Dynamical model based on hydrodynamics for relativistic heavy ion collisions

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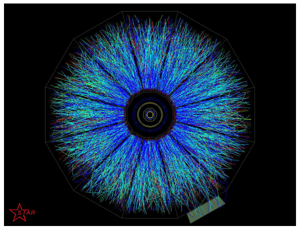
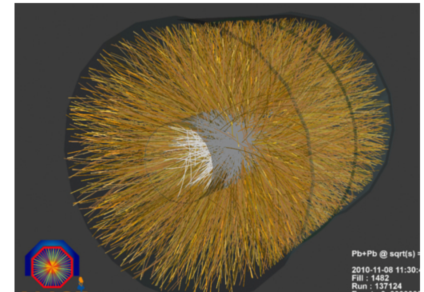
Relativistic Heavy Ion Collisions

RHIC:2000

Strongly interacting QGP
- Relativistic hydrodynamics
- Recombination model
- Jet quenching
- Color Glass Condensate



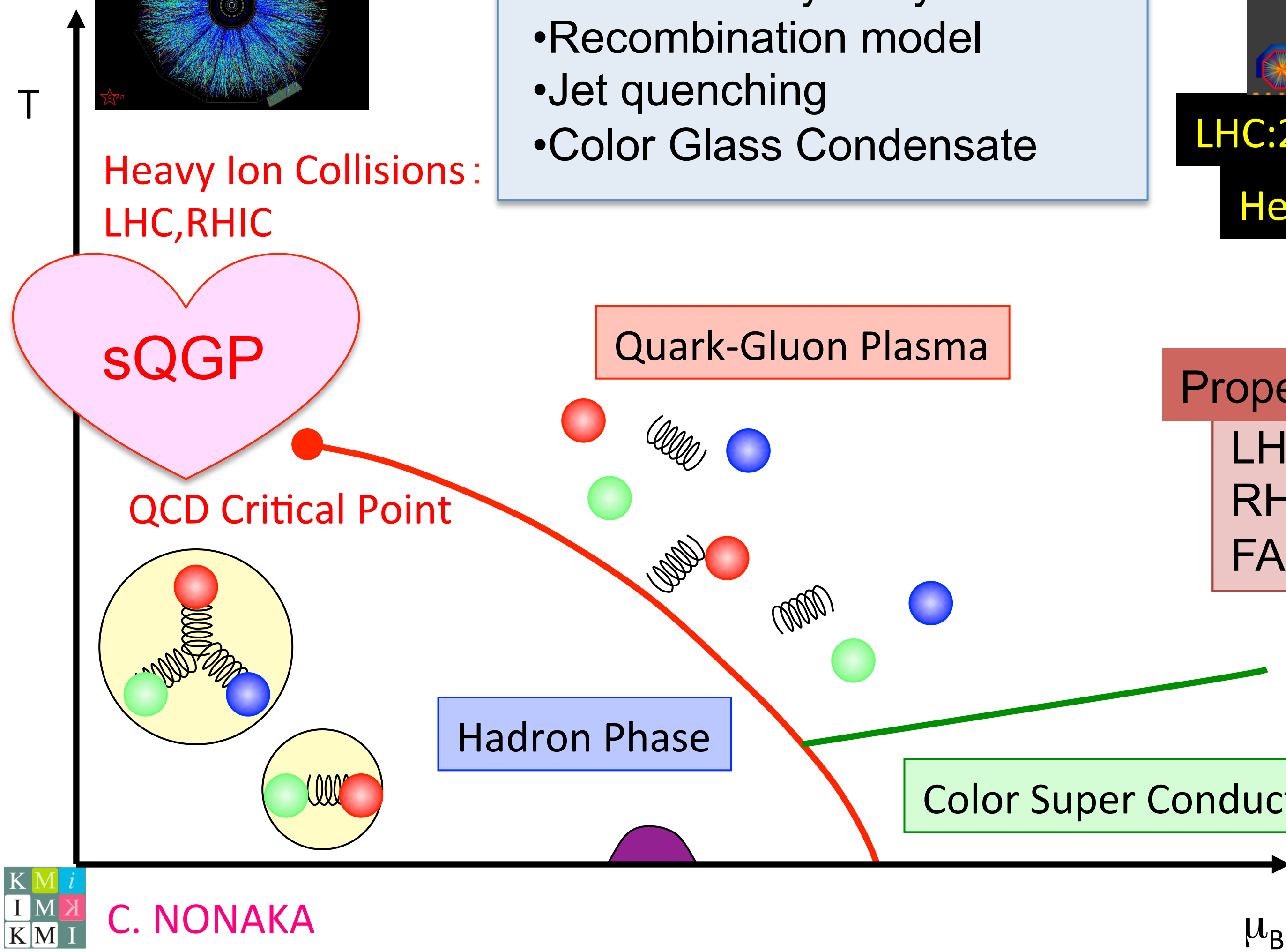


LHC:2010

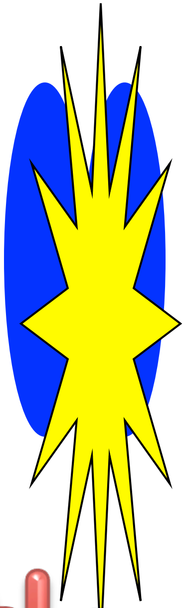
Heavy Ion collisions Start!

Property of QGP

LHC: Energy frontier
RHIC: energy scan
FAIR, NICA : high density



Dynamics of Heavy Ion Collisions



Little Bang

QGP on the earth

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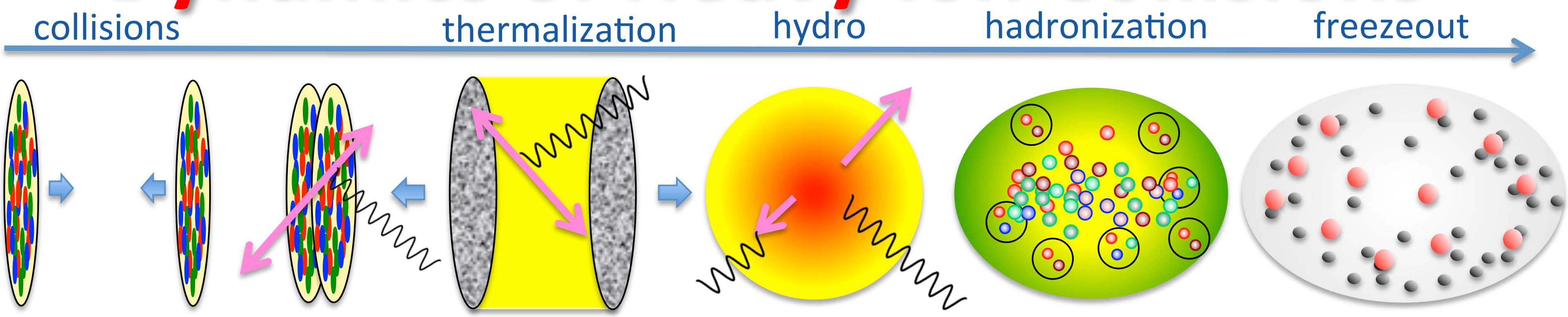
Dynamics of Heavy Ion Collisions



Observables: a lot of experimental data at RHIC and LHC

photons/leptons

bulk property

Jets

heavy quarkonia

Comprehensive understanding: Dynamical model

sQGP

Initial condition → **Hydrodynamic model** → **Freezeout process**

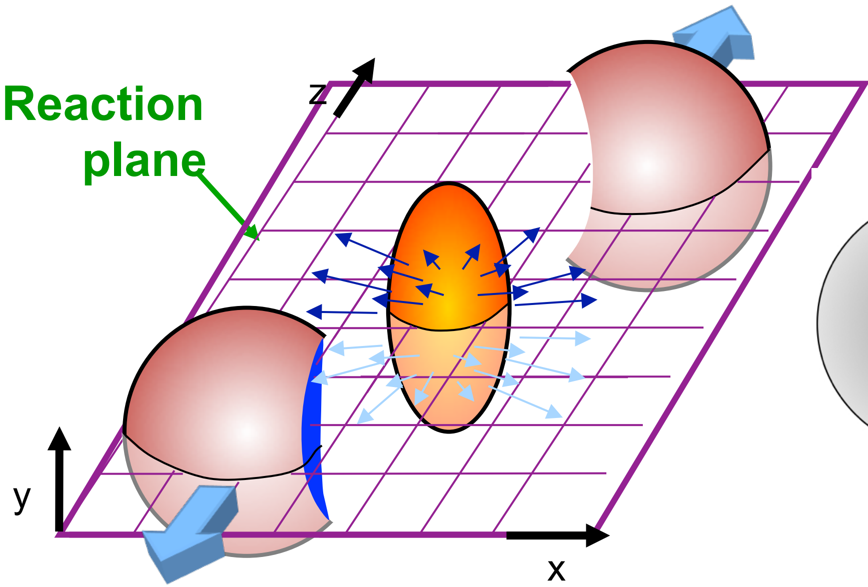
fluctuating Initial condition

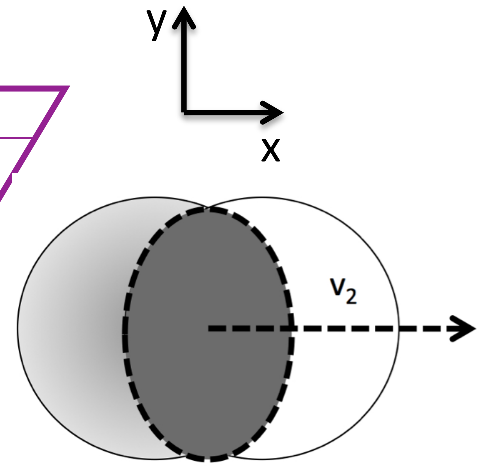
Higher harmonics

viscous, shock wave
Equation of state

final state interactions

Higher Harmonics

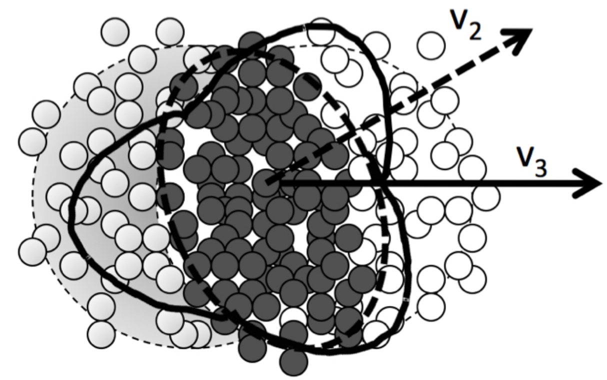




elliptic flow

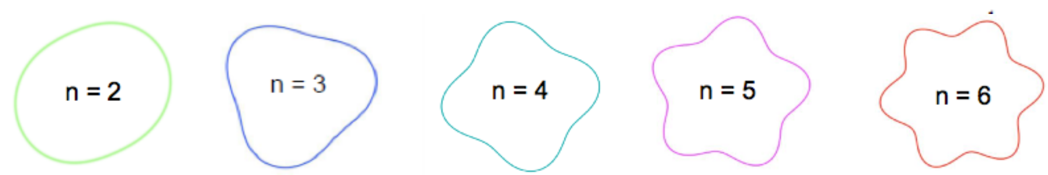
more realistic

event by event
fluctuations



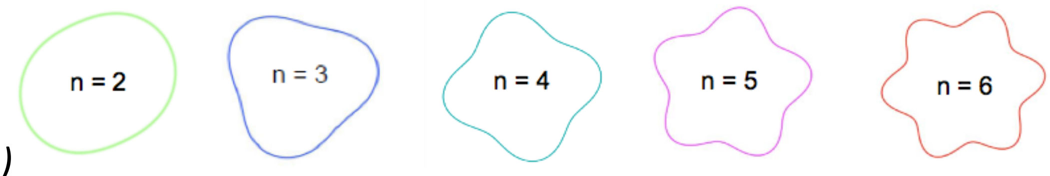
higher harmonics

$$\frac{dN}{dyd\phi} \propto 1+2v_1\cos(\phi-\Theta_1)+2v_2\cos 2(\phi-\Theta_2)+2v_3\cos 3(\phi-\Theta_3)+2v_4\cos 4(\phi-\Theta_4)+\cdots$$

