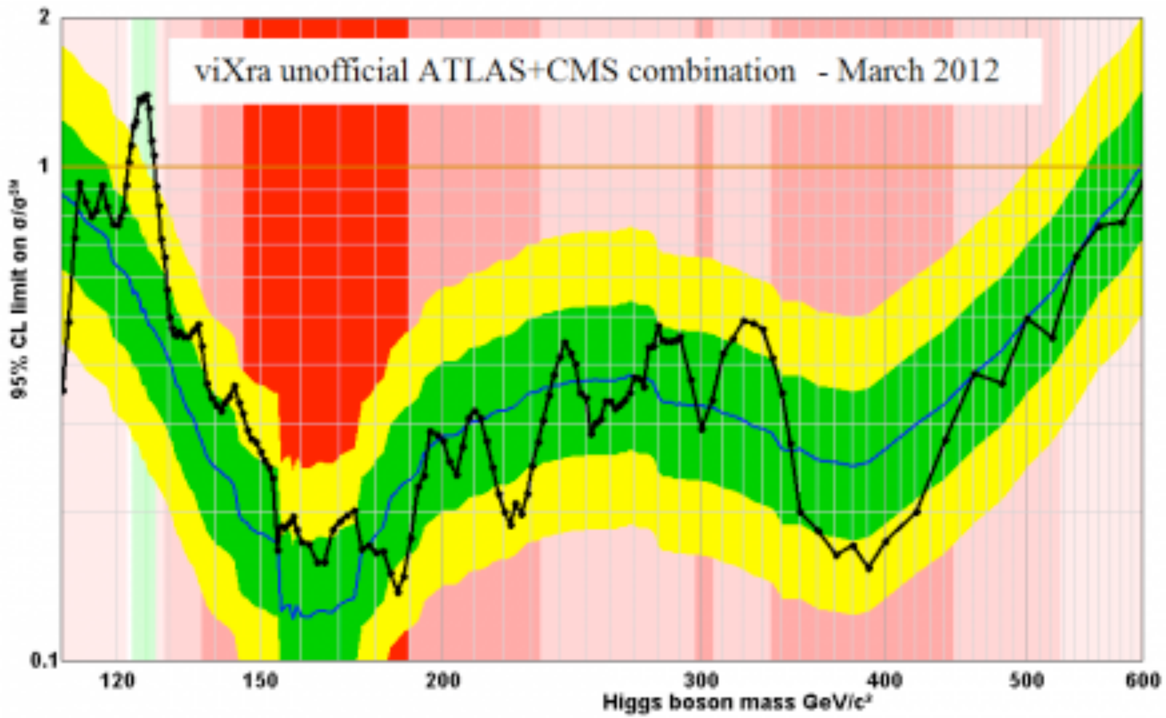
A Very Light Dilaton

with Patipan Uttayarat

arXiv:1105.2370

Grinstein - UCSD
March 19, 2012

Conformality in Strong Coupling Gauge Theories at LFC and Lattice
(SCGT12Mini)


viXra unofficial ATLAS+CMS combination - March 2012
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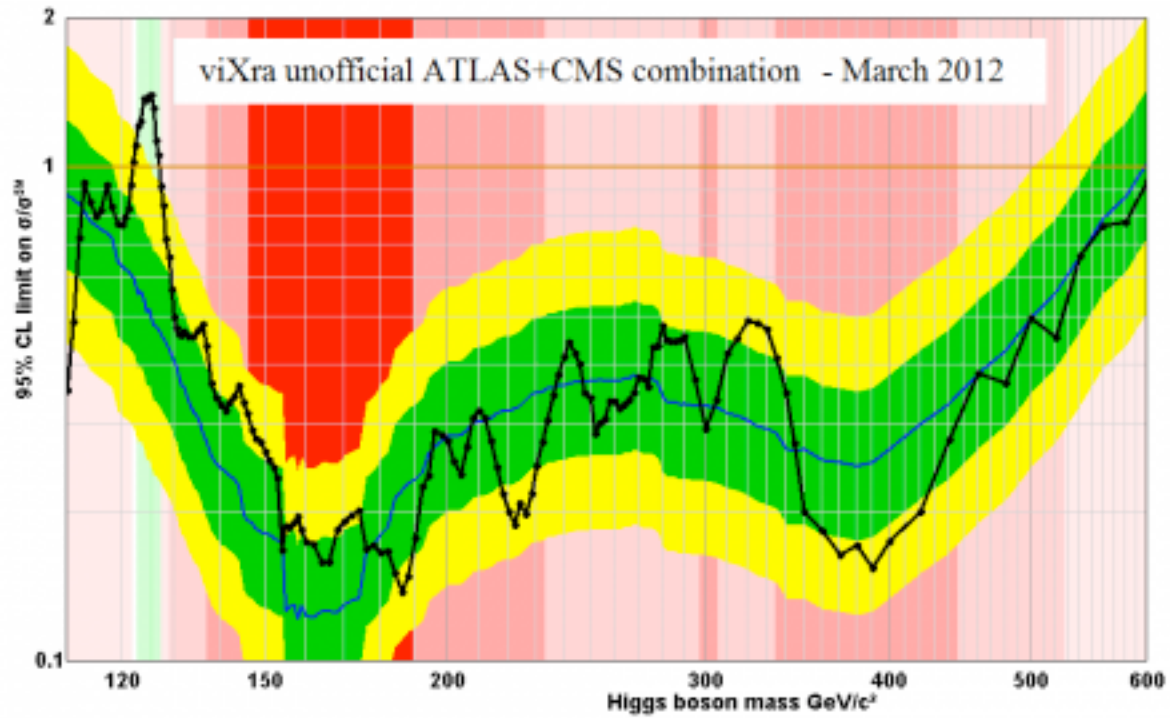
Higgs boson mass GeV/c²

