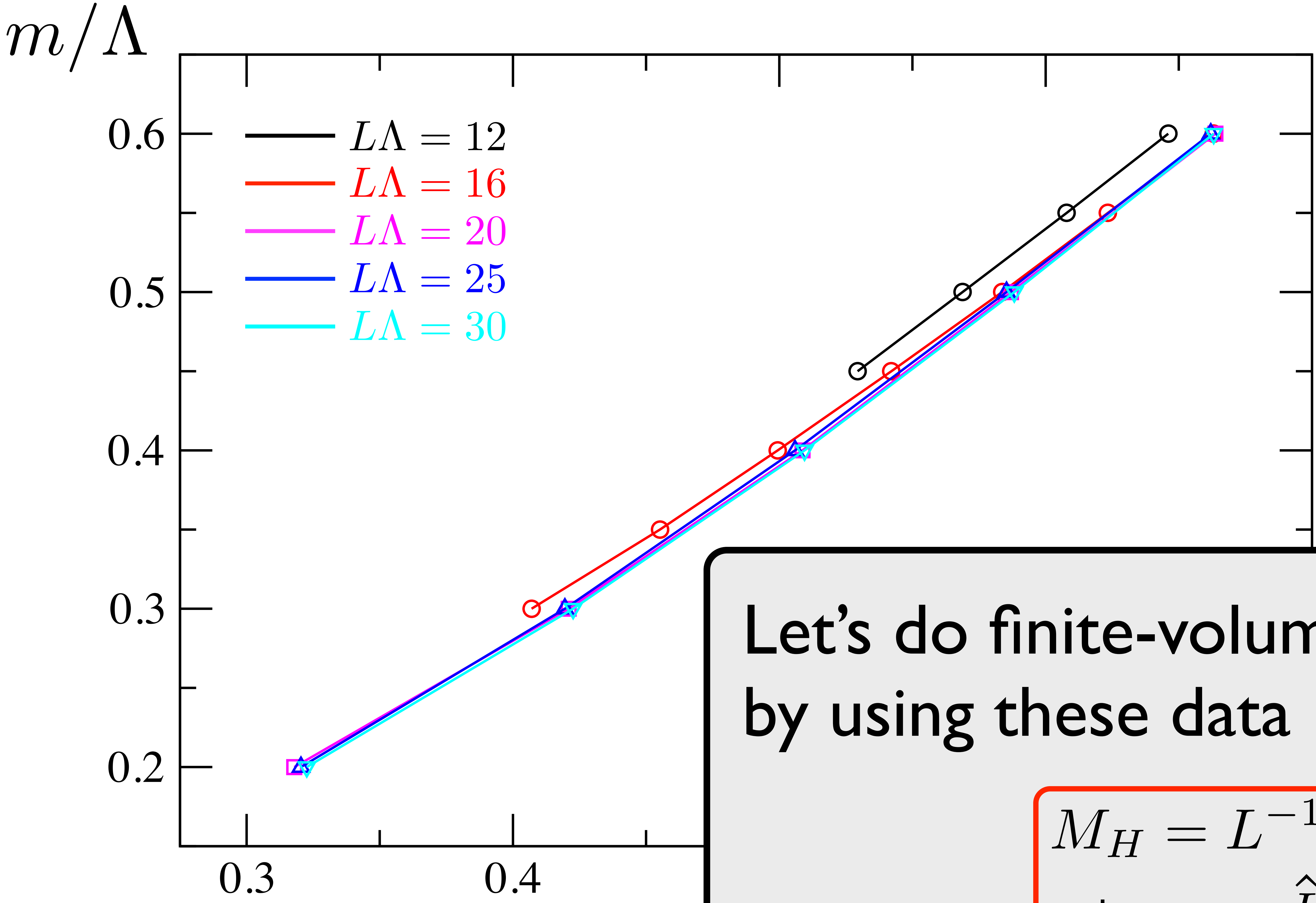
SU(3) gauge theory with 12 fundamental fermion

relation between m_P/Λ and m/Λ for various values of $L\Lambda$



SU(3) gauge theory with 12 fundamental fermion

relation between m_P/Λ and m/Λ for various values of $L\Lambda$



Let's do finite-volume scaling by using these data

$$M_H = L^{-1} f(x)$$

where $x = \hat{L}\ \hat{m}^{1/(1+\gamma_*)}$

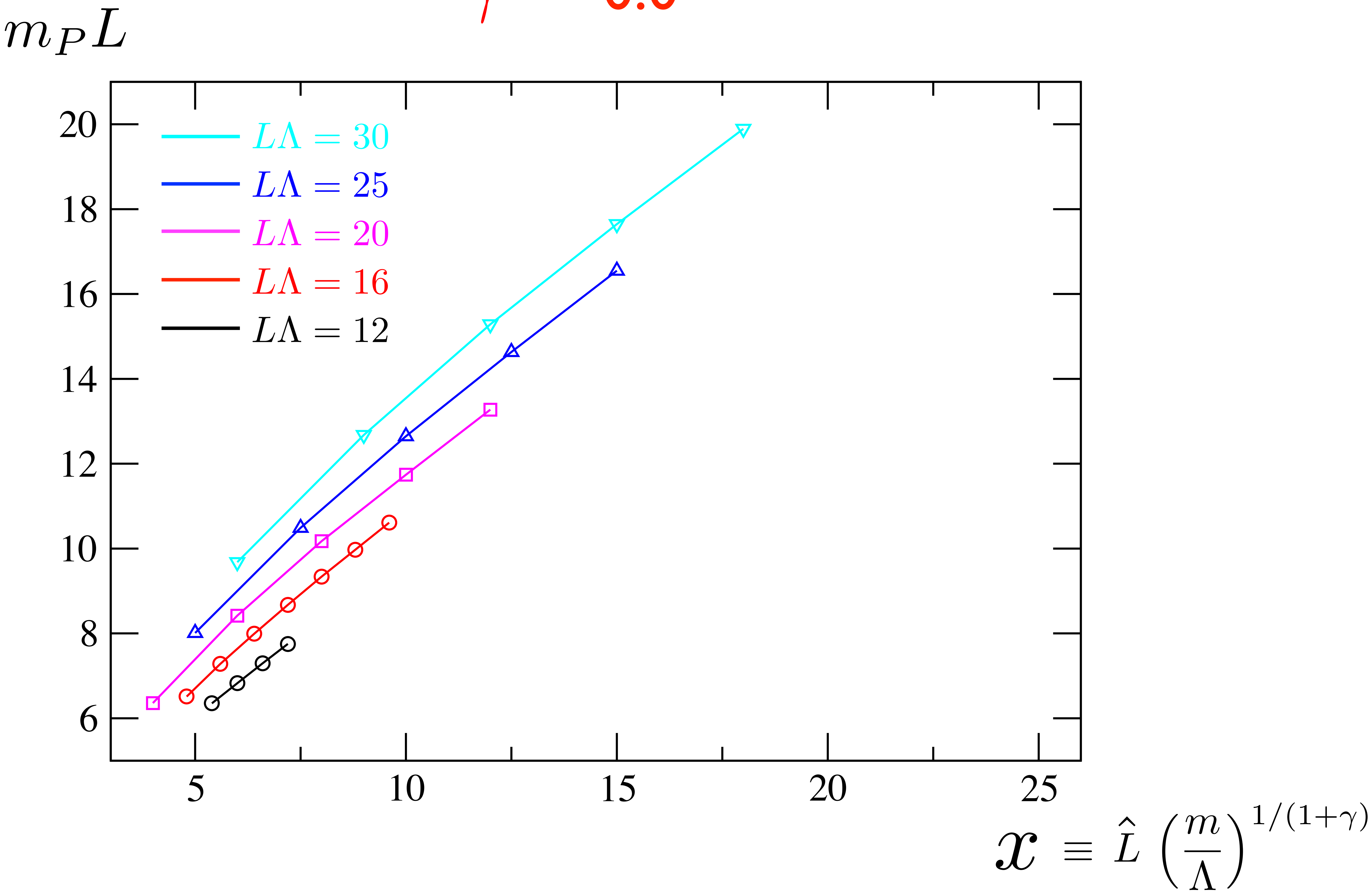
SU(3) gauge theory with 12 fundamental fermion

Note: $\gamma_* \simeq 0.80$ at IRFP

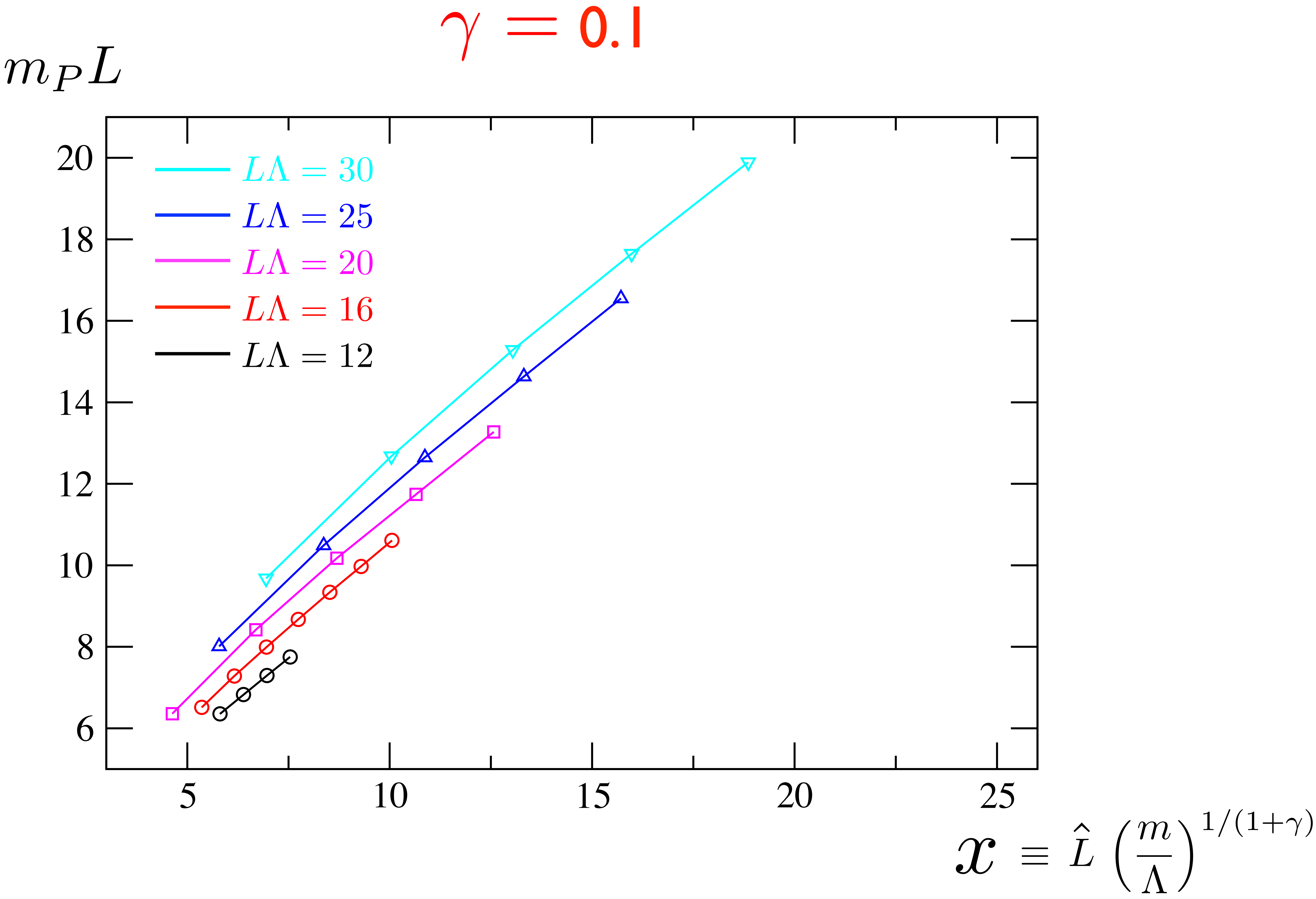
$\gamma_{\text{eff}} = 0.5 \sim 0.6$ for $m_p/\Lambda \gtrsim O(0.1)$

SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.0$

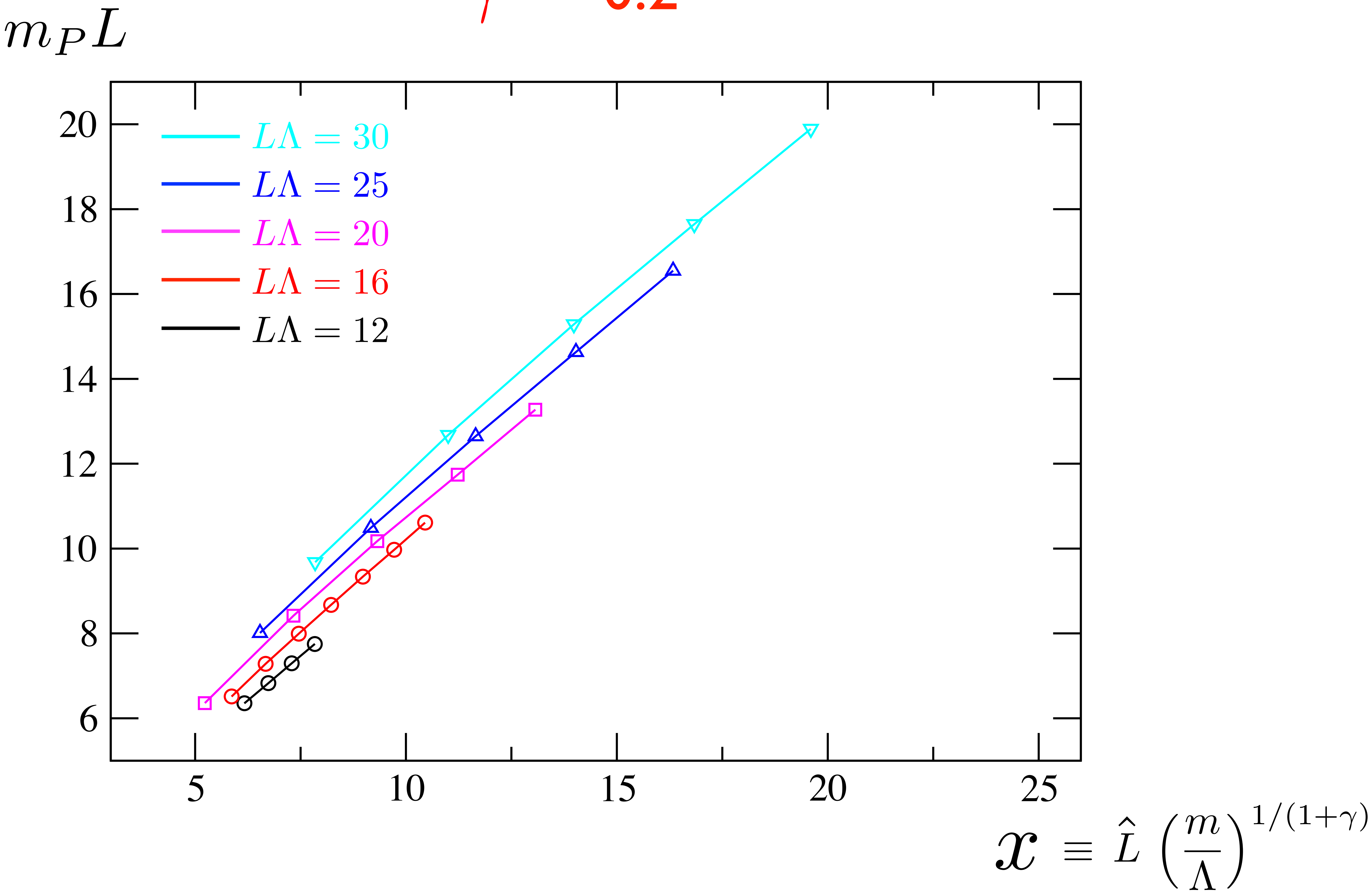


SU(3) gauge theory with 12 fundamental fermion



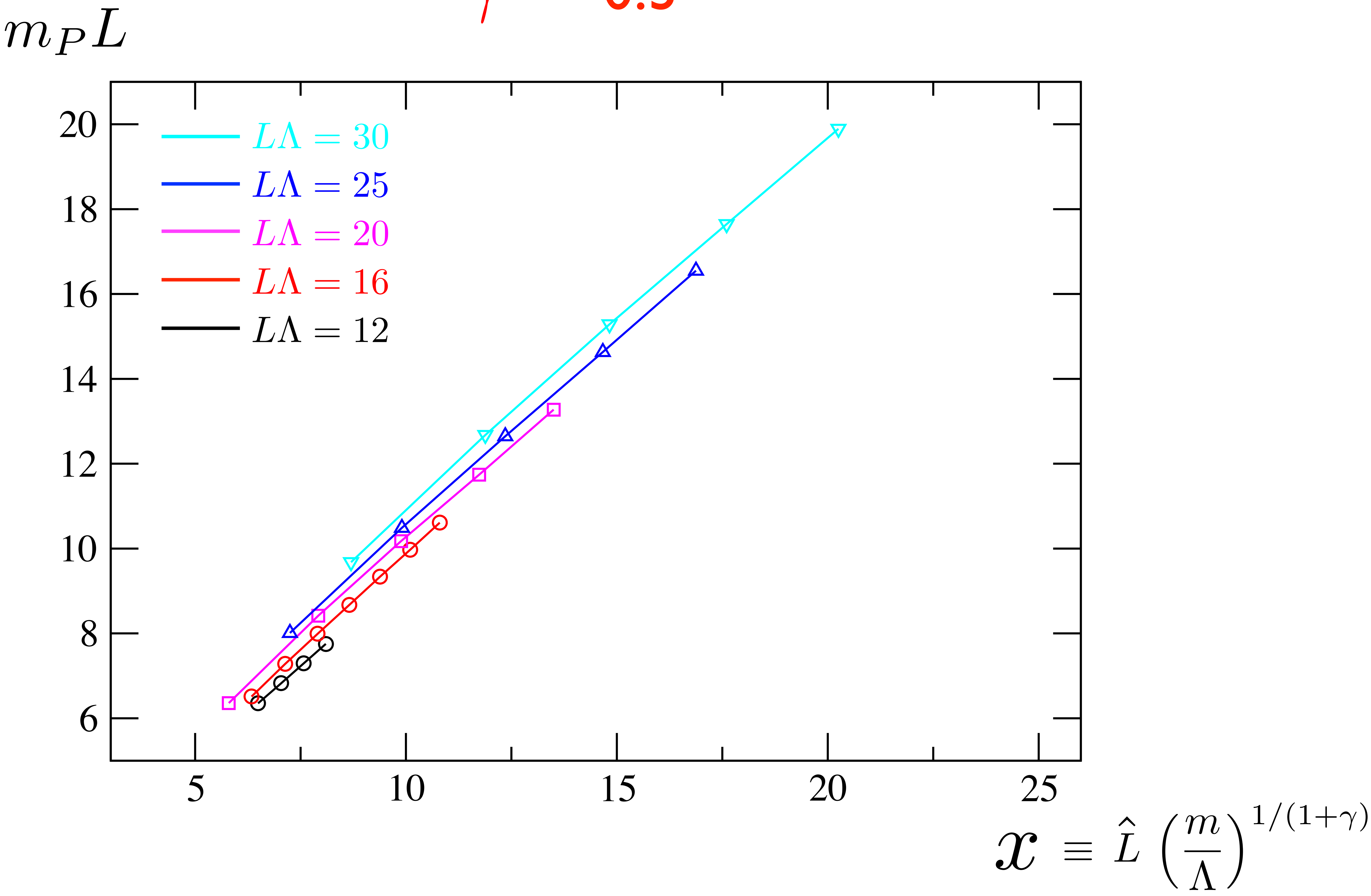
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.2$



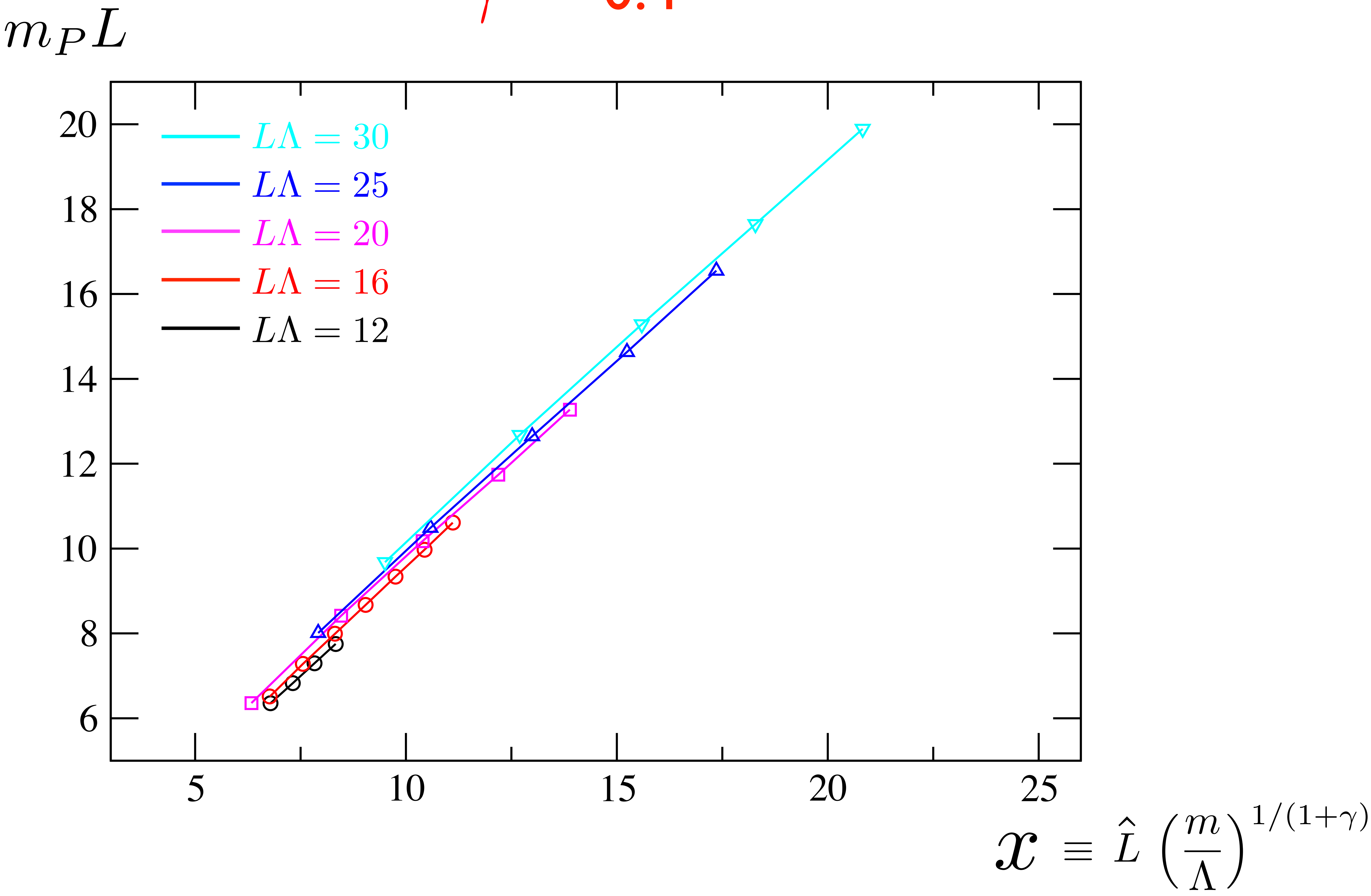
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.3$



