SCGT 12

KMI-GCOE Workshop on Strong Coupling Gauge Theories in the LHC Perspective

Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

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Non-Abelian strings in supersymmetric Yang-Mills: 4D-2D correspondence

A. Yung,

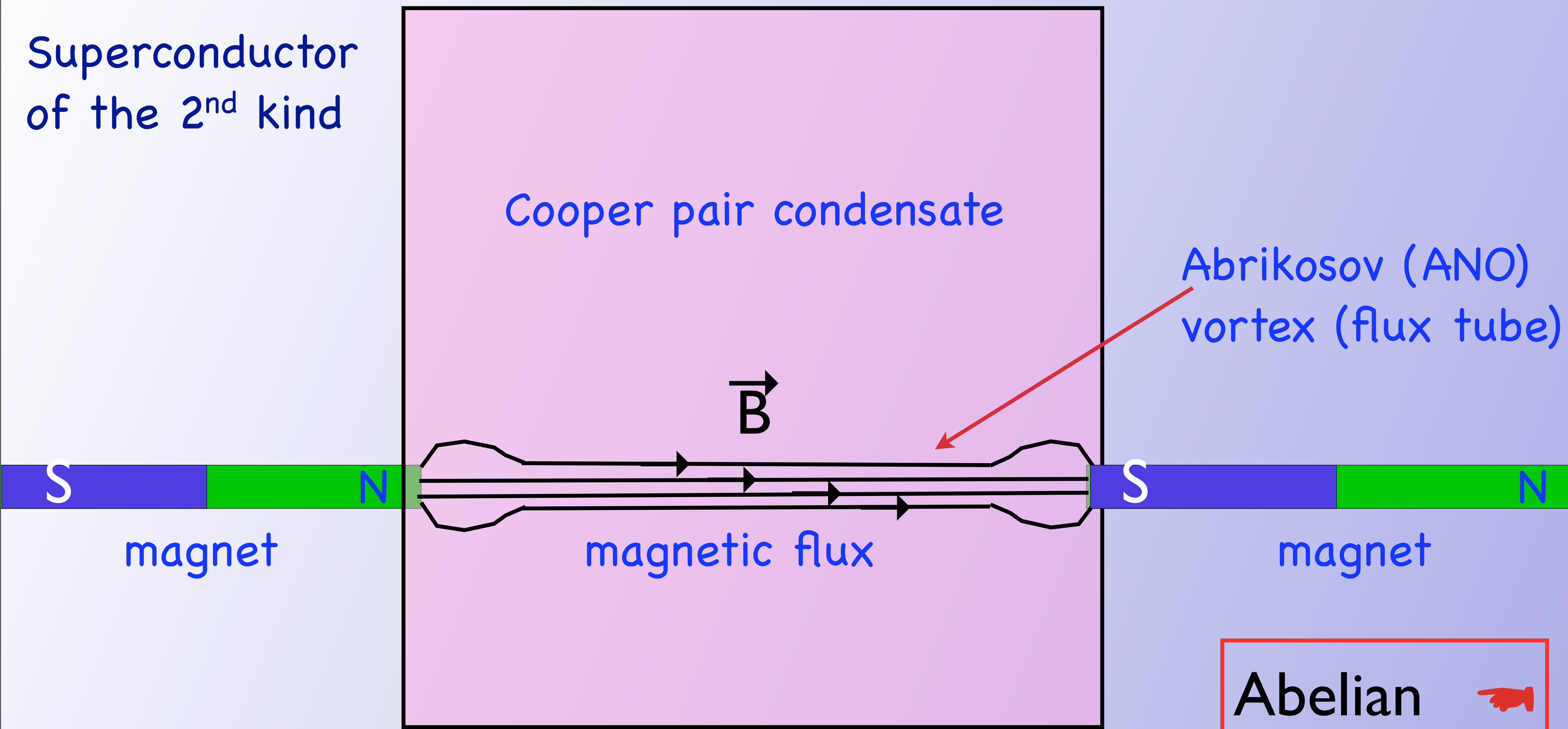
A. Gorsky, ...