vixra log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ROOFING
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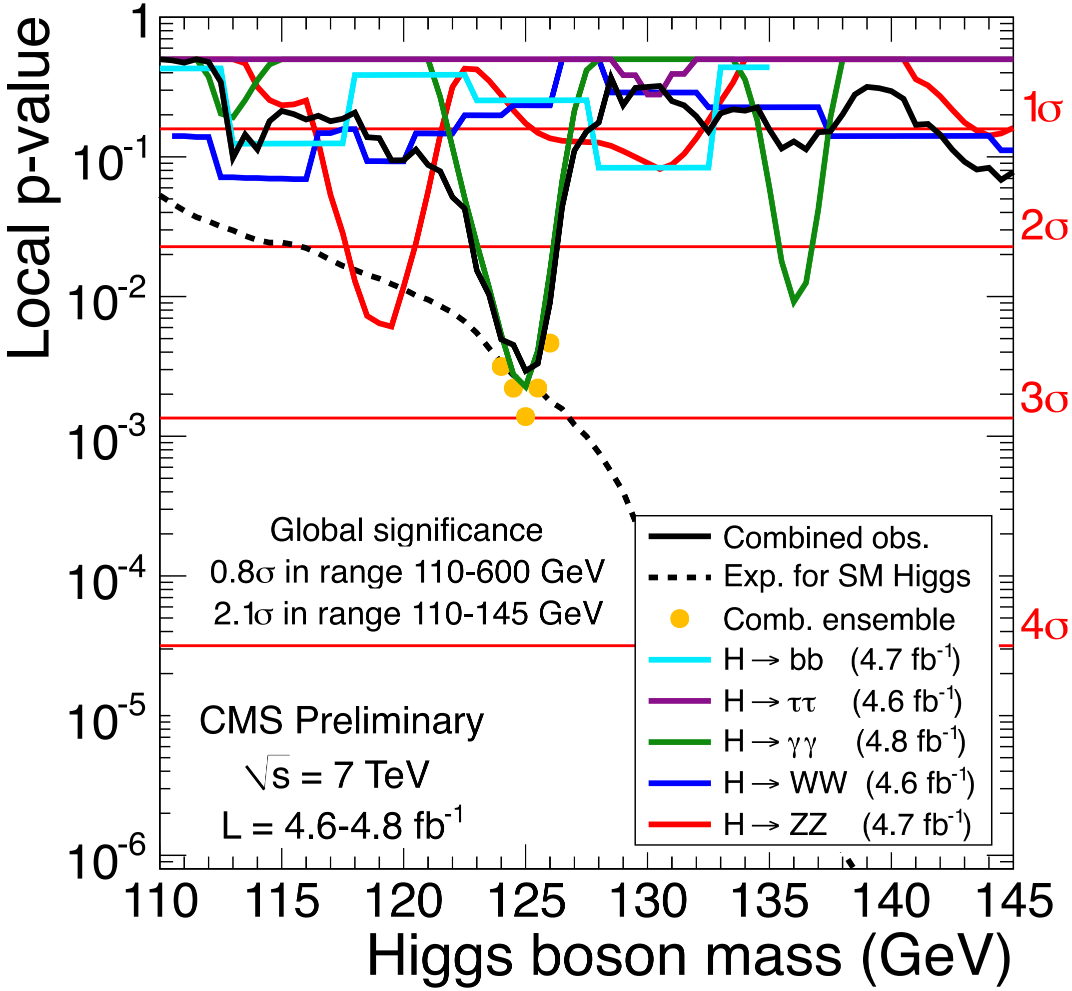
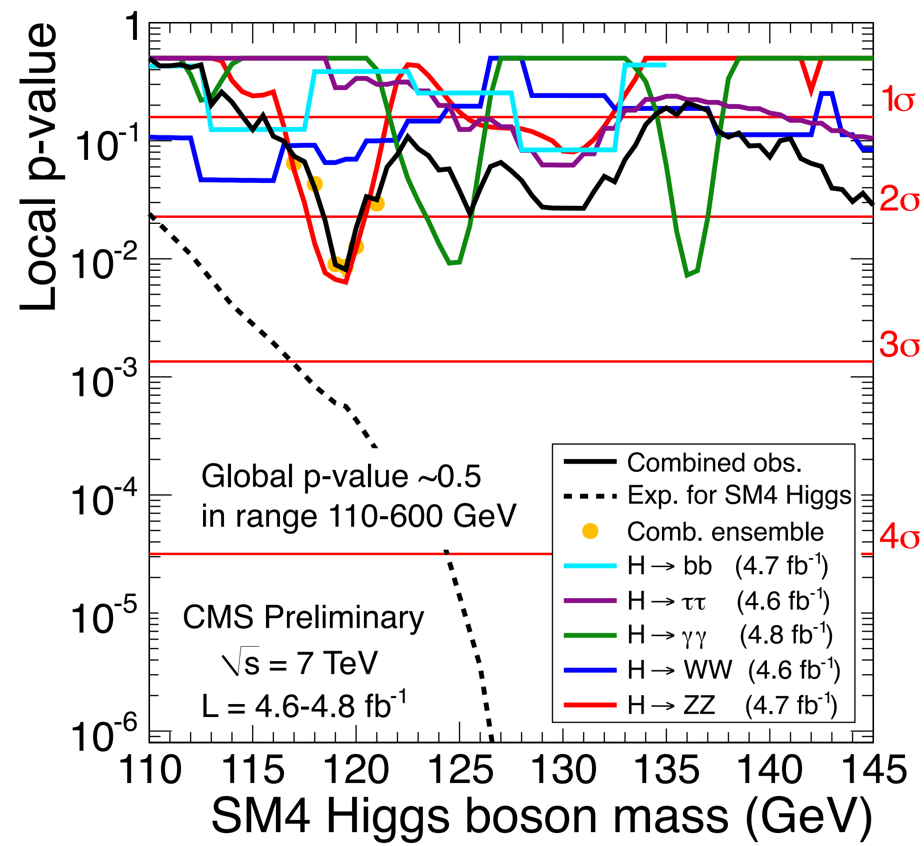


Local p-value
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
1σ
2σ
3σ
4σ
Global significance
0.8σ in range 110-600 GeV
2.1σ in range 110-145 GeV
CMS Preliminary
$\sqrt{s}$ = 7 TeV
L = 4.6-4.8 fb^{-1}
Combined obs.
Exp. for SM Higgs
Comb. ensemble
H → bb (4.7 fb^{-1})
H → ττ (4.6 fb^{-1})
H → γγ (4.8 fb^{-1})
H → WW (4.6 fb^{-1})
H → ZZ (4.7 fb^{-1})
110 115 120 125 130 135 140 145
Higgs boson mass (GeV)



Local p-value
1
10^{-1}
10^{-2}
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10^{-4}
10^{-5}
10^{-6}
1σ
2σ
3σ
4σ
Global p-value ~0.5
in range 110-600 GeV
CMS Preliminary
$\sqrt{s}$ = 7 TeV
L = 4.6-4.8 fb^{-1}
Combined obs.
Exp. for SM4 Higgs
Comb. ensemble
H → bb (4.7 fb^{-1})
H → ττ (4.6 fb^{-1})
H → γγ (4.8 fb^{-1})
H → WW (4.6 fb^{-1})
H → ZZ (4.7 fb^{-1})
110 115 120 125 130 135 140 145
SM4 Higgs boson mass (GeV)

Outline

1. Introduction
2. The Model
3. Dilatation
4. Phase Structure
5. Conclusions

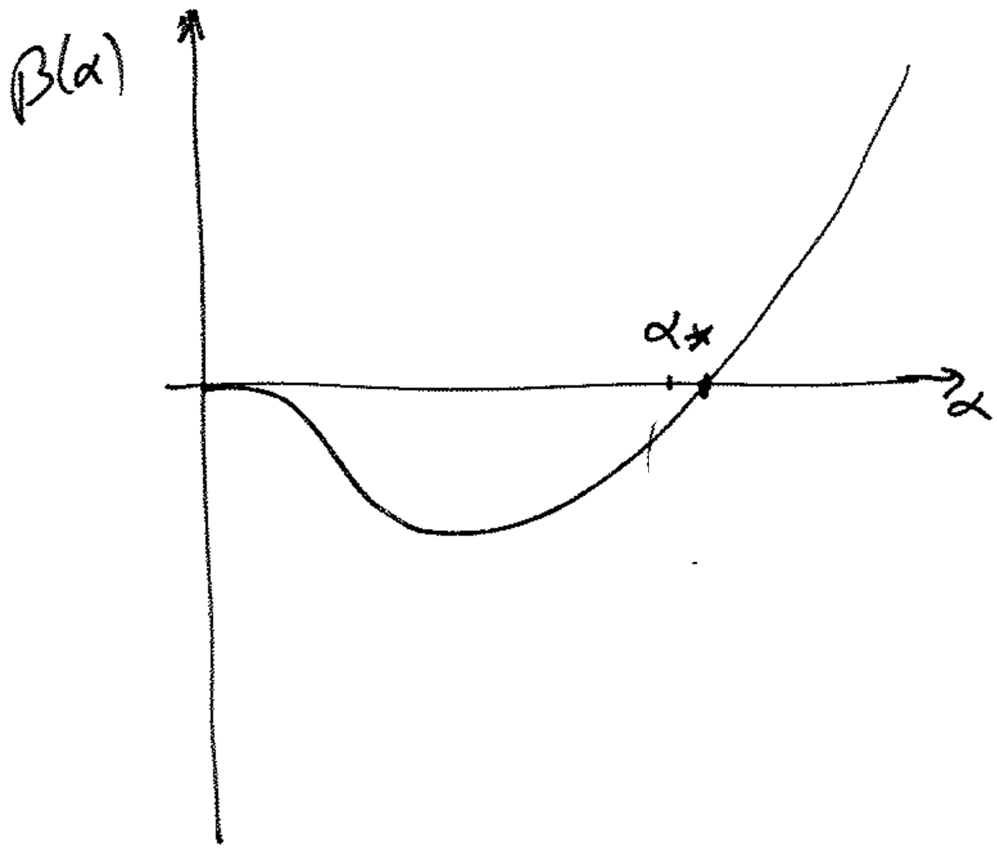
What and why a Dilaton?

A Nambu-Goldstone boson of a spontaneously broken scale (dilatation) symmetry. If scale symmetry is only approximate, we get a pseudo NGB instead, also called dilaton.

Why would one be interested in a light dilaton?

- It can serve as a scalar analog of the graviton. (Sundrum ’03)
- If SM is embedded in a CFT, a dilaton could have properties similar to the Higgs. (Goldberger, BG, Skiba ’08; Vecchi ’10; Campbell, Ellis, Olive ’11; Matsuzaki, Yamawaki ’12)
- It can serve as a force mediator between dark matter and normal matter. (Bai, Carena, Lykken ’09)
- If the dilaton is sufficiently decoupled, can serve as a dark matter candidate.(Choi, Hong, Matsuzaki ’11)

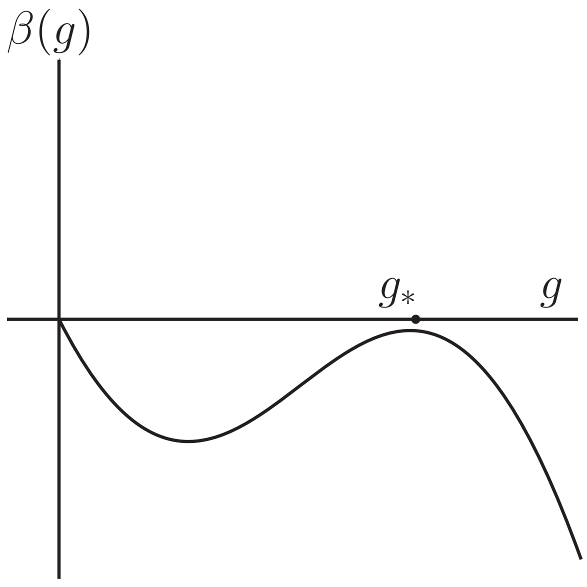
Scale Invariant Theory



Coupling runs from UV fixed point (origin) to IR fixed point
(which it reaches in exponential RG-time).

