Lattice study of conformality in twelve-flavor QCD

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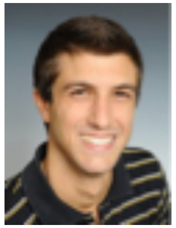

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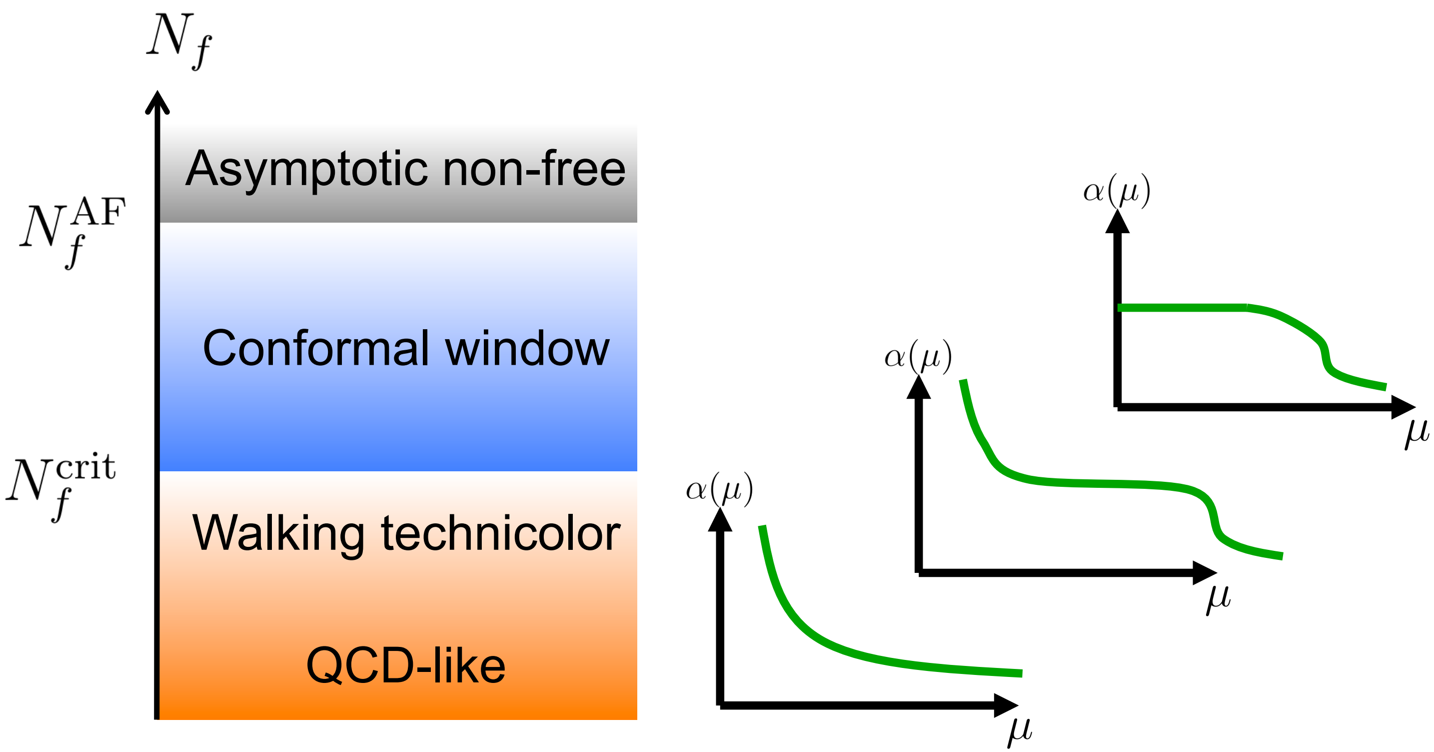
Introduction

Walking (conformal) behavior -> non-perturbative dynamics

Many flavor QCD: benchmark test of walking dynamics



- Walking technicolor (WTC) could be realized just below conformal window.
- What the value of the anomalous dimensions? Preferable value of γ is order 1.
- Rich hadron structures may be observed (LHC).

"Higgs boson"

- Higgs like particle (125 GeV) is found at LHC.

- Consistent with the Standard Model Higgs.
  But true nature is so far unknown.

- Many candidates for beyond the SM.

one possibility

– walking technicolor

  - "Higgs" = dilaton (pNGB) due to breaking of the approximate scale invariance

LatKMI (Nagoya) project (2010-)

Systematic study of flavor dependence in Large Nf QCD using single setup of the lattice simulation

Our goals:

- Understand the flavor dependence of the theory
- Find the conformal window
- Find the walking regime and investigate the anomalous dimension

Status (lattice):

- Nf=16: likely conformal
- Nf=12: controversial
- Nf=8: studies suggests no conformal behavior or walking behavior?
- Nf=4: chiral broken and enhancement of chiral condensate

This talk

talk by K.-i. Nagai (next)

Observables:

- pseudoscalar, vector meson -> a few hundred GeV-TeV in technicolor model
- Glueball (O++), (and/or flavor singlet scalar)

It this lighter than pion? If so, Good candidate of "Higgs" (techni-dilaton).

talk by E. Rinaldi

Our work

- use of improved staggered action

  Highly improved staggered quark action [HISQ]

