Studies on the UV to IR Evolution of Gauge Theories and Quasiconformal Behavior

Robert Shrock

Yale University, on leave from Stony Brook University

Strongly Coupled Gauge Theories in the LHC Perspective (SCGT) 12, Nagoya University, 2012.12.05

Outline

- Renormalization-group flow from UV to IR; types of IR behavior; role of an exact or approximate IR fixed point; conditions for approximately scale-invariant behavior

- Higher-loop calculations of UV to IR evolution, including IR zero of β and anomalous dimension γ_m of fermion bilinear

- Some comparisons with lattice measurements of γ_m

- Higher-loop calcs. of UV to IR evolution for supersymmetric gauge theory

- Study of scheme-dependence in calculation of IR fixed point

- Application to models of dynamical electroweak symmetry breaking

- Conclusions

Some new results covered in this talk are from the following recent papers by T. A. Ryttov and R. Shrock

- Phys. Rev. D 83, 056011 (2011), arXiv:1011.4542
- Phys. Rev. D 85, 076009 (2012), arXiv:1202.1297
- Phys. Rev. D 86, 065032 (2012), arXiv:1206.2366
- Phys. Rev. D 86, 085005 (2012), arXiv:1206.6895

as well as earlier related papers and some new results.

Renormalization-group Flow from UV to IR; Types of IR Behavior and Role of IR Fixed Point

Consider an asymptotically free, vectorial gauge theory with gauge group G and N_f massless fermions in representation R of G.

Asymptotic freedom $\Rightarrow$ theory is weakly coupled, properties are perturbatively calculable for large Euclidean momentum scale μ in deep ultraviolet (UV).

The question of how this theory behaves in the infrared (IR) is of fundamental field-theoretic significance and motivates a detailed study of the UV to IR evolution.

Results are relevant to models of dynamical electroweak symmetry breaking (discussed further below).

Denote running gauge coupling at scale μ as $g = g(\mu)$, and let $\alpha(\mu) = g(\mu)^2/(4\pi)$ and $a(\mu) = g(\mu)^2/(16\pi^2) = \alpha(\mu)/(4\pi)$.

As theory evolves from the UV to the IR, $\alpha(\mu)$ increases, governed by β function

$$\beta_\alpha \equiv \frac{d\alpha}{dt} = -2\alpha \sum_{\ell=1}^{\infty} b_\ell \, a^\ell = -2\alpha \sum_{\ell=1}^{\infty} \bar{b}_\ell \, \alpha^\ell \ ,$$

where $t = \ln \mu$, $\ell =$ loop order of the coeff. b_ℓ, and $\bar{b}_\ell = b_\ell/(4\pi)^\ell$.

Coeffs. b_1 and b_2 in β are indep. of regularization/renormalization scheme, while b_ℓ for $\ell \geq 3$ are scheme-dep.

Asymptotic freedom means $b_1 > 0$, so $\beta < 0$ for small $\alpha(\mu)$, in neighborhood of UV fixed point (UVFP) at $\alpha = 0$.

As the scale μ decreases from large values, $\alpha(\mu)$ increases. Denote α_{cr} (dependent on R) as minimum value for formation of bilinear fermion condensates and resultant spontaneous chiral symmetry breaking (SχSB).

There are two possibilities for the β function and resultant UV to IR evolution:

- There may not be any IR zero in β, so that as μ decreases, $\alpha(\mu)$ increases, eventually beyond the perturbatively calculable region. This is the case for QCD.
- β may have a zero at a certain value (closest to the origin) denoted α_{IR}, so that as μ decreases, $\alpha \to \alpha_{IR}$. In this class of theories, there are two further generic possibilities: $\alpha_{IR} < \alpha_{cr}$ or $\alpha_{IR} > \alpha_{cr}$.

If $\alpha_{IR} < \alpha_{cr}$, the zero of β at α_{IR} is an exact IR fixed point (IRFP) of the ren. group (RG); as $\mu \to 0$ and $\alpha \to \alpha_{IR}$, $\beta \to \beta(\alpha_{IR}) = 0$, and the theory becomes exactly scale-invariant with nontrivial anomalous dimensions (Caswell, Banks-Zaks).

If β has no IR zero, or an IR zero at $\alpha_{IR} > \alpha_{cr}$, then as μ decreases through a scale denoted Λ, $\alpha(\mu)$ exceeds α_{cr} and SχSB occurs - fermions gain dynamical masses $\sim \Lambda$ (e.g., light quarks gain constituent quark masses $\sim \Lambda_{QCD} \simeq 300$ MeV in QCD).

If SχSB occurs, then in low-energy effective field theory applicable for $\mu < \Lambda$, one integrates these fermions out, and β function becomes that of a pure gauge theory, which has no IR zero. Hence, if β has a zero at $\alpha_{IR} > \alpha_{cr}$, this is only an approx. IRFP of RG.

If $\alpha_{IR} > \alpha_{cr}$, effect of approx. IRFP at α_{IR} depends on how close it is to α_{cr}.

If α_{IR} is only slightly greater than α_{cr}, then, as $\alpha(\mu)$ approaches α_{IR}, since $\beta = d\alpha/dt \to 0$, $\alpha(\mu)$ varies very slowly as a function of the scale μ, i.e., there is approximately scale-invariant, i.e. dilatation-invariant or slow-running ("walking") behavior (Yamawaki et al.; Holdom; Appelquist, Wijewardhana...). For these theories, this is equivalent to quasiconformal behavior.

Denote Λ_* as scale μ where $\alpha(\mu)$ grows to O(1) (with Λ the scale where SχSB occurs). In the slow-running case, $\Lambda << \Lambda_*$. The approx. dilatation symmetry applies in this interval $\Lambda < \mu < \Lambda_*$.

The SχSB and attendant fermion mass generation at Λ spontaneously break the approximate dilatation symmetry, plausibly leading to a resultant light Nambu-Goldstone boson, the dilaton (dilaton mass estimates vary, see below). The dilaton is not massless, because β is not exactly zero for $\alpha(\mu) \neq \alpha_{IR}$.

At two-loop (2ℓ) level, $\beta = -[\alpha^2/(2\pi)](b_1 + b_2 a)$, so condition for an IR zero in β is $b_1 + b_2 a = 0$, i.e.,

$$\alpha_{IR,2\ell} = -\frac{4\pi b_1}{b_2}$$

which is physical for $b_2 < 0$. One-loop coefficient b_1 is

$$b_1 = \frac{1}{3}(11 C_A - 4 N_f T_f)$$

(Gross, Wilczek; Politzer), where $C_A \equiv C_2(G)$ is the quadratic Casimir invariant, and $T_f \equiv T(R)$ is the trace invariant. We focus here on $G = \mathrm{SU}(N)$.

As N_f increases, b_1 decreases and vanishes at

$$N_{f,b1z} = \frac{11 C_A}{4 T_f}$$

Hence, for asymp. freedom, require $N_f < N_{f,b1z}$; for fund. rep., this is $N_f < (11/2)N$.

Two-loop coeff. b_2 is

$$b_2 = \frac{1}{3}\left[34C_A^2 - 4(5C_A + 3C_f)N_f\, T_f\right]$$

(Caswell, Jones). For small N_f, $b_2 > 0$; b_2 decreases with increasing N_f and vanishes with sign reversal at $N_f = N_{f,b2z}$, where

$$N_{f,b2z} = \frac{34C_A^2}{4T_f(5C_A + 3C_f)} \ .$$

For arbitrary G and R, $N_{f,b2z} < N_{f,b1z}$, so there is always an interval in N_f for which β has an IR zero, namely

$$I: \quad N_{f,b2z} < N_f < N_{f,b1z}$$

If $R =$ fund. rep., then

$$I: \quad \frac{34N^3}{13N^2 - 3} < N_f < \frac{11N}{2}$$

For example, for $N = 2$, this is $5.55 < N_f < 11$, and for $N = 3$, $8.05 < N_f < 16.5$. (Here, we evaluate these expressions as real numbers, but understand that physical values of N_f are nonnegative integers.)