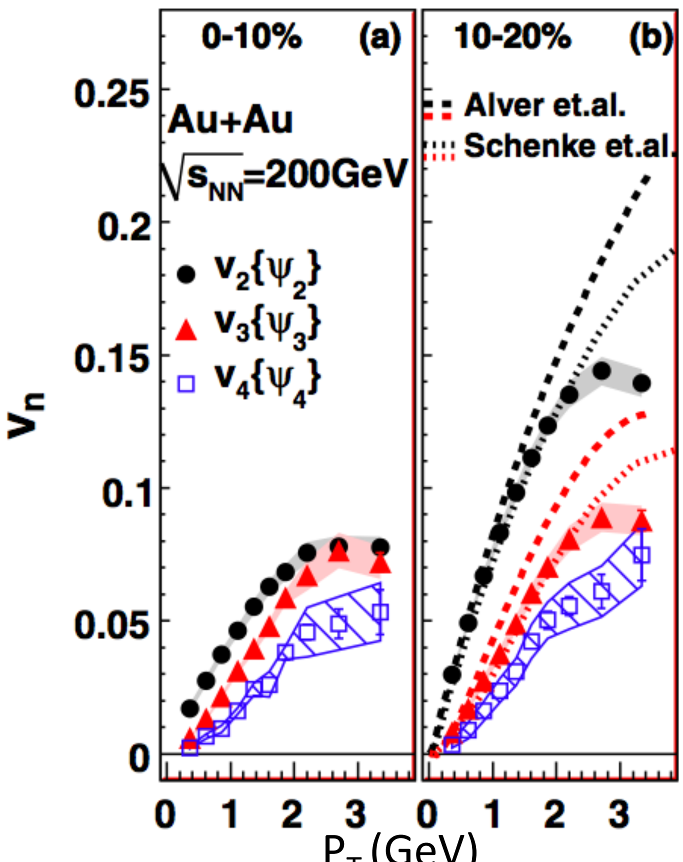


Higher Harmonics @ RHIC & LHC

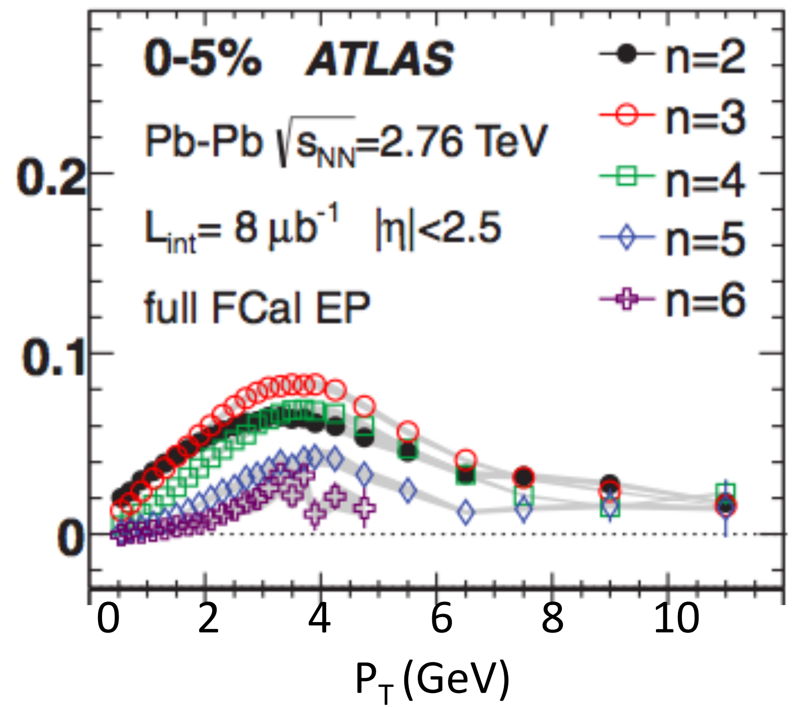
$$\frac{dN}{dyd\phi} \propto 1+2v_1\cos(\phi-\Theta_1)+2v_2\cos 2(\phi-\Theta_2)+2v_3\cos 3(\phi-\Theta_3)+2v_4\cos 4(\phi-\Theta_4)+\cdots$$



PHENIX@RHIC, PRL107,252301(2011)



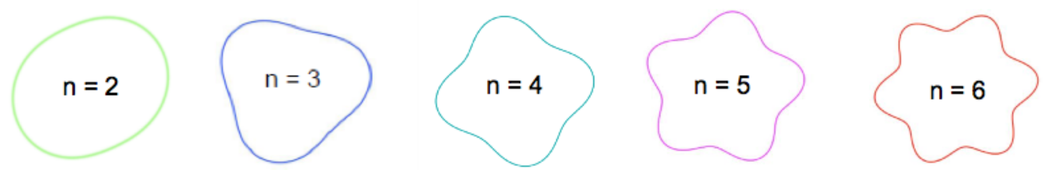
ATLAS@LHC, PRC86,014907(2012)



Challenge to Hydrodynamic Model

- Higher harmonics

$$\frac{dN}{dyd\phi} \propto 1 + 2v_1 \cos(\phi - \Theta_1) + 2v_2 \cos 2(\phi - \Theta_2) + 2v_3 \cos 3(\phi - \Theta_3) + 2v_4 \cos 4(\phi - \Theta_4) + \cdots$$



Challenge to relativistic hydrodynamic model

Initial fluctuations → shock wave

Viscosity effect → viscous hydrodynamics

Higher harmonics → high accuracy calculations

Longitudinal structure → (3+1) dimensional

⇒

State-of-the-art numerical algorithm

- Shock-wave treatment
- Less numerical viscosity

Viscous Hydrodynamic Equation

- Energy and momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = q^\mu u^\nu + q^\nu u^\mu + \tau^{\mu\nu}$$:viscous effect

  – First order in gradient: acausality
  – Second order in gradient:
    - Israel-Stewart
    - Ottinger and Grmela
    - AdS/CFT
    - Grad’s 14-momentum expansion
    - Renomarization group

?

Most stable equation
for relativistic heavy ion collisions

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Numerical Scheme

- Lessons from wave equation
  - First order accuracy: large dissipation
  - Second order accuracy : numerical oscillation

    -> artificial viscosity, flux limiter
- Hydrodynamic equation
  - Shock-wave capturing schemes: Riemann problem
    - Godunov scheme: analytical solution of Riemann problem, Our scheme
    - SHASTA: the first version of Flux Corrected Transport algorithm, Song, Heinz, Chaudhuri
    - Kurganov-Tadmor (KT) scheme, McGill

Current Status of Hydro

Table I. Ideal hydrodynamical models. In the table, we use the following abbreviation. IC: initial condition, G: Glauber model, CGC: color glass condensate, MC-G: Monte Carlo Glauber model, MC-CGC: Monte Carlo CGC, lQCD: lattice QCD inspired EoS, SPH: smoothed particle hydrodynamics, PPM: piecewise parabolic method, CE: continuous emission, Obs: calculated observables, and PD: particle distribution.

Ref.	dim	IC	EoS	scheme	freezeout	Obs
Hama[3)]	3+1	NeXus	Bag model	SPH	CE	PD, v_2, HBT
Hirano[4)]	3+1	G, CGC	Bag model	PPM[**)]	cascade(JAM)	v_2
Nonaka[8)]	3+1	G	Bag model	Lagrange	cascade(UrQMD)	PD, v_2
Hirano[9), 10)]	3+1	MC-G, MC-CGC	lQCD	PPM[**)]	cascade(JAM)	v_2
Petersen[11)]	3+1	UrQMD	hadron gas	SHASTA	cascade(UrQMD)	PD
Holopainen[12)]	2+1	MC-G	lQCD	SHASTA	resonance decay	v_2

Table II. Viscous hydrodynamical models. In the table, we use the following abbreviation. CD: central difference, and KT: Kurganov-Tadmor (KT) scheme.

Ref.	dim	IC	EoS	scheme	freezeout	Obs.
Romatschke[13)]	2+1	G	lQCD	CD	single T_f	v_2
Dusling[14)]	2+1	G	ideal gas	–	viscous correction	v_2
Luzum[15)]	2+1	G, CGC	lQCD	CD	resonance decay	v_2
Schenke[16)]	3+1	MC-G	lQCD	KT	viscous correction	v_2, v_3
Song[17)]	2+1	MC-G, MC-CGC	lQCD	SHASTA	cascade(UrQMD)	v_2
Chaudhuri[18), 19)]	2+1	G	Bag model	SHASTA	viscous correction	v_2
Bozek[20)]	3+1	G	lQCD	–	THERMINATOR2	v_1, v_2, HBT

Our Approach

Takamoto and Inutsuka, arXiv:1106.1732

- Israel-Stewart Theory

(ideal hydro)

1. dissipative fluid dynamics = advection + dissipation

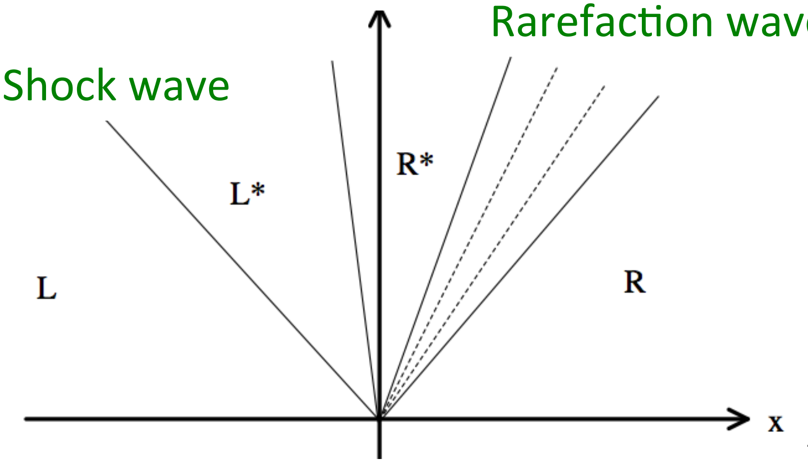
Riemann solver: Godunov method

exact solution

Contact discontinuity

Rarefaction wave

Shock wave



Two shock approximation

Mignone, Plewa and Bodo, Astrophys. J. S160, 199 (2005)

Rarefaction wave ⟶ shock wave

Akamatsu, Nonaka, Takamoto, Inutsuka, in preparation

2. relaxation equation = advection + stiff equation

Numerical Scheme

- Israel-Stewart Theory

Takamoto and Inutsuka, arXiv:1106.1732

1. Dissipative fluid equation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \tau^{\mu\nu}$$
$$= T_{\rm ideal} + T_{\rm dissip}$$

$$\partial_t U + \nabla \cdot F(U) = 0 \qquad U = U_{\rm ideal} + U_{\rm dissip}$$