  - to get nearly continuum results from non-zero lattice spacing
  - to reduce flavor violation for good SU(N) chiral symmetry
  - bound to N_f=4 n
- We use MILC version of HISQ action

  use tree level Symanzik gauge action

  no $(ma)^2$ improvement (no interest to heavy quarks)= HISQ/tree

- Measurement of meson spectrum

  in particular Goldstone pion mass and decay constant varying volume

KMI computer

- non GPU nodes
  - 148 nodes
  - 2x Xenon 3.3 GHz
  - 24 TFlops (peak)
- GPU nodes
  - 23 nodes
  - 3x Tesla M2050
  - 39 TFlops (peak)

simulation setup

- **SU(3), Nf=12 flavor**

- HISQ (Highly Improved Staggered Quarks)
- tree level Symanzik gauge

-> HISQ/tree

- $\beta=6/g^2=3.7$, $V=L^3 x T$: L/T=3/4; L=18, 24, 30, $0.04 \leqq m_f \leqq 0.2$
- $\beta=6/g^2=3.8$, $V=L^3 x T$: L/T=3/4; L=18, 24, 30, $0.04 \leqq m_f \leqq 0.2$ L:T =3:4
- $\beta=6/g^2=4.0$, $V=L^3 x T$: L/T=3/4; L=18, 24, 30, $0.05 \leqq m_f \leqq 0.24$

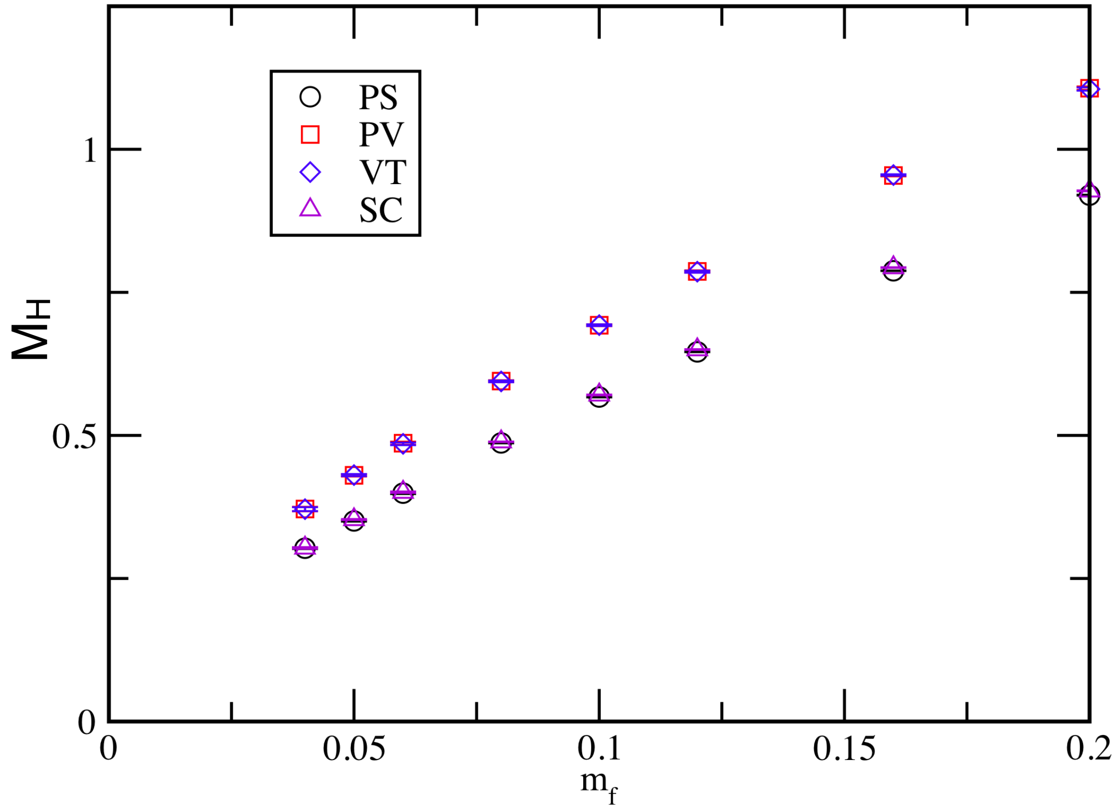
- **Statistics ~ 1000 trajectory**

N_f=12 [fundamental]

[LatKMI collab. PRD86 (2012) 054506]