(Approximate) Scale Invariant Theory

A schematic β function of a WTC-like theory. The coupling constant flows toward the "would be" IR fixed-point, g_*. Close to the fixed-point, the flow is slow and the theory posses approximate scale symmetry.



If the RG-trajectory reaches g_*, scale invariance becomes exact.

However, some order parameter may get a vev. If this happens close to the fixed-point, scale invariance is spontaneously broken and one expects a light dilaton in the spectrum.

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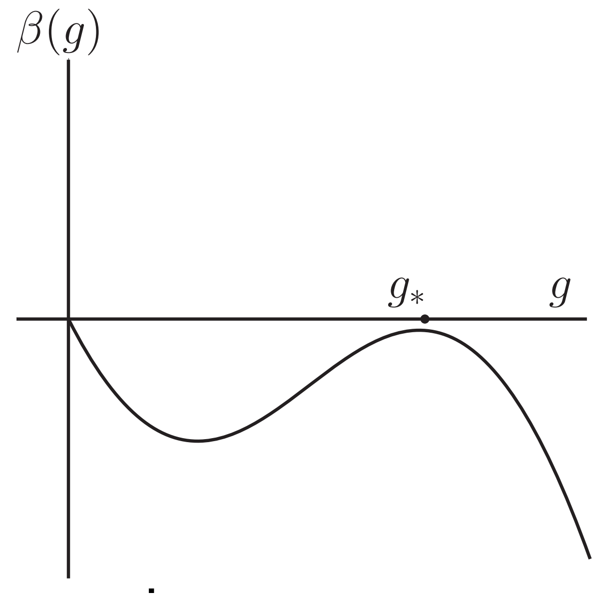

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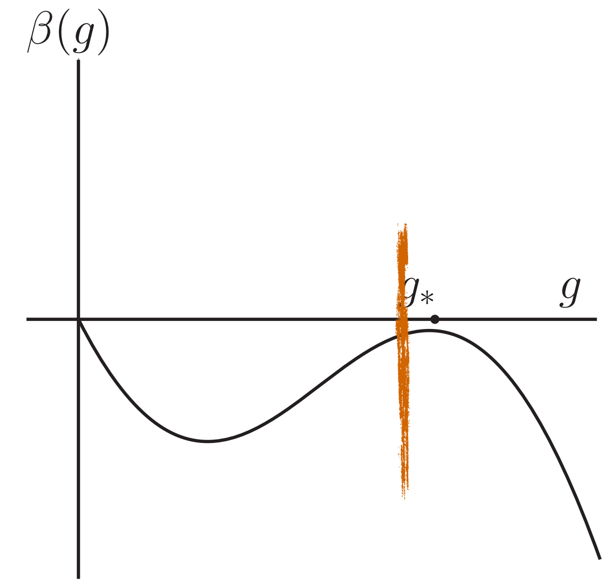

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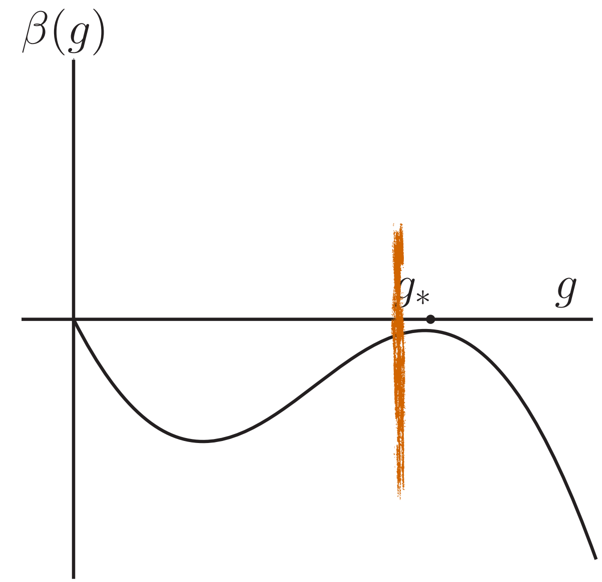

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What does this mean?

A field theory with this behavior is not common!

(Approximate) Scale Invariant Theory (cont.)

In the WTC framework, the fixed-point is strongly interacting.
As a result (wish?), fermion condensate forms and breaks scale invariance.
But the strong interacting nature of the model makes it difficult to analyze analytically.
In particular, the existence of a light dilaton in WTC is not clear and is the subject of a recent rekindled debate

- Yes (Appelquist, Bai '10)
- No (Hashimoto, Yamawaki '10; Vecchi '10)

In a nut-shell: small β vs η'-like (QCD anomaly).

Having a perturbative toy model with the above properties – an interacting fixed-point and an approximate scale invariance which is broken dynamically – would help *me* understand this better.

Can it be done??

Why is it non-straightforward? Try the obvious stuff first:

SSB in CFT?

- Take CFT with moduli space (common in SCFT). Say, for definiteness:
  $\mathcal{N} = 4$ SUSY *SU(N)* ➠ flat directions ➠ expand about a point away from origin.

- EFT = $SU(N) \to SU(N-k) \times SU(k) \times U(1)$.

- But EFT has $\mathcal{N} = 4$ SUSY unbroken, “Dilaton” is exactly massless together with partners ➠ no mass gap.

- Perturbations: flow into ??? (possibly another CFT, interacting), fate of “dilaton?”

Better attempt(?): Coleman-Weinberg abelian-higgs model.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D^{\mu}\phi^{*}D_{\mu}\phi - \lambda|\phi|^{4}$$

- Fine tune mass to zero, classically scale invariant.
- Effective potential develops a minimum away from origin:

$$V = \frac{\lambda}{4!}\varphi_c{}^4 + \left(\cancel{\frac{5\lambda^2}{1152\pi^2}} + \frac{3e^4}{64\pi^2}\right)\varphi_c{}^4\left(\ln\frac{\varphi_c{}^2}{M^2} - \frac{25}{6}\right)$$

- Gauge and scale symmetries spontaneously broken.
- Gauge field acquires mass.
- But would-be-dilaton acquires mass too: trace anomaly spoils scaling symmetry.
- So far, so good. But now:

$$M_{\mathrm{dilaton}}/M_{\mathrm{vector}} \sim e^2/16\pi^2$$

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Want: arbitrarily light dilaton without turning off interactions

(note: Hashimoto and Yamawaki argue that in WTC:
as $M_{\text{dilaton}} \to 0$ then the decay constant $f \to \infty$ and dilaton decouples).

