Twisted reduction in large N QCD with adjoint Wilson fermions

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We study the large N QCD with adjoint fermions using the twisted space-time reduced model.

For two flavor theory ($N_f = 2$) with N=289,
string tension is calculated which seems to vanish at $m_q = 0$,
in a way consistent with the theory governed by an infrared fixed point with $\gamma_* = 0.8 \sim 1.2$.

For one flavor theory ($N_f = 1$), string tension remains finite at $m_q = 0$, indicating a confining theory .

Plan of the talk

- Twisted Eguchi-Kawai model
  for pure SU(N) gauge theory

- large N QCD with two adjoint fermions

- large N QCD with one adjoint fermion

● Eguchi-Kawai model

Eguchi-Kawai model is obtained from the usual SU(N) lattice gauge theory

$$Z_W = \int \prod_{x,\mu} dU_{x,\mu} \exp\left\{ bN \sum_x \sum_{\mu\neq\nu=1}^{d} Tr\left(U_{x,\mu}U_{x+\mu,\nu}U^\dagger_{x+\nu,\mu}U^\dagger_{x,\nu}\right)\right\}$$

by neglecting the space-time dependence of the link variables

$$\boxed{U_{x,\mu} \to U_\mu}$$

$$Z_{EK} = \int \prod_{\mu} dU_{\mu} \exp\left\{ bN \sum_{\mu\neq\nu=1}^{d} Tr\left(U_{\mu}U_{\nu}U^\dagger_{\mu}U^\dagger_{\nu}\right)\right\}, \qquad b = \frac{1}{g^2 N}$$

In the same way, Wilson loop is defined by

$$W_W(C) = \left\langle Tr\left(U_{x,\mu}U_{x+\mu,\nu}\cdots U_{x-\rho,\rho}\right)\right\rangle$$

$$W_{EK}(C) = \left\langle Tr\left(U_{\mu}U_{\nu}\cdots U_{\rho}\right)\right\rangle$$

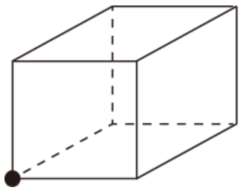
Eguchi and Kawai show that in the large N limit,
the Schwinger-Dyson eqs. satisfied by Wilson loops are identical in both theories provided that the $\left(Z(N)\right)^d$ symmetry

$$\boxed{U_\mu \to e^{i\theta_\mu} U_\mu,} \qquad e^{i\theta_\mu} \in Z(N)$$

of the EK model is not spontaneously broken.

Bhanot, Heller and Neuberger found, however, that this symmetry is broken spontaneously in the weak coupling region.

● Twisted Eguchi-Kawai model



EK model can be viewed as the usual Wilson gauge theory having only one site with the periodic boundary conditions.

We introduce twisted boundary conditions in $SU(N)$, $N = L^2$ theory

$$S_{TEK} = bN \sum_{\mu \neq \nu = 1}^{d} Tr\left(Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \right)$$

$$Z_{\mu\nu} = \exp\left(k \frac{2\pi i}{L} \right), \quad Z_{\nu\mu} = -Z_{\mu\nu}, \qquad \mu > \nu$$

k , L : co-prime, k/L fixed as we go $N = L^2 \rightarrow \infty$

Our model is related to ordinary SU(N) lattice theory on $V = L^4$ space-time volume up to $O(1/N^2)$ corrections.

The number of degree of freedom of SU(N) matrix is $N^2 = L^4$

If the TEK model is correct nonperturbatively,
we should be able to calculate the string tension.