2. Relaxation equation

$$\hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi,$$

$$\hat{D}\pi^{\mu\nu} = \frac{1}{\tau_\pi}(\pi^{\mu\nu}_{NS} - \pi^{\mu\nu}) - I^{\mu\nu}_\pi,$$

$$\hat{D}q^\mu = \frac{1}{\tau_q}(q^\mu_{NS} - q^\mu) - I^\mu_q,$$

$$\hat{D} = u^\mu \partial_u$$

I: second order terms $\quad \tau^{\mu\nu} = \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}$

$$\left(\frac{\partial}{\partial t} + v^j \frac{\partial}{\partial x^j}\right)\Pi = -\frac{I_\Pi}{\gamma}, \quad + \quad \frac{\partial}{\partial t}\Pi = \frac{1}{\gamma\tau_\Pi}(\Pi_{NS} - \Pi),$$

advection + stiff equation

$$\Delta t < \tau_{\rm relax} << \tau_{\rm fluid}$$

Relaxation Equation

Takamoto and Inutsuka, arXiv:1106.1732

- Numerical scheme

$$\hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi,$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + v^j \frac{\partial}{\partial x^j}\right)\Pi = -\frac{I_\Pi}{\gamma}, \quad + \quad \frac{\partial}{\partial t}\Pi = \frac{1}{\gamma\tau_\Pi}(\Pi_{NS} - \Pi),$$

advection

stiff equation

$$\Delta t < \tau_{\mathrm{relax}} << \tau_{\mathrm{fluid}}$$

up wind method

- during Δt Π_{NS} ~constant

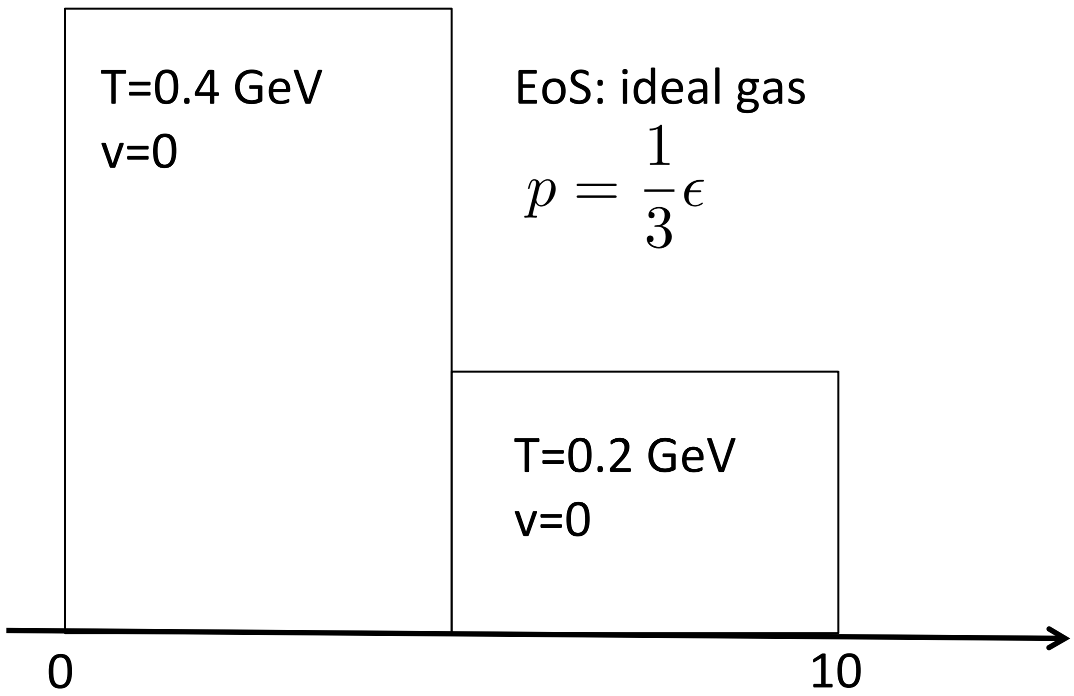
Piecewise exact solution

$$\Pi = (\Pi_0 - \Pi_{NS})\exp\left[-\frac{t - t_0}{\tau_\Pi}\right] + \Pi_{NS}$$

fast numerical scheme

Comparison

- Shock Tube Test : Molnar, Niemi, Rischke, Eur.Phys.J.C65,615(2010)

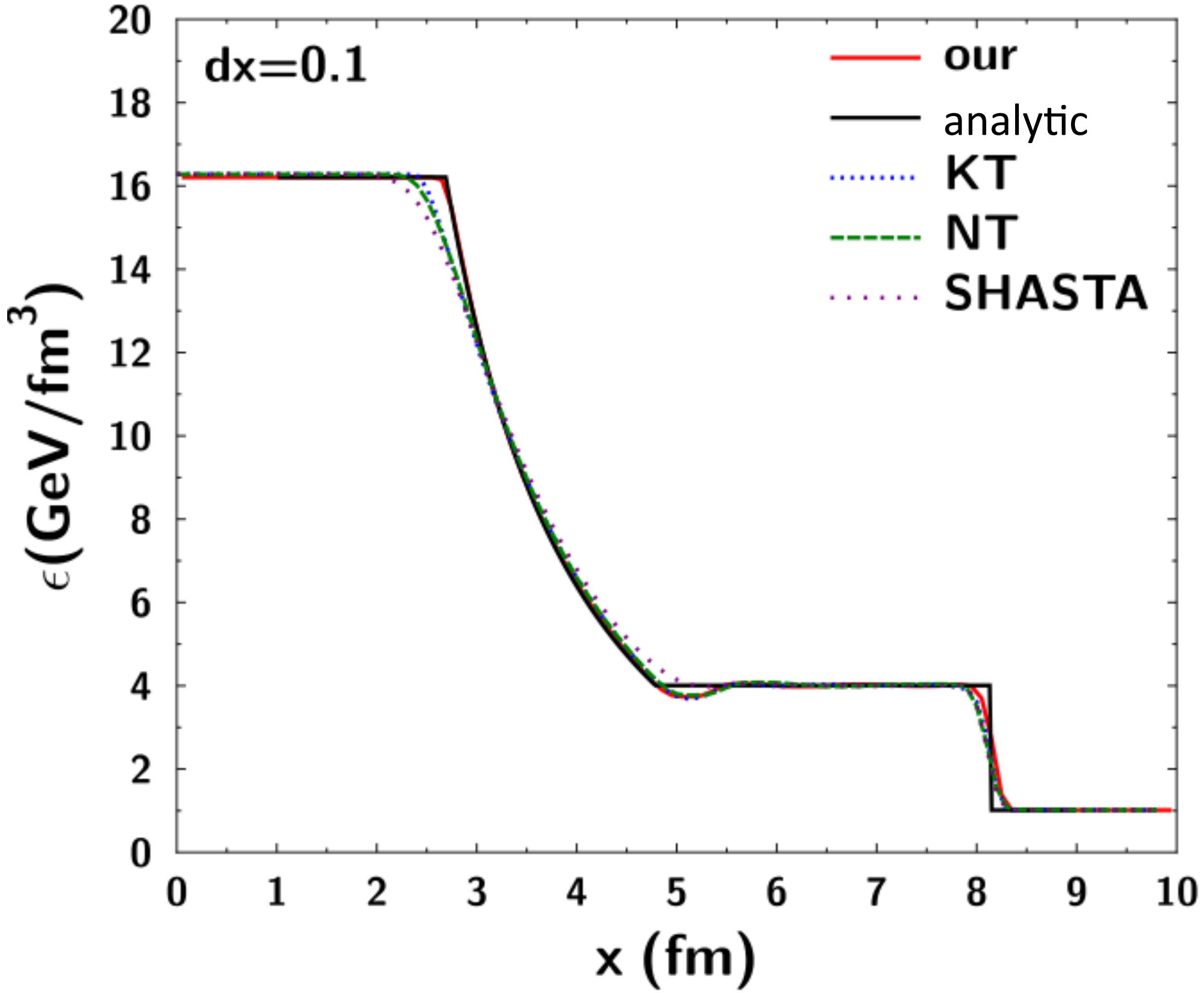


Nx=100, dx=0.1

- Analytical solution
- Numerical schemes
  SHASTA, KT, NT
  Our scheme

Energy Density

t=4.0 fm dt=0.04, 100 steps



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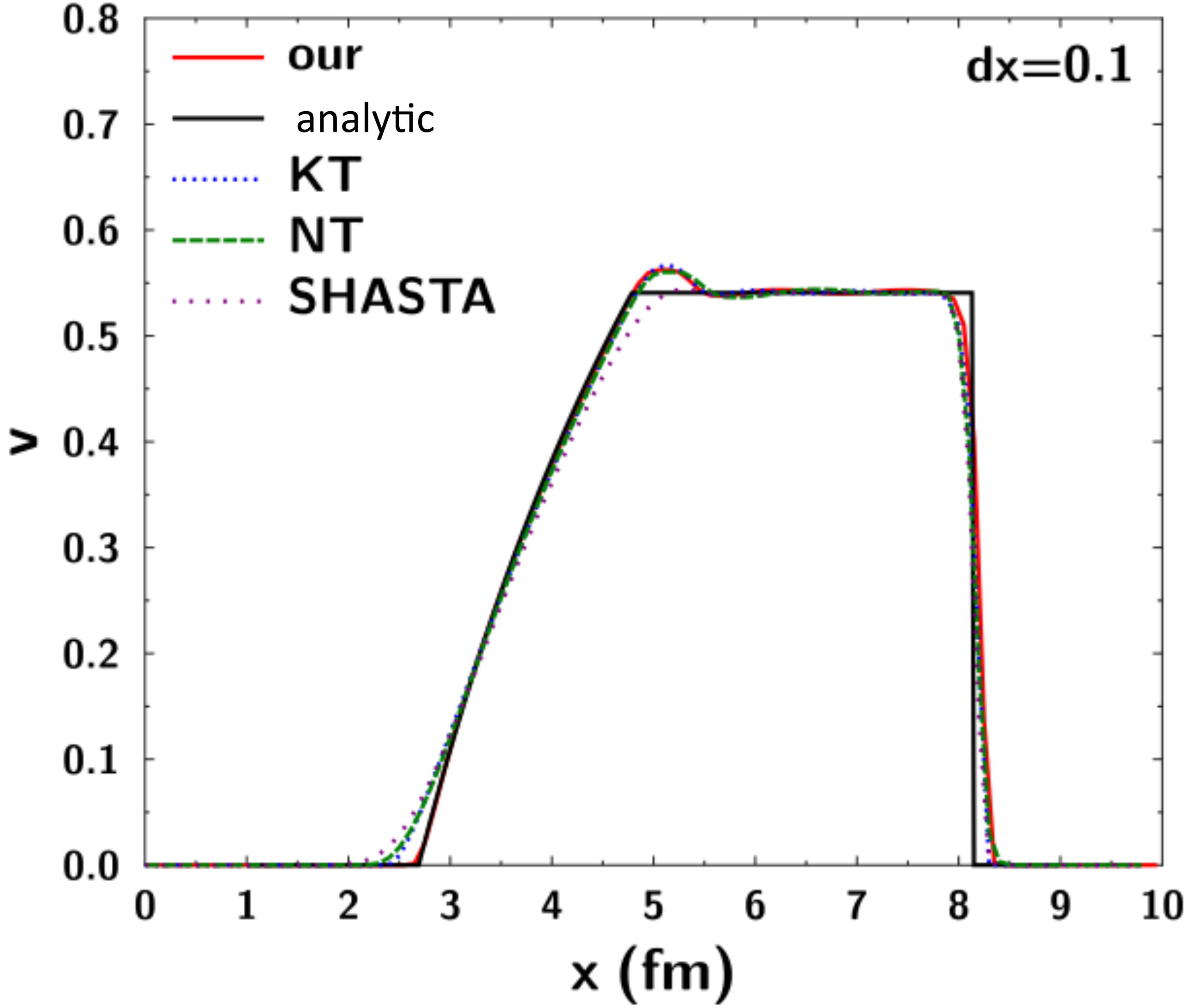
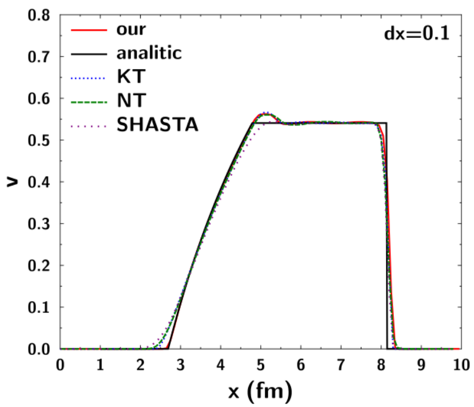
Velocity