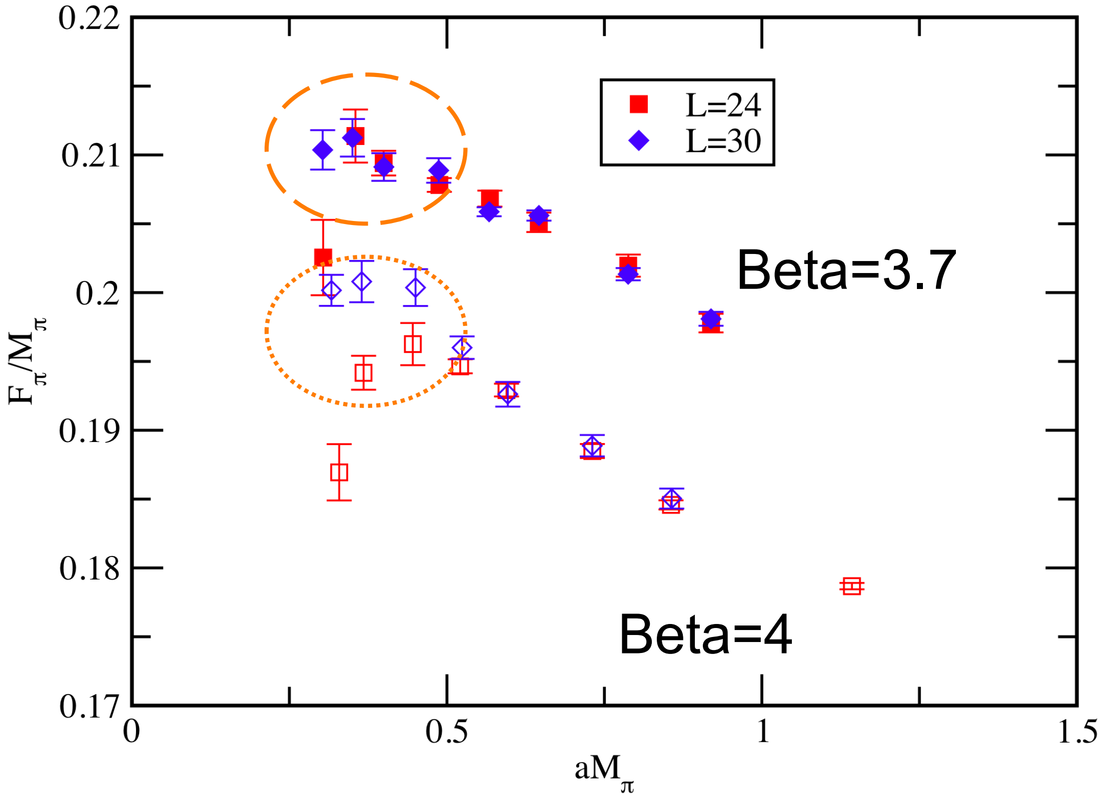
Staggered flavor symmetry for Nf=12 HISQ

Comparing masses with different staggered operators for π and ρ at β=3.7



Good staggered flavor symmetry for HISQ

Primary result, F_π/M_π vs M_π



Beta=3.7

In the small mass region, ratio is going to be flat (consistent with hyper scaling).

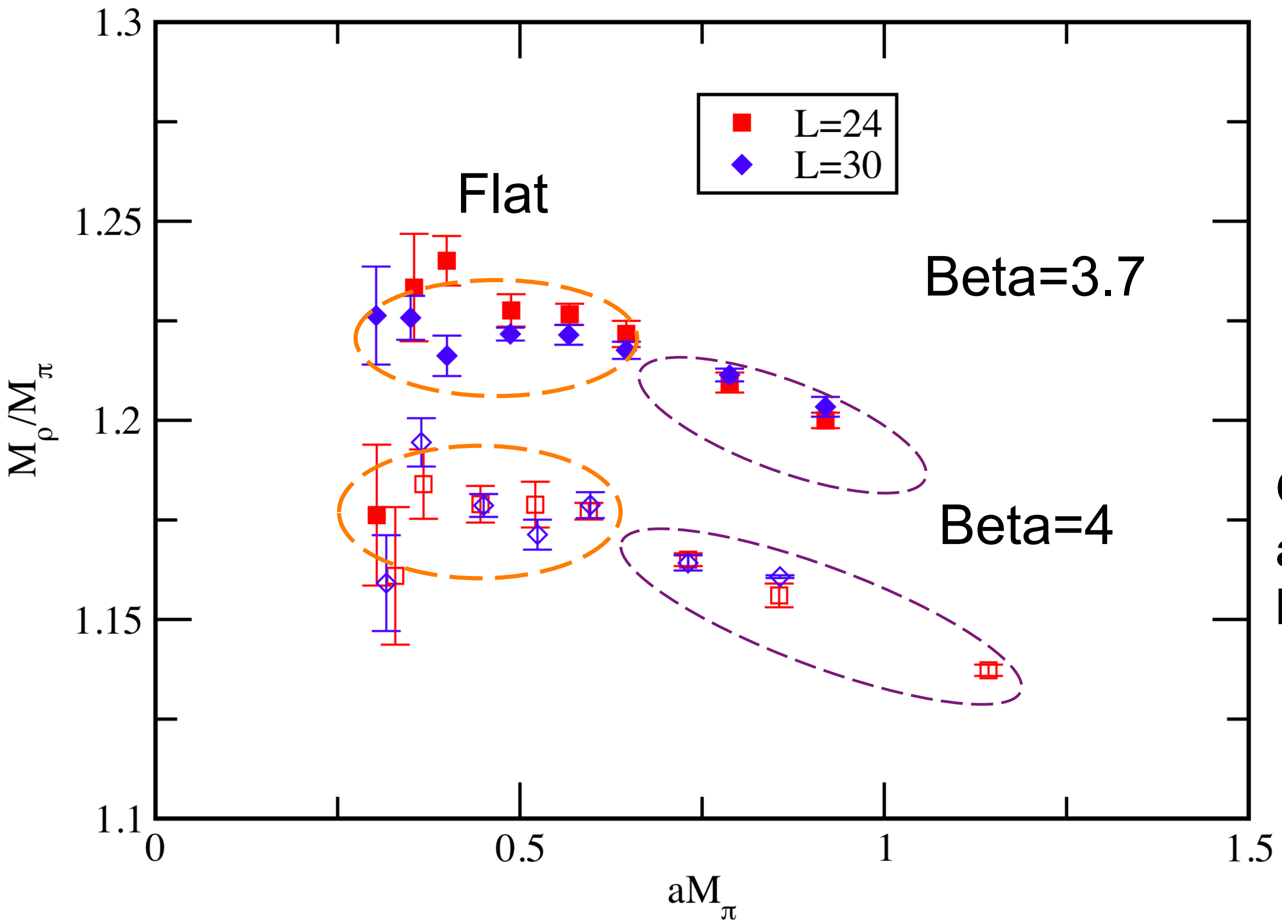
In large mass region, show the mass correction.

