On gauge theory phase diagrams at zero and finite temperature

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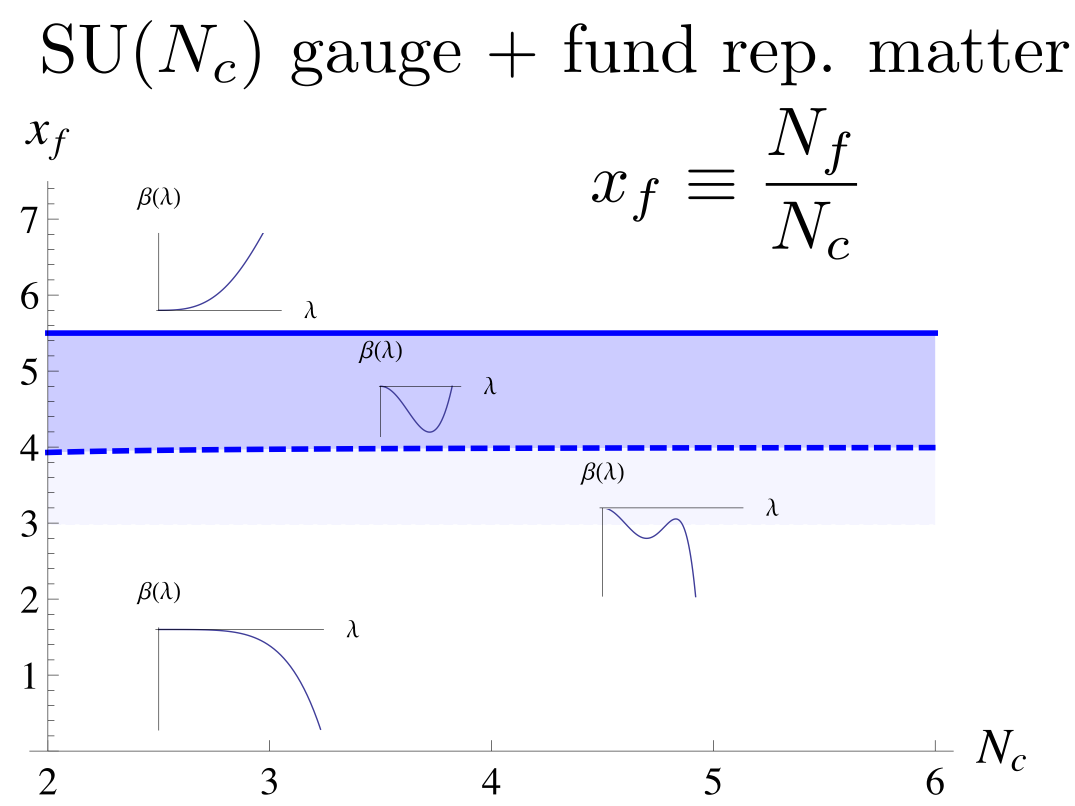
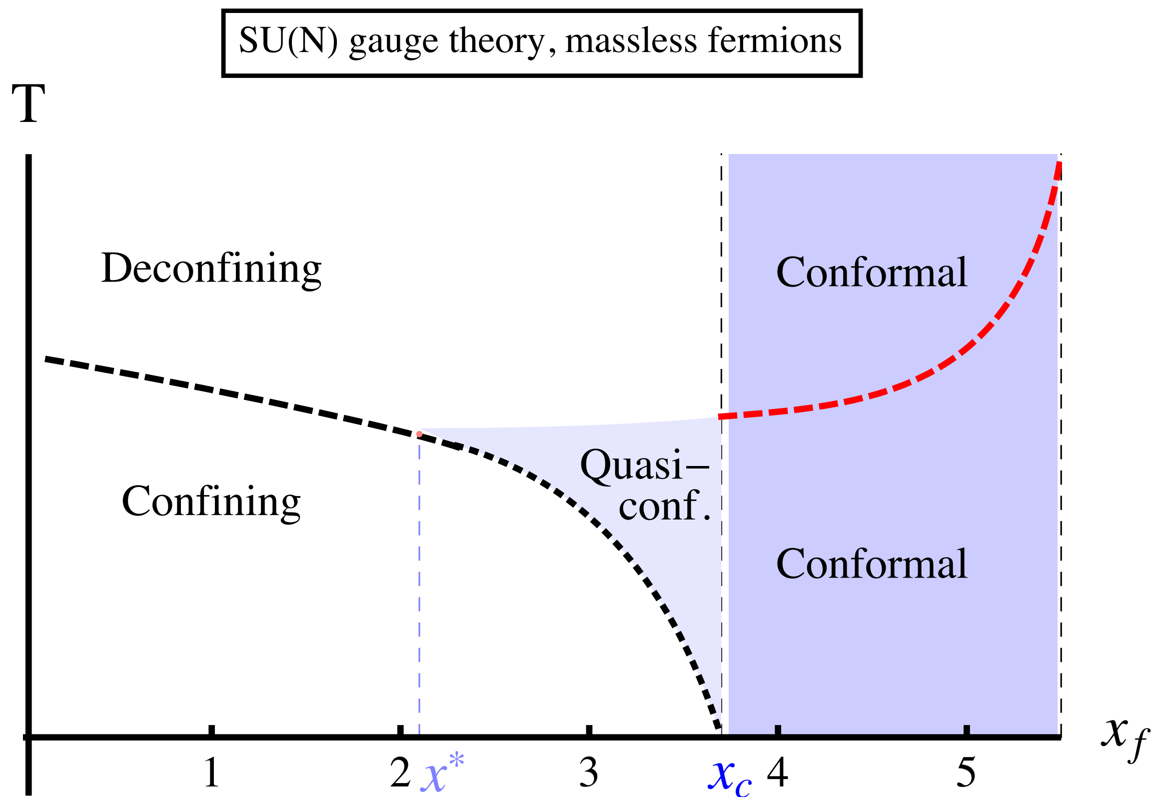
University of Jyväskylä
&
Helsinki Institute of Physics

SCGT 2012, Nagoya.

Outline:

1. Vacuum phase diagrams

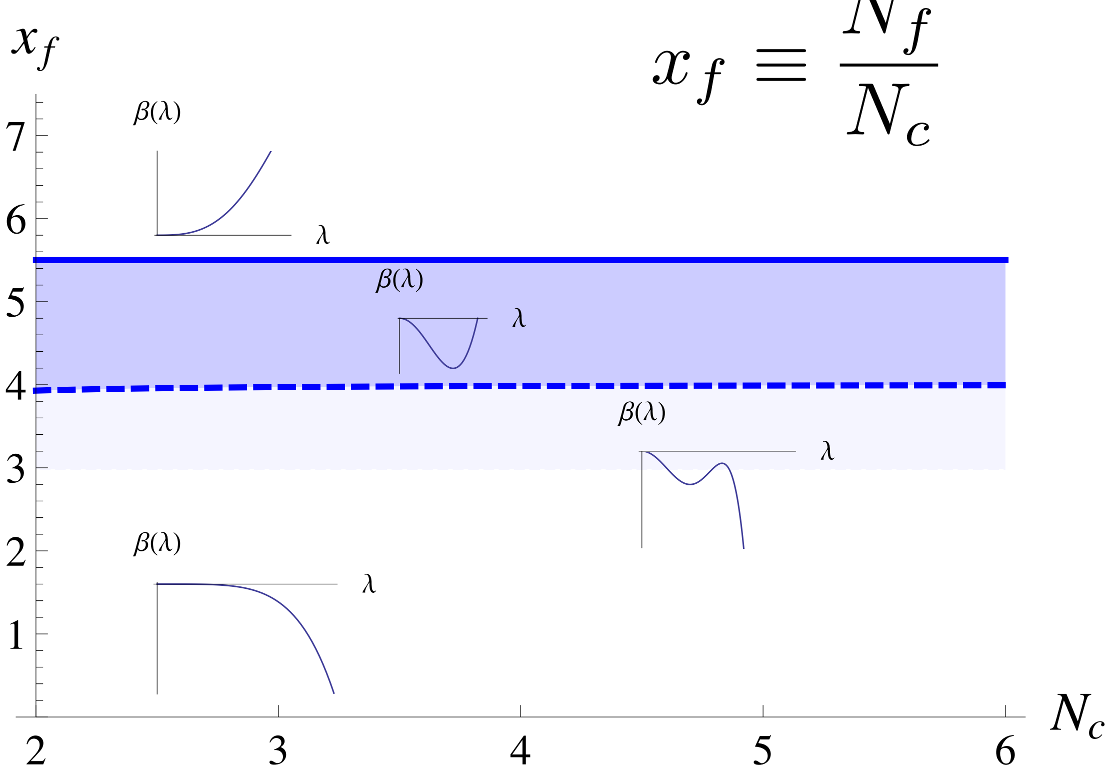




2. Finite T phase diagrams

1. Vacuum Phase diagrams

$$x_f \equiv \frac{N_f}{N_c}$$



Fixed point from 2-loop betaf. $\alpha^* = -\frac{\beta_0}{\beta_1}$

Critical coupling for chiral breaking from SD-equ. $\alpha_c = \frac{\pi}{3C_2(R)}$

Conformal window: $\alpha^* \leq \alpha_c$

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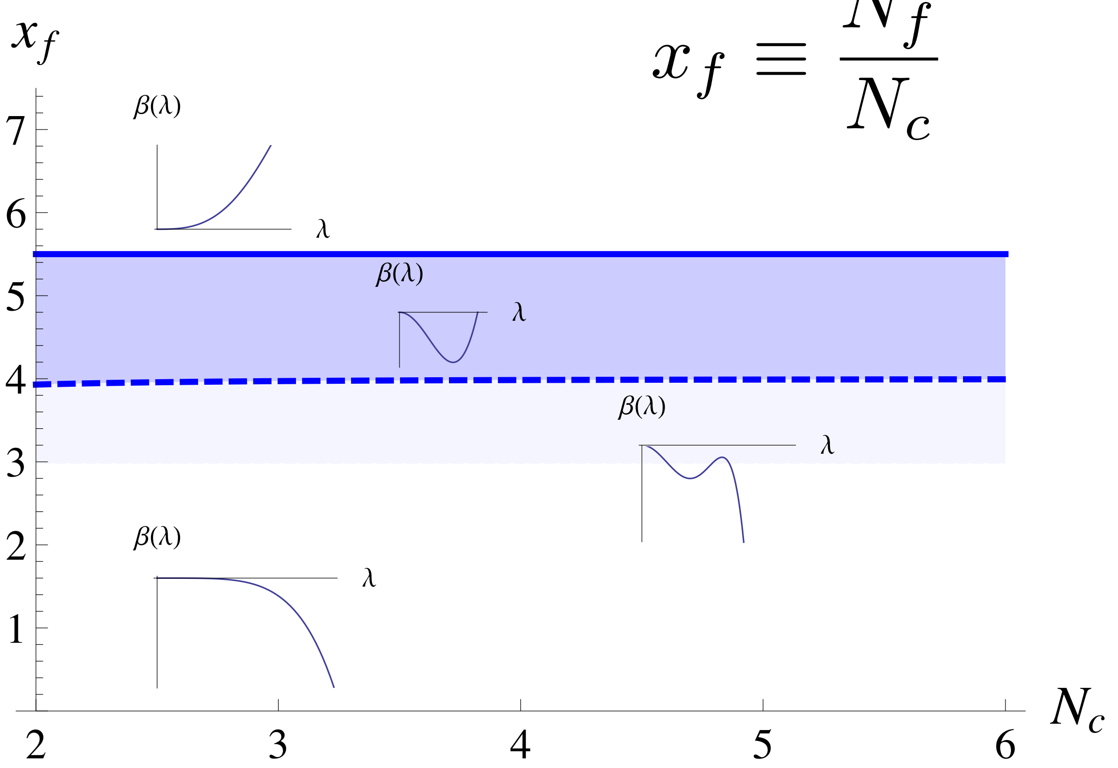
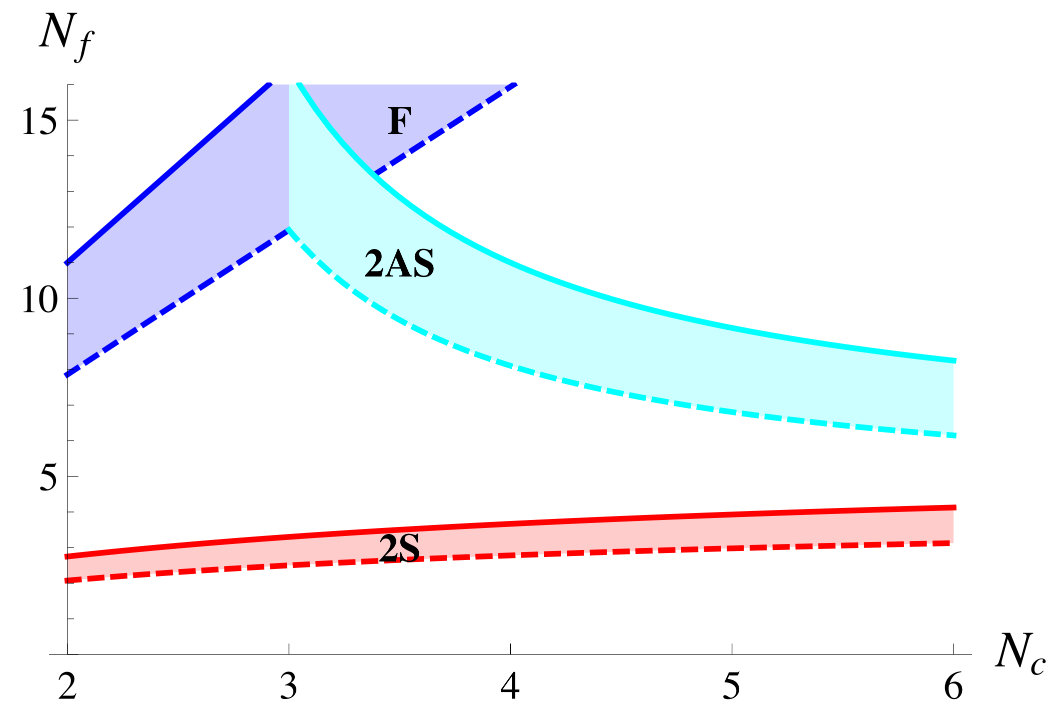


Higher representations:

(Sannino, Tuominen '04)



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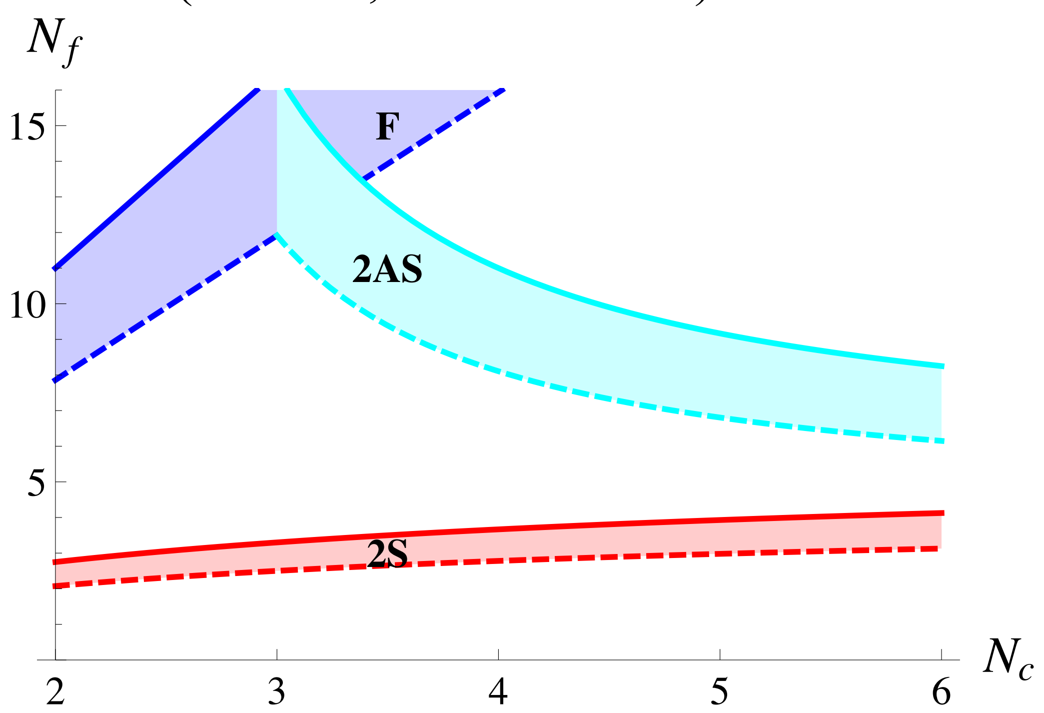

-Walking with less flavors: phenomenologically viable Technicolor models

- Study on the lattice

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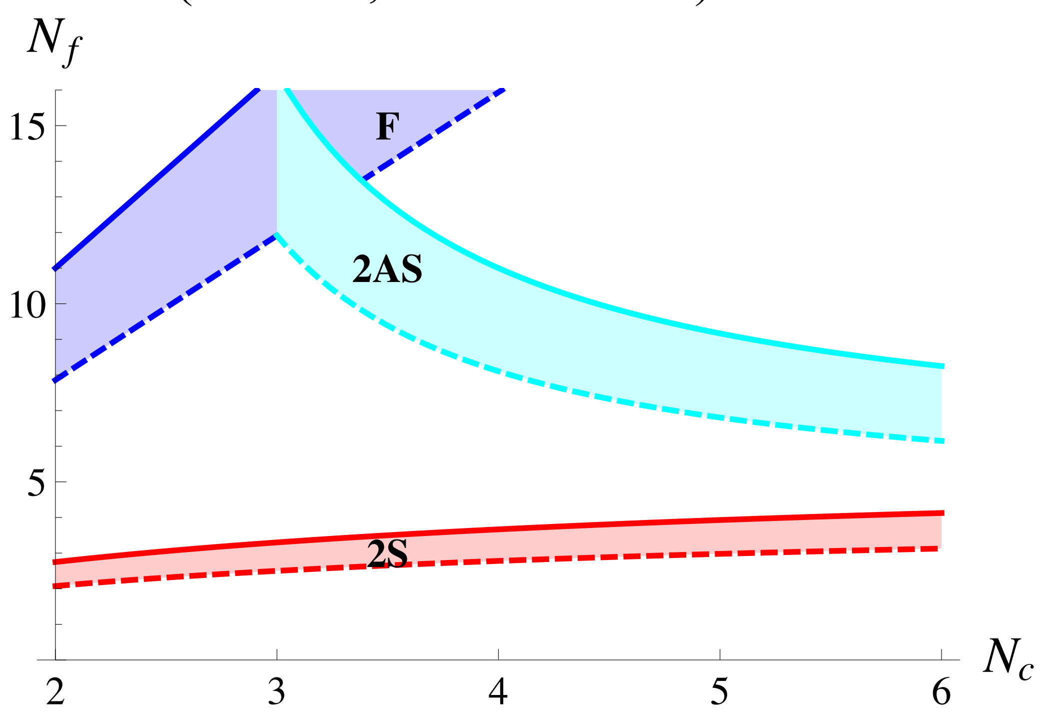

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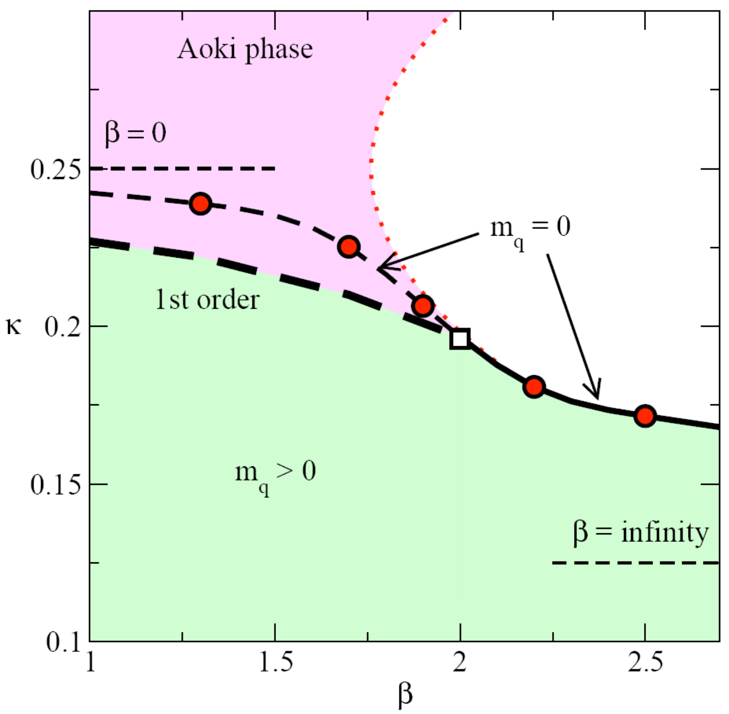
- Study on the lattice

Lots of efforts during last 4...5 years.

SU(2) adjoint: (Catteral et al., Hietanen et al., Del Debbio et al.,...)
SU(2) fundamental: (Del Debbio et al., Karavirta et al.,...)
SU(3) fundamental: (Appelquist et al., Kuti et al.,...)
SU(3) sextet: (De Grand et al.,...)

Some history: SU(2) gauge + 2 adjoint Wilson fermions on the lattice

Lattice phase diagram and spectrum (Hietanen, Rantaharju, Rummukainen, Tuominen, JHEP 0905 (2009).



Strong coupling boundary at $\beta_L \sim 2$

Seems volume independent: Lattice artifact?

Non-QCD like continuum physics?

