From Lattice Strong Dynamics to Phenomenology

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for the LSD Collaboration
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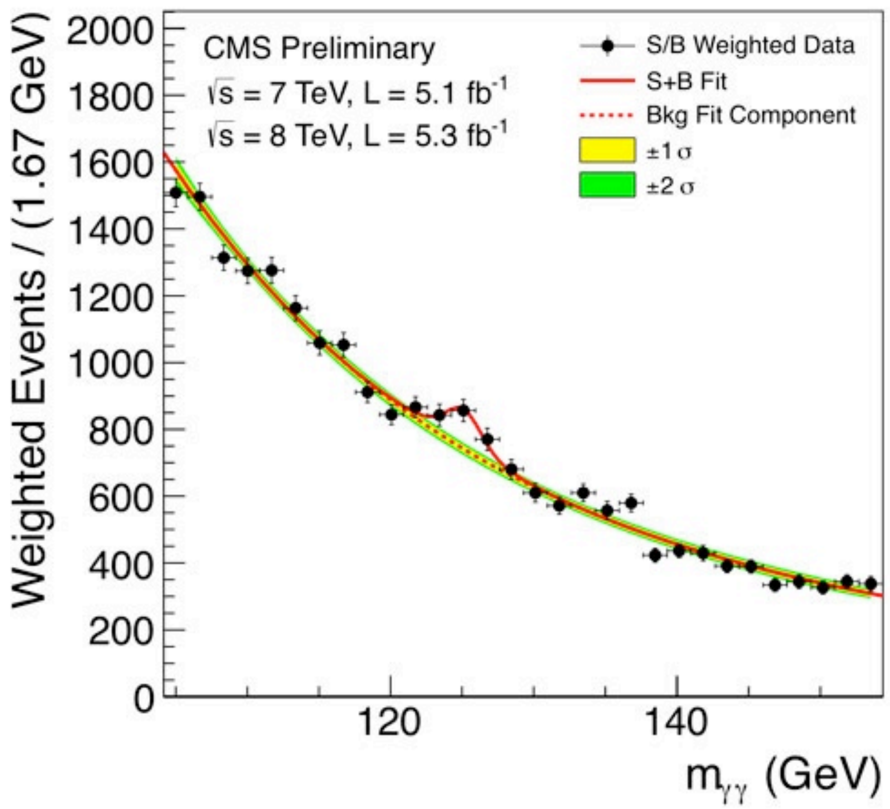
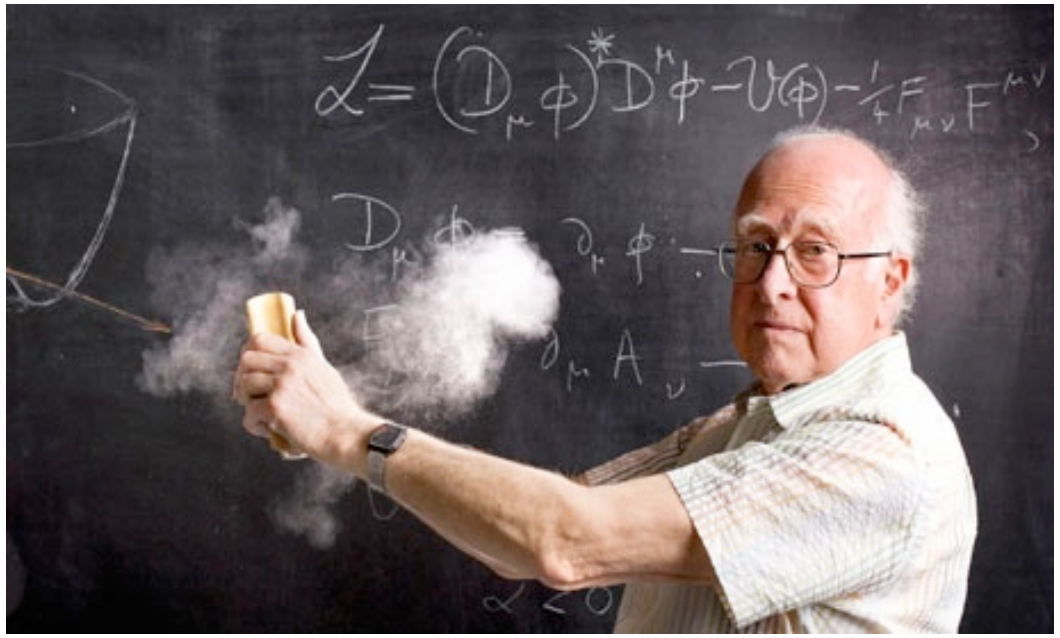
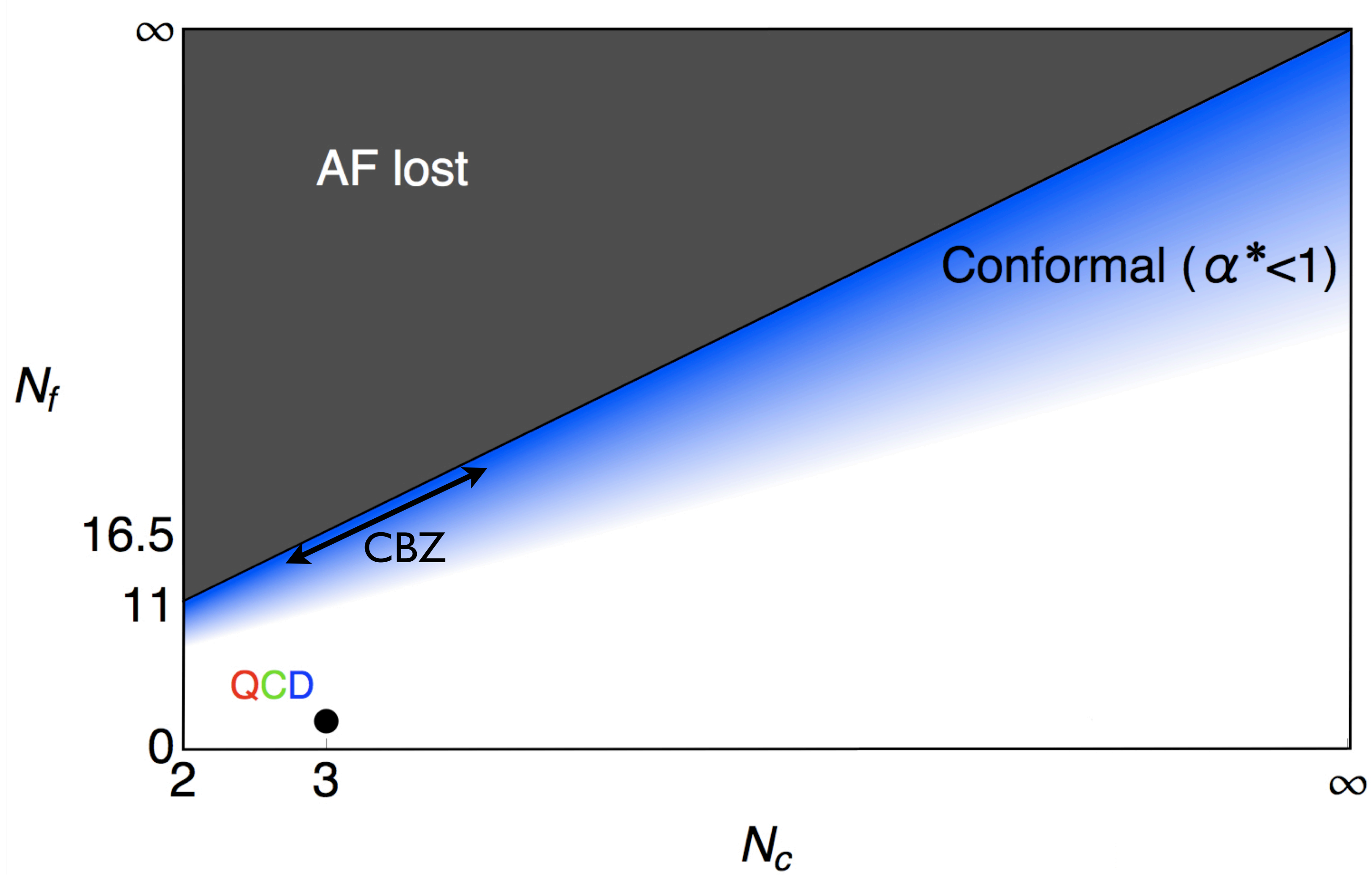
Motivation

- We have a Higgs! Or is it a Higgs impostor? A composite?
- If the new Higgs-like particle is composite, presence of a new strongly-coupled sector should reveal itself dramatically with many new resonances.
- However, scale where the resonances appear may high and difficult to reach directly. First signs of such a sector may appear in low-energy EW physics!
- UV-complete theory determines low-energy effective description, and fixes all low-energy constants (non-perturbative -> lattice!)





Exploring the space



- There exists a large parameter space of theories beyond QCD - cartoon above shows plane for N_f fundamental fermions only
- Many theories in this space can reduce to similar low-energy effective theories of EWSB. How do the coupling constants change in this space? (Lattice!)
- (Not mentioned in my talk, but interesting: bounding the edge of the window, study of IR-conformal theories. See 1204.6000, G. Voronov - PoS Lattice11)

Composite Higgs and setting the scale

- Decay constant *F* gives the EW gauge boson masses, and thus EWSB scale. For simplest case (one EW doublet), identify *v*=*F*=246 GeV.
- For QCD, the higher resonances (ρ,N,...) start around 2πF - separation of scales!

$$W_\mu^{1,2} \equiv A_\mu^{1,2} - \frac{4}{fg}\partial_\mu \phi_{1,2}$$

$$Z_\mu \equiv \frac{g}{\sqrt{g^2+g'^2}}\left(A_\mu^3 - \frac{g'}{g}B_\mu - \frac{4}{fg}\partial_\mu \phi_3\right)$$

- Integrate out --> chiral Lagrangian:

$$\mathcal{L}_{\chi,LO} = \frac{F^2}{4}\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] + \frac{F^2 B}{2}\mathrm{Tr}\left[m(U+U^\dagger)\right]$$

where $U = \exp(2iT^a\pi^a/F)$.

- B is related to mass generation and the chiral condensate:

$$\langle \bar{T}T\rangle \propto F^2 B$$

- Caveat: chiral Lagrangian only has "pion" states - if Higgs is a dilaton, then it needs to be accounted for as well...

The chiral Lagrangian at higher order

[Gasser and Leutwyler, NPB 250 (1985) 465]

$$
\begin{aligned}
\mathscr{L}_2 = {} & L_1\langle \nabla^\mu U^+ \nabla_\mu U\rangle^2 + L_2\langle \nabla_\mu U^+ \nabla_\nu U\rangle\langle \nabla^\mu U^+ \nabla^\nu U\rangle \\
& + L_3\langle \nabla^\mu U^+ \nabla_\mu U \nabla^\nu U^+ \nabla_\nu U\rangle + L_4\langle \nabla^\mu U^+ \nabla_\mu U\rangle\langle \chi^+ U + \chi U^+\rangle \\
& + L_5\langle \nabla^\mu U^+ \nabla_\mu U(\chi^+ U + U^+ \chi)\rangle \\
& + L_6\langle \chi^+ U + \chi U^+\rangle^2 + L_7\langle \chi^+ U - \chi U^+\rangle^2 \\
& + L_8\langle \chi^+ U \chi^+ U + \chi U^+ \chi U^+\rangle \\
& - iL_9\langle F^{\mathrm{R}}_{\mu\nu} \nabla^\mu U \nabla^\nu U^+ + F^{\mathrm{L}}_{\mu\nu} \nabla^\mu U^+ \nabla^\nu U\rangle \\
& + L_{10}\langle U^+ F^{\mathrm{R}}_{\mu\nu} U F^{\mathrm{L}\mu\nu}\rangle + H_1\langle F^{\mathrm{R}}_{\mu\nu} F^{\mu\nu \mathrm{R}} + F^{\mathrm{L}}_{\mu\nu} F^{\mu\nu \mathrm{L}}\rangle + H_2\langle \chi^+ \chi\rangle ,
\end{aligned}
$$

$(\chi = 2Bm)$

- At next order in momentum expansion, many new terms appear. Three- and four-point pion interactions, and interactions with external left/right currents. Once again all LECs fixed by underlying strong dynamics.
- Looking on the electroweak side makes connection to experiment clearer...

Next-to-leading order on the EW side

[Appelquist and Wu, Phys.Rev. D48 (1993) 3235]

$$\mathcal{L}_1 \equiv \frac{1}{2}\alpha_1 g g' B_{\mu\nu} Tr(TW^{\mu\nu}) \qquad \mathcal{L}_2 \equiv \frac{1}{2} i\alpha_2 g' B_{\mu\nu} Tr(T[V^\mu, V^\nu])$$

$$\mathcal{L}_3 \equiv i\alpha_3 g Tr(W_{\mu\nu}[V^\mu, V^\nu]) \qquad \mathcal{L}_4 \equiv \alpha_4 [Tr(V_\mu V_\nu)]^2$$

$$\mathcal{L}_5 \equiv \alpha_5 [Tr(V_\mu V^\mu)]^2 \qquad \mathcal{L}_6 \equiv \alpha_6 Tr(V_\mu V_\nu) Tr(TV^\mu) Tr(TV^\nu)$$

$$\mathcal{L}_7 \equiv \alpha_7\, Tr(V_\mu V^\mu) Tr(TV_\nu) Tr(TV^\nu) \qquad \mathcal{L}_8 \equiv \frac{1}{4}\alpha_8\, g^2\, [Tr(TW_{\mu\nu})]^2$$

$$\mathcal{L}_9 \equiv \frac{1}{2} i\alpha_9 g Tr(TW_{\mu\nu}) Tr(T[V^\mu, V^\nu]) \qquad \mathcal{L}_{10} \equiv \frac{1}{2}\alpha_{10} [Tr(TV_\mu) Tr(TV_\nu)]^2$$

$$\mathcal{L}_{11} \equiv \alpha_{11} g \epsilon^{\mu\nu\rho\lambda}\, Tr(TV_\mu) Tr(V_\nu W_{\rho\lambda}) \qquad \mathcal{L}_1{}' \equiv \frac{1}{4}\beta_1 g^2 f^2 [Tr(TV_\mu)]^2.$$

- Corrections to two-point functions (oblique corrections) should appear first in low-energy experiments.

