Exploring walking behavior in SU(3) gauge theory with 8 HISQ quarks

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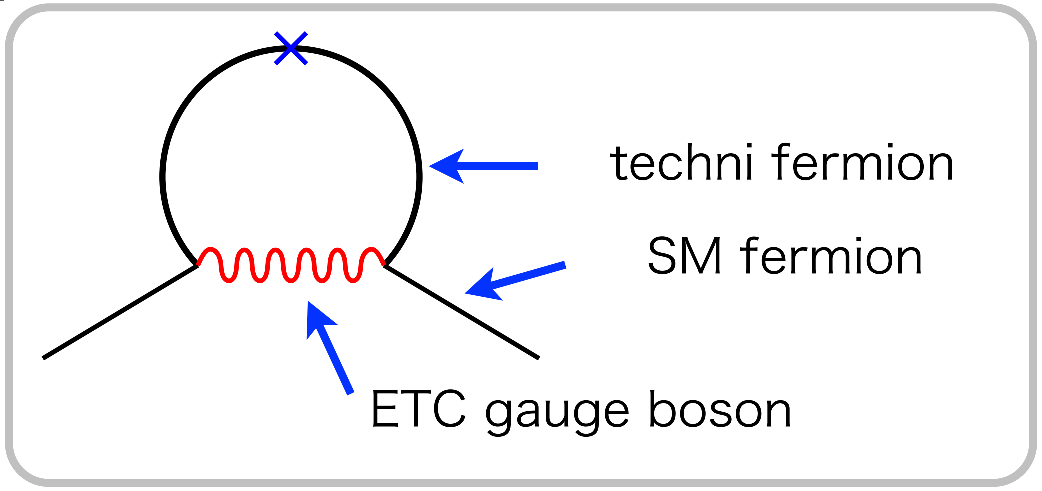
1. Introduction

- LQCD with many fermions
- ⇒ Candidate of the technicolor

Extended Technicolor (ETC)

- fermion masses → extended technicolor (ETC)
- New strong interaction of $SU(N_{ETC})$: $N_{ETC} > N_{TC}$, $T_{ETC} = (T, f)$: $T \in TC$, $f \in SM$
- SSB: $SU(N_{ETC}) \to SU(N_{TC}) \times SM$ @ Λ_{ETC} ($\gg \Lambda_{TC}$)
- $$\frac{1}{\Lambda_{ETC}^2} \overline{T} T \overline{f} f \to m_f = \frac{\langle \overline{T} T \rangle_{ETC}}{\Lambda_{ETC}^2}$$
- $$\frac{1}{\Lambda_{ETC}^2} \overline{f} f \overline{f} f$$ FCNC



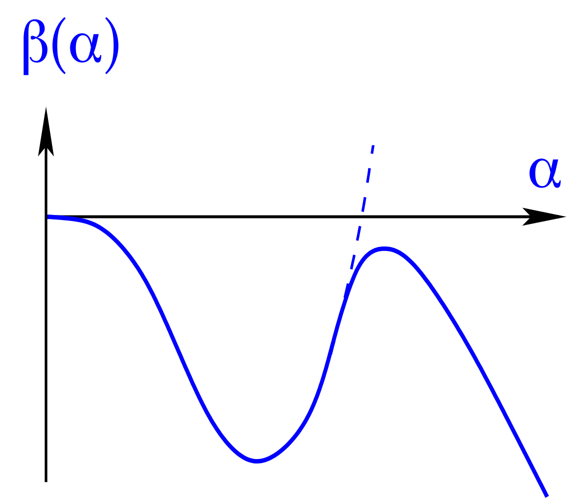
- FCNC should be small ⇔ top or bottom quark mass should be produced

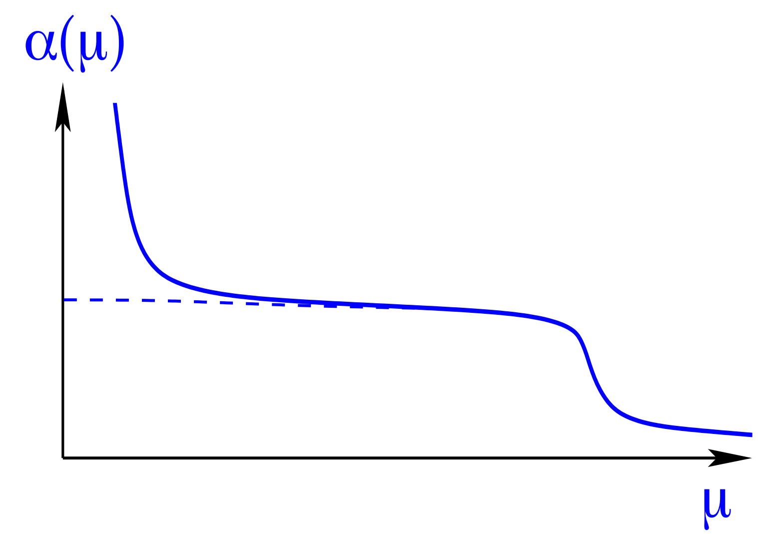
➡ walking TC

Walking Technicolor

- key: to realize suppressed FCNC and appropriate size of fermion masses

[Holdom, Yamawaki-Bando-Matsumoto]



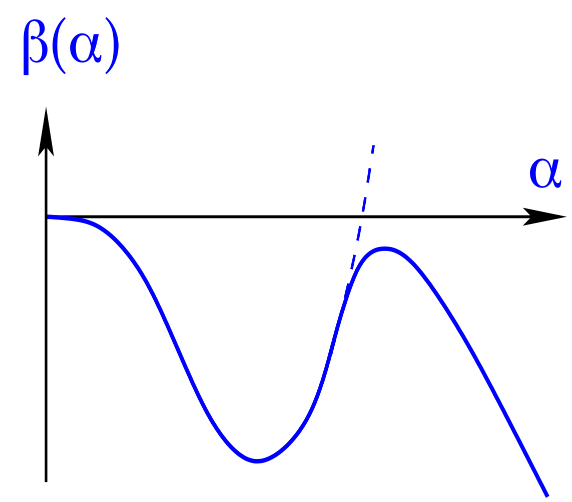


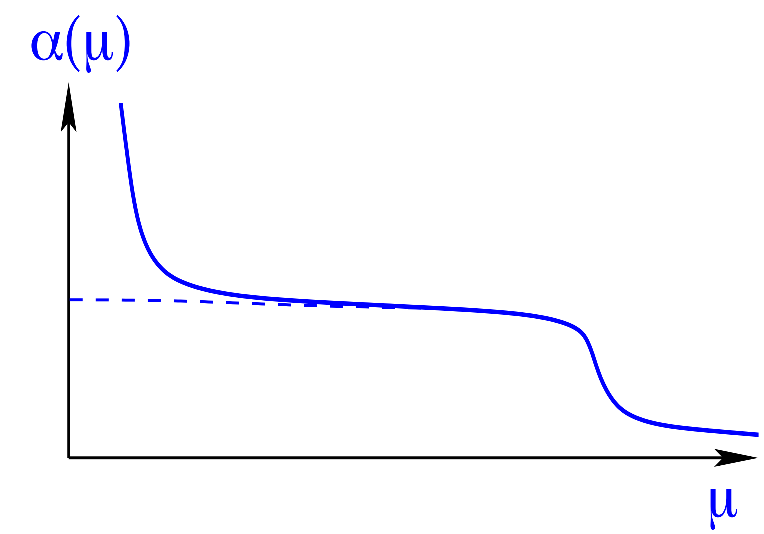
- renormalized gauge coupling
  - to run very slowly (walking)
  - logarithmically divergent at low energies → to produce techni-pions
- mass anomalous dimension
  - large: $\gamma_m \sim 1$

Walking Technicolor

- key: to realize suppressed FCNC and appropriate size of fermion masses

[Holdom, Yamawaki-Bando-Matsumoto]



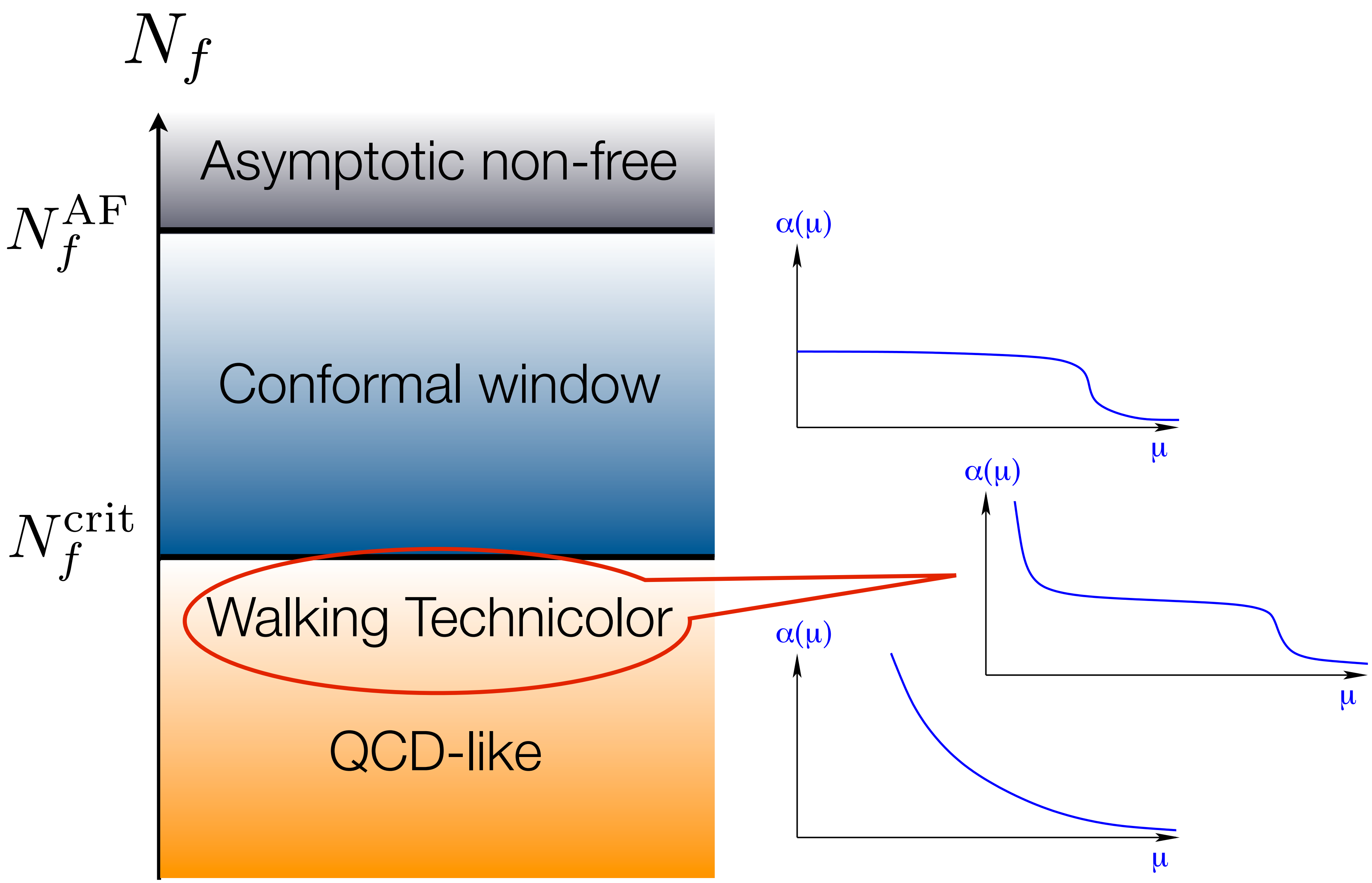


- renormalized gauge coupling
  - to run very slowly (walking)
  - logarithmically divergent at low energies → to produce techni-pions
- mass anomalous dimension
  - large: $\gamma_m \sim 1$