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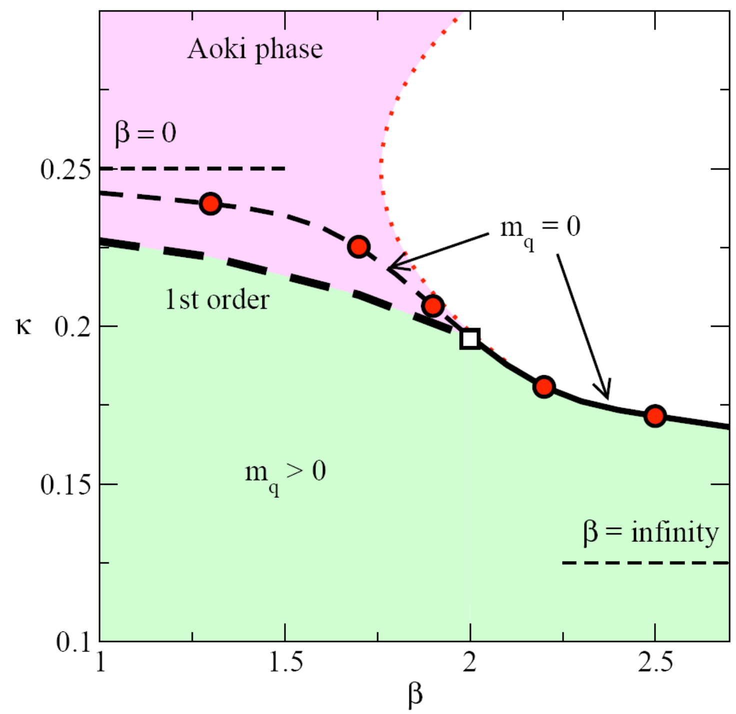

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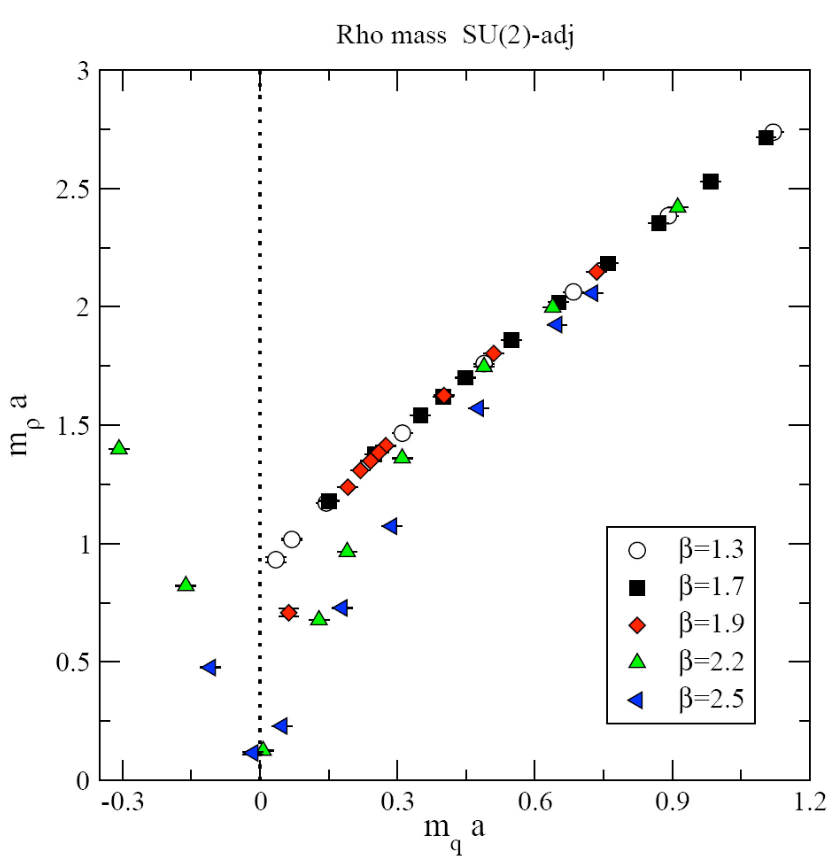
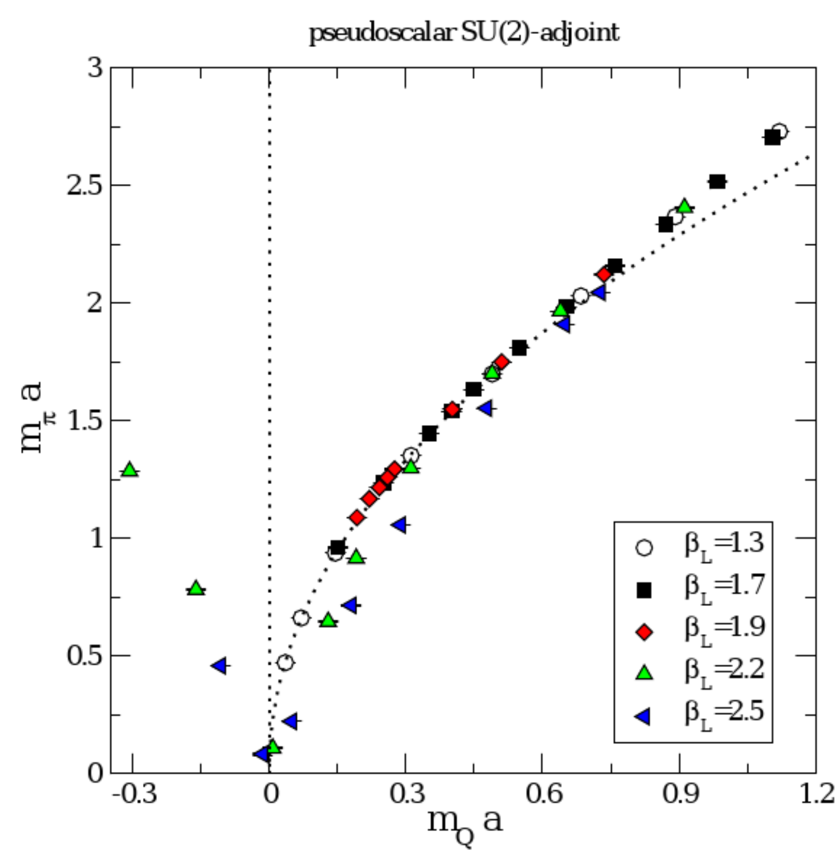


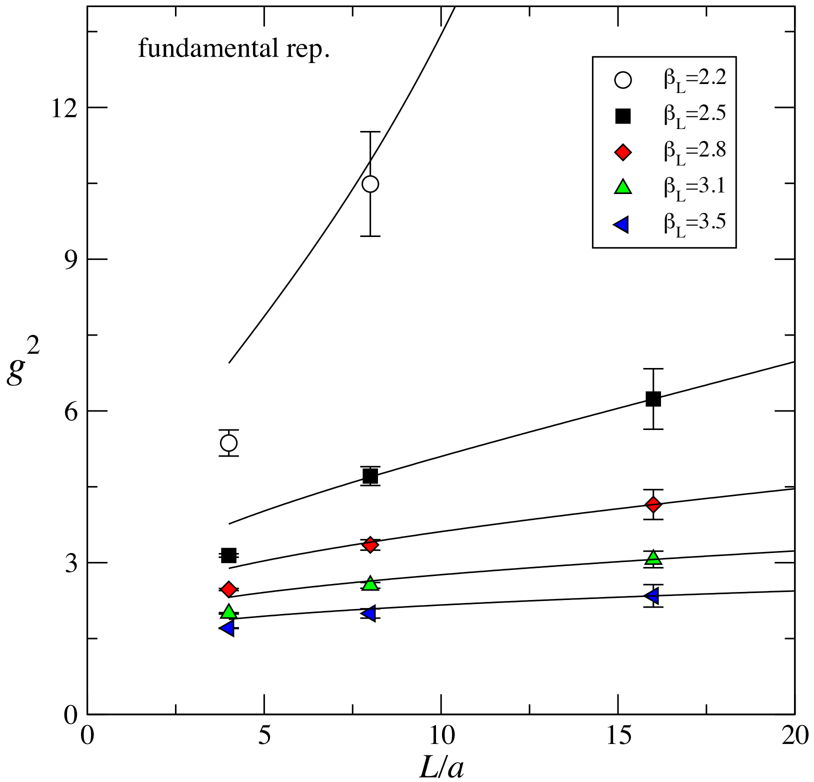


At $\beta \geq 2$
$m_\pi \sim m_\rho \sim m_q$

Conformal?

Measure the coupling at $\beta_L \geq 2$ (Hietanen, Rummukainen, Tuominen, PRD 81 (2009).)

SU(2) with 2 fundamentals
as a control case:



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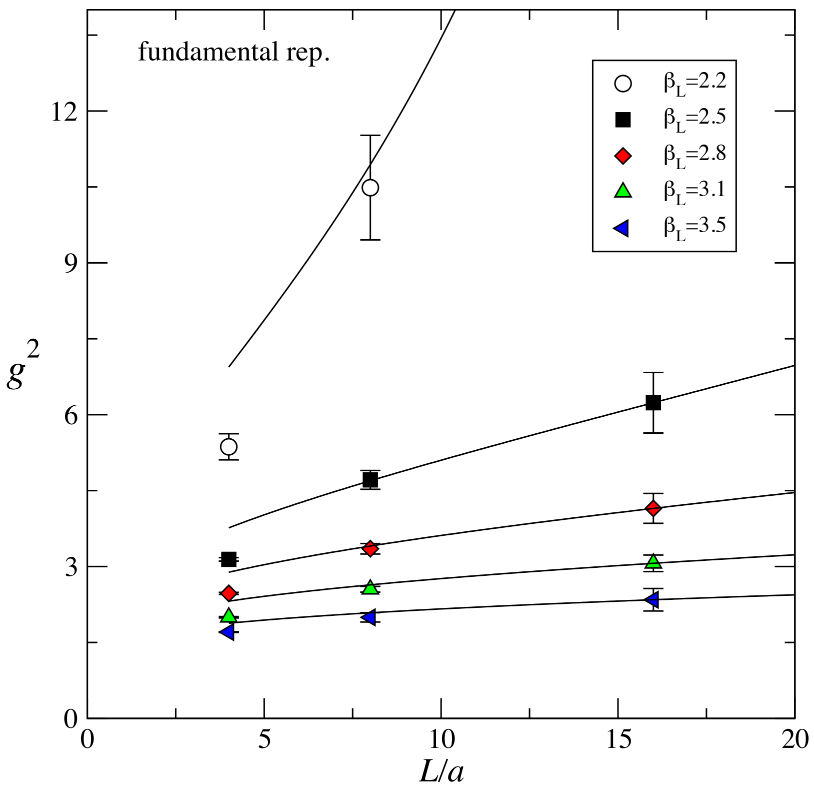

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Del Debbio et al.,
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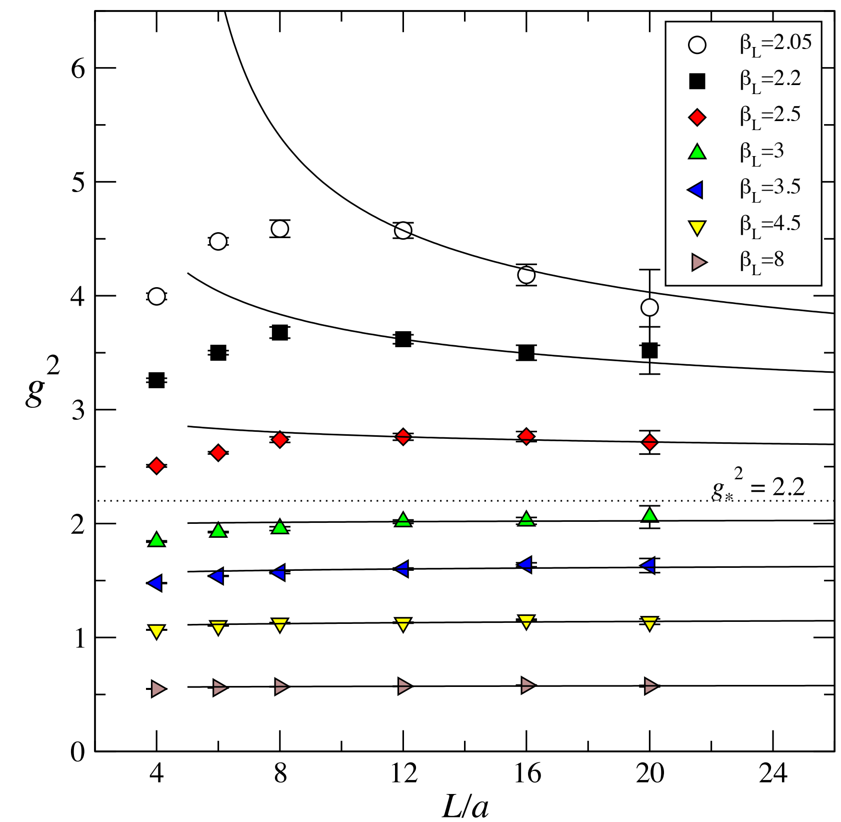

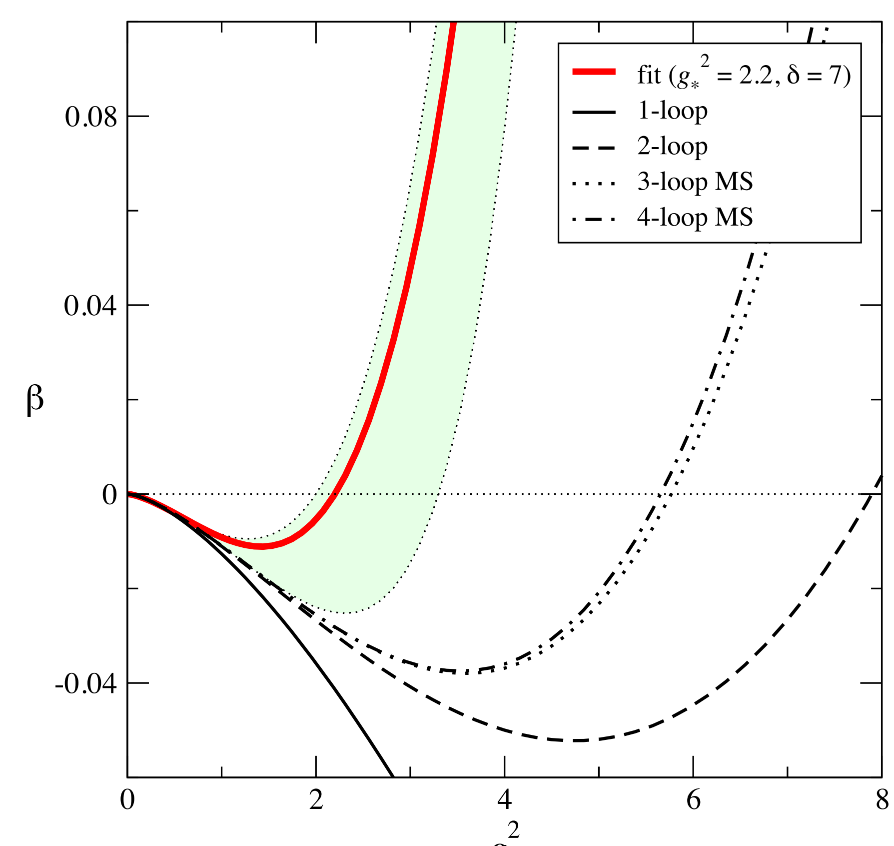

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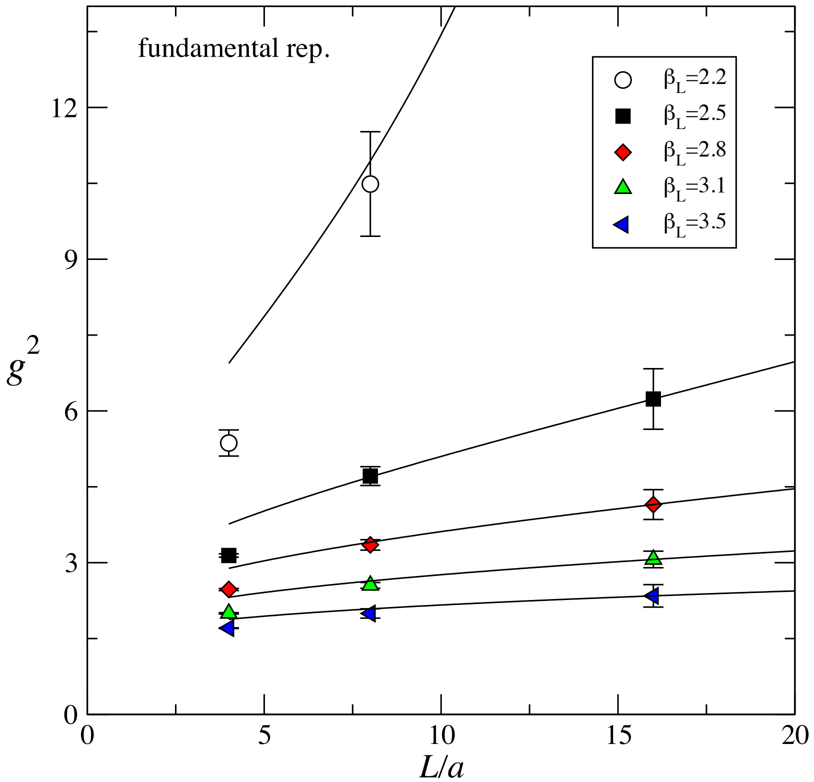

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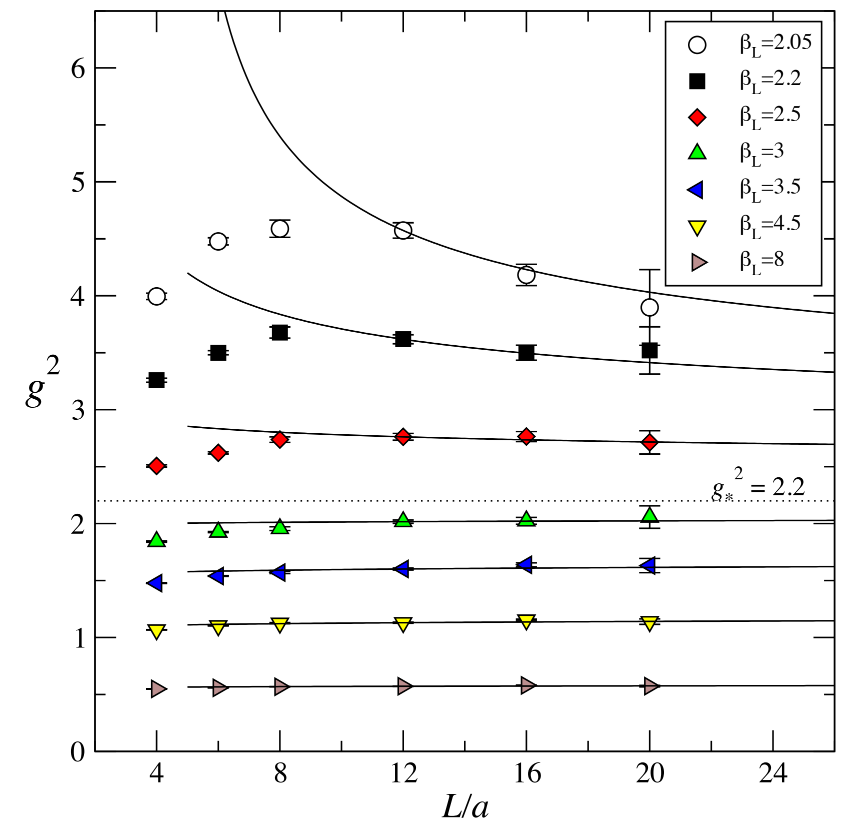

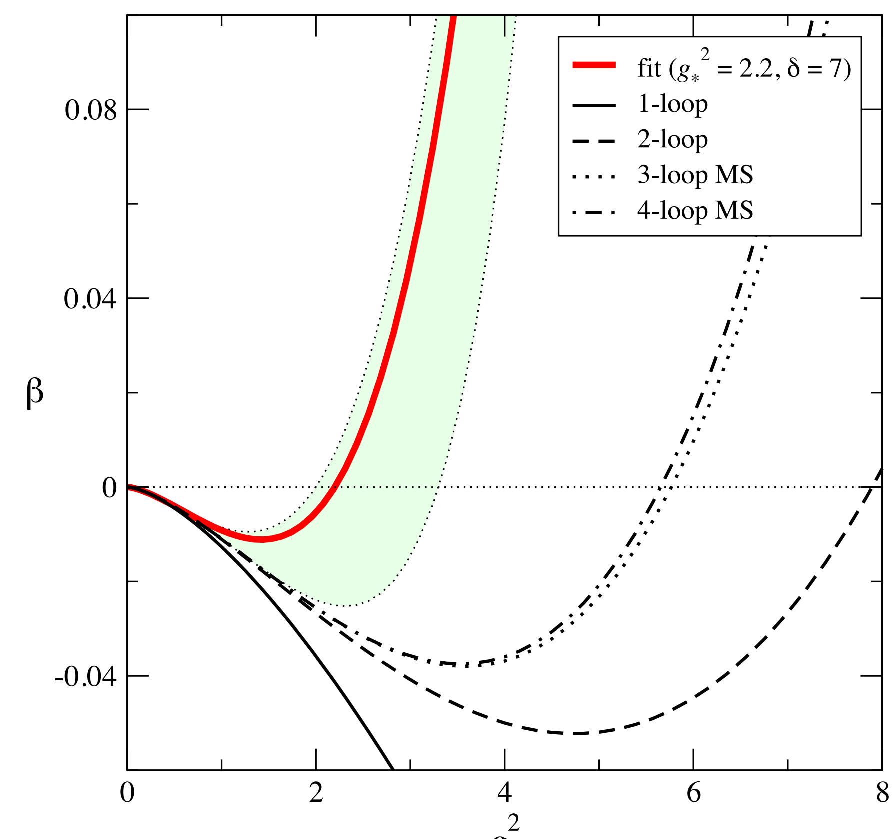

Wilson fermions:
Large $(\mathcal{O}(a))$ lattice artifacts

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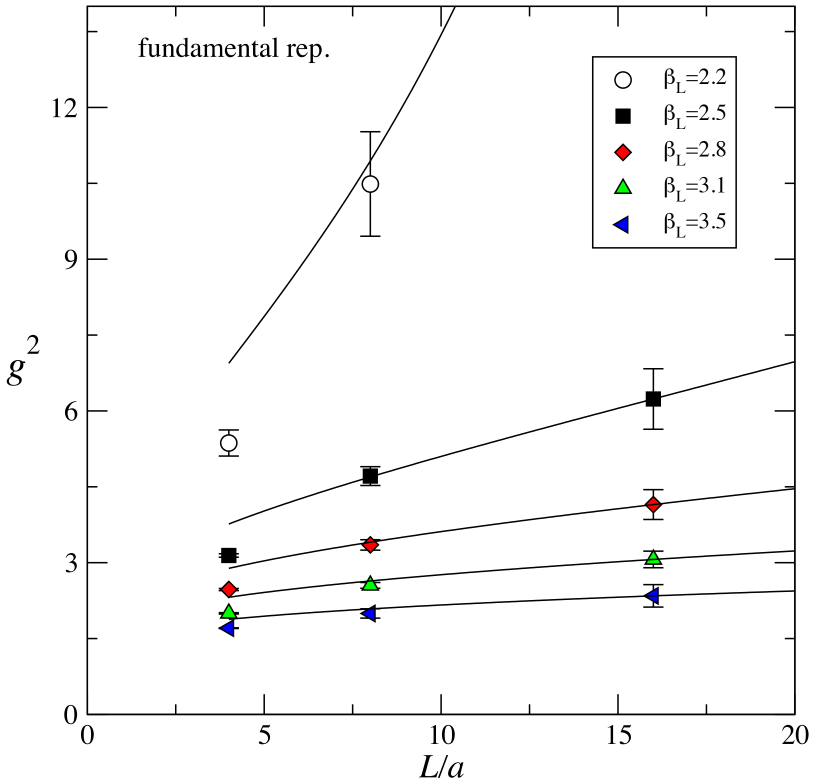

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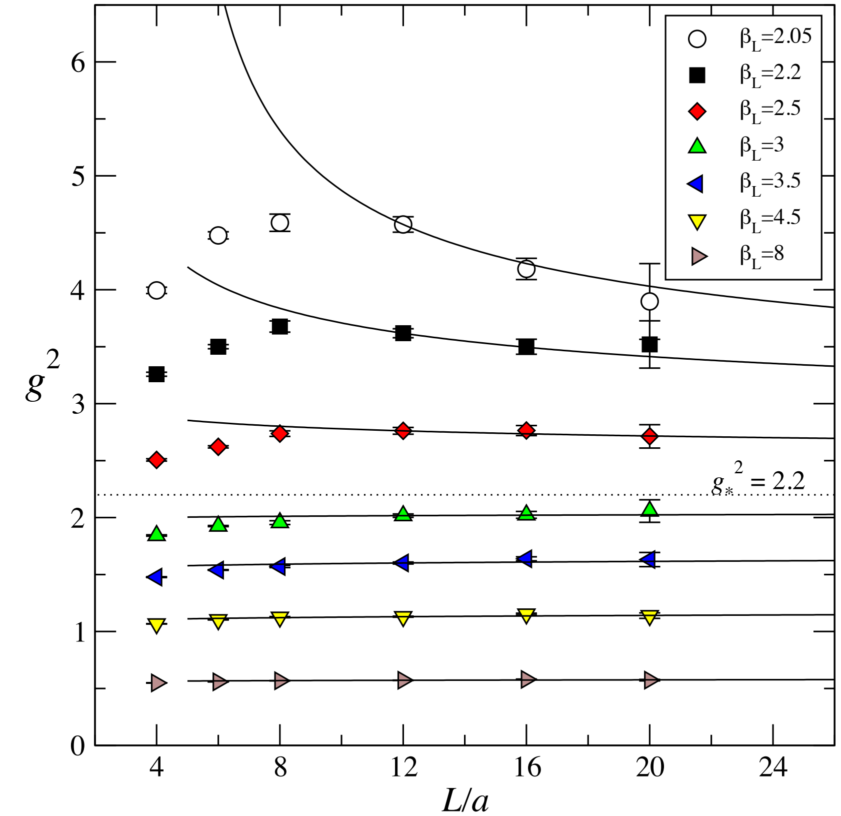

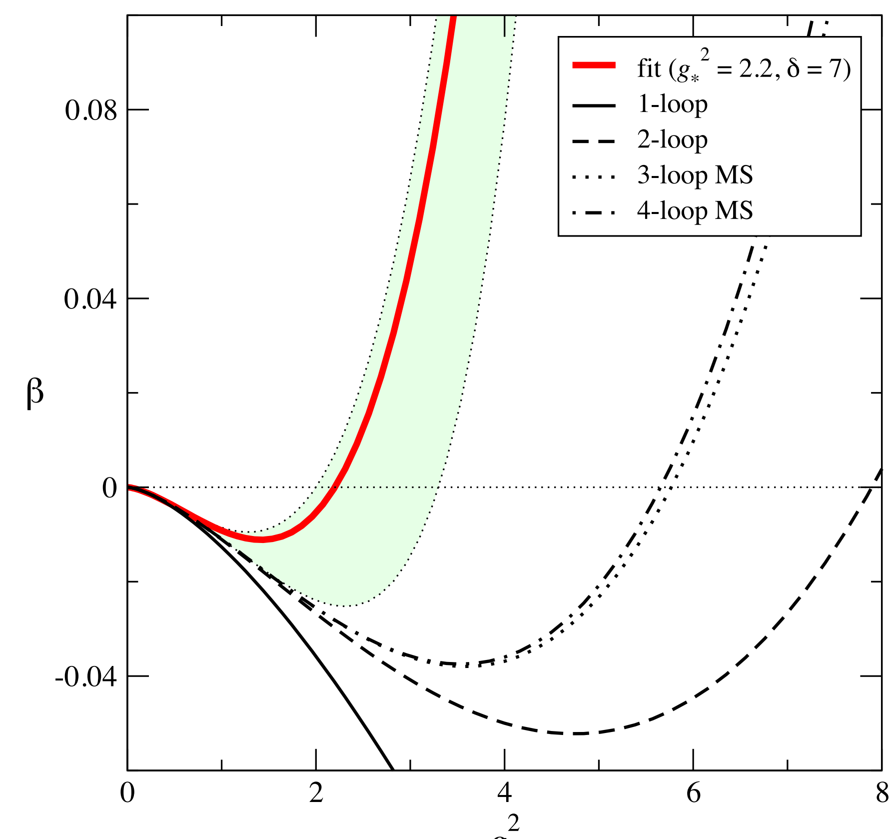

Wilson fermions:
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Need improved actions.