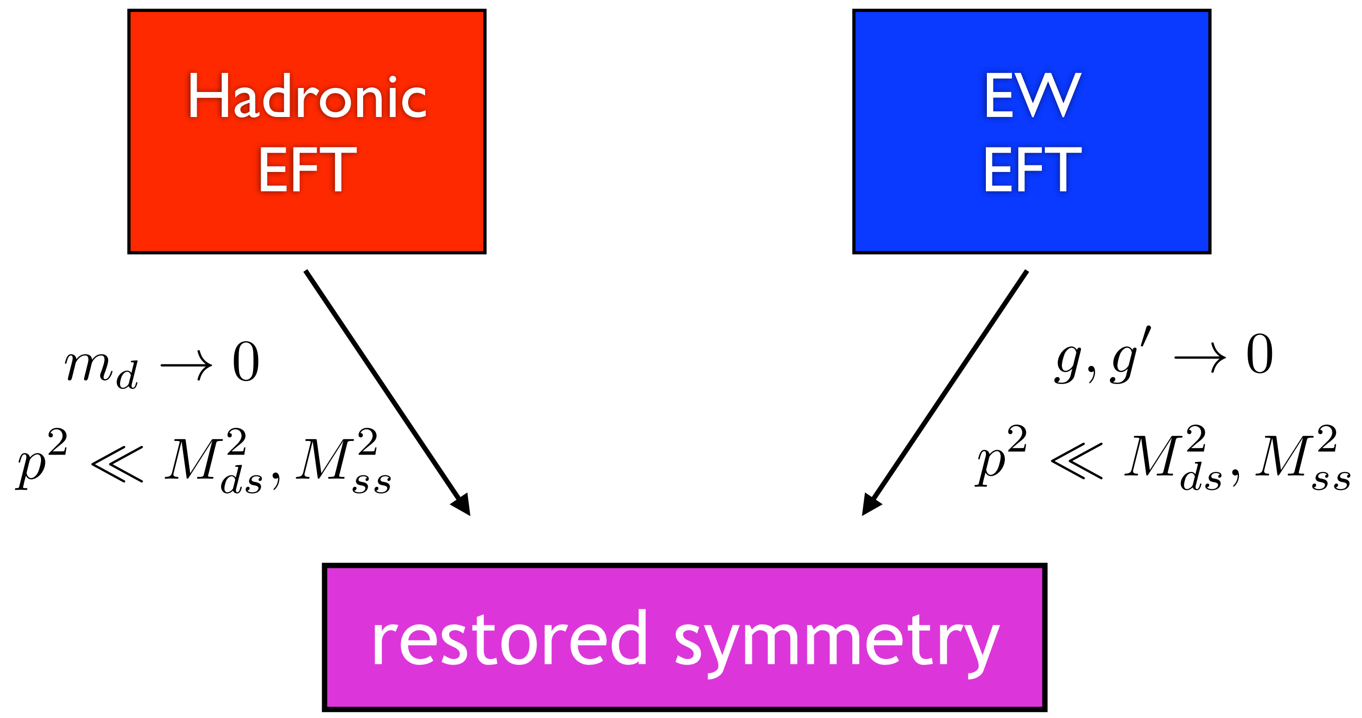
$$S \propto \alpha_1 \qquad T \propto \beta_1 \qquad U \propto \alpha_8$$

- Dominant contributions to *W-W* scattering at NLO from α_4, α_5

A tale of two effective theories

- In lattice simulations, no EW charges - work in terms of hadronic chiral Lagrangian. Zero *g*,*g’*, massive pseudo-Goldstones.

- On the other side, we can write down an electroweak chiral Lagrangian to describe gauge-boson interactions; non-zero *g*,*g’*, massless Goldstones.



- With no Higgs, massless hadronic Goldstones eaten by W/Z, rest taken heavy. With a pion “Higgs impostor”, more complicated matching...

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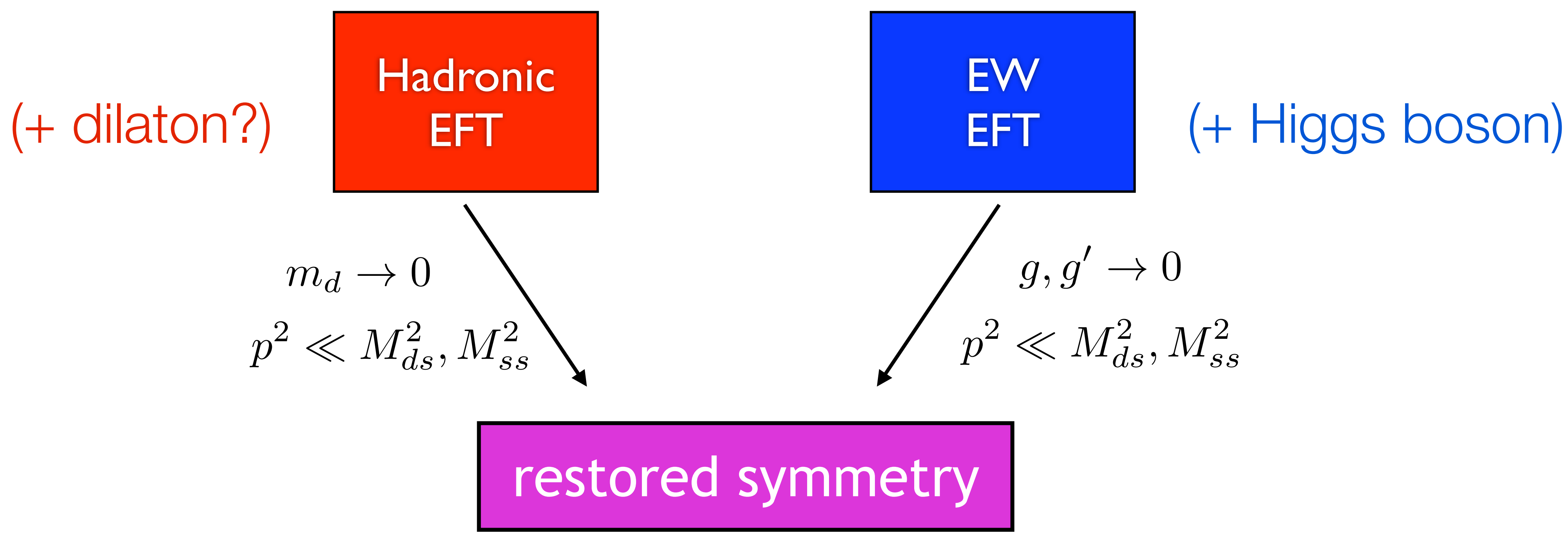

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Lattice Strong Dynamics Collaboration

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(IBM Blue Gene/L supercomputer at LLNL)

(Cray XT5 "Kraken" at Oak Ridge)

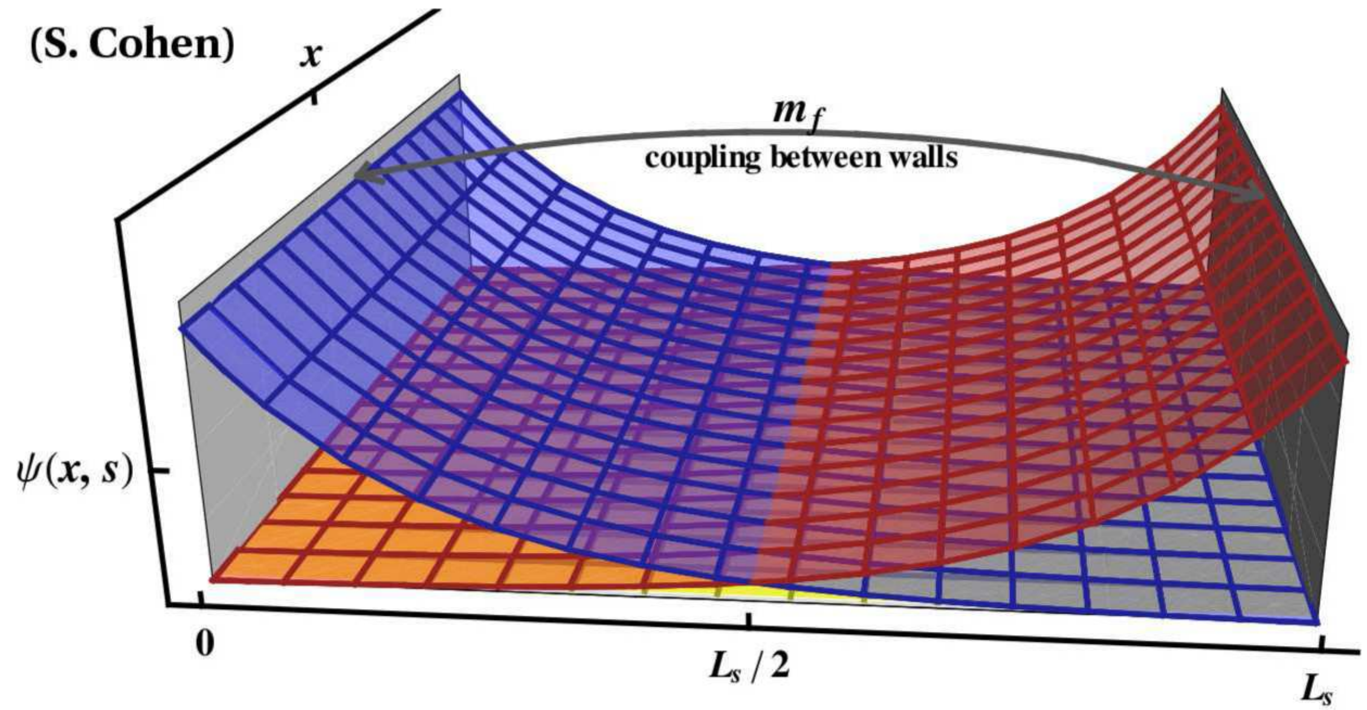
Results to be shown are state-of-the-art for lattice simulation - O(100 million) core-hours for full program

Many thanks to the computing centers and funding agencies (DOE through USQCD and LLNL, NSF through XSEDE)

(Computing cluster "7N" at JLab)

Simulation details

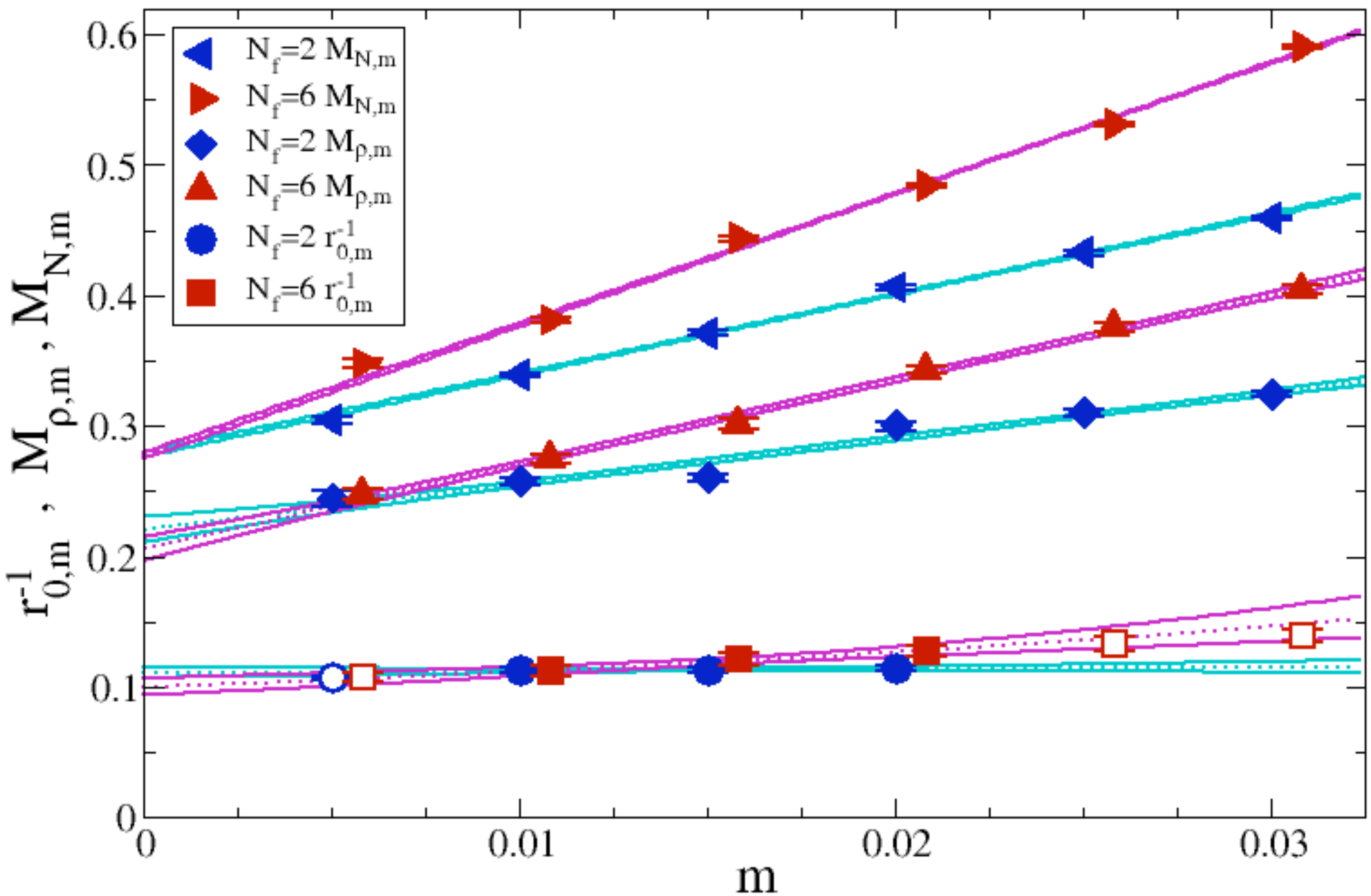
- Iwasaki gauge action + domain-wall fermions, fermion masses from m_f=0.005 to m_f=0.03, one volume (32^3x64).
- Residual chiral symmetry breaking reasonably small, m_{res}~0.002. All chiral extrapolations in $m=m_f+m_{res}$.
- Results also exist for N_f=8 (five ensembles, in progress) and N_f=10 (six ensembles, spectrum may indicate IR-conformality, see 1204.6000)



Runs tuned to $a \sim 5m_\rho$.

	$N_f = 2$		$N_f = 6$	
am_f	"M_π" L	N_{cfg}	"M_π" L	N_{cfg}
0.005	3.5	1430	4.7	1350
0.010	4.4	2750	5.4	1250
0.015	5.3	1060	6.6	550
0.020	6.5	720	7.8	400
0.025	7.0	600	8.8	420
0.030	7.8	400	9.8	360

Scale setting



Chiral condensate

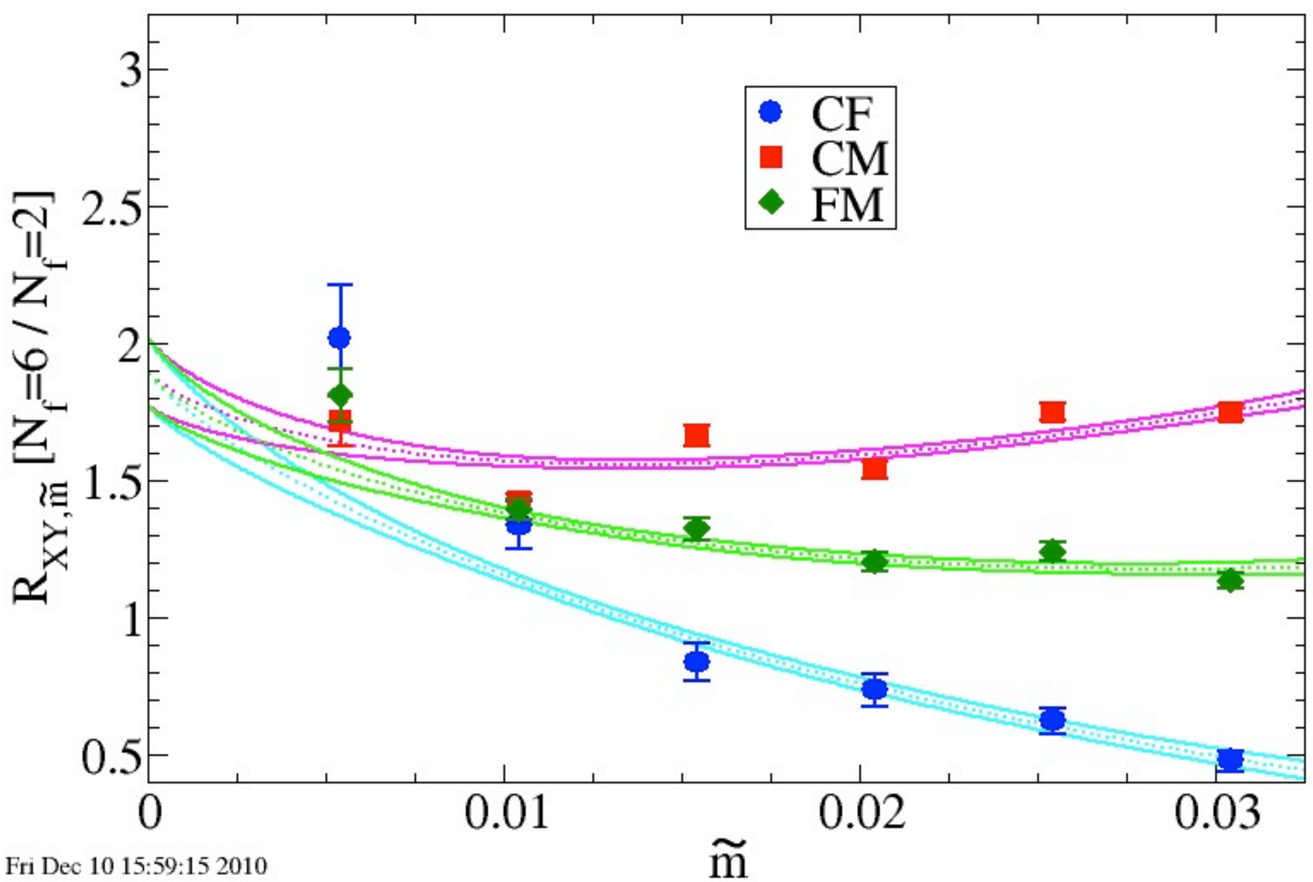
- Condensate fixes other leading-order low-energy constant, **B**. Once overall scale is set by **F**, the ratio **B**/**F** is meaningful.