As $N \to \infty$, interval I is $2.62N < N_f < 4.5N$.

For N_f near lower end of I, $b_2 \to 0$ and $\alpha_{IR,2\ell}$ is too large for calc. to be reliable.

In interval I, α_{IR} is a decreasing fn. of N_f. As N_f decreases below $N_{f,b1z}$ where $b_1 = 0$, α_{IR} increases from 0. As N_f decreases to a value $N_{f,cr}$, α_{IR} increases to α_{cr}, so

$$N_f = N_{f,cr} \quad \text{at} \quad \alpha_{IR} = \alpha_{cr}$$

The value of $N_{f,cr}$ is of fundamental importance in the study of a non-Abelian gauge theory, since it separates two different regimes of IR behavior, viz., an IR conformal phase with no SχSB and an IR phase with SχSB.

$N_{f,cr}$ is not exactly known. To obtain $N_{f,cr}$ for a given gauge group, we need, calcs. of α_{IR} as fn. of N_f and estimate of α_{cr}. To estimate α_{cr}, analyze Schwinger-Dyson (SD) eq. for fermion propagator. For $\alpha > \alpha_{cr}$, this yields a nonzero sol. for a dynamically generated fermion mass.

Ladder approach to SD eq. yields $\alpha_{cr} C_2(R) \simeq 1$. Given the strong-coupling involved, this is only rough estimate. Combining est. of α_{cr} from ladder approx. to SD eq. with 2-loop calc. of $\alpha_{IR} \equiv \alpha_{IR,2\ell}$ yields $N_{f,cr} \simeq 4N$.

Lattice gauge simulations are promising way to determine $N_{f,cr}$ and measurement of anomalous dimension $\gamma \equiv \gamma_m$ describing running of m and bilinear operator, $\bar{F}F$ as fn. of $\ln \mu$.

Higher-Loop Corrections to UV $\longrightarrow$ IR Evolution of Gauge Theories

Because of the strong-coupling nature of the physics at an approximate IRFP, with $\alpha \sim O(1)$, there are significant higher-order corrections to results obtained from the two-loop β function.

This motivates calculation of location of IR zero in β, α_{IR}, and resultant value of γ evaluated at α_{IR} to higher-loop order. We have done this to 3-loop and 4-loop order in Ryttov and Shrock, PRD 83, 056011 (2011), arXiv:1011.4542; see also Pica and Sannino, PRD 83,035013 (2011), arXiv:1011.5917.

Although coeffs. in β at $\ell \geq 3$ loop order are scheme-dependent, results give a measure of accuracy of the 2-loop calc. of the IR zero, and similarly with the value of γ evaluated at this IR zero.

We use $\overline{MS}$ scheme, for which coeffs. of β and γ have been calculated to 4-loop order by Vermaseren, Larin, and van Ritbergen. The value of this sort of higher-loop calcululation using $\overline{MS}$ scheme is demonstrated by the excellent fit of the four-loop $\alpha_s(\mu)$ to data as function of $\mu^2 = Q^2$ in QCD (cf. Bethke).

For 3-loop analysis, we need

$$b_3 = \frac{2857}{54}C_A^3 + T_f N_f \left[2C_f^2 - \frac{205}{9}C_A C_f - \frac{1415}{27}C_A^2 \right] + (T_f N_f)^2 \left[\frac{44}{9}C_f + \frac{158}{27}C_A \right]$$

Coeff. b_3 is quadratic fn. of N_f and vanishes, with sign reversal, at two values of N_f, denoted $N_{f,b3z,1}$ and $N_{f,b3z,2}$. $b_3 > 0$ for small N_f and vanishes first at $N_{f,b3z,1}$, which is smaller than $N_{f,b2z}$, the left endpoint of interval I. Furthermore, $N_{f,b3z,2} > N_{f,b1z}$, the right endpoint of interval I. For example,

$$\text{for} \quad N = 2, \quad N_{f,b3z,1} = 3.99 < N_{f,b2z} = 5.55$$

$$N_{f,b3z,2} = 27.6 > N_{f,b1z} = 11$$

$$\text{for} \quad N = 3, \quad N_{f,b3z,1} = 5.84 < N_{f,b2z} = 8.05$$

$$N_{f,b3z,2} = 40.6 > N_{f,b1z} = 16.5$$

Hence, $b_3 < 0$ in interval I of interest for IR zero of β.

At this 3-loop level,

$$\beta = -\frac{\alpha^2}{2\pi}(b_1 + b_2 a + b_3 a^2)$$

so $\beta = 0$ away from $\alpha = 0$ at two values,

$$\alpha = \frac{2\pi}{b_3}\Big(- b_2 \pm \sqrt{b_2^2 - 4b_1 b_3}\ \Big)$$

Since $b_2 < 0$ and $b_3 < 0$, this is

$$\alpha = \frac{2\pi}{|b_3|}\Big(- |b_2| \mp \sqrt{b_2^2 + 4b_1|b_3|}\ \Big)$$

One of these solutions is negative and hence unphysical; the other is manifestly positive, and is $\alpha_{IR,3\ell}$. Note that if a scheme had $b_3 > 0$ in I, since $b_2 \longrightarrow 0$ at lower end of I, $b_2^2 - 4b_1b_3 < 0$, so this scheme would not have a physical $\alpha_{IR,3\ell}$ in this region.

We find that for any fermion rep. R for which β has a 2-loop IR zero, the value of the IR zero decreases when calculated at the 3-loop level, i.e.,

$$\alpha_{IR,2\ell} > \alpha_{IR,3\ell}$$

Proof:

$$\alpha_{IR,2\ell} - \alpha_{IR,3\ell} = \frac{4\pi b_1}{|b_2|} - \frac{2\pi}{|b_3|}\left(-|b_2| + \sqrt{b_2^2 + 4b_1|b_3|}\ \right)$$
$$= \frac{2\pi}{|b_2 b_3|}\left[2b_1|b_3| + b_2^2 - |b_2|\sqrt{b_2^2 + 4b_1|b_3|}\ \right]$$

The expression in square brackets is positive if and only if

$$(2b_1|b_3| + b_2^2)^2 - b_2^2(b_2^2 + 4b_1|b_3|) > 0$$

This difference is equal to the positive-definite quantity $4b_1^2 b_3^2$, which proves the inequality.

For 4-loop analysis, we use b_4, which is cubic polyn. in N_f. It is positive for $N_f \in I$ for $N = 2, 3$ but is negative in part of I for higher N.

The 4-loop β function is $\beta = -[\alpha^2/(2\pi)](b_1 + b_2 a + b_3 a^2 + b_4 a^3)$, so β has three zeros away from the origin. We determine the smallest positive real zero as $\alpha_{IR,4\ell}$.

We find

- As noted, when one goes from 2-loop level to 3-loop level, there is a decrease in the value of the IR zero of β

- Going from 3-loop to 4-loop level, there is a slight change in the value of the IR zero, but this change is smaller than the decrease from 2-loops to 3-loops, so $\alpha_{IR,4\ell} < \alpha_{IR,2\ell}$.

- Fractional changes in the value of the IR zero of β decrease in magnitude as N_f increases toward its maximum, $N_{f,b1z}$, and all of the values of $\alpha_{IR,n\ell} \to 0$.

Our finding that the fractional change in the location of the IR zero of β is reduced at higher-loop order agrees with the general expectation that calculating a quantity to higher order in perturbation theory should give a more stable and accurate result.

Since $\alpha_{cr} \sim O(1)$ for SχSB, the decrease in α_{IR} at higher-loop order, together with the property that α_{IR} increases as N_f decreases, means that

- one must go to smaller N_f for $\alpha_{IR,n\ell}$ to grow to a given size for $n = 3$ and $n = 4$ loop level as compared with $n = 2$ loop level; since α_{IR} must exceed a given size, α_{cr}, for SχSB, this implies that
- the actual lower boundary of the IR-conformal phase could lie somewhat below the old estimate that $N_{f,cr} \simeq 4N$ from the 2-loop $\alpha_{IR,2\ell}$ plus SD eq.