In our reduced model, the Wilson loop $W(R,T)$ is defined by

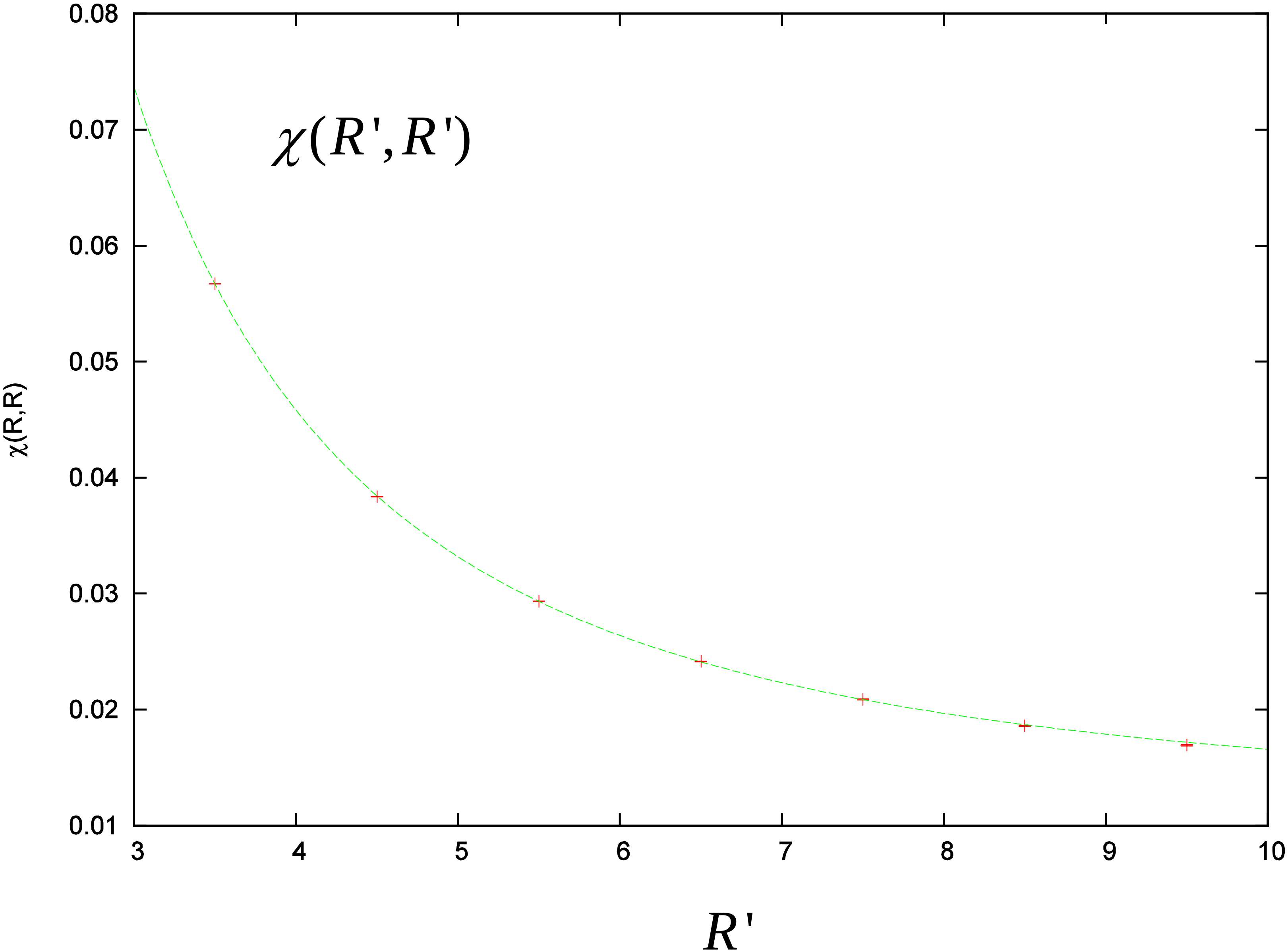
$$W(R,T) = Z_{\mu\nu}^{RT} \left\langle Tr\left(U_\mu^R U_\nu^T U_\mu^{\dagger R} U_\nu^{\dagger T} \right) \right\rangle$$

Then the string tension σ is obtained from Creutz ratio as

$$\chi(R',T') = -\log \frac{W(R'+0.5,T'+0.5)W(R'-0.5,T'-0.5)}{W(R'+0.5,T'-0.5)W(R'-0.5,T'+0.5)}$$

$$\chi(R',R') = \sigma + \frac{2\gamma}{R'^2} + \frac{\eta}{R'^4}$$

with half-integer R',T' .



$\chi(R',R')$
χ(R,R)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
3
4
5
6
7
8
9
10
R'

We calculate the continuum string tension by extrapolating the TEK data with $N = 841 = 29^2,\ k = 9$ at 6 values of b

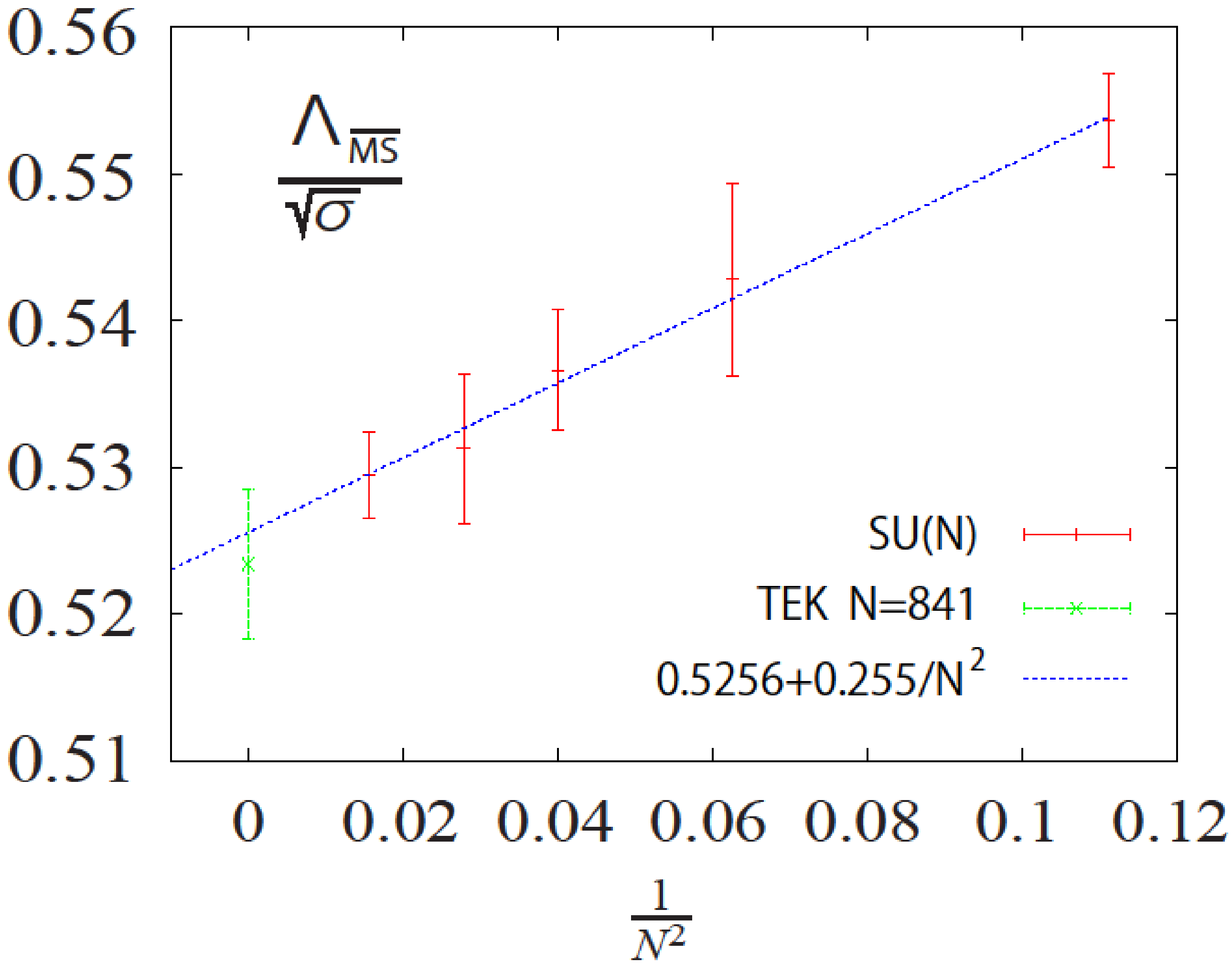
$$b = 0.36,\ 0.365,\ 0.37,\ 0.375,\ 0.38,\ 0.385$$