We construct a model which

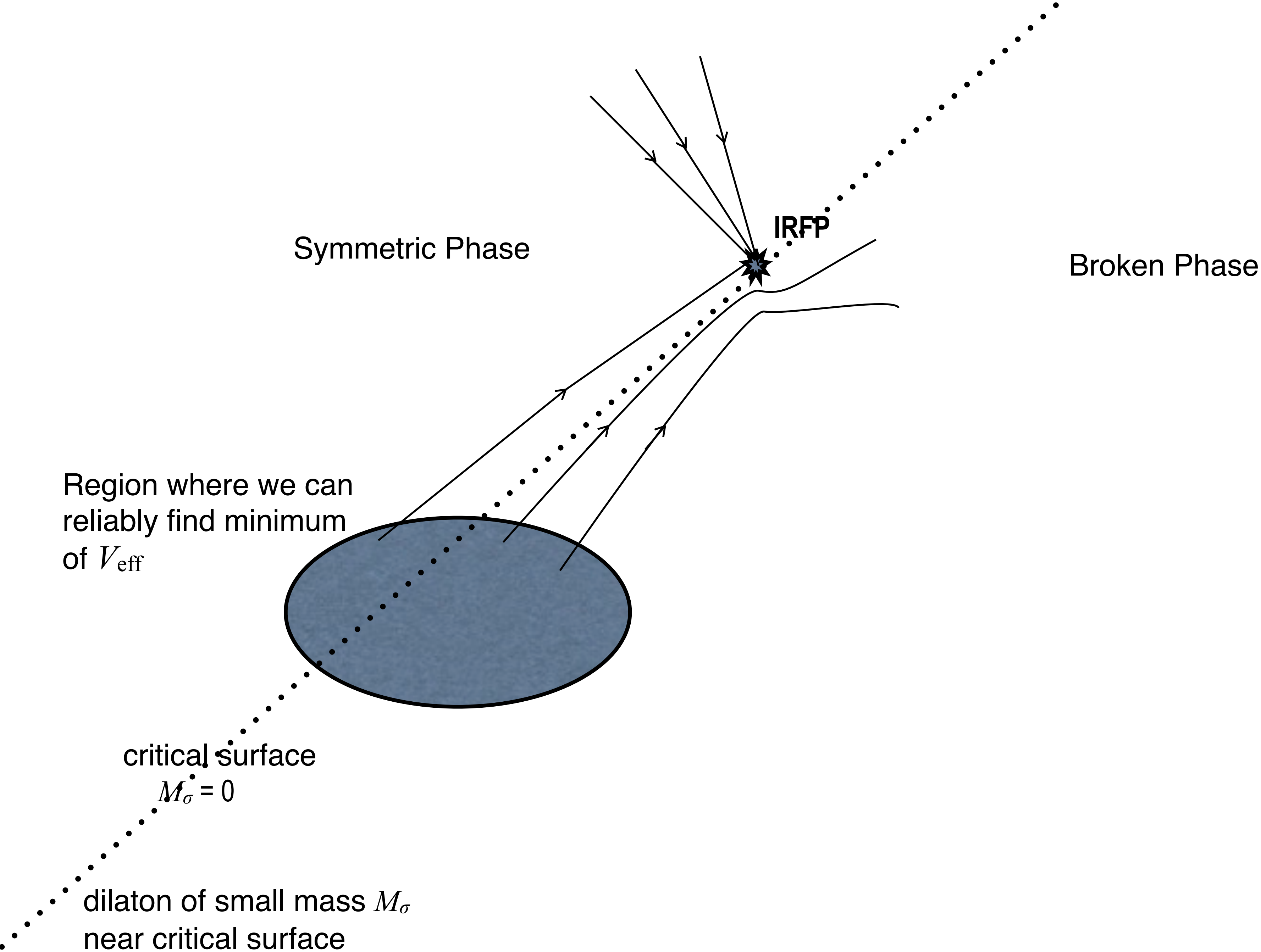
- ☑ Is perturbative (naively)
- ☑ Has a perturbative IR fixed point (naively)
- ☑ Spontaneously breaks approximate scale symmetry
- ☑ Dilaton can be made arbitrarily light (mass gap!) while still interacting

follow-up by Antipin, Mojaza, Sannino ’11
some qualitative differences; unresolved

In the rest of the talk I will explain this model.

First the big picture:

Theory parameter space: couplings at fixed μ_0



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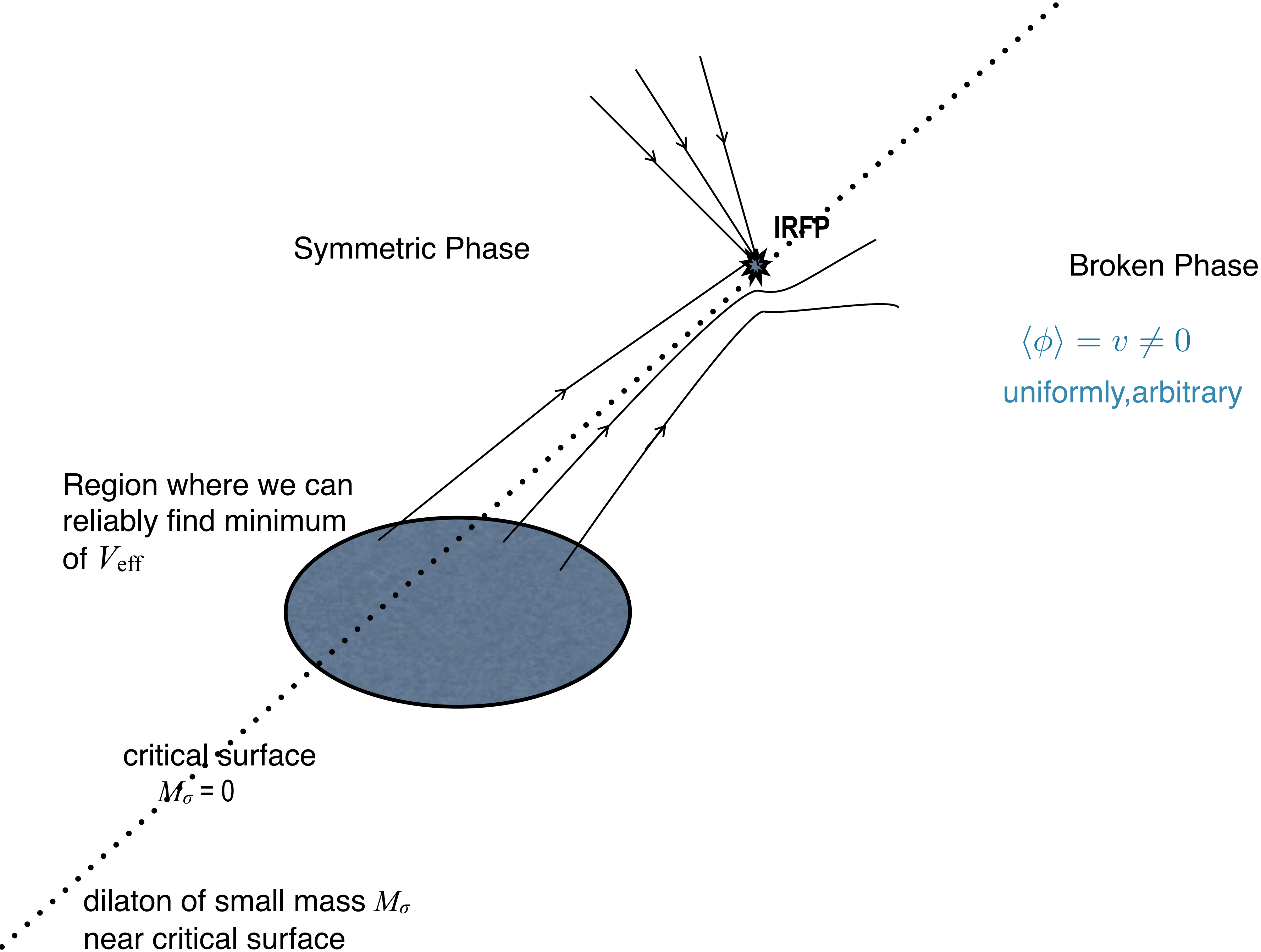


Outline

The Model

SU(N) gauge theory with $n_\psi = n_\chi$ fundamental fermions ψ and χ and two scalar singlets ϕ_1 and ϕ_2.

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu} + \sum_{j=1}^{n_\chi}\left(\bar{\psi}^j i\not{D}\psi_j + \bar{\chi}^j i\not{D}\chi_j\right) + \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2$$
$$- y_1\left(\bar{\psi}\psi + \bar{\chi}\chi\right)\phi_1 - y_2(\bar{\psi}\chi + \bar{\chi}\psi)\phi_2$$
$$- \frac{1}{24}\lambda_1\phi_1^4 - \frac{1}{24}\lambda_2\phi_2^4 - \frac{1}{4}\lambda_3\phi_1^2\phi_2^2$$

This theory is invariant under discrete $\mathbb{Z}_2$ as well as SU(n_χ) symmetry

$$\begin{aligned} \phi_1, \psi &\to \phi_1, \psi \\ \phi_2, \chi &\to -\phi_2, -\chi \end{aligned} \quad \text{and} \quad \begin{aligned} \psi &\to U\psi \\ \chi &\to U\chi \end{aligned}$$

Nota bene

Masses set to zero (I am not solving the hierarchy problem).
Precisely as with Coleman and Weinberg.
Set them to zero and use dimensional regularization.

Theory has Landau pole. This is a UV issue.
We study the IR properties of the model.
We can take it to be a cut-off theory.

This is not the theory of everything.
It is a Toy Model that displays some behavior that mimics some wanted behavior of WTC, may answer some questions and seems to be interesting in its own right (as a field theory).

$\overline{\text{MS}}$ β Functions

For large N with $n_\chi = 11N/4\,(1-\delta/11)$, the leading terms are

$$(16\pi^2)\frac{\partial g}{\partial t} = -\frac{\delta N}{3}g^3 + \frac{25N^2}{2}\frac{g^5}{16\pi^2}$$

$$(16\pi^2)\frac{\partial y_1}{\partial t} = 4y_1y_2^2 + 11N^2y_1^3 - 3Ng^2y_1$$

$$(16\pi^2)\frac{\partial y_2}{\partial t} = 3y_1^2y_2 + 11N^2y_2^3 - 3Ng^2y_2$$

$$(16\pi^2)\frac{\partial \lambda_1}{\partial t} = 3\lambda_1^2 + 3\lambda_3^2 + 44N^2\lambda_1y_1^2 - 264N^2y_1^4$$

$$(16\pi^2)\frac{\partial \lambda_2}{\partial t} = 3\lambda_2^2 + 3\lambda_3^2 + 44N^2\lambda_2y_2^2 - 264N^2y_2^4$$

$$(16\pi^2)\frac{\partial \lambda_3}{\partial t} = \lambda_1\lambda_3 + \lambda_2\lambda_3 + 4\lambda_3^2 + 22N^2\lambda_3y_1^2 + 22N^2\lambda_3y_2^2 - 264N^2y_1^2y_2^2$$

Fixed-point

To get a fixed-point for the gauge coupling, need to balance a 1-loop against a 2-loop.

