Title (even content) has been changed to

Phase structure of many flavor lattice QCD at finite temperature

Norikazu Yamada (KEK/GUAS)

in collaboration with

Shinji Ejiri (Niigata)

KMI/GCOE Workshop on "Strong Coupling Gauge Theories in the LHC Perspective (SCGT 12)"
December 4 (Tuesday) - December 7 (Friday), 2012 @KMI, Nagoya University, Nagoya, Japan

Introduction

- The nature of the chiral phase transition of Many Flavor QCD [MFQCD] depends on the number of flavors and the masses.

  Pisarski and Wilczek, PRD 29, 338 (1984) and many lattice calcs.

- EW baryogenesis in TC models.

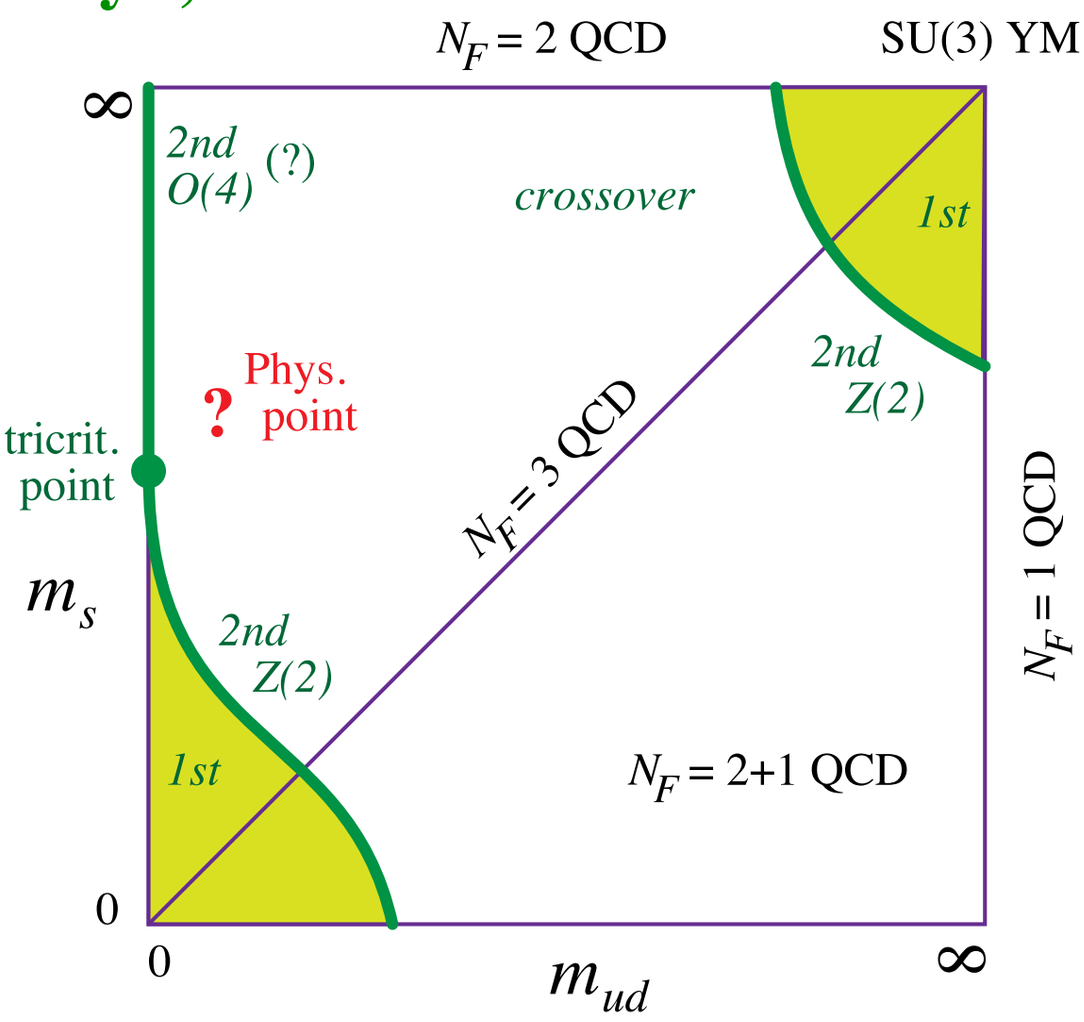
  Appelquist, Schwetz and Selipsky, PRD52, 4741 (1995);
  Kikukawa, Kohda and Yasuda, PRD77 (2008) 015014