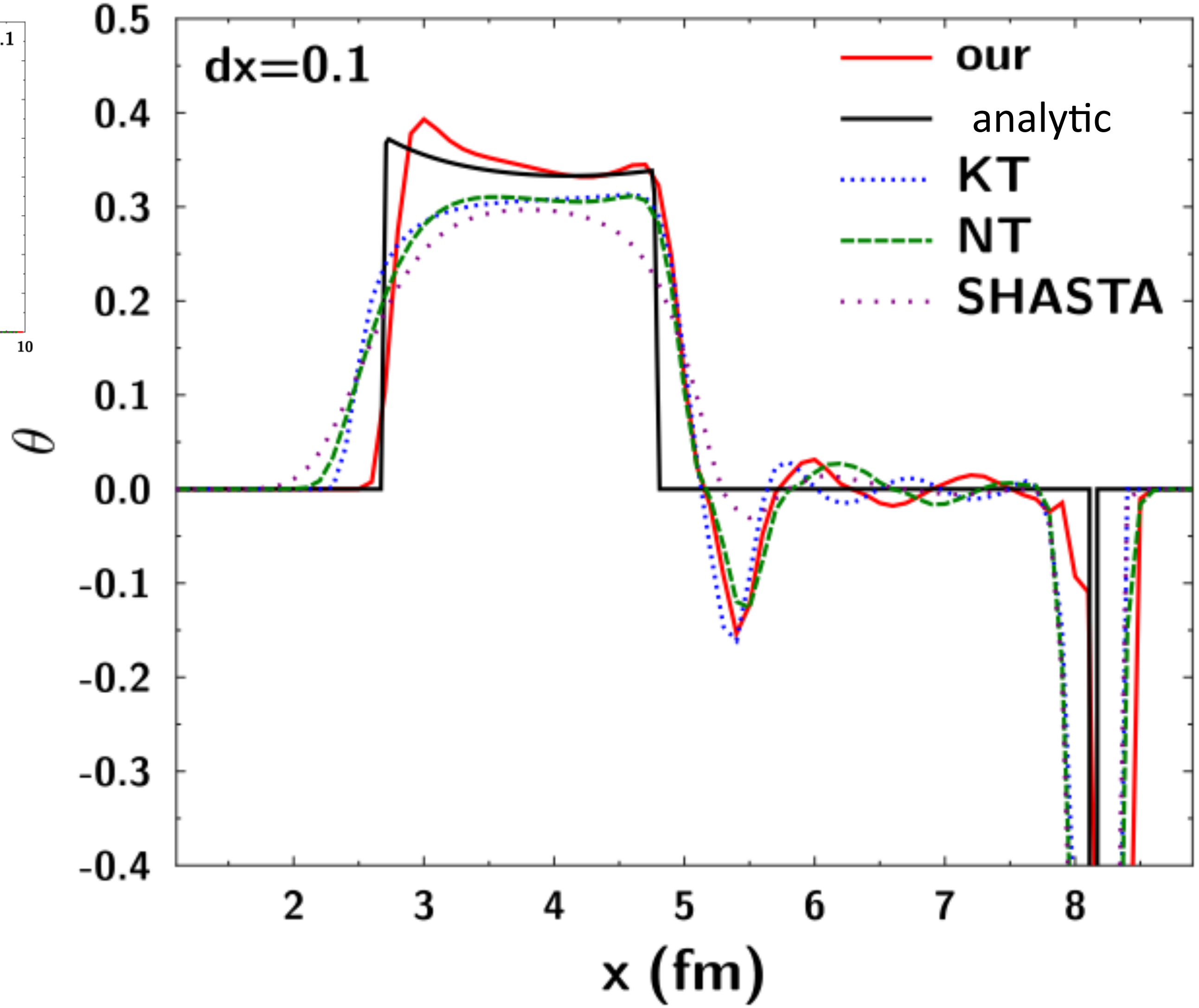
t=4.0 fm dt=0.04, 100 steps



θ

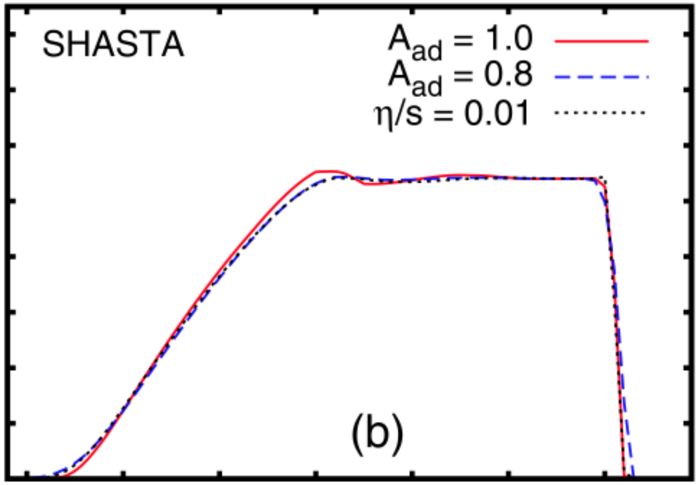
t=4.0 fm dt=0.04, 100 steps

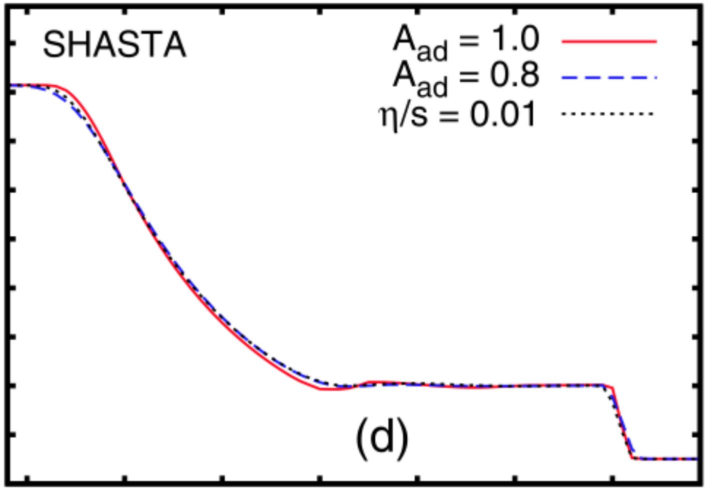


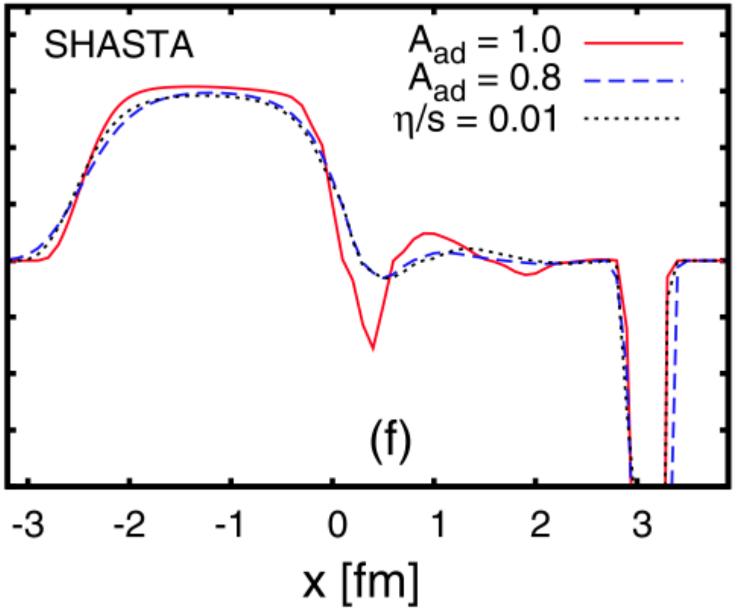


Artificial and Physical Viscosities

Molnar, Niemi, Rischke, Eur.Phys.J.C65,615(2010)







Antidiffusion terms : artificial viscosity   stability

$$U_i^{n+1} = \tilde{U_i} - \tilde{A}_i + \tilde{A_{i-1}}$$
$$A_i = A\mathrm{ad}\tilde{\Delta}_i/8$$

To Multi Dimension

$$\partial_\mu T^{\mu\nu} = 0$$

- Operational split and directional split

$$\partial_t U + \nabla \cdot F(U) = S(U)$$

$$\begin{cases} \partial_t U + CU = SU \\ C \equiv \dfrac{\delta F^i}{\delta U} \cdot \nabla \equiv L^i \partial_i \end{cases}$$

$$U^{n+1} = U^n + \Delta t (S - C) U^n$$

Operational split (C, S)

$$\begin{cases} \partial_t \bar{U} + C\bar{U} = 0 \\ \partial_t \hat{U} = S\hat{U} \end{cases}$$

$$\begin{cases} \bar{U}^{n+1} = \bar{U}^n - \Delta t C \bar{U}^n \\ \hat{U}^{n+1} = \hat{U}^n + \Delta t S \hat{U}^n \end{cases}$$

$$U^{n+1} = \hat{S}^{\Delta t} \hat{C}^{\Delta t} U^n$$

$$= U^n + \Delta t (S - C) U^n$$

To Multi Dimension

$$\partial_\mu T^{\mu\nu} = 0$$

- Operational split and directional split

$$\partial_t U + \nabla \cdot F(U) = S(U)$$

Operational split (C, S)

$$\begin{cases} \partial_t U + CU = SU \\ C \equiv \dfrac{\delta F^i}{\delta U} \cdot \nabla \equiv L^i \partial_i \end{cases} \Longleftrightarrow \begin{cases} \partial_t \bar{U} + C\bar{U} = 0 \\ \partial_t \hat{U} = S\hat{U} \end{cases}$$

$$U^{n+1} = U^n + \Delta t (S - C) U^n$$

$$\begin{cases} \bar{U}^{n+1} = \bar{U}^n - \Delta t C \bar{U}^n \\ \hat{U}^{n+1} = \hat{U}^n + \Delta t S \hat{U}^n \end{cases}$$