W. Vinci, M. Nitta

IPMU, December 3, 2012

★ Discovery of non-Abelian strings in supersymmetric Yang-Mills and applications

★ ★ Beyond supersymmetry

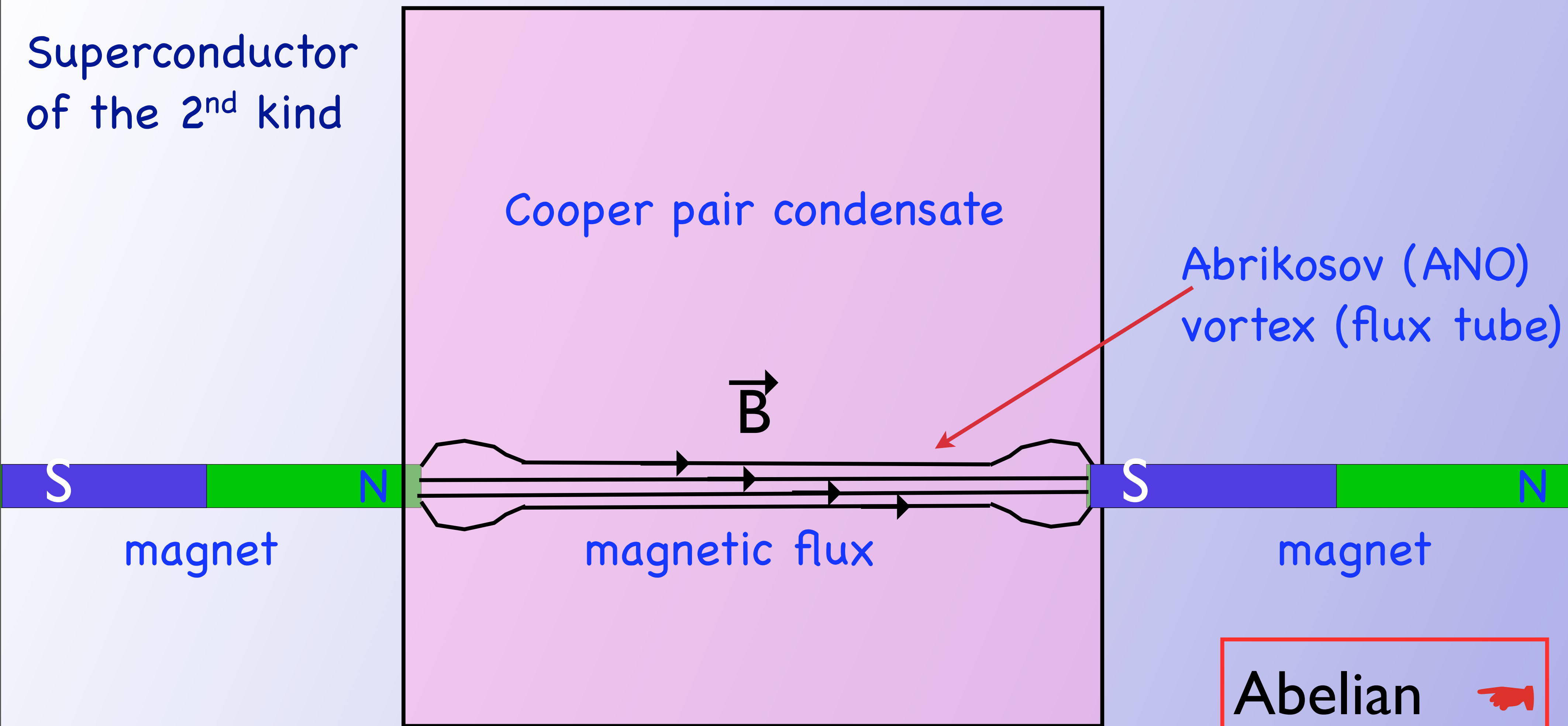


Abelian

DUAL MEISSNER EFFECT (Nambu-'t Hooft-Mandelstam, ~1975)

☞ The Meissner effect: 1930s, 1960s

Superconductor
of the 2nd kind



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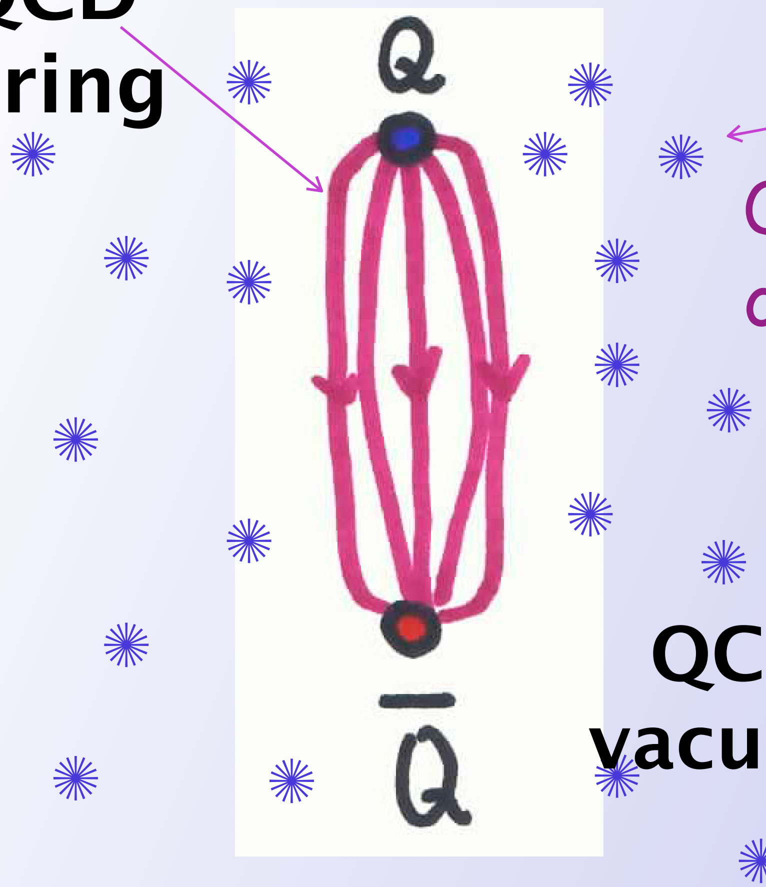
QCD string

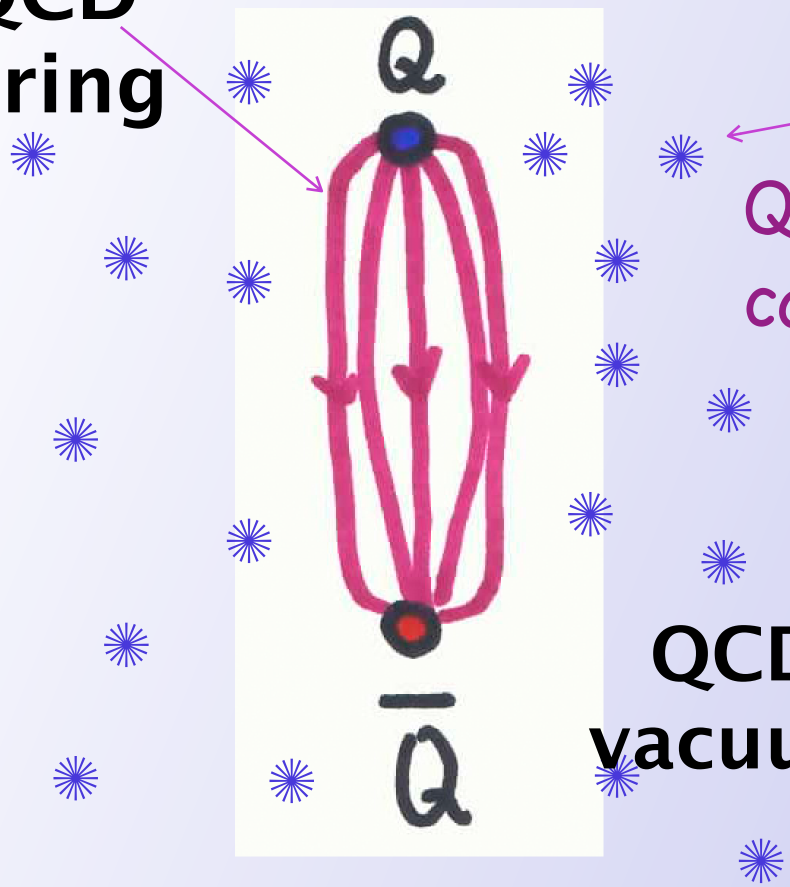
condensed magnetic monopoles

Qualitative explanation of color confinement: Dual Meissner effect:



QCD vacuum

★ Hanany, Strassler, Zaffaroni '97 → SW=Abelian strings, "wrong" confinement...