For example, in the case $N = 3$, these results suggest that the lower boundary of IR-conformal phase could lie somewhat below $4N = 12$.

Some numerical values of $\alpha_{IR,n\ell}$ at the 2-loop, 3-loop, and 4-loop level for fermions in fund. rep., $N_f \in I$, and illustrative groups $G = \mathrm{SU}(2)$ and $G = \mathrm{SU}(3)$:

N	N_f	$\alpha_{IR,2\ell}$	$\alpha_{IR,3\ell}$	$\alpha_{IR,4\ell}$
2	7	2.83	1.05	1.21
2	8	1.26	0.688	0.760
2	9	0.595	0.418	0.444
2	10	0.231	0.196	0.200
3	10	2.21	0.764	0.815
3	11	1.23	0.578	0.626
3	12	0.754	0.435	0.470
3	13	0.468	0.317	0.337
3	14	0.278	0.215	0.224
3	15	0.143	0.123	0.126
3	16	0.0416	0.0397	0.0398

(For N_f values sufficiently close to $N_{f,b2z}$, $\alpha_{IR,n\ell}$ is so large that perturb. calc. not reliable; these are omitted.)

We have performed the corresponding higher-loop calculations for SU(N) gauge theories with N_f fermions in the adjoint, symmetric and antisymmetric rank-2 tensor representations. The general result $\alpha_{IR,3\ell} < \alpha_{IR,2\ell}$ applies. The difference $\alpha_{IR,4\ell} - \alpha_{IR,3\ell}$ tends to be relatively small, but can have either sign.

For example, for $R =$ adjoint, $N_{f,b1z} = 11/4$ and $N_{f,b2z} = 17/16$ (indep. of N), so interval I where β has an IR zero, viz., $N_{f,b2z} < N_f < N_{f,b1z}$, is $1.06 < N_f < 2.75$, which includes only one physical, integral value, $N_f = 2$. For this value of N_f and some illustrative values of N, the results are:

N	$\alpha_{IR,2\ell,adj}$	$\alpha_{IR,3\ell,adj}$	$\alpha_{IR,4\ell,adj}$
2	0.628	0.459	0.450
3	0.419	0.306	0.308
4	0.314	0.2295	0.234

The anomalous dimension $\gamma_m \equiv \gamma$ for the fermion bilinear operator is

$$\gamma = \sum_{\ell=1}^{\infty} c_\ell a^\ell = \sum_{\ell=1}^{\infty} \bar{c}_\ell \alpha^\ell$$

where $\bar{c}_\ell = c_\ell/(4\pi)^\ell$ is the ℓ-loop coeff. The one-loop coeff. c_1 is scheme-independent, the c_ℓ with $\ell \geq 2$ are scheme-dependent, and the c_ℓ have been calculated up to 4-loop level (Vermaseren, Larin, van Ritbergen):

$$c_1 = 6C_f$$

$$c_2 = 2C_f\Big[\frac{3}{2}C_f + \frac{97}{6}C_A - \frac{10}{3}T_f N_f\Big]$$

$$c_3 = 2C_f\Bigg[\frac{129}{2}C_f^2 - \frac{129}{4}C_f C_A + \frac{11413}{108}C_A^2 + C_f T_f N_f(-46 + 48\zeta(3)) - C_A T_f N_f(\frac{556}{27} + 48\zeta(3)) - \frac{140}{27}(T_f N_f)^2\Bigg]$$

and similarly for c_4.

It is of interest to calculate γ at the exact IRFP in IR-conformal phase and the approx. IRFP in phase with SχSB.

We denote γ calculated to n-loop ($n\ell$) level as $\gamma_{n\ell}$ and, evaluated at the n-loop value of the IR zero of β, as

$$\gamma_{IR,n\ell} \equiv \gamma_{n\ell}(\alpha = \alpha_{IR,n\ell})$$

N.B.: In the IR conformal phase, an all-order calc. of γ evaluated at an all-order calc. of α_{IR} would be an exact property of the theory, but in the broken phase, just as the IR zero of β is only an approximate IRFP, so also, the γ is only approx., describing the running of $\bar{\psi}\psi$ and the dynamically generated fermion mass near the zero of β:

$$\Sigma(k) \sim \Lambda\Big(\frac{\Lambda}{k}\Big)^{2-\gamma}$$

In both phases, γ is bounded above as $\gamma < 2$. At the 2-loop level we calculate $\gamma_{IR,2\ell} =$

$$\frac{C_f(11C_A - 4T_fN_f)[455C_A^2 + 99C_AC_f + (180C_f - 248C_A)T_fN_f + 80(T_fN_f)^2]}{12[-17C_A^2 + 2(5C_A + 3C_f)T_fN_f]^2}$$

Our analytic expressions for $\gamma_{IR,n\ell}$ at the 3-loop and 4-loop level are too complicated to list here. Illustrative numerical values of $\gamma_{IR,n\ell}$ at the 2-, 3-, and 4-loop level are given below for fermions in the fund. rep. and for the illustrative values $N = 2, \ 3$.

N	N_f	$\gamma_{IR,2\ell}$	$\gamma_{IR,3\ell}$	$\gamma_{IR,4\ell}$
2	7	(2.67)	0.457	0.0325
2	8	0.752	0.272	0.204
2	9	0.275	0.161	0.157
2	10	0.0910	0.0738	0.0748
3	10	(4.19)	0.647	0.156
3	11	1.61	0.439	0.250
3	12	0.773	0.312	0.253
3	13	0.404	0.220	0.210
3	14	0.212	0.146	0.147
3	15	0.0997	0.0826	0.0836
3	16	0.0272	0.0258	0.0259

(Two-loop values in parentheses for N_f are unphysically large, reflect inadequacy of lowest-order perturb. calc. if α too large.)