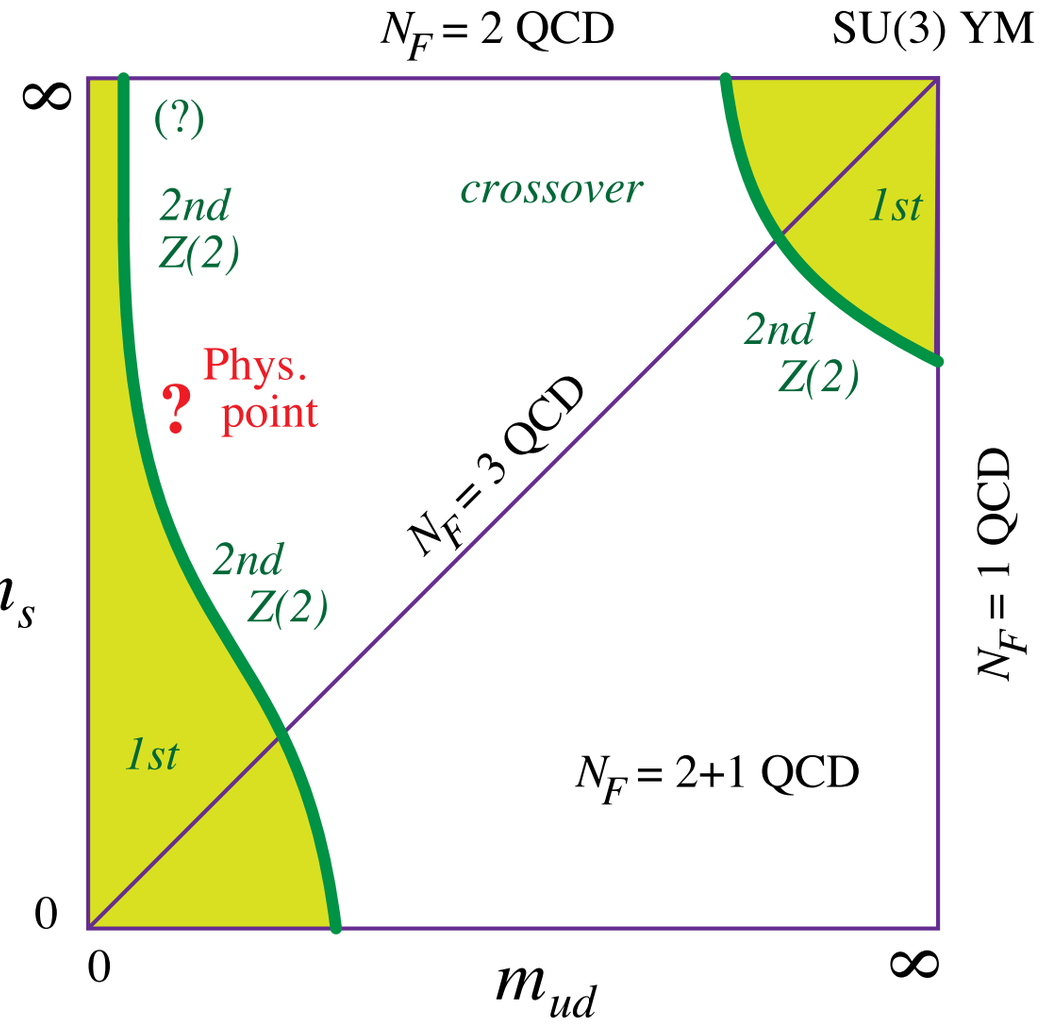
- Towards the study of this possibility,
  we consider 2(light)+N_f(heavy)-flavor QCD, propose an easy way to explore the phase structure and demonstrate the feasibility of the method.

Columbia plot for 2+1 QCD

Brown, Butler, Chen, Christ, Dong, Schaffer, Unger, and Vaccarino (90),
N.H. Christ, Z. Dong (92) and N.H. Christ(92)

Kanaya, Lattice 2010



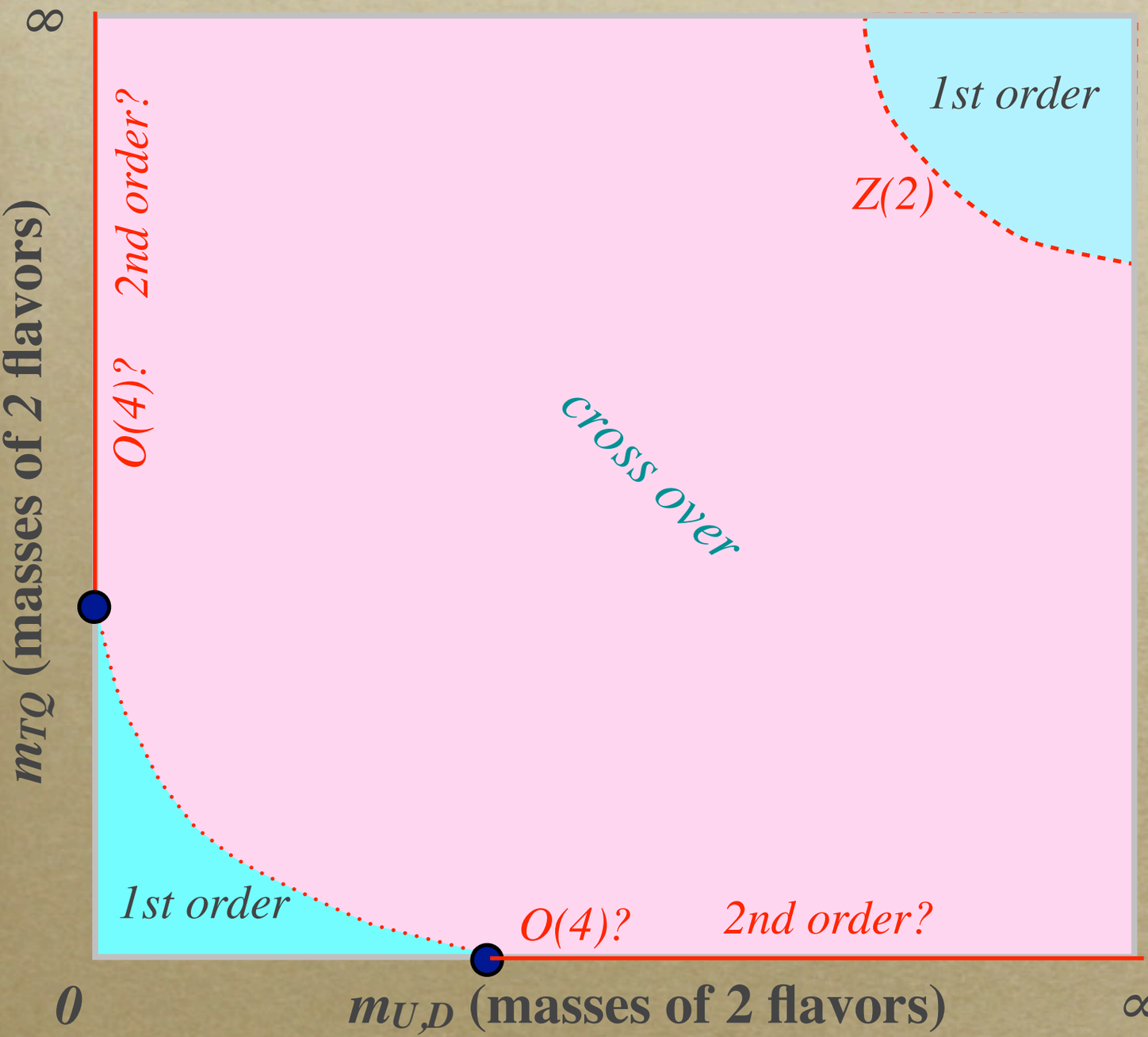


When $N_F \cong 3$ (or 2?), Chiral Phase Transition around the massless limit is 1st order.