Beta=4.0

Data at L=30 seems to be flat, but L=24 shows large finite volume effect.

This behavior is contrast to the result for ordinary QCD system
(c.f. 4,8 flavors QCD talk by K.-I. Nagai)

$M\rho/M_\pi$ vs M_π



From naïve scale matching, one can obtain the relation

- $a(\beta=3.7) > a(\beta=4.0)$

Our result suggests asymptotically free region for beta=3.7-4.

In both beta=3.7 and 4,
Ratio is almost flat in small mass region (wider than $F\pi/M\pi$)
-> consistent with hyper scaling
Volume dependence is smaller than $F\pi/M\pi$.

In the large mass region, large mass effects show up.
$M\rho/M_\pi$ should be 1, as mf -> infinity.

Test of Finite size hyper scaling

Conformal hypothesis

- Universal behavior for all hadron masses (hyperscaling)

Mass dependence is determined by scaling dimension
(mass anomalous dimension in mass-deformed CFT.)

$$M_H \propto m_f^{\frac{1}{1+\gamma^*}} \qquad F_\pi \propto m_f^{\frac{1}{1+\gamma^*}}$$

- Finite size scaling in L^4 theory [DeGrand; Del debbio]

$$x = Lm^{1/(1+\gamma_*)}$$

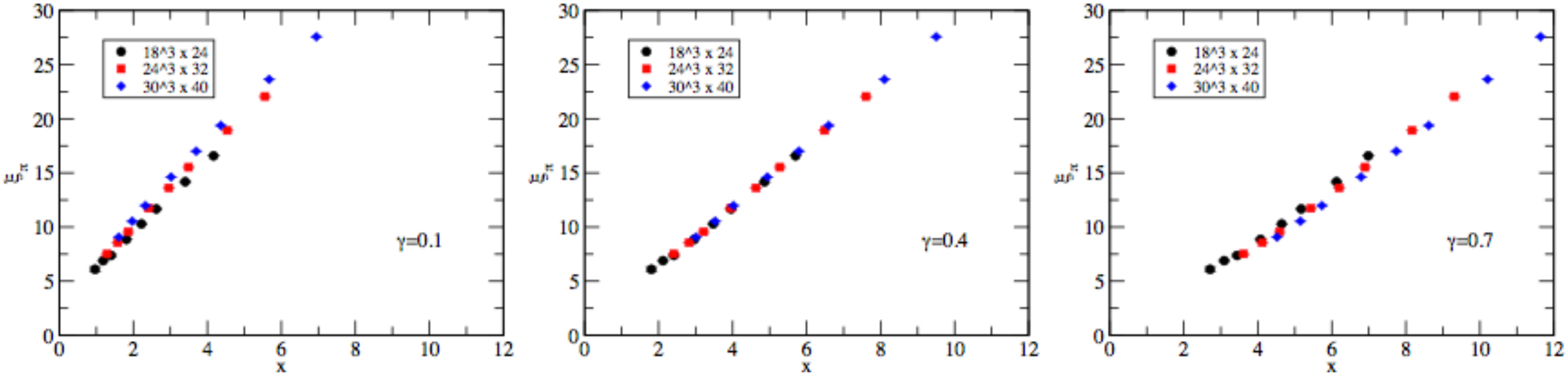
- Scaling variable:

$$L \cdot M_H = f_H(x) \qquad L \cdot F_H = f_F(x)$$

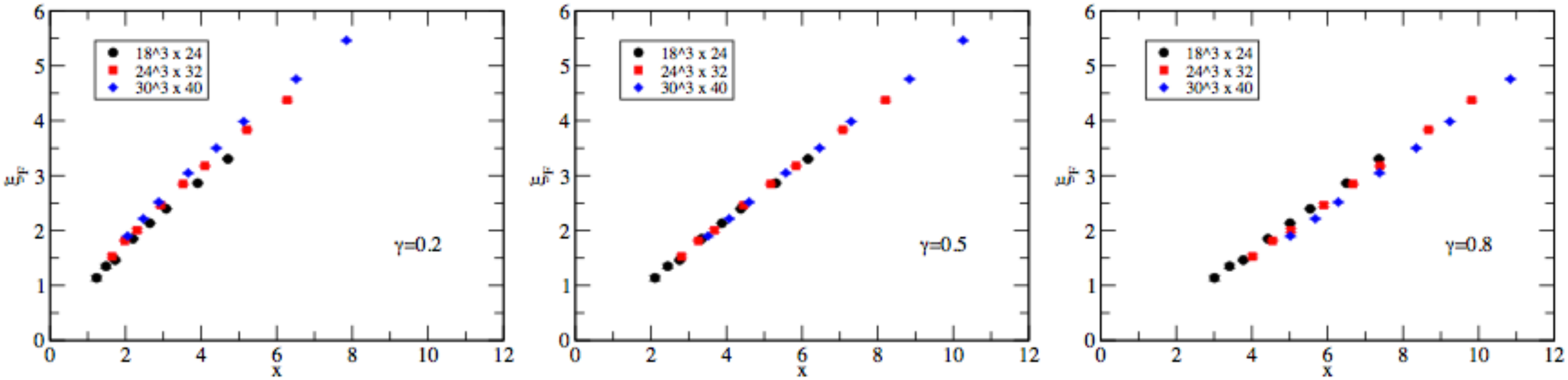
We test the finite hyper-scaling for our data at L=18, 24, 30.
If the theory is inside the conformal window,
the data should be described by one scaling parameter x.