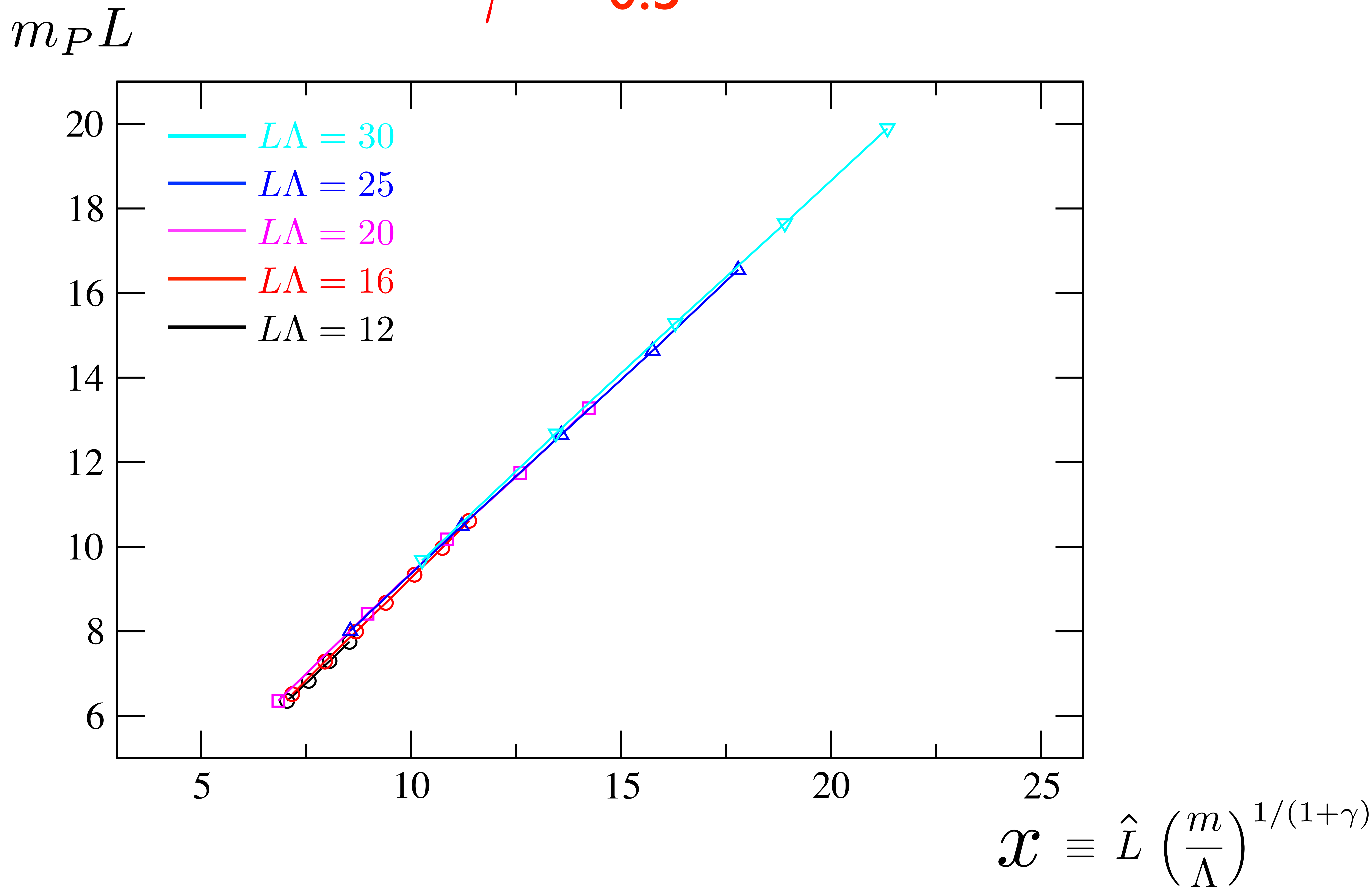
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.4$



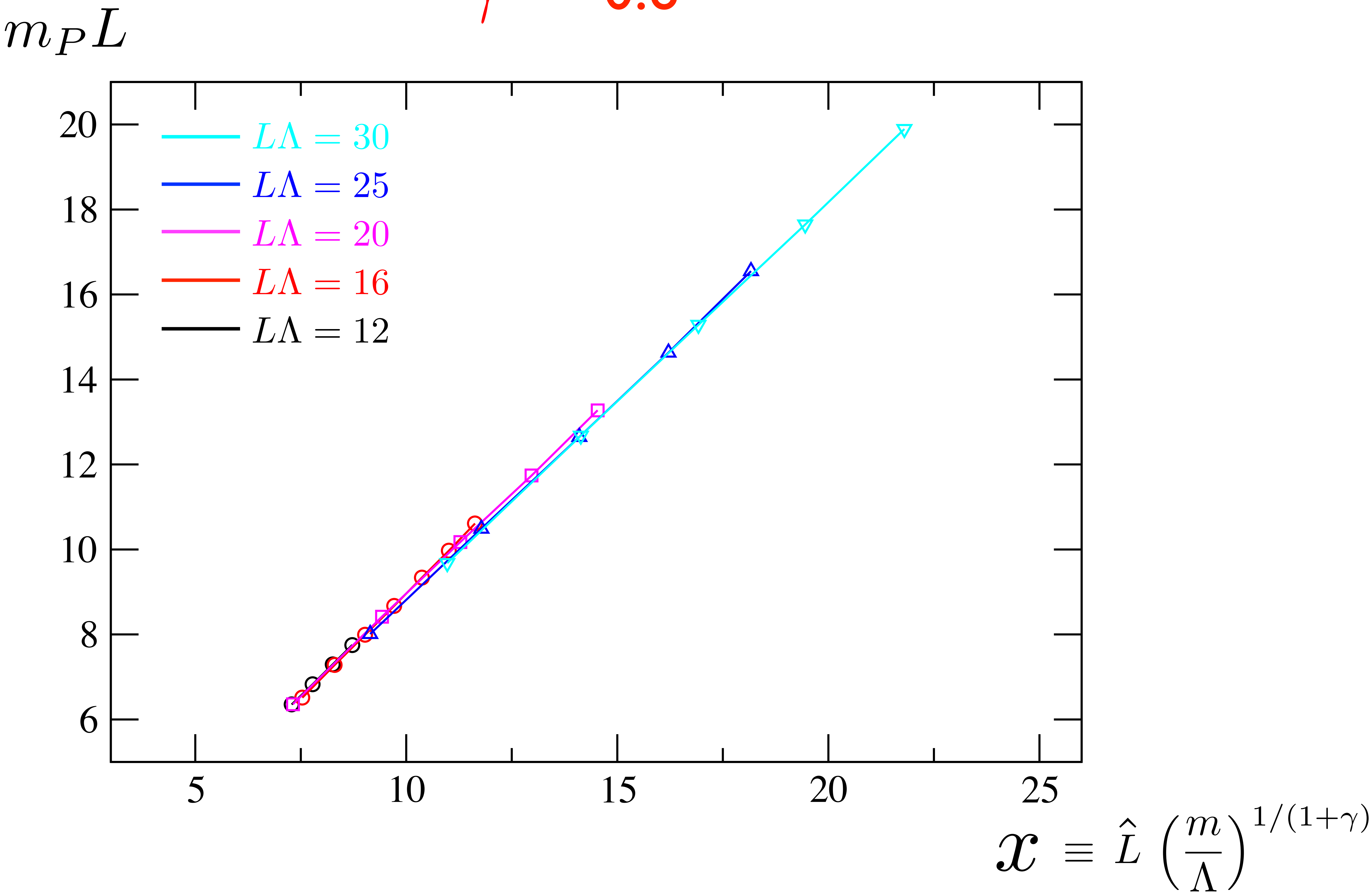
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.5$