Pisarski and Wilczek, PRD 29, 338 (1984); Iwasaki et al. (1997). See also Aoki, Fukaya, Taniguchi (2012)

Extend Columbia plot to 2+N_f QCD.

N_f=2 (2+2< N_f^{crit})



In TC, two flavors must be exact massless.

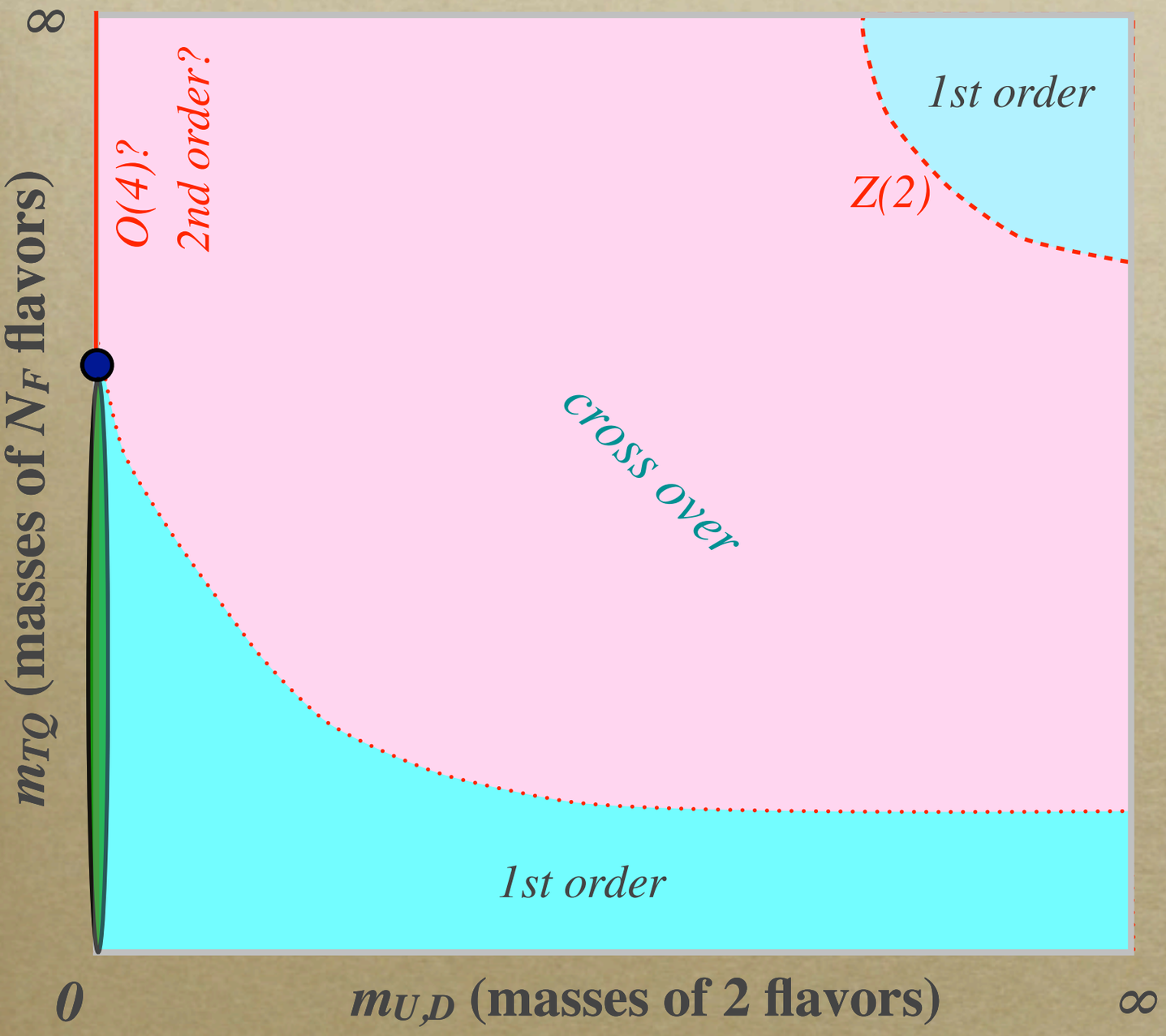
N_f^{crit} separating confining and conformal theories

Symmetric under reflection w.r.t. diagonal line.

Running of g^2 is not slow enough.

Less interesting.

$3 \leqq N_f\,(2{+}N_f < N_f^{\rm crit})$



1st order persists to $m_{U,D} = \infty$ for sufficiently small m_{TQ}.