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Measuring the coupling using the Schrödinger functional (=background field)

Chromoelectric background field from fixed boundaries:

$$U_k(x)|_{(x_0=0)} = \exp(aC_k(\eta)), \quad U_k(x)|_{(x_0=L)} = \exp(aC'_k(\eta))$$

$$C_k = \frac{i}{L}\mathrm{diag}(\phi_1(\eta), \ldots, \phi_n(\eta)) \qquad C'_k = \frac{i}{L}\mathrm{diag}(\phi'_1(\eta), \ldots, \phi'_n(\eta))$$

Coupling defined as response to changes of the background field:

$$\frac{g_0^2}{g^2} = \frac{\partial S}{\partial \eta} \Big/ \frac{\partial S^{\mathrm{cl.}}}{\partial \eta}$$

$$\mu \sim 1/L$$

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Fundamental rep.

$\eta = \pi/4, \quad \rho = \pi$

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SU(3):

$$\phi_1 = \eta - \rho \qquad \phi'_1 = -\phi_1 - 4\rho$$
$$\phi_2 = \eta(\nu - 1/2) \qquad \phi'_2 = -\phi_3 + 2\rho$$
$$\phi_3 = -\eta(\nu + 1/2) + \rho \qquad \phi'_3 = -\phi_2 + 2\rho$$

Fundamental rep.
$\eta = 0, \quad \rho = \pi/3, \quad \nu = 0$

Improvement: Luscher, Narayanan, Weisz, Wolff (hep-lat/9207009)
Developed and tested for QCD (fundamental rep. fermions).

$$S_{\mathrm{impr}} = S_0 + a^5 c_{\mathrm{sw}} \sum_x \bar{\psi}(x) \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu}(x) \psi(x) + \underbrace{\delta S_{G,b}}_{c_t} + \underbrace{\delta S_{F,b}}_{\tilde{c}_t}$$

Two counterterms due to nontrivial boundaries

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c_t $\tilde{c}_t$

Two counterterms due to nontrivial boundaries

$$c_{\mathrm{sw}} = 1$$

$$c_t = 1 + c_t^{(1)} g_0^2$$

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$$\Sigma(u, s, L/a) = g^2(g_0, sL/a)|_{g^2(g_0,L/a)=u}$$

$$= u + [\Sigma_{1,0}(s, L/a) + \Sigma_{1,1}(s, L/a) N_f] u^2$$

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$$b_{0,0} = 11 Nc/(48\pi^2)$$

$$b_{0,1} = N_f T_R/(12\pi^2)$$

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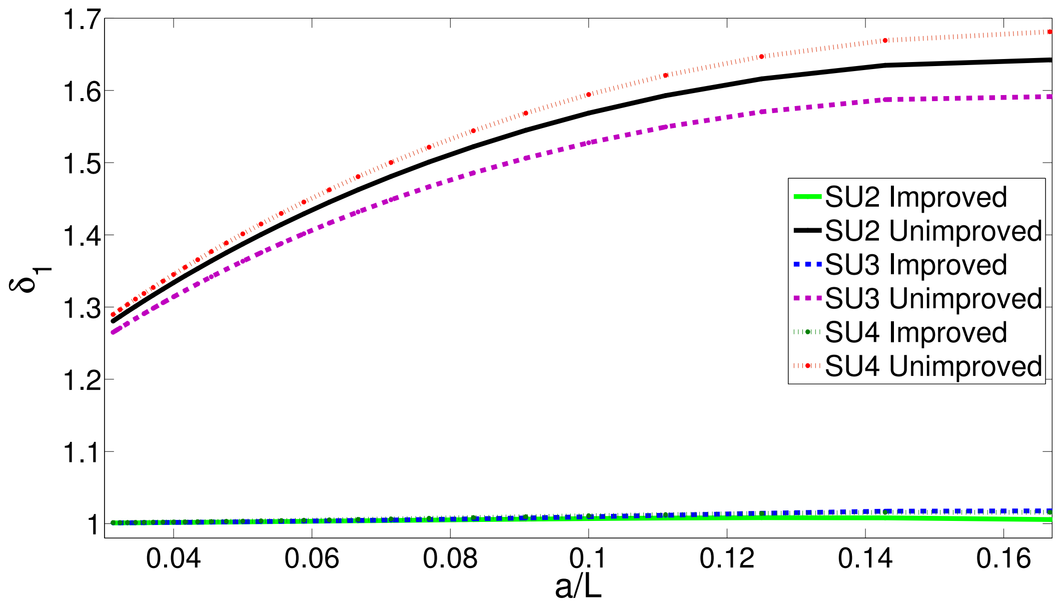

SU(2) gauge + $N_f = 2$ fundamental

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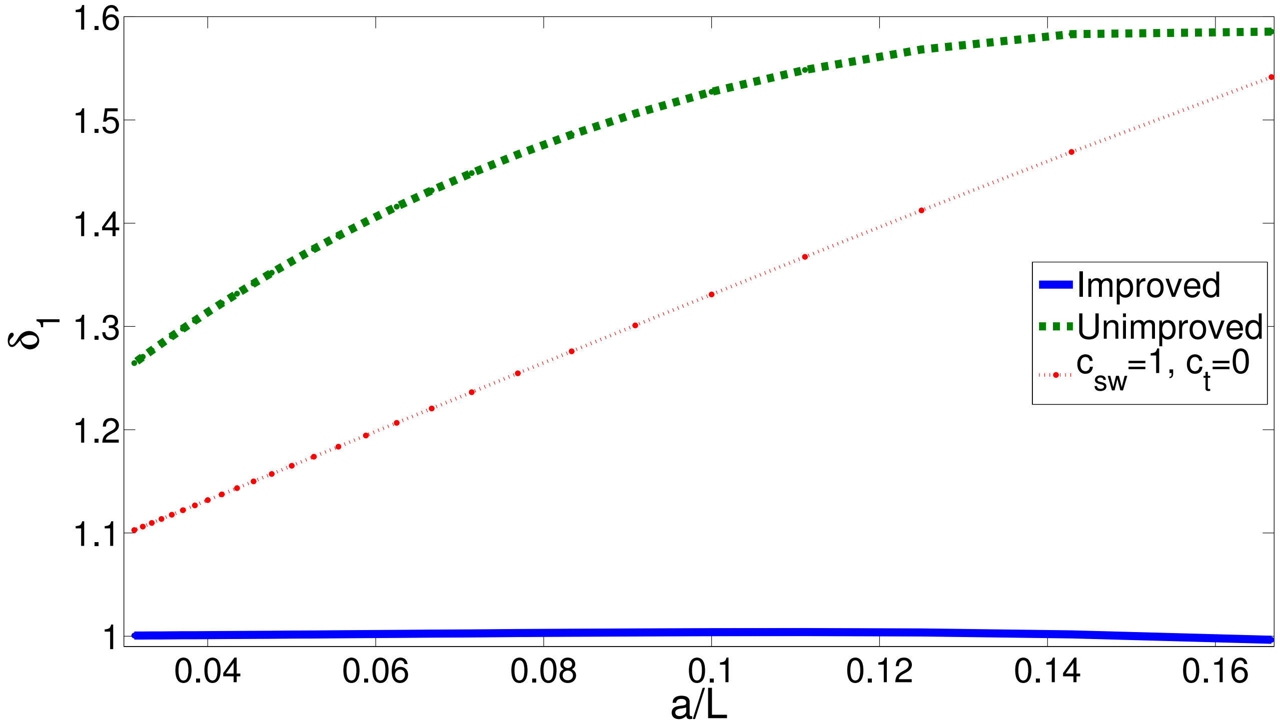
Nontrivial calculation of the counterterms required for higher representations.

N_c	rep.	$c_t^{(1,0)}$	$c_t^{(1,1)}$	$\tilde{c}_t^{(1)}$
2	**2**	$-0.0543(5)$	$0.0192(2)$	$-0.0101(3)$
2	**3**	$-0.0543(5)$	$0.0766(2)$	$-0.0270(2)$
3	**3**	$-0.08900(5)$	$0.0192(4)$	$-0.0180(1)$
3	**8**	$-0.08900(5)$	$0.1148(3)$	$-0.0405(3)$
3	**6**	$-0.08900(5)$	$0.09571(2)$	$-0.0450(3)$

(T. Karavirta et al. JHEP 1106 (2011), 1101.0154
T. Karavirta et al. PRD85 (1012), 1201.1883)

1. Need to include all coefficients consistently:



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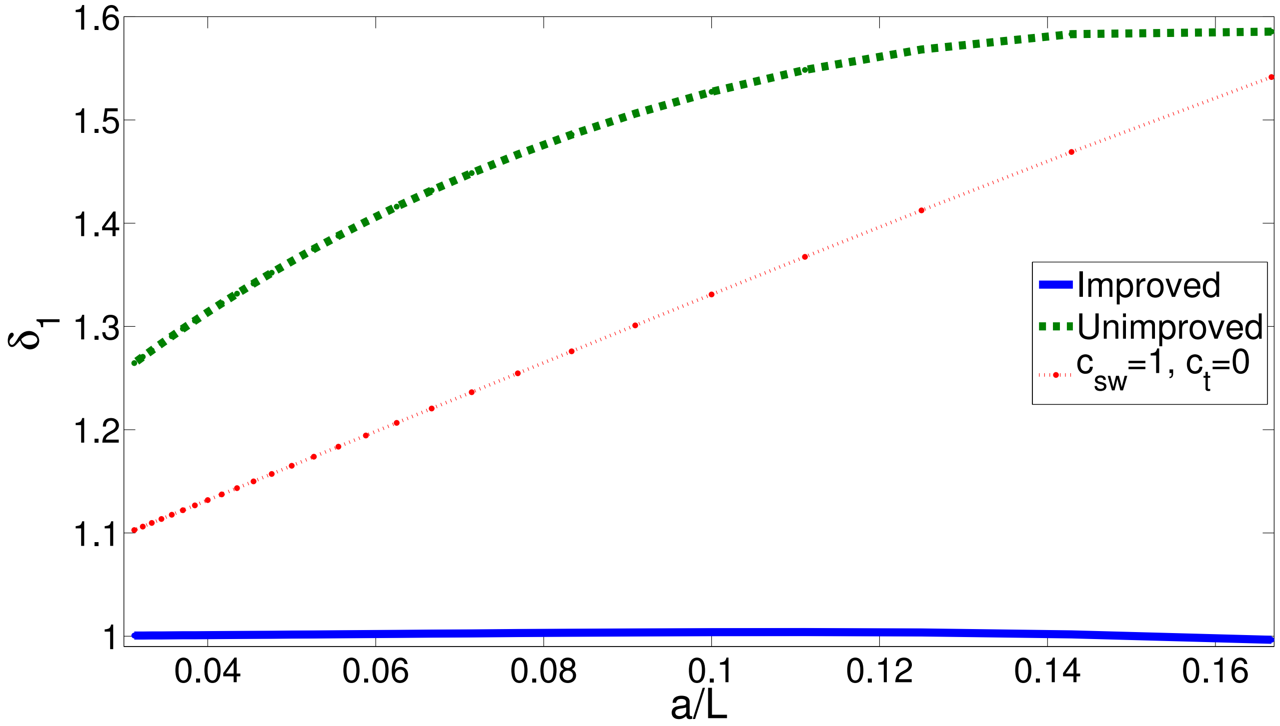

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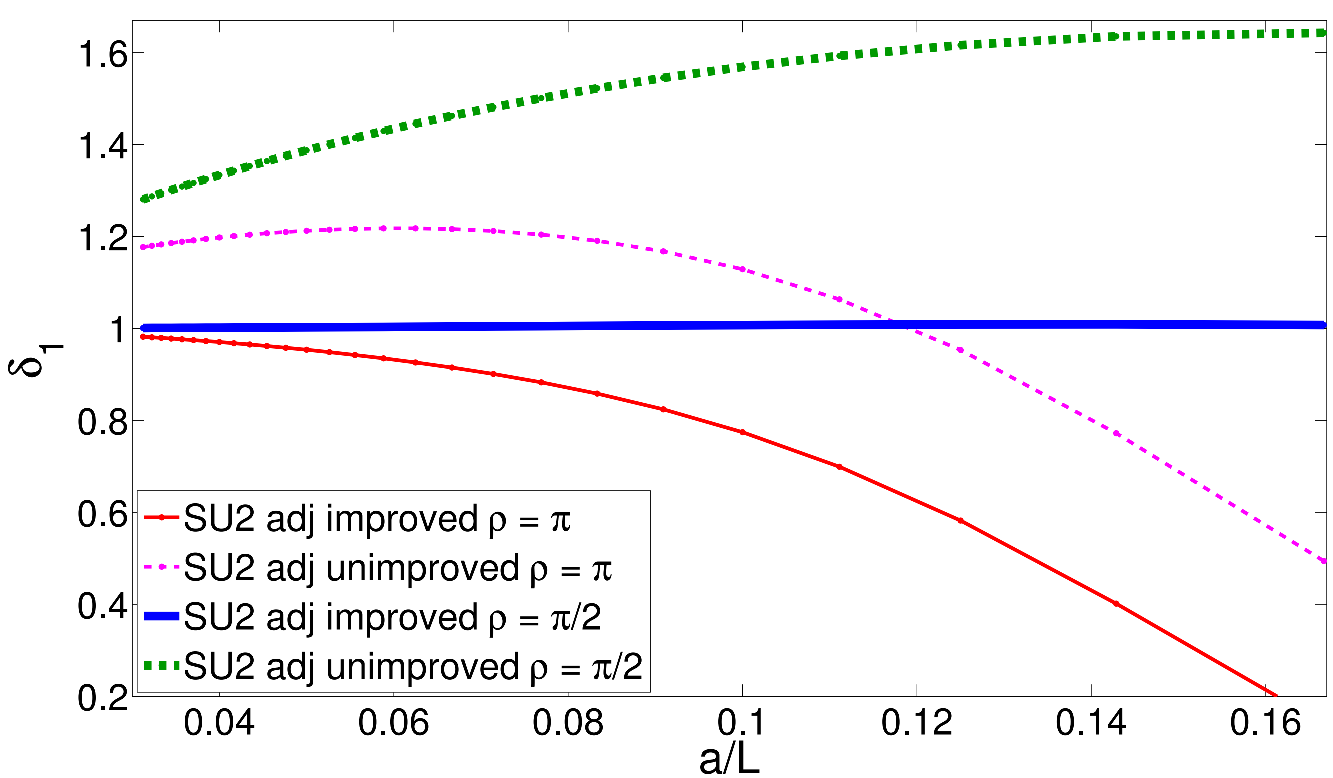
2. Also, the choice of boundary fields is nontrivial.



SU(2) gauge

$N_f = 2$ adjoint

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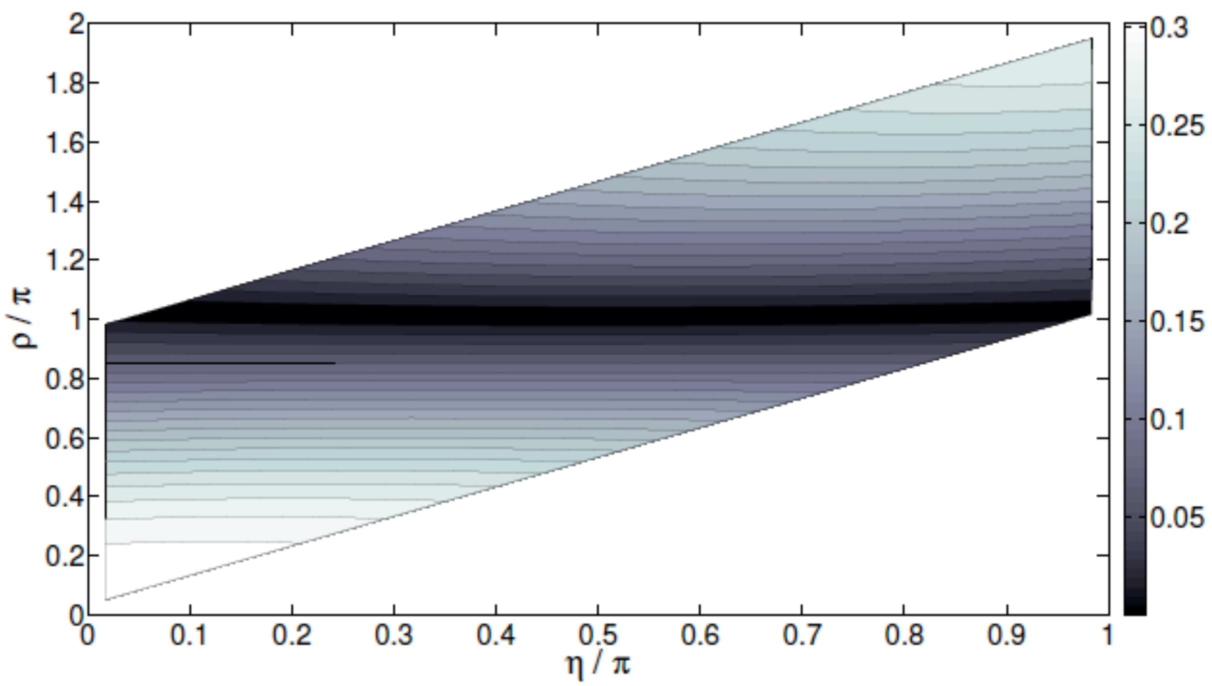
$\delta_1 - 1$ contours: SU(2) fundamental



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OK!

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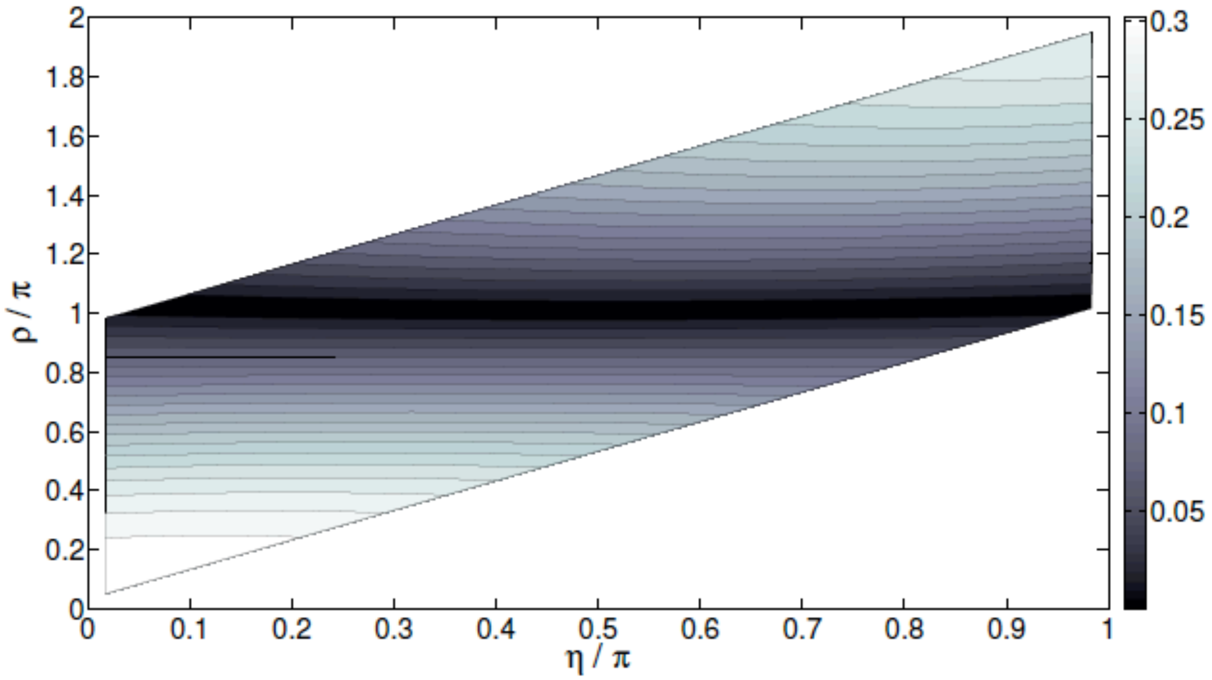

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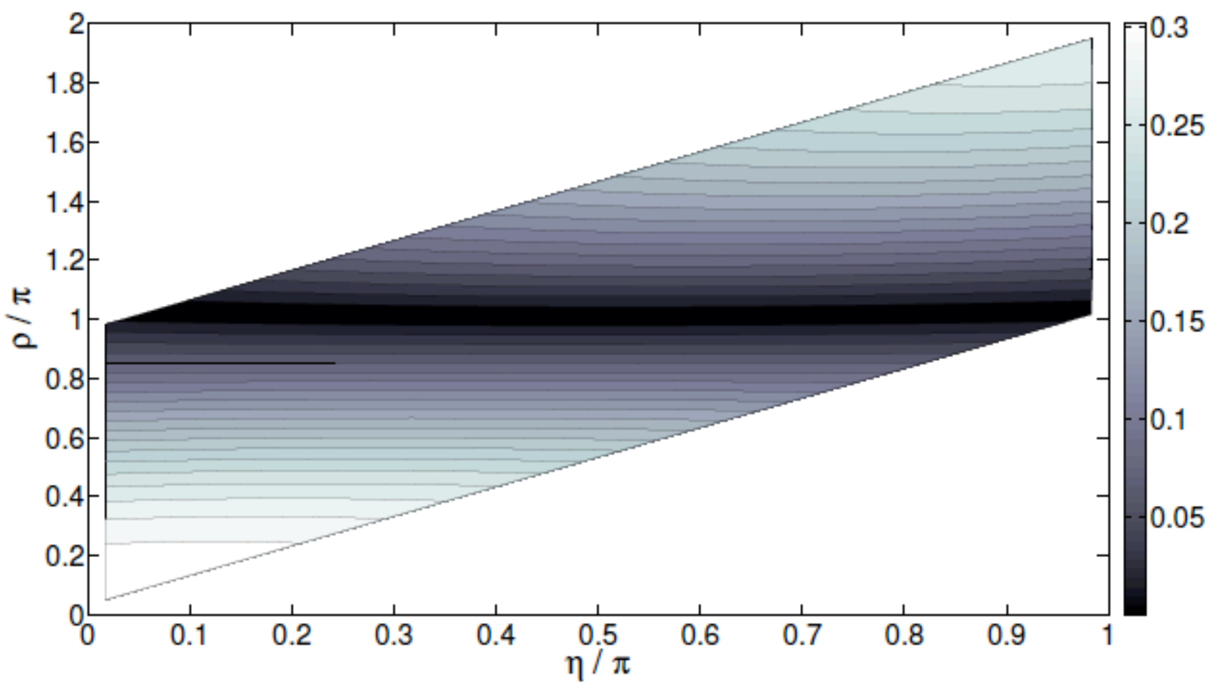

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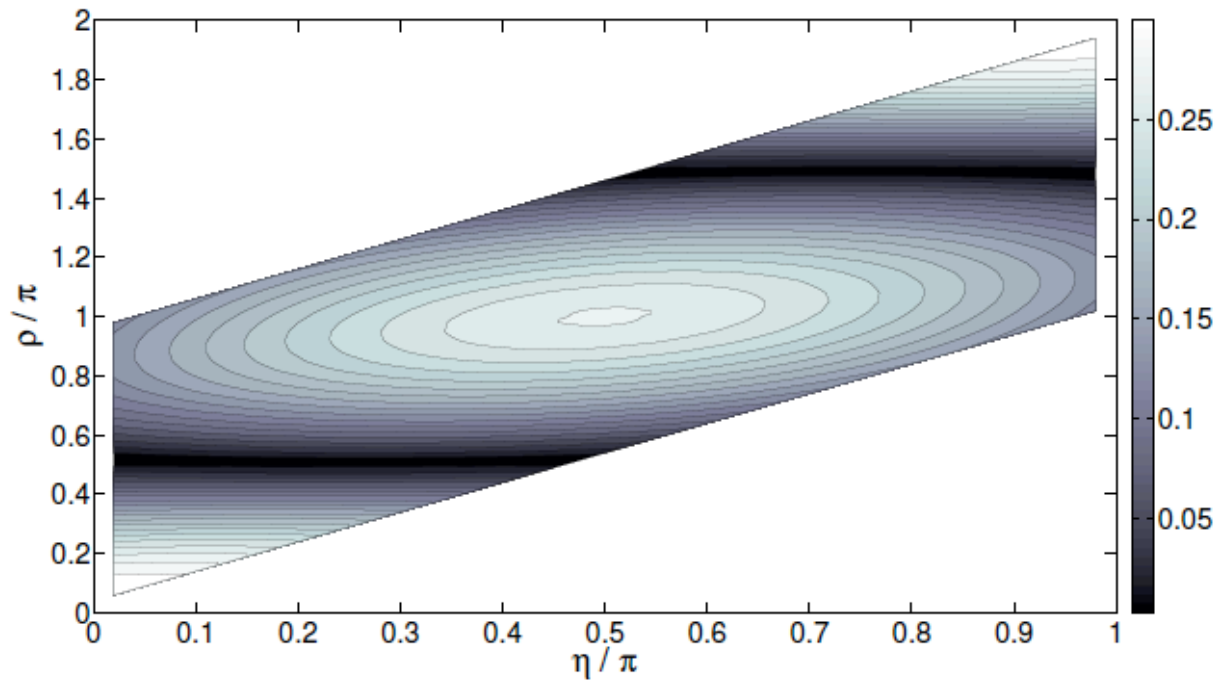
SU(2) adjoint: $\eta = \pi/8, \quad \rho = \pi/2$