Data alignment at a certain γ

$$\xi = LM_\pi$$



$$LF_\pi$$



$$x = L \cdot m^{1/(1+\gamma)}$$

■To quantify the alignment and the optimal γ

We define a function P(γ) to quantify how much the data "align" as a function of x

$$P(\gamma) = \frac{1}{\mathcal{N}} \sum_{L} \sum_{j \notin K_L} \frac{\left|\xi^j - f^{(K_L)}(x_j)\right|^2}{\left|\delta\xi^j\right|^2}$$

Squared error of ξ

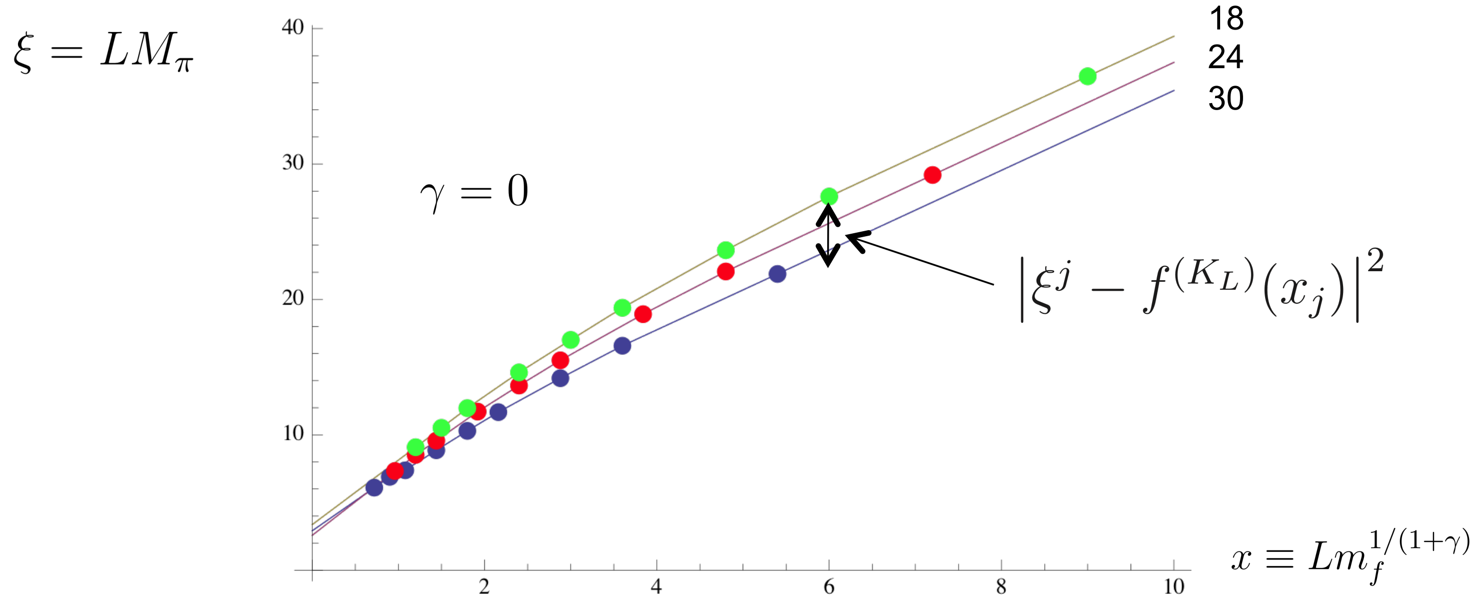
•Optimal value of γ for alignment will minimize P(γ)

•$\xi_p = LM_p$ for p=π, ρ; $\xi_F = LF_\pi$

• f(xj) : interpolation function Linear (quadratic for a systematic error)

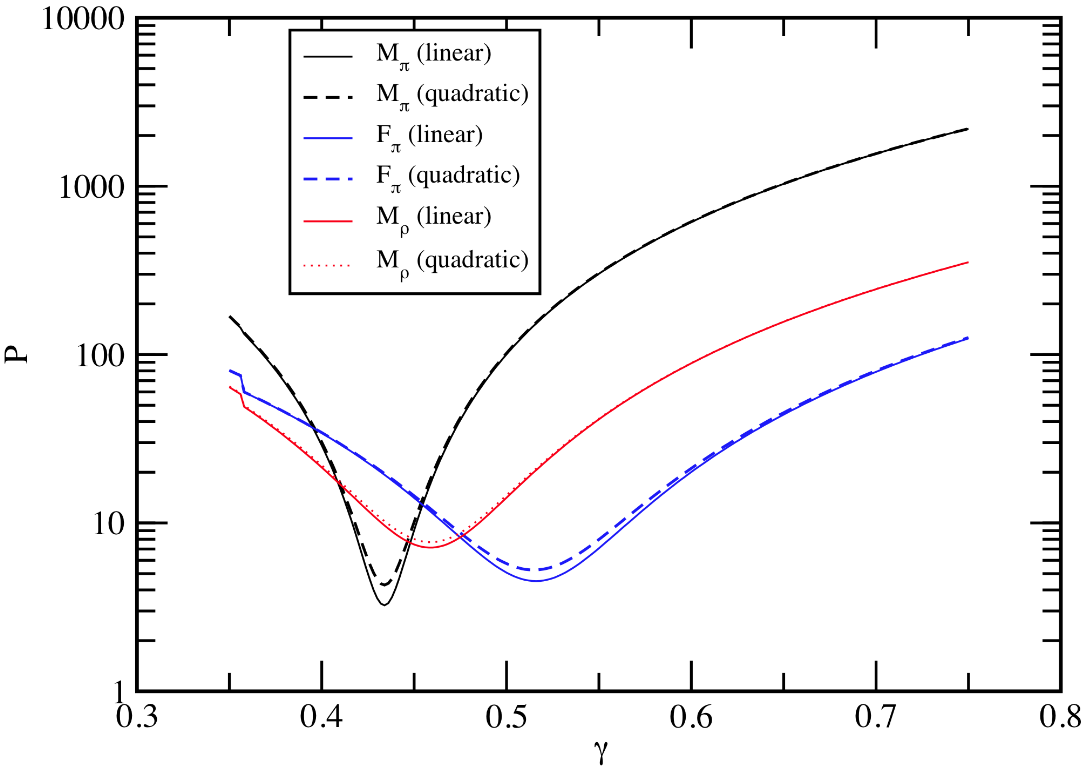
• if ξ^j is away from $f(x_j)$ by $\delta\,\xi^j$ as average → P=1