Is it possible to construct such a theory ?

conformal window and walking coupling

- non-Abelian gauge theory with N_f massless fermions -



- Walking Techinicolor could be realized just below the conformal window
- crucial information: N_f^{crit} & mass anomalous dimension around N_f^{crit}

One-family Extended TechniColor model

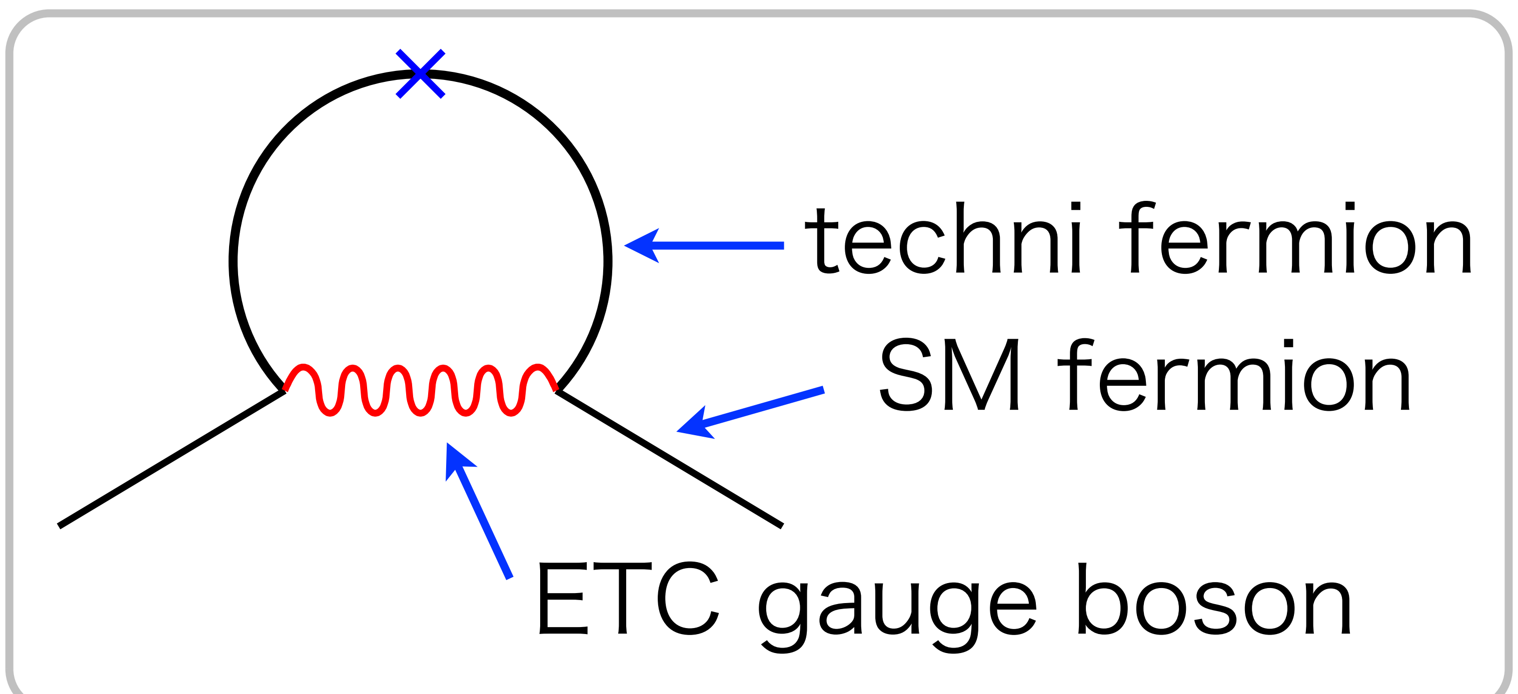
$$
\begin{pmatrix} u \\ d \\ u \\ d \\ u \\ d \\ e \\ \nu_e \end{pmatrix}
\begin{pmatrix} c \\ s \\ c \\ s \\ c \\ s \\ \mu \\ \nu_\mu \end{pmatrix}
\begin{pmatrix} t \\ b \\ t \\ b \\ t \\ b \\ \tau \\ \nu_\tau \end{pmatrix}
\begin{pmatrix} U_1 \\ D_1 \\ U_1 \\ D_1 \\ U_1 \\ D_1 \\ E_1 \\ N_1 \end{pmatrix}
\cdots
\begin{pmatrix} U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ E_{N_{\rm TC}} \\ N_{N_{\rm TC}} \end{pmatrix}
$$

$SU(3+N_{\rm TC})$

$\Lambda_1 \to SU(2+N_{\rm TC})$

$\Lambda_2 \to SU(1+N_{\rm TC})$

$\Lambda_3 \to SU(N_{\rm TC})$



8-flavor $\mathrm{SU}(N_{\rm TC})$ technicolor

We consider $N_{\rm TC} = 3$

Nf=12 is consistent with the conformal with $\gamma \sim 0.5$.

(Nf=12, H.Ohki's talk) [LatKMI collab. PRD86 (2012) 054506]

Nf=8?

Perturbation : 2-loop running coupling in large Nf QCD

RGE $\mu \frac{d}{d\mu}\alpha(\mu) = \beta(\alpha) = -b\,\alpha^2(\mu) - c\,\alpha^3(\mu)$

$(N_c = 3)$	$N_f < 8.05$	$8.05 < N_f < 16.5$	$16.5 < N_f$
$b = \frac{1}{6\pi}(33 - 2N_f)$	+	+	−
$c = \frac{1}{12\pi^2}(153 - 19N_f)$	+	−	−

$N_f = 8$ is QCD-like theory??

Q:

Is Nf=8 walking theory to construct the one-family model in ETC ?

(non-perturbative)

Strong coupling dynamics ⇒ Lattice simulation

1. Lattice simulation

LatKMI collaboration

- Y.Aoki, T.Aoyama, M.Kurachi, T.Maskawa, K.-i.Nagai, H.Ohki,

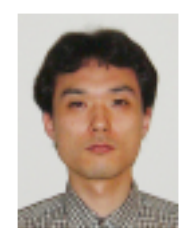





E. Rinaldi, K.Yamawaki, T.Yamazaki

名古屋大学





KMi IMK KMI

- K. Hasebe, A.Shibata

愛知大学 AICHI UNIVERSITY


KEK

Simulation details (Nf=8 ⇒ large Nf QCD)

lattice action (8 flavor HMC simulation)

- Tree-level Symanzik gauge action
- Highly Improved Staggered Quarks (HISQ)

parameter set

- $\beta\,(\equiv 6/g^2) =$ 3.8, (3.7, 3.9 4.0)

V	12^3 x 16	18^3 x 24	24^3 x 32	30^3 x 40	36^3 x 48
mf	0.04~0.16	0.04~0.1	0.02~0.1	0.02~0.07	0.015~0.03

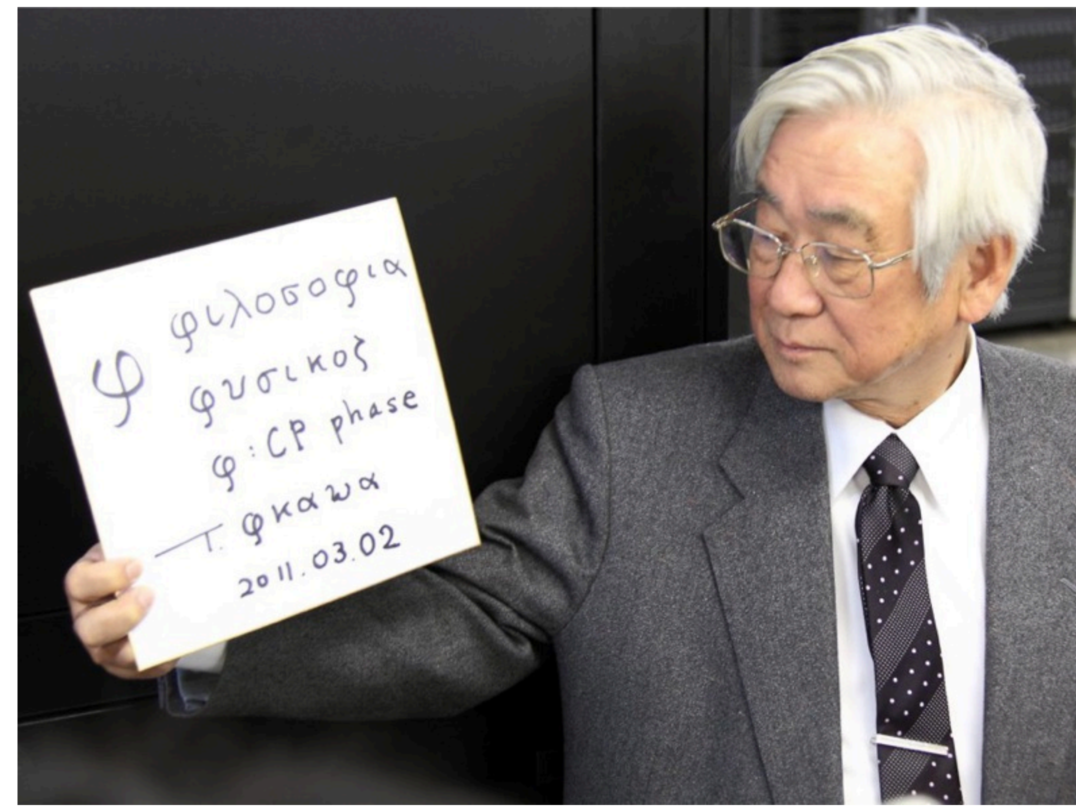
(In preparation.)