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$$C'_k = \frac{i}{L}\mathrm{diag}(\phi'_1(\eta), \ldots, \phi'_n(\eta))$$

SU(2):

$$\phi_1 = -\eta \qquad \phi'_1 = \eta - \rho$$

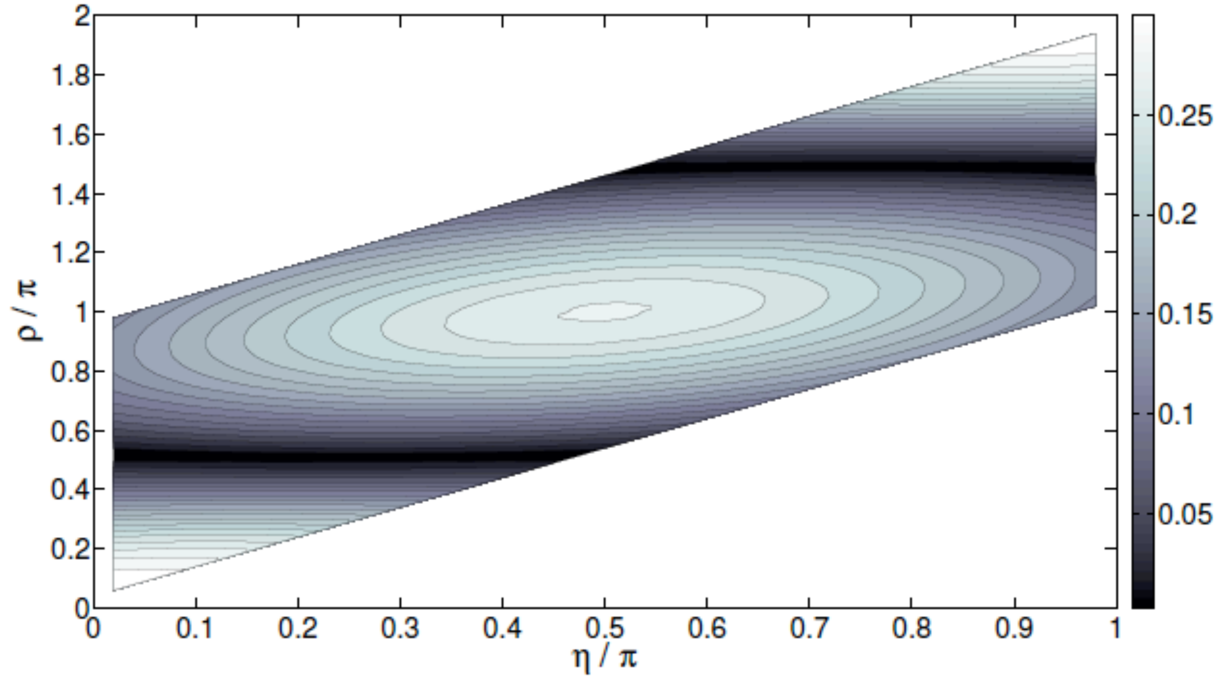
$$\phi_2 = \eta \qquad \phi'_2 = \rho - \eta$$

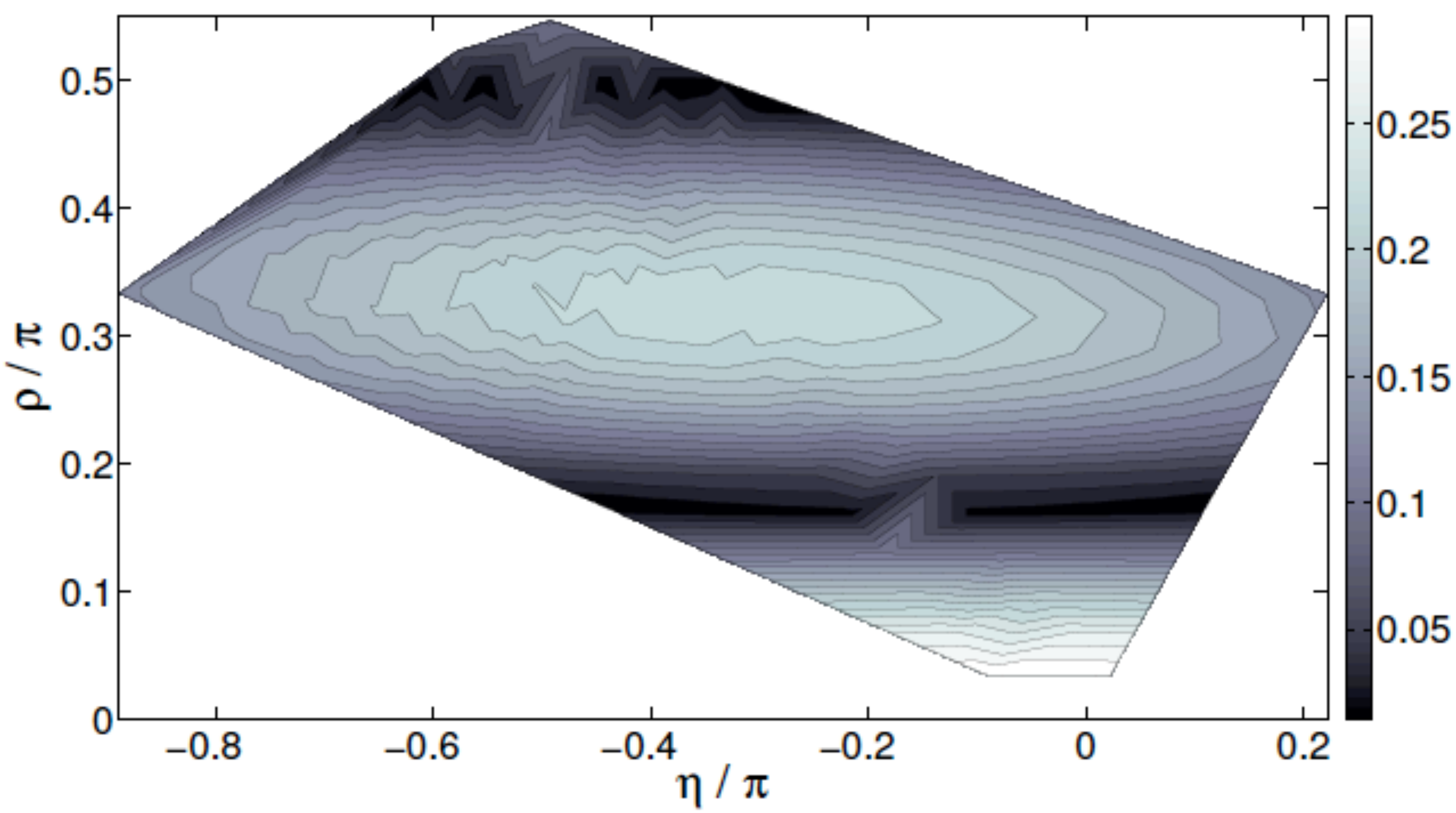
Fundamental rep.

$\eta = \pi/4, \quad \rho = \pi$

"Half of the field"

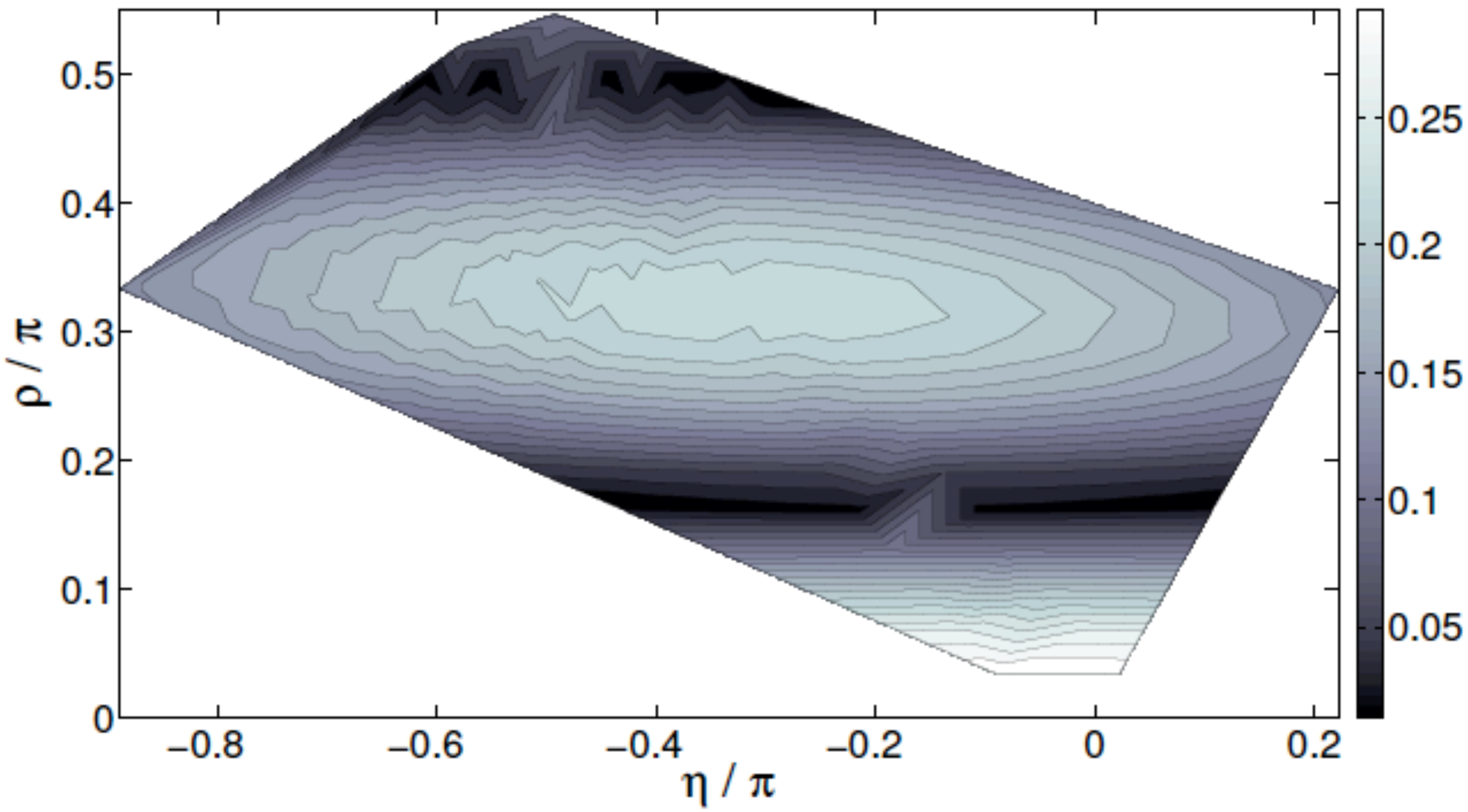
SU(2) adjoint: $\eta = \pi/8, \quad \rho = \pi/2$





SU(3) adjoint

$$\rho = \pi/6, \quad \eta = -\pi/9$$

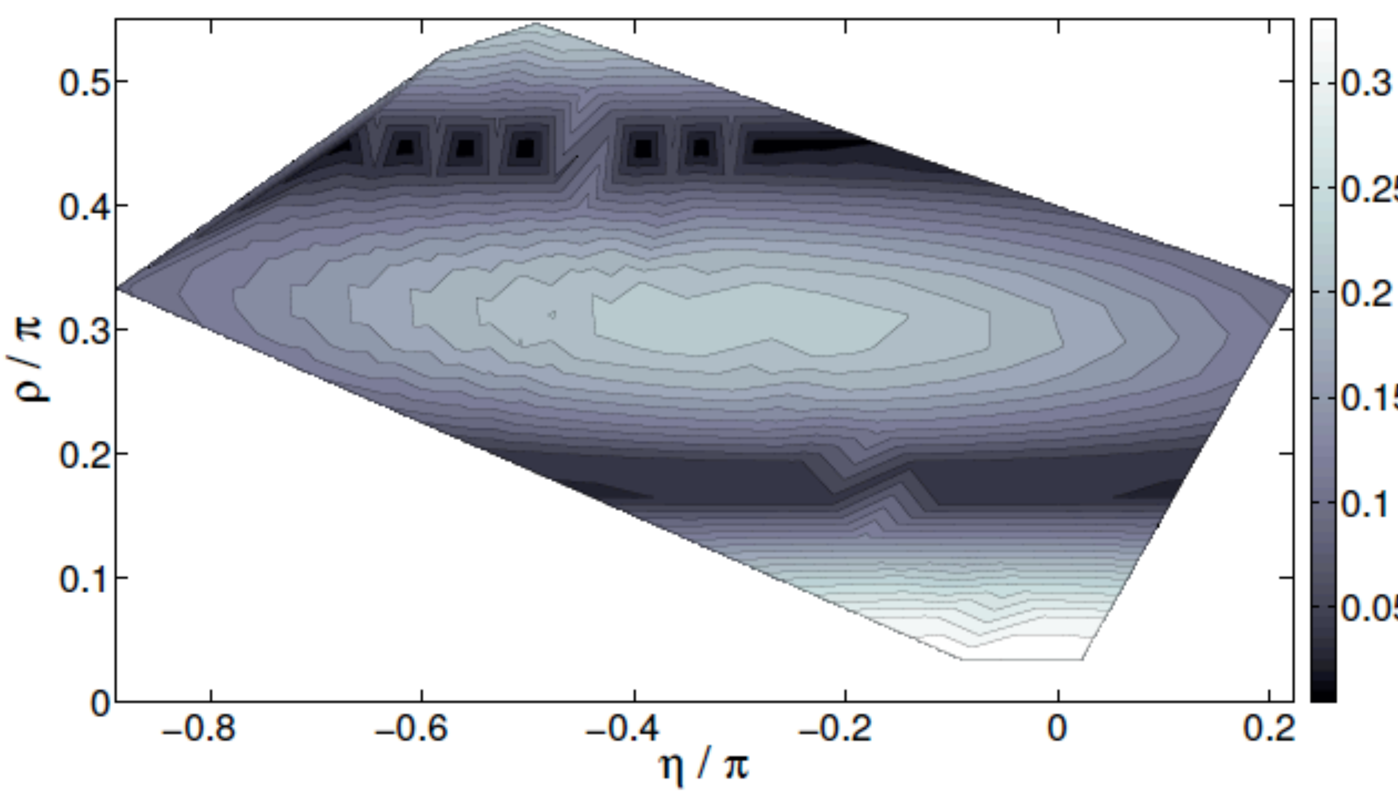


SU(3) adjoint

$$\rho = \pi/6, \quad \eta = -\pi/9$$

SU(3) sextet

$$\rho = 67\pi/150, \quad \eta = -\pi/3$$



The lessons:

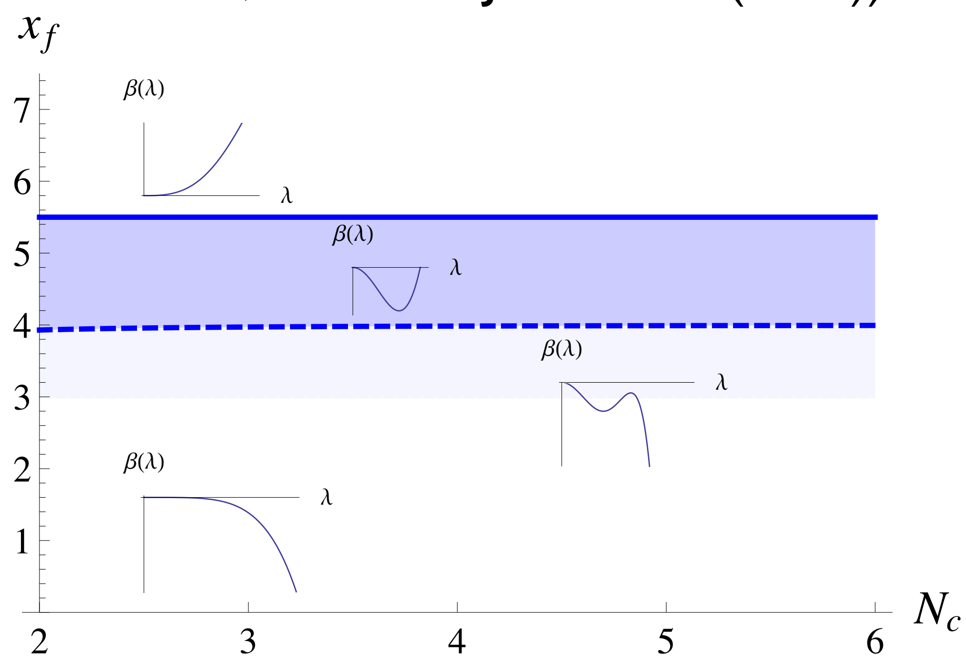
- **Wilson fermions must be improved.**
- **Improvement must done consistently:** all coefficients must be fixed to their correct values.
- **Optimization needed:** pull between gauge and fermion contributions; maintain large enough background field for measurements.

Alternative approaches:
S. Sint and P.Vilaseca, 1211.0411;
S. Sint, 1008.4857

Current status: SU(2) with improved fundamental Wilson fermions

(Karavirta, Rantaharju, Rummukainen, Tuominen JHEP 1205 (2012))

See also: F. Bursa et al. 1010.0901 and H.Ohki et al. 1011.0373



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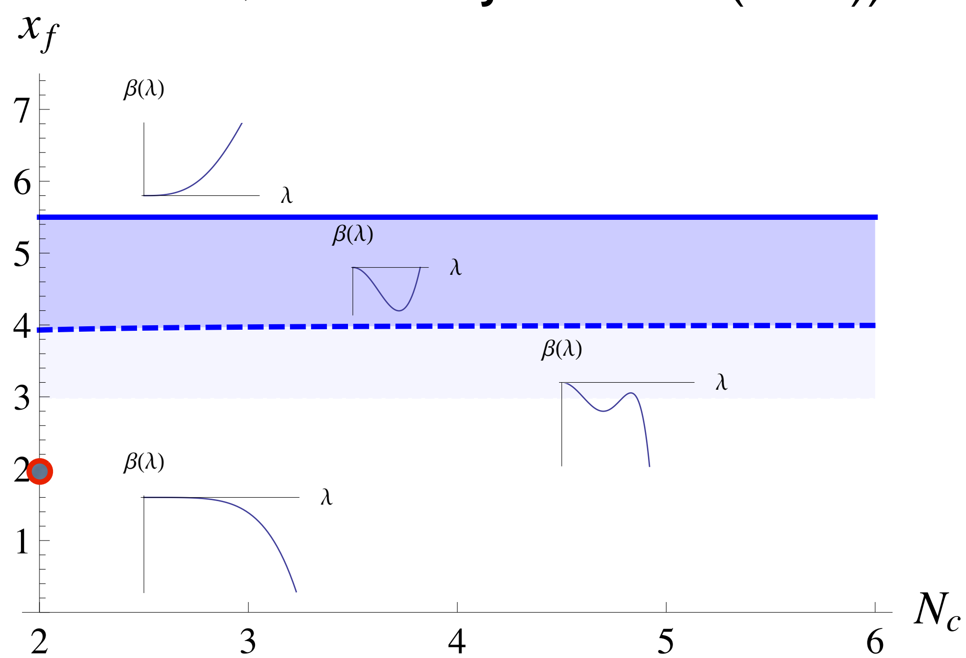
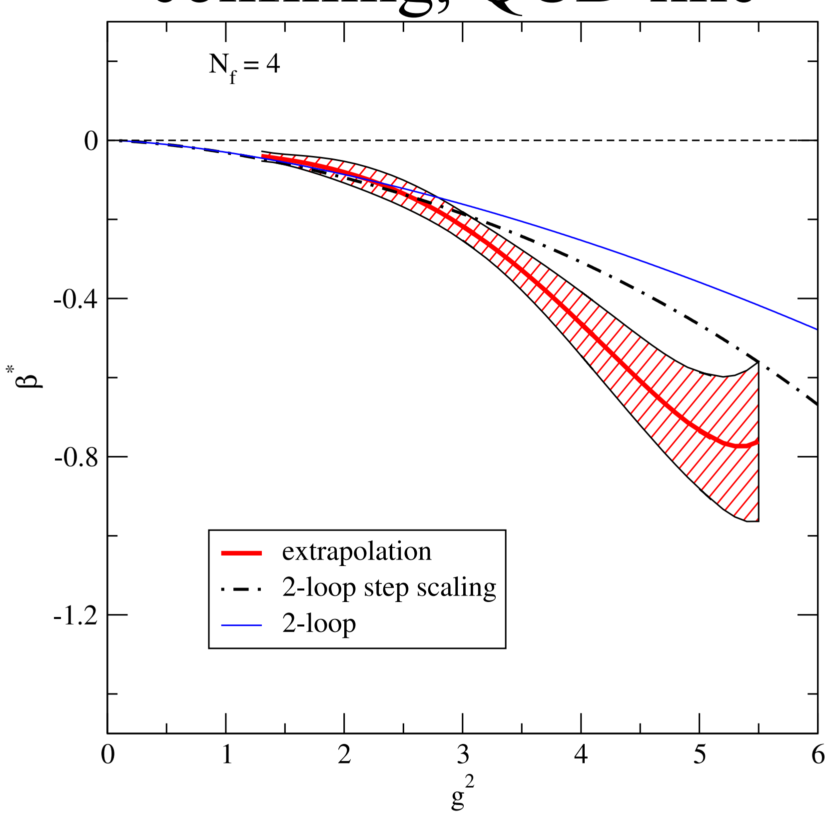