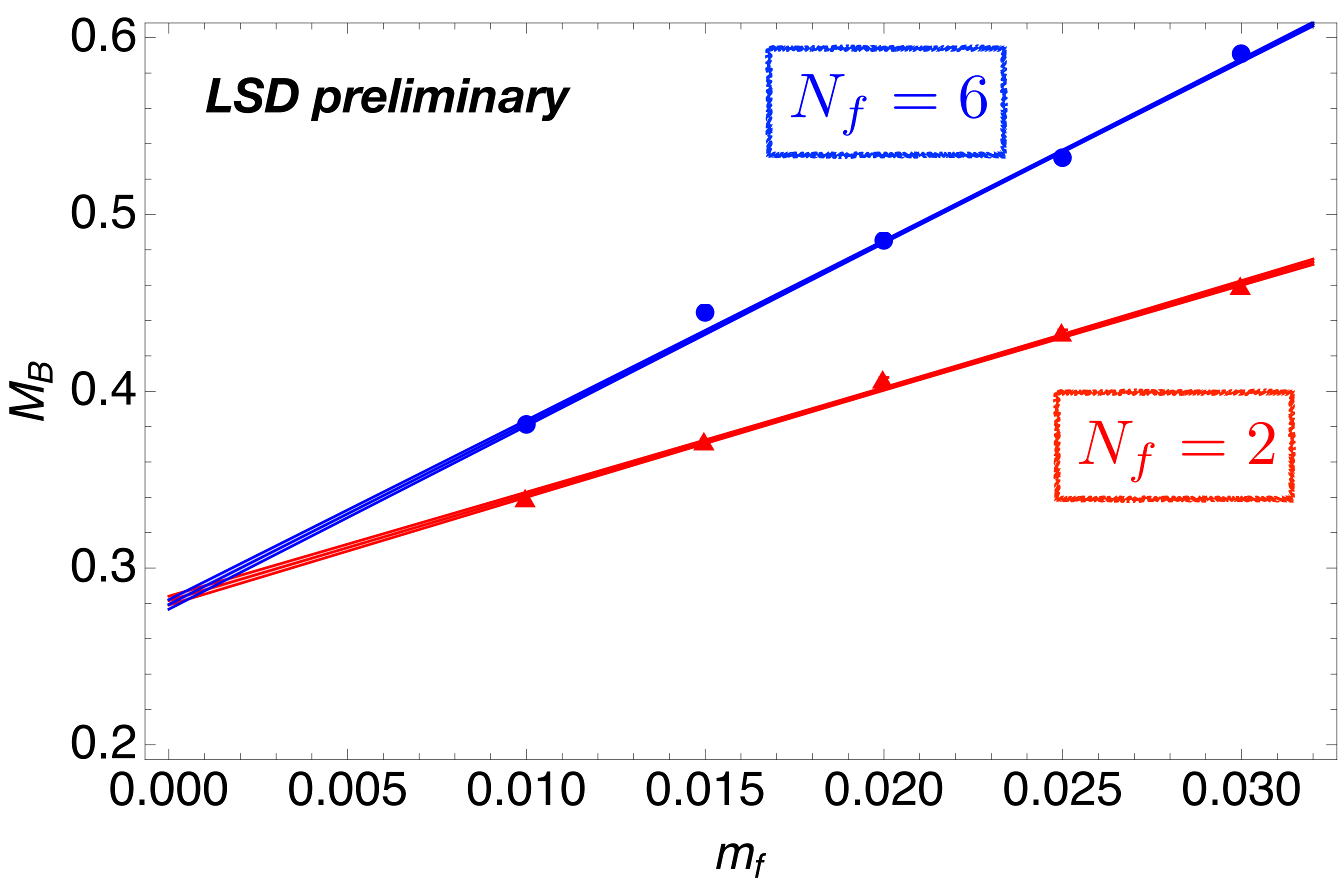
- In a composite Higgs theory, mass terms arise from four-fermion operators and the condensate:

$$y_f H \bar{f} f \to \frac{c_f}{\Lambda^2} \bar{f} f \bar{\psi} \psi$$

- Generically, standard model four-fermi operators also generated are a problem (FCNC!) Viable models tend to require small coupling and large **B**/**F**.

Condensate enhancement results





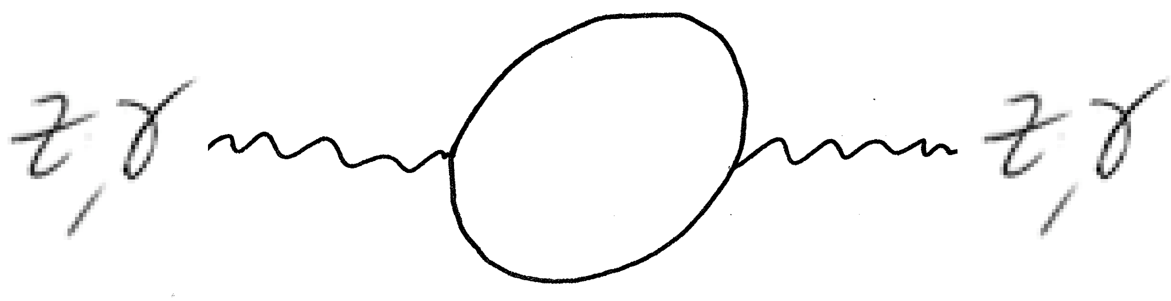
$$\frac{B}{F} \leftarrow_{m\to 0} \quad \frac{(M_m^2/2m)^{3/2}}{\langle\overline{\psi}\psi\rangle_m^{1/2}} \quad \frac{M_m^2}{2mF_m} \quad \frac{\langle\overline{\psi}\psi\rangle_m}{F_m^3}$$

$$\sigma_f \equiv \langle B|\bar{f}f|B\rangle|_{q^2\to 0} = m_f \frac{\partial M_B}{\partial m_f}$$

$$\frac{(B/F)_6}{(B/F)_2} = 1.9 \pm 0.1$$

$$\frac{\sigma_6}{\sigma_2} = 1.71(4)$$

Overview: The S-parameter



- We measure the current correlators at fixed m and q^2, and fit. Operator product expansion constrains the form at large momentum:

- As stated previously, S measures corrections from new physics to gauge boson 2-pt functions

$$S = 16\pi(\Pi'_{33}(0) - \Pi'_{3Q}(0))$$
$$= -4\pi(\Pi'_{VV}(0) - \Pi'_{AA}(0))$$

(note: model assumption!)

$$\Pi_{V-A}(q^2) \xrightarrow{q^2 \to \infty} \frac{N_{TC}}{8\pi^2} m^2 + \frac{m\langle\overline{\psi}\psi\rangle}{q^2} + \mathcal{O}(\alpha) + \mathcal{O}(q^{-4})$$

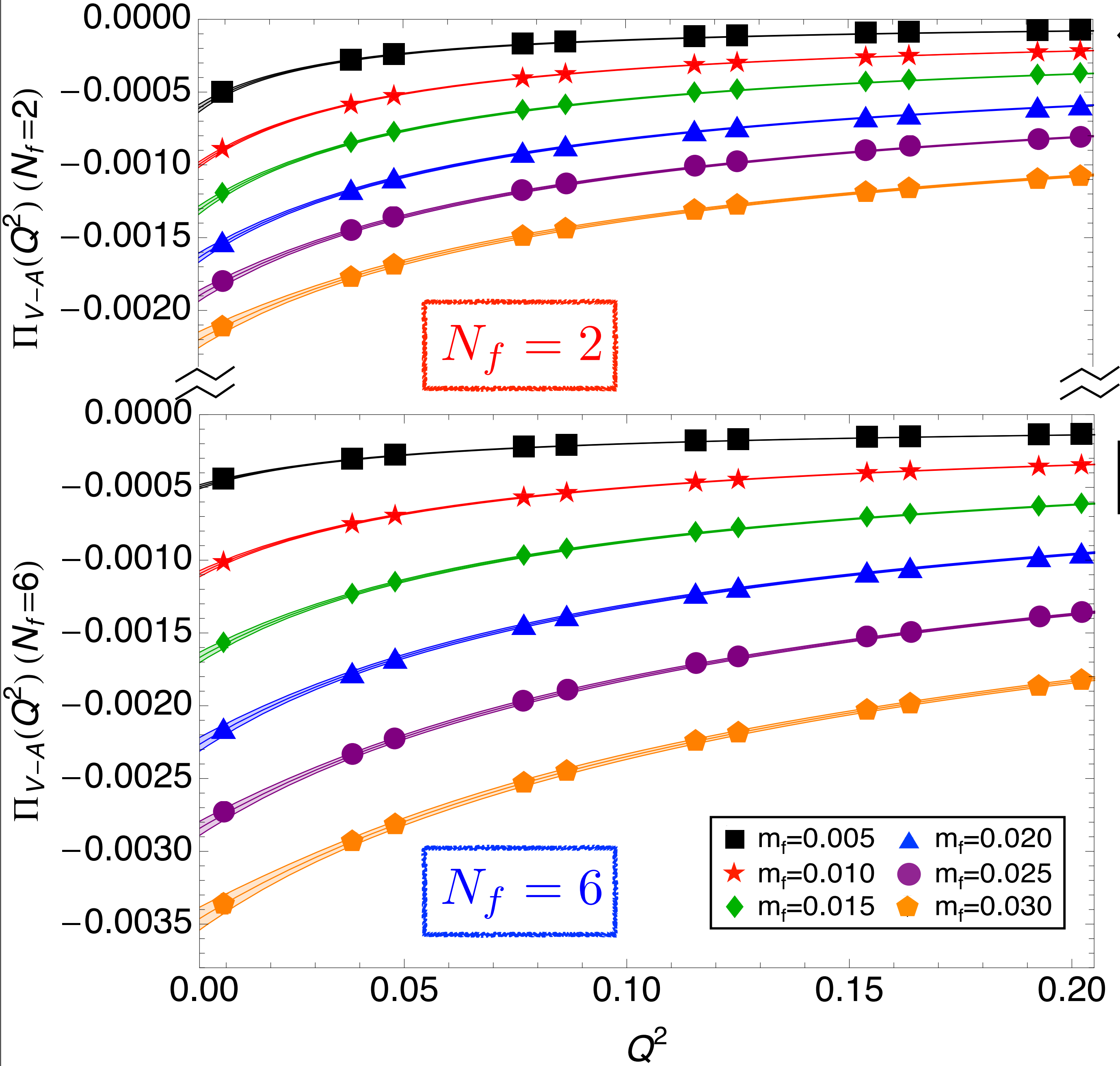
[M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. B 147 (1979)]

- Fit using Pade approximants:
(Pade (1,2) gives best fit.)

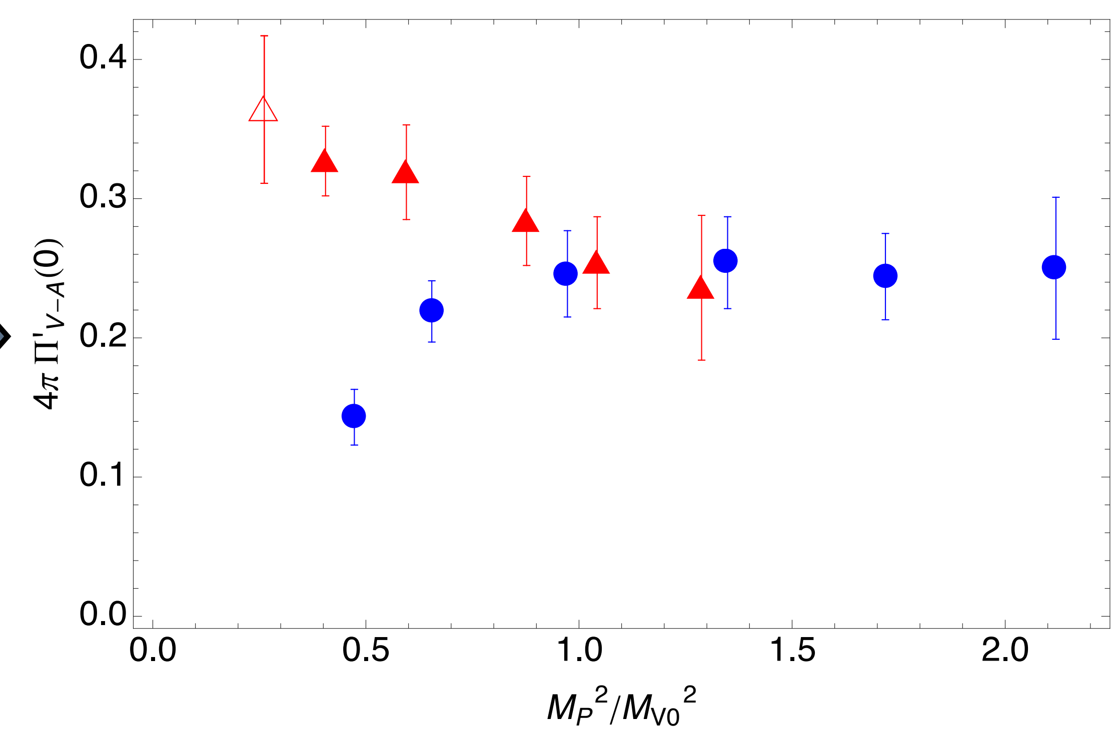
$$\Pi_{V-A}(q^2) = \frac{\sum_m a_m q^{2m}}{\sum_n b_n q^{2n}}$$

Momentum/mass fits

(m=1, n=2)