Measurements

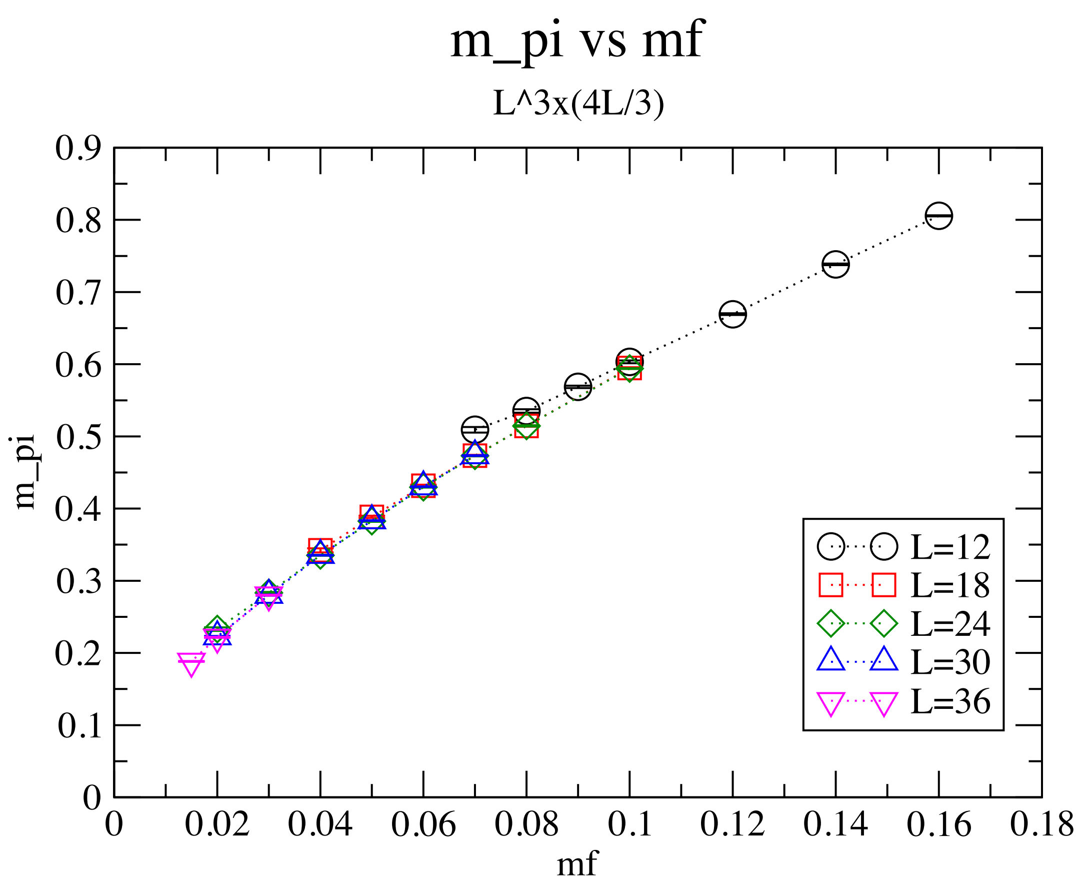
- $m_\pi,\ f_\pi,\ m_\rho$

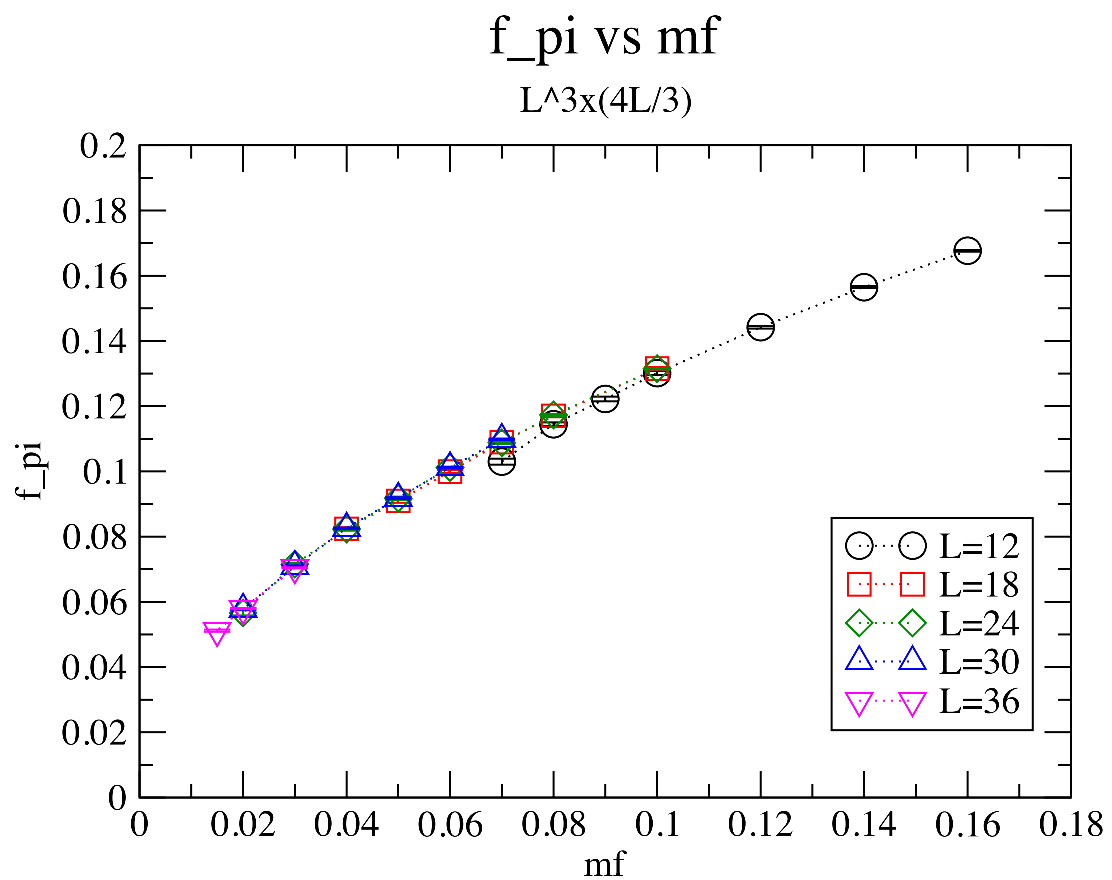
KMI computer, φ

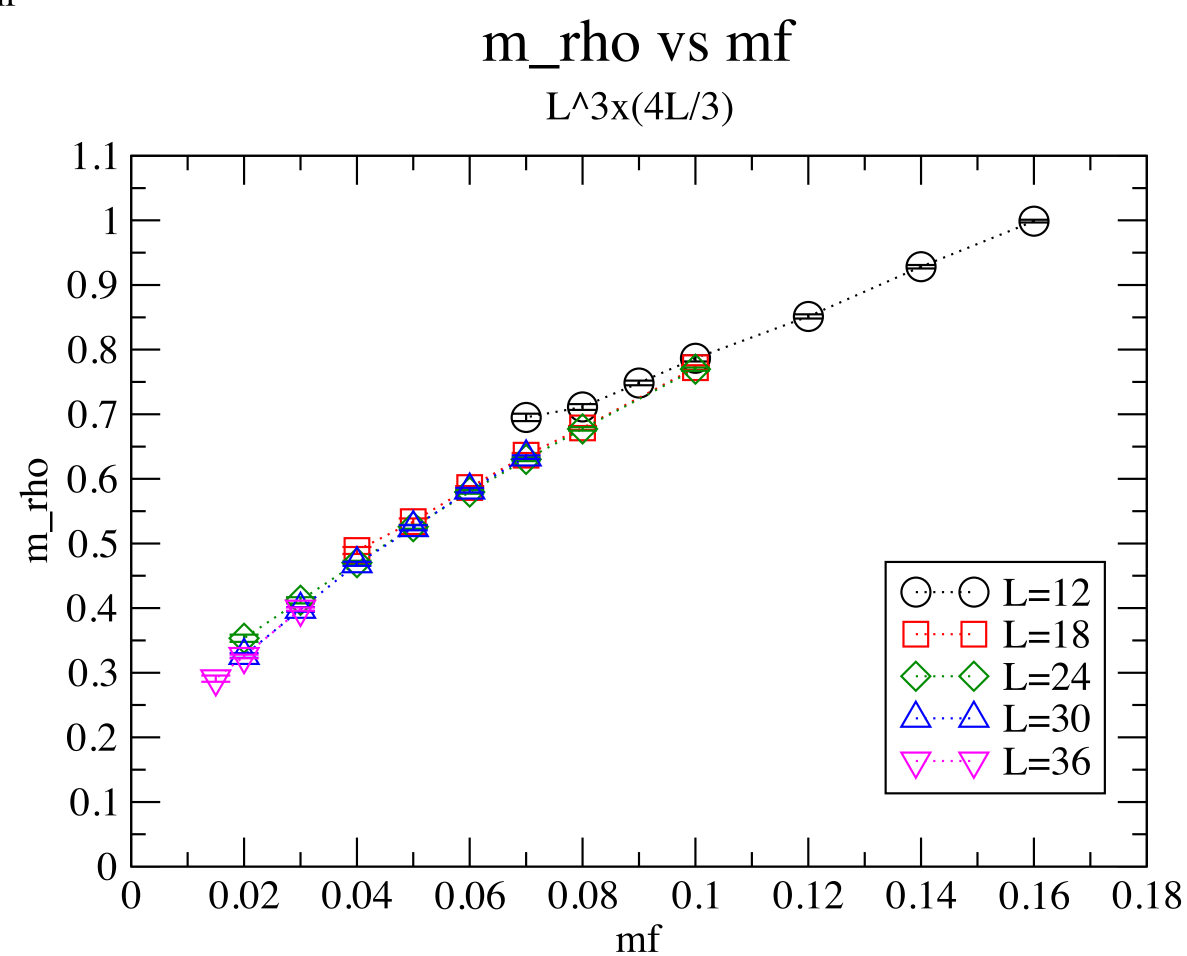
- non GPU nodes
  - 148 nodes
  - 2x Xenon 3.3 GHz
  - 24 TFlops (peak)
- GPU nodes
  - 23 nodes
  - 3x Tesla M2050
  - 39 TFlops (peak)



Spectroscopy (raw data) at β=3.8







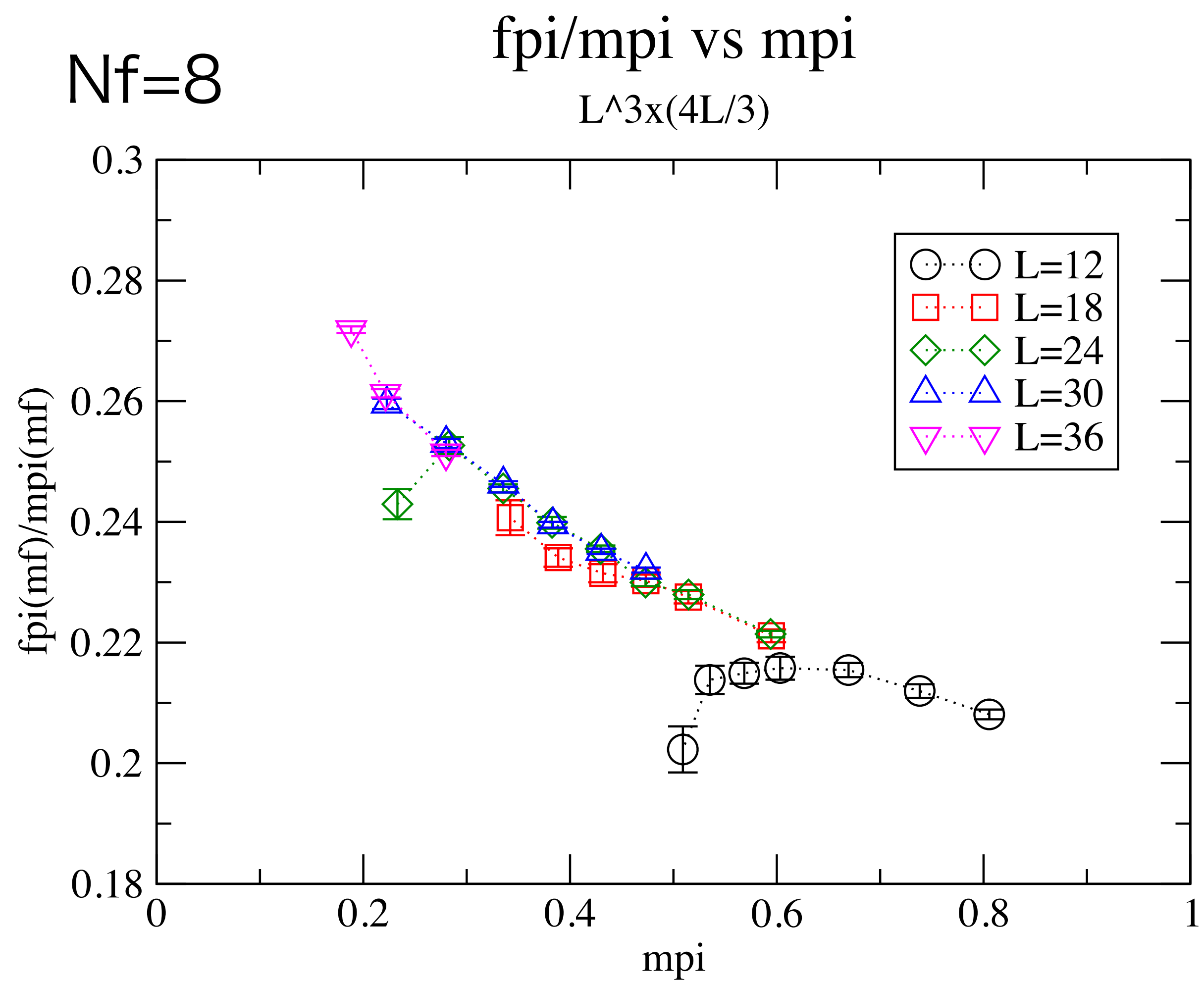
2. ChPT analysis (preliminary)