Our system should be related to the lattice theory with $V = 29^4$

For comparison, we also calculate the continuum string tension using ordinary SU(N) lattice gauge theory with $N = 3, 4, 5, 6, 8$ on a $V = 32^4$ lattice

Comparison of the continuum string tension $\Lambda_{\overline{MS}} / \sqrt{\sigma}$

TEK model with $N = 841 = 29^2$ and LGT with $N = 3, 4, 5, 6, 8$



Plan of the talk

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- large N QCD with one adjoint fermion

Motivation for $N_f = 2$ adjoint fermions

SU(N) LGT with two adjoint fermions is thought to be conformal or nearly conformal for any value of N since the first two coefficient of beta functions expressed in term of 't Hooft coupling is independent of N.

$$b_0 = \frac{4N_f - 11}{24\pi^2}, \quad b_1 = \frac{16N_f - 17}{192\pi^4}$$

$$b_0 < 0 \;\rightarrow\; N_f < \frac{11}{4} = 2.75$$ asymptotic free

$$b_1 > 0 \;\rightarrow\; N_f > \frac{17}{16} = 1.08$$ infrared fixed point

- Twisted reduced model of large N QCD with two adjoint Wilson fermions

We consider gauge group $SU(N),\ N = L^2$

$$S = bN \sum_{\mu \neq \nu = 1}^{d} Tr\left(Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \right) + \sum_{j=1}^{N_f} \bar{\psi}_j D_W \psi_j$$

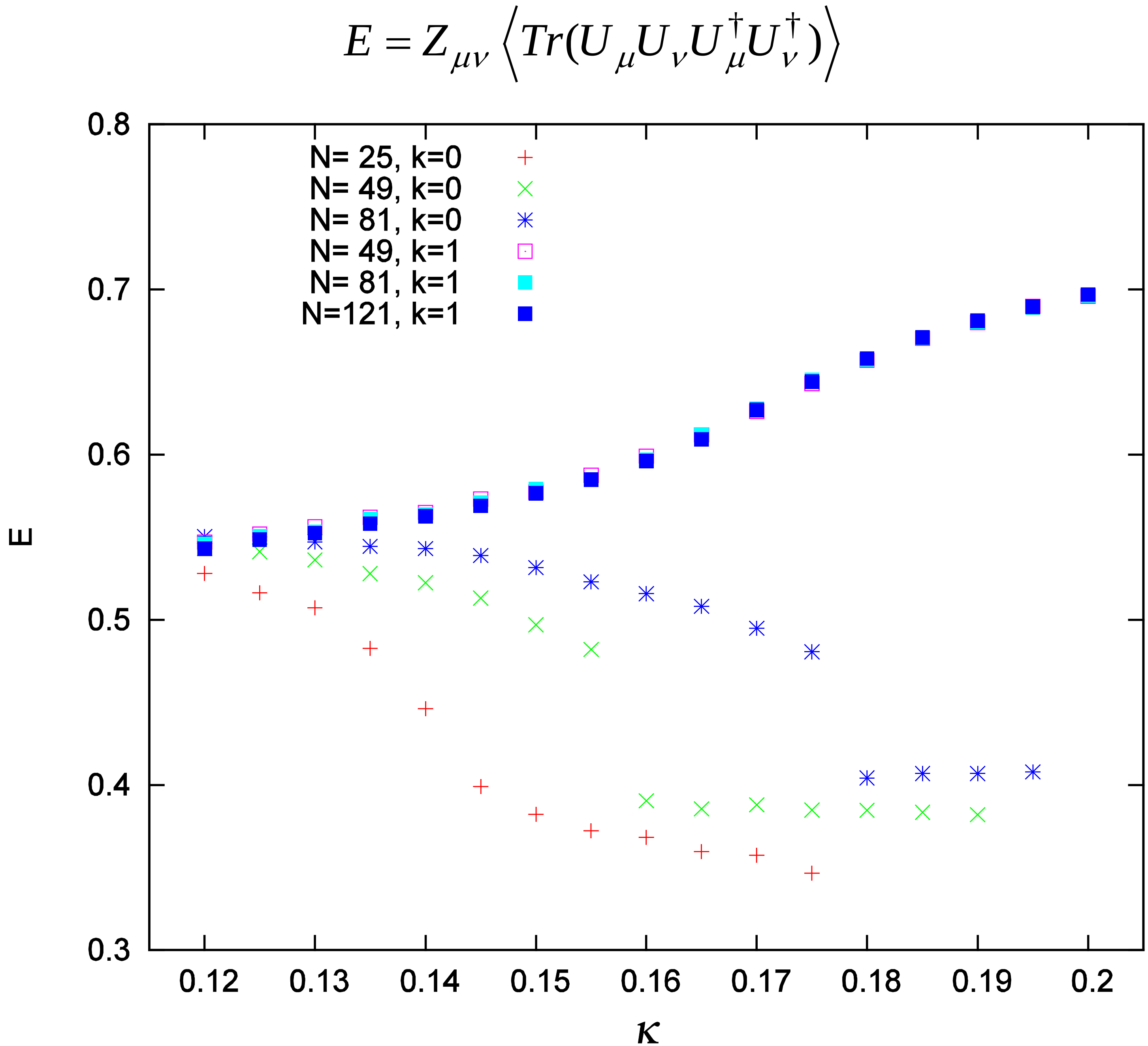
$$Z_{\mu\nu} = \exp\left(k \frac{2\pi i}{L} \right), \quad Z_{\nu\mu} = Z_{\mu\nu}^*, \qquad \mu > \nu$$