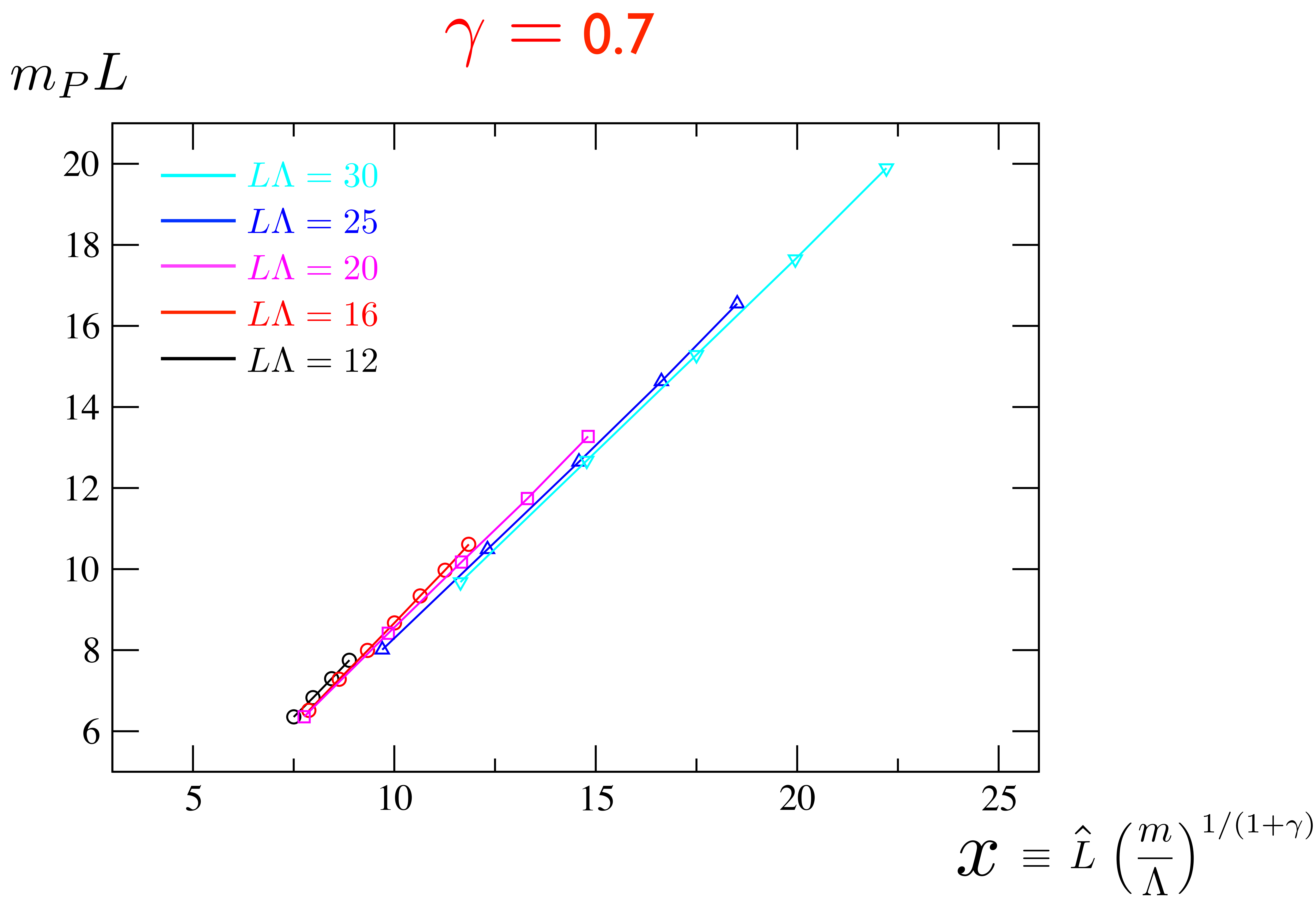


SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.6$

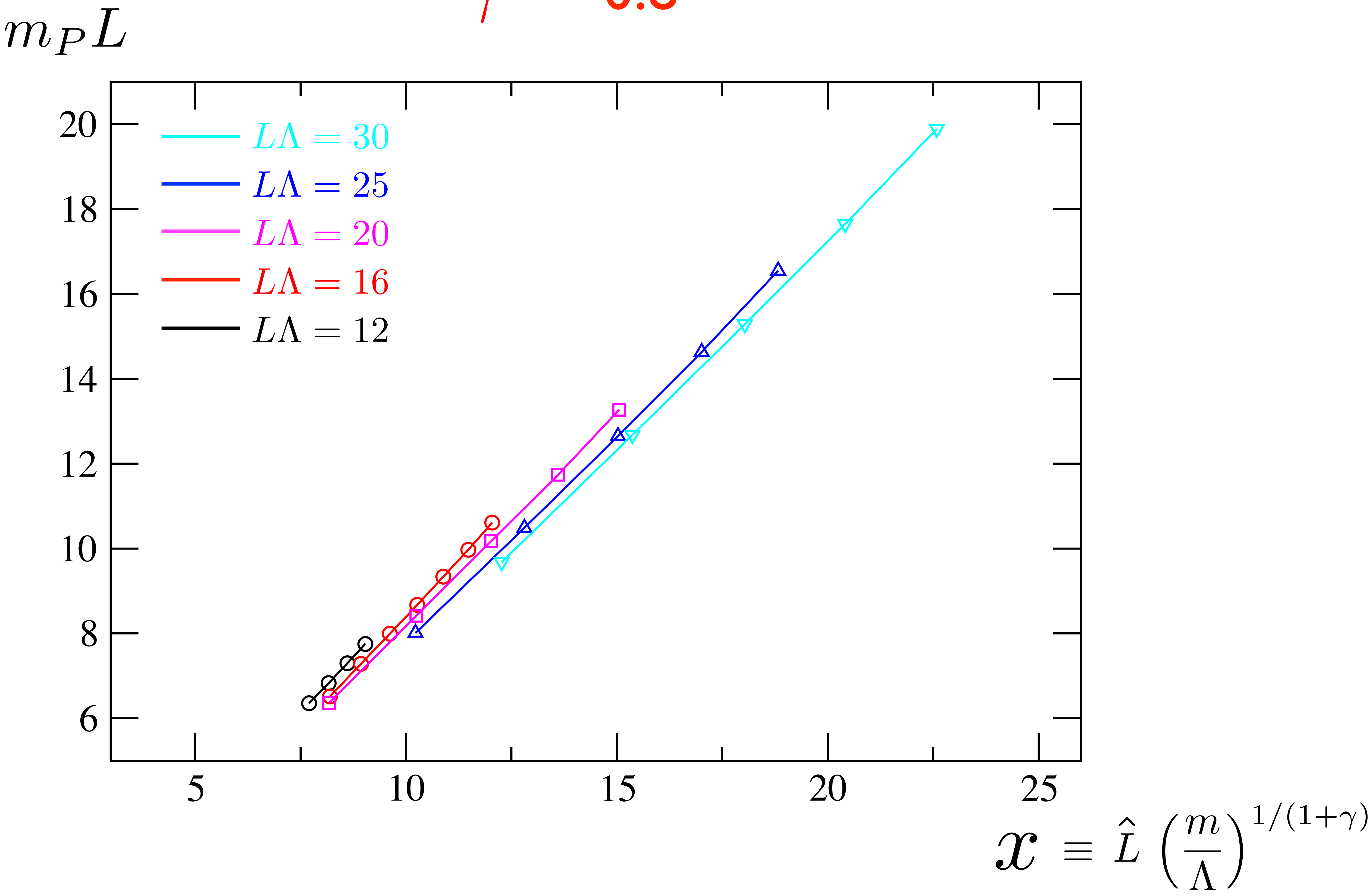


SU(3) gauge theory with 12 fundamental fermion



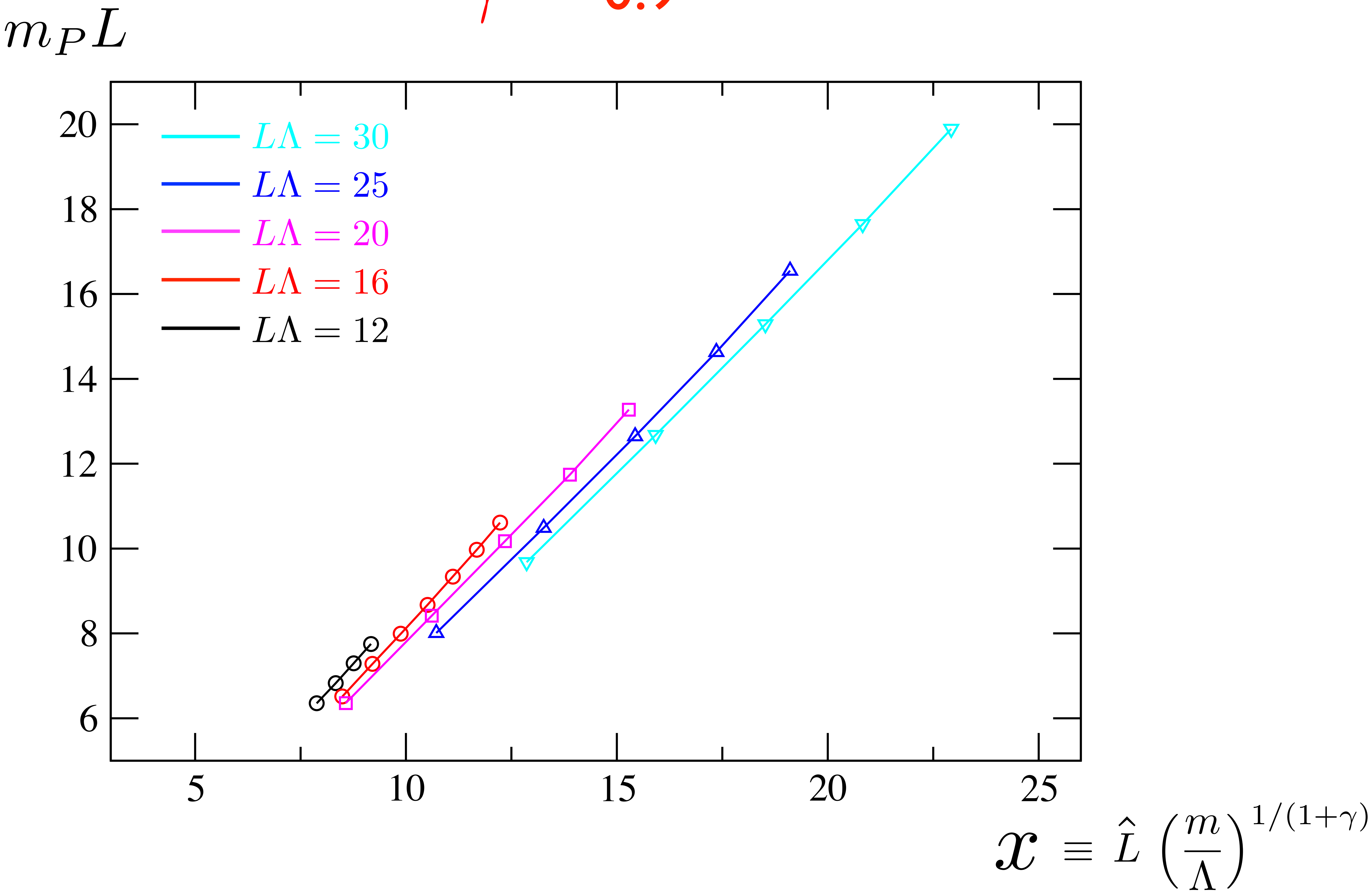
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.8$