f_π / M_π

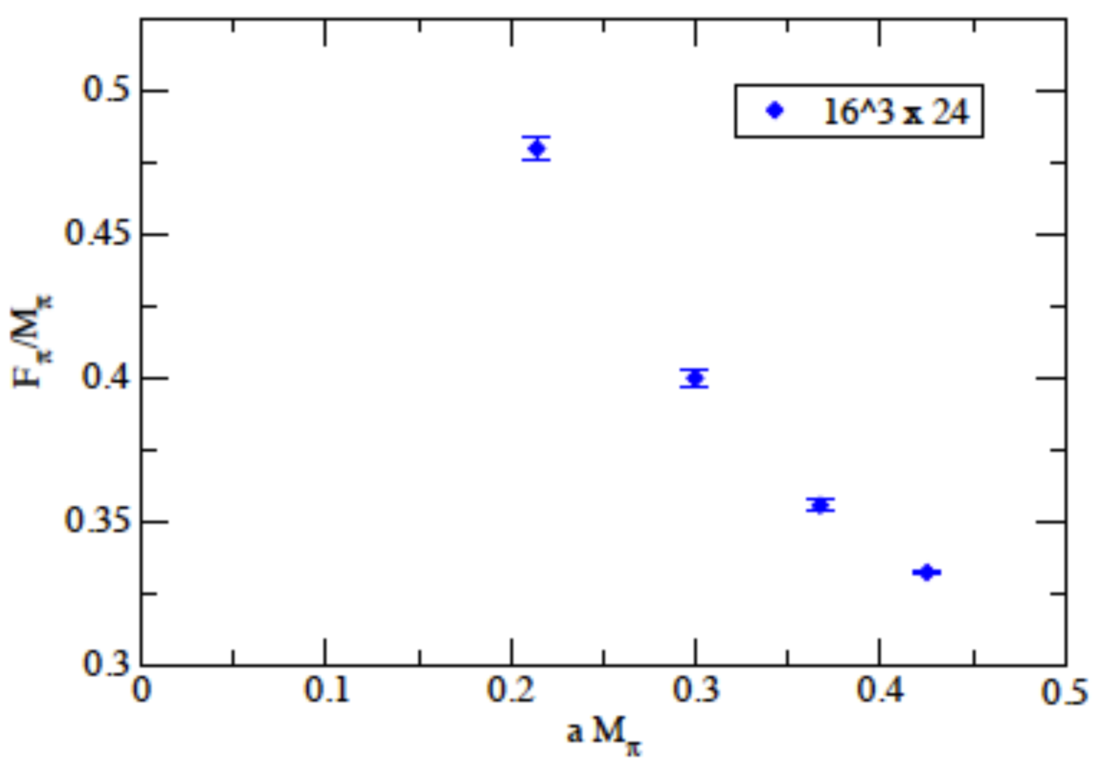
Conformal: flat behavior

SχSB: divergent

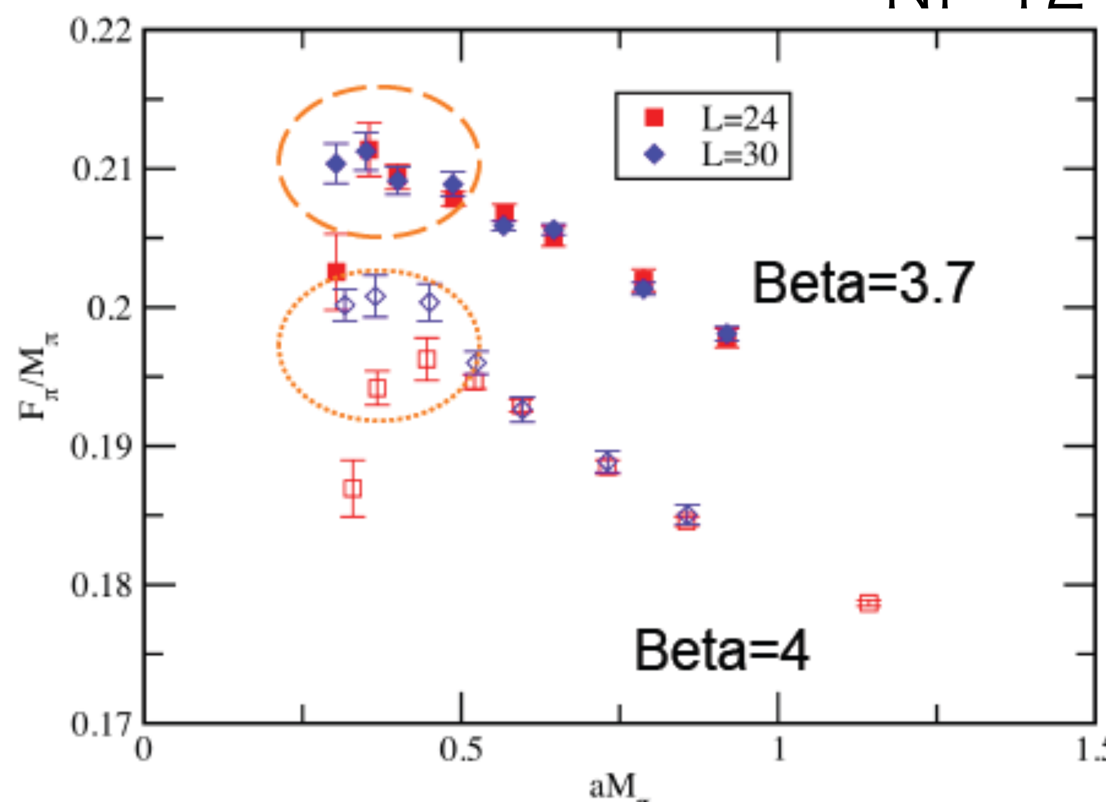
Nf=8



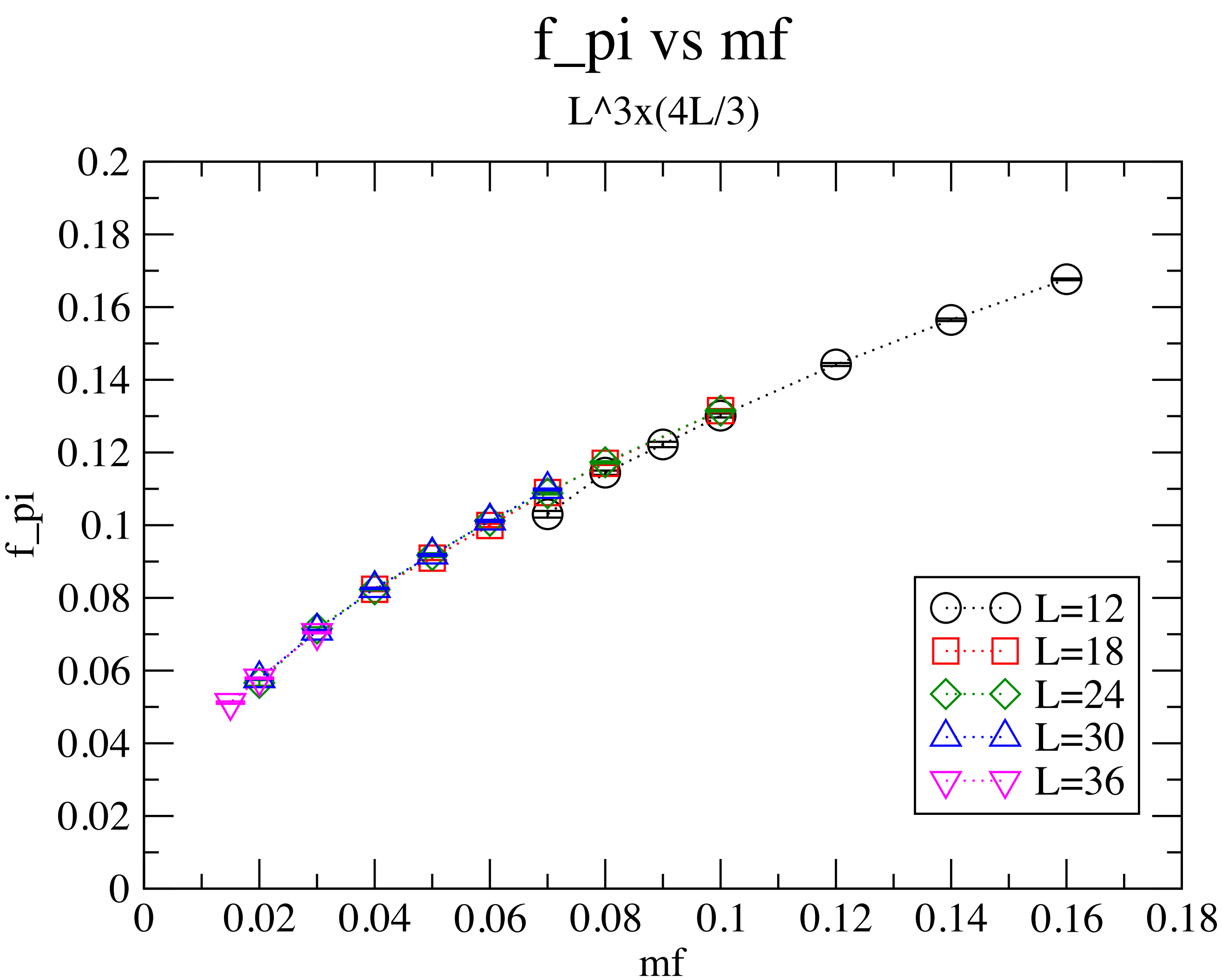
Nf=4, β=3.7

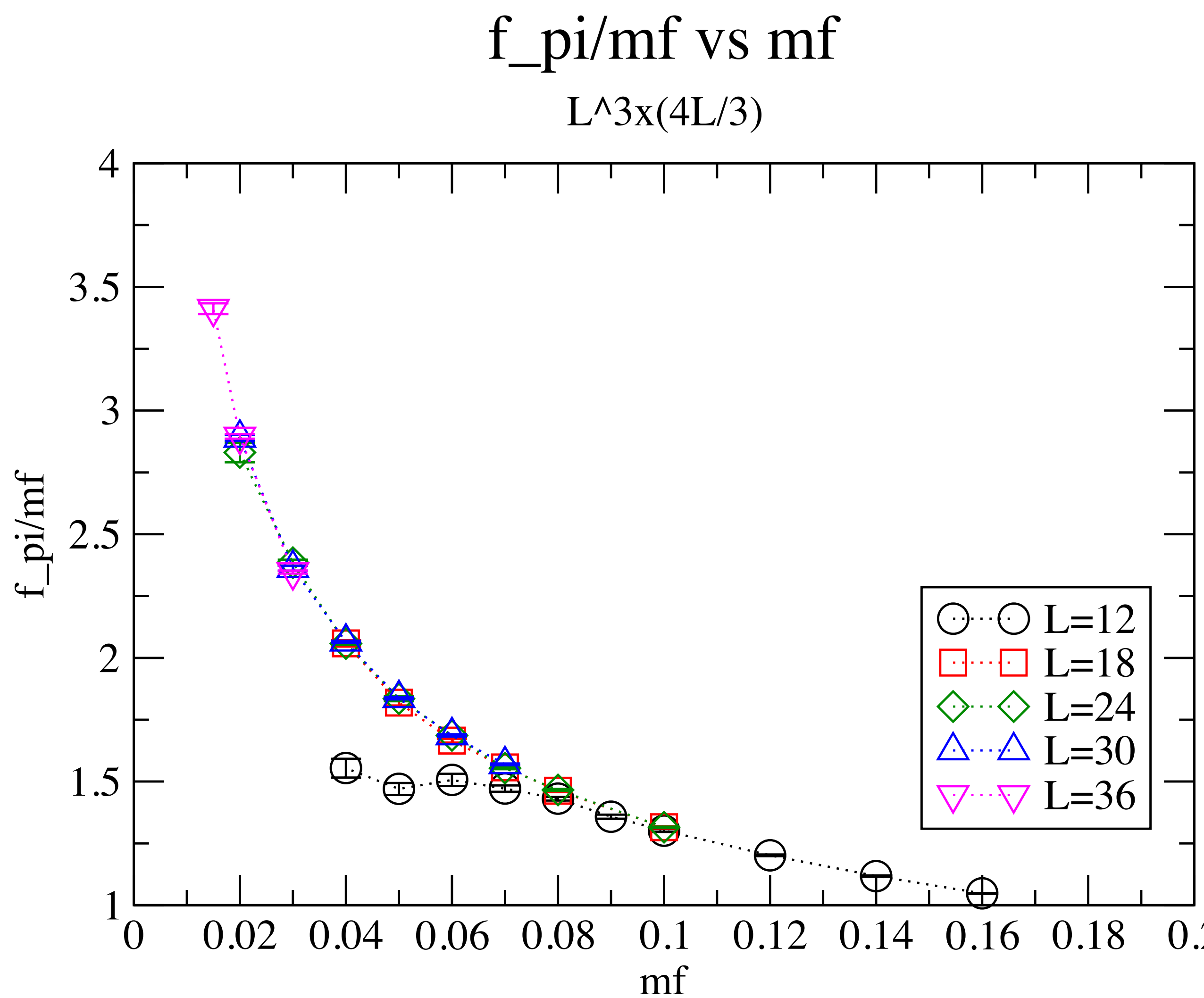


Nf=12



$f_\pi = 0$ or $\neq 0$? as $m_f \to 0$





There is the constant term or α<1 in mf^α behavior.

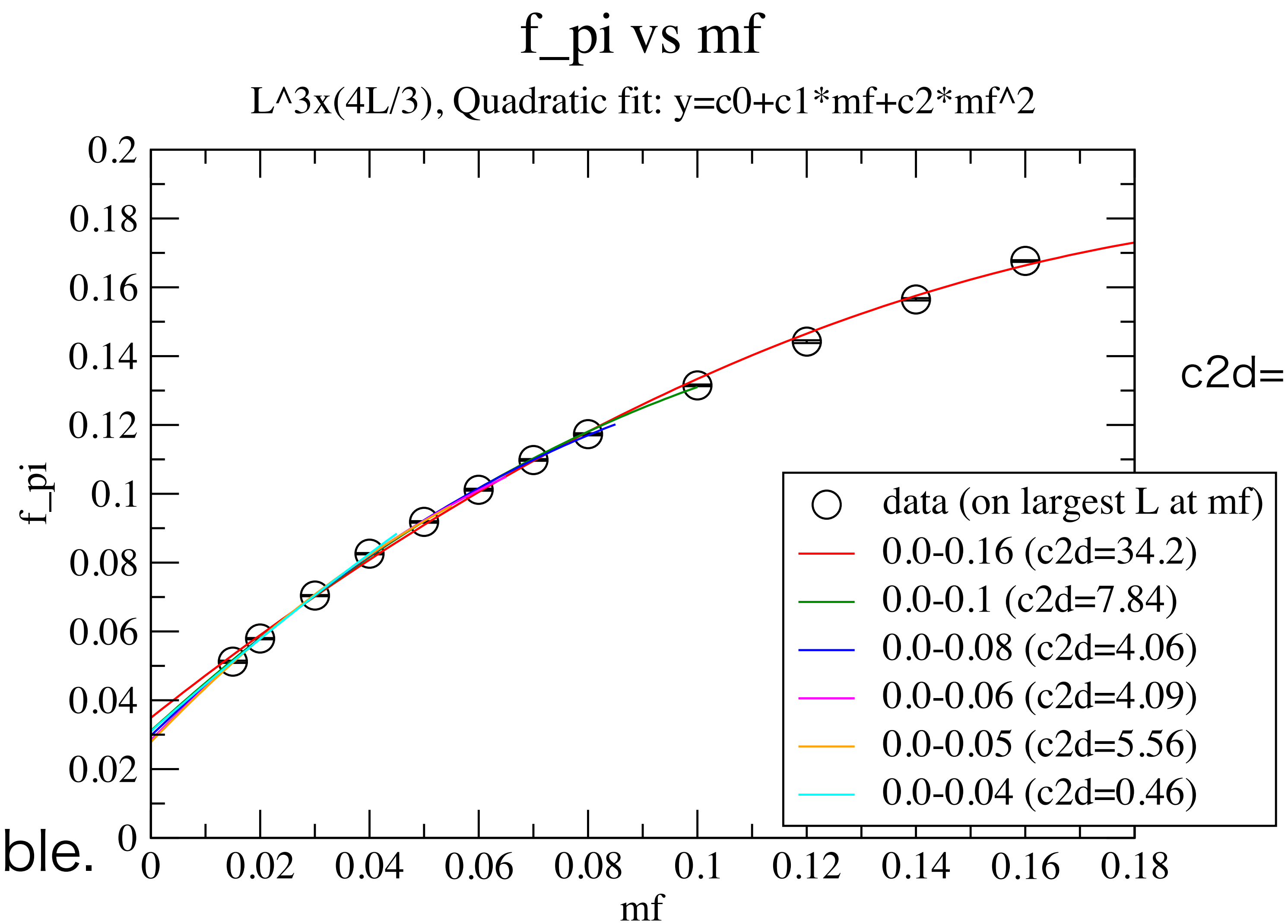
ChPT analysis (quadratic fit) of fπ

$$f_\pi \neq 0 \text{ as } m_f \to 0$$