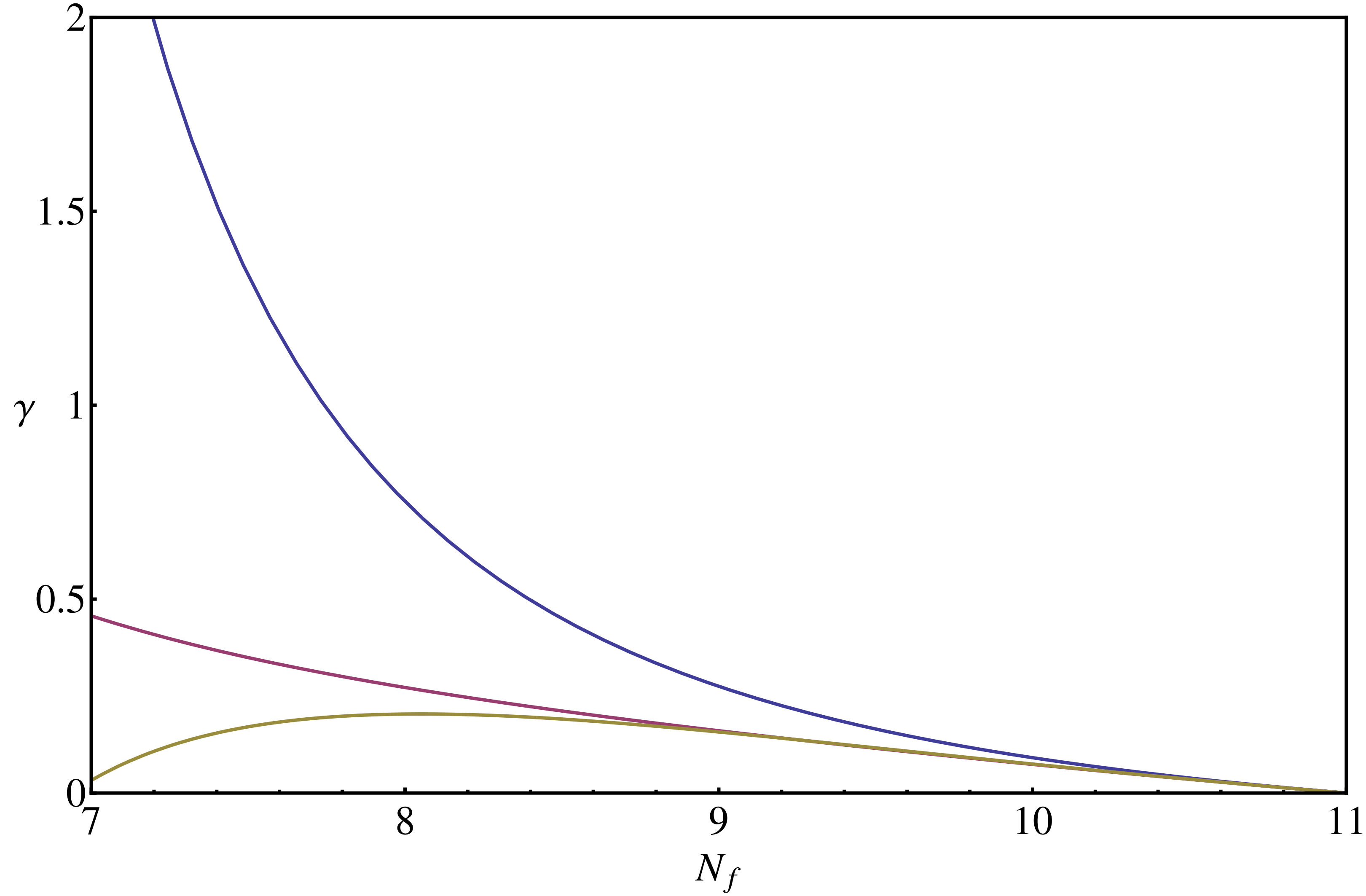

Figure 1: Anomalous dimension $\boldsymbol{\gamma_m \equiv \gamma}$ for SU(2) for $\boldsymbol{N_f}$ fermions in the fundamental representation; (i) blue: $\boldsymbol{\gamma_{IR,2\ell}}$; (ii) red: $\boldsymbol{\gamma_{IR,3\ell}}$; (iii) brown: $\boldsymbol{\gamma_{IR,4\ell}}$.

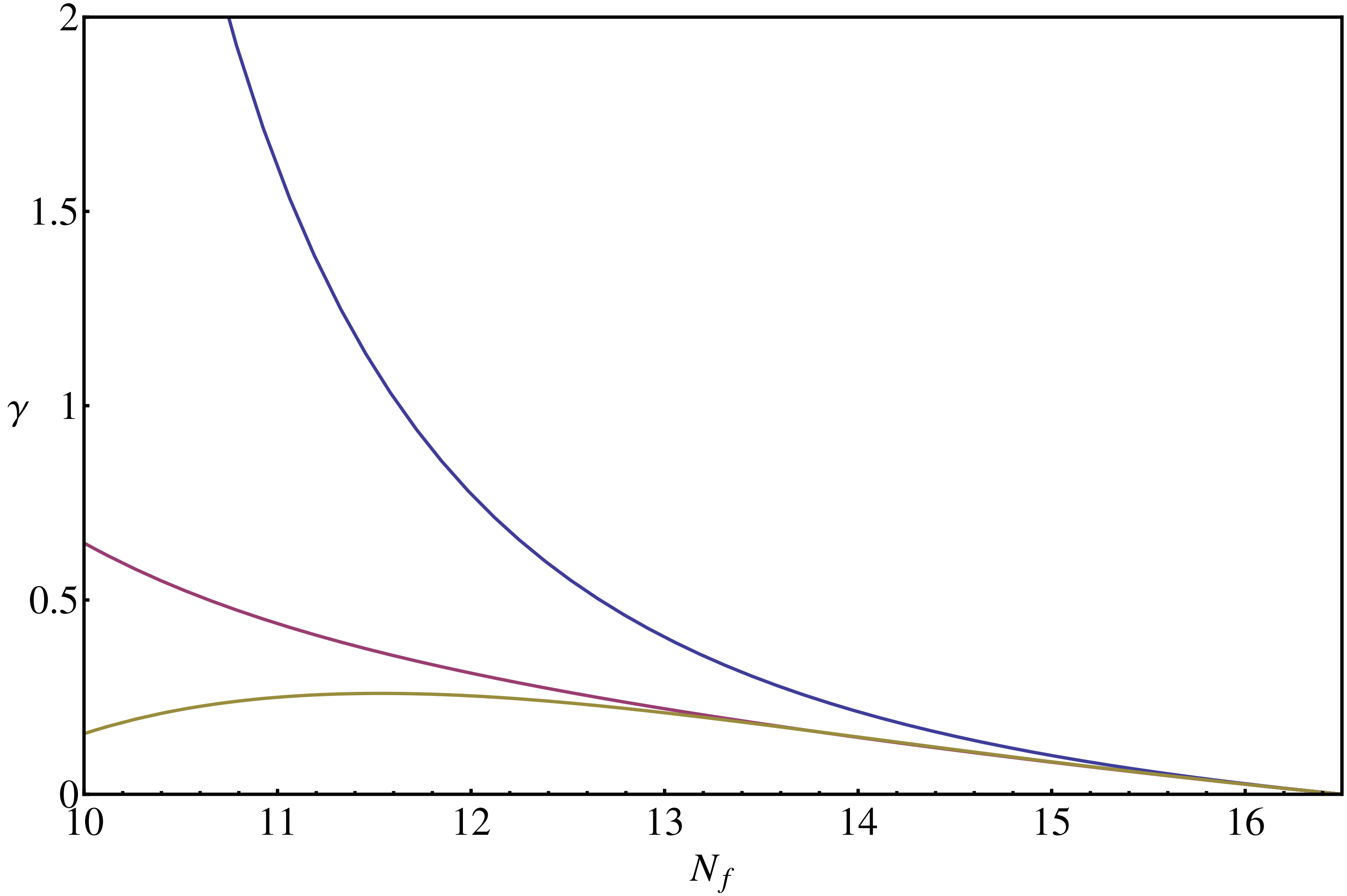

Figure 2: Anomalous dimension $\boldsymbol{\gamma_m \equiv \gamma}$ for SU(3) for $\boldsymbol{N_f}$ fermions in the fundamental representation; (i) blue: $\boldsymbol{\gamma_{IR,2\ell}}$; (ii) red: $\boldsymbol{\gamma_{IR,3\ell}}$; (iii) brown: $\boldsymbol{\gamma_{IR,4\ell}}$.

We have also performed these higher-loop calculations for higher fermion reps. R. In general, we find that, for a given N, R, and N_f, the values of $\gamma_{IR,n\ell}$ calculated to 3-loop and 4-loop order are smaller than the 2-loop value.

The value of these higher-loop calcs. to 3-loop and 4-loop order is evident from the figures. A necessary condition for a perturb. calc. to be reliable is that higher-order contribs. do not modify the result too much. One sees from the tables and figures that, especially for smaller N_f, there is a substantial decrease in $\alpha_{IR,n\ell}$ and $\gamma_{IR,n\ell}$ when one goes from 2-loop to 3-loop order, but for a reasonable range of N_f, the 3-loop and 4-loop results are close to each other.

Thus, our higher-loop calcs. of α_{IR} and γ allow us to probe the theory reliably down to smaller values of N_f and thus stronger couplings.