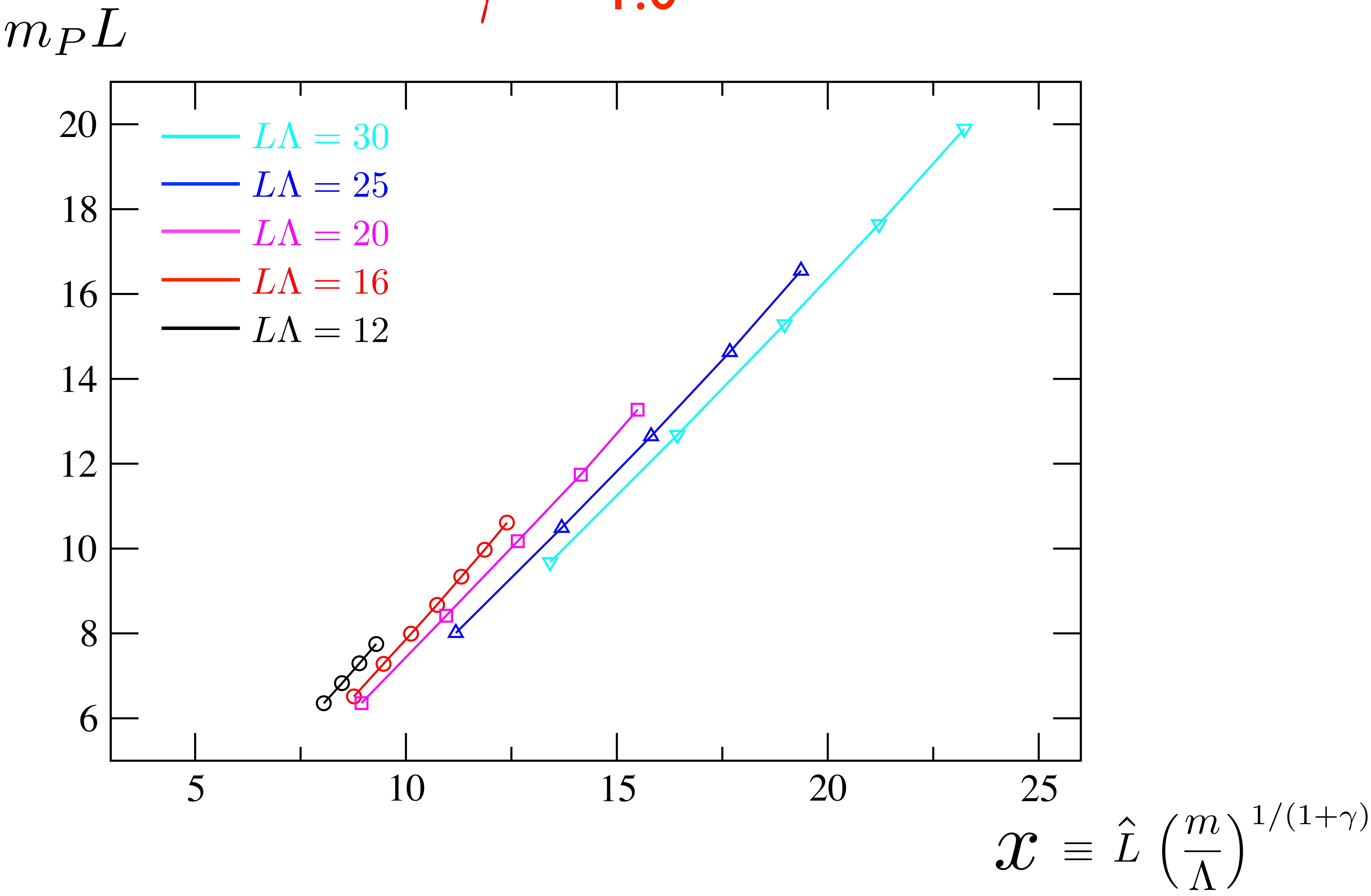
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.9$

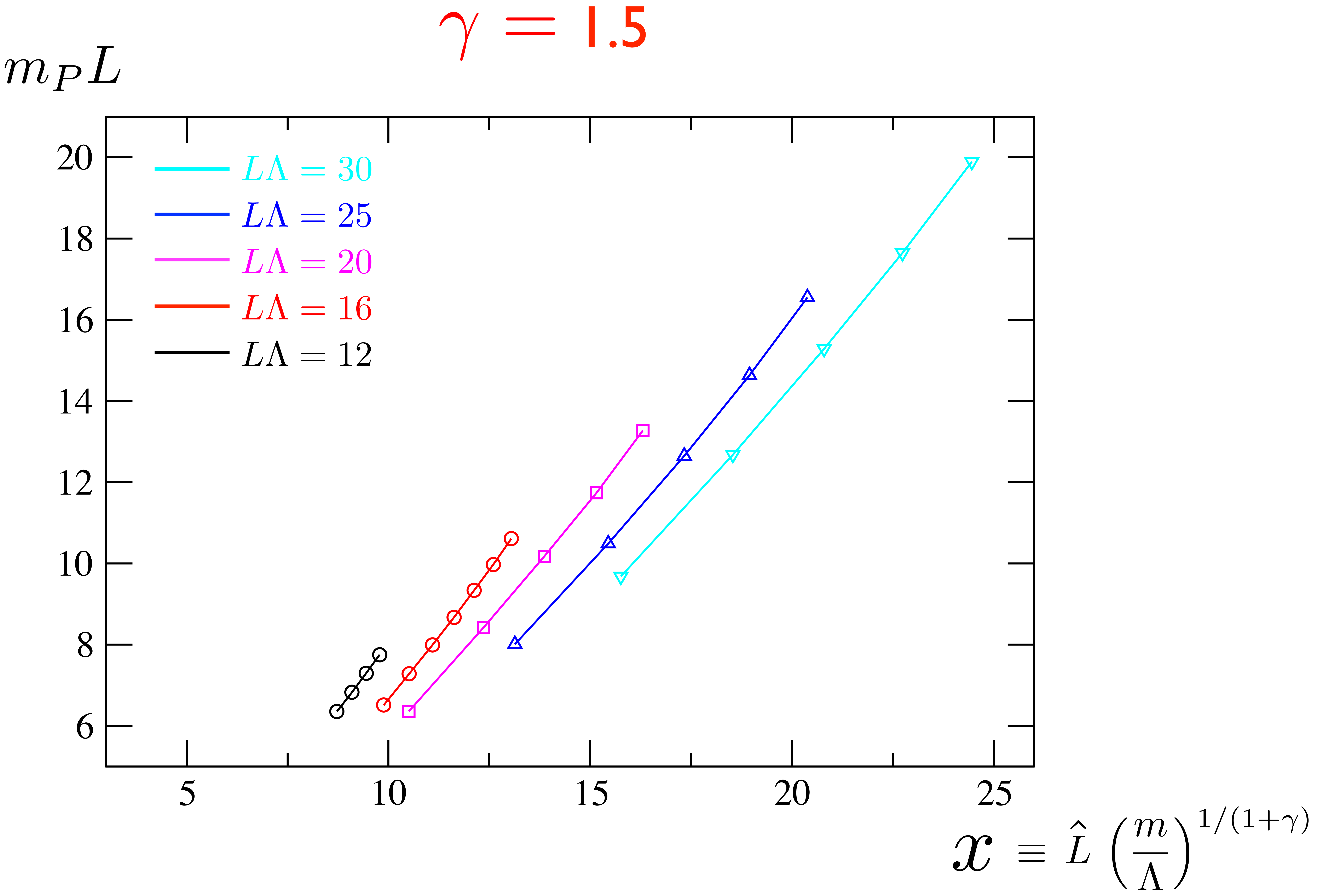


SU(3) gauge theory with 12 fundamental fermion

$\gamma = 1.0$

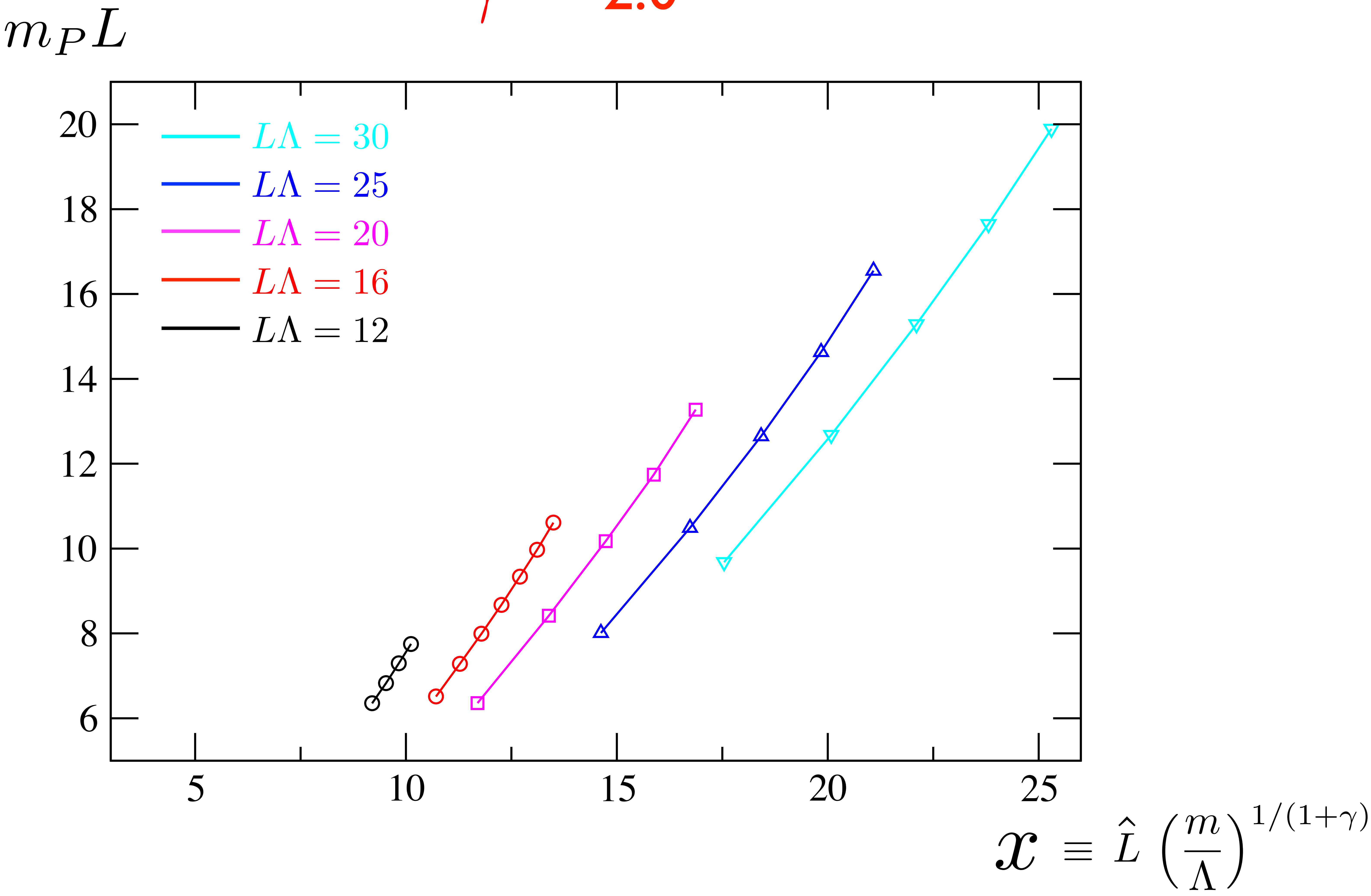


SU(3) gauge theory with 12 fundamental fermion



SU(3) gauge theory with 12 fundamental fermion

$\gamma = 2.0$



SU(3) gauge theory with 12 fundamental fermion

Note: $\gamma_* \simeq 0.80$ at IRFP

$\gamma_{\rm eff} = 0.5 \sim 0.6$ for $m_p/\Lambda \gtrsim O(0.1)$

Scaling is good with $\gamma_{\rm eff}$ rather than γ_*

SU(3) gauge theory with 12 fundamental fermion

Note: $\gamma_* \simeq 0.80$ at IRFP

$\gamma_{\rm eff} = 0.5 \sim 0.6$ for $m_p/\Lambda \gtrsim O(0.1)$

Scaling is good with $\gamma_{\rm eff}$ rather than γ_*

This is also an indication that the correction coming from a finite volume effect is negligible