$$U^{n+1} = \hat{S}^{\Delta t} \hat{C}^{\Delta t} U^n$$

$$= U^n + \Delta t (S - C) U^n$$

2d $U^{n+1} = L_x^{1/2} L_y L_x^{1/2} U^n$

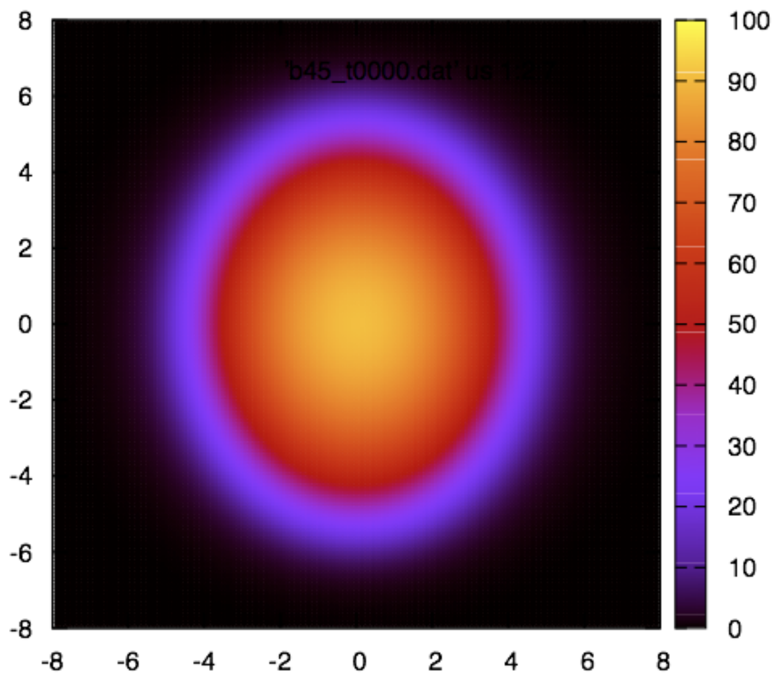
3d $U^{n+1} = L_x^{1/6} L_y^{1/6} L_z^{1/3} L_y^{1/6} L_x^{1/3} L_z^{1/6} L_y^{1/3} L_x^{1/6} L_z^{1/3} L_x^{1/6} L_y^{1/3} L_z^{1/6} L_x^{1/6} U^n$

L_i : operation in i direction

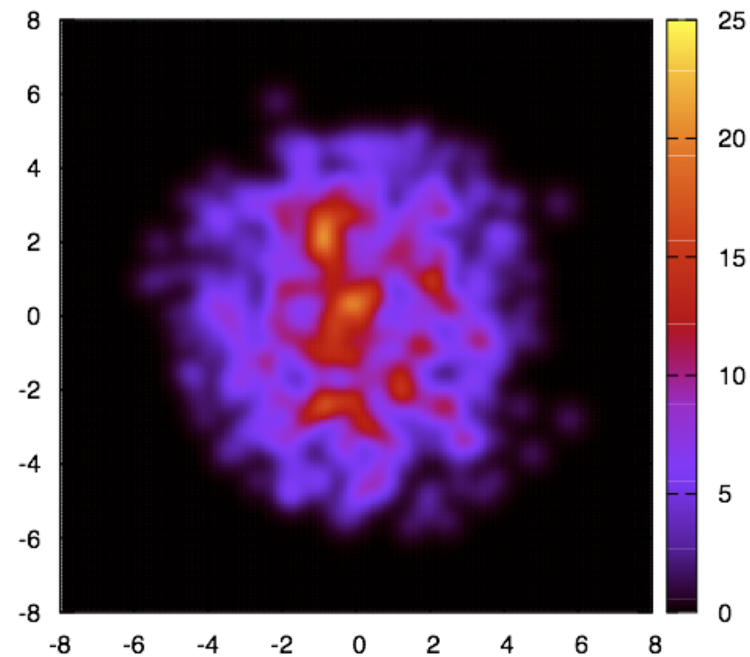
Higher Harmonics

- Initial conditions
  - Gluaber model

smoothed

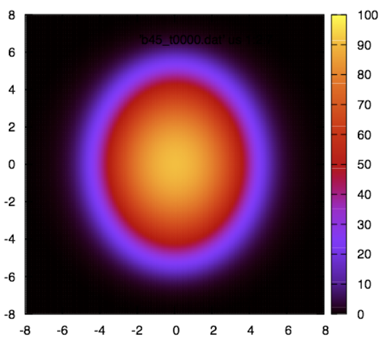


fluctuating



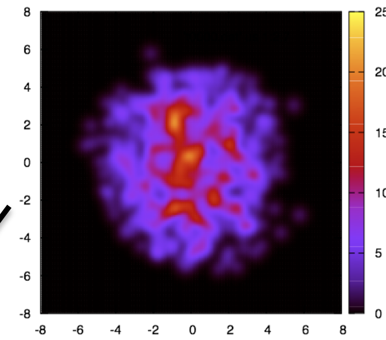
Higher Harmonics

- Initial conditions at mid rapidity
  - Gluaber model

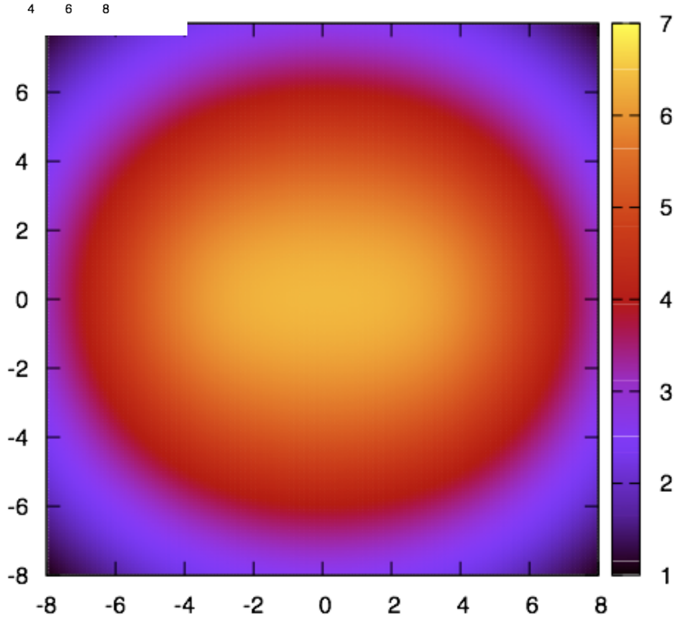


smoothed

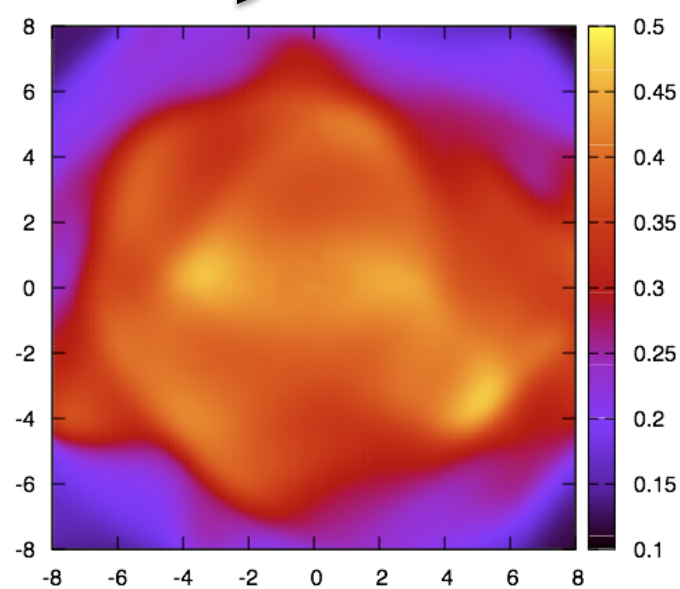
fluctuated



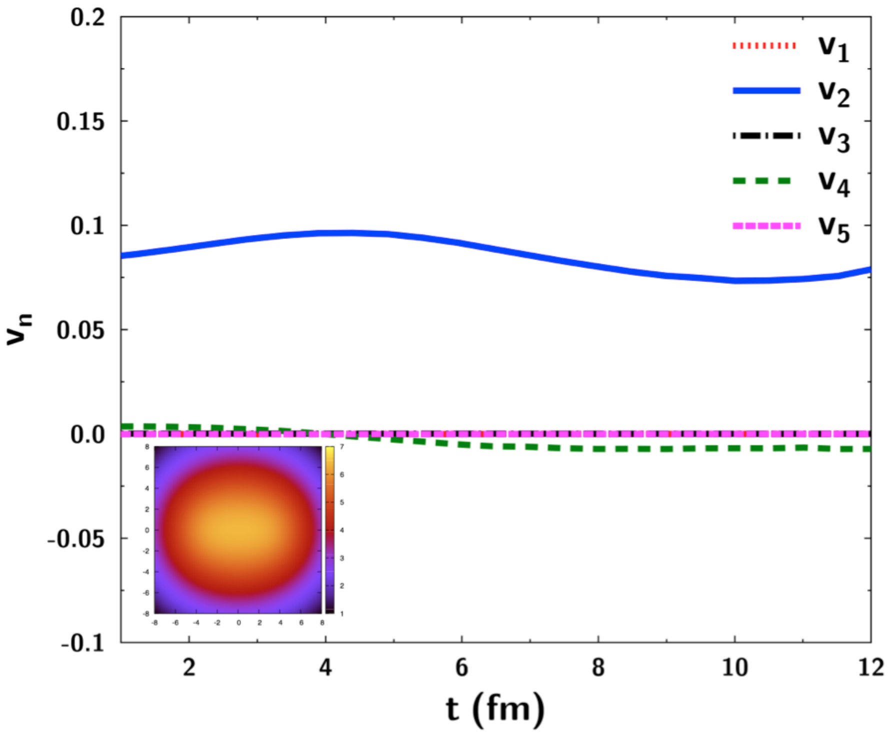
t=10 fm



t=10 fm

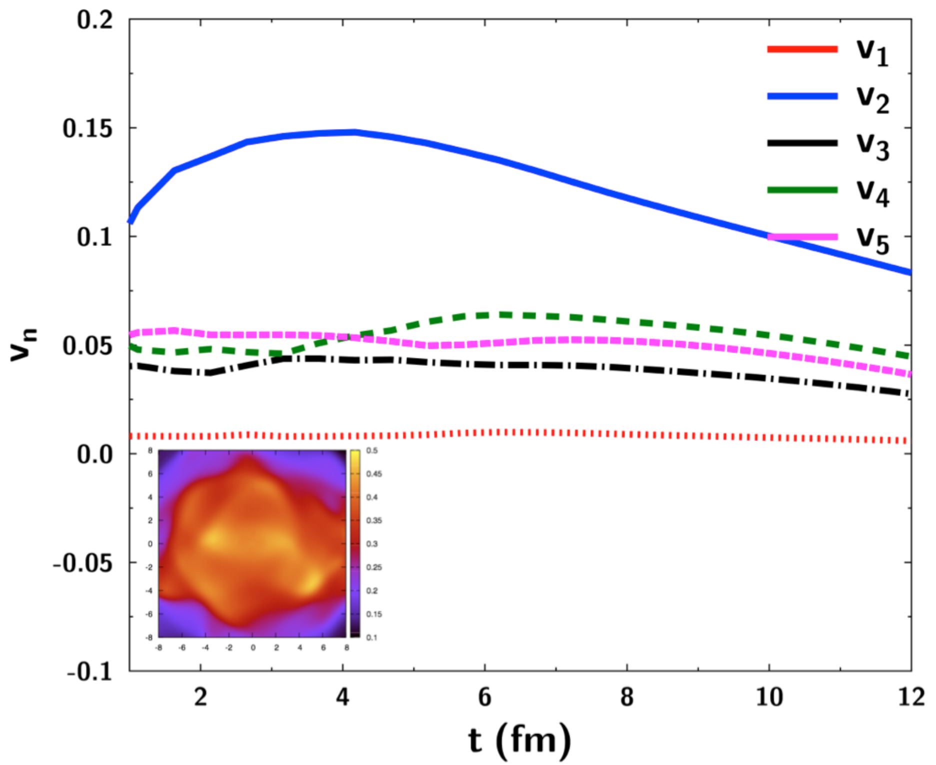


Time Evolution of v_n



Smoothed IC

v_2 is dominant.