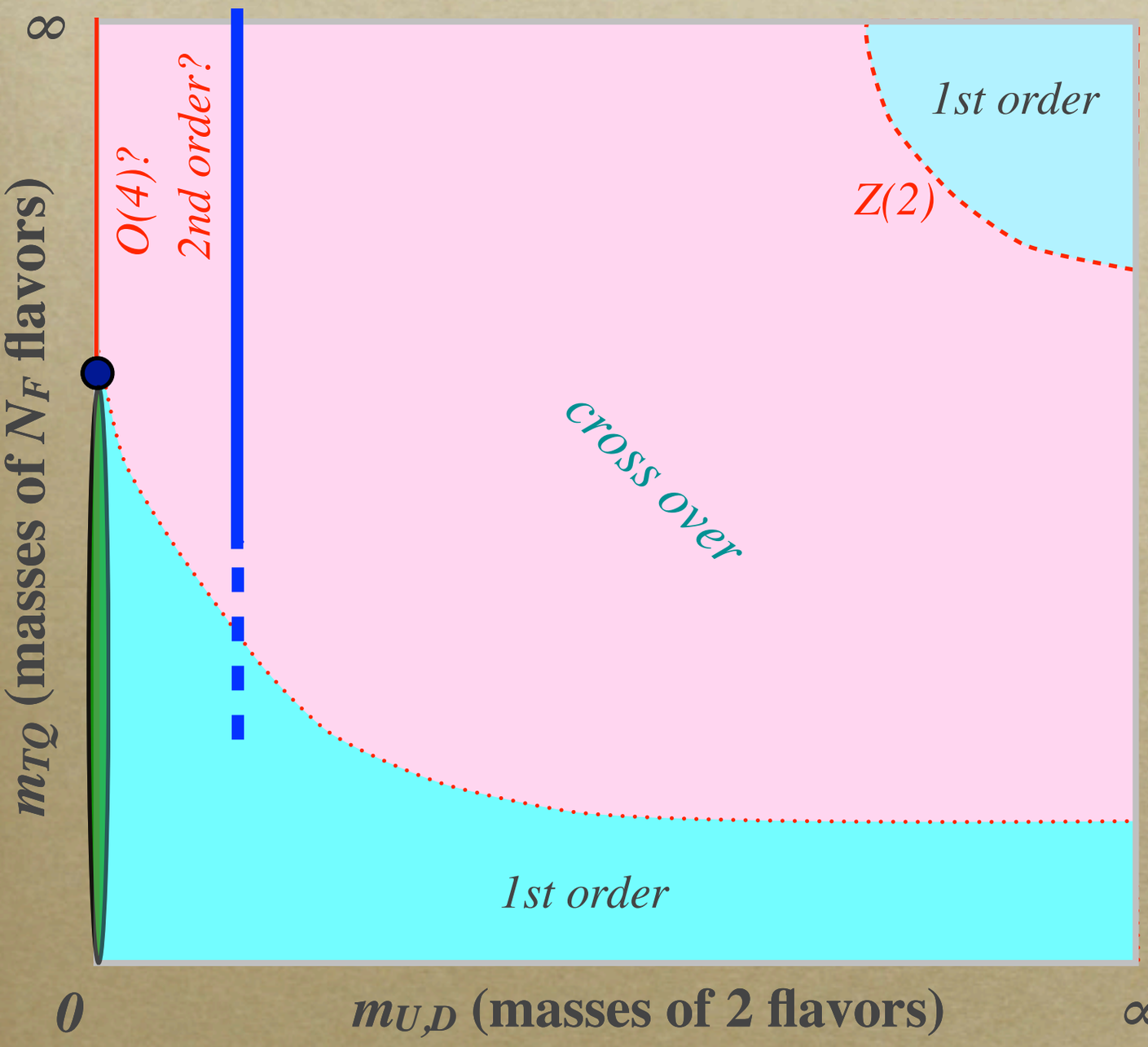
Walking with γ_m~O(1) is expected at an appropriate N_f.

For EW baryogenesis, important to identify the location of the 1st order region and its strength.

Appelquist, Schwetz and Selipsky, PRD52, 4741 (1995).
Kikukawa, Kohda and Yasuda, PRD77 (2008) 015014

Phenomenologically interesting!

$3 \leqq N_f\,(2{+}N_f < N_f^{\rm crit})$



1st order persists to $m_{U,D} = \infty$ for sufficiently small m_{TQ}.

Walking with γ_m~O(1) is expected at an appropriate N_f.

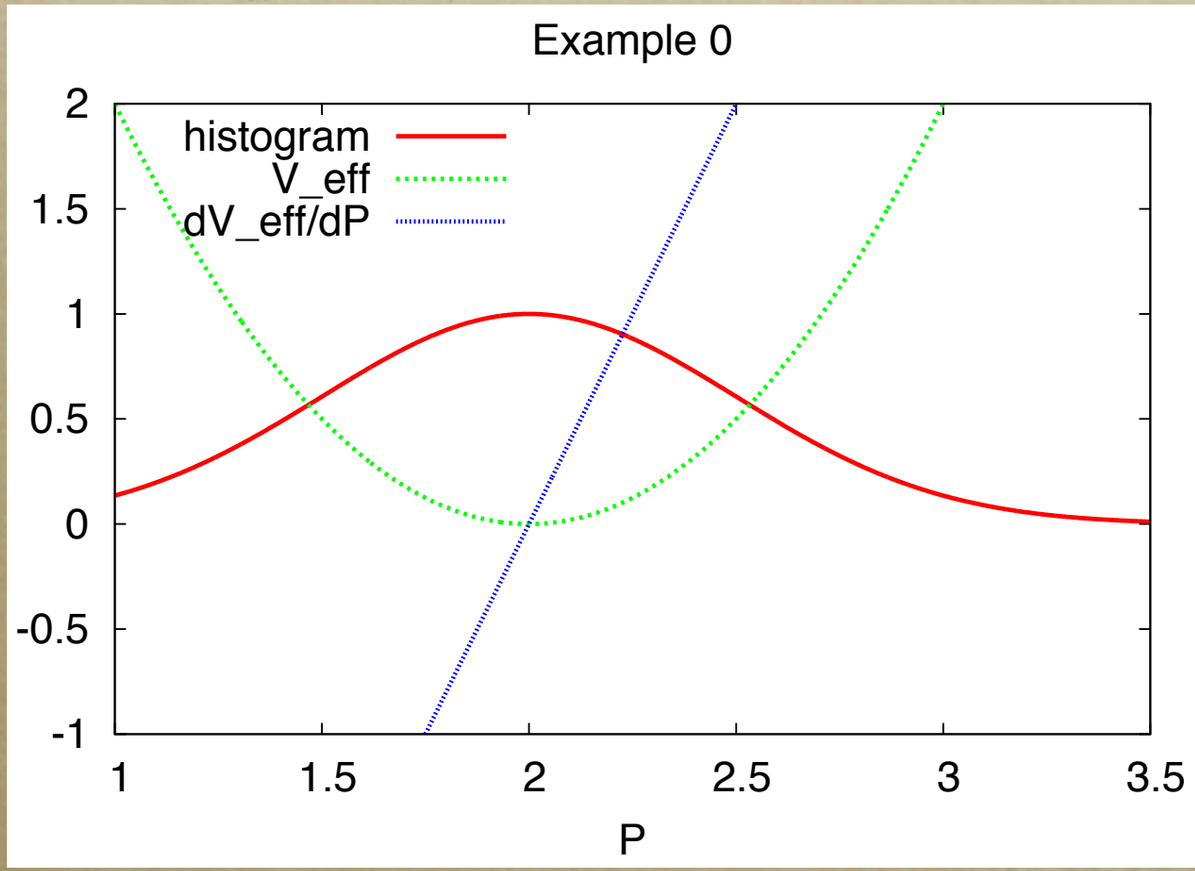
For EW baryogenesis, important to identify the location of the 1st order region and its strength.