confining, QCD like



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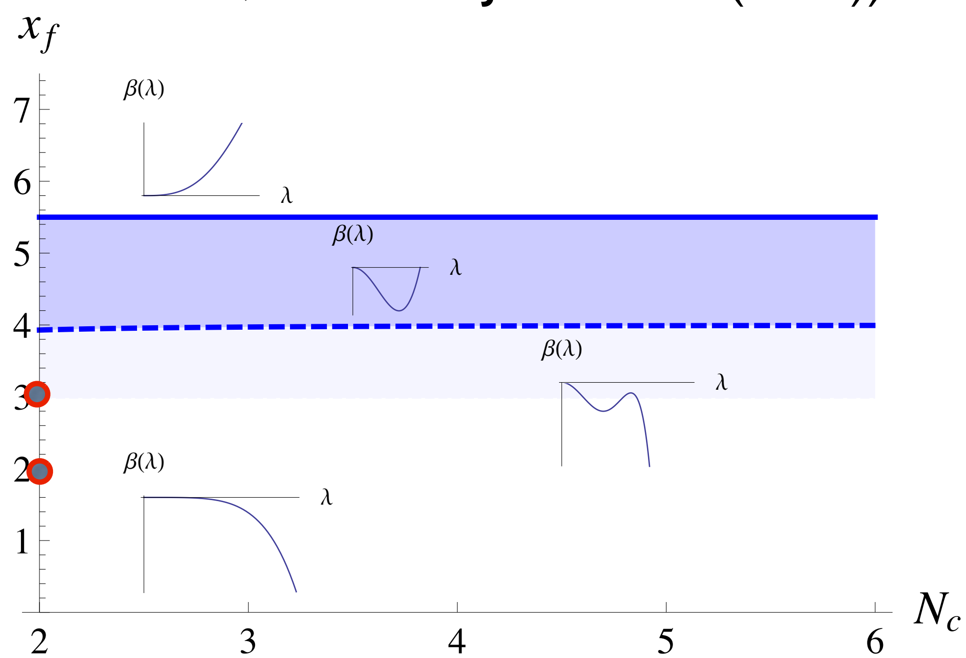
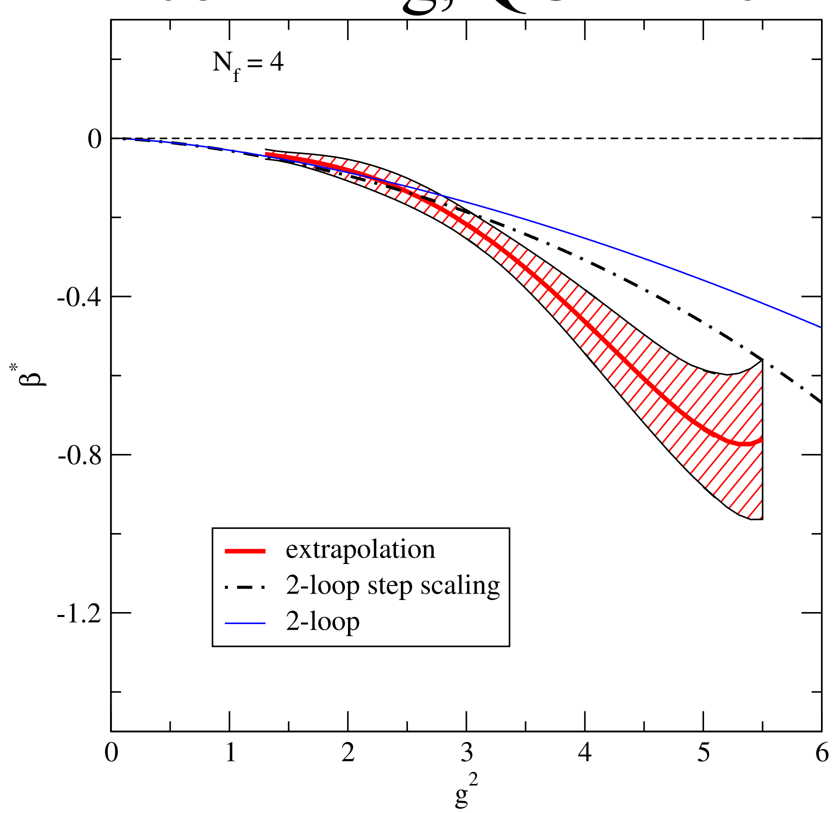
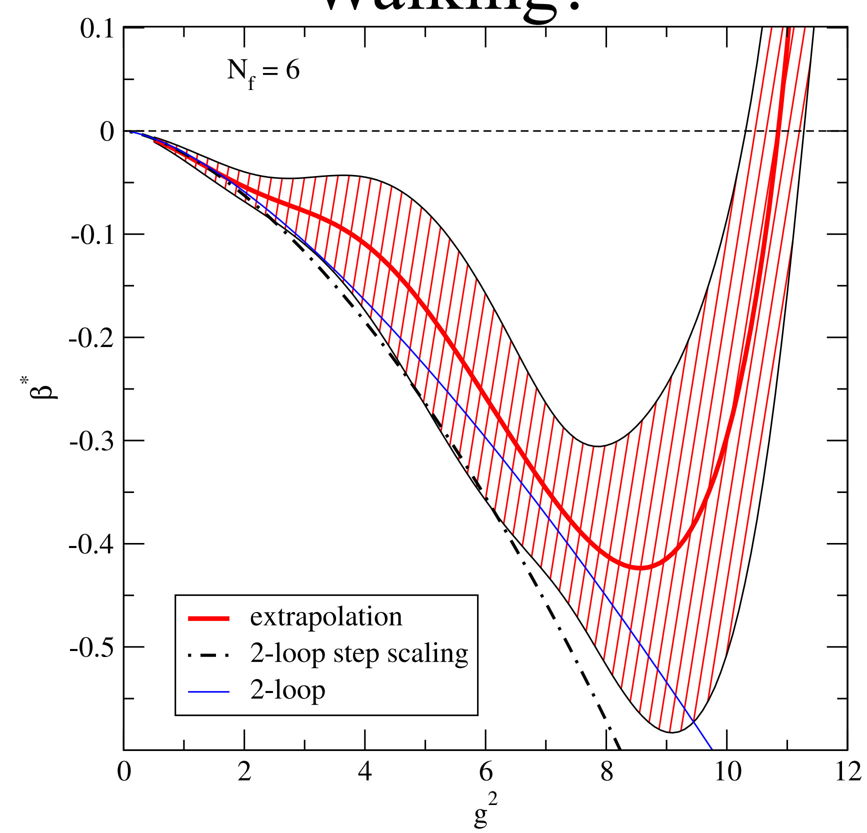


confining, QCD like



Walking?



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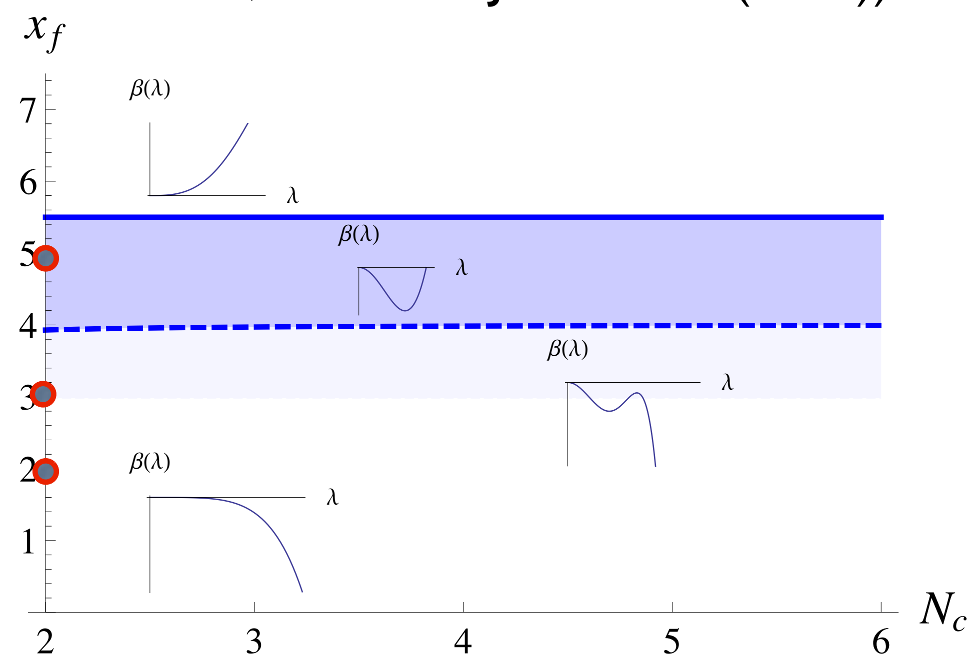

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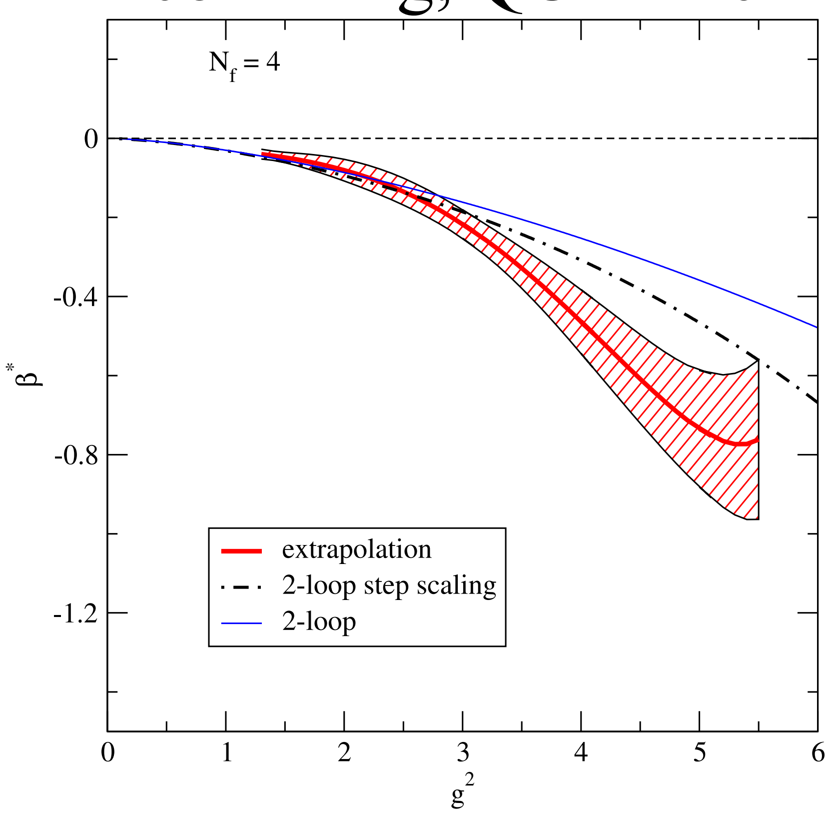
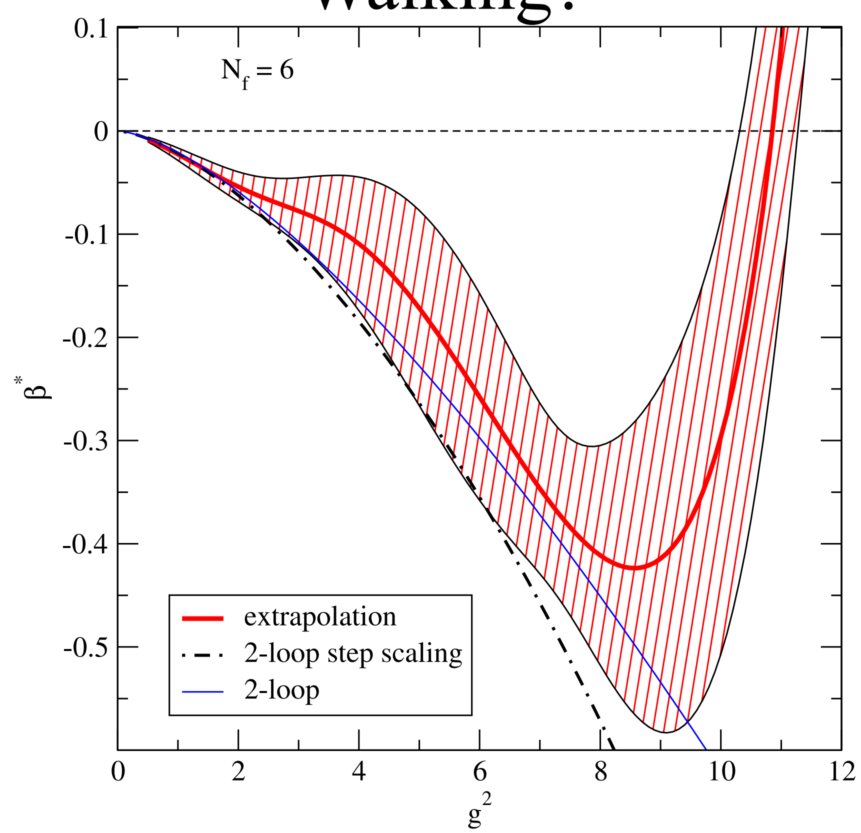
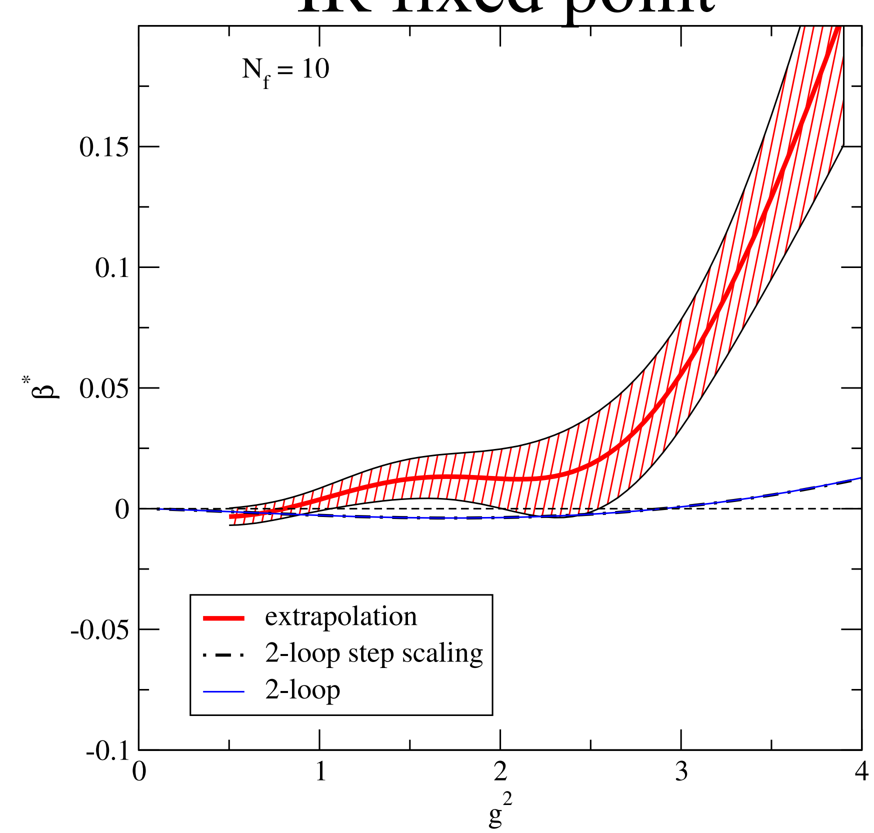


Walking?



IR fixed point



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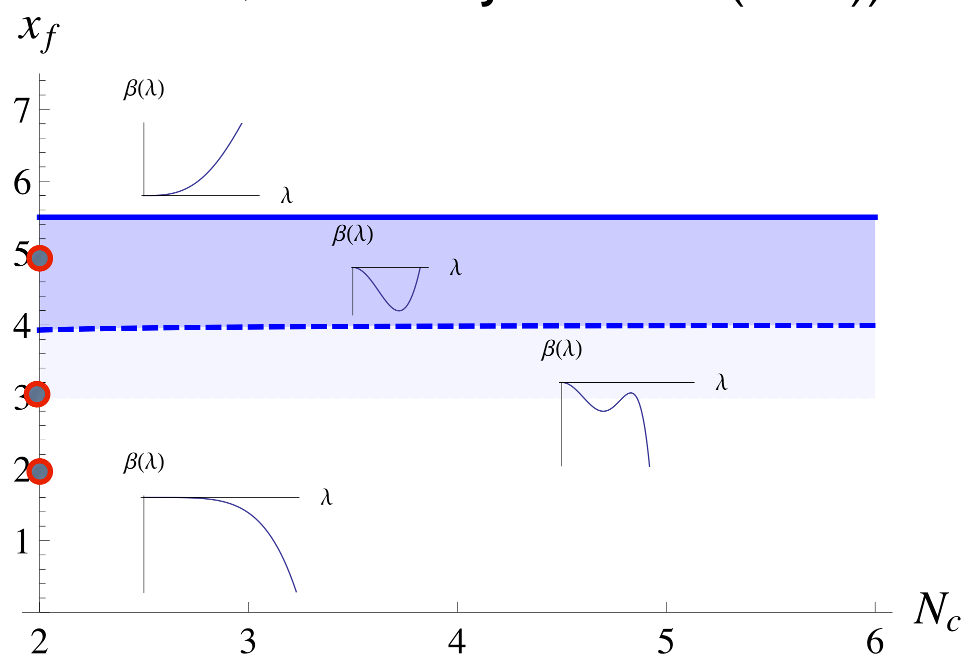

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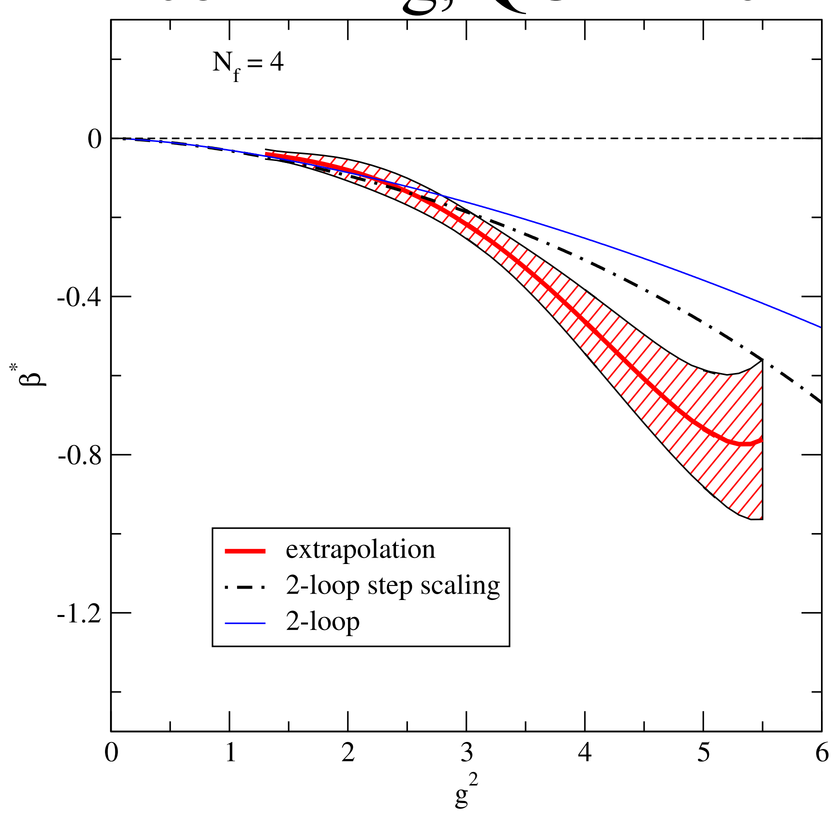
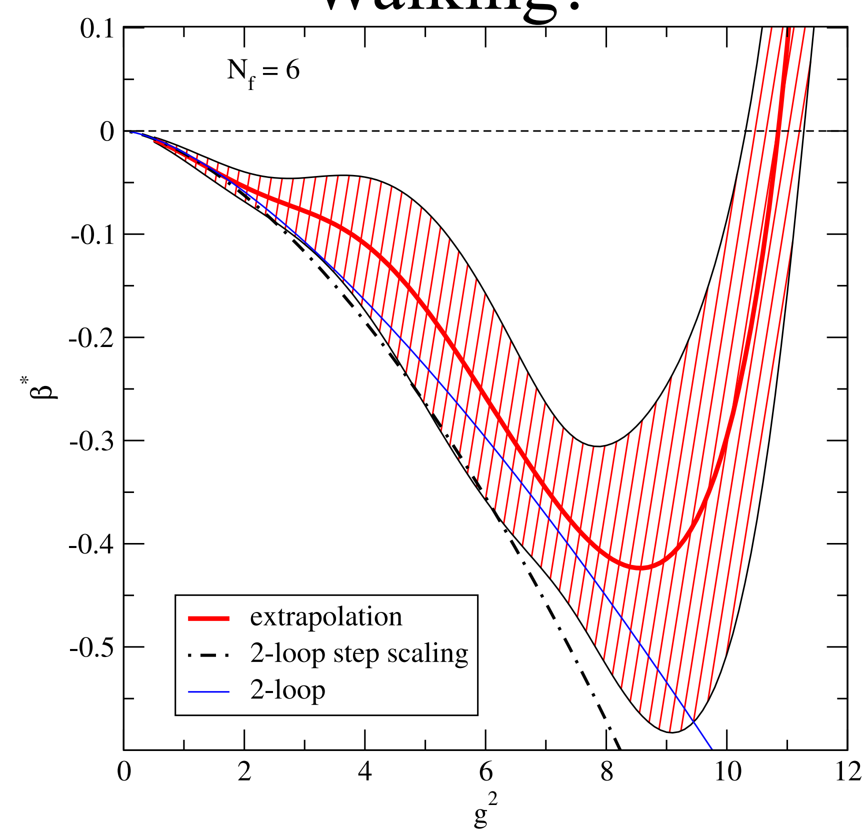


Walking?



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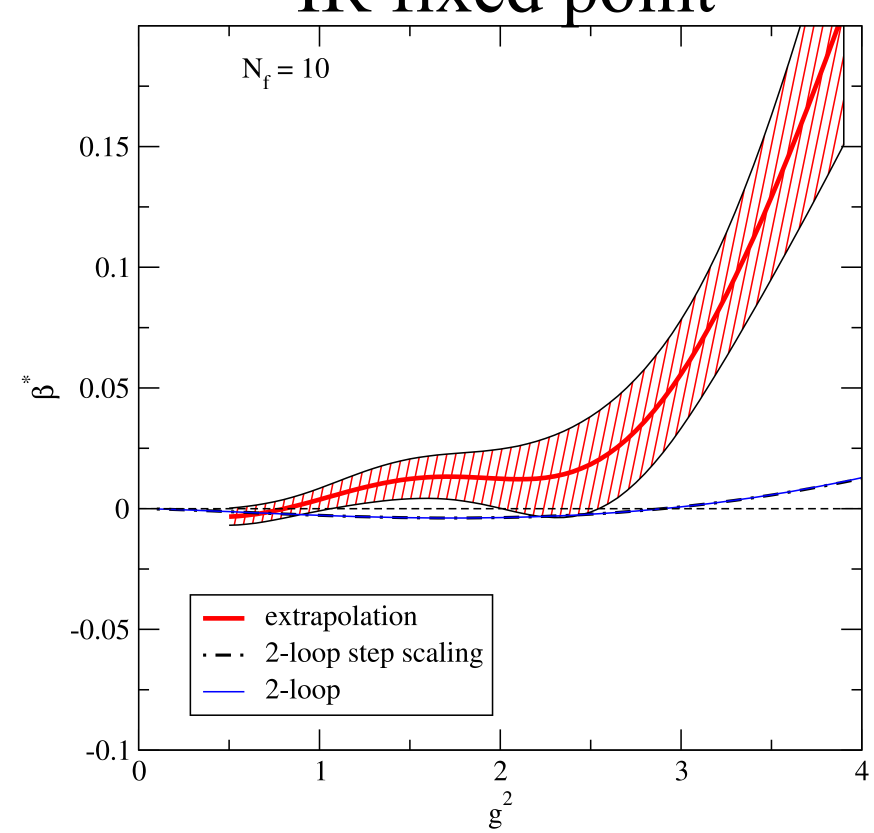
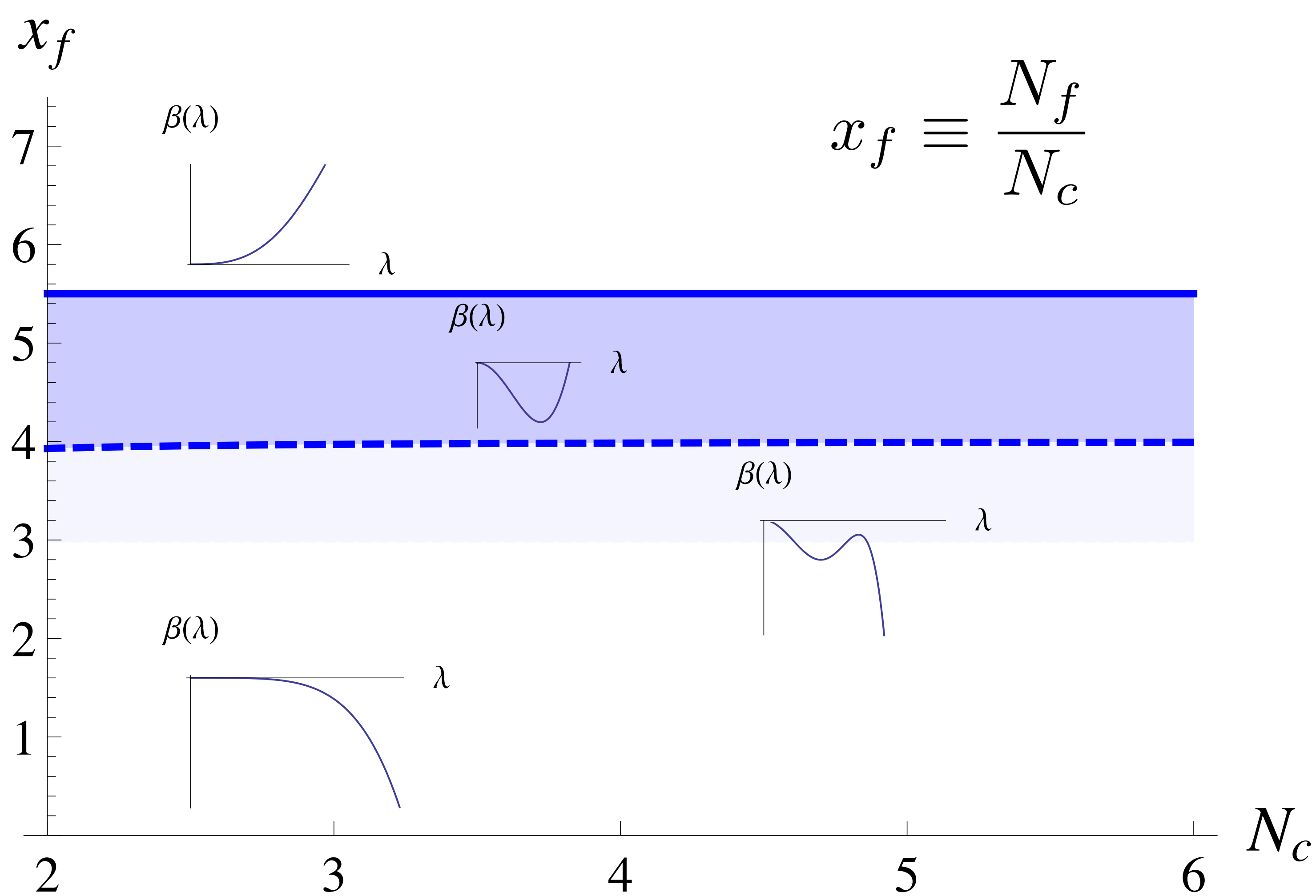