Fluctuating IC

v_n becomes finite.

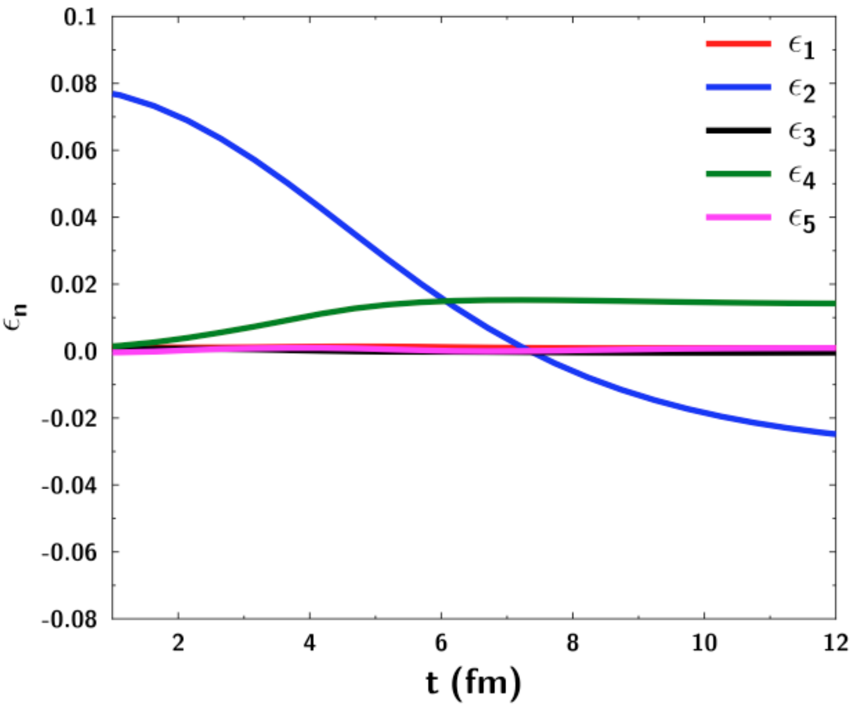
Time Evolution of Higher Harmonics

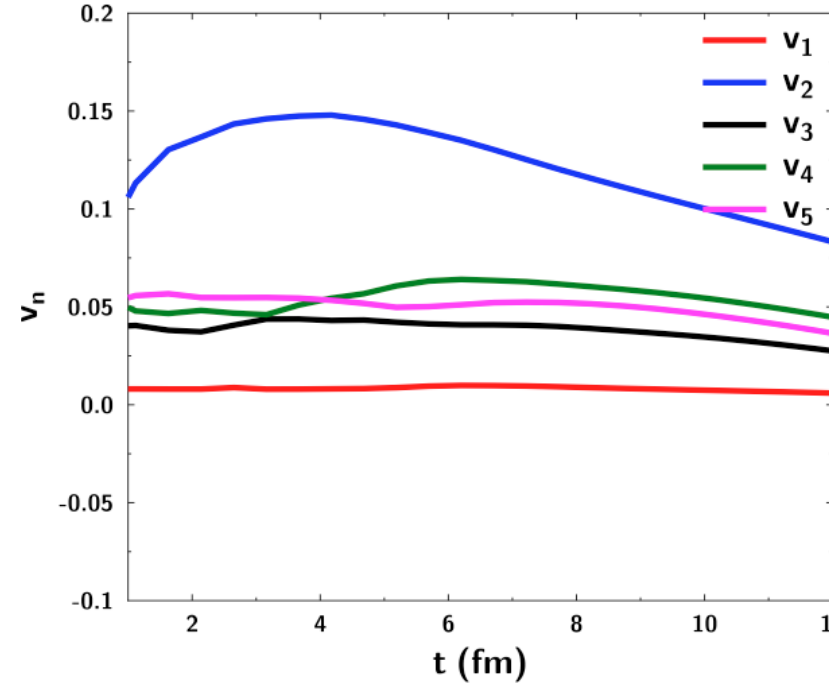
Petersen et al, Phys.Rev. C82 (2010) 041901

$$\epsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}}{\langle r^n \rangle}$$

$$v_n = \langle \cos(n(\phi_p - \Psi_n))\rangle$$

$$\Phi_n = \frac{1}{n}\arctan\frac{\langle r^n \sin(n\phi)\rangle}{\langle r^n \cos(n\phi)\rangle}$$





Ideal hydrodynamic calculation
at mid rapidity
ε_n, v_n: Sum up with entropy density weight
EoS: ideal gas

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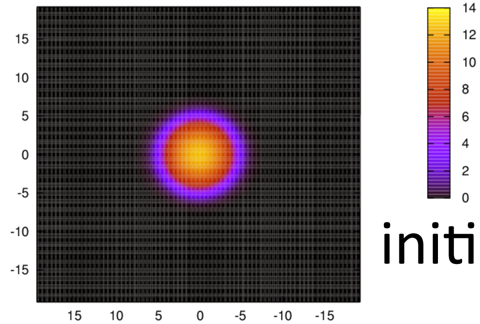
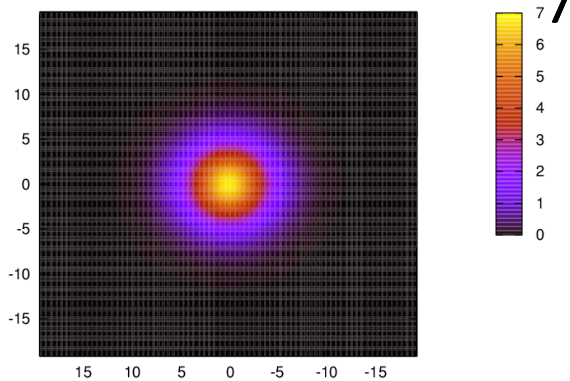
Viscosity Effect