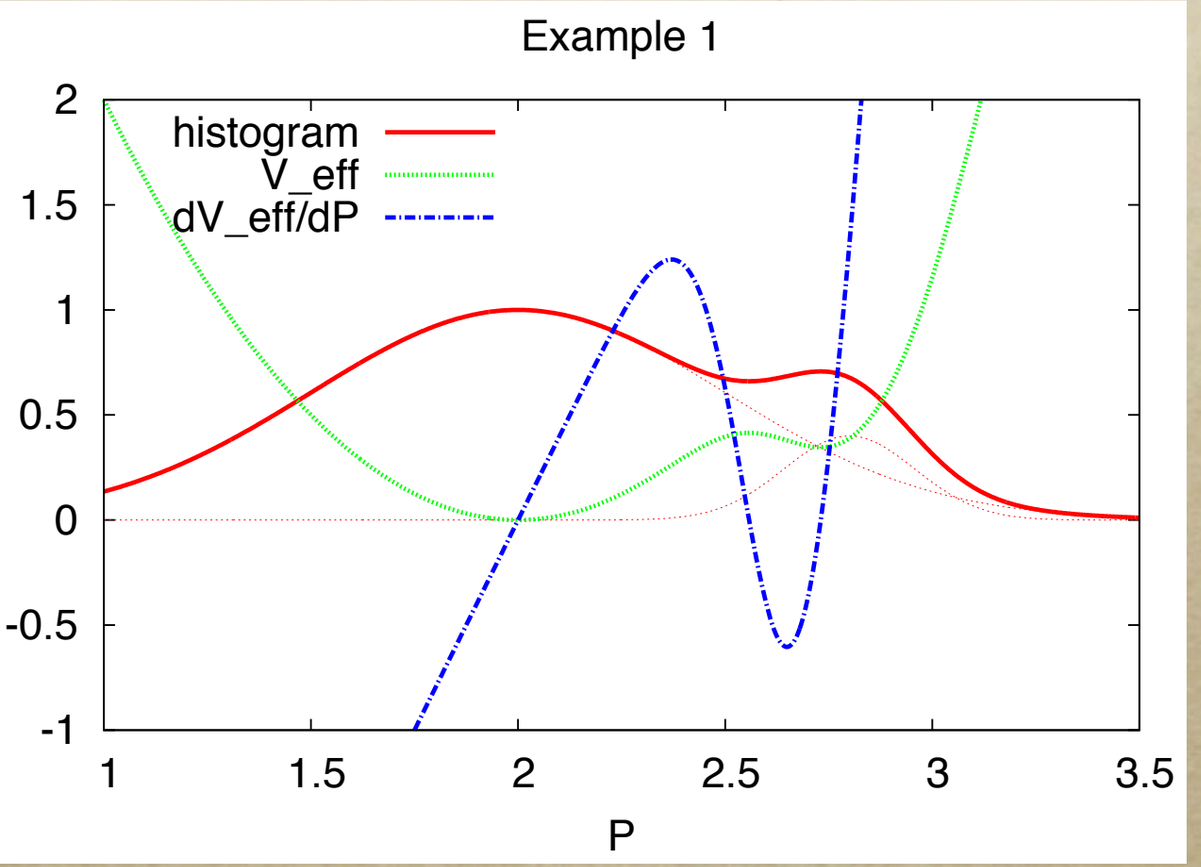
Appelquist, Schwetz and Selipsky, PRD52, 4741 (1995).
Kikukawa, Kohda and Yasuda, PRD77 (2008) 015014

Phenomenologically interesting!

Effective potential through histogram

S. Ejiri, PRD77, 014508 (2008);Saito et al, [WHOT-QCD], PRD84, 054502 (2011)





$$
\begin{aligned}
w(P,\beta) &= \langle \delta(P-\hat{P}) \rangle_\beta \\
V_{\rm eff}(P,\beta) &= -\ln w(P,\beta) \\
&= -\ln w(P,\beta_0) + 6(\beta-\beta_0) N_{\rm site} P
\end{aligned}
$$

Double well potential
=1st order PT
$\Rightarrow$look for $\partial^2 V_{\rm eff}/\partial P^2 \leq 0$

Re-weighting method

- Start with the partition function with 2+N_f-flavor QCD.

$$Z(\beta, m_f, \mu_f) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S_q - S_g} = \int \mathcal{D}U\; e^{6\beta N_{\rm site}\hat{P}} \prod_{f=1}^{N_{\rm f}+2} (\det M(m_f, \mu_f))$$

$$\prod_{f=1}^{N_{\rm f}+2} (\det M(m_f, \mu_f)) \to (\det M(m_{\rm l}, 0))^2 \left(\frac{\det M(m_{\rm l}, \mu)}{\det M(m_{\rm l}, 0)}\right)^2 \prod_{h=1}^{N_{\rm f}} \left(\frac{\det M(m_{\rm h}, 0)}{\det M(\infty, 0)}\right)$$

- N_f flavors are heavy and the chemical potential for u and d quarks are small.

$$\ln\left[\frac{\det M(m_{\rm l}, \mu)}{\det M(m_{\rm l}, 0)}\right] = \sum_{n=1}^{N_\mu} \frac{1}{n!}\left[\frac{\partial^n(\ln\det M)}{\partial(\mu/T)^n}\right]\left(\frac{\mu}{T}\right)^n$$

$$\ln\left[\frac{\det M(\kappa_{\rm h}, 0)}{\det M(0, 0)}\right] = 288 N_{\rm site}\kappa_{\rm h}^4 P + 12 N_s^3 (2\kappa_{\rm h})^{N_t}\Omega + \cdots$$