SU(2) with adjoint fermions: see the poster by Jarno Rantaharju

2. Finite T phase diagrams

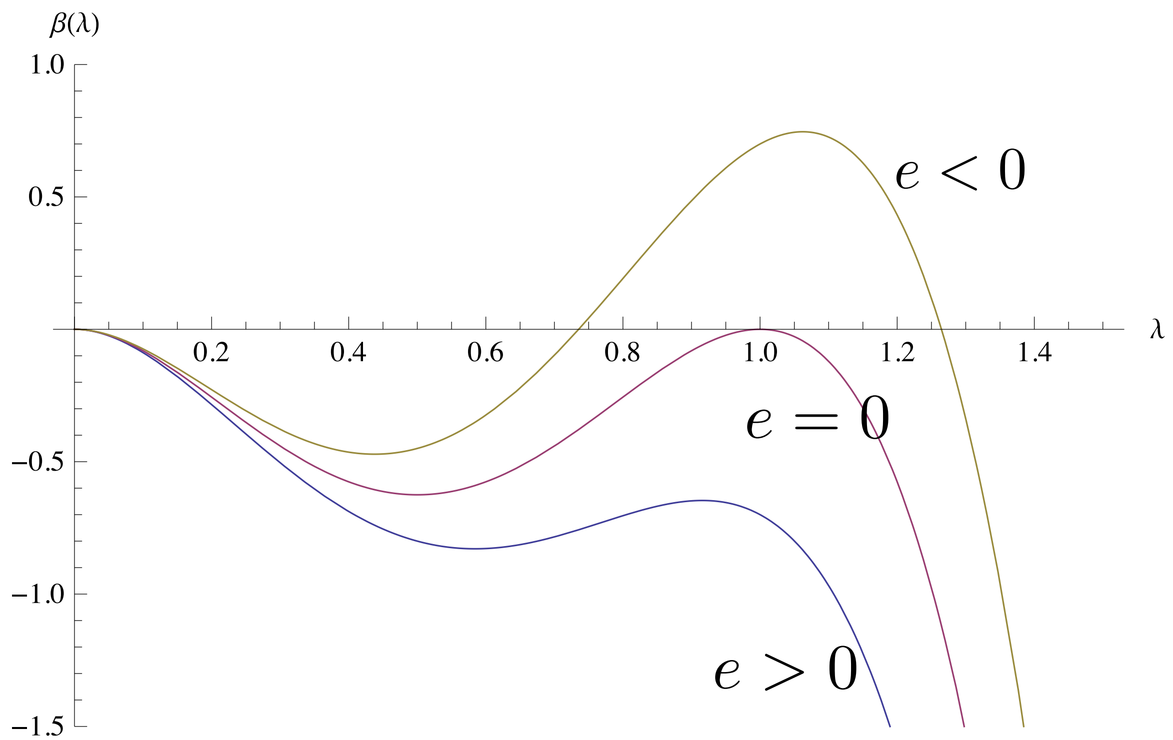
What happens when you heat these theories?



Suppose the theory is confining, walking/jumping or IR conformal and put it in finite T

(F. Sannino: 1205.4246)

$$\beta(\lambda) \sim -\lambda^2 \left[(\lambda-1)^2 + e\right], \qquad e \propto N_f^{\rm crit} - N_f$$



IR and UV fixed points annihilate.

A second order zero at the lower boundary of the conformal window

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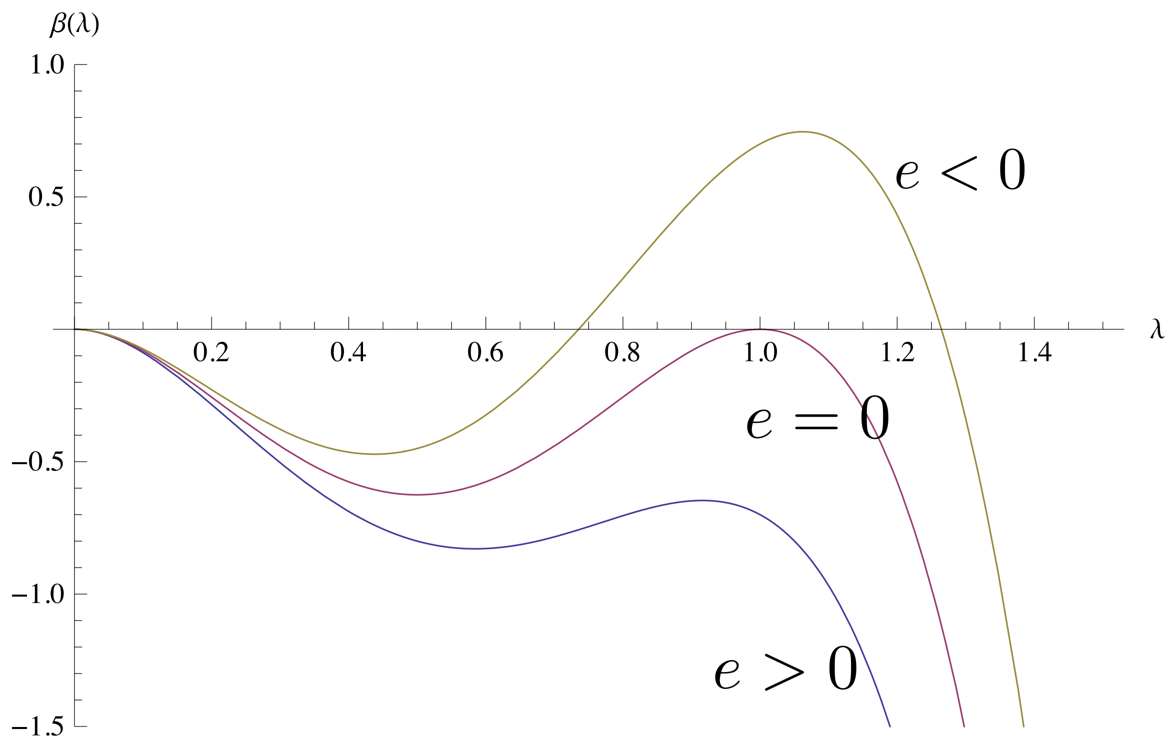
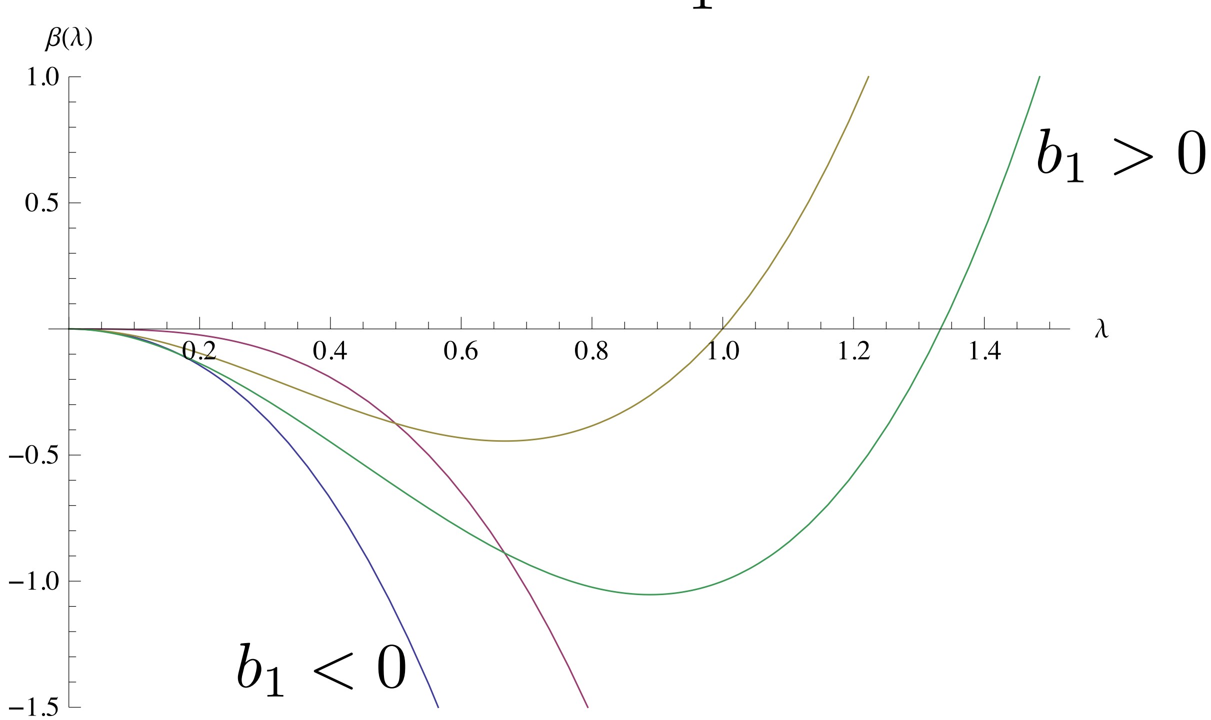


IR and UV fixed points annihilate.

A second order zero at the lower boundary of the conformal window

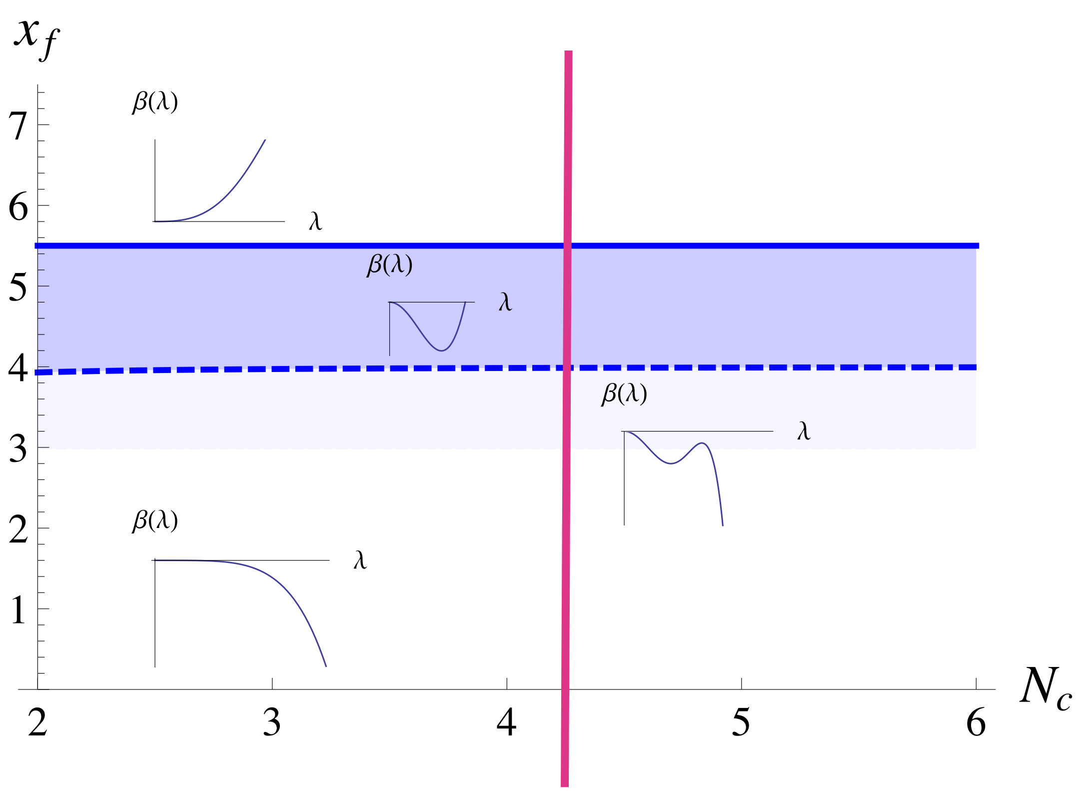
$$\beta(\lambda) = -b_1\lambda^2(\lambda+\frac{b_0}{b_1}), \qquad b_0<0, \quad b_1 \propto N_f^{\mathrm{crit}} - N_f$$

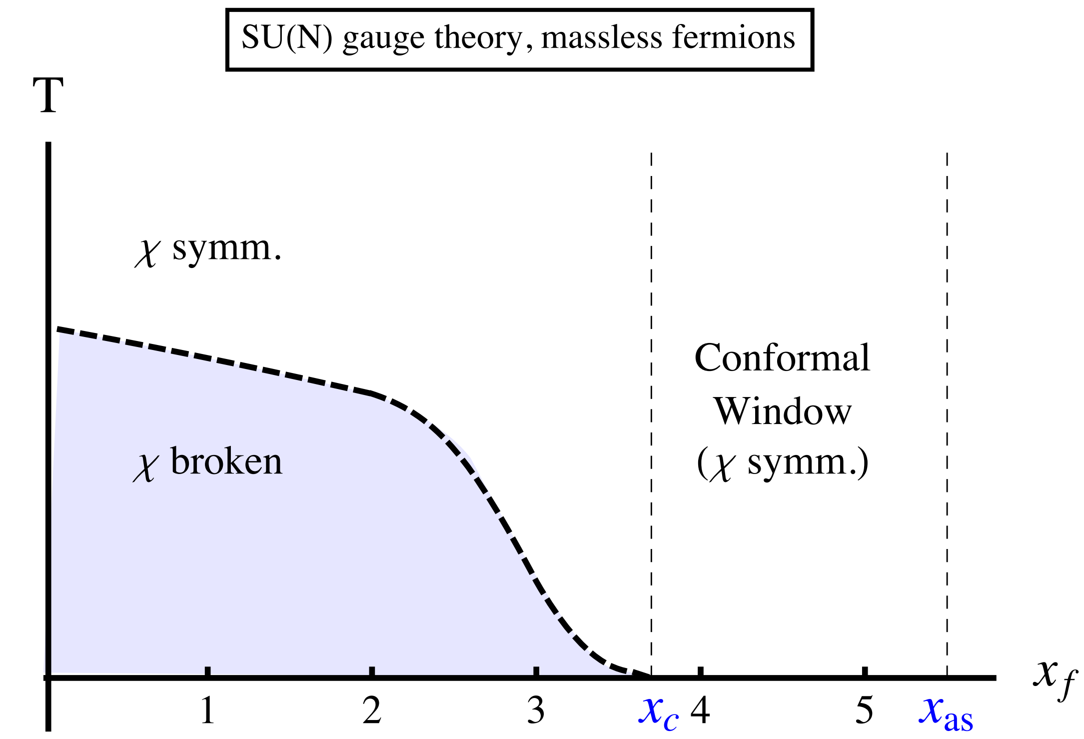


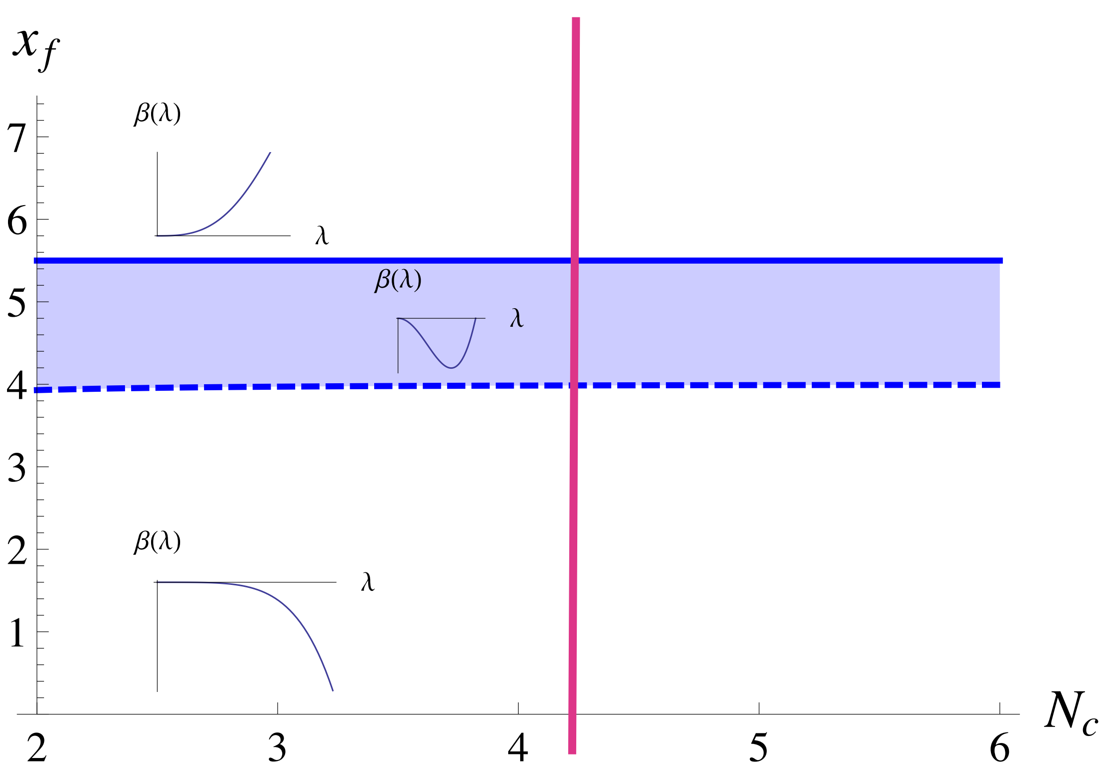
A first order zero at the lower boundary of the CW.