$$D_W = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{adj} + (1+\gamma_\mu) U_\mu^{\dagger adj} \right], \quad U_\mu^{adj} \psi_j = U_\mu \psi_j U_\mu^\dagger$$

k , L : co-prime

$k = 0$ corresponds to periodic boundary condition

$k \neq 0$ corresponds to twisted boundary condition



$E = Z_{\mu\nu} \left\langle Tr(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger) \right\rangle$
N= 25, k=0
N= 49, k=0
N= 81, k=0
N= 49, k=1
N= 81, k=1
N=121, k=1
0.8
0.7
0.6
0.5
0.4
0.3
E
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
κ

We calculate the string tension with $N = 289 = 17^2,\ k = 5$
at 2 values of $b = 0.35,\ 0.36$ for various values of κ

Our system should be related to the lattice theory with $V = 17^4$

- For $N_f = 2$, we use the Hybrid Monte Carlo method.

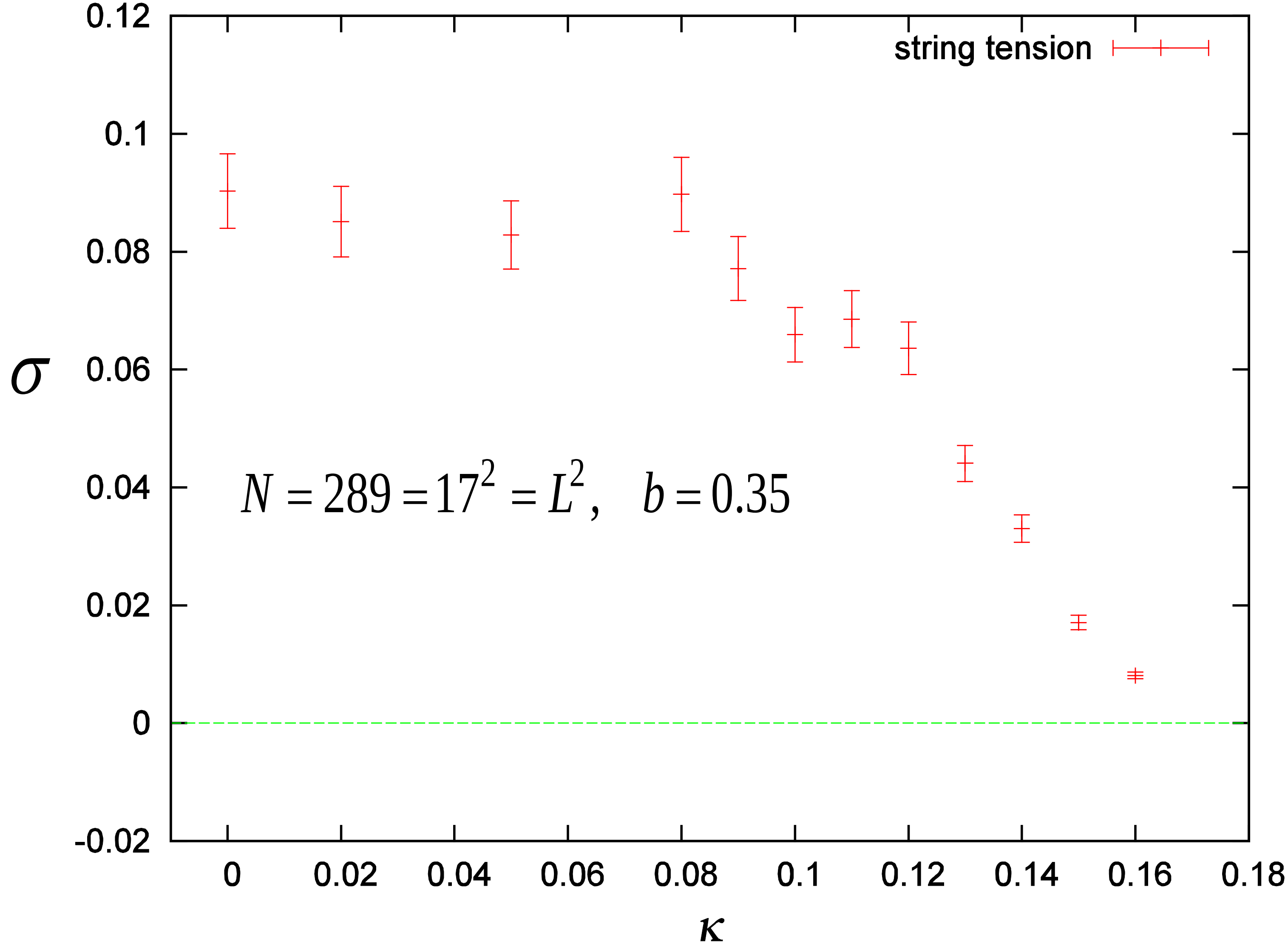
Simulations have been done on Hitachi SR16000 at KEK