Some Comparisons with Lattice Measurements

For SU(3) with $N_f = 12$, from table above,

$$\gamma_{IR,2\ell} = 0.77, \qquad \gamma_{IR,3\ell} = 0.31, \qquad \gamma_{IR,4\ell} = 0.25$$

Some lattice results (N.B.: some error estimates do not include all syst. uncertainties)

$\gamma = 0.414 \pm 0.016$ (Appelquist, Fleming, Lin, Neil, Schaich, PRD 84, 054501 (2011), arXiv:1106.2148, analyzing data of Kuti et al., PLB 703, 348 (2011), arXiv:1104.3124, inferring conformality [Kuti et al. find SχSB])

$\gamma \sim 0.35$ (DeGrand, PRD 84, 116901 (2011), arXiv:1109.1237, also analyzing data of Kuti et al., finding conformality)

$0.2 \lesssim \gamma \lesssim 0.4$ (Fodor, Holland, Kuti, Nogradi, Schroeder, Wong (method-dep.), arXiv:1205.1878, arXiv:1211.3548, 1211.6164, finding SχSB)

$\gamma = 0.4 - 0.5$ (Y. Aoki et al., (LatKMI) PRD 86, 054506 (2012), arXiv:1207.3060, finding IR-conformality)

$\gamma = 0.27 \pm 0.03$ (Hasenfratz, Cheng, Petropoulos, Schaich, arXiv:1207.7162, finding IR-conformality)

So here the 2-loop value is larger than, and the 3-loop and 4-loop values closer to, these lattice measurements. Thus, our higher-loop calcs. of γ yield better agreement with these lattice measurements than two-loop calc.

This SU(3) theory with $N_f = 12$ fermions in fund. rep. was found to be in the IR-conformal phase by Appelquist et al. (PRL, 100, 171607 (2008)); other studies by Deuzeman, Lombardo, Pallante; Hasenfratz et al.; Degrand et al.; Aoki et al. also find IR-conformality, while Kuti et al. and Jin and Mawhinney find SχSB.

For SU(3) with $N_f = 10$ fermions in fund. rep., Appelquist et al., LSD Collab., arXiv:1204.6000 get $\gamma_{IR} \sim 1$, consistent with idea that $\gamma_{IR} \simeq 1$ at lower end of IR-conformal phase. Also LATKMI get $\gamma \simeq 1$ for SU(3) with $N_f = 8$.

Similar comparisons can be carried out for SU(2) with N_f fermions in fund. rep. Lattice studies indicate that for SU(2), $N_f = 10$ is in IR-conformal phase and $N_f = 4$ is in SχSB phase; $N_f = 6,\ 8$ are also being considered, e.g., Bursa et al., PRD 84, 034506 (2011), arXiv:1104.4301; Karavirta, Rantaharju, Rummukainen, Tuominen, JHEP 1205, 003 (2012), arXiv:1111.4104; Hayakawa, Ishikawa, Osaki, Takeda, Yamada, arXiv:1210.4985; G. Voronov and LSD Collab., in progress.

Our results for some higher fermion reps.: For $R =$ adj. rep., interval I contains only the integer $N_f = 2$. For this we get

N	$\gamma_{IR,2\ell,adj}$	$\gamma_{IR,3\ell,adj}$	$\gamma_{IR,4\ell,adj}$
2	0.820	0.543	0.500
3	0.820	0.543	0.523
4	0.820	0.543	0.532

For SU(2) with $N_f = 2$ fermions in the adjoint rep., lattice results include (N.B.: various groups quote uncertainties differently):

$\gamma = 0.31 \pm 0.06$ DeGrand, Shamir, Svetitsky, PRD 83, 074507 (2011), arXiv:1102.2843

$\gamma = 0.17 \pm 0.05$ (Appelquist et al., PRD 84, 054501 (2011) (analyzing data of Bursa, Del Debbio et al.), arXiv:1106.2148)

$-0.6 < \gamma < 0.6$ (Catterall, Del Debbio, Giedt, Keegan, PRD 85, 094501 (2012), arXiv:1108.3794)

Case of SU(N) with fermions in symmetric rank-2 tensor rep. (for SU(2), this is equiv. to adjoint rep.) Here,

$$N_{f,b1z} = \frac{11N}{2(N+2)} , \quad N_{f,b2z} = \frac{17N^2}{(N+2)(8N+3-6N^{-1})}$$

and interval I is $N_{f,b2z} < N_f < N_{f,b1z}$;

$$N = 3 : \quad 1.22 < N_f < 3.30 , \quad \Longrightarrow \quad N_f = 2, \ 3$$

$$N = 4 : \quad 1.35 < N_f < 3.67 , \quad \Longrightarrow \quad N_f = 2, \ 3$$

(as $N \to \infty$, $2.125 < N_f < 4.5, \quad \Longrightarrow \quad N_f = 3, \ 4$).

Analytic expressions are given in our paper; here, only list numerical values.

N	N_f	$\alpha_{IR,2\ell,S2}$	$\alpha_{IR,3\ell,S2}$	$\alpha_{IR,4\ell,S2}$
3	2	0.842	0.500	0.470
3	3	0.085	0.079	0.079
4	2	0.967	0.485	0.440
4	3	0.152	0.129	0.131

N	N_f	$\gamma_{IR,2\ell,S2}$	$\gamma_{IR,3\ell,S2}$	$\gamma_{IR,4\ell,S2}$
3	2	(2.44)	1.28	1.12
3	3	0.144	0.133	0.133
4	2	(4.82)	(2.08)	1.79
4	3	0.381	0.313	0.315

Some lattice results for $N_f = 2$ fermions in this symmetric rank-2 tensor rep.:

e.g., SU(3), $N_f = 2$: here, need to resolve a difference between two groups on the presence or absence of SχSB and value of γ before comparison with our continuum higher-loop calcs.

$\gamma \lesssim 0.45$ (Degrand, Shamir, Svetitsky, arXiv:1201.0935, find IR-conformality)

$\gamma \sim 1$ (method-dep.) (Fodor, Holland, Kuti, et al., arXiv:1205.1878, arXiv:1211.3548, finding SχSB)

Higher-Loop Calculations of UV to IR Evolution for an $\mathcal{N} = 1$ Supersymmetric Gauge Theory

It is of interest to carry out a similar analysis in an asymptotically free $\mathcal{N} = 1$ supersymmetric gauge theory with vectorial chiral superfield content $\Phi_i, \ \tilde{\Phi}_i$, $i = 1, ..., N_f$ in the $R, \ \bar{R}$ reps., respectively.

We have done this in Ryttov and Shrock, Phys. Rev. D 85, 076009 (2012), arXiv:1202.1297.

An appeal of this analysis: exact results on the IR properties of the theory are known from work of Seiberg (1994).

One goal of this study: to compare results from higher-loop perturb. calcs. with exact results, in particular, for $N_{f,cr}$.

1-loop and 2-loop coeffs. in β function (Jones), which are scheme-indep. :

$$b_1 = 3C_A - 2T_f N_f$$

$$b_2 = 6C_A^2 - 4T_f N_f (C_A + 2C_f)$$

To maintain asympt. freedom, require $b_1 > 0$ and hence

$$N_f < N_{f,b1z} = \frac{3C_A}{2T_f}$$

For $R =$ fund. rep., $N_f < N_{f,b1z} = 3N$.

Condition for 2-loop β fn. to have IR zero: $b_2 < 0$.

Qualitative behavior of b_2 similar to that in the non-SUSY theory; b_2 is a decreasing fn. of N_f, which is positive for small N_f and decreases through zero to negative values as N_f increases through

$$N_{f,b2z} = \frac{3C_A^2}{2T_f(C_A + 2C_f)}$$

For arbitrary G and R, $N_{f,b1z} > N_{f,b2z}$, as shown by

$$N_{f,b1z} - N_{f,b2z} = \frac{3C_A C_f}{T_f(C_A + 2C_f)} > 0$$

So again, there is always an interval in N_f for which the 2-loop β fn. has an IR zero, namely $I : N_{f,b2z} < N_f < N_{f,b1z}$.

If $R =$ fund. rep., then

$$I : \quad \frac{3N^3}{2N^2 - 1} < N_f < 3N$$

For example,

For $N = 2$, $\quad I : \quad 3.43 < N_f < 6$, so $N_f = 4,\ 5$

For $N = 3$, $\quad I : \quad 4.76 < N_f < 9$, so $N_f = 5,\ 6,\ 7,\ 8$.