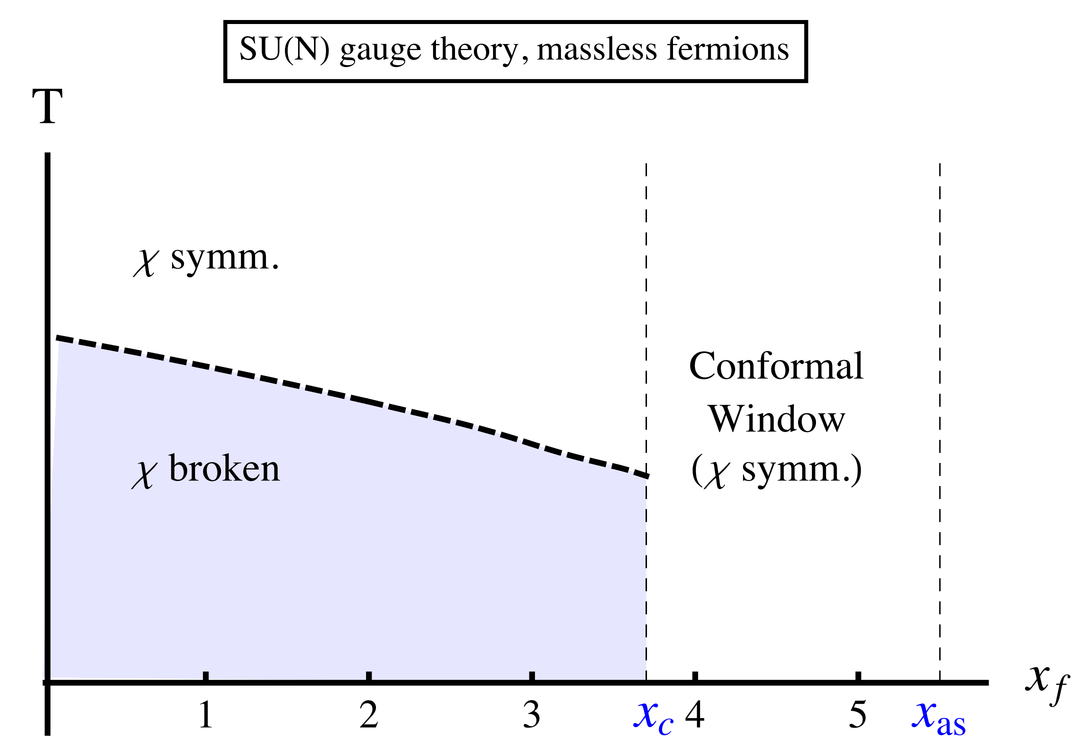
Discontinuity

General features: 1) exact symmetries









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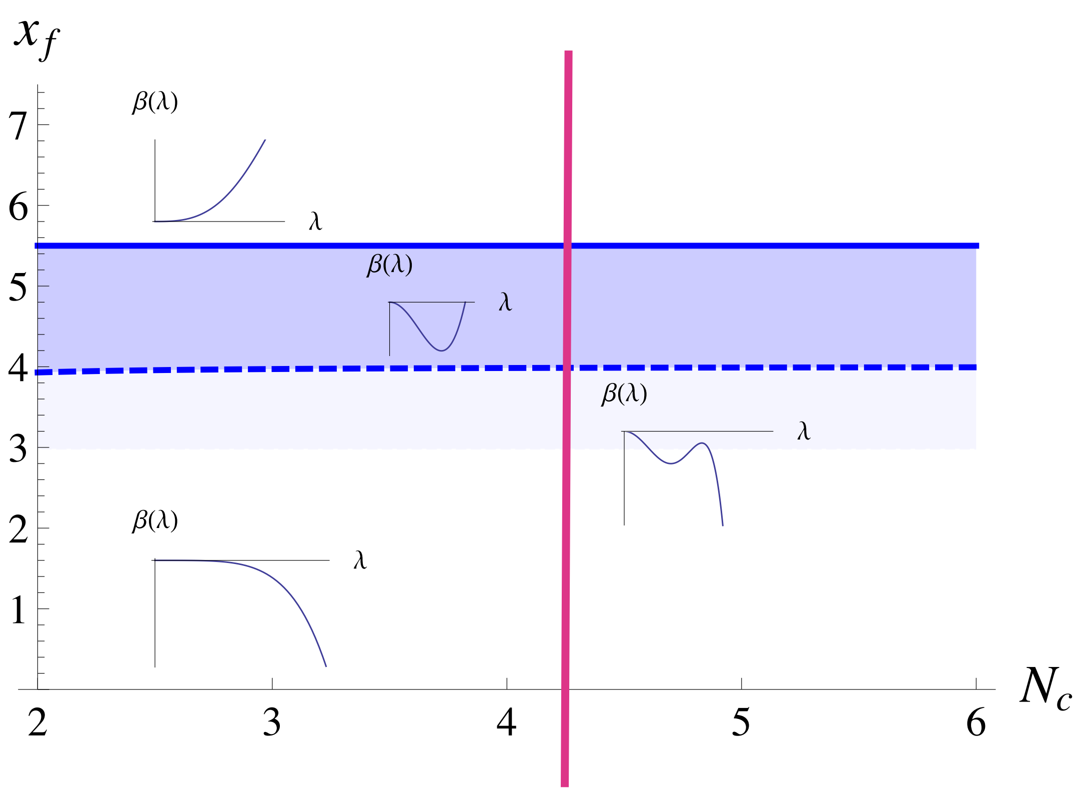
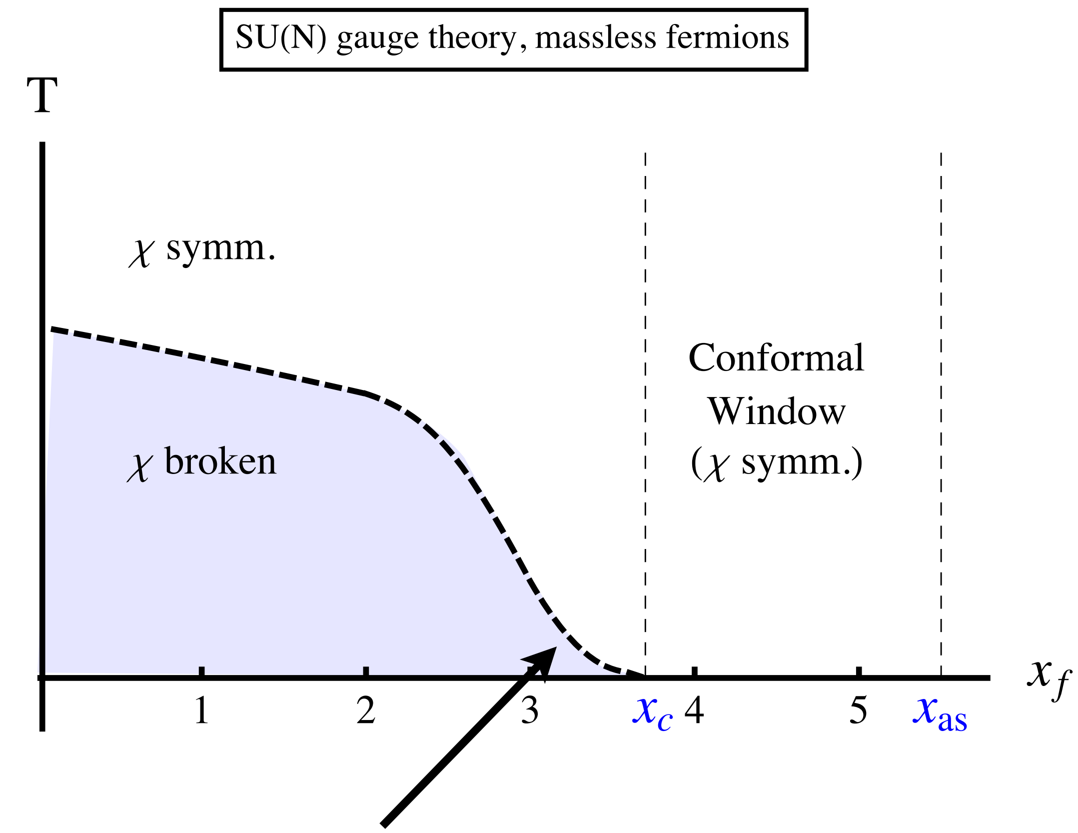
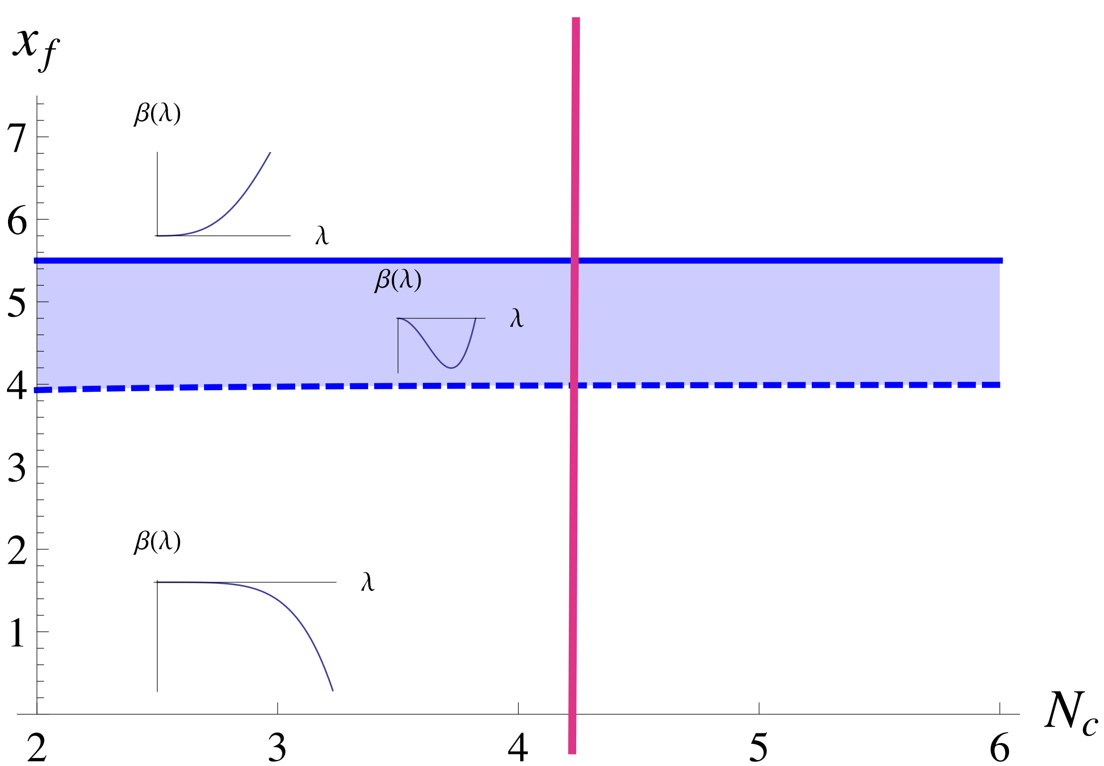
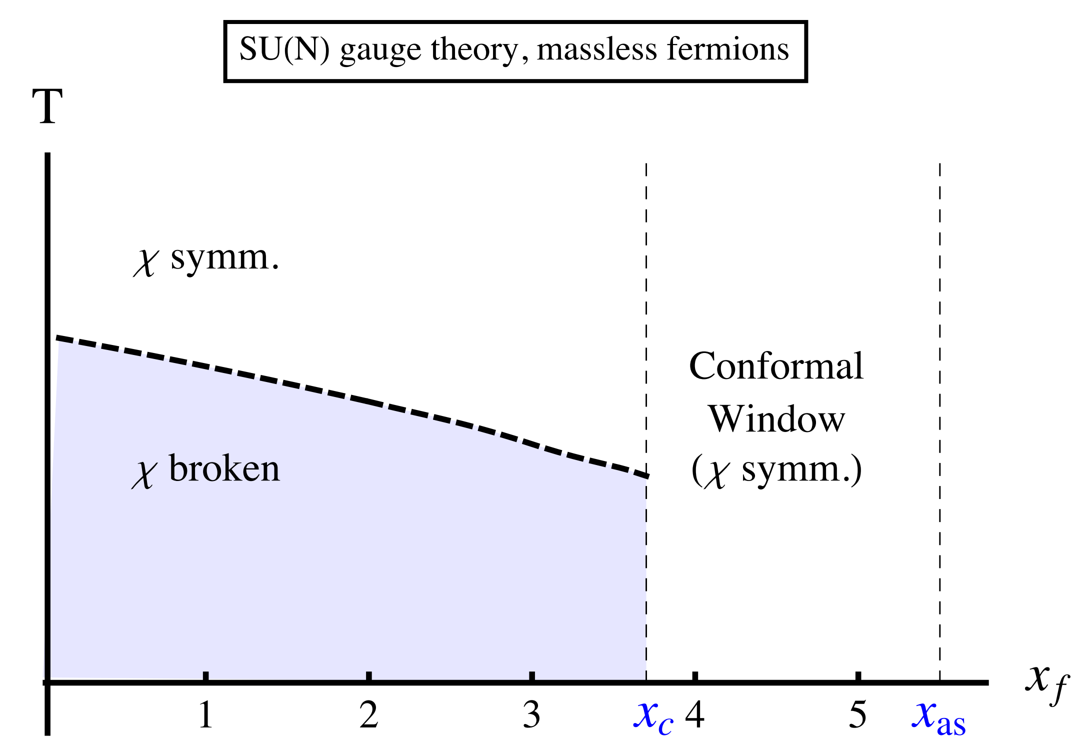




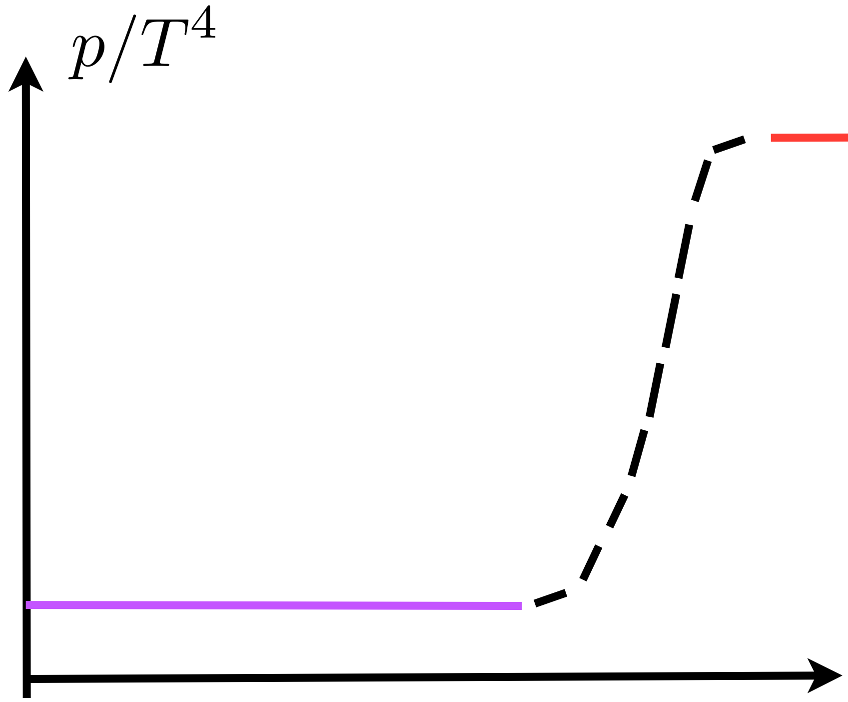
Miransky scaling





General features: 2) relevant scales and dofs

Inside the conformal window:

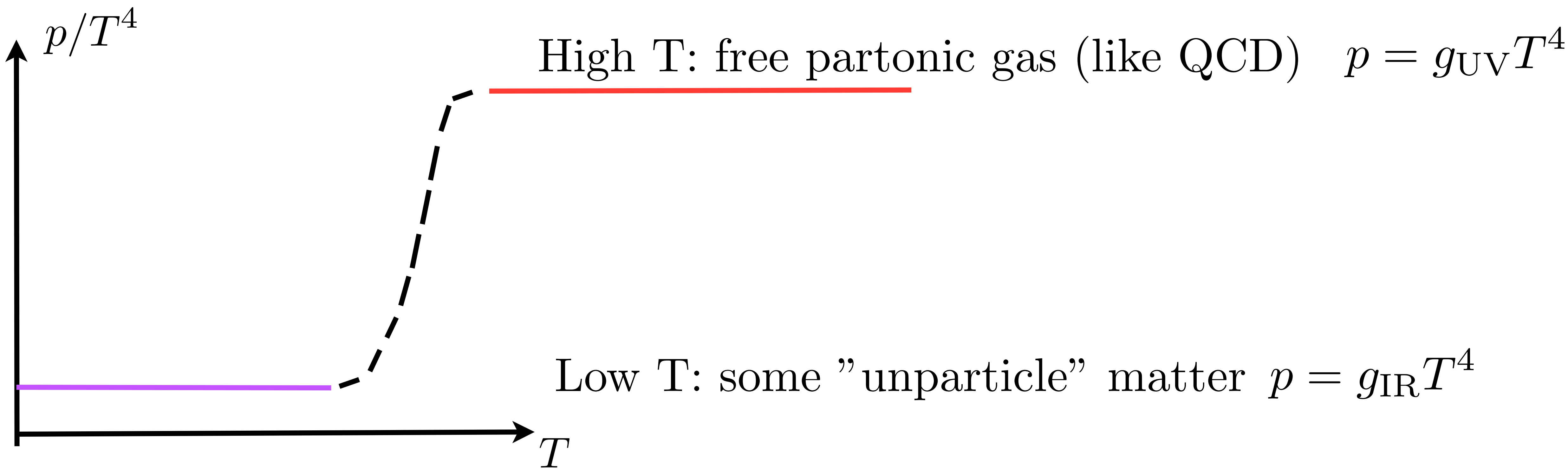


High T: free partonic gas (like QCD) $p = g_{\mathrm{UV}} T^4$

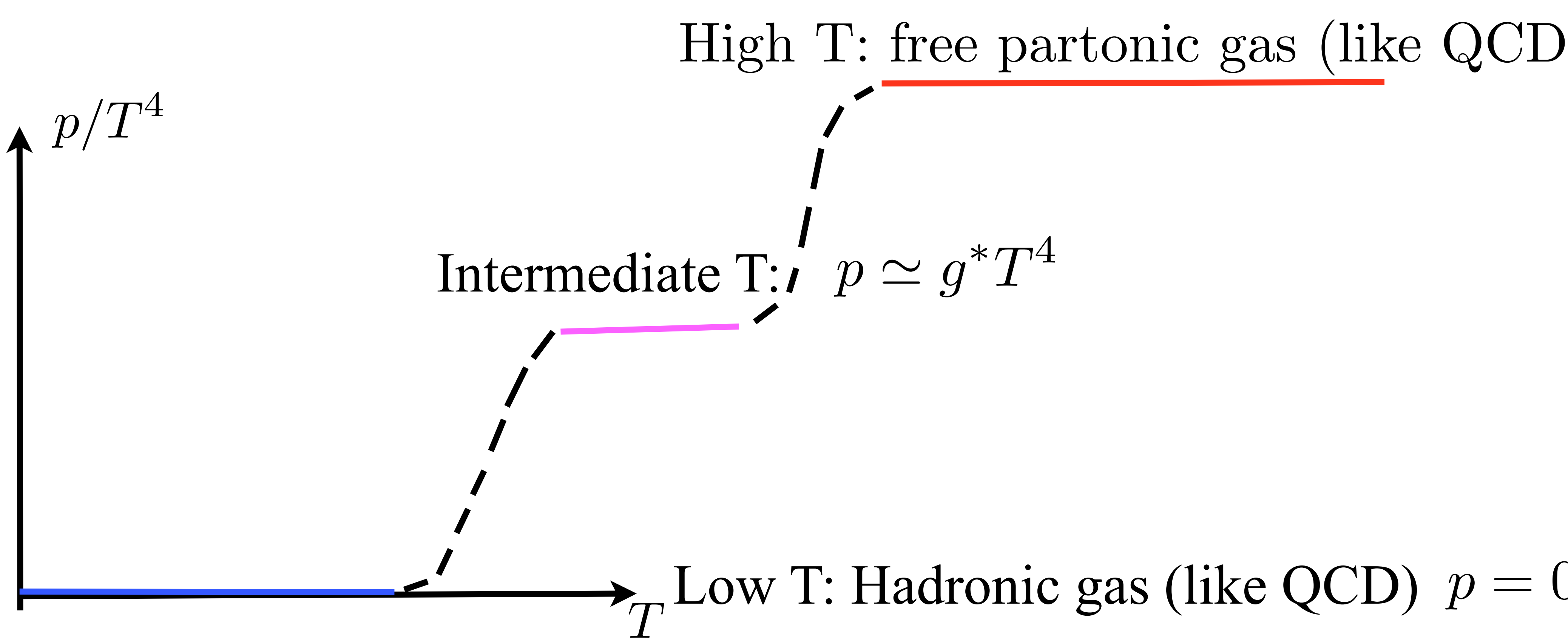
Low T: some "unparticle" matter $p = g_{\mathrm{IR}} T^4$

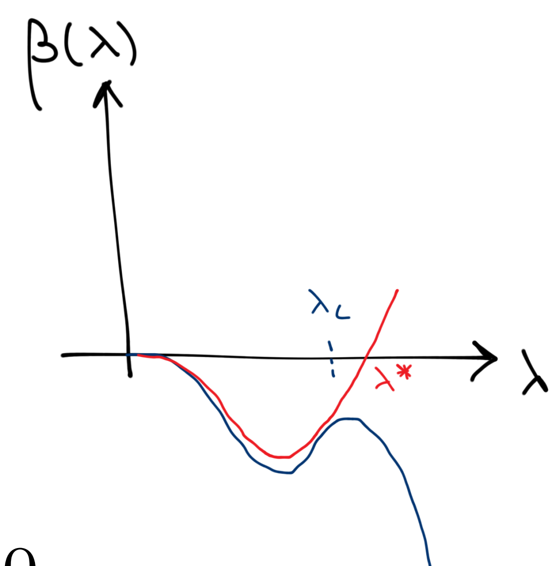
General features: 2) relevant scales and dofs

Inside the conformal window:

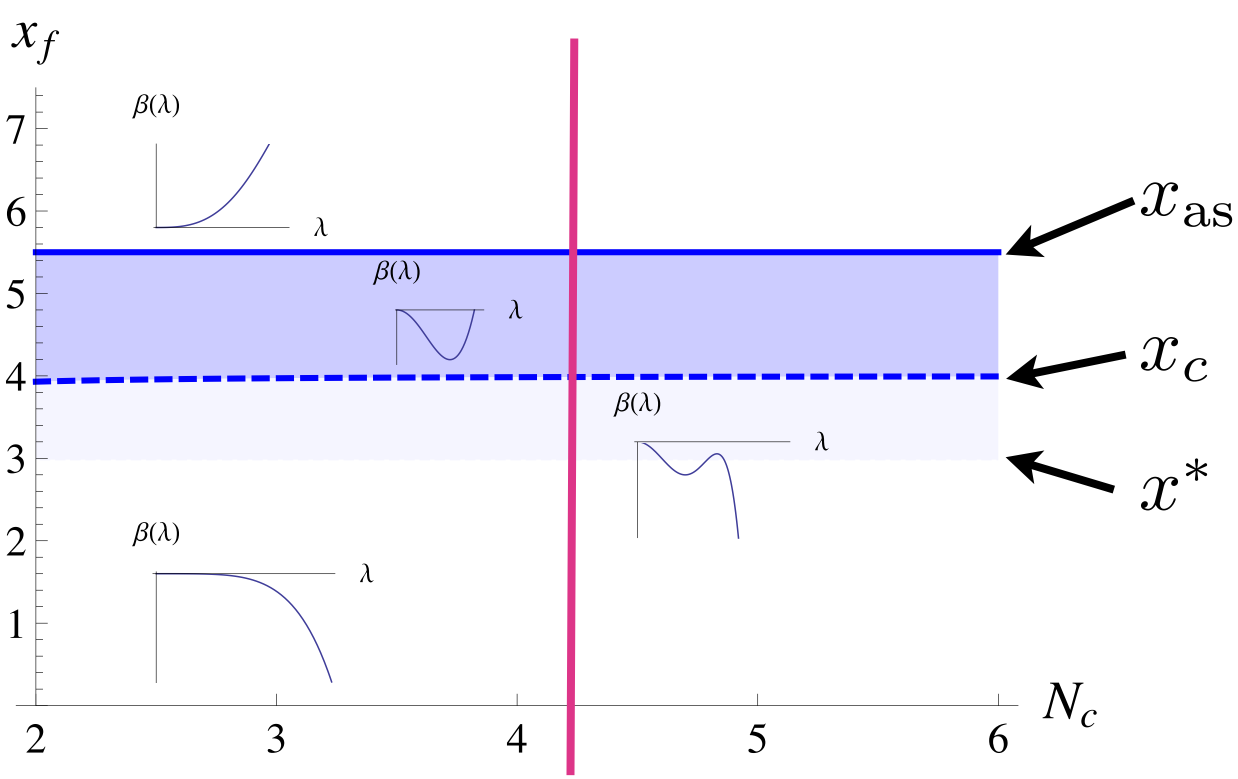


Walking region:

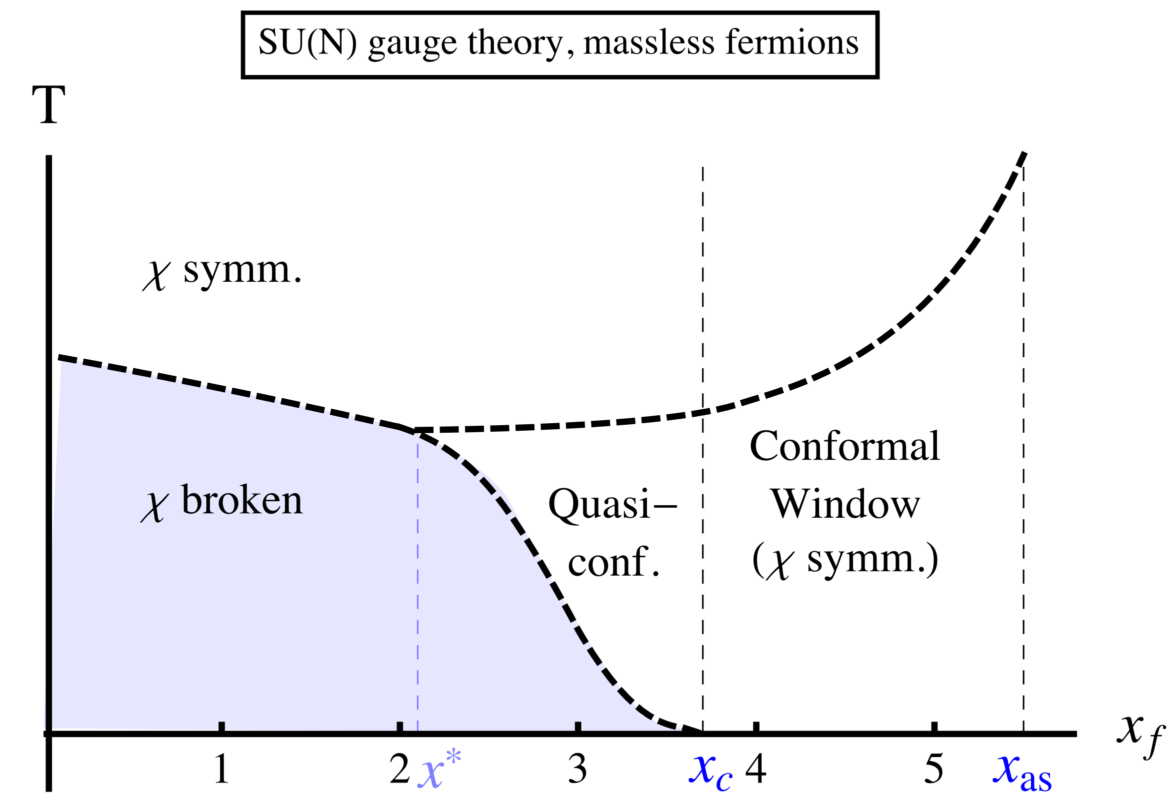




General picture (K.T. 1206.5772)



fixed N_c



$x^* < x_f < x_c$ "walking window"

$x_c < x_f < x_{as}$ conformal window

A holographic model with fermion backreaction

(Järvinen, Kiritsis 1112.1261; Alho, Järvinen, Kajantie, Kiritsis, K.T. 1210.4516)

Veneziano limit: $N_c \to \infty$ and $x_f \equiv \frac{N_f}{N_c}$ fixed

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R + \left[-\frac{4}{3}(\partial_\mu \phi)^2 + V_g(\lambda) \right] - x_f V_f(\lambda, \tau) \sqrt{1 + g^{zz}\kappa(\lambda(z))\dot{\tau}^2} \right),$$

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$$ds^2 = b^2(z) \left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)} \right] \qquad b(z) \sim \frac{\mathcal{L}}{z} \qquad f(z) \sim 1 + \frac{z^4}{z_h^4}$$

AdS_5 black hole; i.e. conformal N=4.

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The dilaton $\lambda(z)$: $\beta(\lambda) = b\dfrac{d\lambda}{db}$, $\lambda(z) = \dfrac{1}{b_0 \ln(1/\Lambda z)} + \cdots$ Breaks conformality

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The tachyon $\quad \tau(z) = m_q (\ln \dfrac{1}{\Lambda z})^a z + \langle \bar{q} q \rangle (\ln \dfrac{1}{\Lambda z})^{-a} z^3 + \cdots$ leading running of quark mass

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$V_g(\lambda) \sim$ pure YM case

Fix $V_f(\lambda, \tau)$ and $\kappa(\lambda)$ by

- matching to the known UV behaviors and
- fixing the IR divergence of the tachyon (several possibilities)

$$V_f(\lambda, \tau) = x_f V_{f0}(\lambda) e^{-a(\lambda)\tau^2}$$

The four functions, $b(z)$, $f(z)$, $\lambda(z)$, $\tau(z)$, solved from Einstein eqs.
The action is tuned to reproduce required UV behaviors + confinement at small x_f.

As in lattice MC, the effort is to go to the limit $m_q \to 0$

Phases: black hole solutions. Stable phase: one with smallest action.