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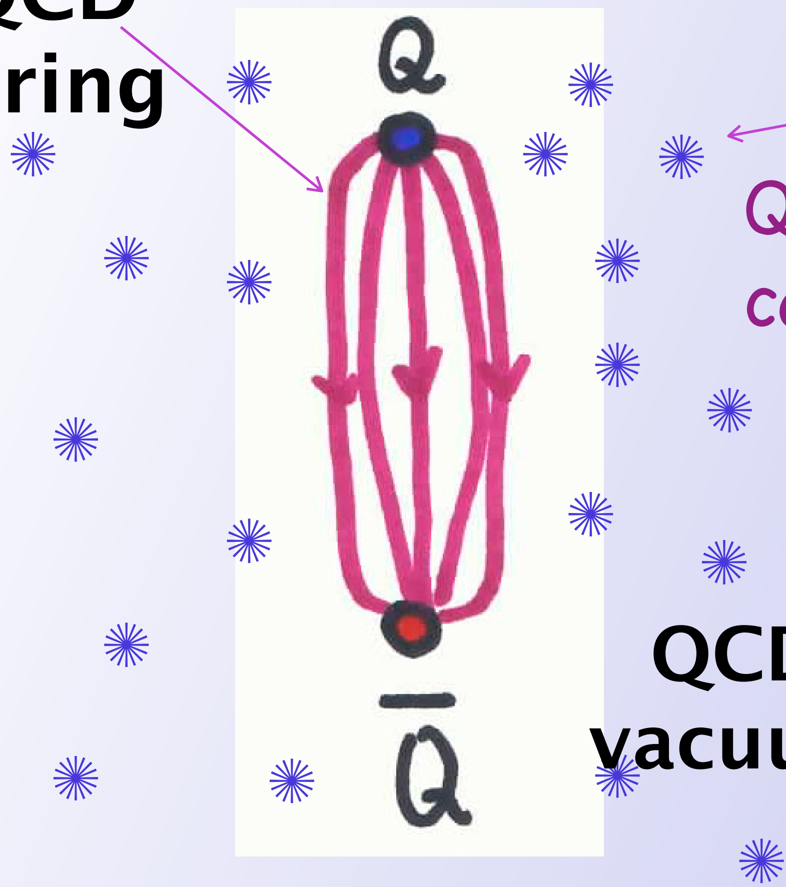

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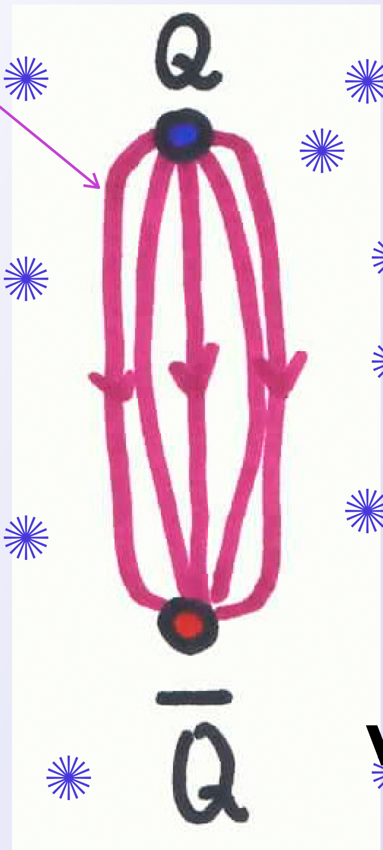

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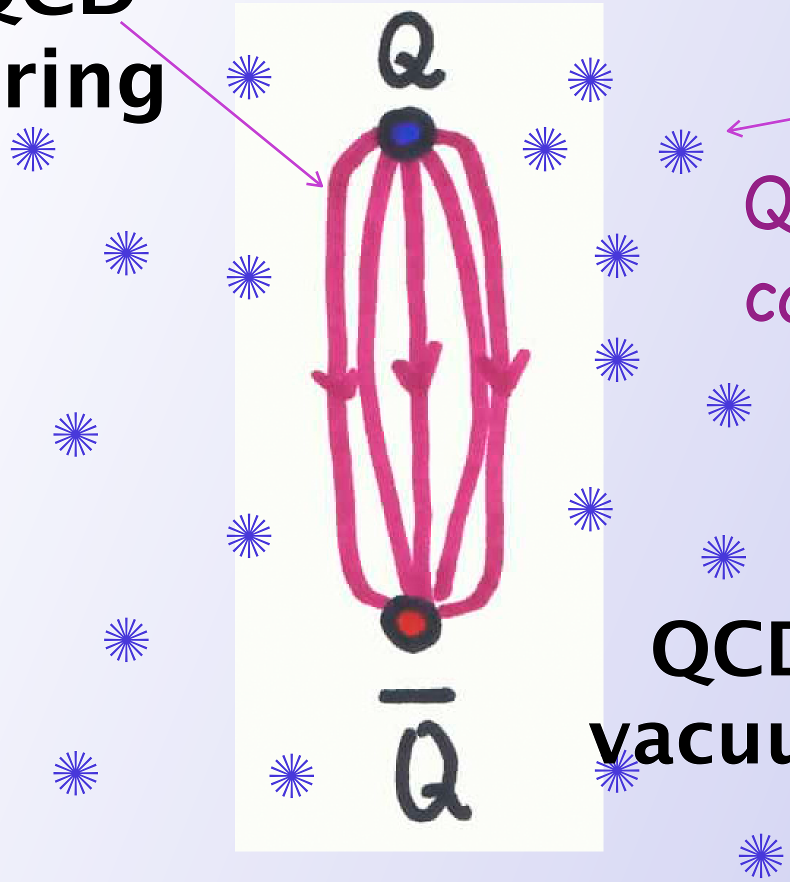

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☆ Non-Abelian theory, but Abelian flux tube

☹

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"...[monopoles] turn to develop a non-zero vacuum expectation value. Since they carry color-magnetic charges, the vacuum will behave like a superconductor for color-magnetic charges. What does that mean? Remember that in ordinary electric superconductors, magnetic charges are connected by magnetic vortex lines ... We now have the opposite: it is the color charges that are connected by color-electric flux tubes."
G. 't Hooft (1976)

$\mathcal{N}$=2 $\Rightarrow$ add the second gluino + add a scalar gluon φ^a (a complex scalar field in the adjoint)

$$V(\varphi^a) = |\varepsilon^{abc}\,\varphi^b\,\varphi^c|^2$$

In the vacuum $\varphi^3 \neq 0$ while $\varphi^1 = \varphi^2 = 0 \quad \Rightarrow$

$SU(2)_{gauge} \to U(1) \Rightarrow$

Georgi-Glashow model

$\Rightarrow$ 't Hooft-Polyakov monopoles

If $|\varphi^3| \gg \Lambda$, then monopoles are very heavy!