•Systematic uncertainties due to the small L and large mass estimated by studying the x and L dependence



Result of P(γ)

- P(γ) has minimum at a certain value of γ, from which we evaluate the optimal value of γ.
- At minimum, P(γ) is close to 1.



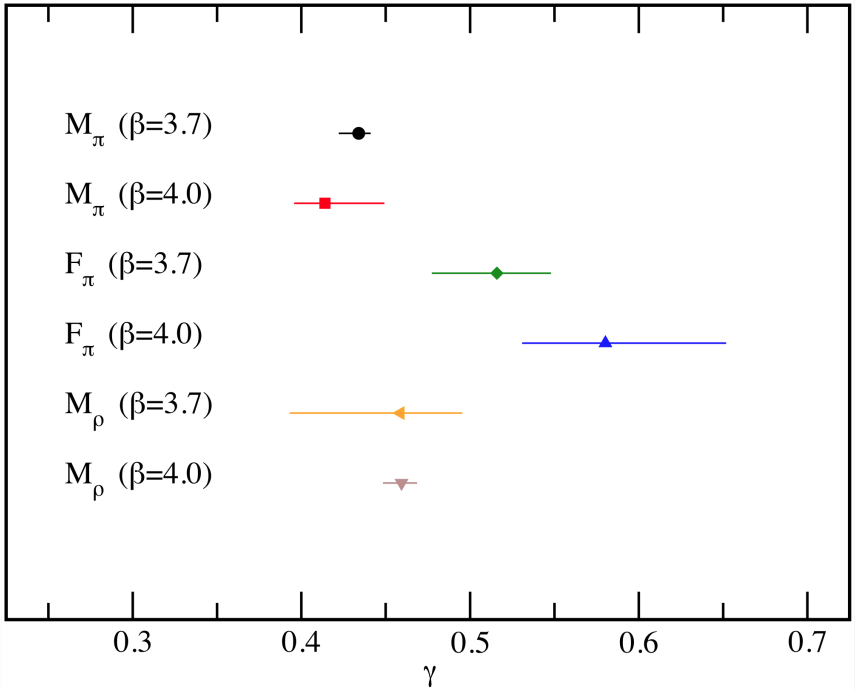
X and L dependence of γ (beta=3.7)

TABLE VII. Summary of the optimal values of γ. See the text for details.

			x			L		
quantity	β	all	range 1	range 2	range 3	(18,24)	(18,30)	(24,30)
M_π	3.7	0.434(4)	0.425(9)	0.436(6)	0.437(4)	0.438(6)	0.433(4)	0.429(8)
F_π	3.7	0.516(12)	0.481(19)	0.512(19)	0.544(14)	0.526(18)	0.514(11)	0.505(24)
M_ρ	3.7	0.459(8)	0.411(17)	0.461(10)	0.473(8)	0.491(15)	0.457(8)	0.414(18)

- γ(Mπ) is stable against the change of the mass(x) and size.
- smaller mass (range3->range1) : closer value to γ(Mπ)
- Larger volume (18,24->24,30): closer value to γ(Mπ)

■Summary of gamma



- The error -> both statistical & systematic errors
  <- estimation by changing volume & mass ranges of fit analysis

•Note: <u>Fpi data (β=4) seems to be out of scaling region due to finite mass & volume corrections. Flat range is smaller than Mρ/Mπ.</u>

•The universal hyper scaling is good; γ=0.4~0.5. except Fpi.

■Simultaneous fit with common γ for $\xi = LM_\pi,\ LF_\pi,\ LM_\rho$

•We consider simultaneous fit with finite mass (volume) correction.