SU(3) gauge theory with 12 fundamental fermion

Note: $\gamma_* \simeq 0.80$ at IRFP

$\gamma_{\rm eff} = 0.5 \sim 0.6$ for $m_p/\Lambda \gtrsim O(0.1)$

Scaling is good with $\gamma_{\rm eff}$ rather than γ_*

We have also confirmed similar results for $N_f = 14, 16$

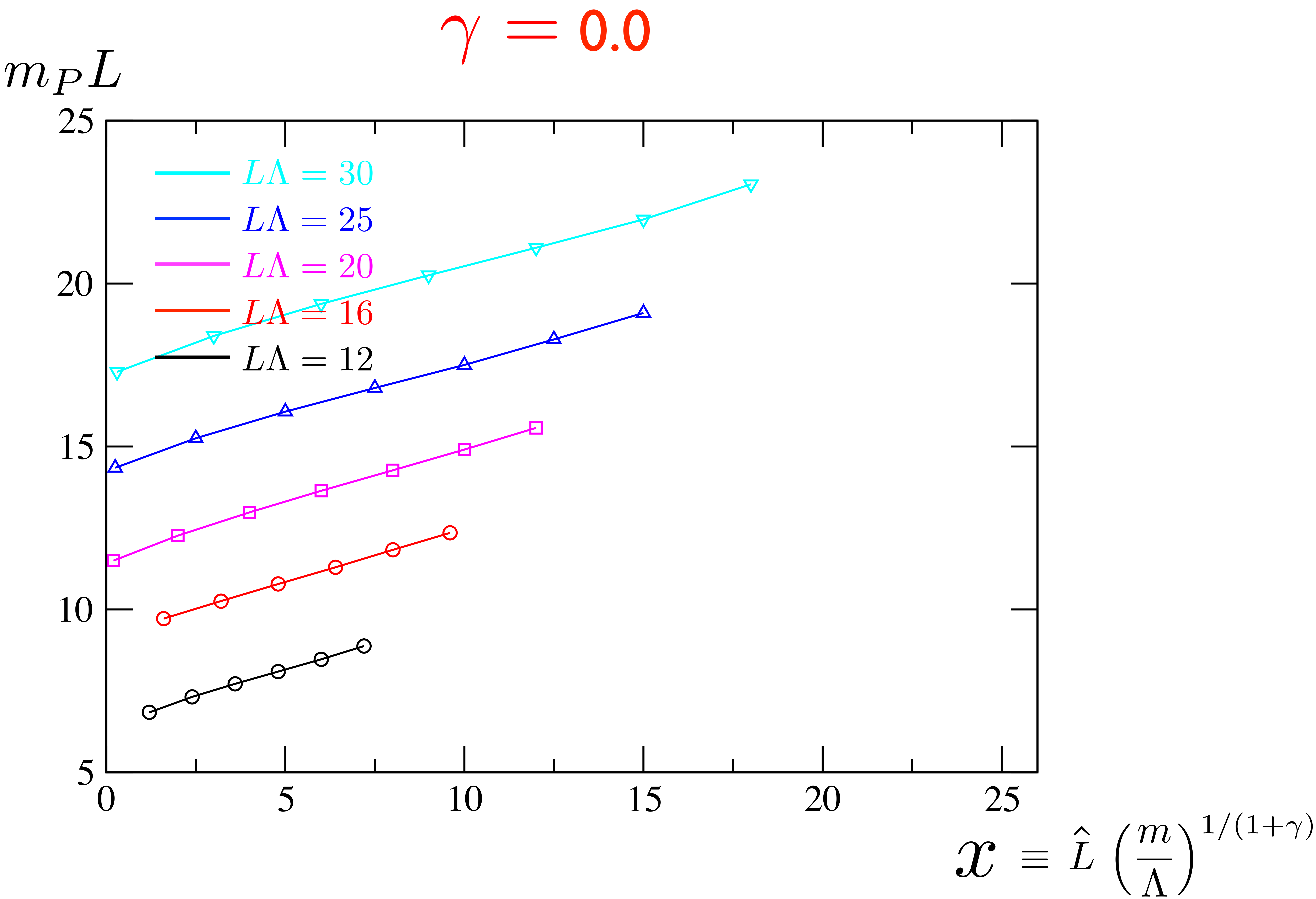
Another question:

when the theory does not have an IRFP
(namely, in the chiral symmetry breaking phase),
how and how much the scaling relation is violated?

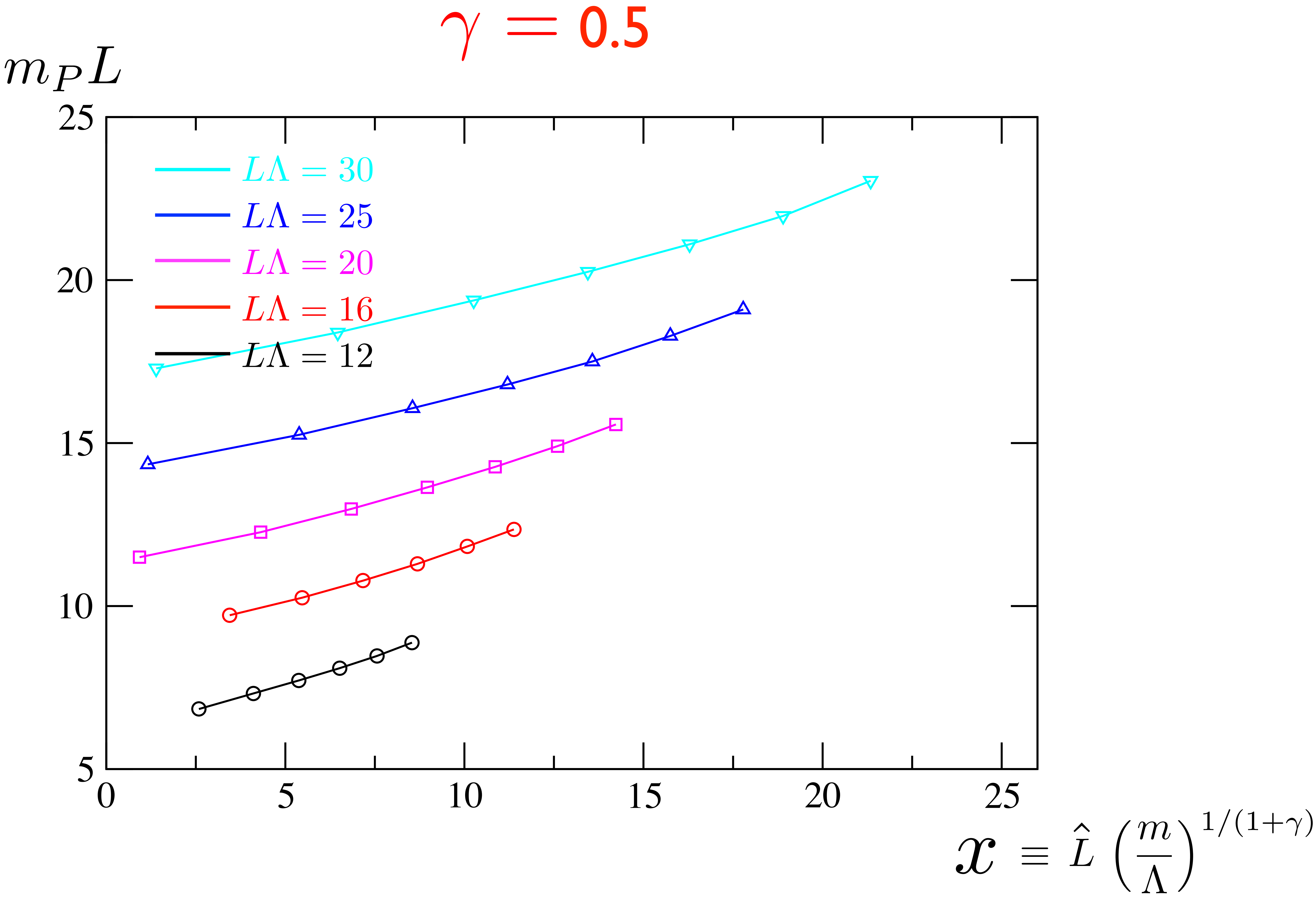
We show two examples:

SU(3), 9 flavor: deeply broken
SU(3), 11 flavor: close to the critical flavor

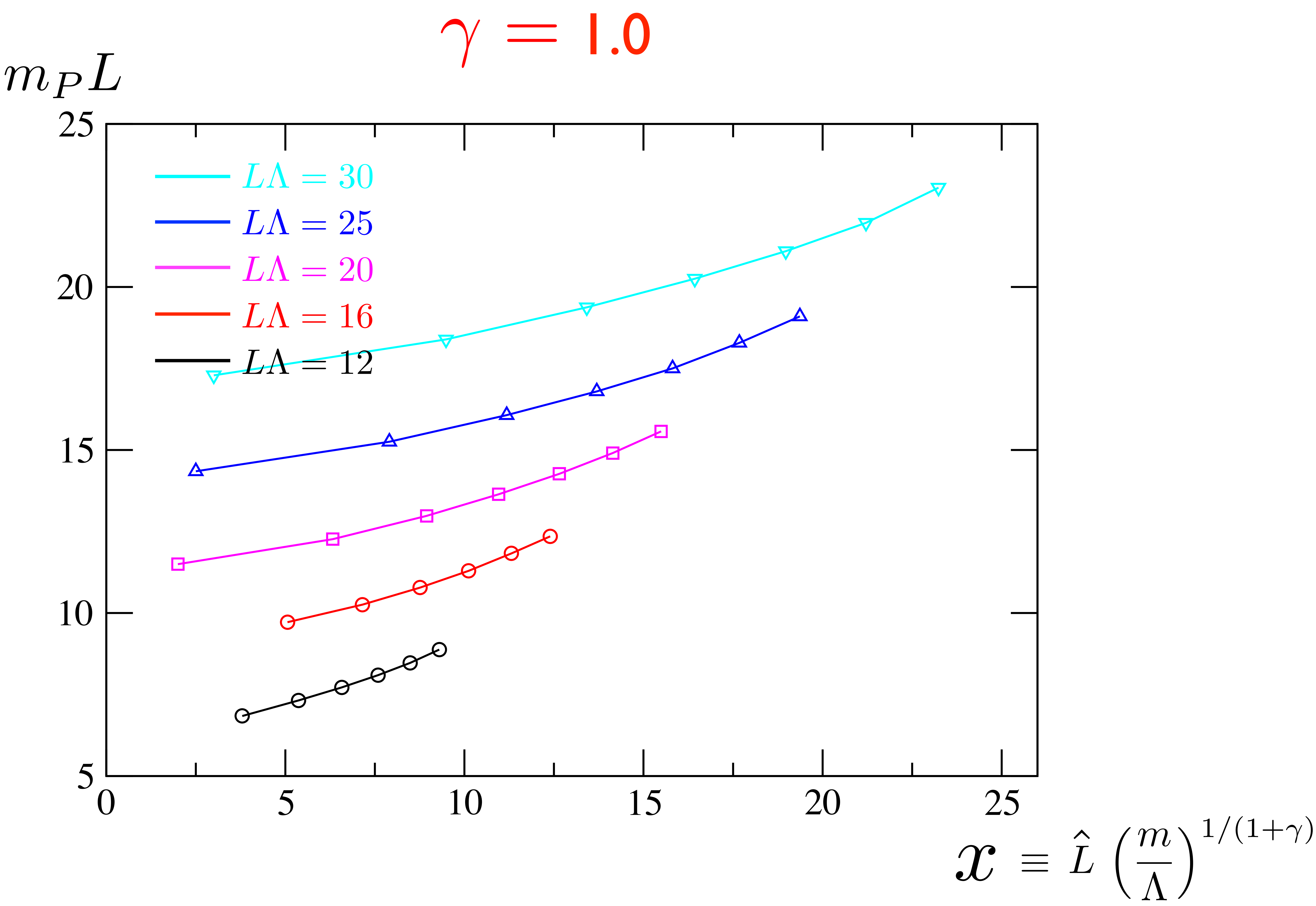
SU(3) gauge theory with 9 fundamental fermion