☺ First demonstration of the dual Meissner effect: Seiberg & Witten, 1994 ☺

- gluons+complex scalar superpartner
- two gluinos
- Georgi-Glashow model built in

$\longmapsto$ analytic continuation

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SU(2) $\to$ U(1), monopoles $\rightarrow$

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Monopoles become light if $|\varphi^3| \lesssim \Lambda$ ➔ At two points, massless!

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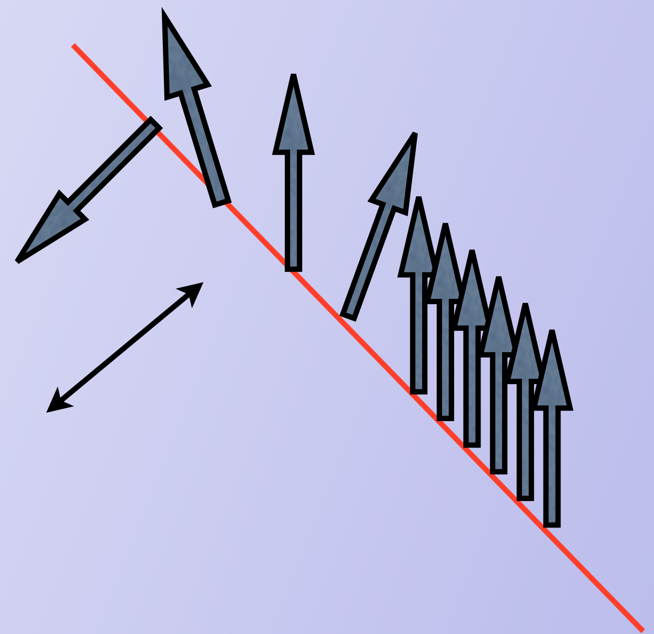
☹ ☹ Dynamical Abelization ... dual Abrikosov string

⟼ analytic continuation

☞ Non-Abelian Strings, 2003 → Now

"Non-Abelian" string is formed if all non-Abelian degrees of freedom participate in dynamics at the scale of string formation



2003: Hanany, Tong
Auzzi et al.
Yung + M.S.

classically gapless excitation

SU(2)/U(1) = CP(1)~O(3) sigma model

Prototype model

$$S = \int d^4x \left\{ \frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + \frac{1}{g_2^2}\left|D_\mu a^a\right|^2 \right.$$

$$+ \; \mathrm{Tr}\left(\nabla_\mu \Phi\right)^\dagger \left(\nabla^\mu \Phi\right) + \frac{g_2^2}{2}\left[\mathrm{Tr}\left(\Phi^\dagger T^a \Phi\right)\right]^2 + \frac{g_1^2}{8}\left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right) - N\xi\right]^2$$

$$\left. + \; \boxed{\frac{1}{2}\mathrm{Tr}\left|a^a T^a \Phi + \Phi\sqrt{2}M\right|^2} + \frac{i\,\theta}{32\,\pi^2}\, F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu} \right\},$$

$$\Phi = \begin{pmatrix} \varphi^{11} & \varphi^{12} \\ \varphi^{21} & \varphi^{22} \end{pmatrix}$$

$$M = \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}$$

$$\Phi = \sqrt{\xi} \times I$$

Basic idea:

- **Color-flavor locking in the bulk → Global symmetry G;**
- **G is broken down to H on the given string;**
- G/H coset; **G/H sigma model on the world sheet.**

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U(2) gauge group, 2 flavors of (scalar) quarks

SU(2) Gluons A^a_μ + U(1) photon + gluinos+ photino

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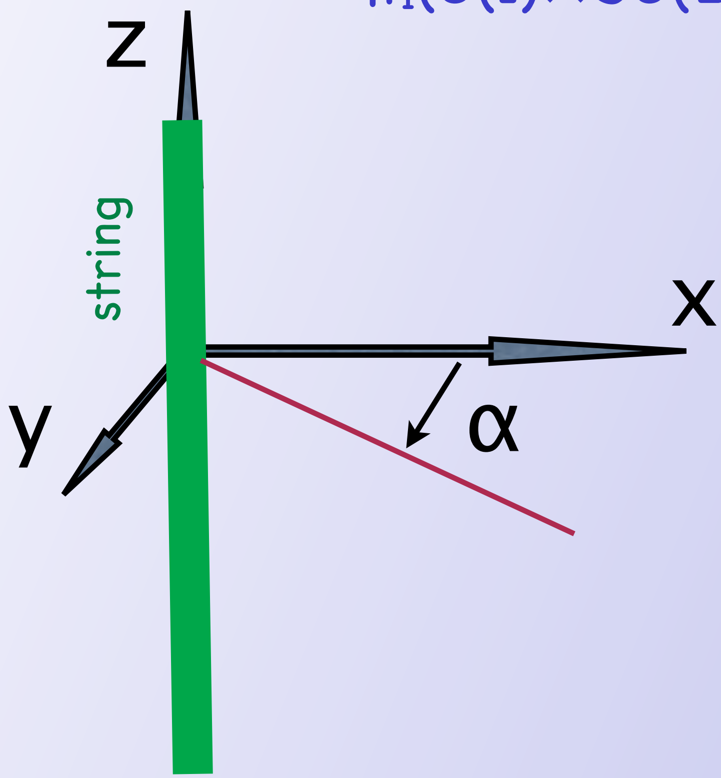
☆ ANO strings are there because of U(1)!