One node: 32 cores power 7,
peak speed 980 GFlops
256 GB shared memory

Sustained speed of our code in one node is
600 Gflops at N=289

We thank to Hitachi system engineers !

string tension, SU(289), k=5, b=0.35

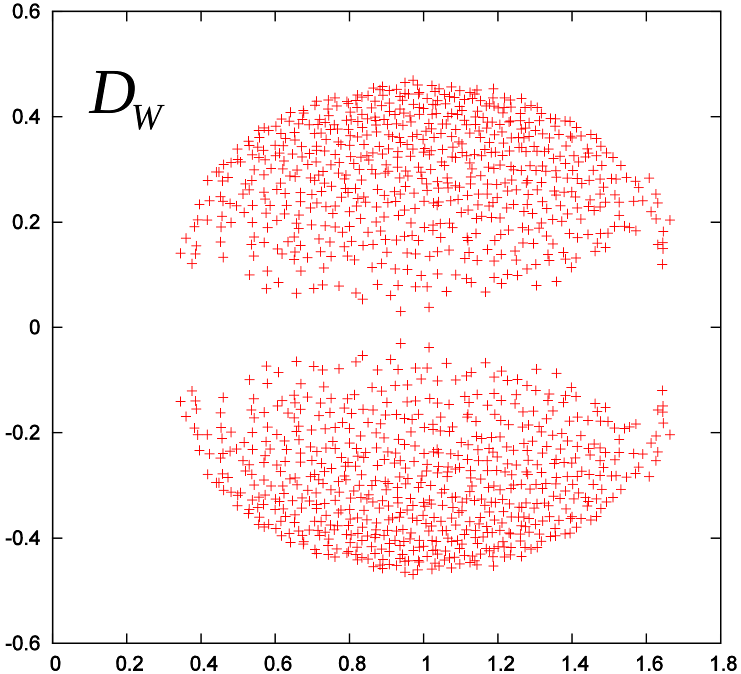


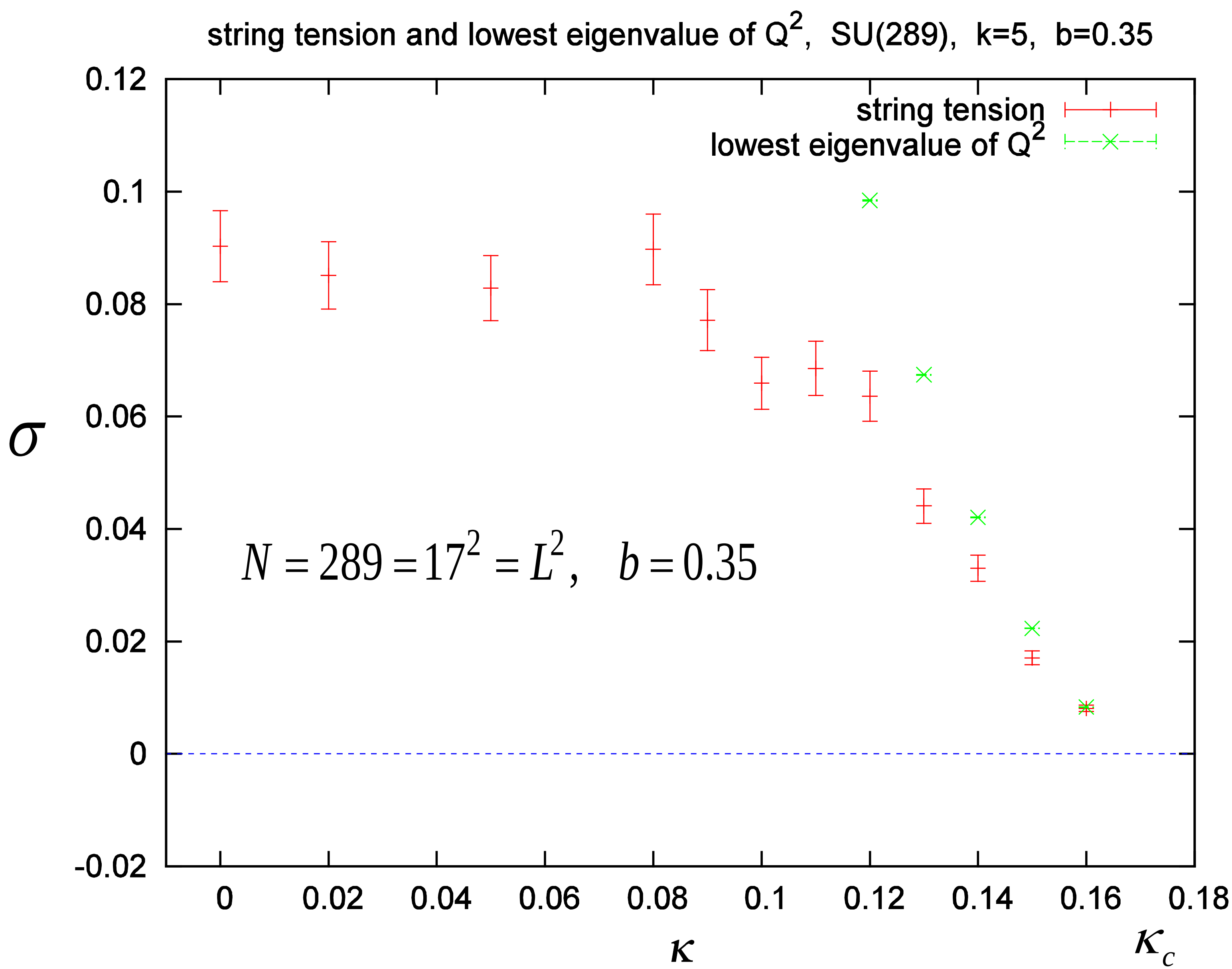
$$m_q = 1/\kappa - 1/\kappa_c$$

So far, we have not calculated any hadronic spectrum.

However, it is straightforward to calculate the lowest eigenvalue of positive hermitian Wilson Dirac operator $Q^2 = (D_W \gamma_5)^2$, which should be related to the physical quark mass square.

$$Q^2 = (D_W \gamma_5)^2, \quad D_W = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{adj} + (1+\gamma_\mu) U_\mu^{\dagger adj} \right]$$





$$m_q = 1/\kappa - 1/\kappa_c$$

We can fit both the string tension and the lowest eigenvalue of Q^2 with the following fitting form

$$a\left(1/\kappa - 1/\kappa_c\right)^b$$

Requiring that both the string tension and the lowest eigenvalue of Q^2 vanish at the same critical κ_c, we have $\kappa_c = 0.1694(7)$ and