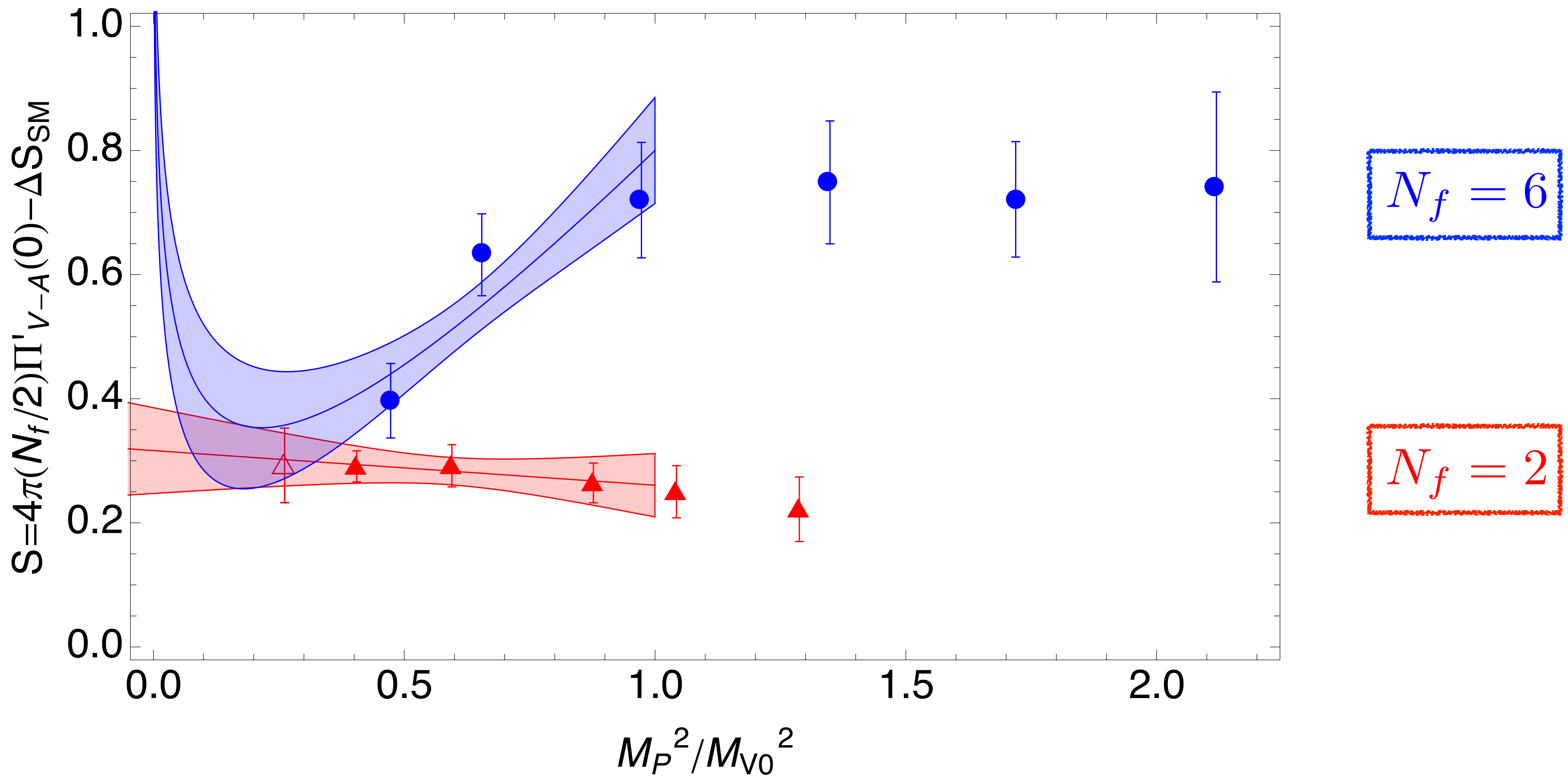


$$\Pi_{V-A}(q^2) = \frac{\sum_m a_m q^{2m}}{\sum_n b_n q^{2n}}$$



(above quantity gives LEC L_{10}.)

S-parameter results



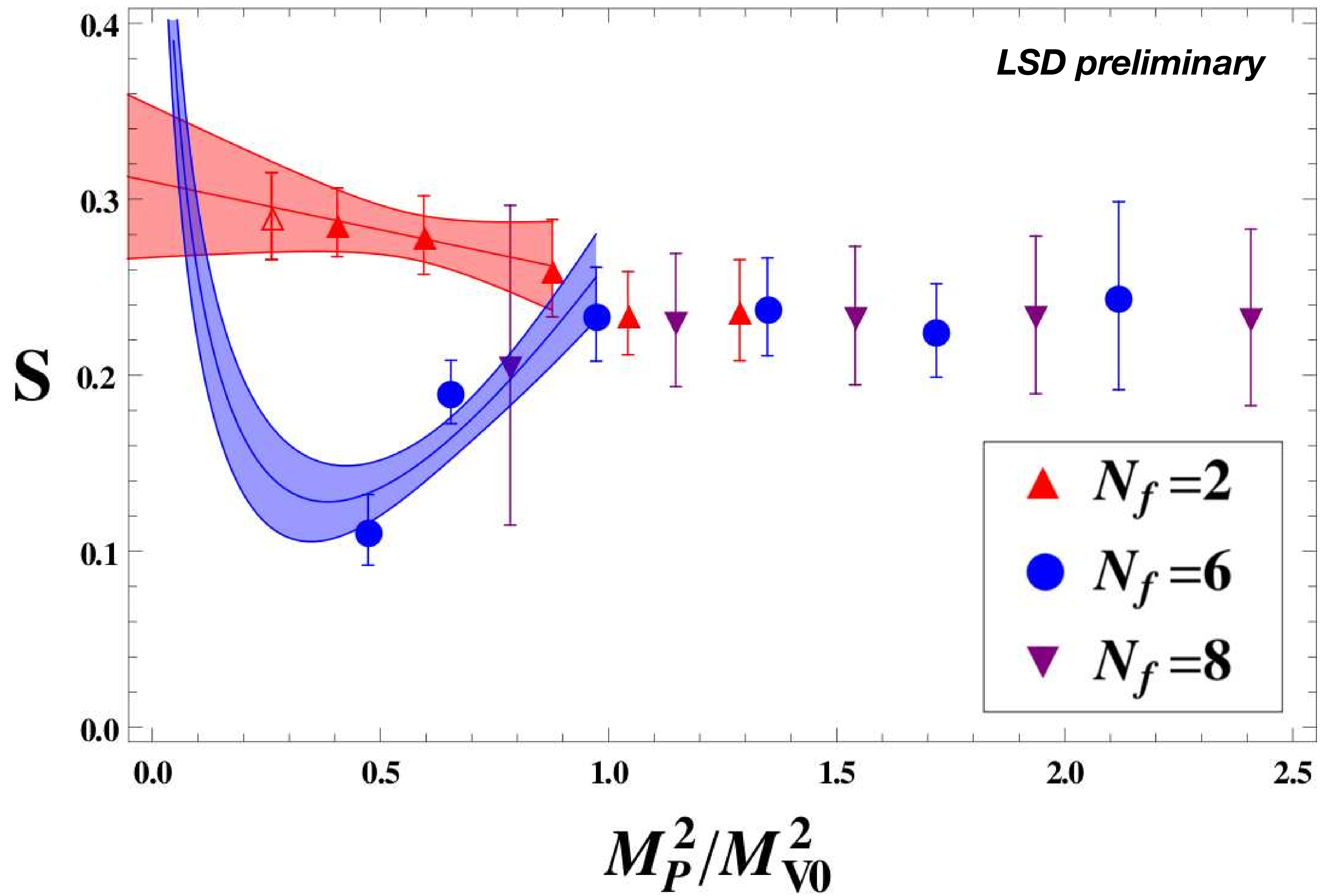
S_{2f} (m=0) = 0.35(6) - agrees with other determinations

For 6f, divergence due to PNGBs: $S(x) = A + Bx + \frac{1}{12\pi}\left(\frac{N_f^2}{4} - 1\right)\log(1/x)$

S_{6f} drops far below naive scaling estimate at light masses! Still above conjectured bound:

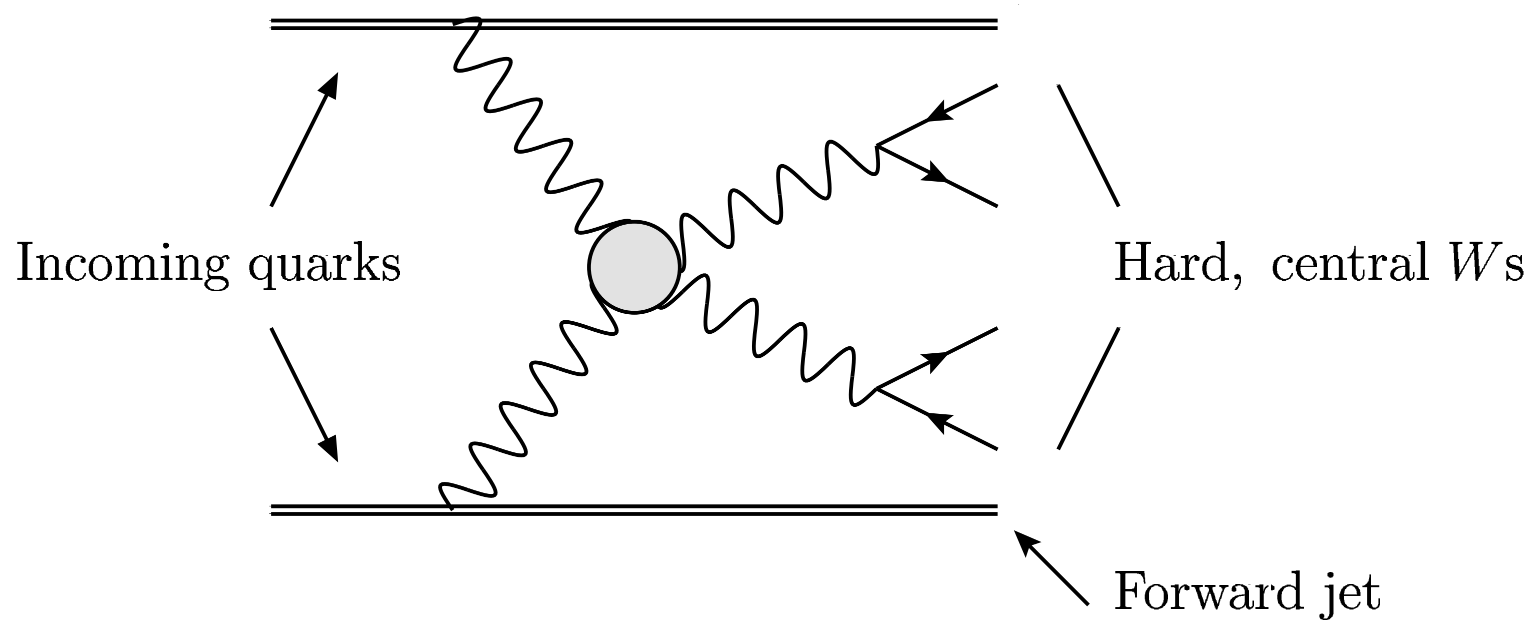
$$S \geq \frac{N_D}{2\pi}$$ (F. Sannino, arXiv:1006.0207)

Preview: S-parameter at Nf=8



Overview: WW scattering

- Direct probe of EW symmetry breaking physics. Unitarized by the Higgs boson in SM.
- Experimental process as shown (VBF). Relatively clean signal, especially with Z's, but low rates for large momentum transfer!
- At low energy, corrections appear through LECs α_4, α_5:



Estimates for 99% CL bounds for 100 inverse fb: $\mu \sim 2$ TeV

$$-7.7 \times 10^{-3} < \alpha_4 < 15 \times 10^{-3}$$

$$-12 \times 10^{-3} < \alpha_5 < 10 \times 10^{-3}$$

Eboli et. al. 2006

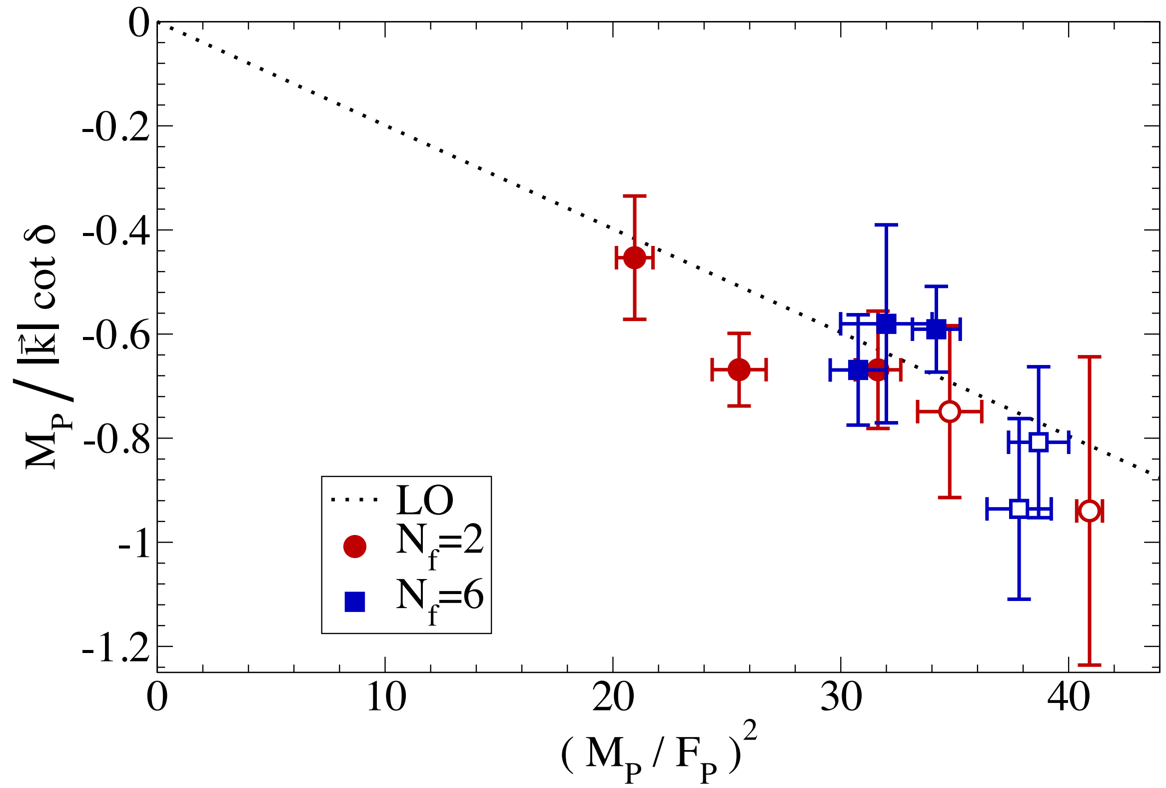
On the lattice: pi-pi scattering

- We measure I=2 ("maximal isospin") pion scattering - identified with WW scattering on the electroweak side.

- Finite-volume scaling of two-particle energy used to extract scattering phase shift (Luscher method.) Then, fit mass dependence to get LECs:

$$M_P a_{PP}^{I=2} = -\frac{M_P^2}{8\pi F_P^2}\left\{1 + \frac{M_P^2}{16\pi^2 F_P^2}\left[3\log\left(\frac{M_P^2}{\mu^2}\right) - 1 - \ell_{PP}^{I=2}(\mu)\right]\right\}$$

- Plotted on right: $M_P a_{PP}$ vs. mass for N_f=2,6. Good agreement in both cases with zero-parameter LO prediction - triumph of Weinberg.