$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)} \quad \cdots \text{fit a},$$

$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)} + \underline{c_2 L m_f^{\alpha}} \quad \cdots \text{fit b}.$$

• 3 (4) fit parameters : c0, c1, γ, (c2) for fit-a (b).
• fit b is hyper scaling with correction term.

two possibilities

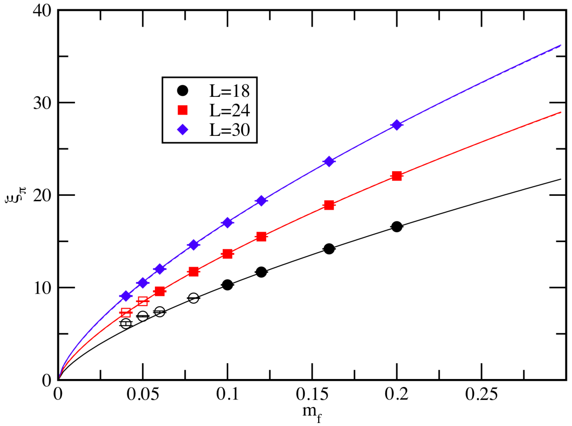
fit b-1: ladder Scwinger-Dyson eq. analysis $\alpha = (3-2\gamma)/(1+\gamma)$

(See LatKMI PRD85(2012)074502)

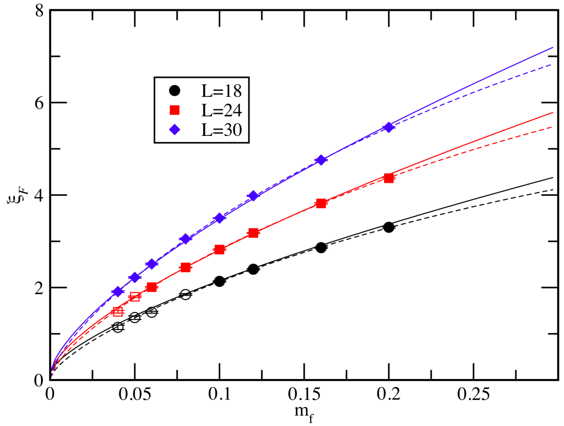
fit b-2: lattice (am)^2 artifact $\alpha = 2$

■Results (beta=3.7)

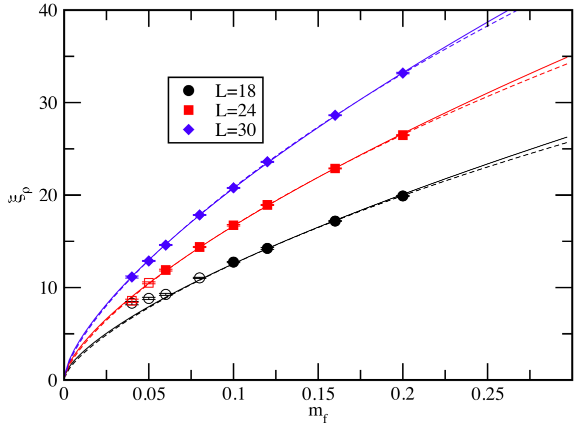
LM_π



LF_π



$LM\rho$



- The data with empty symbols are not used in the fit

$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)} \quad \cdots \text{fit-A},$$

$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)} + c_2 L m_f^{\alpha} \quad \cdots \text{fit-B}.$$

	γ	α	χ^2/dof
fit-A	0.449(3)	-	4.52
fit-B1	0.411(9)	$\frac{(3-2\gamma)}{(1+\gamma)}$	1.23
fit-B2	0.423(7)	[2]	1.15

Fit results

- **Simultaneous fit with hyper scaling (fit-A) is not bad.**
- **The mass correction fits (fit-B) for both α work to improve chi^2**
- **The resultin γ is consistent with previous analysis (P(γ))**

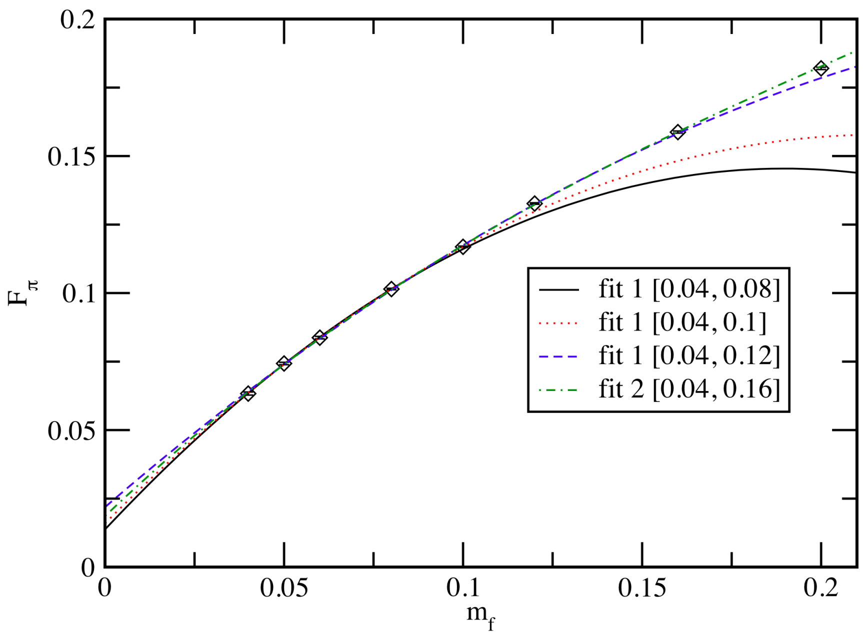
Short summary

- β=3.7-4.0: consistent with being in the asymptotically free regime
- $M\pi$, $F\pi$, $M\rho$ show conformal hyper scaling
- The resulting γ's for different quantities and different lattice spacing (β) are consistent except $F\pi$
- $F\pi$: large mass corrections in our mass parameters, likely too heavy mf to be neglect.
- The mass correction terms for simultaneous fit work to improve the accuracy. The universal γ can be obtained for $M\pi$, $F\pi$, $M\rho$.