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As in lattice MC, the effort is to go to the limit $m_q \to 0$

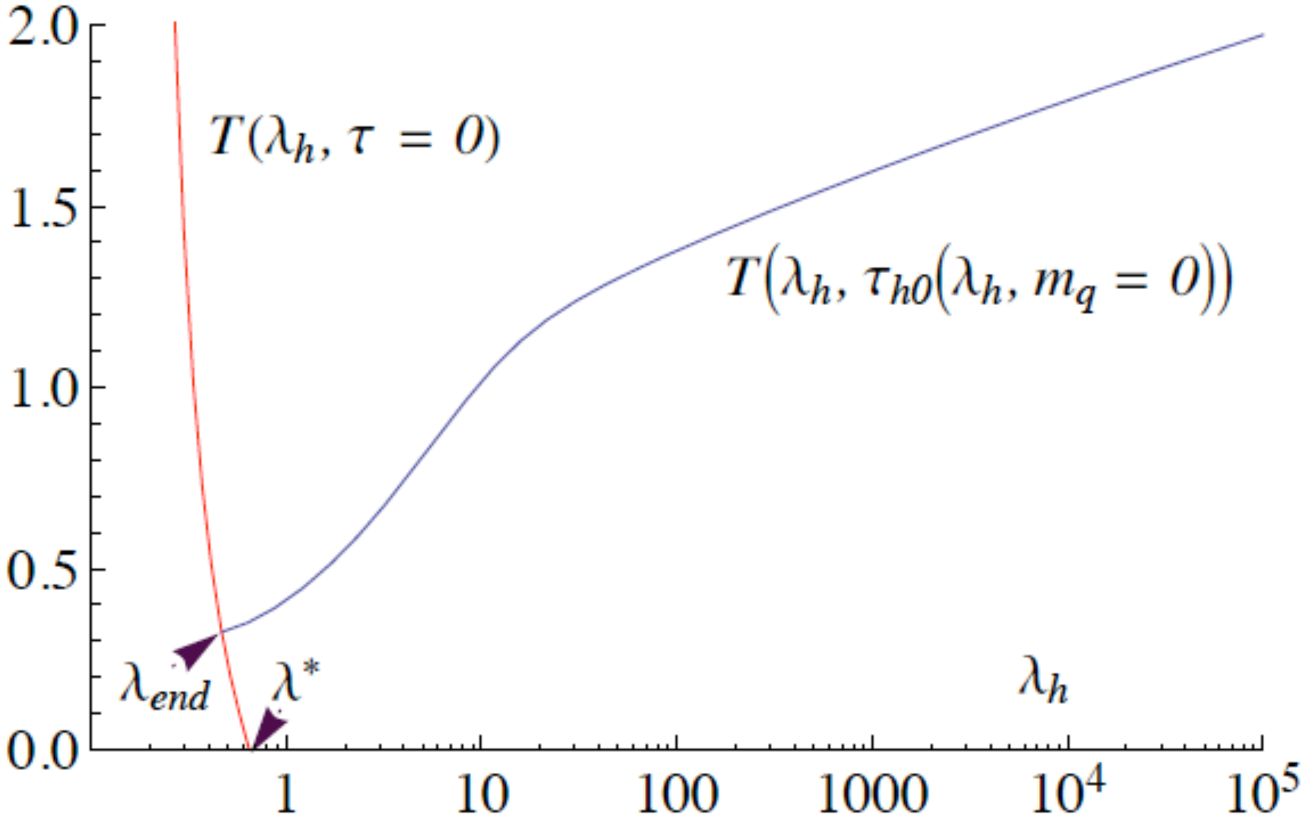
Phases: black hole solutions. Stable phase: one with smallest action.

The solutions are parametrized by $\{\lambda_h, \tau_h(\lambda_h)\}$

$$T = -\frac{1}{4\pi} f'(r_h), \quad s = \frac{1}{4G_5} b^3(r_h)$$

$$p_b(T) = \frac{1}{4G_5} \int_{\lambda_h(T)}^{\infty} d\lambda_h (-T_b'(\lambda_h))\, b_b^3(\lambda_h) + p_b(\infty)$$

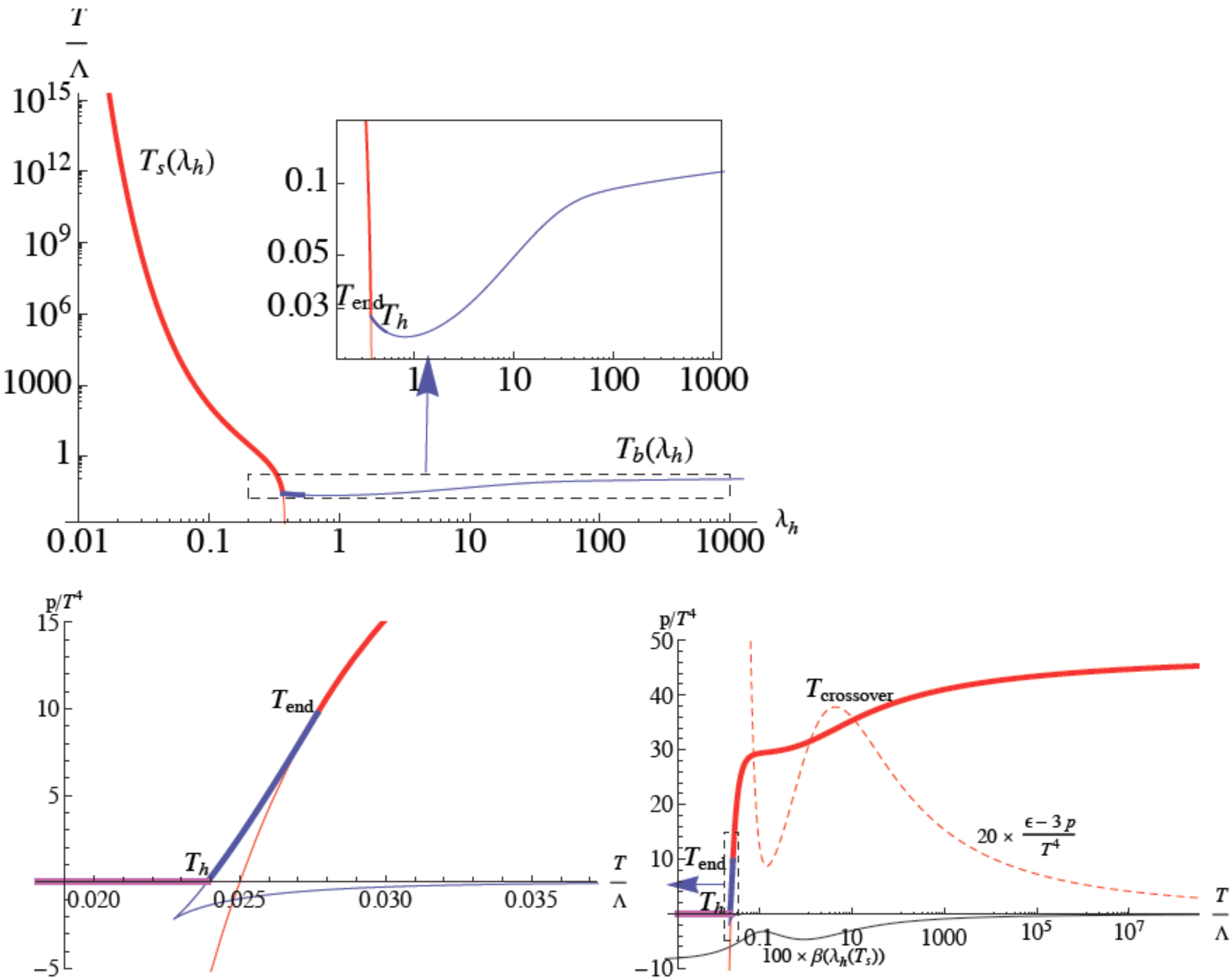
$$p_s(T) = \frac{1}{4G_5} \int_{\lambda_h(T)}^{\lambda_*} d\lambda_h (-T_s'(\lambda_h))\, b_s^3(\lambda_h) + p_s(\lambda_*)$$



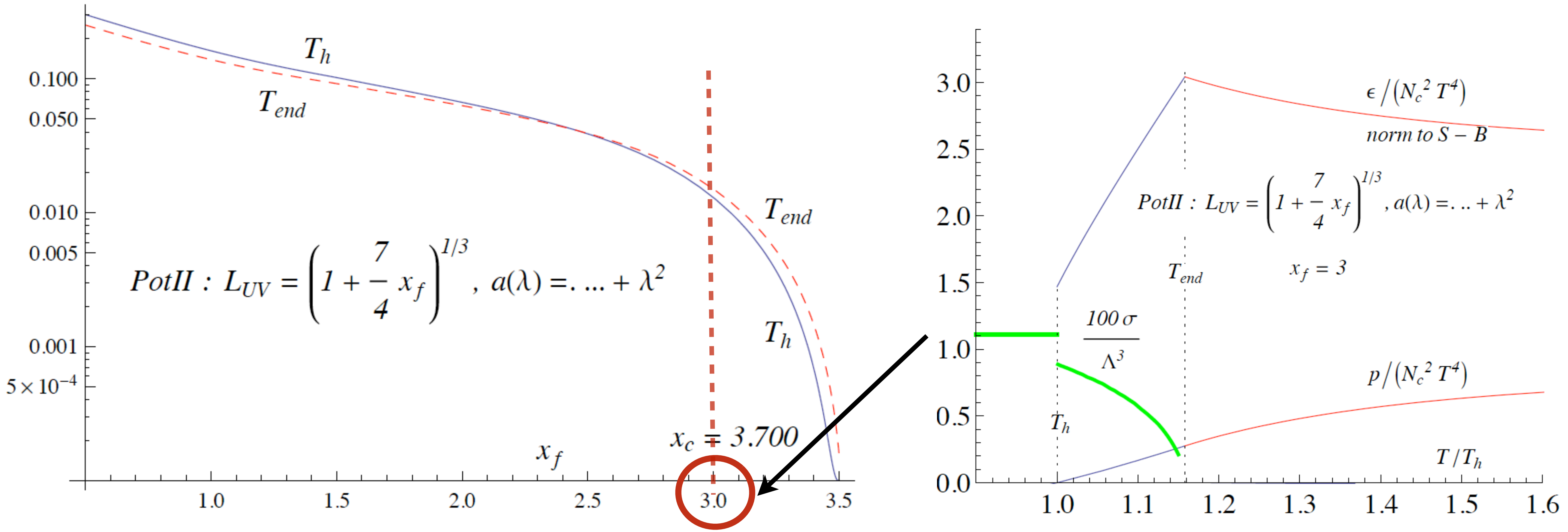
Two distinct branches of solutions:

$\tau = 0$

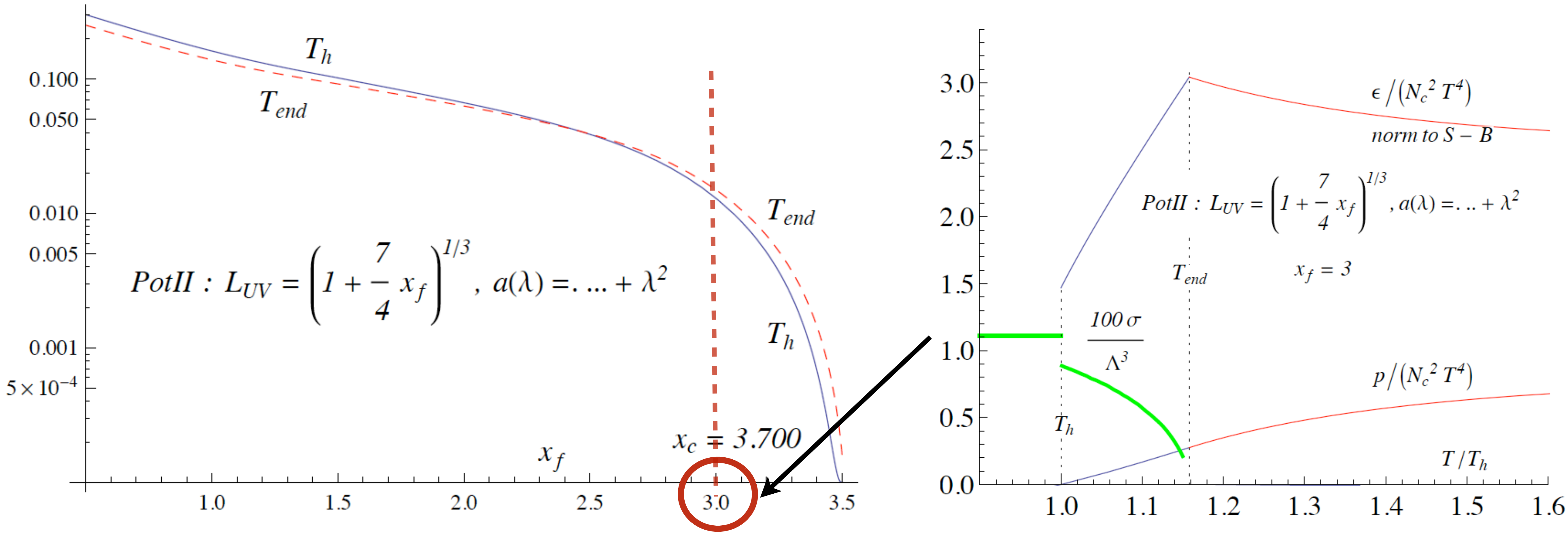
$\tau \neq 0$



$\frac{T}{\Lambda}$
10^{15}
10^{12}
10^{9}
10^{6}
1000
1
$T_s(\lambda_h)$
0.1
0.05
0.03
T_{end}
T_h
1
10
100
1000
$T_b(\lambda_h)$
0.01
0.1
1
10
100
1000
λ_h
p/T^4
15
10
5
−5
T_{end}
T_h
0.020
0.025
0.030
0.035
$\frac{T}{\Lambda}$
p/T^4
50
40
30
20
10
−10
$T_{\mathrm{crossover}}$
$20 \times \frac{\epsilon - 3p}{T^4}$
T_{end}
T_h
0.1
10
1000
10^5
10^7
$100 \times \beta(\lambda_h(T_s))$
$\frac{T}{\Lambda}$



T_h: Transition to hadron gas $\quad$ $T_{\rm end}$: $2^{\rm nd}$ order endpoint (chiral restoration)



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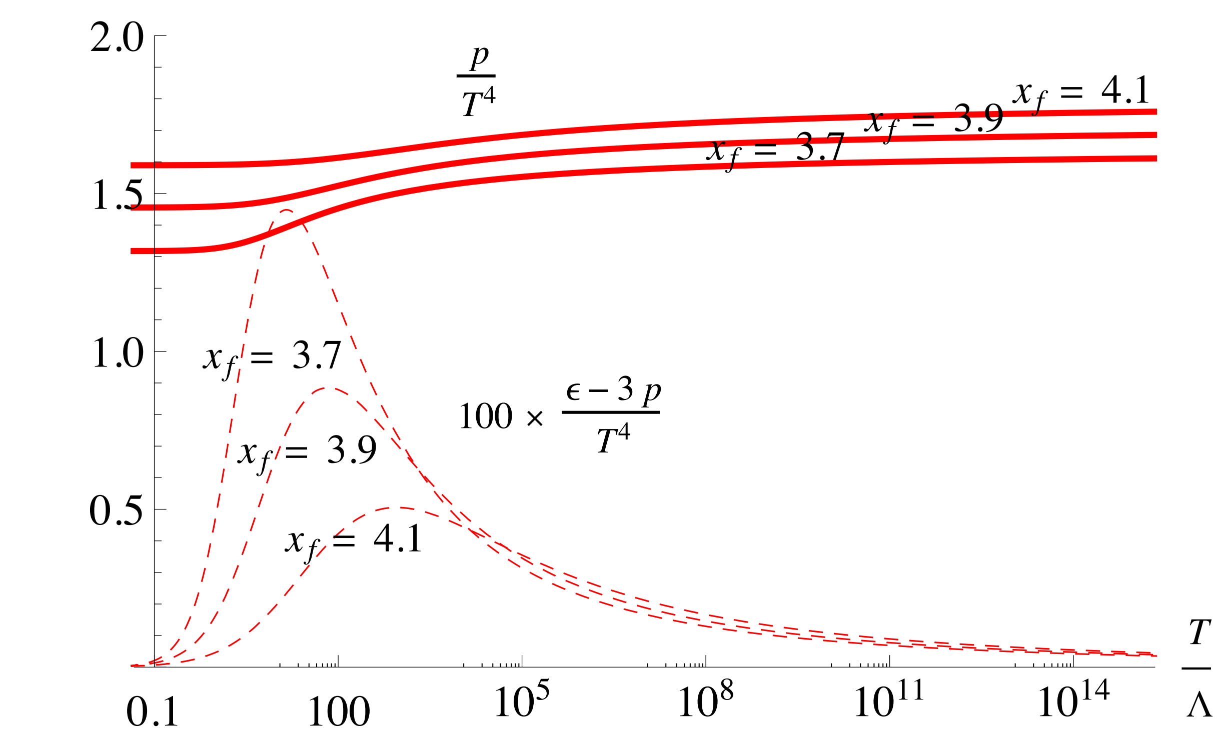
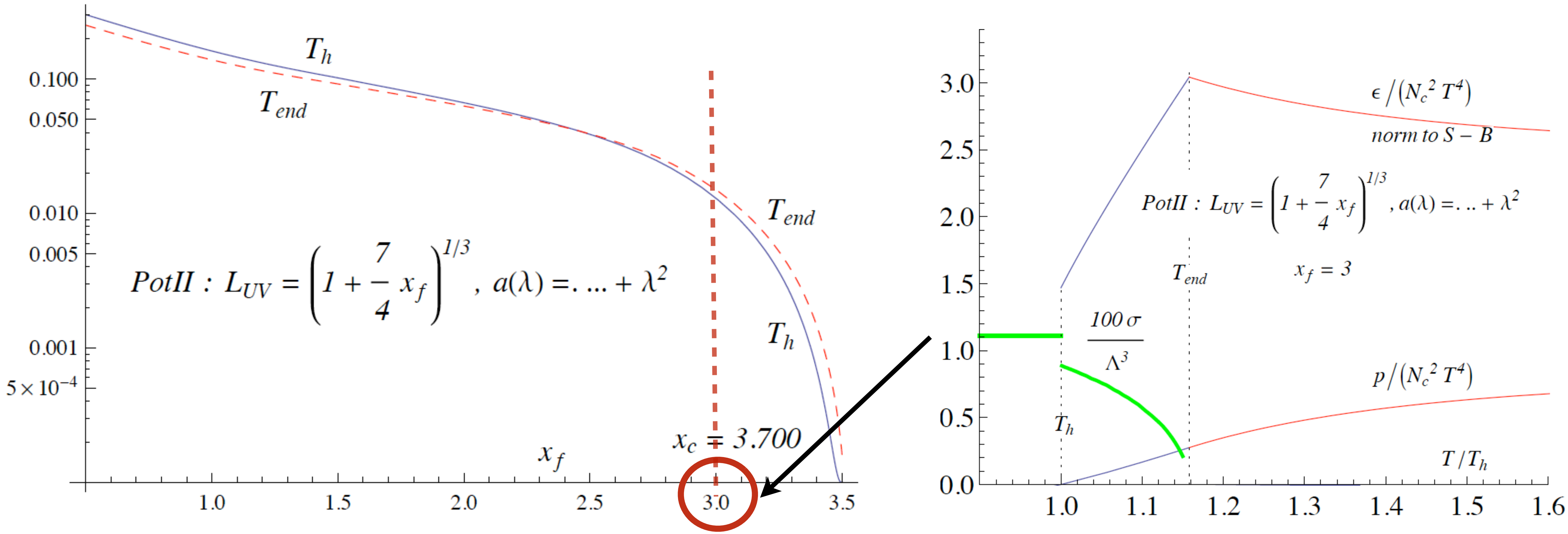
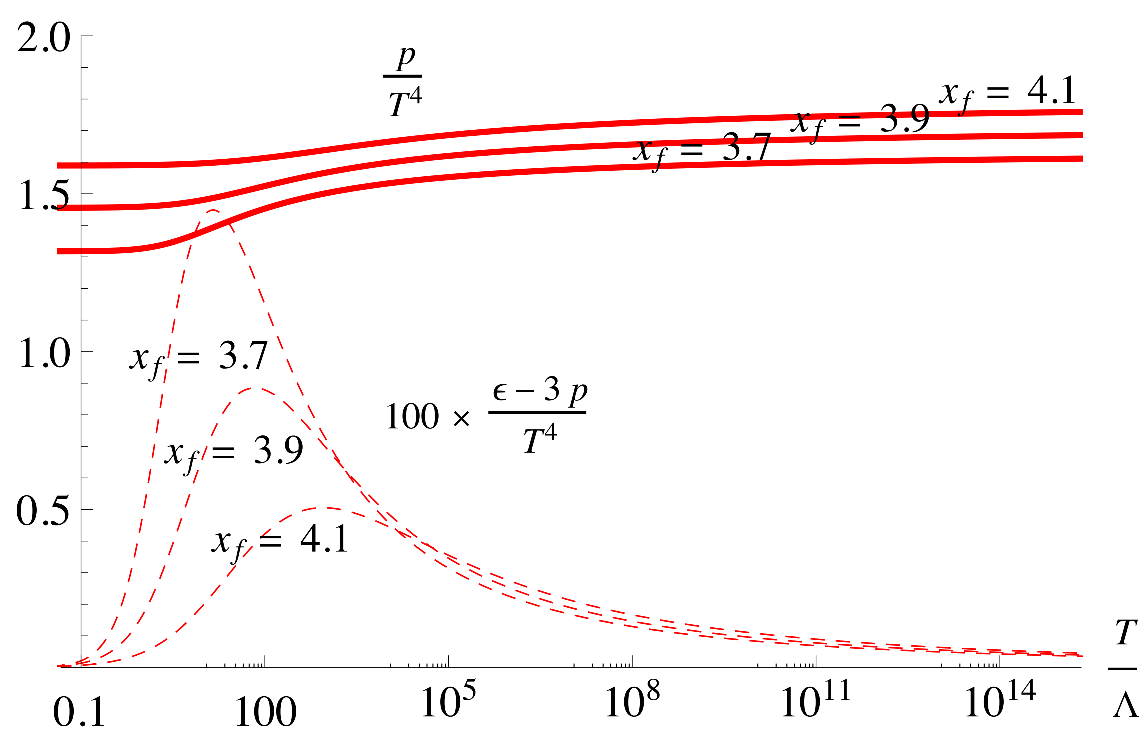
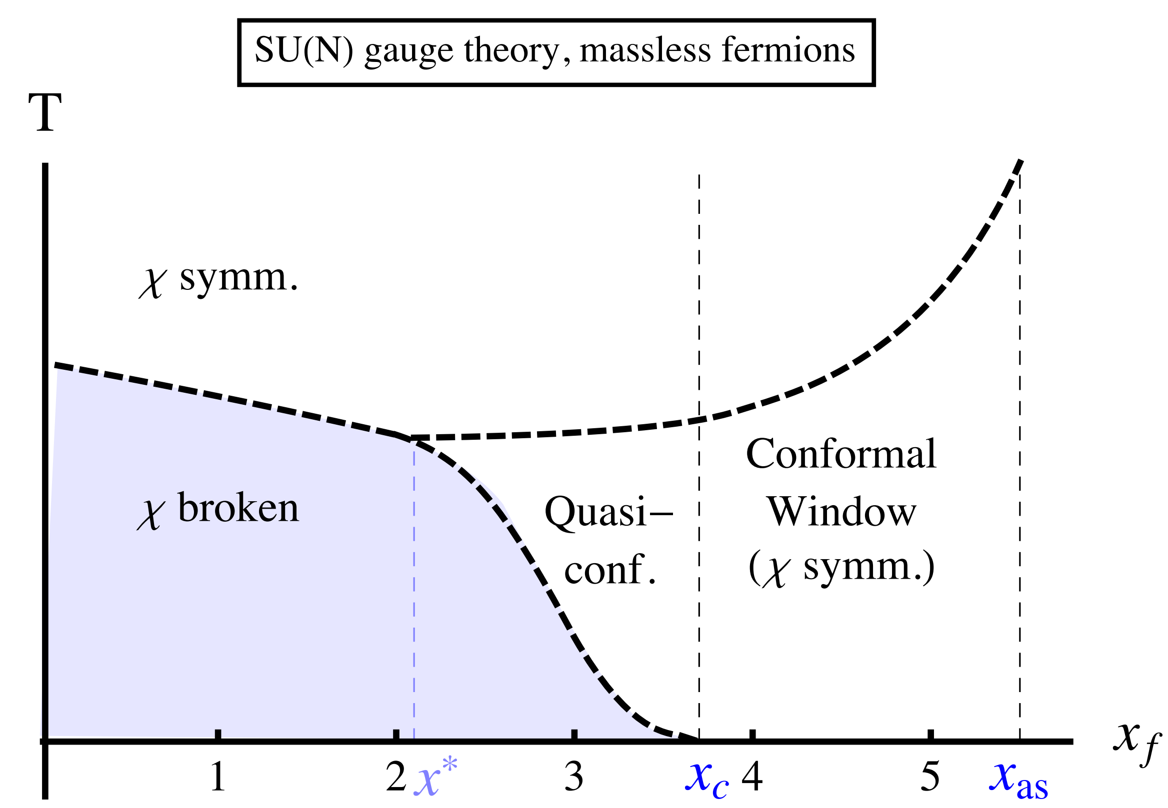




T_h: Transition to hadron gas $\quad T_{\mathrm{end}}$: 2$^{\mathrm{nd}}$ order endpoint (chiral restoration)

Inside the conformal window:





Conclusions

Unfolding strong dynamics: Lattice and holography.

Improving precision for of non-QCD like theories.

Refining holographic models.

Combine the two for coherent picture.