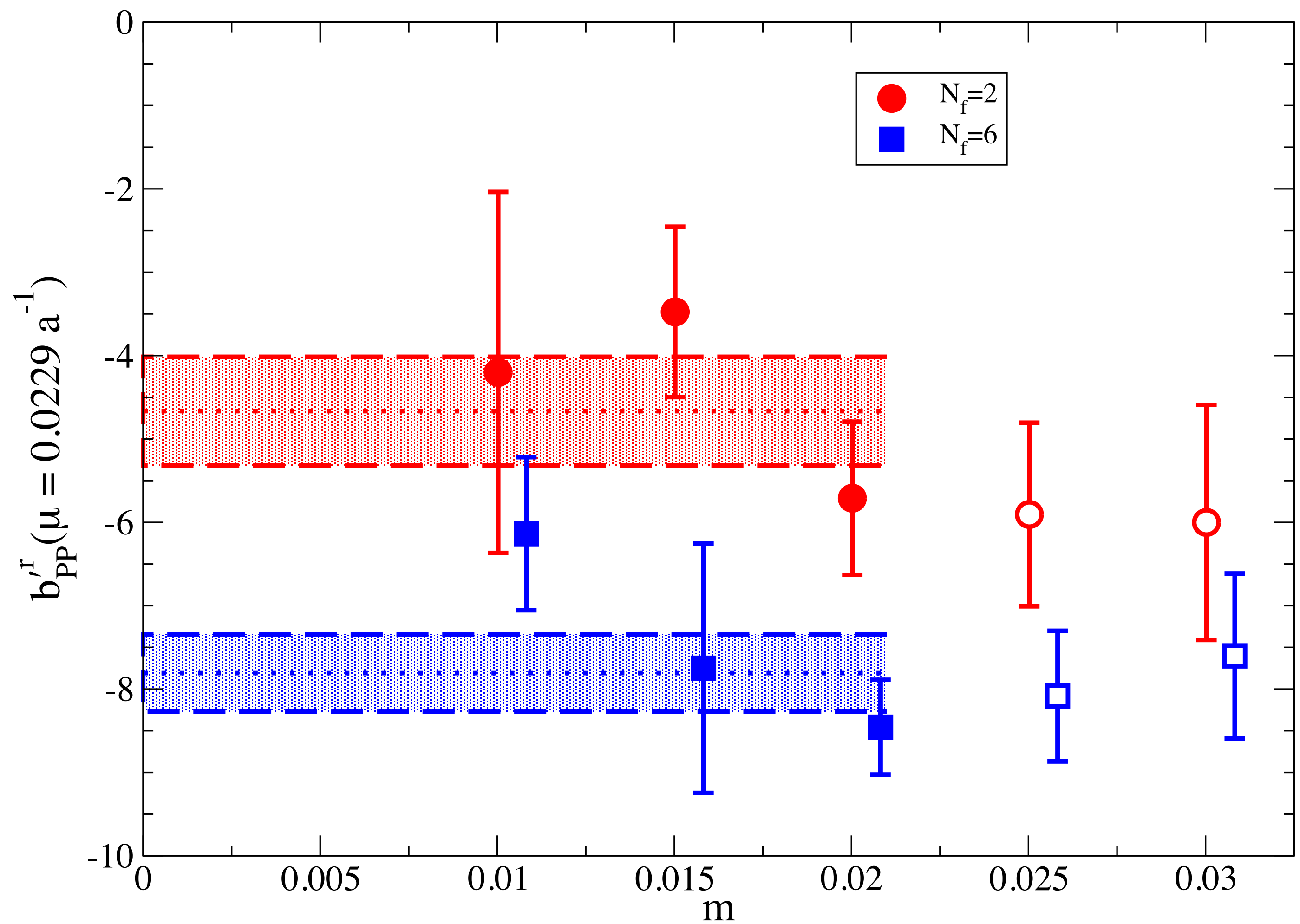


Getting the LECs

- Can't isolate α_4 and α_5 for N_f=6 yet - with only I=2 scattering, entangled with other LECs.

$$
\begin{aligned}
b'^r_{PP}(\mu) \;=\; & -256\pi^2\big[L_0^r(\mu) + 2L_1^r(\mu) + 2L_2^r(\mu) \\
& + L_3^r(\mu) - 2L_4^r(\mu) - L_5^r(\mu) + 2L_6^r(\mu) \\
& + L_8^r(\mu)\big]\,.
\end{aligned}
$$

- Still, comparison with N_f=2 shows a hopeful trend...



- At N_f=2 some of the extra LECs don't exist, and we can get the linear combination $\alpha_4+\alpha_5$ by itself.

For 2 flavors: $$\alpha_4+\alpha_5 = \begin{cases} (3.43 \pm 0.31)\times 10^{-3} & \mu \sim 246 \text{ GeV} \\ (0.15 \pm 0.31)\times 10^{-3} & \mu \sim 2 \text{ TeV} \end{cases}$$

Switching gears: composite dark matter

- Composite Higgs models tend to have a natural dark matter candidate - lightest "baryon" can be stable and electroweak neutral.

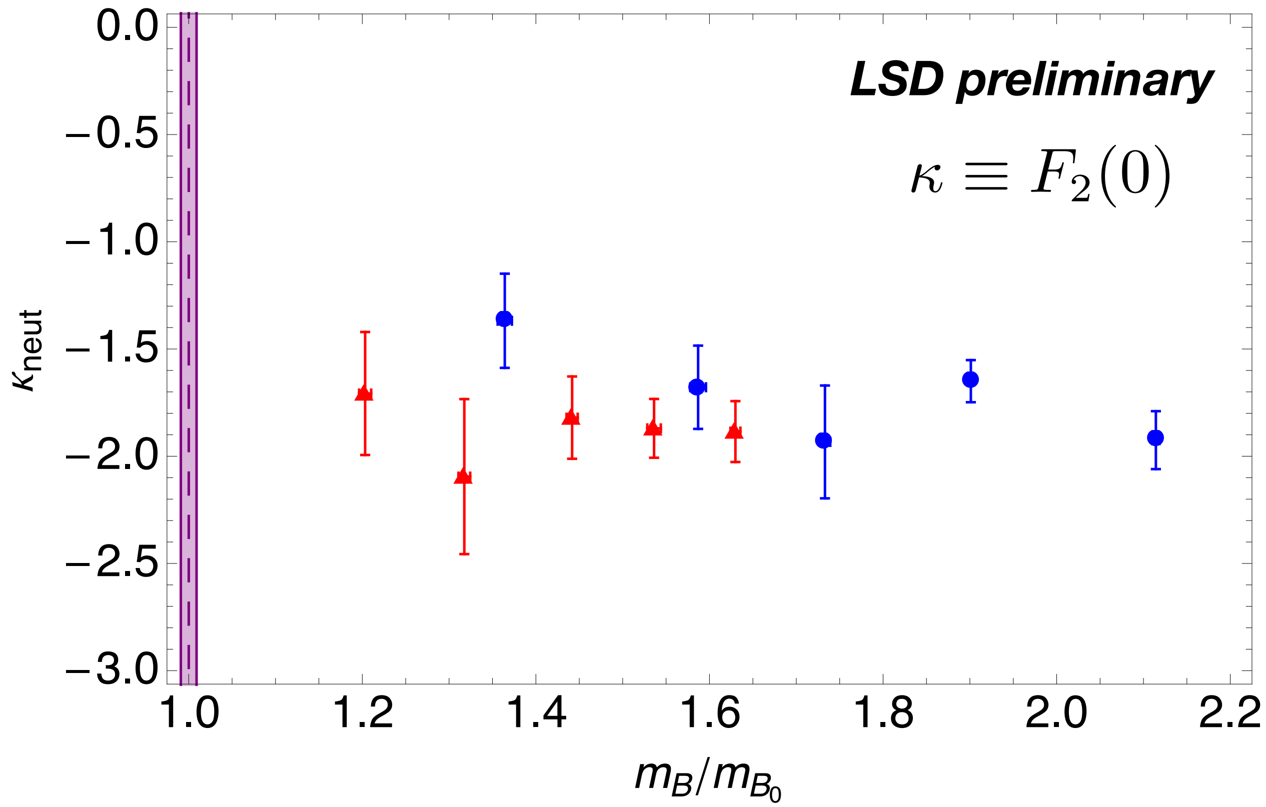
- Composite dark matter is interesting even without a direct EWSB connection! Allows balance between EW interactions (relic density) and lack thereof (direct detection.)

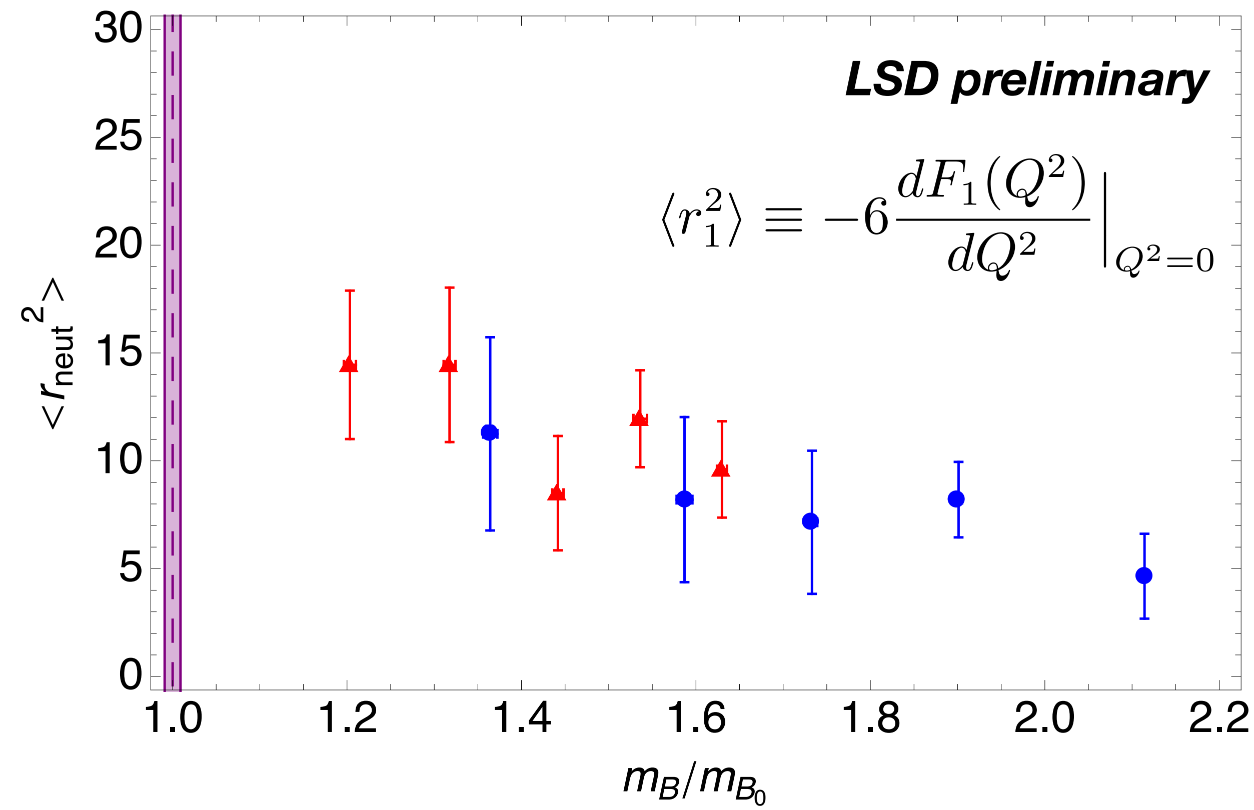
- Lattice can contribute in several ways: spectrum, pion-nucleon interactions, etc. A major application is baryon form factors, which determine recoil rates in direct-detection experiments.

- No longer working with a chiral Lagrangian - baryons will be heavy*. But now connection to experiment is more obvious: compute baryon form factors, take appropriate combination for EM current.

- *(exception: PNGB dark matter - see 1209.6054 and references therein)

Simulation results: form factors





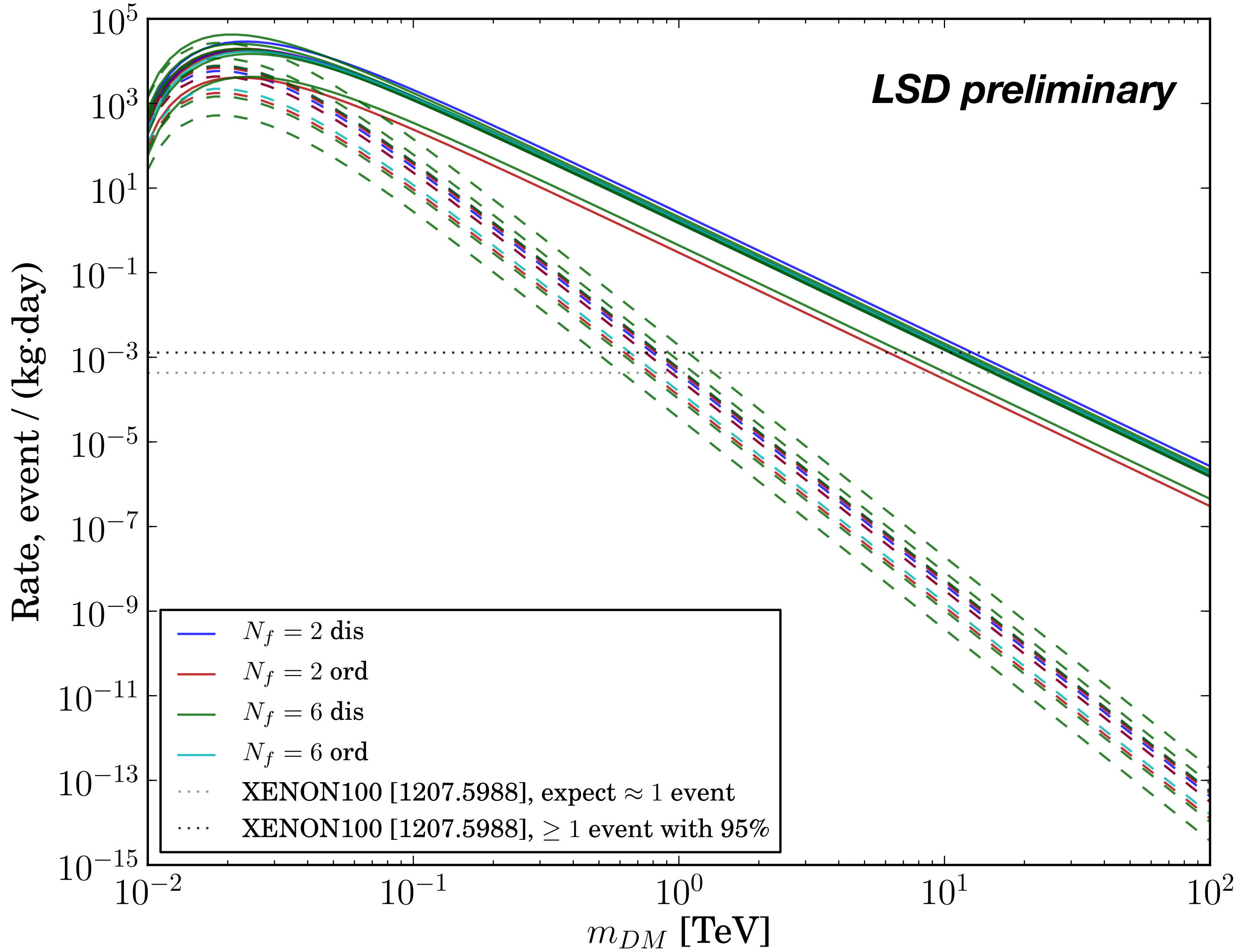
- Form factors $F_i(Q^2)$ computed from three-point function (right). Fit and extract κ, $<r^2>$.

$$\langle N(p')|\overline{q}\gamma^\mu q|N(p)\rangle$$
$$= \overline{u}_{p'}\left[F_1^q(Q^2)\gamma^\mu + F_2^q(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_B}\right]u_p$$

- Results shown for N_f=2,6 theories. "Neutron" charges assumed (+2/3, -1/3), with hypercharge only (no net weak charge allowed.)