$$f_\pi = F + A m_f + B m_f^2$$

Natural chiral expansion parameter

$$\chi = N_f \left(\frac{M_\pi}{4\pi F} \right)^2$$



c2d=χ^2/dof

Use of ChPT less questionable.

fit range(mf)	F	χ (mf=min)	χ (mf=max)
0.015-0.04	0.0310(13)	1.87	5.90
0.015-0.05	0.0278(8)	2.32	9.64
0.015-0.06	0.0284(6)	2.22	11.6
0.015-0.08	0.0296(4)	2.05	15.3
0.015-0.10	0.0311(3)	1.85	18.5
0.015-0.16	0.0349(2)	1.47	27.0

ChPT fit is not good.

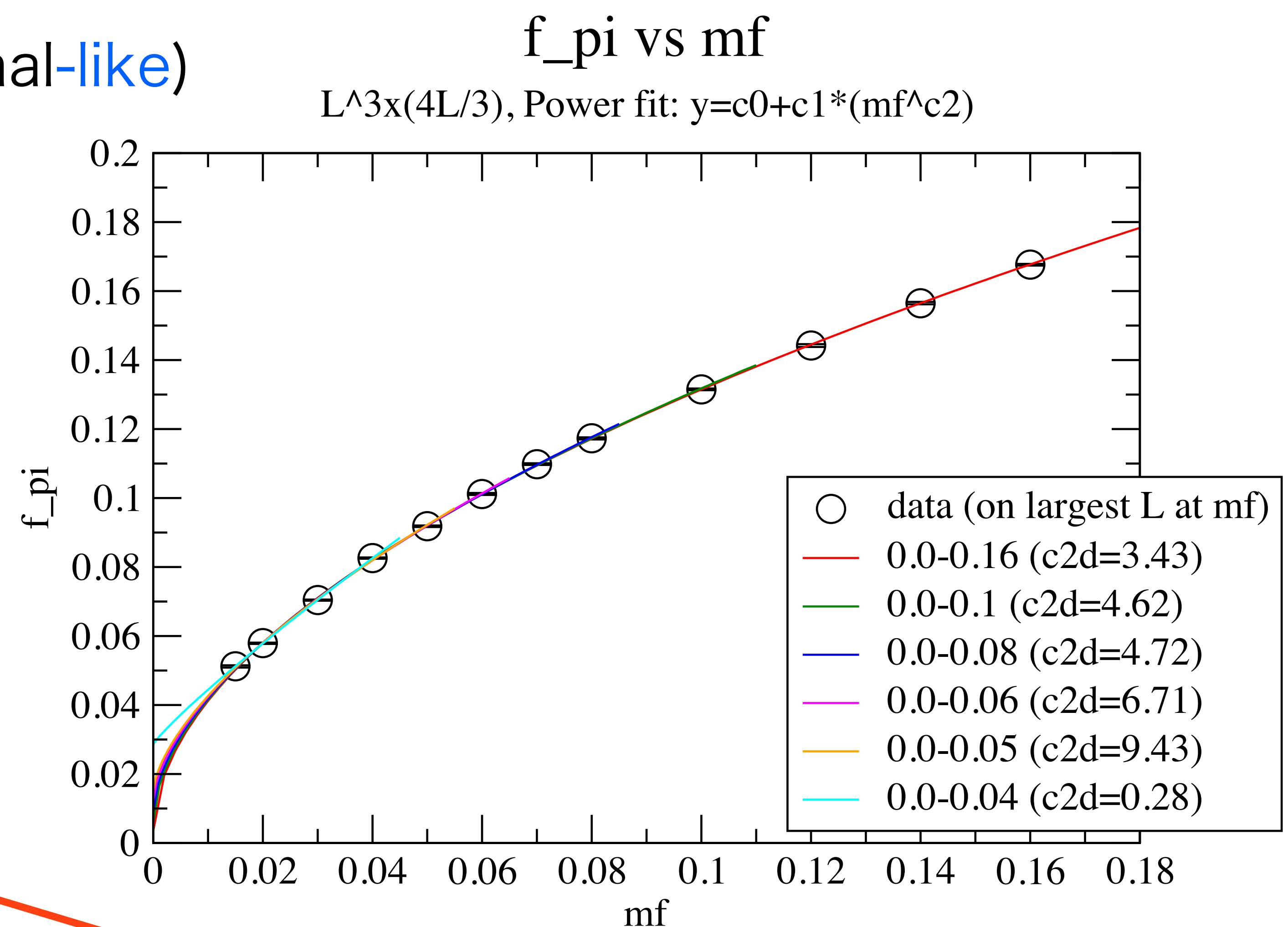
Detour: (sideways)