M:quark matrix
P :plaquette
Ω:Re[Polyakov loop]

Re-weighting method

Expectation value in 2+N_f-flavor QCD and its μ- and $m_{\rm h}$-dependence can be calculated by ensemble average over two-flavor QCD configurations upto truncation errors.

$$V_{\rm eff}(P,\beta,m_{\rm l},m_{\rm h},\mu) \;=\; -\ln w^{(2+N_{\rm f})}(P,\beta,m_{\rm l},m_{\rm h},\mu)$$

Re-weighting method

Expectation value in 2+N_f-flavor QCD and its μ- and $m_{\rm h}$-dependence can be calculated by ensemble average over two-flavor QCD configurations upto truncation errors.

$$V_{\rm eff}(P,\beta,m_{\rm l},m_{\rm h},\mu) = -\ln w^{(2+N_{\rm f})}(P,\beta,m_{\rm l},m_{\rm h},\mu)$$

$$V_0(P,\beta) = V_{\rm eff}(P,\beta,m_{\rm l},\infty,0) = -\ln w^{(2+N_{\rm f})}(P,\beta,m_{\rm l},\infty,0)$$

Re-weighting method

Expectation value in 2+N_f-flavor QCD and its μ- and $m_{\rm h}$-dependence can be calculated by ensemble average over two-flavor QCD configurations upto truncation errors.

$$V_{\rm eff}(P,\beta,m_{\rm l},m_{\rm h},\mu) = -\ln w^{(2+N_{\rm f})}(P,\beta,m_{\rm l},m_{\rm h},\mu)$$

$$V_0(P,\beta) = V_{\rm eff}(P,\beta,m_{\rm l},\infty,0) = -\ln w^{(2+N_{\rm f})}(P,\beta,m_{\rm l},\infty,0)$$

$$V_{\rm eff}(P,\beta,m_{\rm h},\mu) = V_0(P,\beta_0) - \ln R(P;\beta,m_{\rm l},m_{\rm h},\mu;\beta_0)$$

Two-flavor part
Always single well

Effect of additional heavy flavors

Re-weighting method

Expectation value in 2+N_f-flavor QCD and its μ- and $m_{\rm h}$-dependence can be calculated by ensemble average over two-flavor QCD configurations upto truncation errors.

$$V_{\rm eff}(P,\beta,m_{\rm l},m_{\rm h},\mu) = -\ln w^{(2+N_{\rm f})}(P,\beta,m_{\rm l},m_{\rm h},\mu)$$

$$V_0(P,\beta) = V_{\rm eff}(P,\beta,m_{\rm l},\infty,0) = -\ln w^{(2+N_{\rm f})}(P,\beta,m_{\rm l},\infty,0)$$

$$V_{\rm eff}(P,\beta,m_{\rm h},\mu) = V_0(P,\beta_0) - \ln R(P;\beta,m_{\rm l},m_{\rm h},\mu;\beta_0)$$

Two-flavor part
Always single well

Effect of additional heavy flavors

$$\ln R(P;\beta,m_{\rm h},\mu;\beta_0) = 6(\beta-\beta_0)N_{\rm site}P$$

$$+\ln \frac{\left\langle \delta(P-\hat{P}) \left(\frac{\det M(m_{\rm l},\mu)}{\det M(m_{\rm l},0)}\right)^2 \prod_{h=1}^{N_{\rm f}} \left(\frac{\det M(m_{\rm h},0)}{\det M(\infty,0)}\right) \right\rangle_{\rm two-flavors,\beta}}{\left\langle \delta(P-\hat{P}) \right\rangle_{\rm two-flavors,\beta}}$$

Heavy quark mass and N_f dependence

$$\ln R(P;\beta,\kappa_{\rm h},0;\beta_0) = \ln \bar{R}(P;\kappa_{\rm h},0) + (\text{plaquette term}) + O(\kappa_{\rm h}^{N_t+2})$$

$$\bar{R}(P;\kappa_{\rm h},0) = \frac{\left\langle \delta(P-\hat{P}) \exp[6hN_s^3\Omega] \right\rangle_\beta}{\left\langle \delta(P'-P) \right\rangle_\beta}$$