☆ New strings:

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



x_0 ← string center in perp. plane

ANO $\quad \sqrt{\xi}\, e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$T = 4\pi\xi$

Non-Abelian $\quad \sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}$

$T_{U(1)} \pm T^3_{SU(2)}$

$T = 2\pi\xi$

SU(2)/U(1) ←orientational moduli; O(3) σ model

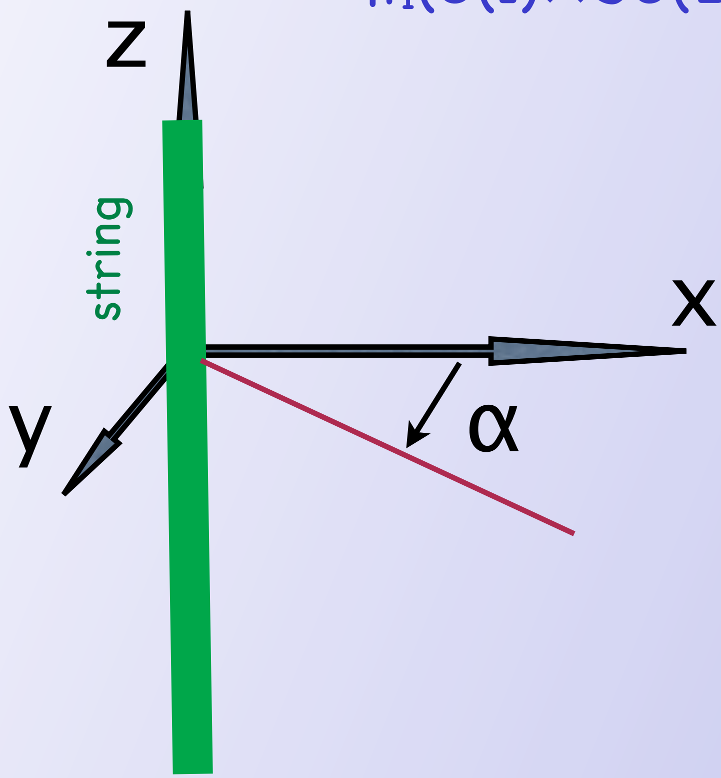
☆ ANO strings are there because of U(1)!

☆ New strings:

$\pi_1(SU(2)\times U(1)) = Z_2$: **rotate by π around 3-d axis in SU(2)**

→ -1; another -1 rotate by π in U(1)

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



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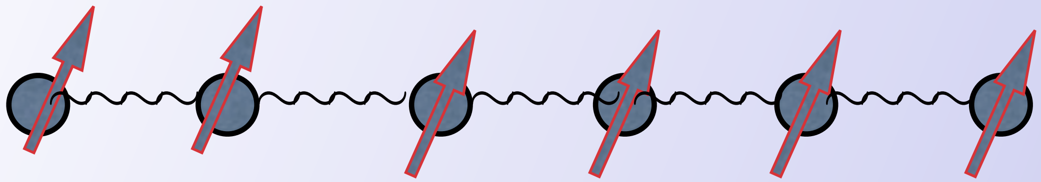
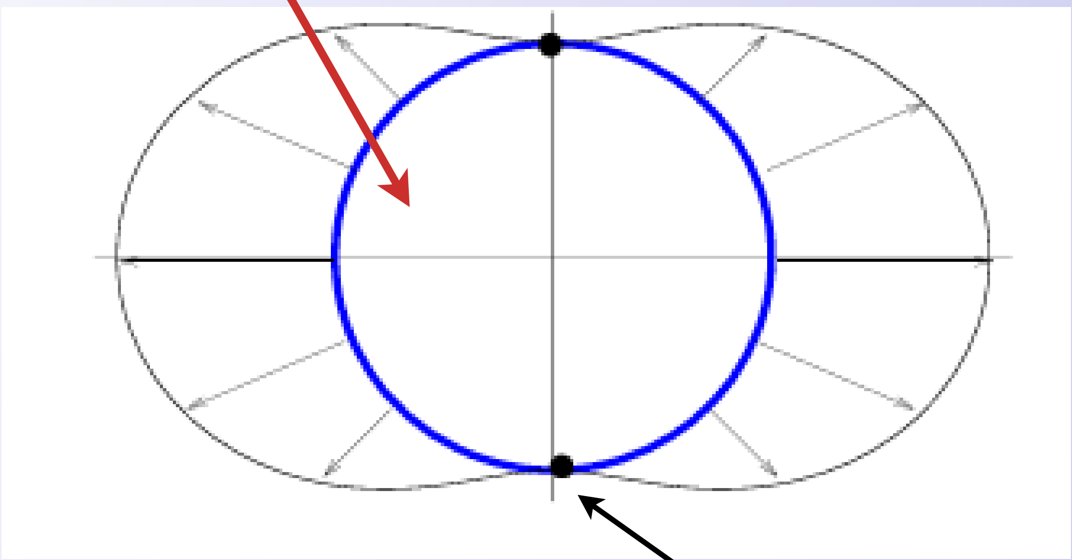
Worldsheet theory



S_2



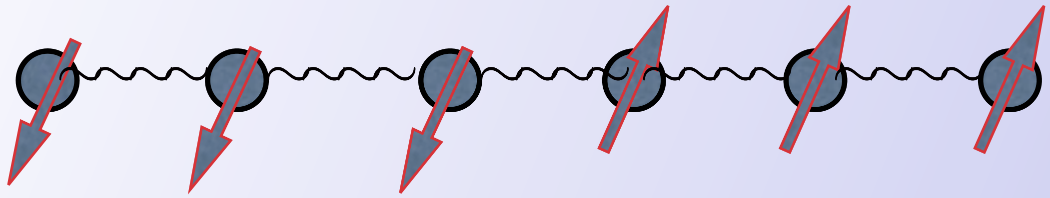
Global SU(2) is gone!