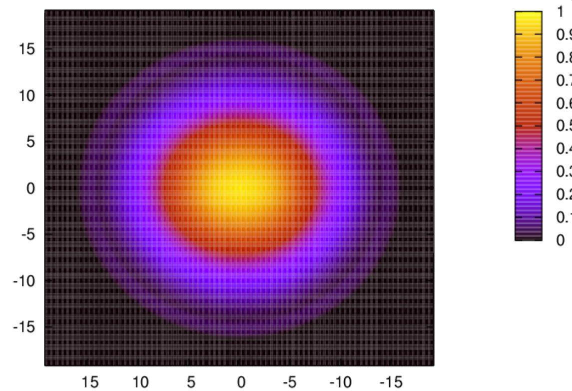
initial

Pressure distribution

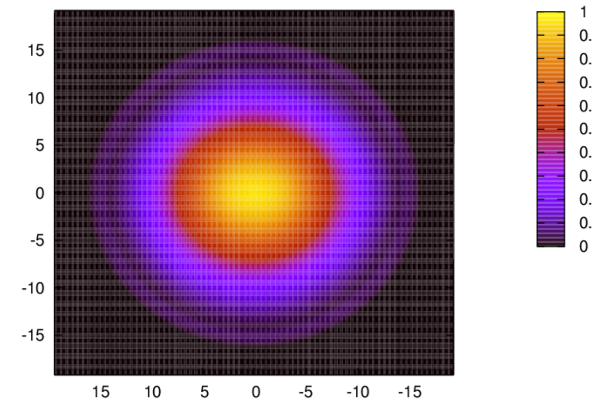
Ideal t~5 fm



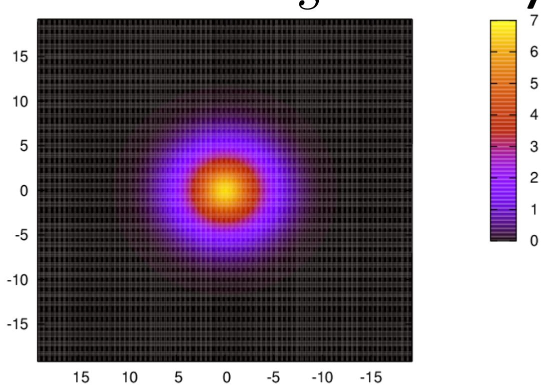
t~10 fm

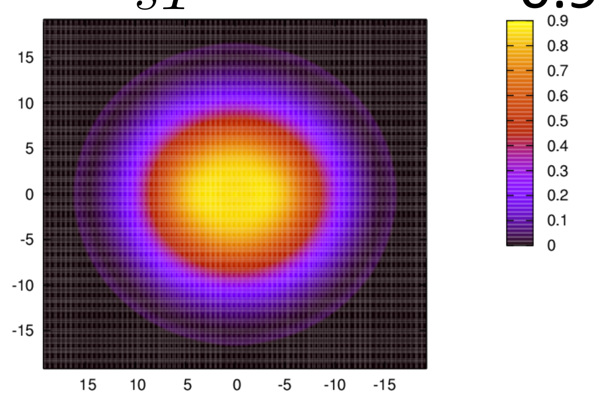


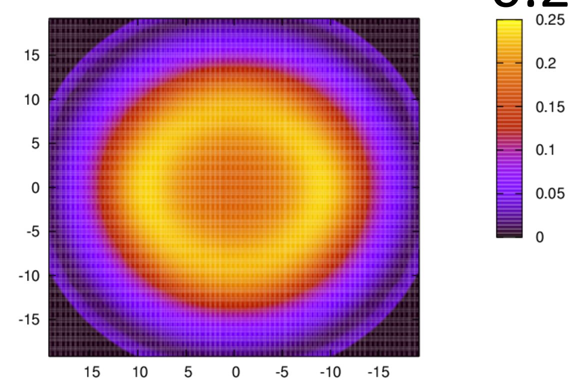
t~15 fm



Viscosity $\frac{\eta}{s} = 1.0$ $\tau = 10\frac{\eta}{sT}$



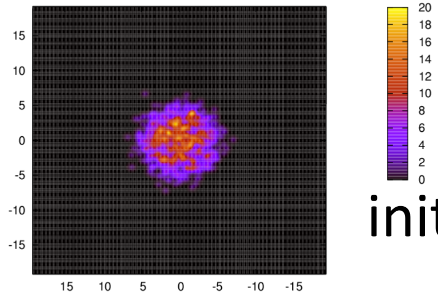




Viscous Effect

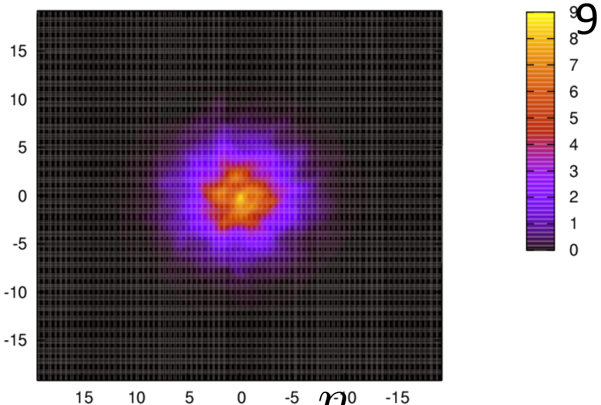
Pressure distribution

initial fm^{-4} 20

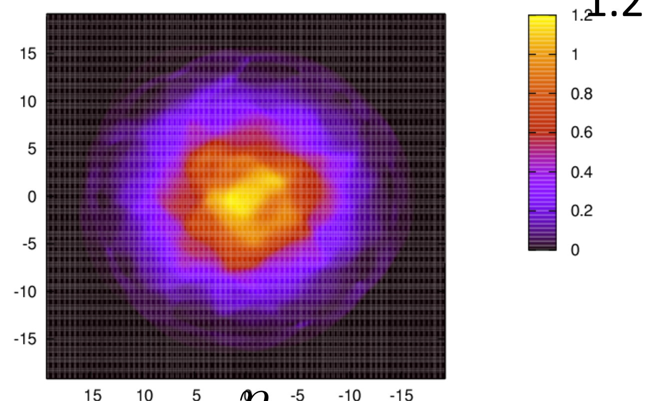


Ideal

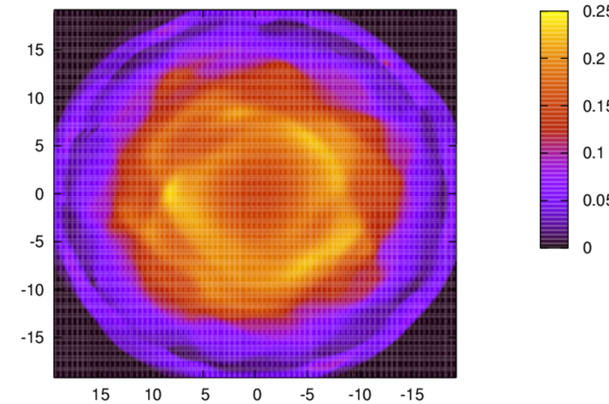
t~5 fm fm^{-4} 9



t~10 fm 1.2



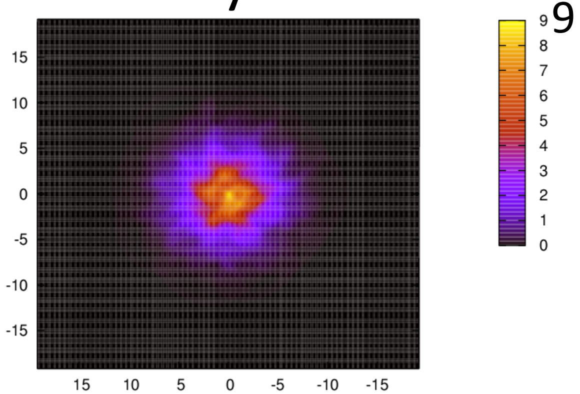
t~15 fm 0.25



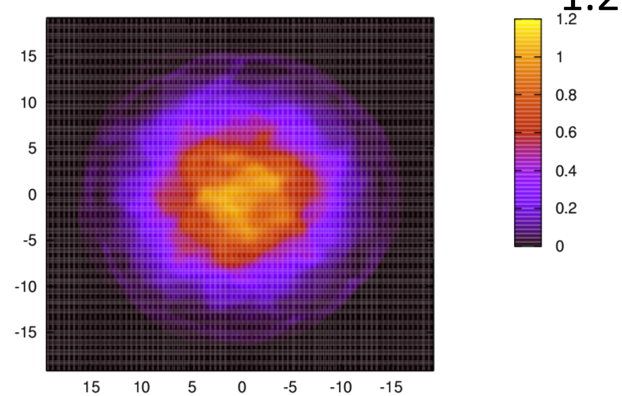
Viscosity

$$\frac{\eta}{s} = 1.0 \qquad \tau = 100\frac{\eta}{sT}$$

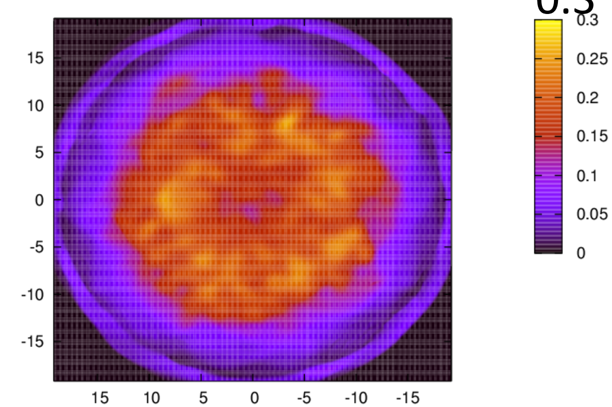
9



1.2



0.3



Summary

- We develop a state-of-the-art numerical scheme
  - Viscosity effect
  - Shock wave capturing scheme: Godunov method

Our algorithm

  - Less artificial diffusion: crucial for viscosity analyses
  - Fast numerical scheme
- Higher harmonics
  - Time evolution of ε_n and v_n
- Work in progress
  - Comparison with experimental data
  - Construction of dynamic model

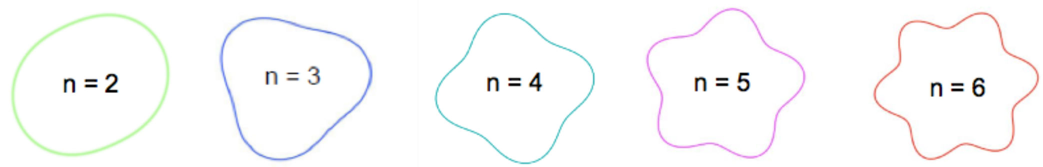
Nagoya-Duke-Texas A&M

Backup

Challenge to Hydrodynamic Model

- Higher harmonics and Ridge structure

$$\frac{dN}{dy\,d\phi} \propto 1+2v_1\cos(\phi-\Theta_1)+2v_2\cos 2(\phi-\Theta_2)+2v_3\cos 3(\phi-\Theta_3)+2v_4\cos 4(\phi-\Theta_4)+\cdots$$



Mach-Cone-Like structure, Ridge structure

Challenge to relativistic hydrodynamic model

Viscosity effect ⟶ from initial ε_n to final v_n

Longitudinal structure ⟶ (3+1) dimensional

Higher harmonics ⟶ high accuracy calculations

State-of-the-art numerical algorithm

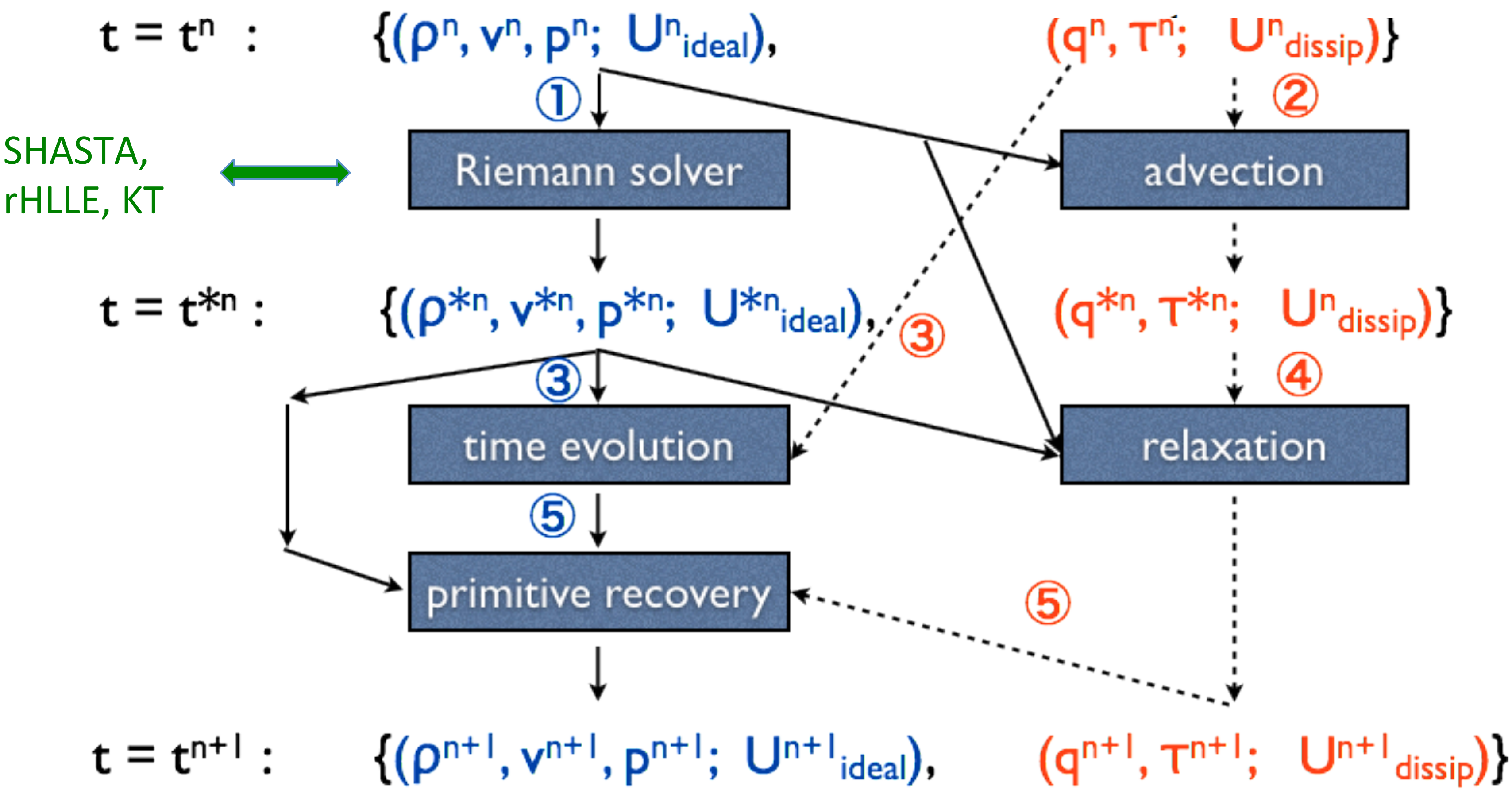
- Shock-wave treatment
- Less numerical viscosity

Numerical Method

Takamoto and Inutsuka, arXiv:

$$\partial_\mu T^{\mu\nu} = 0$$

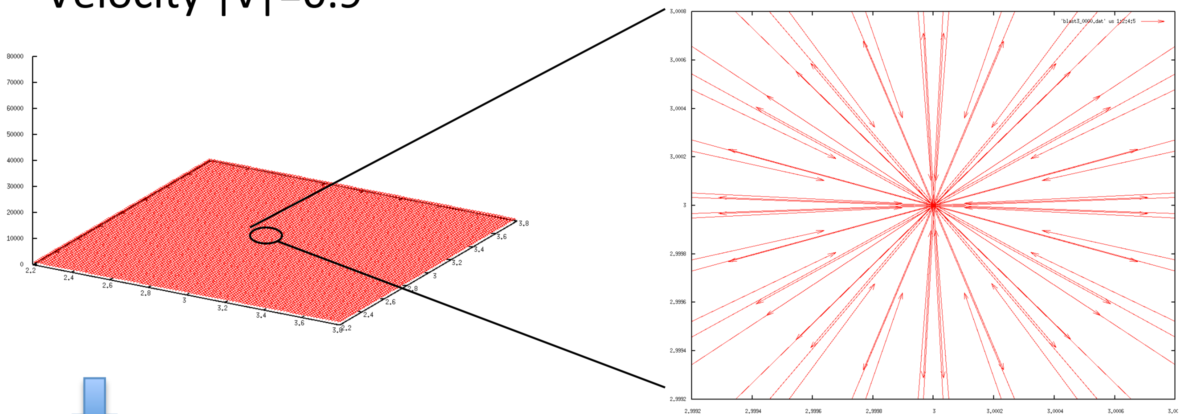
$$\hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi,$$