Form factors independent of N_f at this precision (for these masses)!

Connecting to experiment



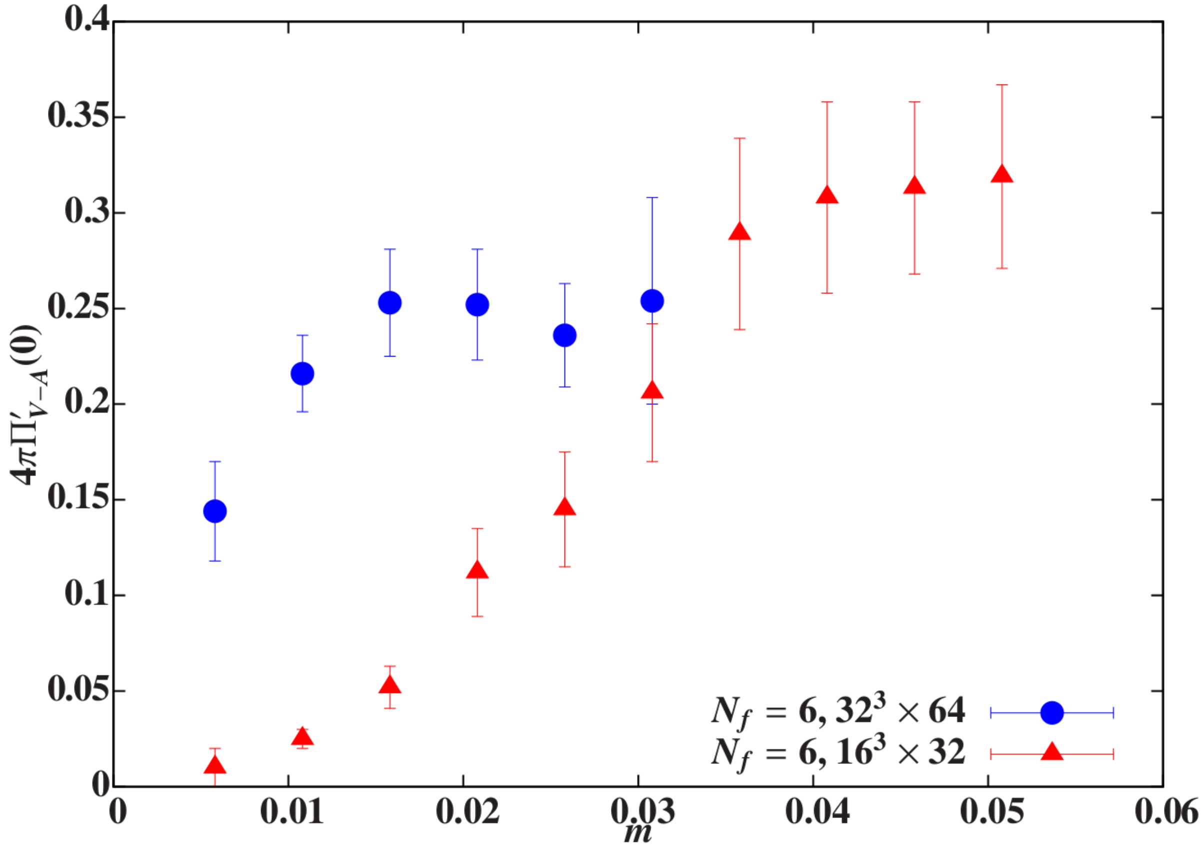
- Computed event rate for XENON100 latest results. Dominated by magnetic moment interaction κ, exclusion for DM up to 5-10 TeV in this model.
- Dashed lines show bound from charge radius operator only (e.g. even N_c?)

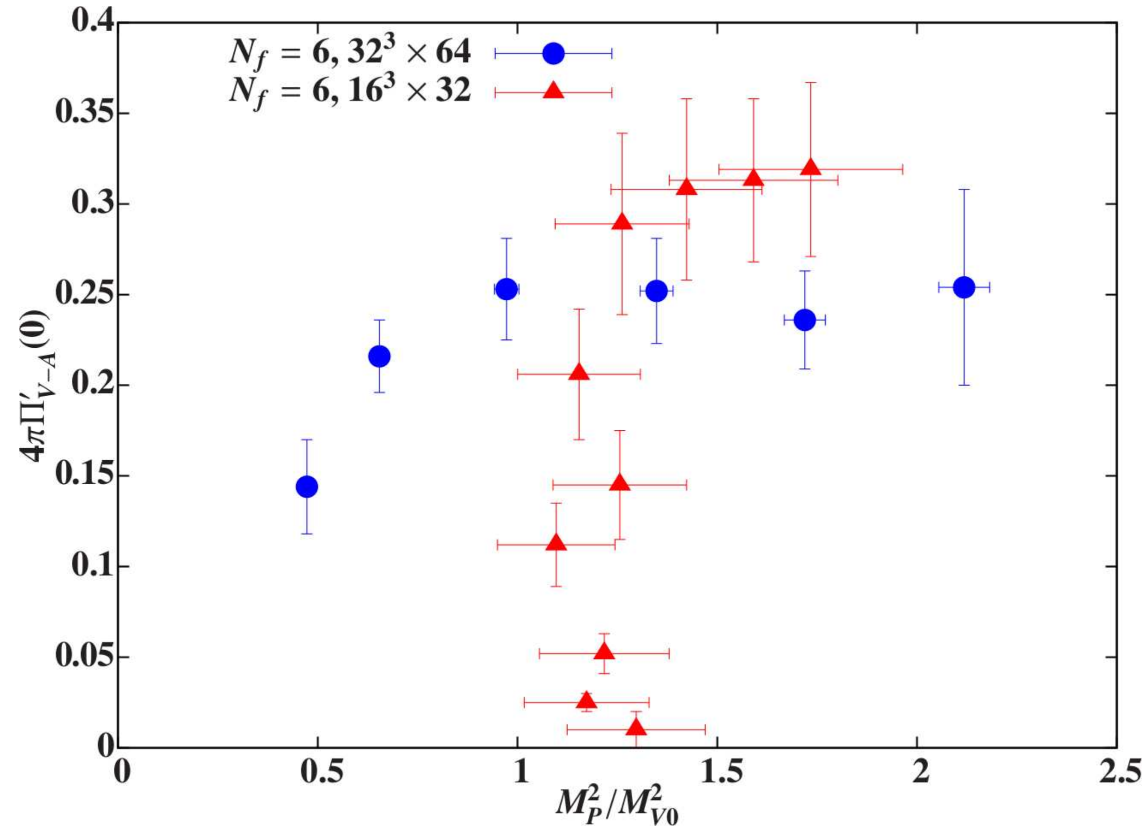
Conclusion

- A composite Higgs sector may reveal itself first through low-energy effects - deviations in EW precision, WW scattering, etc.
- Many UV theories can reduce to one effective theory, but low-energy constants determined by strong dynamics. Lattice lets us explore these constants and how they evolve in the large parameter space.
- LSD program focused on SU(3) thus far, N_f=2 to N_f=6. Hints of interesting trends for chiral condensate, S-parameter, WW scattering length. N_f=8,10 in progress - stay tuned
- Part of getting the low-energy theory right is getting the states right, so priority focus now on other light states, in particular light scalar! Scalar meson and glueball calculations in progress on all our lattices.

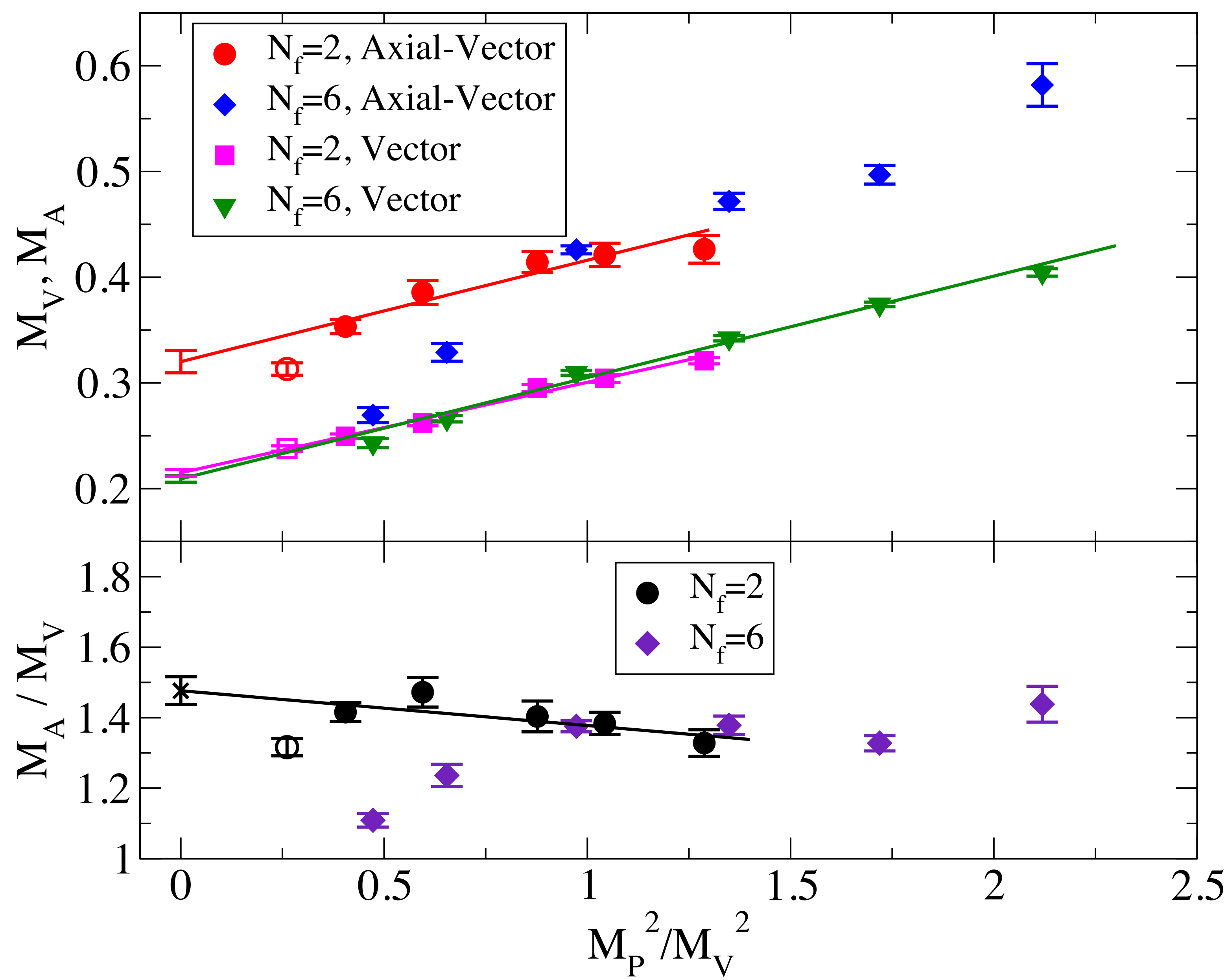
Backup slides

Finite-volume issues and S?

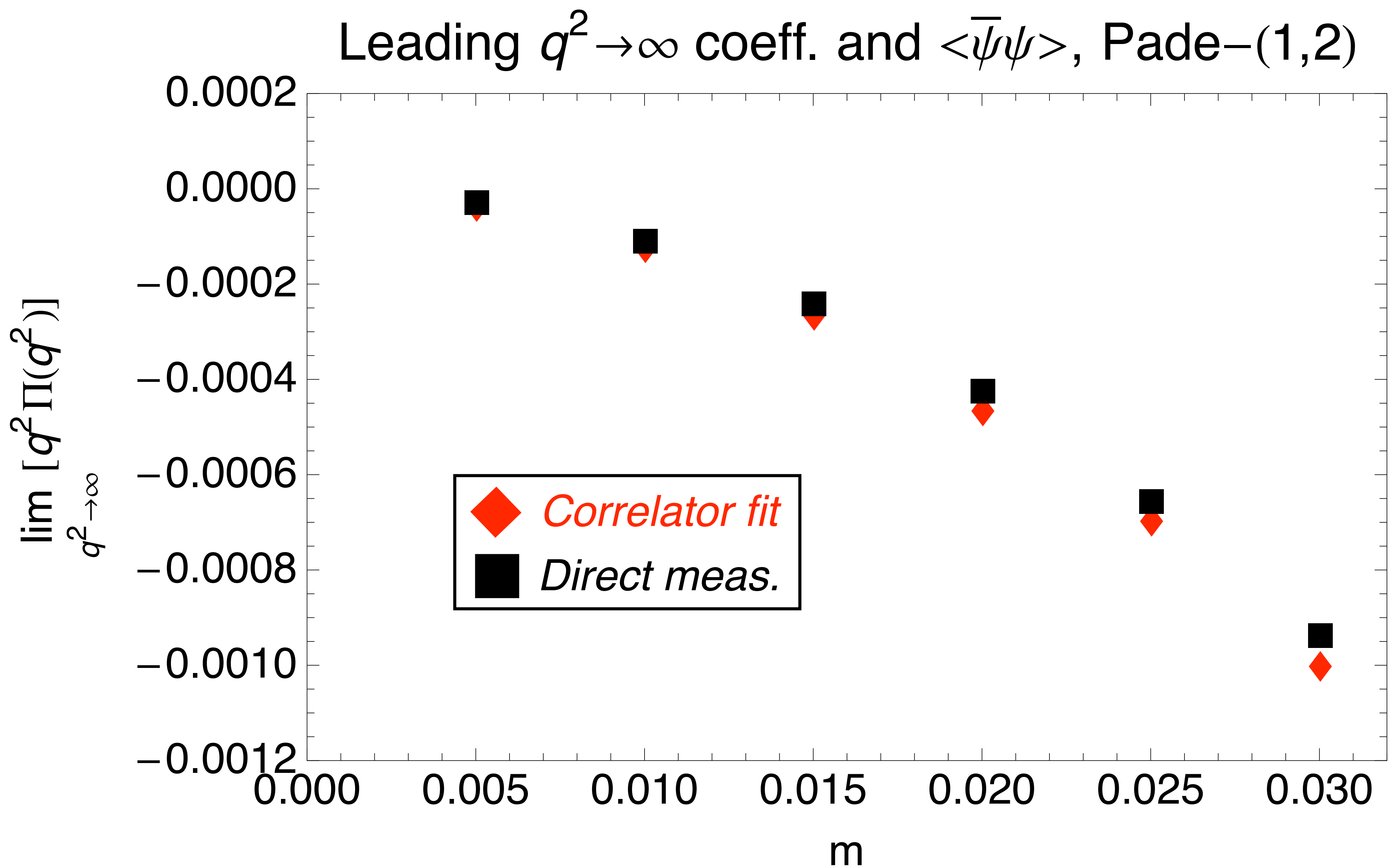




Comparing with the spectrum



OPE and extrapolation to large q^2



From slope to S

assumes all technifermions carry EW charge!