This is possible because for large N, δ can be made small by a carefully chosen n_χ.

The fixed-point to leading order in $1/N$ is

$$g_*^2 = 16\pi^2 \frac{2}{75}\frac{\delta}{N}$$

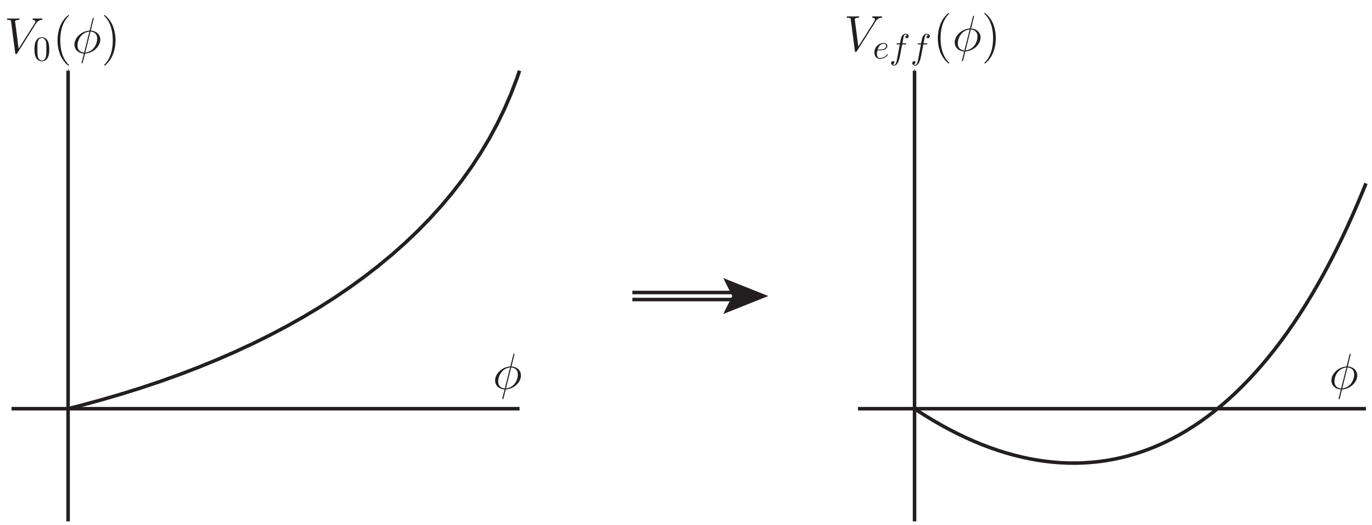
$$y_{1*}^2 = y_{2*}^2 = \frac{3}{11}\frac{g_*^2}{N}$$

$$\lambda_{1*} = \lambda_{2*} = \lambda_{3*} = \frac{18}{11}\frac{g_*^2}{N} = 6y_{1*}^2$$

Effective Potential

At tree-level, $\langle \phi_i \rangle = 0$ and all the particle are massless. The theory flows to the IR fixed-point.

However, quantum effect could drastically change the structure of the vacuum (Coleman, Weinberg '73)



The non-trivial vev gives mass to both fermions and scalars and alters the RG trajectory.

Effective Potential (cont.)

The effective potential in $\overline{\text{MS}}$ is

$$\begin{aligned}
V_{\text{eff}} = &-\frac{1}{24}\lambda_1\phi_1^4 - \frac{1}{24}\lambda_2\phi_2^4 - \frac{1}{4}\lambda_3\phi_1^2\phi_2^2 \\
&- \frac{11N^2M_{f+}^4}{(64\pi^2)}\left(\ln\frac{M_{f+}^2}{2\mu^2} - \frac{3}{2}\right) - \frac{11N^2M_{f-}^4}{(64\pi^2)}\left(\ln\frac{M_{f-}^2}{2\mu^2} - \frac{3}{2}\right) \\
&+ \frac{M_{s+}^4}{(64\pi^2)}\left(\ln\frac{M_{s+}^2}{\mu^2} - \frac{3}{2}\right) + \frac{M_{s-}^4}{(64\pi^2)}\left(\ln\frac{M_{s-}^2}{2\mu^2} - \frac{3}{2}\right)
\end{aligned}$$

$$M_{f\pm} = y_1\phi_1 \pm y_2\phi_2,$$

$$\begin{aligned}
M_{s\pm}^2 = &\frac{(\lambda_1+\lambda_3)\phi_1^2 + (\lambda_2+\lambda_3)\phi_2^2}{4} \\
&\pm \frac{\sqrt{(\lambda_1-\lambda_3)^2\phi_1^4 + (\lambda_2-\lambda_3)^2\phi_2^4 - 2(\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2\lambda_3 - 7\lambda_3^2)\phi_1^2\phi_2^2}}{4}
\end{aligned}$$

Effective Potential (cont.)

Minimizing the potential analytically is difficult. But easy to identify some local minima. Focus on minimum which preserves discrete $\mathbb{Z}_2$ (i.e. $\langle\phi_2\rangle = 0$).

The potential reduces to

$$V_{\text{eff}} = \frac{\lambda_1}{24}\phi_1^4 + \frac{(\lambda_1\phi_1^2)^2}{256\pi^2}\left(\ln\frac{\lambda_1\phi_1^2}{2\mu^2} - \frac{3}{2}\right) + \frac{(\lambda_3\phi_1^2)^2}{256\pi^2}\left(\ln\frac{\lambda_3\phi_1^2}{2\mu^2} - \frac{3}{2}\right) - \frac{22N^2y_1^4\phi_1^4}{64\pi^2}\left(\ln\frac{y_1^2\phi_1^2}{\mu^2} - \frac{3}{2}\right)$$

The extremum, $\partial/\partial\phi_1 V_{\text{eff}}(\langle\phi_1\rangle) = 0$, is at

$$-\frac{\lambda_1}{6} = \frac{\lambda_1^2}{64\pi^2}\left(\ln\frac{\lambda_1\langle\phi_1\rangle^2}{2\mu^2} - 1\right) + \frac{\lambda_3^2}{64\pi^2}\left(\ln\frac{\lambda_3\langle\phi_1\rangle^2}{2\mu^2} - 1\right) - \frac{88N^2y_1^4}{64\pi^2}\left(\ln\frac{y_1^2\langle\phi_1\rangle^2}{\mu^2} - 1\right)$$

Vacuum Expectation

λ_1 can be traded with $\langle\phi_1\rangle$ as a free parameter. For consistency, $\frac{\lambda_1}{16\pi^2}\ln\frac{\langle\phi_1\rangle^2}{\mu^2} \ll 1$. Stability of the vev is determined from the eigenvalues of second derivative matrix

$$\frac{\partial^2}{\partial\phi_1^2}V_{\text{eff}}(\langle\phi_1\rangle, 0) = \frac{\lambda_3^2 - 88N^2y_1^4}{32\pi^2}\langle\phi_1\rangle^2$$
$$\frac{\partial^2}{\partial\phi_2^2}V_{\text{eff}}(\langle\phi_1\rangle, 0) = \frac{\lambda_3}{2}\langle\phi_1\rangle^2 + \mathcal{O}(\text{1-loop})$$

Evaluate V_{eff} at $\langle\phi_1\rangle$ yields

$$V_{\text{eff}}(\langle\phi_1\rangle) = -\frac{\lambda_3^2 - 88N^2y_1^4}{512\pi^2}\langle\phi_1\rangle^4$$

Thus when $\varepsilon \equiv \lambda_3^2 - 88N^2y_1^4 \geq 0$, there is a non-trivial minimum.

Role of ϕ_2

Note that ϕ_2 never enters any calculations above.
Moreover, one can get an attractive IR fixed-point with just one scalar singlet.
This raises the question: what is the purpose of the second singlet?

Role of ϕ_2

Note that ϕ_2 never enters any calculations above. Moreover, one can get an attractive IR fixed-point with just one scalar singlet.
This raises the question: what is the purpose of the second singlet?