2D Blast Wave Check

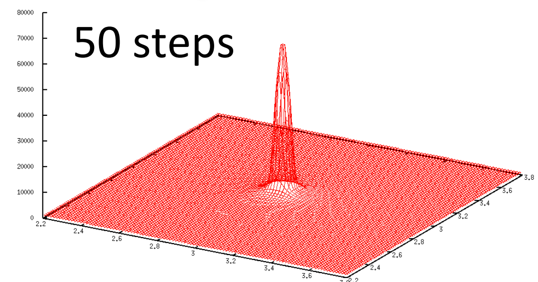
t=0 Pressure const.
Velocity |v|=0.9

Velocity vectors (t=0)

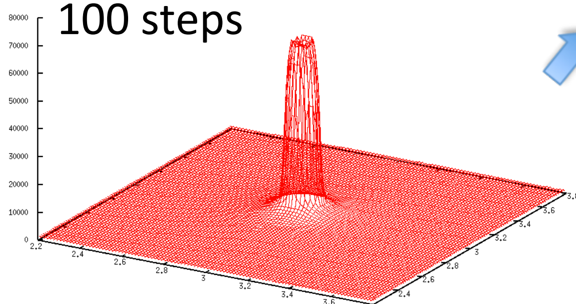


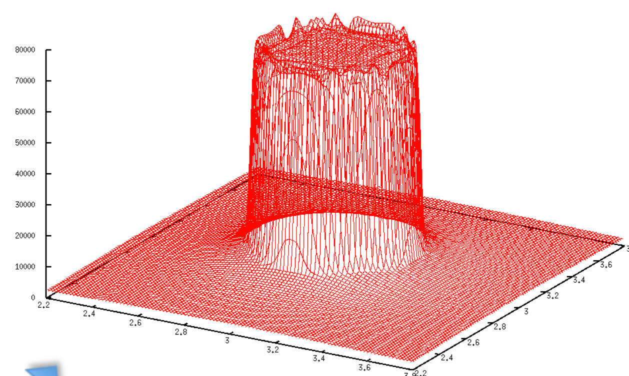
Shock wave

50 steps

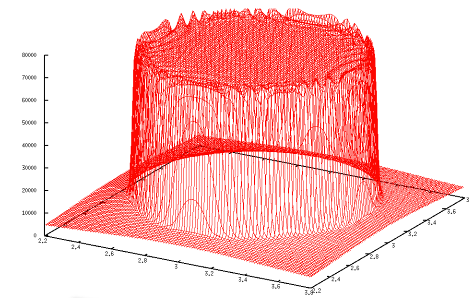


100 steps





500 steps



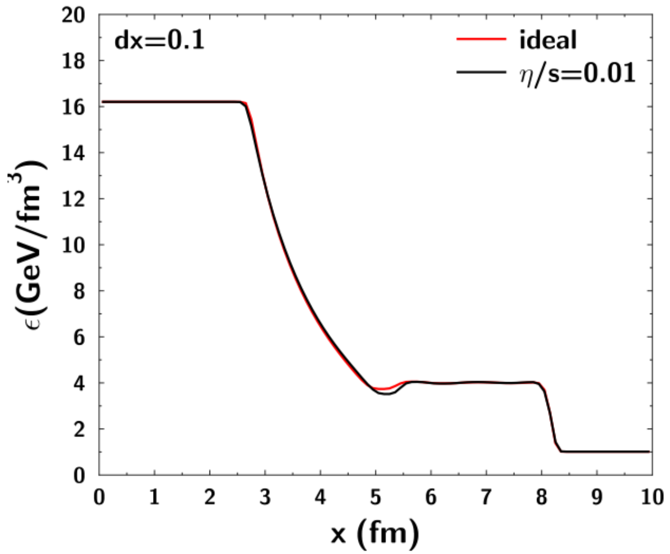
1000 steps

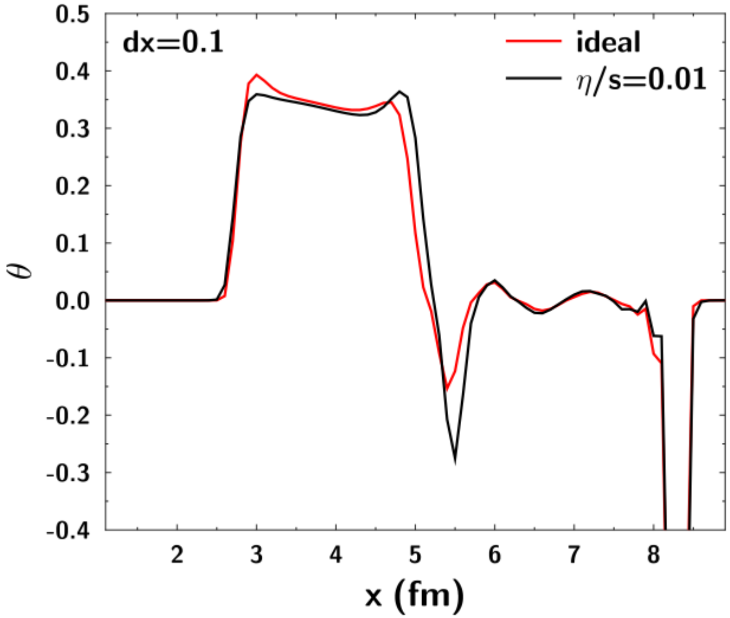
Numerical scheme, in preparation
Akamatsu, Nonaka and Takamoto

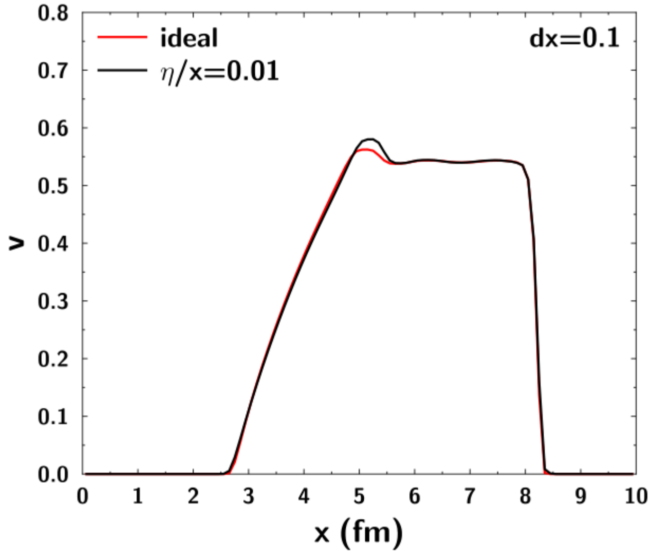
Application to Heavy Ion collisions
At QM2012!!

C. NONAKA

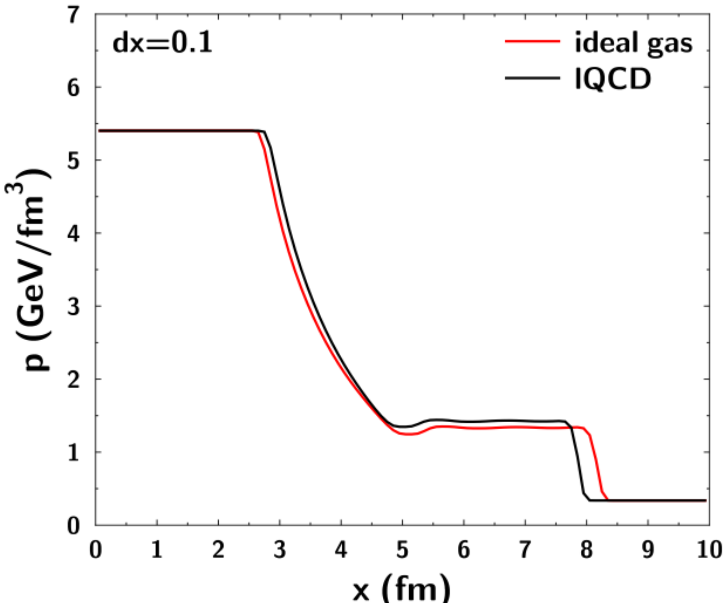
Viscosity Effect

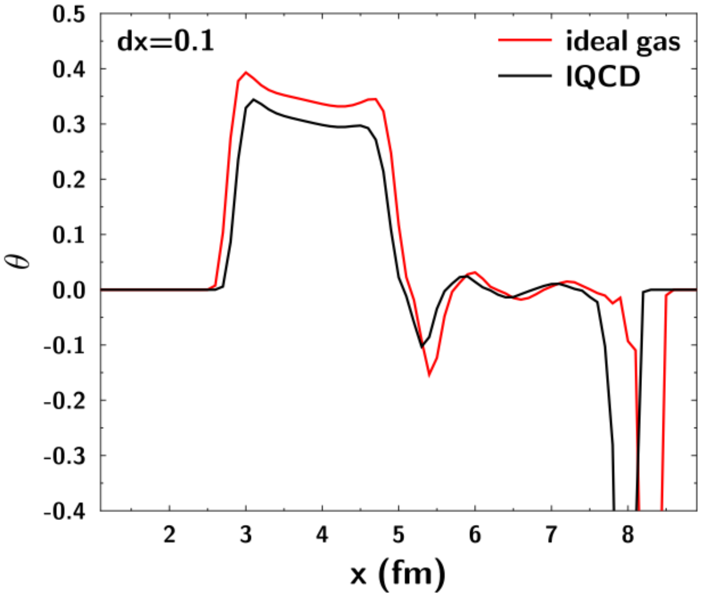


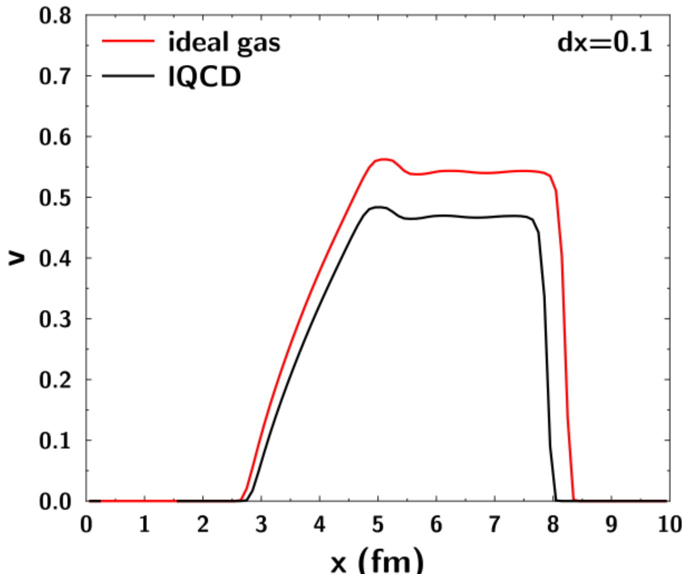




EoS Dependence







rHLLE vs SHASTA

Schneider et al. J. Comp.105(1993)92