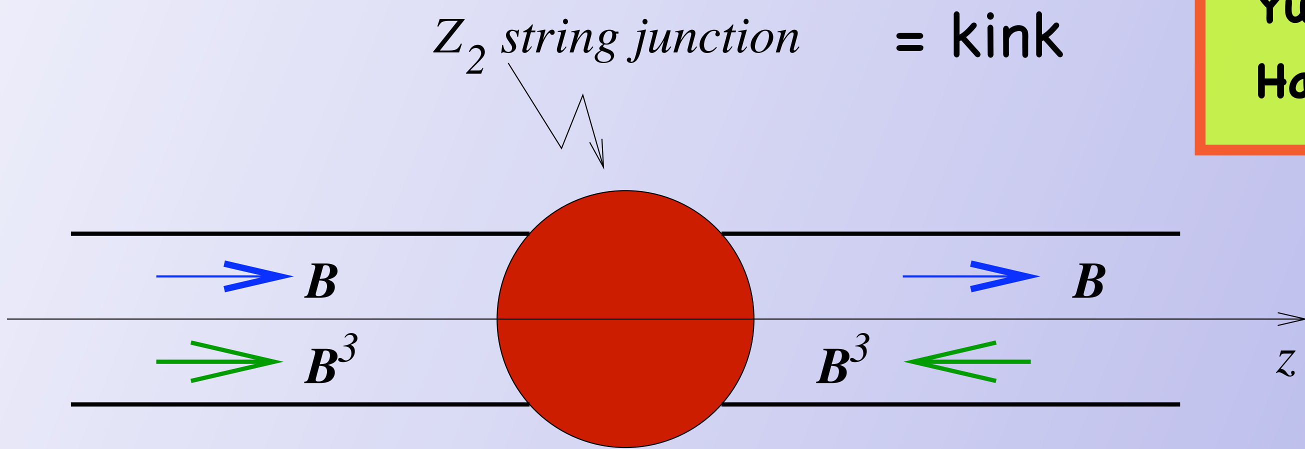
U(1) remains intact

Two vacua= 2 degenerate strings

CP(1) model with twisted mass

$$S = \int d^2x \left\{ \frac{2}{g^2} \frac{\partial_\mu \bar{\phi} \partial^\mu \phi - (\Delta m)^2 \bar{\phi}\phi}{(1+\bar{\phi}\phi)^2} + fermions \right\}$$

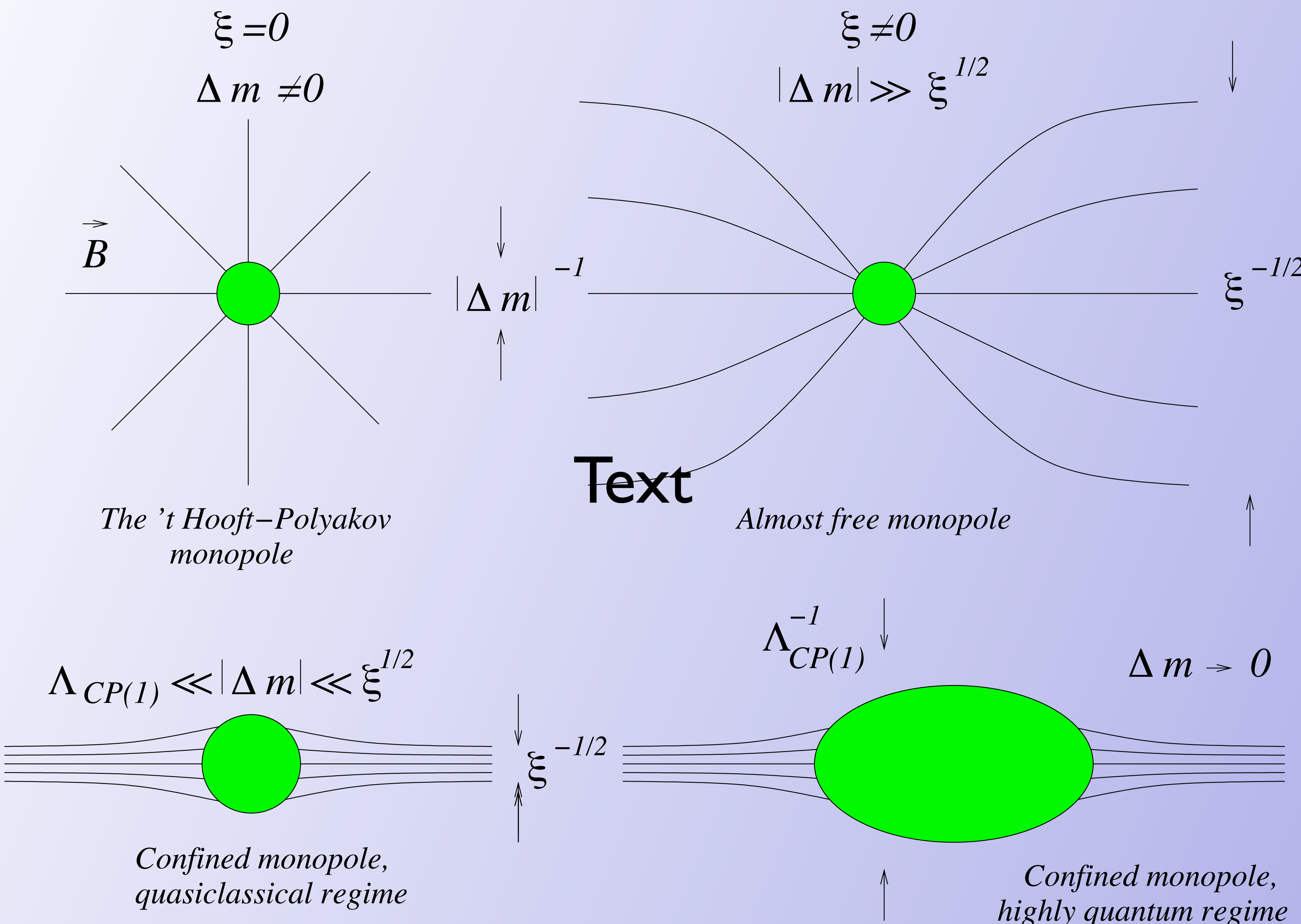




= kink

Yung + M.S.
Hanany, Tong

Evolution in dimensionless parameter m^2/ξ


$\xi = 0$
$\Delta m \neq 0$
$\vec{B}$
$|\Delta m|^{-1}$
$\xi \neq 0$
$|\Delta m| \gg \xi^{1/2}$
$\xi^{-1/2}$
Text
The 't Hooft−Polyakov monopole
Almost free monopole
$\Lambda_{CP(1)} \ll |\Delta m| \ll \xi^{1/2}$
$\xi^{-1/2}$
$\Lambda^{-1}_{CP(1)}$
$\Delta m \to 0$
Confined monopole, quasiclassical regime
Confined monopole, highly quantum regime

✳ Kinks are confined in 4D (attached to strings).