- It allows us to introduce more couplings, in particular the cross-coupling λ_3.
- Without the second singlet, the extremum found by perturbative analysis would have been the maximum.
  - The scalar potential appears to be unbounded from below.
  - Possible to have non-trivial minimum at higher scale which is inaccessible to perturbative analysis.

Pole mass in Broken Phase

The explicit 1-loop pole masses are

$$M_\psi(\mu) = M_\chi(\mu) = y_1 v \left[1 - \frac{g^2}{16\pi^2}\frac{N}{2}\left(3\ln\frac{y_1^2 v^2}{\mu^2} - 4\right)\right]$$

$$\begin{aligned}
M_{\phi_1}^2 &= \frac{\lambda_1 v^2}{2} + \frac{3\lambda_1^2 v^2}{64\pi^2}\left(\ln\frac{\lambda_1 v^2}{2\mu^2} - \frac{5}{3} + \frac{2\pi}{3\sqrt{3}}\right) + \frac{3\lambda_3^2 v^2}{64\pi^2}\left(\ln\frac{\lambda_3 v^2}{2\mu^2} - \frac{1}{3} - \frac{2\lambda_1}{3\lambda_3}\right) \\
&\quad + \frac{22N^2 y_1^2}{16\pi^2}\left[y_1^2 v^2 - \frac{\lambda_1 v^2}{12} - 3\left(y_1^2 v^2 - \frac{\lambda_1 v^2}{12}\right)\left(\ln\frac{y_1^2 v^2}{\mu^2}\right)\right. \\
&\quad \left. - 3\int_0^1 \mathrm{d}x\left(y_1^2 v^2 - \frac{x(1-x)}{2}\lambda_1 v^2\right)\ln\left(1 - x(1-x)\frac{\lambda_1}{2y_1^2}\right)\right] \\
&\simeq \frac{\lambda_3^2 - 88N^2 y_1^4}{32\pi^2}v^2 = \frac{\varepsilon}{32\pi^2}v^2
\end{aligned}$$

Since $v = \langle\phi_1\rangle$, it has the same anomalous dimension as ϕ_1.
Using the anomalous dimension and the β functions, one can verify that the masses are RG invariant at 1-loop.

Outline

Dilatation Current

The dilatation current, $\mathcal{D}^\mu$, is constructed from the improved energy-moementum tensor, $\Theta^{\mu\nu}$, of Callan, Coleman and Jackiw.

$$\mathcal{D}^\mu = x_\nu \Theta^{\mu\nu}$$

$$\Theta^{\mu\nu} = -F^{a\mu\lambda}F^{a\nu}_{\lambda} + \frac{1}{2}\bar{\chi}i(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\chi + \frac{1}{2}\bar{\psi}i(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi$$
$$+ \partial^\mu\phi_i\partial^\nu\phi_i - g^{\mu\nu}\mathcal{L} - \frac{1}{2}\kappa(\partial^\mu\partial^\nu - g^{\mu\nu}\partial^2)\phi_i^2$$

κ is the improvement term. It is a total derivative.
The CCJ improved tensor is the one with $\kappa = 1/3$.

- The improvement term does not change the charges constructed from $\Theta^{\mu\nu}$.
- The matrix elements of $\Theta^{\mu\nu}$arefinite, it doesn't get renormalized.

Trace Anomaly

The divergence of the dilatation current is the trace of the improved energy-momentum tensor.
Classically Θ^μ_μ vanishes for theory without any dimensional couplings.
Quantum effects make Θ^μ_μ non-zero, this is known as trace anomaly.
For the theory under consideration

$$\Theta^\mu_\mu = \gamma_{\phi_1}\phi_1\partial^2\phi_1 + (4\gamma_{\phi_1}\lambda_1 - \beta_{\lambda_1})\frac{\phi_1^4}{24} + \ldots$$

Terms involving other fields are omitted.

Terms proportional to γ_{ϕ_1} are usually omitted.
They cancel when EOM is applied but can contribute to off-shell matrix element and Green functions.
Also these terms are needed to make the trace RG-invariant.

Dilaton: The particle (state)

We may look for the dilaton state, σ, by using the following generic criteria:

- spinless state
- couples strongly/linearly to the energy-momentum tensor
- lightest such state

Clearly ϕ_1 satisfies all of the above.

- It is the only state whose mass start at 1-loop (modulo gauge fields)
- It is the only state which couples linearly to the energy-momentum tensor when expanded about $\phi_1\rangle = v$, $\langle\phi_2\rangle = 0$

Thus we identify σ with a single particle state created by ϕ_1.

Decay Constant

Define the decay constant f_σ by

$$\langle 0|\Theta^{\mu\nu}(x)|\sigma\rangle = \frac{f_\sigma}{3}\left(p^\mu p^\nu - g^{\mu\nu}p^2\right)e^{ip\cdot x}$$

where p is the momentum of $|\sigma\rangle$. The form of the right hand side is constrained by conservation of $\Theta^{\mu\nu}$. The factor $1/3$ comes from

$$\langle 0|\partial_\mu \mathcal{D}^\mu|\sigma\rangle = \langle 0|\Theta^\mu_\mu|\sigma\rangle = -f_\sigma M_\sigma^2 e^{ip\cdot x}$$

Note that $\Theta^{\mu\nu} = -1/3v\partial^\mu\partial^\nu\phi_1 + \cdots$.
Thus to lowest order $f_\sigma = v + \cdots$.
The RG invariant expression is easy to guess

$$f_\sigma = vZ_{\phi_1}^{-1/2}$$

where Z_{ϕ_1} is the wavefunction renormalization factor.

Dilaton Mass

Having determined the decay constant f_σ, the mass of the dilaton can be obtained from the trace anomaly.
To lowest order, the mass is

$$
\begin{aligned}
M_\sigma^2 &= \frac{\lambda_1^2 + \lambda_3^2 - 88N^2 y_1^4}{32\pi^2} v^2 \\
&= \frac{\varepsilon}{32\pi^2} v^2 = M_{\phi_1}^2
\end{aligned}
$$

where λ_1^2 term is dropped for consistency.
RG invariance of M_σ can be inferred from M_{ϕ_1}.

- Given the vev, we can tune ε to make the dilaton light by comparison

Outline

1. Intorduction
2. The Model
3. Dilatation
4. Phase Structure
   - Broken/Symmetric Phase
   - Numerical Analysis
5. Conclusion

Broken Phase

Recall the theory admits a non-trivial minimum provided

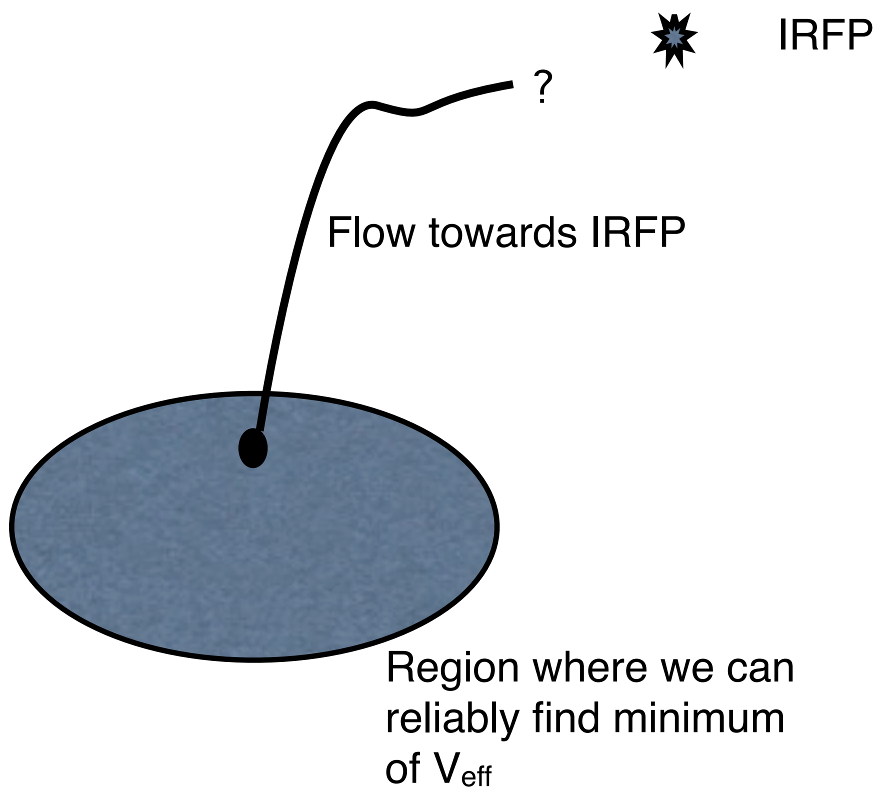
- λ_1 is much smaller than other couplings,
- $\varepsilon \equiv \lambda_3^2 - 88N^2 y_1^4 \geq 0.$

We want to study symmetry breaking close to the IR fixed-point. However, near the fixed-point these conditions are not satisfied.