As $N \to \infty$, $\quad I$: $\quad \frac{3N}{2} < N_f < 3N$

Exact result for $R =$ fund. rep. (Seiberg): for $(3/2)N < N_f < 3N$, theory evolves from UV to an IR-conformal (non-Abelian Coulomb) phase, so

$$N_{f,cr} = \frac{3N}{2} = \frac{N_{f,b1z}}{2} \quad \text{(for fund. rep.)}$$

(This is only formal for odd N, since then $N_{f,cr}$ is not an integer.)

Note that $N_{f,b2z} > N_{f,cr}$;

$$N_{f,b2z} - N_{f,cr} = \frac{3N}{2(2N^2-1)} > 0$$

so here, $N_{f,b2z}$, the lower end of the interval I, lies within the IR-conformal phase.

For $N_f \in I$, the 2-loop β function has an IR zero at

$$\alpha_{IR,2\ell} = -\frac{4\pi b_1}{b_2} = \frac{2\pi(3C_A - 2T_f N_f)}{2T_f N_f(C_A + 2C_f) - 3C_A^2}$$

Since $b_2 = 0$ at $N_f = N_{f,b2z}$, $\alpha_{IR,2\ell}$ diverges within the IR-conformal phase, this restricts the range in N_f where our perturbative analysis can be applied.

The b_ℓ have been calculated up to $\ell = 3$ loop order.

For the analysis of the 3-loop β function, we need b_3 (from Jack, Jones, North, 1996, in $\overline{DR}$ scheme):

$$b_3 = 21C_A^3 + 4T_fN_f(-5C_A^2 - 13C_AC_f + 4C_f^2) + 4(T_fN_f)^2(C_A + 6C_f)$$

b_3 is positive for small N_f and vanishes at two values of N_f, again denoted $N_{f,b3z,1}$ and $N_{f,b3z,2}$. As in the non-SUSY case, we find that $N_{f,b3z,1} < N_{f,b2z}$ and $N_{f,b3z,2} > N_{f,b1z}$, so $b_3 < 0$ for $N_f \in I$.

For example, for $R = fund.$ rep., $N = 2$, $N_{f,b3z,1} = 3.09 < N_{f,b2z} = 3.43$, and $N_{f,b3z,2} = 8.38 > N_{f,b1z} = 6$.

Since $b_3 < 0$ for all $N_f \in I$, we find, by the same type of proof as for the non-SUSY case, that for any G, R, and $N_f \in I$ (i.e., where the 2-loop β function has an IR zero),

$$\alpha_{IR,2\ell} > \alpha_{IR,3\ell}$$

Some numerical values of $\alpha_{IR,2\ell}$ and $\alpha_{IR,3\ell}$ below for $R =$ fund. rep. and illustrative values of N and N_f:

N_c	N_f	$\alpha_{IR,2\ell}$	$\alpha_{IR,3\ell}$
2	4	(6.28)	2.65
2	5	1.14	0.898
3	5	(18.85)	(3.05)
3	6	2.69	1.40
3	7	0.992	0.734
3	8	0.343	0.308
4	7	(5.03)	1.64
4	8	1.795	0.984
4	9	0.867	0.615
4	10	0.426	0.357
4	11	0.169	0.158

For fixed N, we find that $\alpha_{IR,n\ell}$ increases monotonically with decreasing N_f at both the 2-loop and 3-loop level.

Next, analyze anomalous dim. $\gamma_m \equiv \gamma$ of the (gauge-invariant) superfield operator product $\Phi\tilde{\Phi}$ containing the bilinear fermion product in term $\theta\theta\psi\tilde{\psi}$ (recall component-field decomposition of superfield, $\Phi = \phi + \theta\psi + \theta\theta F$).

In a conformally invariant field theory (whether supersymmetric or not), unitarity yields a lower bound on the dim. $D_{\mathcal{O}}$ of a spin-0 operator $\mathcal{O}$ (other than the identity): $D_{\mathcal{O}} \ge (d-2)/2$, where $d =$ spacetime dim.; so $D_{\mathcal{O}} \ge 1$ here (Mack, 1977; see also Grinstein, Intriligator, Rothstein, PLB 662, 367 (2008); arXiv:0801.1140).

In the non-SUSY theory, with $\dim(\bar{\psi}\psi) = 3 - \gamma_m$, this constraint is $D_{\bar{\psi}\psi} = 3 - \gamma_m > 1$, so $\gamma_m < 2$.

In the SUSY theory, with $\dim(\theta) = -1/2$ and $\dim(\psi\tilde{\psi}) = 3 - \gamma_m$, the constraint is $D_{\Phi\tilde{\Phi}} = -1 + 3 - \gamma_m > 1$, so $\gamma_m < 1$.

Perturbative expansion: $\gamma_m = \sum_{\ell=1}^{\infty} c_\ell a^\ell = \sum_{\ell=1}^{\infty} \bar{c}_\ell \alpha^\ell$, where, as before, 1-loop coeff. is scheme-indep., higher-loop coeffs. are scheme-dep.

values of coeffs: $c_1 = 4C_f$

and, to 3-loop order (from Jack, Jones, North (1996); Harlander, Mihaila, Steinhauser (2009), in $\overline{DR}$ scheme):

$$c_2 = 4C_f(-2C_f + 3C_A - 2T_fN_f)$$

$$c_3 = 8C_f\Big[4C_f^2 + 3C_A(C_A - C_f) + T_fN_f\Big\{(-8+12\zeta(3))C_f + (1-12\zeta(3))C_A\Big\} - 2T_f^2N_f^2\Big]$$

We evaluate the n-loop expression for γ at the n-loop value of the IR zero of β. At the 2-loop level,

$$\gamma_{IR,2\ell} = \frac{C_f(3C_A - 2T_fN_f)(2T_fN_f - C_A)(2T_fN_f - 3C_A + 6C_f)}{[2(C_A + 2C_f)T_fN_f - 3C_A^2]^2}$$

Some numerical values:

N_c	N_f	$\gamma_{IR,2\ell}$	$\gamma_{IR,3\ell}$
2	5	0.260	0.0802
3	7	0.399	0.0584
3	8	0.139	0.104
4	9	0.490	0.0219
4	10	0.239	0.127
4	11	0.0970	0.0835

$\gamma_{IR,2\ell}$ increases monotonically as N_f decreases, while $\gamma_{IR,3\ell}$ is a non-monotonic function of N_f.

We find $\gamma_{IR,3\ell} < \gamma_{IR,2\ell}$, as in non-SUSY case.

To get a perturbative estimate of $N_{f,cr}$, assume that the upper bound $\gamma_m \le 1$ is saturated as $N_f \searrow N_{f,cr}$ and solve eq. $\gamma_{IR,n\ell} = 1$ for $N_{f,cr}$.

The 2-loop condition $\gamma_{IR,2\ell} = 1$ is a cubic eq. in N_f; for the unique, physical root for the estimated (est.) $N_{f,cr,est.}$, we find

$N = 2 \;\Rightarrow\; N_{f,cr,est.} = 4.24$, factor 1.41 larger than exact $N_{f,cr} = 3$

$N = 3 \;\Rightarrow\; N_{f,cr,est.} = 6.15$, factor 1.37 larger than exact $N_{f,cr} = 4.5$

As $N \to \infty$, $N_{f,cr,est.} \to 2N$, factor (4/3) larger than exact $N_{f,cr} = (3/2)N$

From this analysis of $\mathcal{N} = 1$ supersymmetric gauge theory we conclude:

Perturbative calc. slightly overestimates the value of $N_{f,cr}$, i.e., slightly underestimates the size of the IR-conformal phase.

- similar to conclusion from our analysis of the non-SUSY theory.

Study of Scheme-Dependence in Calculation of IR Fixed Point

Since the coeffs. in β at 3-loops and higher are scheme-dependent, so is the resultant value of $\alpha_{IR,n\ell}$ calculated to a (finite-loop order) of $n \geq 3$ loops - important to assess quantitatively the uncertainty due to this scheme dependence.

A way to do this is to perform scheme transformations and determine how much of a change there is in $\alpha_{IR,n\ell}$. We have carried out this study in Ryttov and Shrock, PRD 86, 065032 (2012), arXiv:1206.2366; PRD 86, 085005 (2012), arXiv:1206.6895.