$\sim 4\pi\Pi'_{V-A}(0)$

$$S = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left\{(N_f/2)\left[R_V(s) - R_A(s)\right] - \frac{1}{4}\left[1 - \left(1 - \frac{m_h^2}{s}\right)^3 \Theta(s - m_h^2)\right]\right\}$$

ref. Higgs mass; we take $m_h \equiv M_{V0}$ (=1 TeV, roughly)

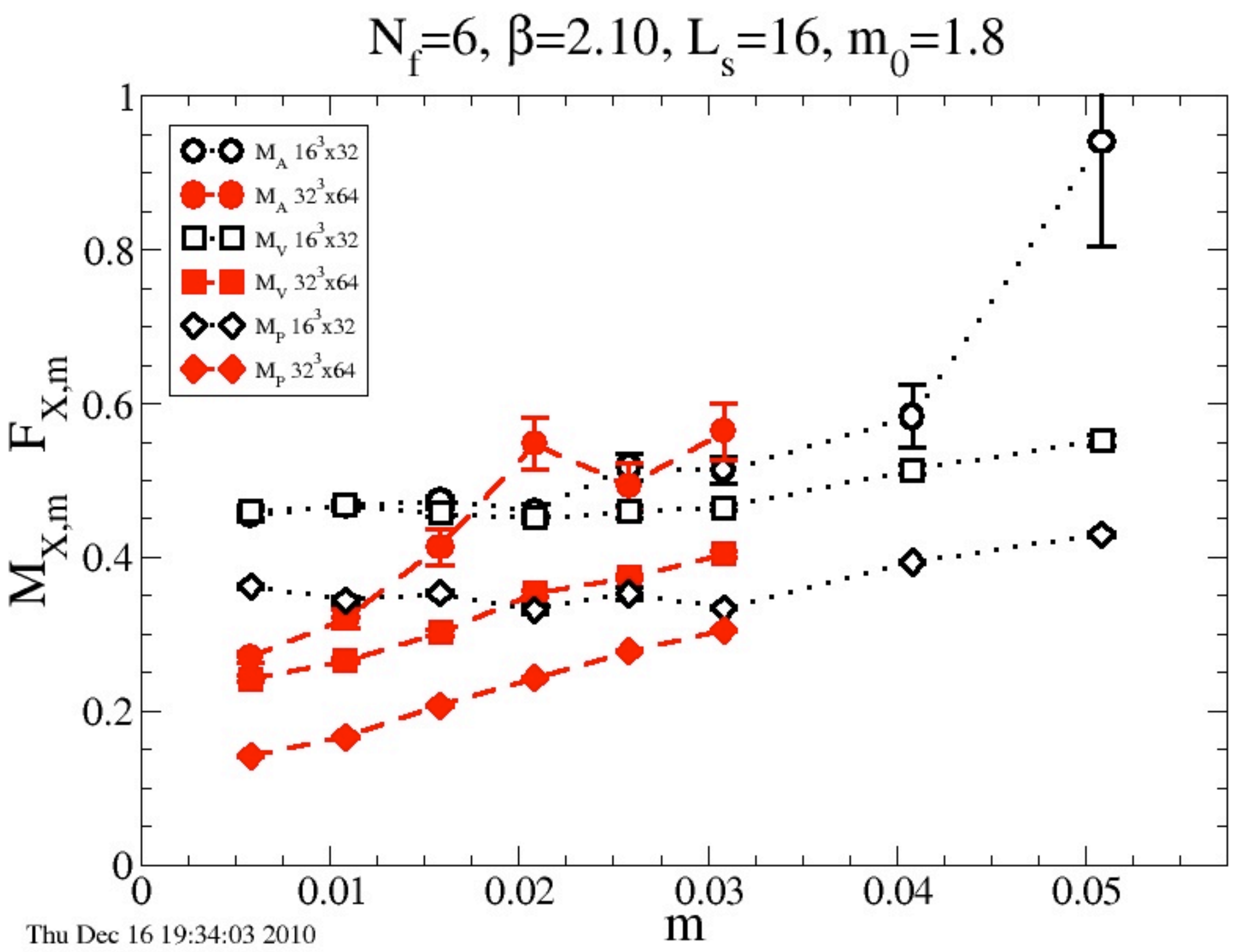
Standard model subtraction:

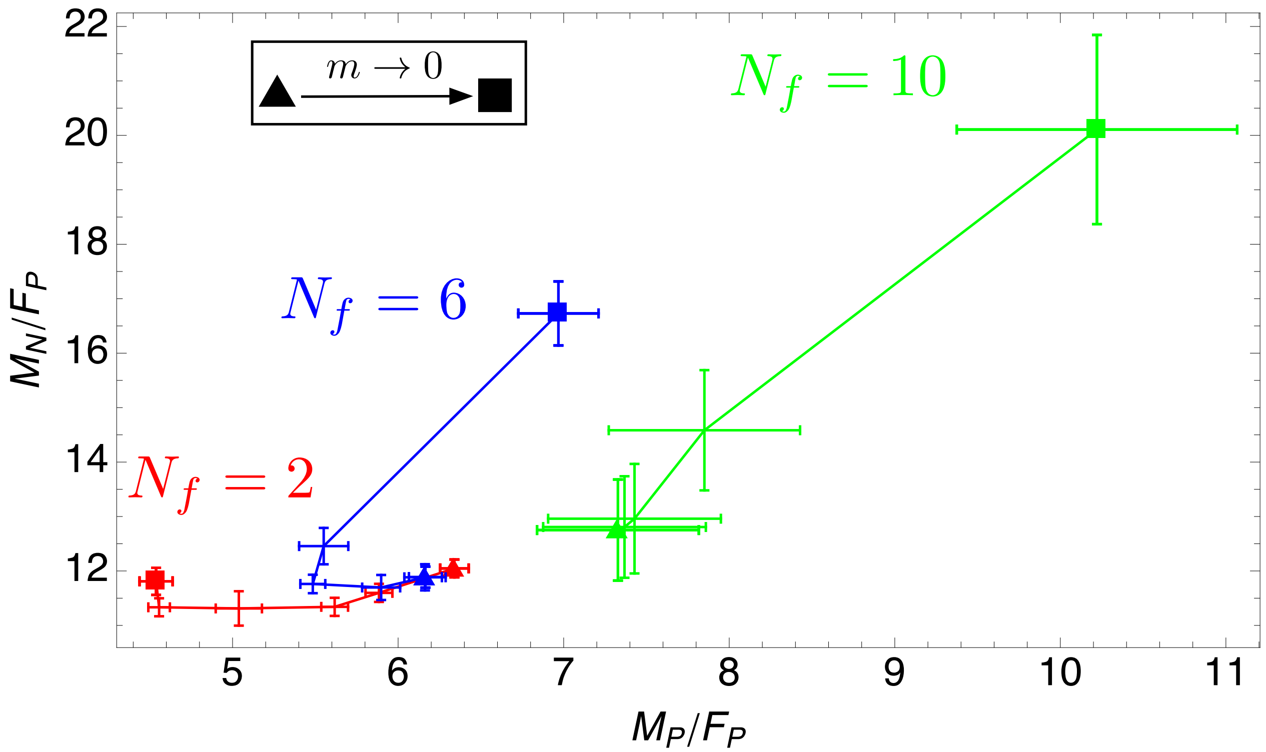
$$\Delta S_{SM} = \frac{1}{12\pi}\left[\frac{11}{6} + \log\left(\frac{M_{V0}^2}{4M_P^2}\right)\right]^*$$

- SM subtraction removes Higgs scalar contribution to S, cancels IR divergence

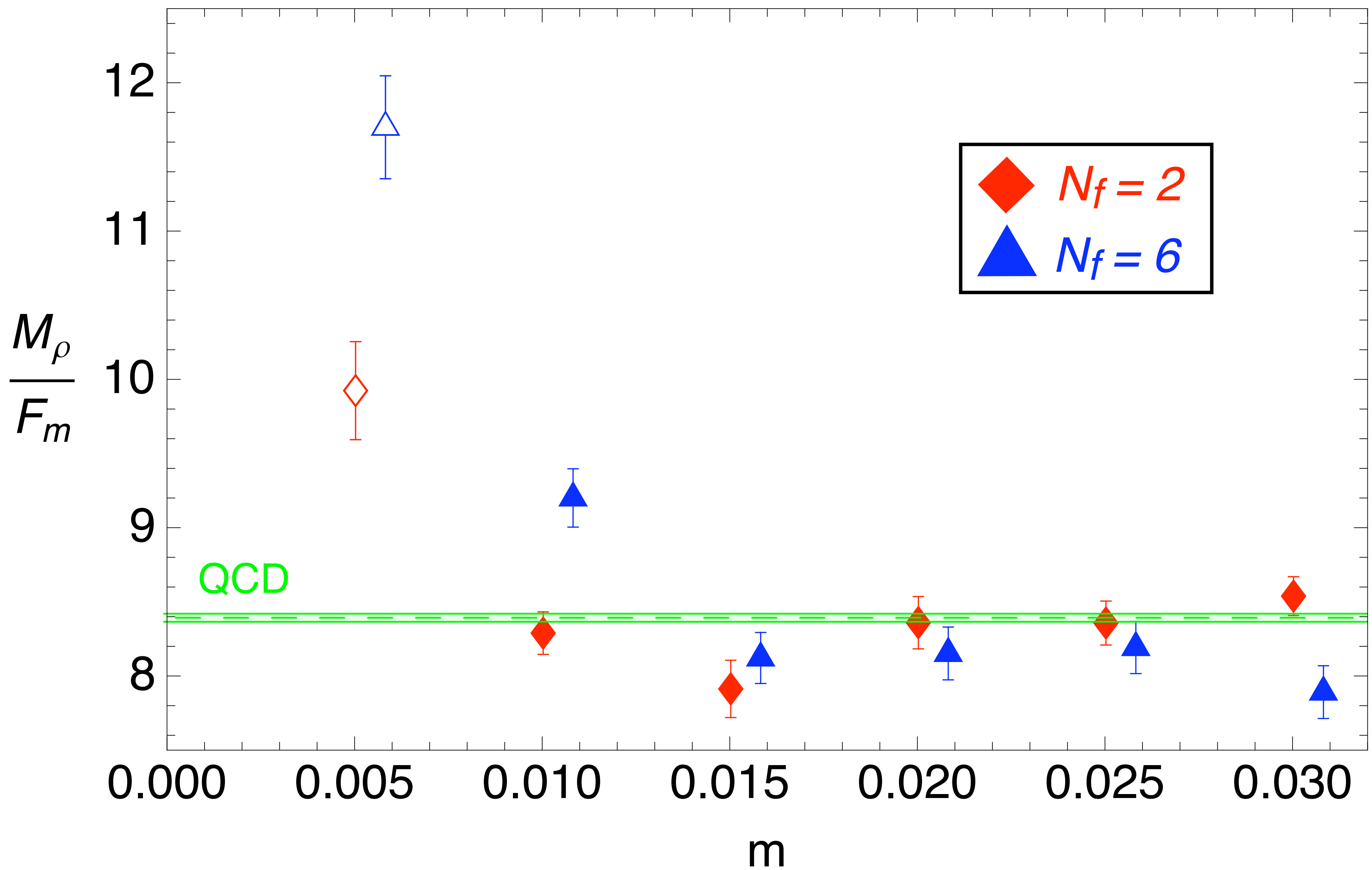
$* \left(\frac{M_{V0}^2}{M_P^2} < 1/4\right)$