String tension

$$b = 1.10(4) = \frac{2}{1+\gamma_*} \qquad \therefore \gamma_* = 0.81(8)$$

The lowest eigenvalue of Q^2

$$b = 1.28(1) \overset{?}{=} f(\gamma_*)$$

We repeat the same analysis at b=0.36

$$a\left(1/\kappa - 1/\kappa_c\right)^b$$

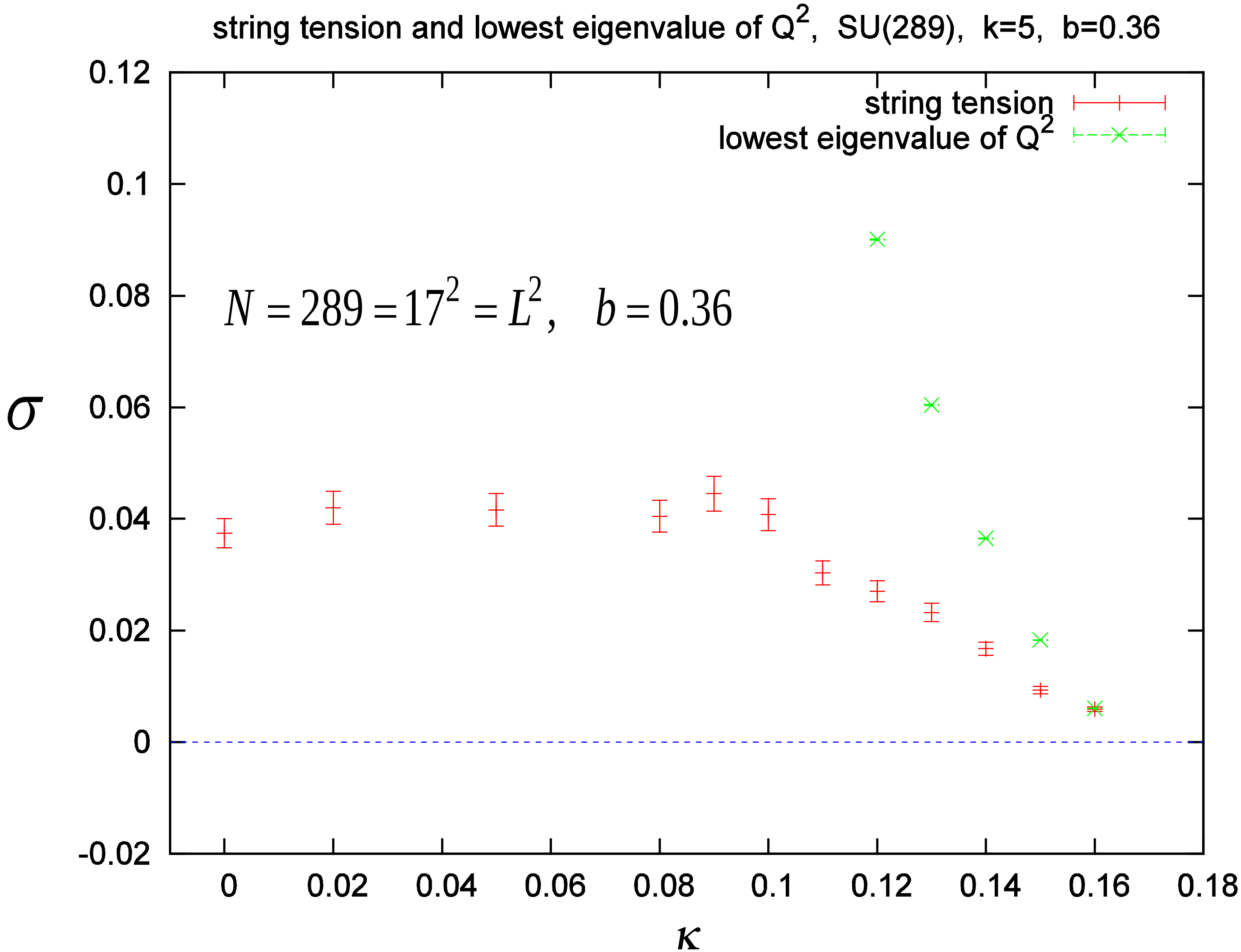
String tension

$$b = 0.92(9) = \frac{2}{1+\gamma_*} \qquad \therefore \gamma_* = 1.17(21)$$

The lowest eigenvalue of Q^2

$$b = 1.33(4)$$

with common $\kappa_c = 0.168(1)$



$m_q = 1/\kappa - 1/\kappa_c$

Plan of the talk

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- We can simulate $N_f \neq 2$ theory by using the Rational Hybrid Monte Carlo method

$$\left(Q^2\right)^{-s} = \sum_i \frac{\beta_i}{Q^2 - \alpha_i}$$