SU(3) gauge theory with 9 fundamental fermion

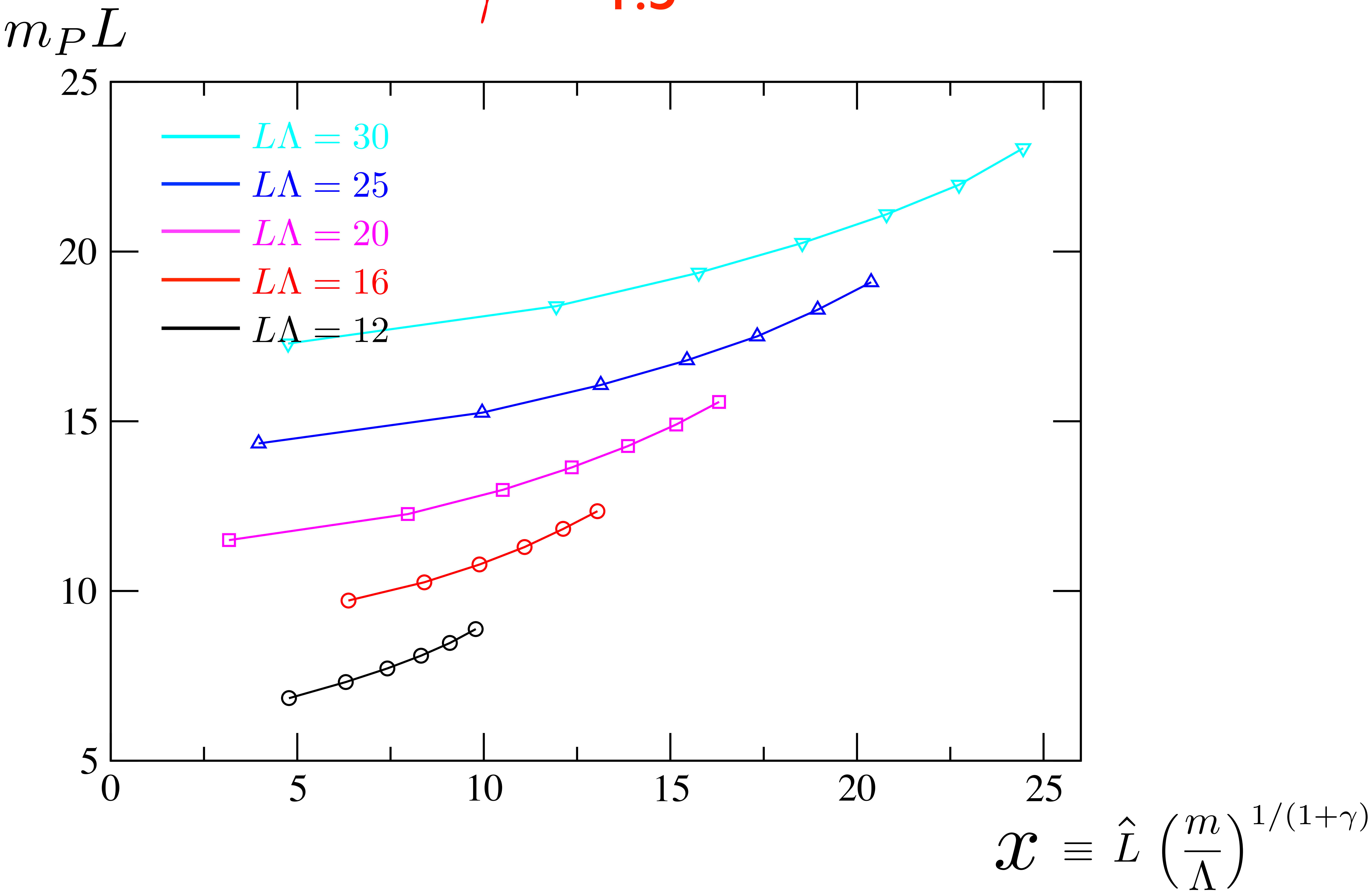


SU(3) gauge theory with 9 fundamental fermion



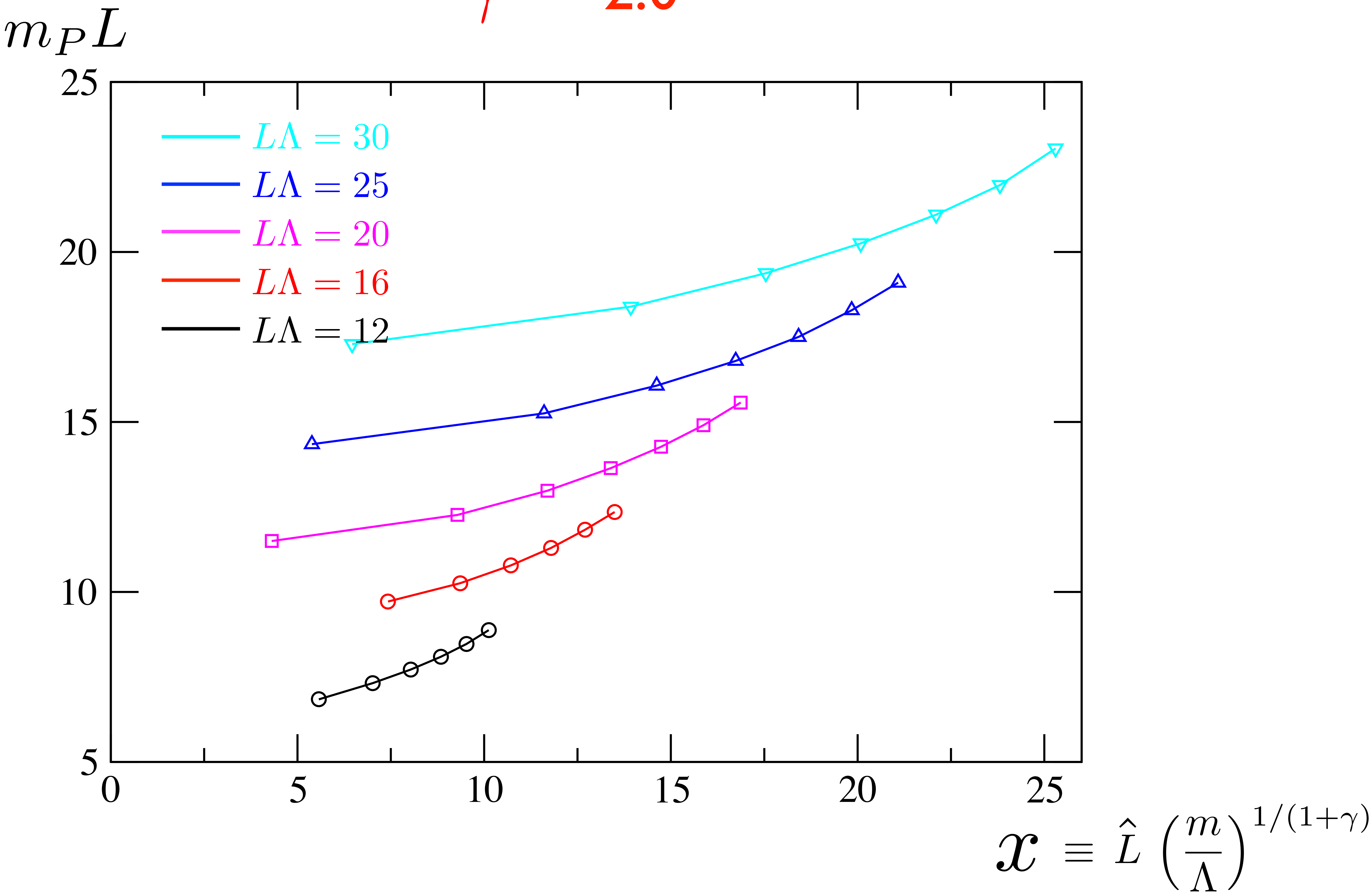
SU(3) gauge theory with 9 fundamental fermion

$\gamma = 1.5$



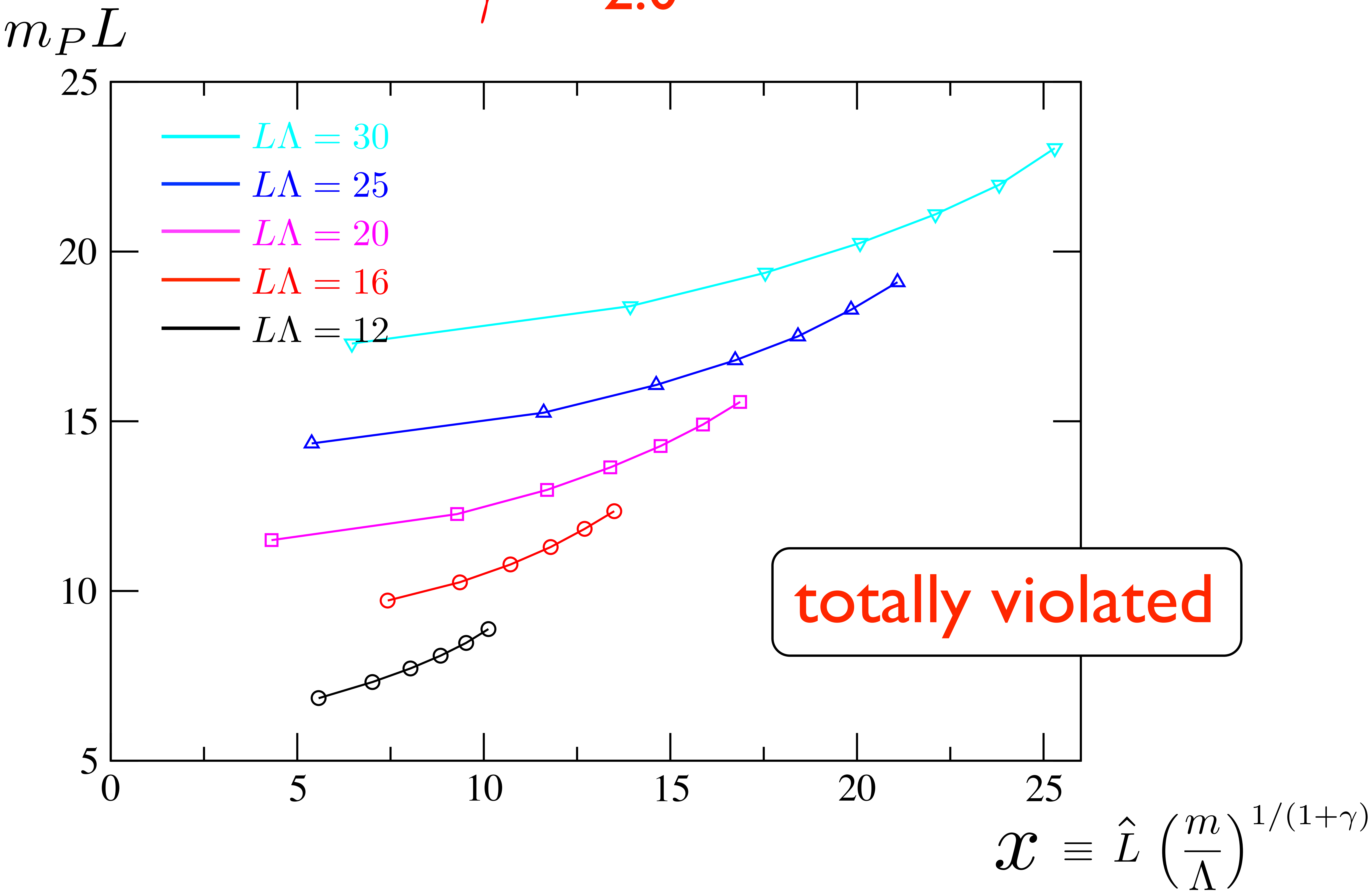
SU(3) gauge theory with 9 fundamental fermion