A scheme transformation (ST) is a map between α and α' or equivalently, a and a', where $a = \alpha/(4\pi)$, which can be written as

$$a = a' f(a')$$

with $f(0) = 1$ to keep the UV properties unchanged. Considering STs analytic about $a = 0$, we write

$$f(a') = 1 + \sum_{s=1}^{s_{max}} k_s (a')^s = 1 + \sum_{s=1}^{s_{max}} \bar{k}_s (\alpha')^s \ ,$$

where the k_s are constants, $\bar{k}_s = k_s/(4\pi)^s$, and s_{max} may be finite or infinite.

Hence, the Jacobian $J = da/da' = d\alpha/d\alpha'$ satisfies $J = 1$ at $a = a' = 0$. We have

$$\beta_{\alpha'} \equiv \frac{d\alpha'}{dt} = \frac{d\alpha'}{d\alpha}\frac{d\alpha}{dt} = J^{-1}\beta_\alpha \ .$$

This has the expansion

$$\beta_{\alpha'} = -2\alpha' \sum_{\ell=1}^{\infty} b'_\ell (a')^\ell = -2\alpha' \sum_{\ell=1}^{\infty} \bar{b}'_\ell (\alpha')^\ell \ ,$$

where $\bar{b}'_\ell = b'_\ell/(4\pi)^\ell$.

Using these two equiv. expressions for $\beta_{\alpha'}$, one can solve for the b'_ℓ in terms of the b_ℓ and k_s. This leads to the well-known result that

$$b'_1 = b_1 \ , \quad b'_2 = b_2$$

i.e, the one-loop and two-loop terms in β are scheme-indep.

To assess the scheme-dependence of an IRFP, we have calculated the relations between the b'_ℓ and b_ℓ for higher ℓ values. For example, for $\ell = 3, \ 4, \ 5$, we obtain

$$b'_3 = b_3 + k_1 b_2 + (k_1^2 - k_2) b_1 \ ,$$

$$b'_4 = b_4 + 2k_1 b_3 + k_1^2 b_2 + (-2k_1^3 + 4k_1 k_2 - 2k_3) b_1$$

$$b'_5 = b_5 + 3k_1 b_4 + (2k_1^2 + k_2) b_3 + (-k_1^3 + 3k_1 k_2 - k_3) b_2$$
$$+ (4k_1^4 - 11k_1^2 k_2 + 6k_1 k_3 + 4k_2^2 - 3k_4) b_1$$

Since β function coefficients b_ℓ with $\ell \geq 3$ are scheme-dep.,, there should exist a ST in which one can make these coeffs. zero ('t Hooft). We constructed an explicit ST that can does this at a UVFP.

To be physically acceptable, a ST must satisfy several conditions, C_i. For finite s_{max}, the ST is is an algebraic eq. of degree $s_{max} + 1$ for α' in terms of α. We require that at least one of the $s_{max} + 1$ roots must satisfy these conditions. For $s_{max} = \infty$, the eq. for α' in terms of α is generically transcendental, and again we require that the relevant sol. must satisfy these conditions, which are:

- C_1: the ST must map a real positive α to a real positive α', since a map taking $\alpha > 0$ to $\alpha' = 0$ would be singular, and a map taking $\alpha > 0$ to a negative or complex α' would violate the unitarity of the theory.
- C_2: the ST should not map a moderate value of α, for which pert. theory may be reliable, to an excessively large value of α' where pert. theory is inapplicable

- C_3: J should not vanish in the region of α and α' of interest, or else there would be a pole in the relation between β_α and $\beta_{\alpha'}$.
- C_4: The existence of an IR zero of β is a scheme-independent property, depending (in an AF theory) only on the condition that $b_2 < 0$. Hence, a ST should satisfy the condition that β_α has an IR zero if and only if $\beta_{\alpha'}$ has an IR zero.

These four conditions can always be satisfied by STs near a UV fixed point, and hence in applications to pert. QCD calcs., since α is small, and one can choose the k_s to be small also, so $\alpha' \simeq \alpha$.

However, these conditions C1-C4 are not automatically satisfied, and are a significant constraint, on a ST applied in the vicinity of an IRFP, where α may be O(1).

For example, consider the ST

$$\alpha = \tanh(\alpha')$$

with inverse

$$\alpha' = \frac{1}{2}\ln\Big(\frac{1+\alpha}{1-\alpha}\Big)$$

If $\alpha << 1$, as at a UVFP, this is acceptable, but if α exceeds 1, even if by a small amount, then it is unacceptable, since it maps a real positive α to a complex α'.

We have studied scheme dependence of the IR zero of β using several STs; e.g., the ST (depending on a parameter r)

$$S_{sh,r}: \quad a = \frac{\sinh(ra')}{r}$$

Since $\sinh(ra')/r$ is an even fn. of r, we take $r > 0$ with no loss of generality.

This has the inverse

$$a' = \frac{1}{r} \ln\left[ra + \sqrt{1+(ra)^2}\,\right]$$

and the Jacobian

$$J = \cosh(ra')$$

For this ST,

$$f(a') = \frac{\sinh(ra')}{ra'} \, .$$

This has a series expansion with $k_s = 0$ for odd s and for even s,

$$k_2 = \frac{r^2}{6} \, , \quad k_4 = \frac{r^4}{120}$$

$$k_6 = \frac{r^6}{5040} \, , \quad k_8 = \frac{r^8}{362880} \, ,$$

etc. for higher s.

Substituting these results for k_s into the general eq. for b'_ℓ, we obtain

$$b'_3 = b_3 - \frac{r^2 b_1}{6}$$
$$b'_4 = b_4$$
$$b'_5 = b_5 + \frac{r^2 b_3}{6} + \frac{31 r^4 b_1}{360}$$

etc. for higher ℓ.

We apply this S_{sh_r} ST to the β function in the $\overline{MS}$ scheme, calculated up to $\ell = 4$ loop level. For N_f in the interval I where the 2-loop β function has an IR zero, we then calculate the resultant IR zeros in $\beta_{\alpha'}$ at the 3- and 4-loop order and compare the values with those in the $\overline{MS}$ scheme.

We list some numerical results for illustrative values of r and for $N = 2,\ 3$. We denote the IR zero of $\beta_{\alpha'}$ at the n-loop level as $\alpha'_{IR,n\ell} \equiv \alpha'_{IR,n\ell,r}$.

For example, for $N = 3$, $N_f = 10$, $\alpha_{IR,2\ell} = 2.21$, and:

$$\alpha_{IR,3\ell,\overline{MS}} = 0.764, \quad \alpha'_{IR,3\ell,r=3} = 0.762, \quad \alpha'_{IR,3\ell,r=6} = 0.754,$$
$$\alpha'_{IR,3\ell,r=9} = 0.742, \quad \alpha'_{IR,3\ell,r=4\pi} = 0.723$$
$$\alpha_{IR,4\ell,\overline{MS}} = 0.815, \quad \alpha'_{IR,4\ell,r=3} = 0.812, \quad \alpha'_{IR,4\ell,r=6} = 0.802,$$
$$\alpha'_{IR,4\ell,r=9} = 0.786, \quad \alpha'_{IR,4\ell,r=4\pi} = 0.762$$

In general, the effect of scheme dependence tends to be reduced (i) for a given N and N_f, as one calculates to higher-loop order, and (ii) for a given N, as $N_f \longrightarrow N_{f,b1z}$, so that the value of $\alpha_{IR} \longrightarrow 0$.

The results provide a quantitative measure of scheme dependence of the location of an IR zero of β.

Application of Quasiconformal Gauge Theories to Models of Dynamical Electroweak Symmetry Breaking and Implications for LHC Data

Models with dynamical electroweak symmetry breaking (EWSB) have been of interest as one way to avoid the hierarchy (fine-tuning) problem with the Standard Model (SUSY is another).