Use RGE to trace back the RG trajectory to large RG time where perturbative analysis of effective potential yields a non-trivial minimum.

Alternatively, define the theory at scale μ_0 where perturbative analysis yields a non-trivial minimum. Moreover, if the vev is well below μ_0, RG flows will get close to the fixed-point before the massive particles decouple.

Theory parameter space: couplings at fixed μ_0



Symmetric Phase

For a point in parameter space where $\varepsilon < 0$

- $V_{eff}(\langle\phi_1\rangle)$ becomes positive and the non-trivial minimum disappears,
- the effective potential seems to be unbounded from below along ϕ_1 direction for large ϕ_1.

The second point threatens the validity of the model.

However, at large ϕ_1 perturbative analysis breaks down.
Can extend the range of perturbativity using the improved effective potential which effectively re-sum large logarithms.

$$V_{eff}^{\mathrm{imp}} = \frac{1}{24}\bar{\lambda}_1(t)e^{-4\int_0^{t'}\gamma_{\phi_1}\mathrm{d}t'}\phi_1^4$$

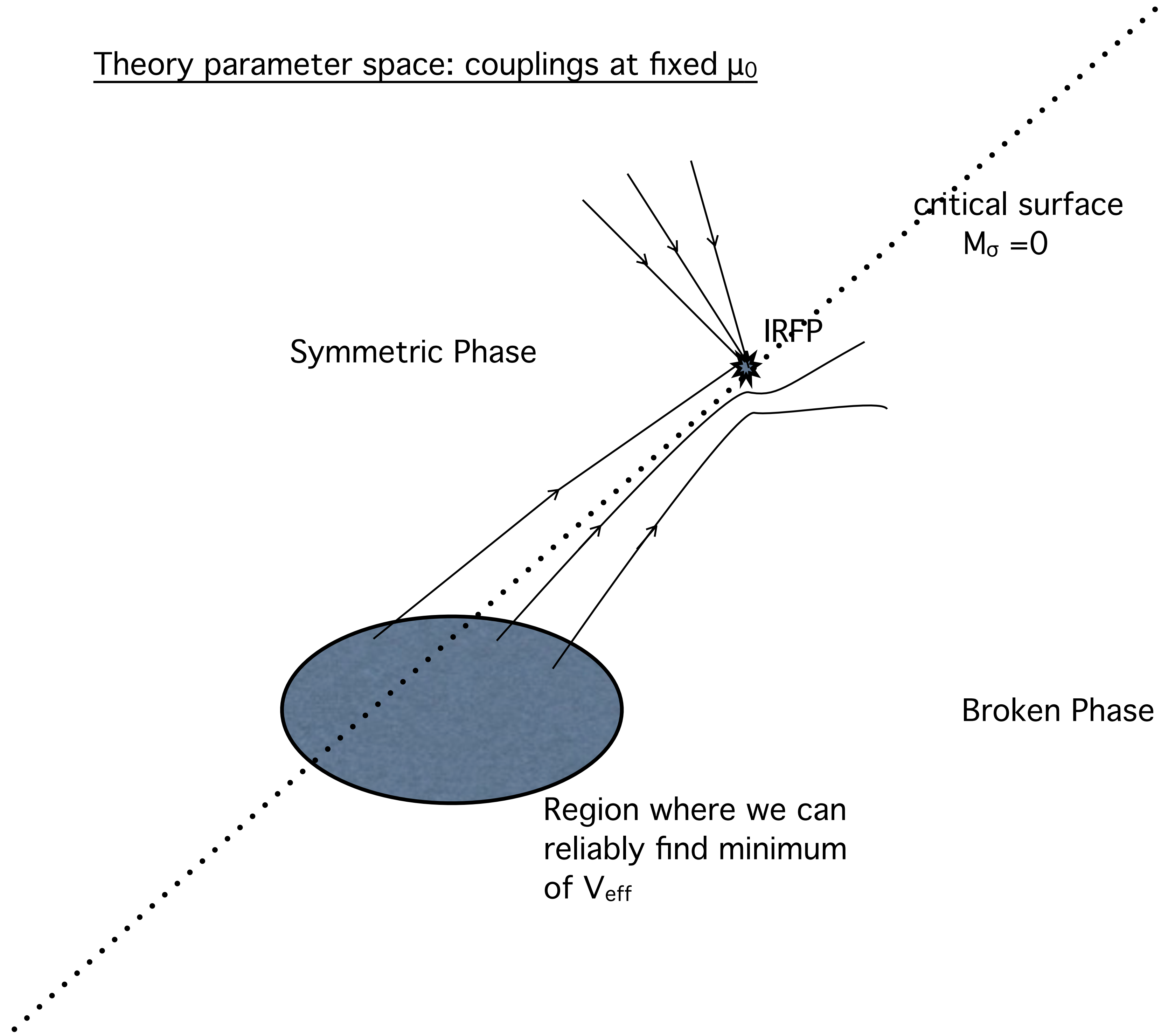
Here $t = \ln\phi_1/\mu_0$. This form is valid as long as $\bar{\lambda}_1(t)$ is perturbative.

Symmetric Phase (cont.)

For points in parameter space closed to the IR fixed-point, gauge coupling drives the Yukawa coupling to 0 in the UV.
Thus in the far UV, the β-function for λ_1 has a Landau pole.

- The effective potential is bounded from below because $\bar{\lambda}_1(t) > 0$ for large ϕ_1.
- The theory need a UV cutoff.
  - One can view the model as being a low energy effective theory of some UV completed models.
  - Since the cutoff will be many order of magnitude above the scale of symmetry breaking, one can (safely) ignore it.

Theory parameter space: couplings at fixed μ_0



Numerical Value

$N = 20$, $n_f = 11/2\,N$, $\delta = 0.2$,
$g(\mu_0) = \frac{4}{9} g_*$, $y_1(\mu_0) = 0.32 y_{1*}$, $y_2(\mu_0) = \frac{1}{5} y_{2*}$,
$\lambda_1(\mu_0) = \frac{1}{30}\lambda_{2*}$, $\lambda_2(\mu_0) = 3\lambda_{2*}$, $\lambda_3(\mu_0) = 5.2\lambda_{3*}$.
These condition corresponds to $\varepsilon \gtrsim 0$.
The vev is at

$$\ln \frac{\langle \phi_1 \rangle}{\mu_0} \simeq -29$$

and the spectrum are

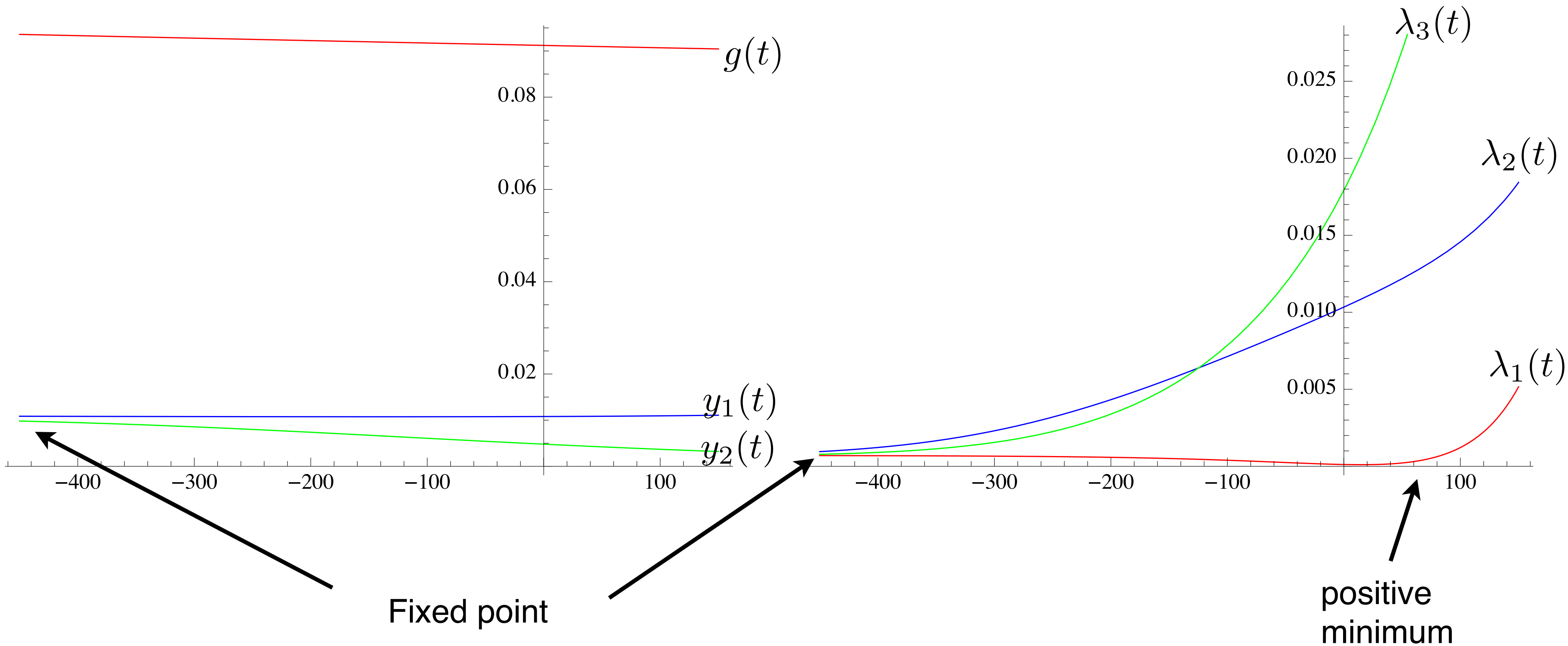
$$\frac{M_{\psi,\chi}}{v} \simeq 8.5 \times 10^{-3}, \quad \frac{M_{\phi_1}}{v} \simeq 7.9 \times 10^{-4}, \quad \frac{M_{\phi_2}}{v} \simeq 9.5 \times 10^{-2}.$$