$N_f = 1$

In the large N limit, equivalent to $N_f = 2$ fundamental fermion in rank two anti-symmetric rep. For N=3, it is just two flavor QCD

$N_f = 1/2$

supersymmetric gauge theory (one majorana fermion) ?

$N_f = 3/2$

conformal or confining ?

$N_f = 5/2$

conformal or confining ?

Motivation for $N_f = 1$ adjoint fermion

In the large N limit, $N_f = 1$ adjoint fermion is equivalent to $N_f = 2$ fundamental fermion in rank two anti-symmetric rep.

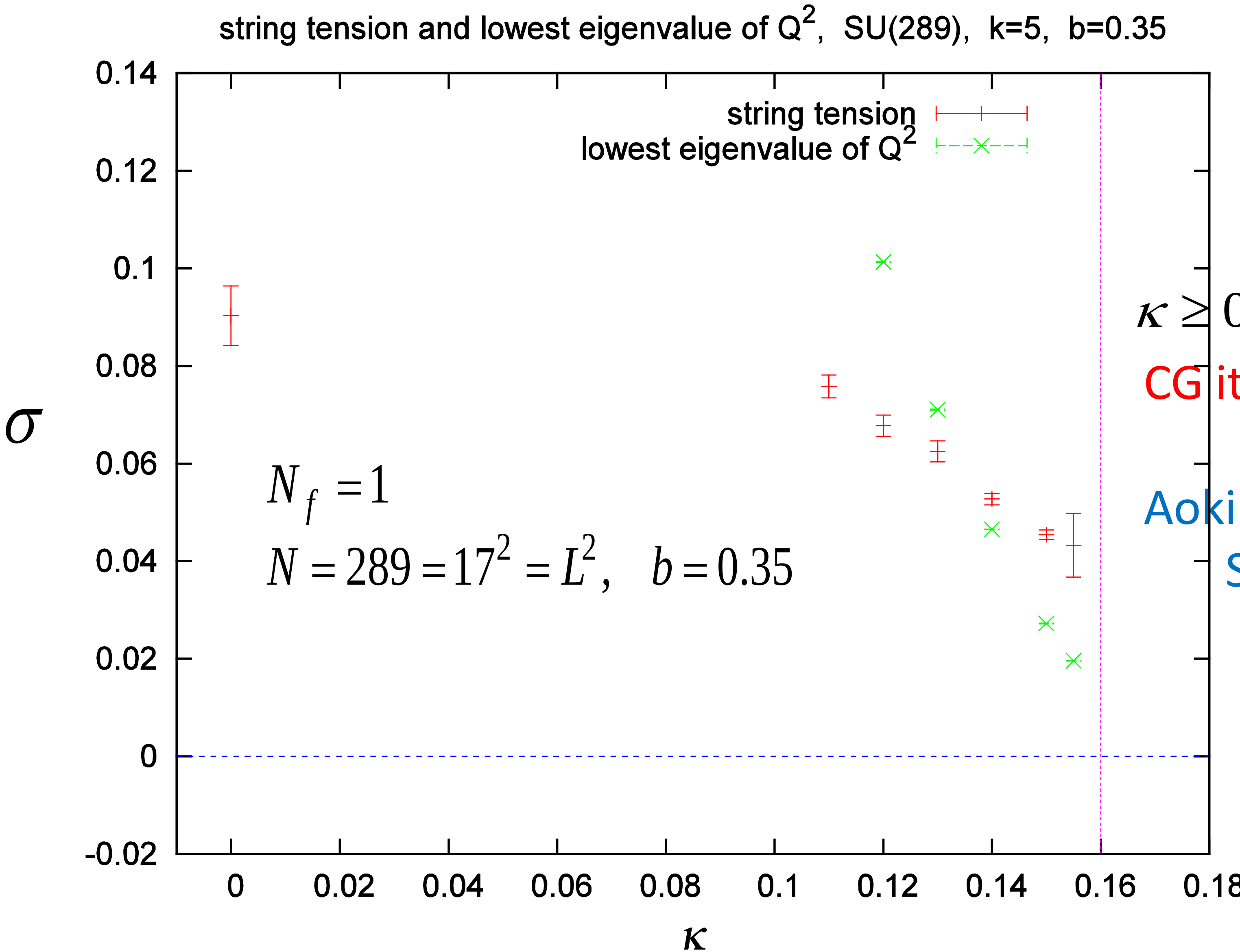
(orientifold equivalence, Kovtun, Unsal, Yaffe)

For N=3, the latter theory is just two flavor QCD and our model corresponds to Corrigan-Ramond large-N limit.