✳ ✳ Kinks are confined in 2D:

Kink = Confined Monopole

4D $\leftrightarrow$ 2D Correspondence

☛ World-sheet theory $\leftrightarrow$ strongly coupled bulk theory inside

Dewar flask

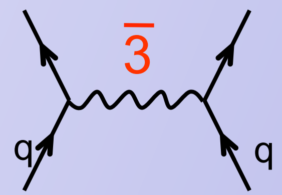
1) Confined monopoles in dense QCD

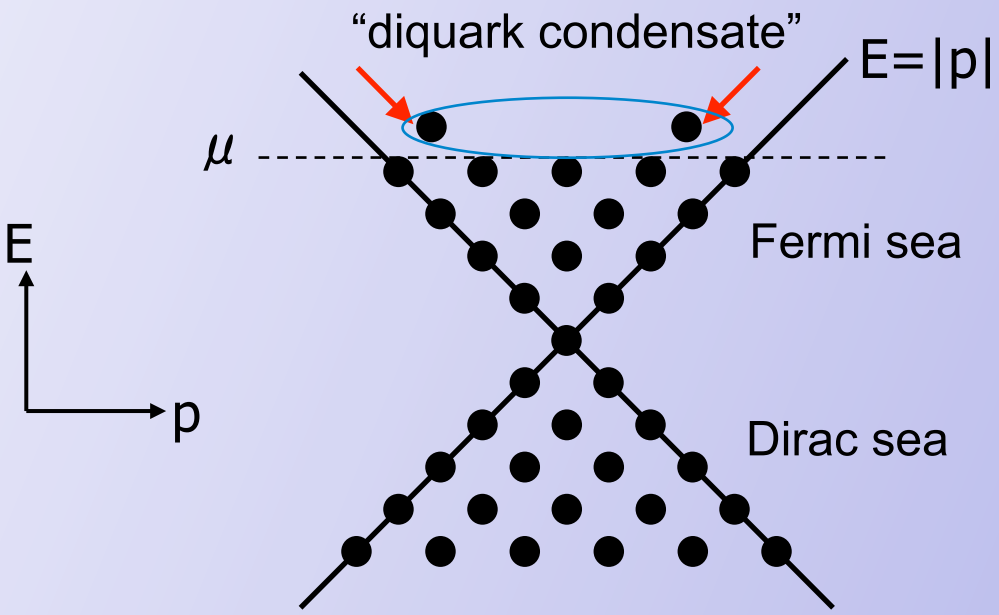
Color Superconductivity (CSC)

- QCD at high density → Fermi surface, weak-coupling
- Attractive channel → Cooper instability

  $[3]_C \times [3]_C = [6]_S + [\bar{3}]_A$

$$(\tau_a)_{ij}(\tau_a)_{kl} = \frac{2}{3}(\tau_S)_{ik}(\tau_S)_{lj} - \frac{4}{3}(\tau_A)_{ik}(\tau_A)_{lj}$$





Neutron stars?

2 Ginsburg–Landau effective description

At large μ QCD is in the CFL phase. Diquark condensate

3 colors and 3 flavors

$$\Phi^{kC} \sim \varepsilon_{ijk}\, \varepsilon_{ABC} \left(\psi_\alpha^{iA}\, \psi^{jB\,\alpha} + \tilde{\bar{\psi}}^{iA\,\dot\alpha}\, \tilde{\bar{\psi}}^{jB}_{\dot\alpha} \right)$$

At $T \to T_c$ gap fluctuations become important.

Chiral fluctuations (π-mesons) are considered less important

$$S = \int d^4x \left\{ \frac{1}{4g^2} \left(F^a_{\mu\nu}\right)^2 + 3\,\mathrm{Tr}\,(\mathcal{D}_0\Phi)^\dagger\,(\mathcal{D}_0\Phi) \right.$$

$$\left. + \quad \mathrm{Tr}\,(\mathcal{D}_i\Phi)^\dagger\,(\mathcal{D}_i\Phi) + V(\Phi) \right\}$$

with the potential

$$V(\Phi) = -m_0^2\,\mathrm{Tr}\,\left(\Phi^\dagger\Phi\right) + \lambda \left(\left[\mathrm{Tr}\,\left(\Phi^\dagger\Phi\right)\right]^2 + \mathrm{Tr}\,\left[\left(\Phi^\dagger\Phi\right)^2\right] \right)$$

$$\Phi_{\rm vac} = v\,{\rm diag}\,\{1,1,1\}$$

where

$$v^2 = \frac{m_0^2}{8\lambda} = \frac{4\pi^2}{3}\,\frac{T_c - T}{T_c}\,\mu^2$$

The symmetry breaking pattern

$$SU(3)_C \times SU(3)_F \times U(1)_B \to SU(3)_{C+F}$$

9 symmetries are broken.

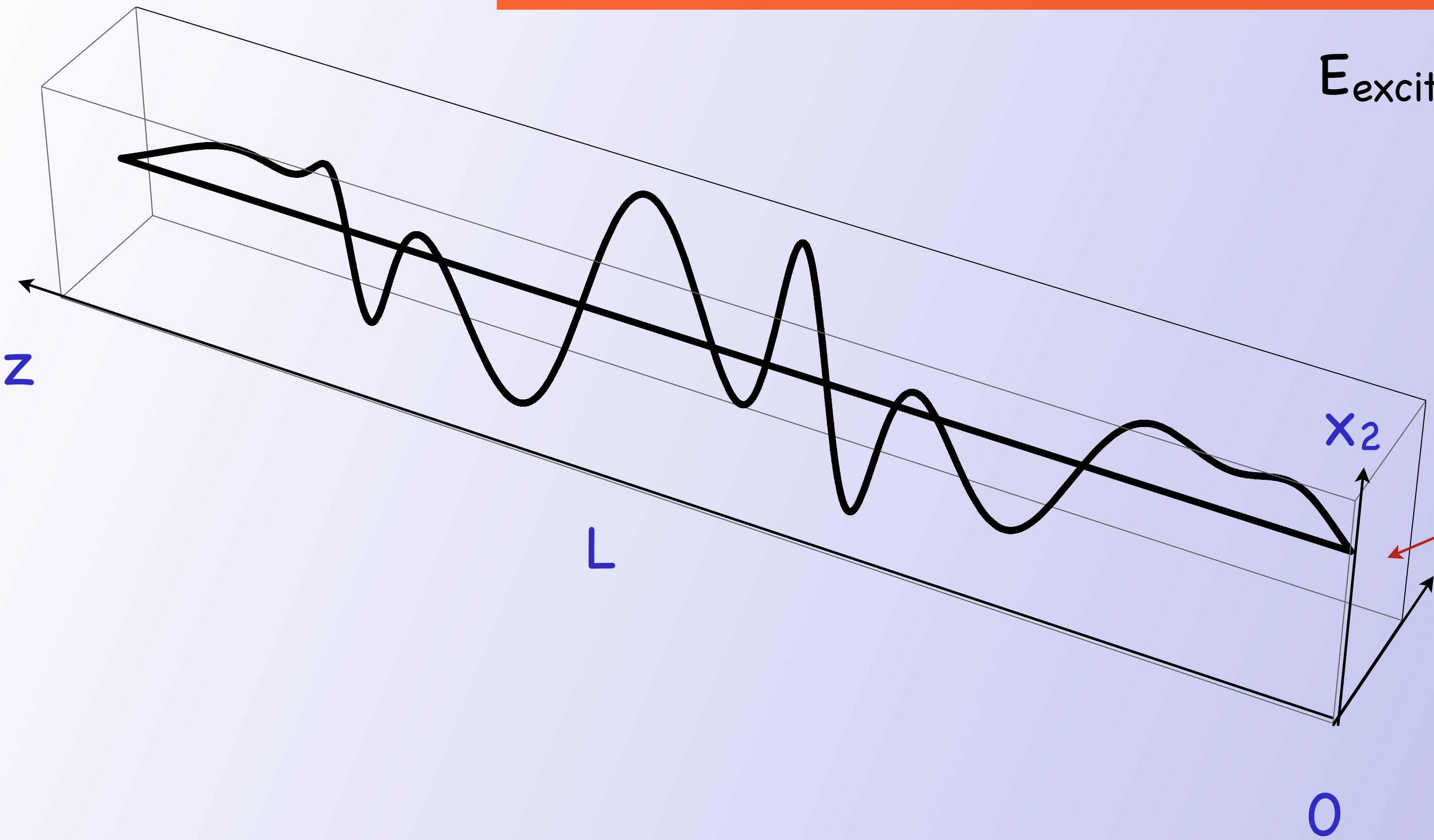
8 are eaten by Higgs mechanism.