These models use an asymp. free vectorial gauge interaction, technicolor (TC), with a set of massless technifermions $\{F\}$ and a gauge coupling $\alpha_{TC}(\mu)$ that gets large at TeV scale, producing condensates $\langle \bar{F}F \rangle = \langle \bar{F}_L F_R \rangle + h.c. \sim \Lambda_{TC}^3$.

These dynamically break EW symmetry, since the technifermions include a left-handed SU(2)$_L$ doublet with corresponding right-handed SU(2)$_L$ singlets. Their condensates transform as EW $I = 1/2$, $Y = 1$, same as SM Higgs, and give masses to W and Z satisfying $m_W^2/(m_Z^2 \cos^2\theta_W) = 1$ to leading order.

Indeed quark condensates $\langle \bar{q}q \rangle$ also dynamically break EW symmetry (at much smaller scale, Λ_{QCD}), also transform as $I = 1/2$, $Y = 1$.

The TC theory is embedded in extended technicolor (ETC) to give masses to SM fermions via exchanges of ETC gauge bosons, which take SM fermion $\leftrightarrow$ technifermions.

Resultant SM fermion mass matrices

$$M_{ii}^{(f)} \sim \frac{\eta\,\Lambda_{TC}^3}{\Lambda_{ETC,i}^2}$$

where $i = 1, 2, 3$ is generation index, $\Lambda_{ETC,i}$ is a corresponding ETC mass scale, and

$$\eta_i = \exp\left[\int_{\Lambda_{TC}}^{\Lambda_i} \frac{d\mu}{\mu}\gamma(\alpha_{_{TC}}(\mu))\right] \quad \text{is RG factor}$$

Typical values: $\Lambda_1 \simeq 10^3$ TeV, $\Lambda_2 \simeq 50 - 100$ TeV, $\Lambda_3 \simeq$ few TeV. Hierarchy in ETC symmetry breaking scales $\Lambda_{ETC,1} > \Lambda_{ETC,2} > \Lambda_{ETC,3}$ produces inverse generational hierarchy in SM fermion masses.

The running mass $m_{f_i}(p)$ of a SM fermion of generation i is constant up to the ETC scale $\Lambda_{ETC,i}$ and has the power-law decay (Christensen and Shrock, PRL 94, 241801 (2005))

$$m_{f_i}(p) \propto p^{-2} \quad \text{for} \;\; p >> \Lambda_{ETC,i}$$

Original TC models were scaled-up versions of QCD and were excluded by their inability to produce sufficiently large SM fermion masses without having ETC scales so low as to cause excessively large flavor-changing neutral current (FCNC) effects.

TC models after mid 1980s have been built to have a coupling that gets large but runs very slowly (walking, quasiconformal TC, WTC) (Yamawaki et al.; Holdom; Appelquist, Wijewardhana...). This quasiconformal behavior arises naturally from an approx. IR zero of the TC β function, with α_{IR} slightly greater than α_{cr}.

If γ_{IR} is approx. const. near this IRFP, then, e.g., third-gen. SM fermion masses are increased by factor

$$\eta_3 \simeq \left(\frac{\Lambda_3}{\Lambda_{TC}}\right)^{\gamma_{IR}}$$

which could give significant enhancement. Hence, one can raise ETC scales Λ_i, reducing FCNC effects.

Further, studies of reasonably UV-complete ETC models showed that approximate residual generational symmetries suppress FCNC effects (Appelquist, Piai, Shrock, PRD 69, 015002 (2004); PLB 593, 175 (2004); PLB 595, 442 (2004); Appelquist, Christensen, Piai, Shrock, PRD 70, 093010 (2004).)

ETC models still face challenges in trying to reproduce all features of SM fermion masses, such as $m_t >> m_b$, etc. Here focus on TC.

TC models that include color-nonsinglet technifermions, such as the one-family TC model, in which technifermions comprise one SM family, are disfavored at present, for several reasons, including (i) possibly excessive contributions to precision electroweak S parameter; (ii) prediction of pseudo-NGB's (PNGB's), some of which are color-nonsinglets, with O(100) GeV masses that they should have been observed at LHC; (ii) color-octet techni-vector mesons, with masses of order TeV, in tension with the current lower bound of ~ 2.5 TeV set by ATLAS and CMS.

But TC models need not have any color-nonsinglet technifermions; a TC model may have a minimal EW-nonsinglet technifermion content of one SU(2)$_L$ doublet with corresponding right-handed SU(2)$_L$ singlets, all of which are color-singlets.

TC models of this type can exhibit quasiconformal behavior. For models in which technifermions are in fund. rep. of TC group, one may add SM-singlet technifermions to get N_f slightly less than $N_{f,cr}$ (Christensen and Shrock, Phys. Lett. B632, 92 (2006); Ryttov and Shrock, Phys. Rev. D84, 056009 (2011), arXiv:1107.3572). Alternatively, one can use higher-dim. TC reps. (Dietrich, Tuominen, Ryttov; e.g., Dietrich, Sannino, and Tuominen, PRD 72, 055001 (2005); Sannino, arXiv:0911.0931).

In these minimal TC models, all NGBs with EW quantum numbers are eaten, so no left-over EW-nonsinglet NGBs, in contrast with one-family TC. Also, S parameter may be sufficiently reduced (also by walking) to satisfy precision EW constraints.

As noted, because quasiconformal TC has approx. scale invariance, dynamically broken by $\langle \bar{F}F \rangle$, this could plausibly lead to a light approx. NGB, the techidilaton (Yamawaki..Goldberger, Grinstein, and Skiba.. Fan; Sannino...; Appelquist and Bai; Elander, Nunez, and Piai; for different estimates of χ mass, see Bardeen, Leung, Love; Holdom and Terning). Approx. Bethe-Salpeter calc. finds $m_S/m_V \sim 0.3$ in WTC (Kurachi, Shrock, JHEP 12, 034 (2006)). Much recent work on estimates of the dilaton mass; e.g., Matsuzaki and Yamawaki, PRD85, 095020 (2012); arXiv:1201.4722. arXiv:1209.2017; Lawrance and Piai, arXiv:1207.0427; Elander and Piai, arXiv:1208.0546; Bellazzini, Csáki, Hubisz, Serra and Terning, arXiv:1209.3299. A technidilaton might be as light as 125 GeV.

Eventually, lattice gauge measurements may be able to determine the mass of a dilaton in a quasiconformal theory (a difficult calculation). Important progress from the LATKMI reported at this conf. (Rinaldi's talk).

N.B. Technicolor gauge fields are color-singlets and all technifermions may be color-singlets as well, in which case a technidilaton χ may have no color-nonsinglet constituents, affecting couplings to gluons.

The boson discovered at the LHC by ATLAS and CMS with mass of ~ 125 GeV is consistent with being the SM Higgs, although the diphoton rate is slightly high. However, it might also be explained as a technidilaton, χ, resulting from a quasiconformal TC theory; further experimental and theoretical work should settle this decisively.

A general TC collider signature is resonant scattering of longitudinally polarized W and Z bosons, via techni-vector mesons in s-channel. A decisive search at LHC may require $\int \mathcal{L} dt \sim 50 - 100$ fb^{-1} at $\sqrt{s} = 14$ TeV.

Conclusions

- Understanding the UV to IR evolution of an asymptotically free gauge theory and the nature of the IR behavior is of fundamental field-theoretic interest

- Our higher-loop calculations give new information on this UV to IR flow and on determination of $\alpha_{IR,n\ell}$ and $\gamma_{IR,n\ell}$; valuable to compare and combine results from higher-loop continuum calcs. with lattice measurements

- Higher-loop study of UV to IR flow for supersymmetric gauge theories yields further insights

- Quantitative study of scheme-dependence in higher-loop calculations, noting that scheme transformations are subject to constraints that are easily satisfied at a UVFP but are quite restrictive at IRFP

- Application of quasiconformal gauge theories to models of dynamical EWSB; role of a light dilaton

- Importance of these calculations in deciding the outstanding question of whether dynamical EWSB is realized in nature