Power fit trial: (conformal-like)

$$f_\pi = F + A m_f^{\alpha}$$

Suppose:

$$\alpha = \frac{1}{1+\gamma}$$

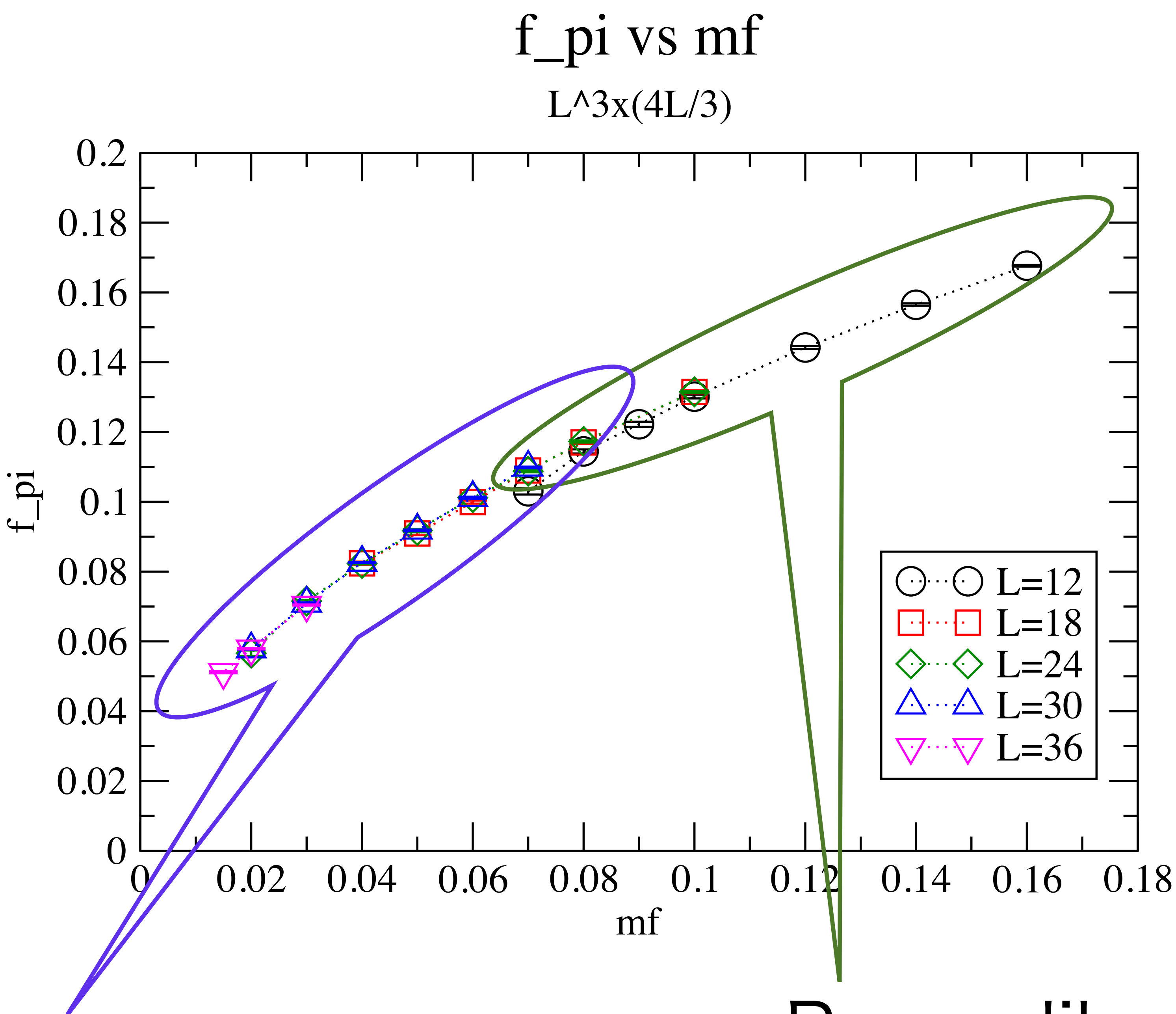


almost linear behavior

fit range (mf)	F	α	γ
0.015-0.04	0.0286(31)	0.88(7)	0.13
0.015-0.05	0.0149(36)	0.64(4)	0.56
0.015-0.06	0.0121(30)	0.60(3)	0.67
0.015-0.08	0.0089(22)	0.57(2)	0.75
0.015-0.10	0.0058(18)	0.55(1)	0.82
0.015-0.16	0.0035(11)	0.53(1)	0.89

better χ^2/dof than that in the quadratic fit

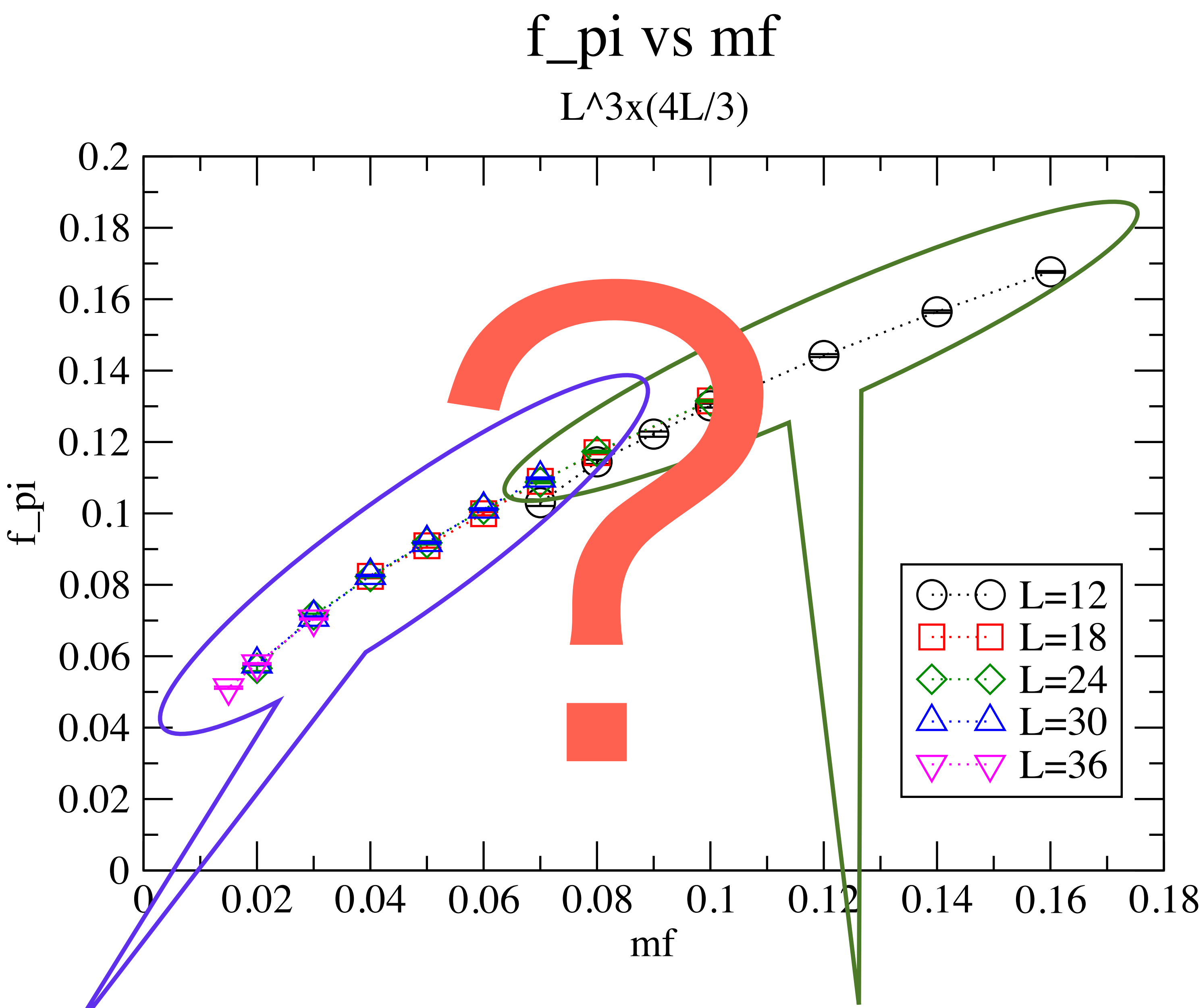
fit range:



Polynomial-like behavior?
(ChPT-like)

Power-like behavior ?
(Remnant of conformality ?)

fit range:

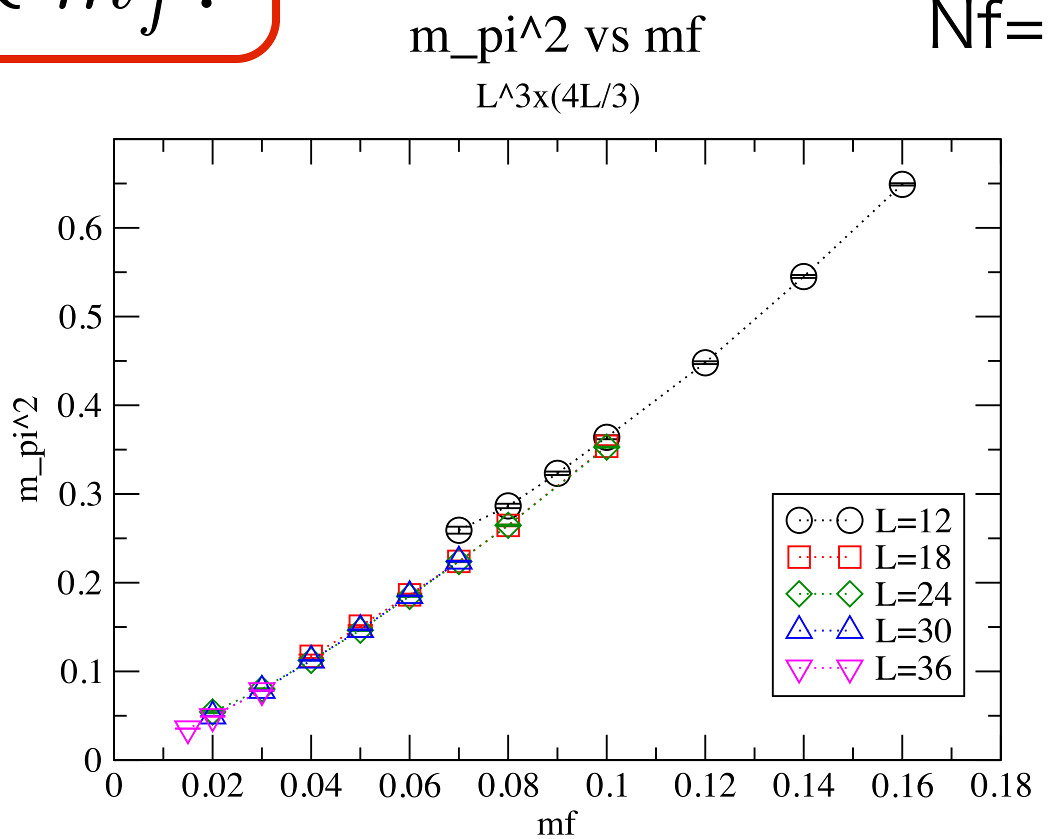


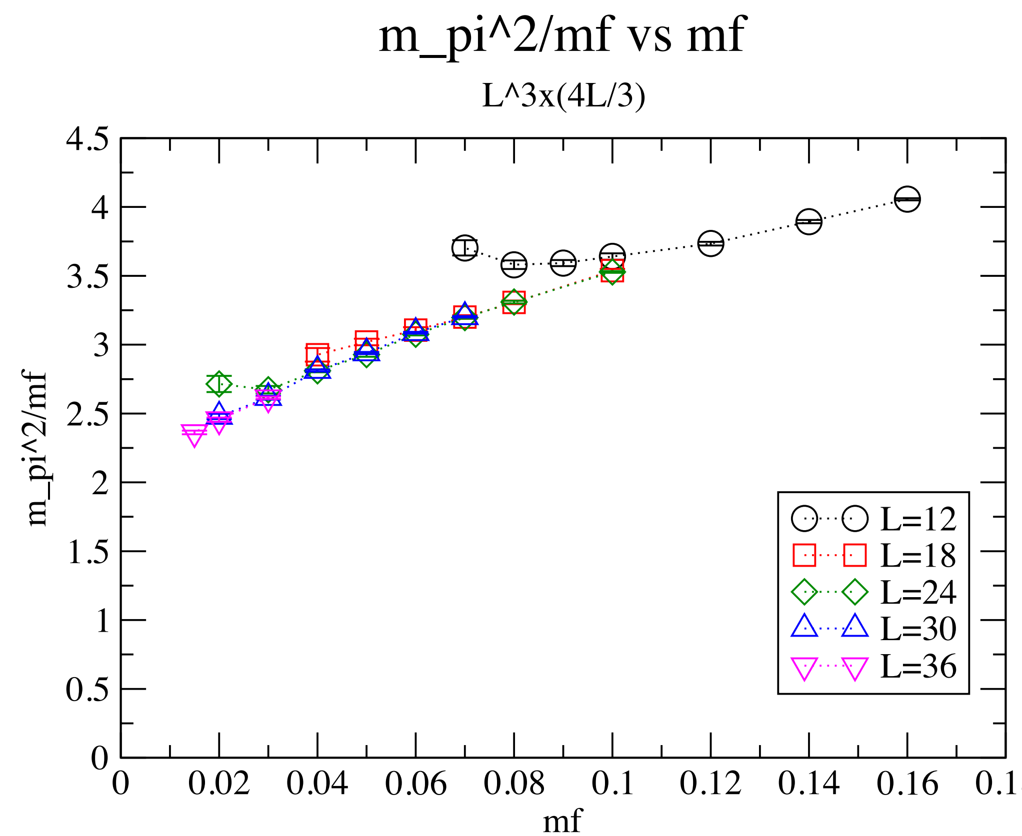
Polynomial-like behavior?
(ChPT-like)

Power-like behavior ?
(Remnant of conformality ?)

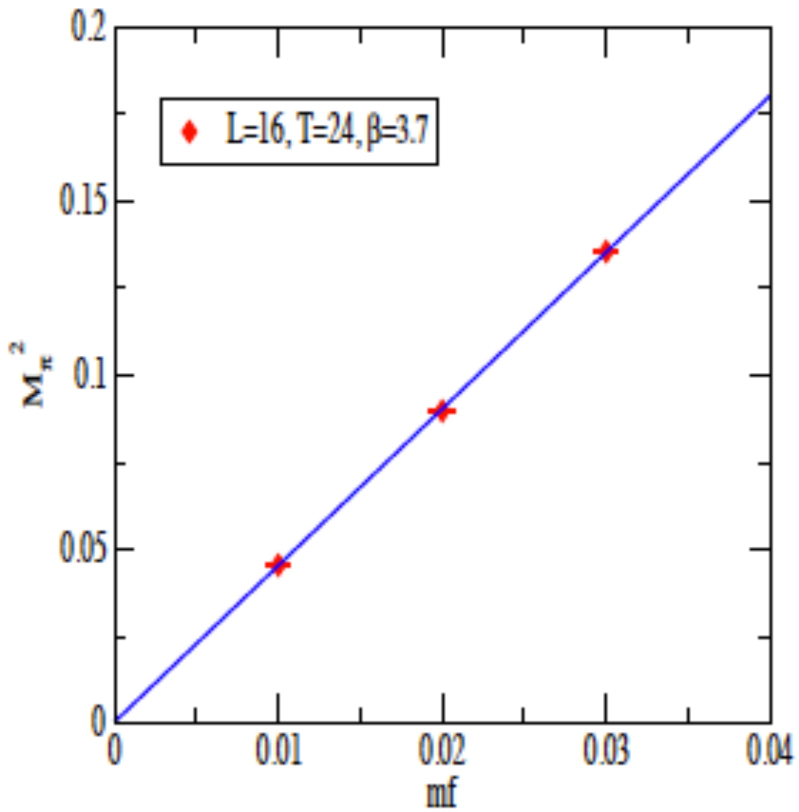
$m_\pi^2 \propto m_f?$

Nf=8

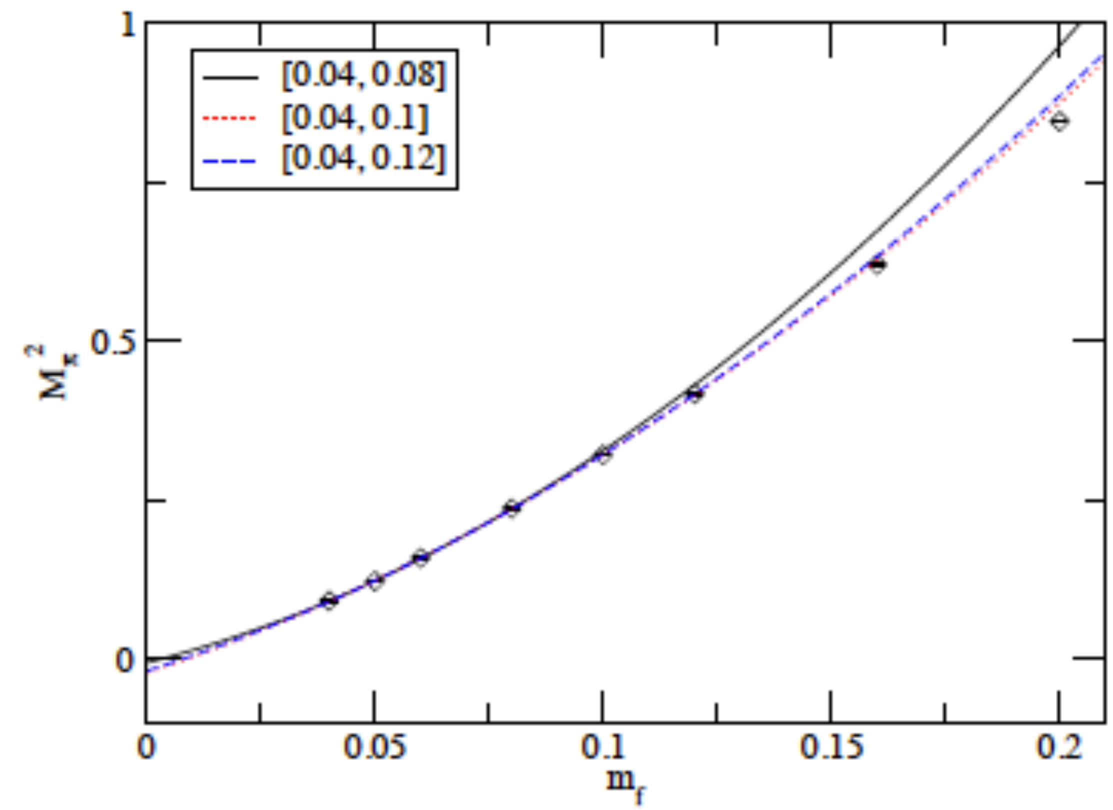




$N_f = 4$ $SU(3)$ gauge theory on $16^3 \times 24$ at $\beta = 3.7$



Nf=12 M_π^2 at $\beta = 3.7$ using the data on $L = 30$.



Summary-1, ChPT analysis (Preliminary)

- In the chiral limit, $f\pi>0$ in the quadratic fit.
- Also, $M\rho>0$ in the chiral limit.
- Nf=8 is in SχSB phase.
- The expansion parameter, χ~O(1).
- $M\pi$^2 is not proportional to mf, which is similar to Nf=12 case.
- ⇒ Remnant of conformal?