$\gamma = 2.0$



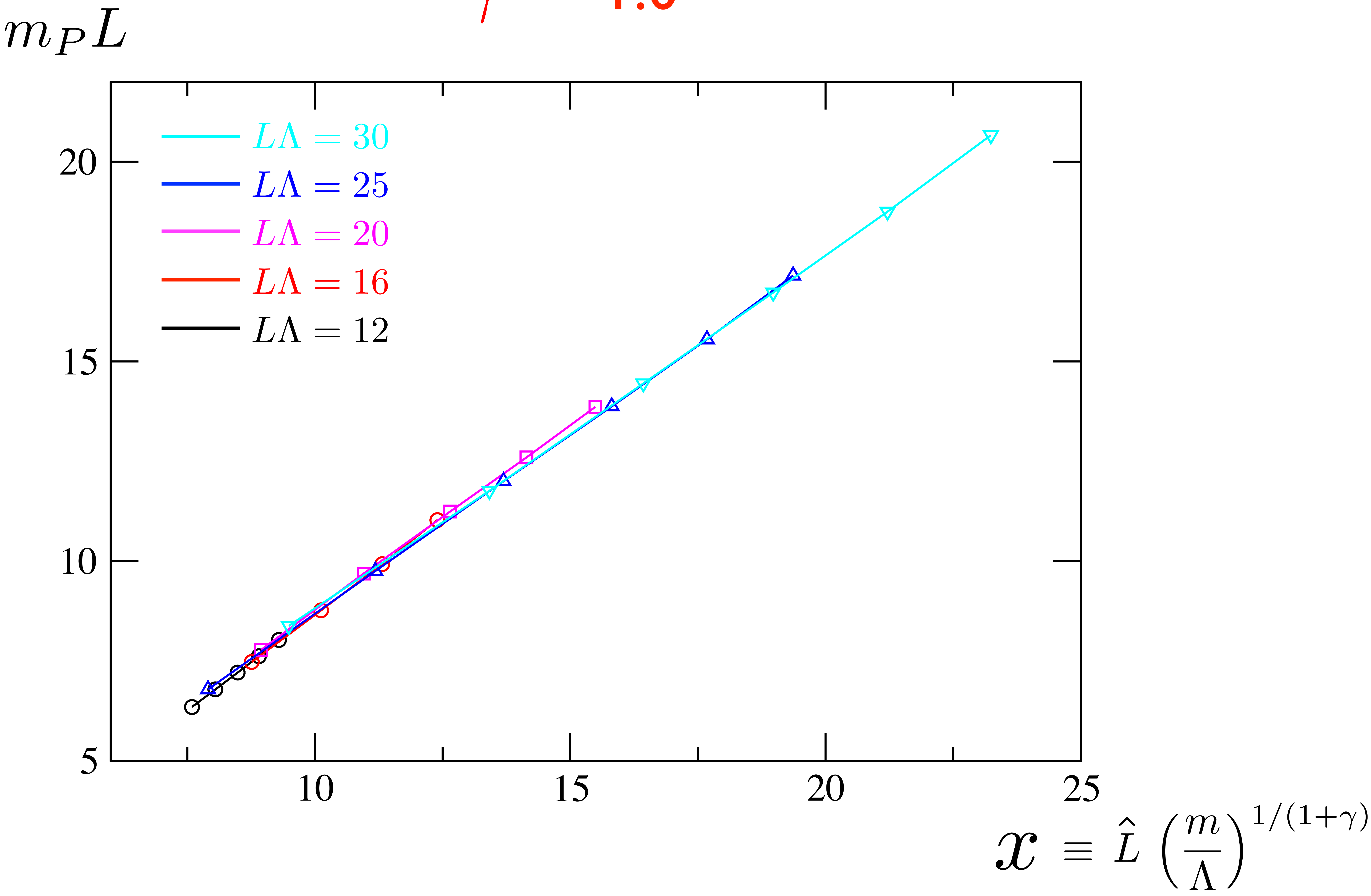
SU(3) gauge theory with 9 fundamental fermion

$\gamma = 2.0$



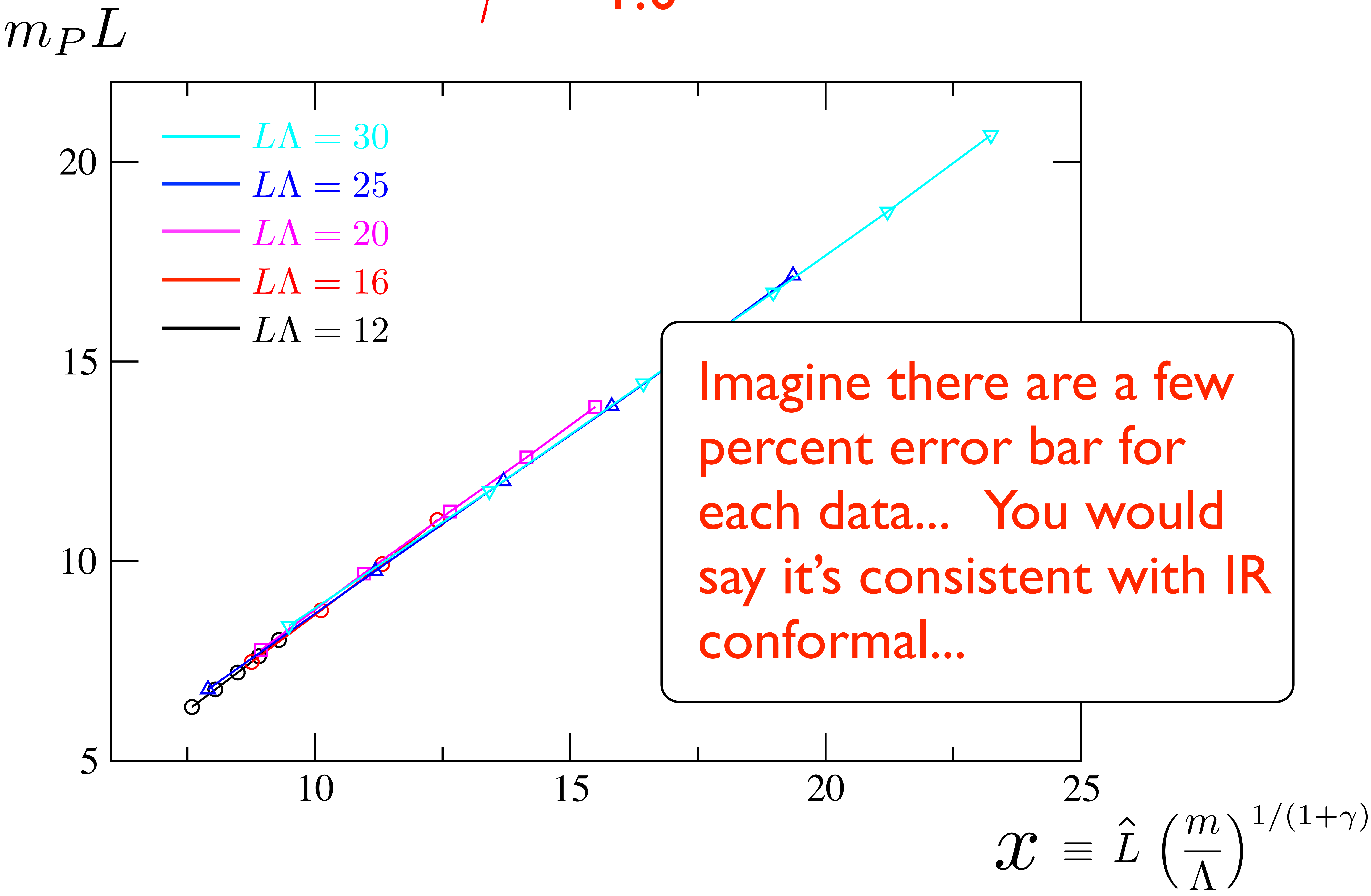
SU(3) gauge theory with 11 fundamental fermion

$\gamma = 1.0$



SU(3) gauge theory with 11 fundamental fermion

$\gamma = 1.0$



Imagine there are a few percent error bar for each data... You would say it's consistent with IR conformal...

Summary

From the analytic form of relation between the bare quark mass and the physical quantity obtained from the SD equation, we showed the importance of the mass correction to the hyperscaling analysis

SD equation in a finite size spacetime was formulated, and finite-volume scaling was studied by using numerical data in a self consistent manner

Summary

Possible scenarios (and possible confusions)

case 1: when the theory is deep in the hadronic phase

⇨ There is no confusion

Summary

Possible scenarios (and possible confusions)

case 2: when the theory is in the hadronic phase, but close to the edge of the conformal window

⇨ One might observe approximate scaling behavior, and conclude that the theory is IR conformal...

Summary

Possible scenarios (and possible confusions)

case 2: when the theory is in the hadronic phase, but close to the edge of the conformal window

One might observe approximate scaling behavior, and conclude that the theory is IR conformal...

Confusing!!

Summary

Possible scenarios (and possible confusions)

case 3: when the theory is in the conformal window, and data are taken in a wide range of input bare mass

One might observe misalignment in the hyperscaling plot, because effective γ is different for different mass regions... and those might look like not consistent with IR conformality

Summary

Possible scenarios (and possible confusions)

case 3: when the theory is in the conformal window, and data are taken in a wide range of input bare mass

One might observe misalignment in the hyperscaling plot, because effective γ is different for different mass regions... and those might look like not consistent with IR conformality

Confusing!!

Summary

Possible scenarios (and possible confusions)

case 4: when the theory is in the conformal window, and data are taken in a rather small range of large bare mass region

⇨ One might observe good alignment in the hyperscaling plot, and obtain an effective value of γ. However, it is very possible that mass corrections to the hyperscaling relations for different physical quantities are different, so one might obtain non-universal values of γ for hyperscaling plots with different physical quantities...

Summary

Possible scenarios (and possible confusions)

case 4: when the theory is in the conformal window, and data are taken in a rather small range of large bare mass region

One might observe good alignment in the hyperscaling plot, and obtain an effective value of γ. However, it is very possible that mass corrections to the hyperscaling relations for different physical quantities are different, so one might obtain non-universal values of γ for hyperscaling plots with different physical quantities...

Confusing!!

Summary

It might be natural that our comunity is confused by the hyperscaling results

Of course, the mass correction shown here might be just one of possible sources of confusion, but it is definitely important to keep this in mind especially when one's simulations are limited in a large mass region

To judge the conformality, a combination of several analysis (ChPT, running coupling, etc.) is important