We then expect the reduced model of $N_f = 1$ adjoint fermion as

- confining theory

- there might exist
  either Aoki phase or Sharpe-Singleton phase transition
  since we use Wilson fermion.



$$N_f = 1$$

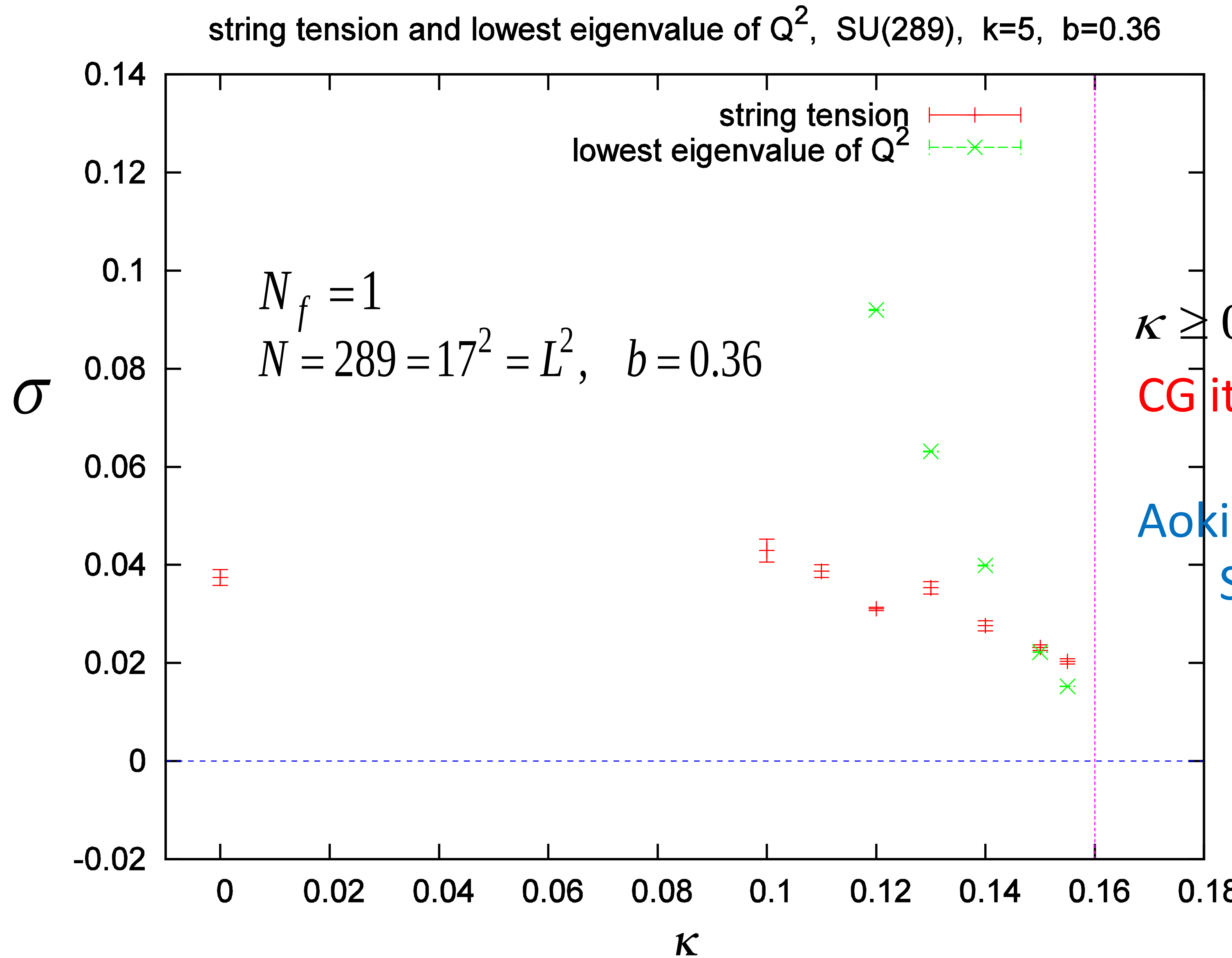
$$N = 289 = 17^2 = L^2, \quad b = 0.35$$

$\kappa \geq 0.16$

CG iteration diverges

Aoki? Sharpe-Singleton?

$$m_q = 1/\kappa - 1/\kappa_c$$



$\kappa \geq 0.16$

CG iteration diverges

Aoki? Sharpe-Singleton?

$$m_q = 1/\kappa - 1/\kappa_c$$

Conclusion

We have demonstrated that the twisted reduced model of large N QCD with adjoint Wilson fermions works quite well.

$N_f = 2$

String tension is calculated at N=289, which clearly decreases as we increase kappa and seems to vanish around kappa ∼ 0.17 in a way consistent with the theory governed by an infrared fixed point with $\gamma_* = 0.8 \sim 1.2$.

$N_f = 1$

String tension is calculated at N=289, which clearly remains finite around kappa ∼ 0.16 strongly suggesting this is the confining theory. We also find that cg iteration does not converge for kappa > 0.16.

Aoki phase or Sharpe-Singleton phase transition?

Remaining important problems

- We need to understand the finite N (finite volume) effects

  make simulation with larger $N = 529 = 23^2$

- calculate hadronic correlators

On going project

Wilson fermions with $N_f = 1/2,\ \ 1,\ \ 2$

overlap fermions with $N_f = 2$

with Garcia-Perez, Gonzalez-Arroyo, Ishikawa, Keegan