One Goldstone boson associated with broken $U(1)_B$.

Broken→SU(2)×U(1) ➟ CP(2) model on the string w.-s. !

Low-energy excitations (gapless modes)

$E_{excit} \ll m_\gamma \sim e\eta$



Perpendicular plane

◇◇ $\Delta H_{GL} = (T/2)(\partial_z x_{perp}\, \partial_z x_{perp})$ + h.d. ➡ time derivatives can be rel. or non-relat.

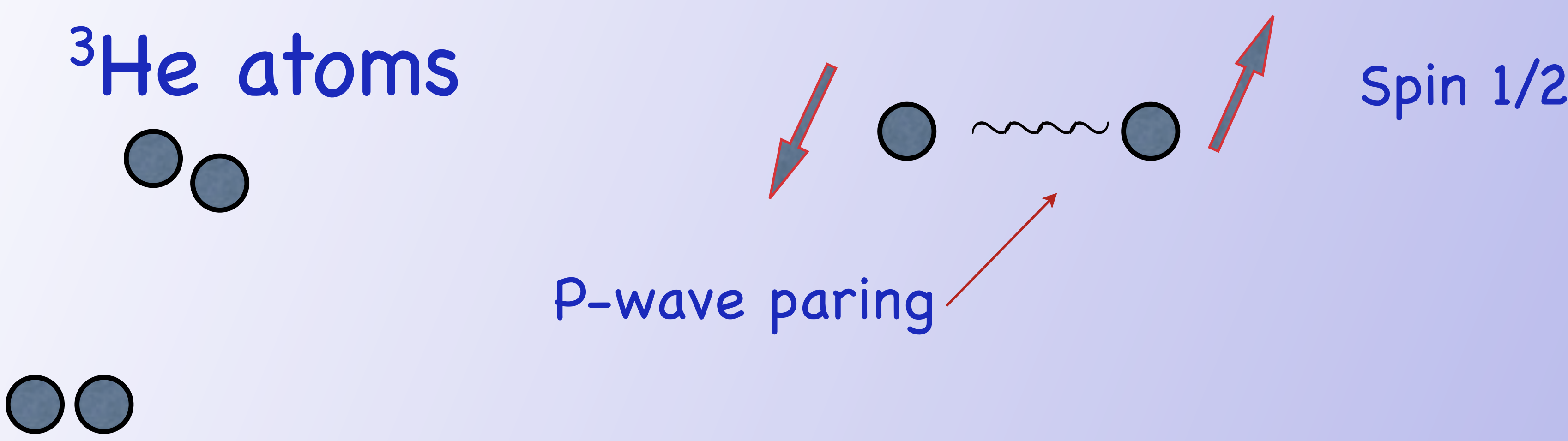
Nambu-Goto → String Theory

Kelvin modes or Kelvons
2 NG gapless modes in relat.
1 NG gapless mode in non-rel.

$E_{str} = TL + C/L$

Counts # of gapless modes !

^{3}He-B example?



L=1, S=1 ➟ Cooper pair order parameter $e_{\mu i}$ ⟵ 3×3 matrix

Spin-orbit small, symmetry of H is $G = U(1)_p \times SO_S(3) \times SO_L(3)$

In the ground state $U_p(1) \times SO_S(3) \times SO_L(3) \to H_{\rm B} = SO(3)_{S+L}$

Hence, contrived NG modes in the bulk!

Amending Abrikosov to non-Abelian

◇ $\Delta H_{GL} = D_k\phi^\dagger D_k\phi + \lambda(\phi^\dagger\phi-\eta^2)^2$ ➟ time derivatives can be rel. or non-relat.

◇◇ $\Delta H_{NA} = \partial_k n^i\, \partial_k n^i + (-\mu^2+\phi^\dagger\phi)n^i n^i + \beta(n^i n^i)^2$ + time derivatives

with $\eta^2>\mu^2$

✱ In ground state $\phi^\dagger\phi_{gr.st}= \eta^2$, hence the mass term of $n^i = \eta^2-\mu^2>0$ and O(3) is unbroken

✱✱ Inside Abrikosov $\phi^\dagger\phi_{gr.st.}= 0$ hence the mass term of $n^i = -\mu^2<0$ and O(3) is broken down to O(2), while $n^i n^i = \mu^2/2\beta$

✱✱✱ Classically O(3) sigma model on vortex, 2 gapless interacting modes

Conclisions

★ Non-Abelian strings in N=1 SUSY →

heterotic CP(n-1) models on string; poorly explored.

★ ★ Unexpected applications in condensed matter (not explored).