Finite-volume again from the spectrum

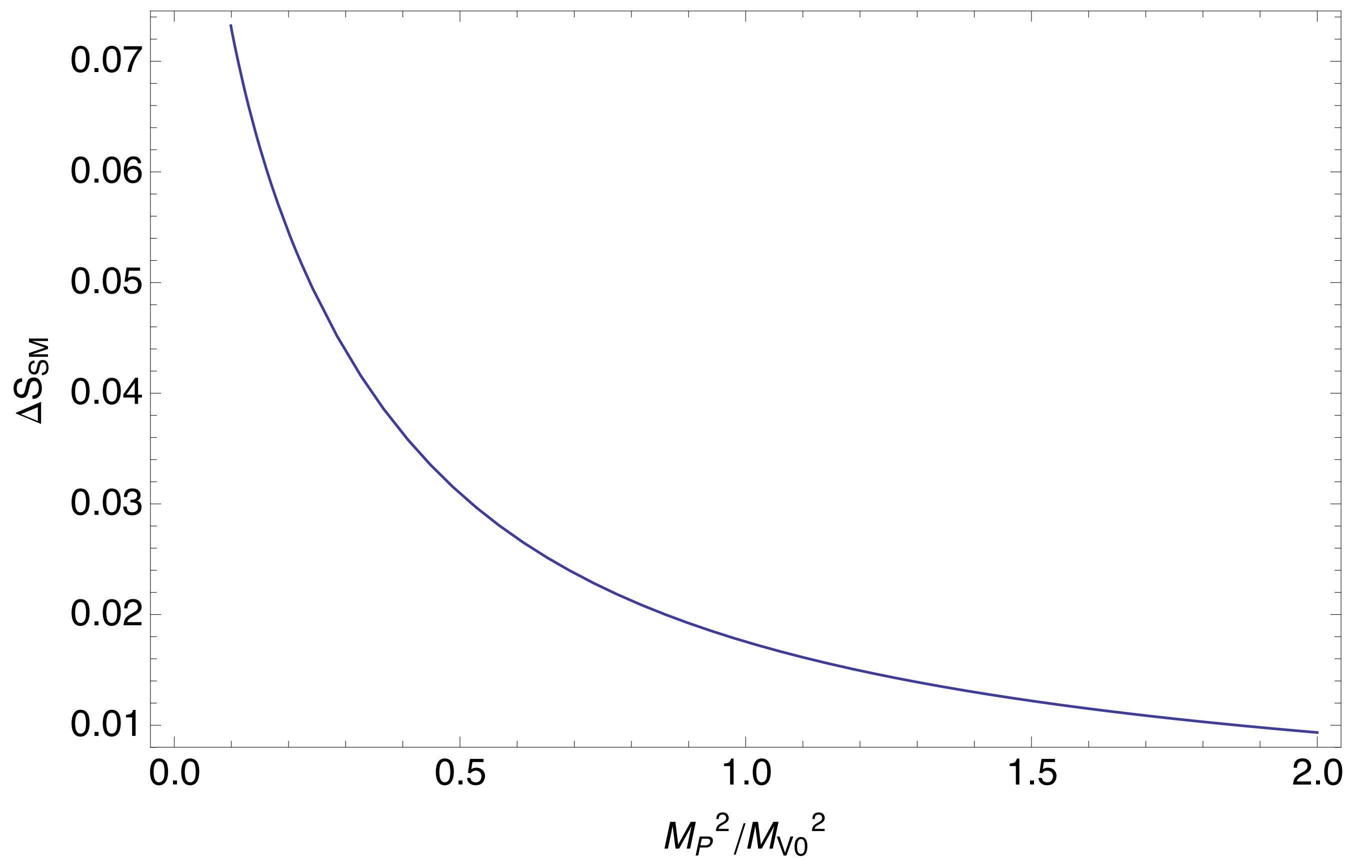




Scale setting: rho vs. F



S-parameter SM subtraction

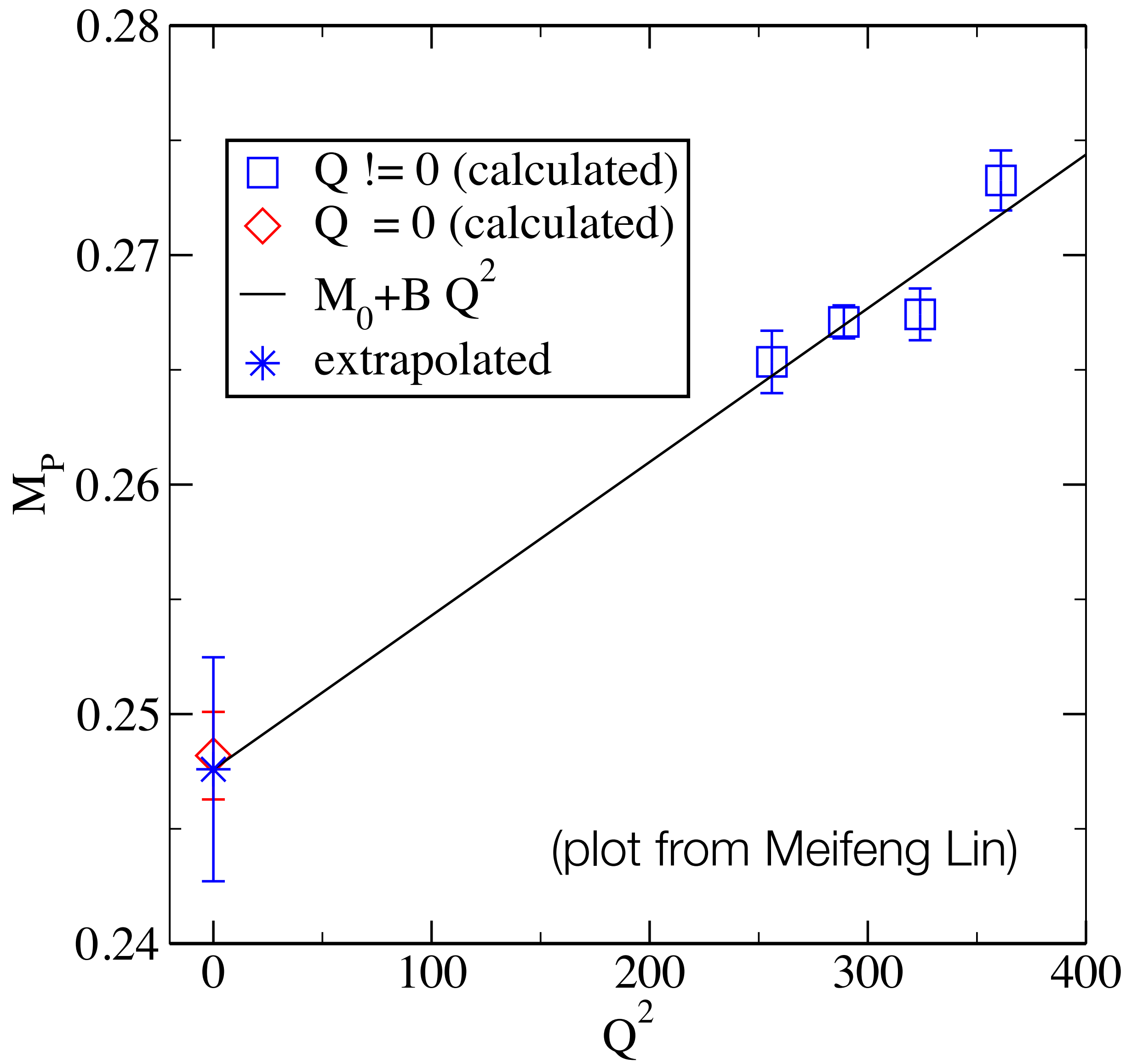


As a function of $x \equiv M_P^2/M_{V0}^2$, then, the SM subtraction is

$$\Delta S_{SM} = \begin{cases} \frac{1}{12\pi}\left[\frac{11}{6} + \log\left(\frac{1}{4x}\right)\right], & x < 1/4, \\ \frac{1}{12\pi}\left(\frac{3}{4x} - \frac{3}{32x^2} + \frac{1}{192x^3}\right), & x \geq 1/4. \end{cases}$$

Ordered vs. disordered at 10 flavors

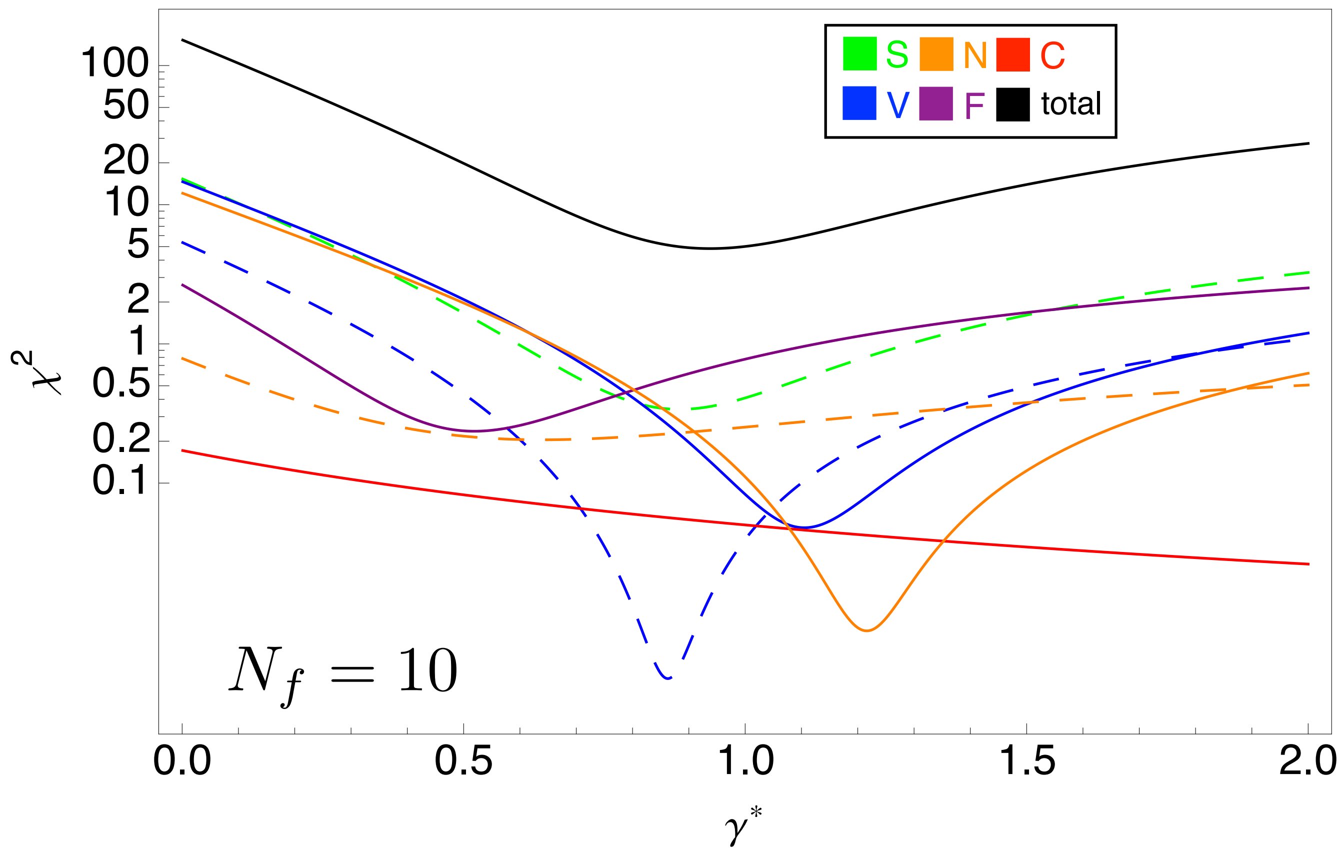
- Internal analysis has revealed that frozen topological charge can explain the discrepancy between our two starts:



- Current plan is to measure topological susceptibility (slope of the Q-dependence) and correct our results

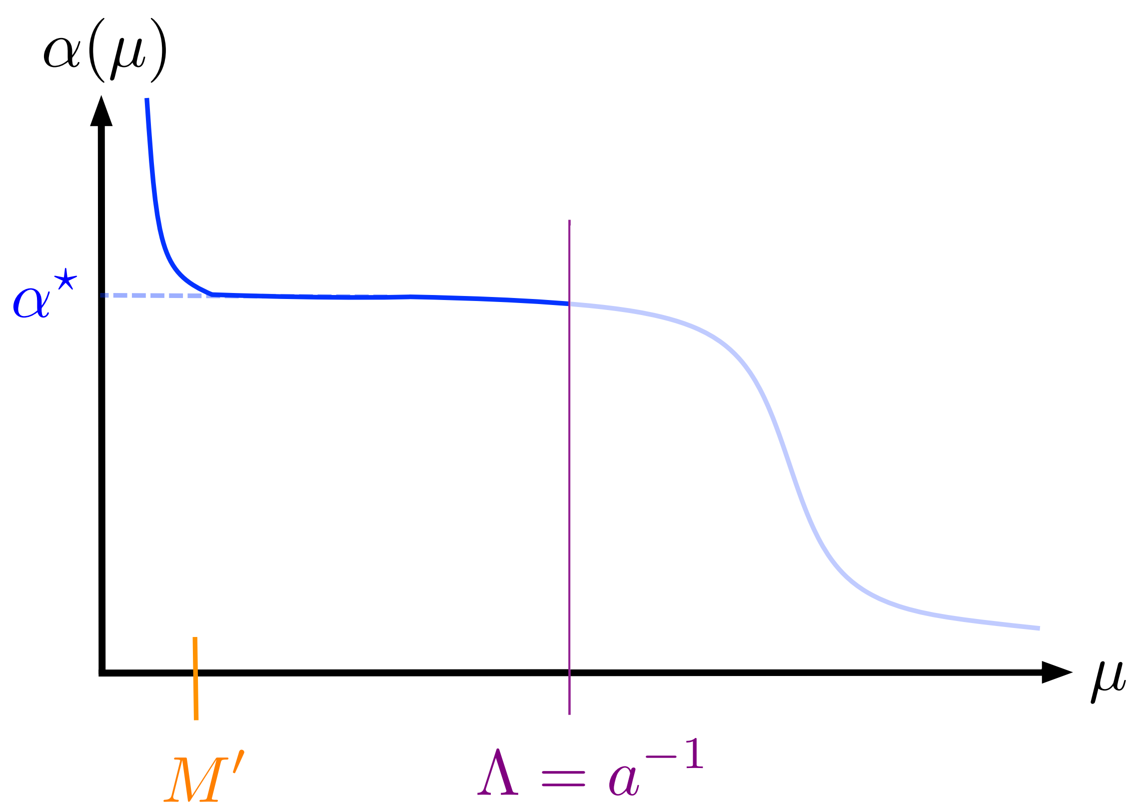
Scaling fit results, Nf=10

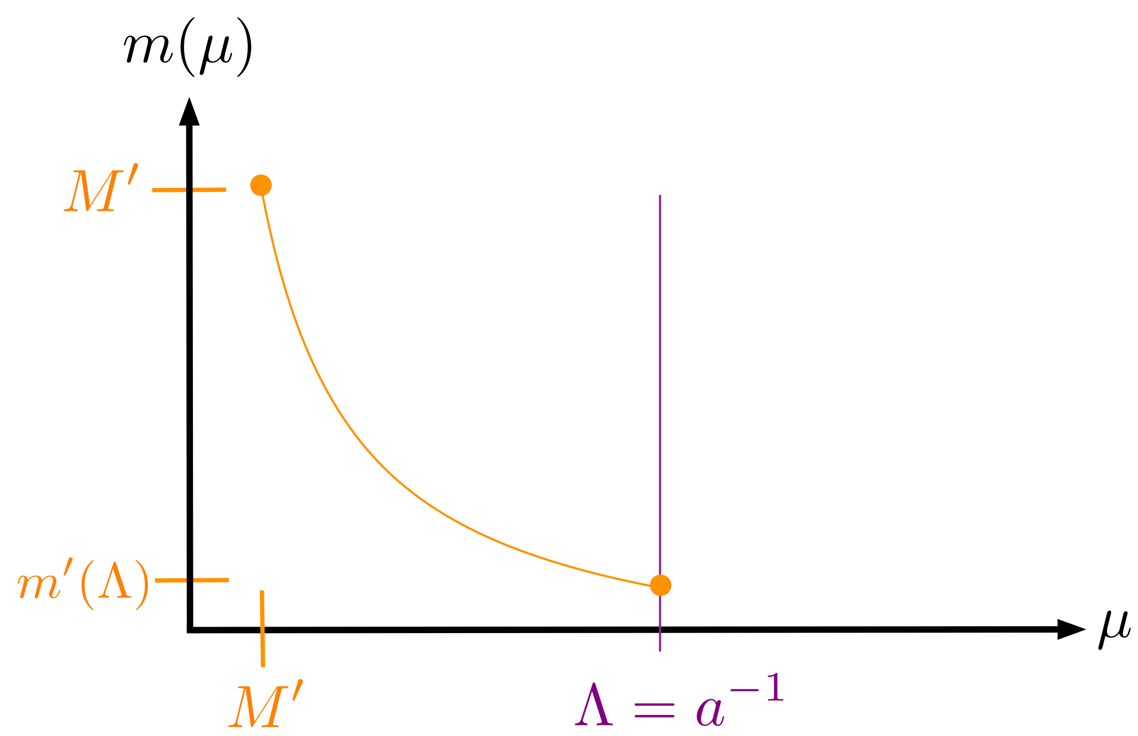
(shown for mf>=0.015



Obs.	$m_f \geq 0.010$	$m_f \geq 0.015$	$m_f \geq 0.020$
$\gamma^\star$	*1.69(16)*	1.10(17)	1.35(47)
[68% CI]	*[1.54,1.86]*	[0.95,1.27]	[1.06,1.73]
[95% CI]	*[1.40,2.06]*	[0.82,1.46]	[0.83,2.27]
C_P	*0.98(9)*	1.44(21)	1.21(37)
C_V	*1.17(10)*	1.70(25)	1.42(44)
C_A	*1.43(13)*	2.14(32)	1.79(56)
C_N	*1.75(16)*	2.53(37)	2.10(65)
$C_{N^\star}$	*2.23(25)*	3.35(55)	2.87(92)
C_{FP}	*0.121(12)*	0.190(28)	0.164(51)
C_{FV}	*0.165(15)*	0.238(35)	0.195(60)
C_{FA}	*0.136(13)*	0.192(28)	0.154(48)
χ^2/d.o.f.	*69/31*	14/23	3.1/15

Mass deformation





4) Bound-state masses are set by M, as in QCD-like theory. Three major differences here:

- No Goldstones - PS state scales like everything else.
- M is <u>controlled</u> by m: $M \sim m^{1/(1+\gamma^\star)}$
- Expansion in *am*, as opposed to $aM_\pi^2/(4\pi F_\pi)^2$ for χPT