Fractional correction to the effective potential from higher order terms are approximated to be

$$\left| \frac{N g_*^2}{16\pi^2} \ln\left(\frac{y_1^2 v^2}{\mu^2} \right) \right| \simeq 0.2.$$

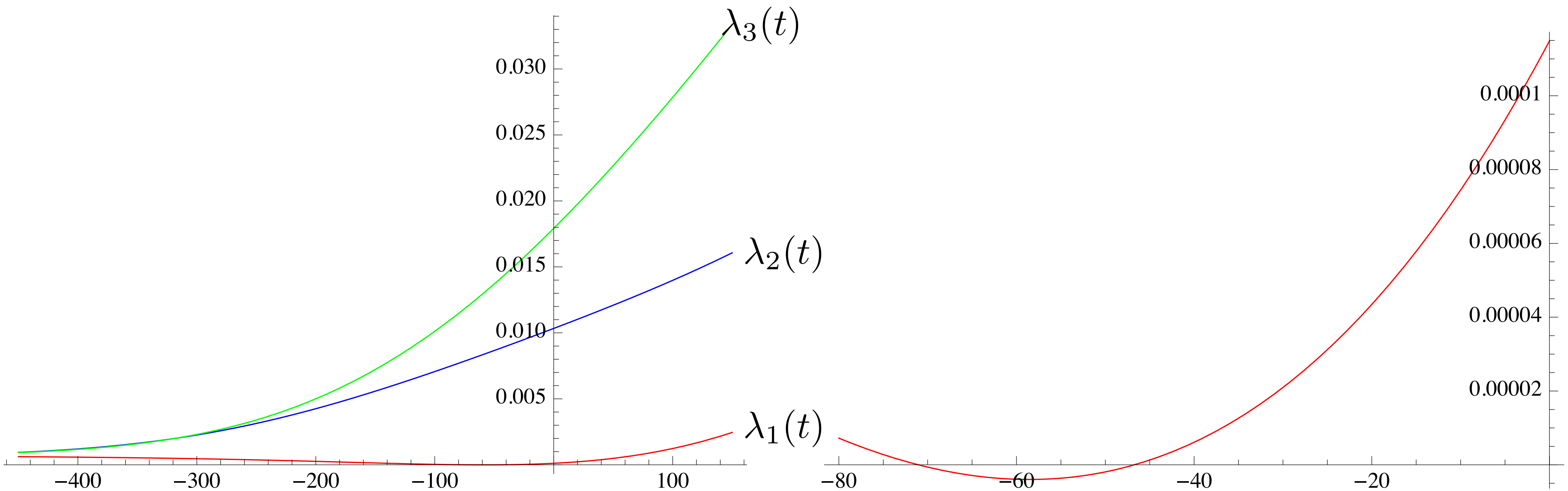
Numerical: Couplings Evolution

$N = 20$, $n_f = 11/2\,N$, $\delta = 0.2$,
$g(\mu_0) = \frac{4}{9} g_*$, $y_1(\mu_0) = 0.45 y_{1*}$, $y_2(\mu_0) = \frac{1}{5} y_{2*}$,
$\lambda_1(\mu_0) = \frac{1}{30}\lambda_{1*}$, $\lambda_2(\mu_0) = 3\lambda_{2*}$, $\lambda_3(\mu_0) = 5.2\lambda_{3*}$. These condition corresponds to $\varepsilon < 0$.



Numerical: Broken Phase

$y_1(\mu_0) = 0.32 y_{1*}$. This corresponds to a positive ε.



The coupling λ_1 becomes negative during the flow.
This agrees with our expectation from the improved effective potential.

Gauge coupling walks. Eventually runs again. Endgame: glueballs.

Outline

Summary

- We construct a perturbative model which a non-trivial IR fixed point.
- There are two distinct phases in our model:
  - Symmetric phase: the flow reaches the fixed point (in infinite RG time.)
  - Broken phase: the vev is dynamically generated. this phase somewhat mimics the behavior of walking technicolor.
- The broken phase can be used to study the dilaton:
  - The mass of the dilaton can be made arbitrary small compared to the vev by tuning the parameter $\varepsilon = \lambda_3^2 - 88N^2 y_1^4$.
- The model can be used to verify results/conjectures in literature which are obtained via indirect argument.

Applications

We can use this model to verify various results in the literature. For a specific example we will verify the dilaton potential in nearly conformal theory (Goldberger, BG, Skiba) . Taking $\mathcal{L} = \mathcal{L}_{\text{CFT}} + \sum_n \lambda_n \mathcal{O}_n$, GGS arrives, via indirect argument, at the dilaton potential

$$V_{\text{eff}}(\chi) = \frac{M_\sigma^2}{4 f_\sigma^2} \chi^4 \left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4} \right] + \mathcal{O}(\gamma^2).$$

If $$\mathcal{L} = \mathcal{L}_{\rm CFT} + \sum_n \lambda_n \mathcal{O}_n \,.$$ and

$|\gamma_n| \ll 1$ then $$V_{\rm eff}(\chi) = \frac{M_\sigma^2}{4 f_\sigma^2}\chi^4 \left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4}\right] + \mathcal{O}(\gamma^2)\,.$$

or

$|\lambda_n| \ll 1$ $$V_{\rm eff}(\chi) = \frac{M_\sigma^2}{f_\sigma^2 \gamma}\chi^4 \left[\frac{1}{4+\gamma}\left(\frac{\chi}{f_\sigma}\right)^\gamma - \frac{1}{4}\right] + \mathcal{O}(\lambda^2)\,,$$

$$V_{\rm eff}(\chi) = \frac{M_\sigma^2}{f_\sigma^2}\chi^4 \sum_n \left\{ x_n \left[\frac{1}{4+\gamma_n}\left(\frac{\chi}{f_\sigma}\right)^{\gamma_n} - \frac{1}{4}\right]\right\} + \mathcal{O}(\lambda^2)\,, \qquad \textstyle\sum_n \gamma_n x_n = 1.$$

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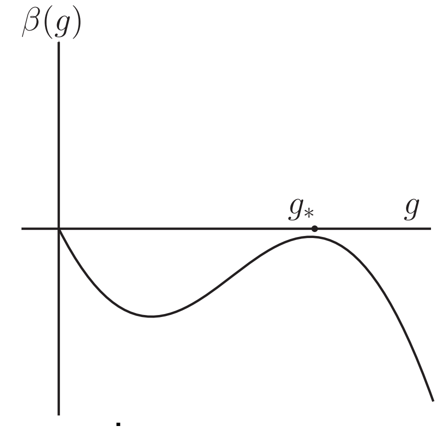
To compare our model with GGS, we view our model as $\mathcal{L}(g) = \mathcal{L}(g_*) + (\mathcal{L}(g) - \mathcal{L}(g_*))$. In our model the dilaton field is identified with ϕ_1 and the anomalous dimensions are small. Our effective potential for ϕ_1 turns out to be exactly the same as GGS.

Dilaton in WTC?

AB say:

$$M_\sigma^2 \simeq \frac{s(\alpha_* - \alpha_c)}{\alpha_c}\Lambda^2 \simeq \frac{N_f^c - N_f}{N_f^c}\Lambda^2,$$



First equation: In our model the critical coupling is a critical surface, the IRFP is on critical surface, 0=0, correct but not interesting and not what is intended

Second equation: $(N^c - N)/N$ plays role of ε, measures distance to critical surface, and equation is qualitatively correct!