ChPT fit

■Fit result on Fpi (beta=3.7)

$$\begin{cases} c_0 + c_1 m_f + c_2 m_f^2 & \cdots \text{fit1} \\ c_0 + c_1 m_f + c_2 m_f^2 + c_3 m_f^3 & \cdots \text{fit2} \end{cases}$$

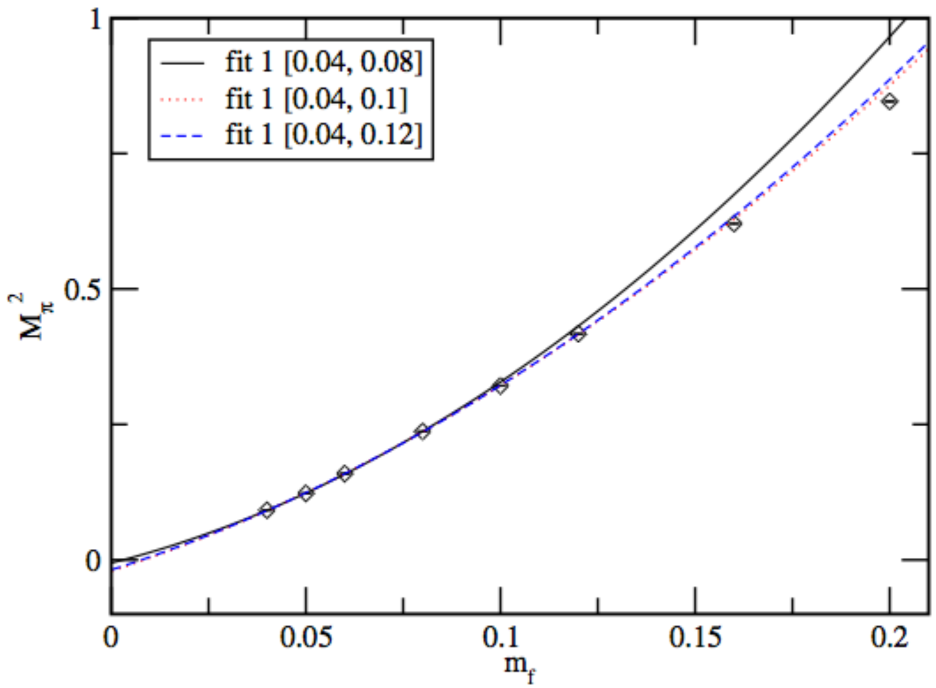


fit range	c_0	c_1	c_2	χ^2/dof
$[0.04, 0.08]$	$0.0190(52)$	$1.21(18)$	-2.2(1.5)	0.29
$[0.04, 0.1]$	$0.0162(30)$	$1.31(85)$	-3.01(58)	0.37
$[0.04, 0.12]$	$0.0231(18)$	$1.093(48)$	-1.51(29)	3.30

Fπ

- **Polynomial fit with second order is reasonable for small fermion mass range.**
- **Fπ in the chiral limit is tiny non-zero (c0>0).**

■Fit result on pion mass (beta=3.7)



$$M_\pi^2 = \begin{cases} c_0 + c_1 m_f + c_2 m_f^2 & \cdots \text{fit1} \\ c_0 + c_1 m_f + c_2 m_f^2 + c_3 m_f^3 & \cdots \text{fit2} \end{cases}$$

fit range	c_0	c_1	c_2	χ^2/dof
[0.04, 0.08]	-0.0057(91)	1.82(32)	15.2(2.6)	1.35
	[0]	1.62(3)	16.76(45)	0.88
[0.04, 0.1]	-0.0209(48)	2.37(15)	10.6(1.1)	2.59
	[0]	1.729(21)	14.99(25)	8.33
[0.04, 0.12]	-0.0183(31)	2.28(87)	11.21(55)	1.90
	[0]	1.780(17)	14.28(17)	10.29

Fit results

- **Negative constant C0.**
  For the smallest mass range, it is consistent with 0.

Note on CHPT fit in many flavor QCD

- **Natural chiral expansion parameter is** $$\boxed{\chi = N_f \left(\frac{M_\pi}{4\pi F}\right)^2}$$

[M. Soldate and R. Sundrum, Nucl.Phys.B340,1 (1990)],
[R. S. Chivukula, M. J. Dugan and M. Golden, Phys. Rev. D47,2930 (1993)]

The parameter χ should be less than 1 to be consistent with ChPT expansion.
χ ~ 2 at our lightest mass point and χ >39 using F in the chiral limit.

->It is difficult to tell real chiral behavior. e.g. Fπ in the chiral limit, if it exists.

c.f. large volume data [Fodor et. al., Phys. Lett. B703, 348(2011), arXiv:1104.3124]

- Their value of χ ~ 33
  using Fπ=0.00784 at the chiral limit and Mπ=0.1647 at the smallest mass.

Summary

•Large Nf SU(3) gauge theory is being investigated in LatKMI project.
•We focus on the Nf=12 case.

•We measure the pion (NG-boson) mass, decay constant and rho meson mass.
•Finite size hyper scaling is observed.
•Nf=12 is consistent with conformal gauge theory.
•The mass correction terms improve the fit.
•Scalar bound-state is important for the Higgs search.
-> Glueball (flavor singlet meson) spectrum, talk by E. Rinaldi

•ChPT expansion is not valid, expandsion parameter is much larger than 1. (Not yet exclude chiral broken scenario (very small Fπ))

•The resulting universal γ ~0.4-0.5 (not favored as WTC)
How about other # of fermions??
-> e.g. 8 flavor case, talk by K.-i. Nagai (next!)

END

Thank you