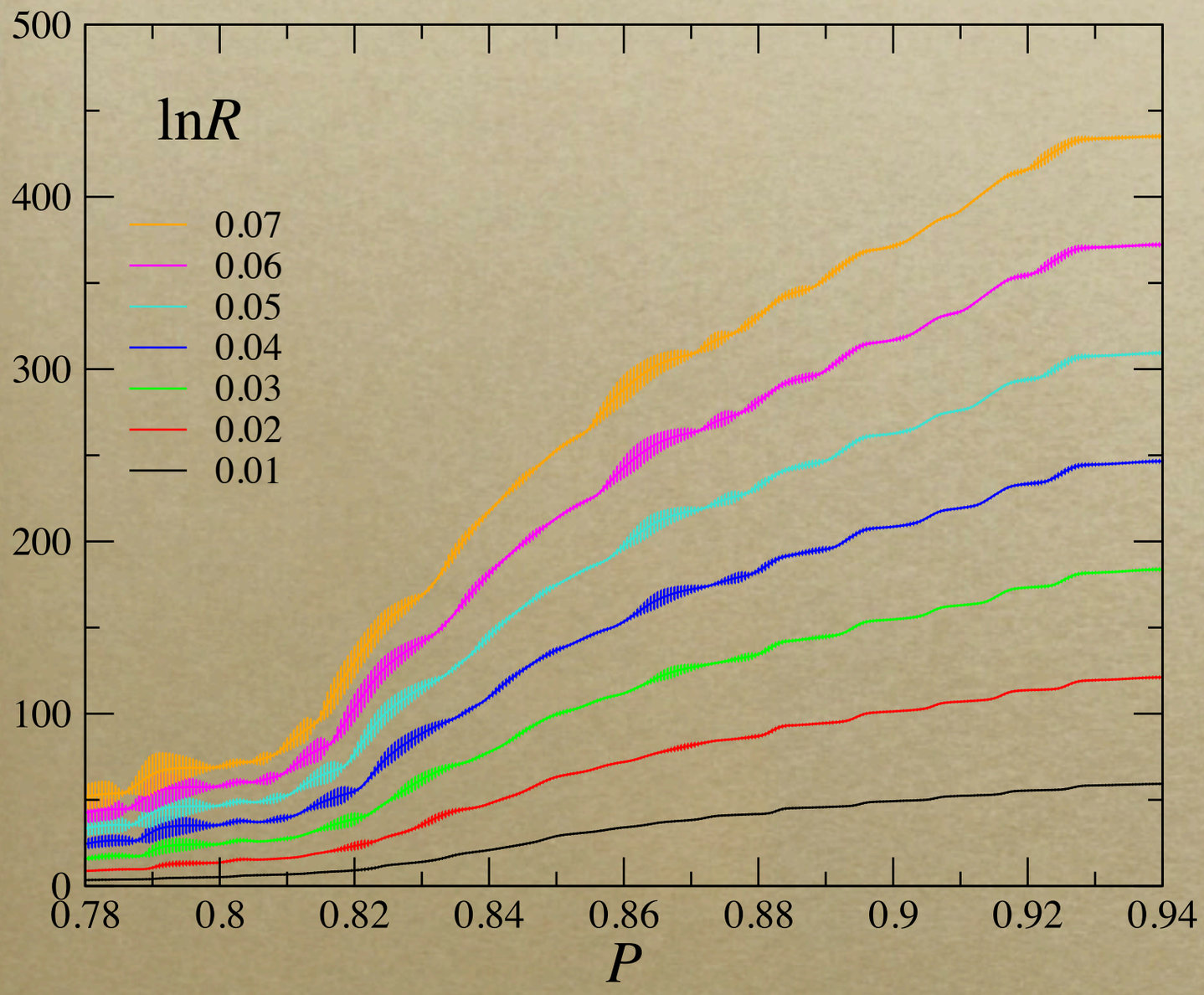
where

$h = 2N_{\rm f}(2\kappa_{\rm h})^{N_t}$ for the Wilson quark action

$h = N_{\rm f}/(4 \times (2m_{\rm h})^{N_t})$ for the staggered quark action

Heavy quark dependence and N_f-dependence can be parameterized in a single parameter h.

Result: ln R



Two-flavor QCD configurations:
p4-improved staggered quark &
the standard plaquette gauge
$a\,m_l = 0.1$,
10,000-40,000 trajs.
$V=16^3\times4$,
16 β in [3.52, 4.00],
T/T_c=[0.76,1.98],
M_{PS}/M_V~0.7

[C.R. Allton, et al., PRD71,054508 (2005)]

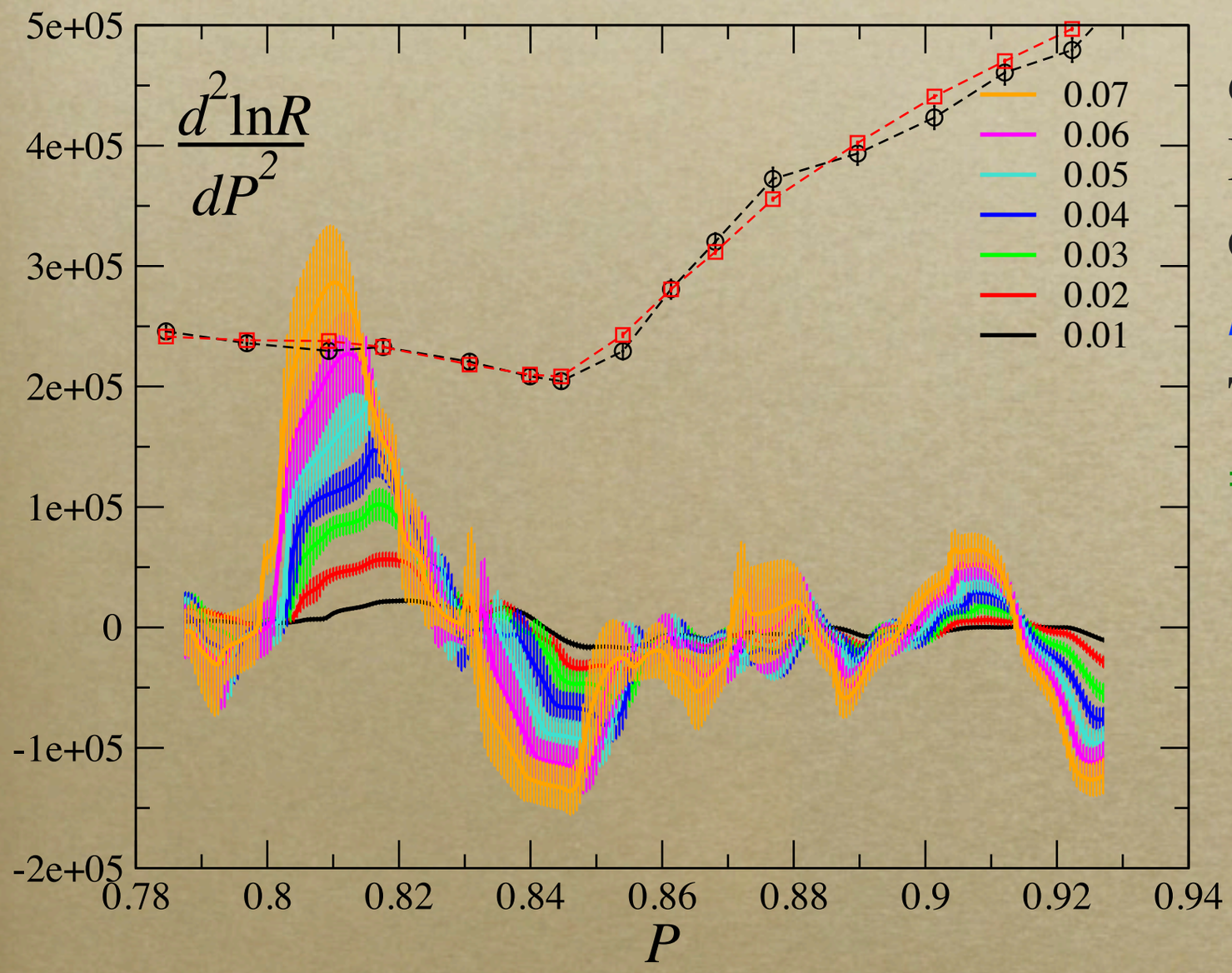
Calculated with h = [0.01, 0.07]

- ln$R$ increases with h.
- Rapid increase@P~0.81

⇒large curvature

Result:

$$\partial^2 V_{\rm eff}/\partial P^2 = \partial^2 V_0/\partial P^2 - \partial^2(\ln R)/\partial P^2$$



1st term is calculate in two different ways (black and red). Maximum of the 2nd term exceeds 1st term at P~0.81 for $h \gtrsim 0.06$.

There $\partial^2 V_{\rm eff}/\partial P^2$ is negative.

⇒ **1st order phase transition**

$h_c = 0.0614(69)$

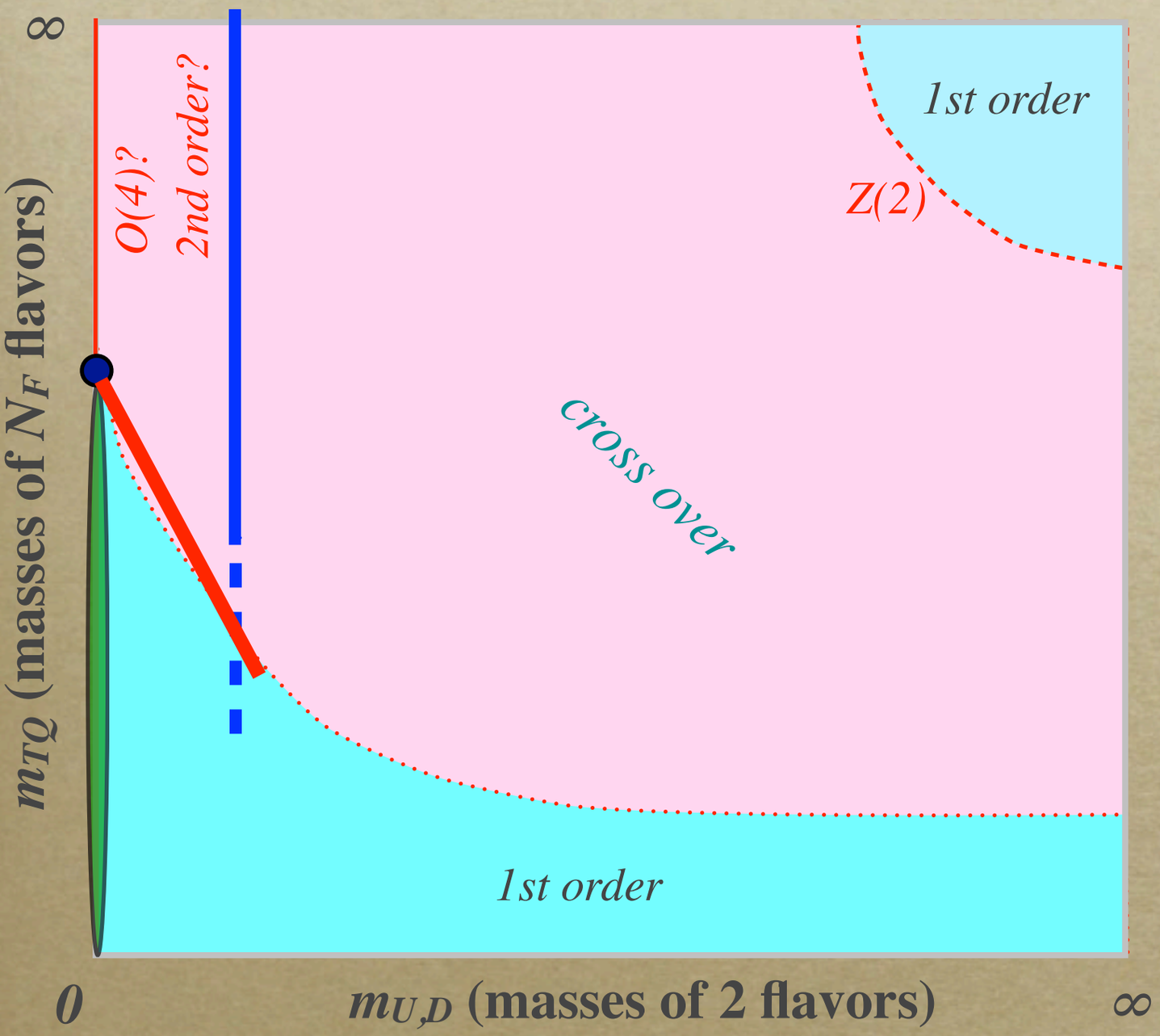
Comments

- $h_c = 0.0614(69) = 2\,N_{\rm f}\,(2K_{\rm hc})^{Nt}$
- Critical kappa $K_{\rm hc}$ becomes small as $N_{\rm f}$ increases.
- For example, for $N_{\rm f}$=10, $K_{\rm hc} \sim 0.118$.
- Convergence of the HPE is studied in quenched theory in [Ejiri et al. in preparation], and found to be (LO)~(NLO) @$K_{\rm h} \sim 0.18$.
- Study of phase structure at finite density can be done in the same footing. [S. Ejiri, PRD77, 014508 (2008)]
- We found that a finite μ also makes $K_{\rm hc}$ small.

Summary and outlooks

- ✓ In general, QCD with many flavors is computationally demanding.
- ✓ We proposed an easy way to explore the phase structure of MFQ, and gave the critical mass.
- ✓ Future works:
  - Quantify the strength of 1st order transition
  - Check the universal power behavior at the tri-critical point

Structure around the tri-critical point



Meanfield approximation predicts

$$m_{lc} \sim (m_h - m_{hc})^{5/2}.$$

The power (5/2) is independent of N_f.