3. Hyperscaling analysis (preliminary)

Hyperscaling test = Conformal test
⇒ Finite size Hyperscaling

If the theory is in conformal phase,

$$LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)$$

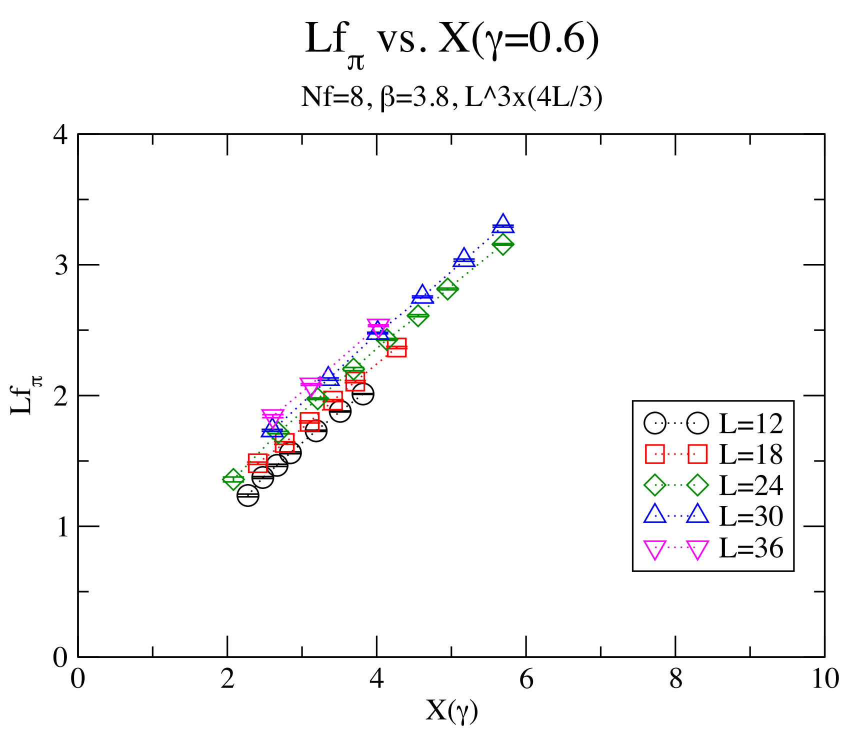
$$x \equiv L\, m^{1/1+\gamma}$$

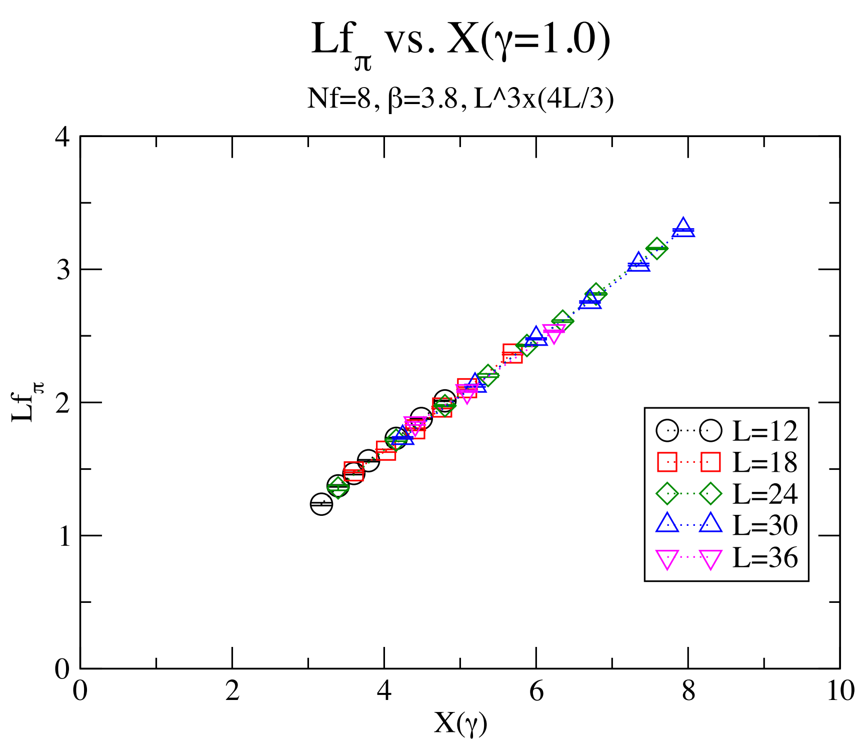
mass anomalous dimension (universal value)

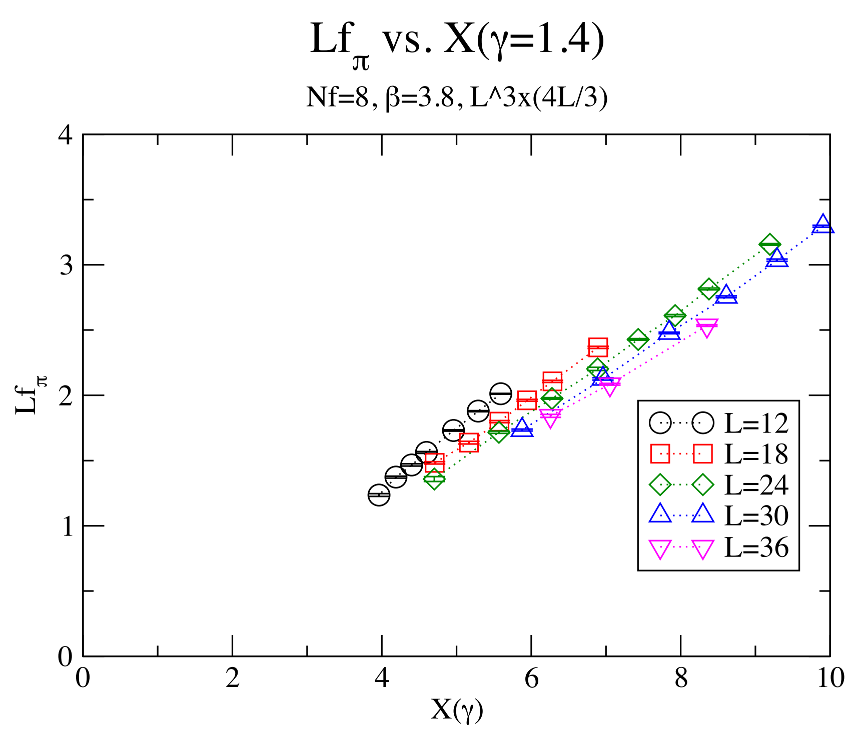
c.f. : Finite Size Scaling (FSS) of the correlation length in the 2nd order phase transition

$$\xi_L(T) = L f_\xi\left(\frac{L}{\xi_\infty}\right). \quad \xi_\infty \propto \left|\frac{T_c - T}{T_c}\right|^{-\nu}$$

Finite size hyperscaling of f_π







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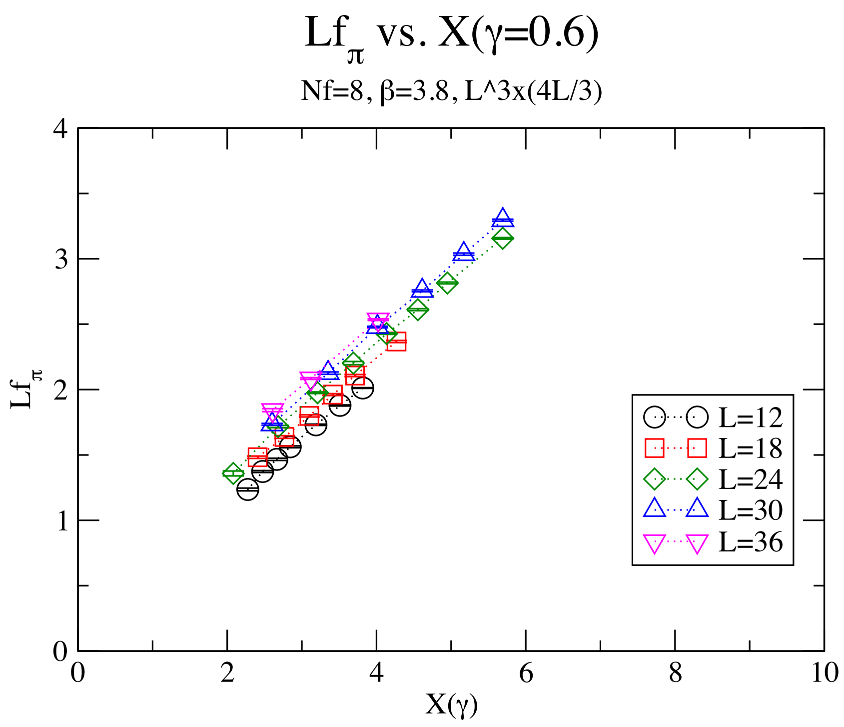
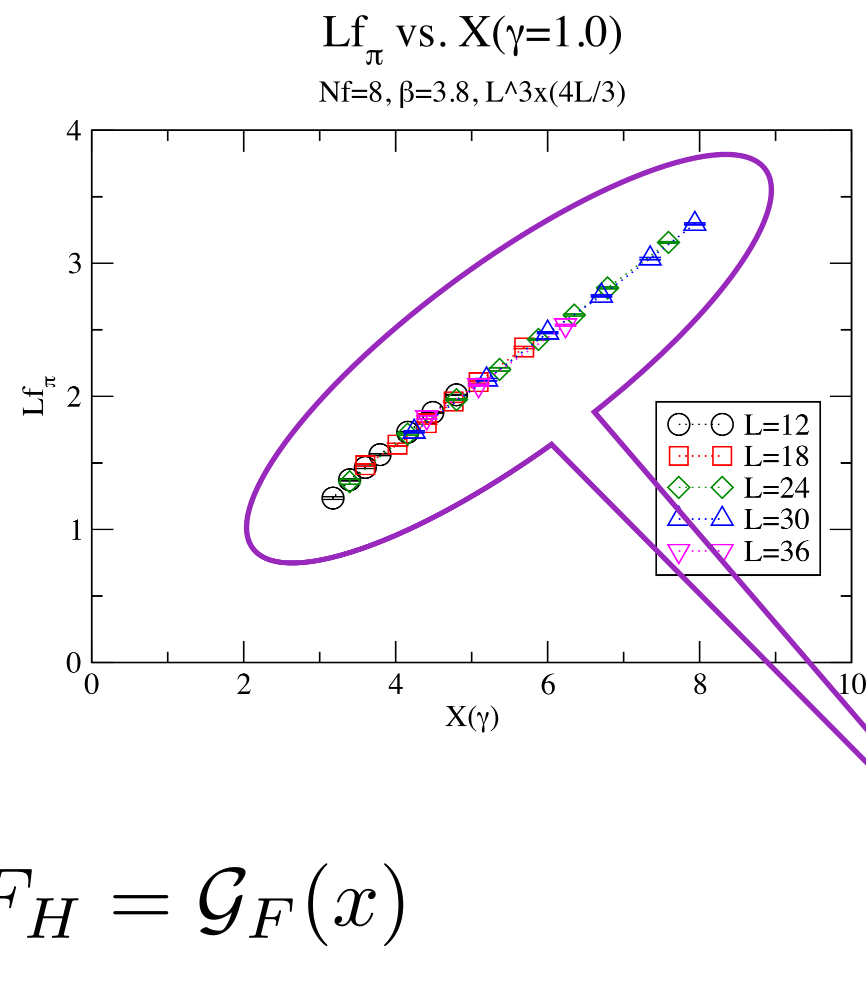
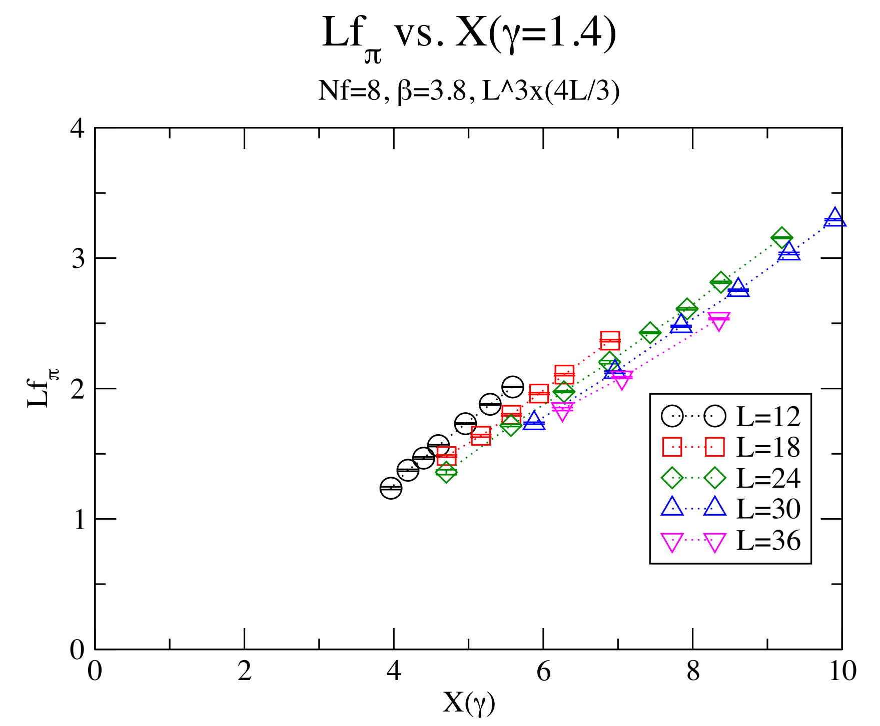
Finite size hyperscaling of f_π







$$LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)$$

$$x \equiv L\, m^{1/1+\gamma}$$

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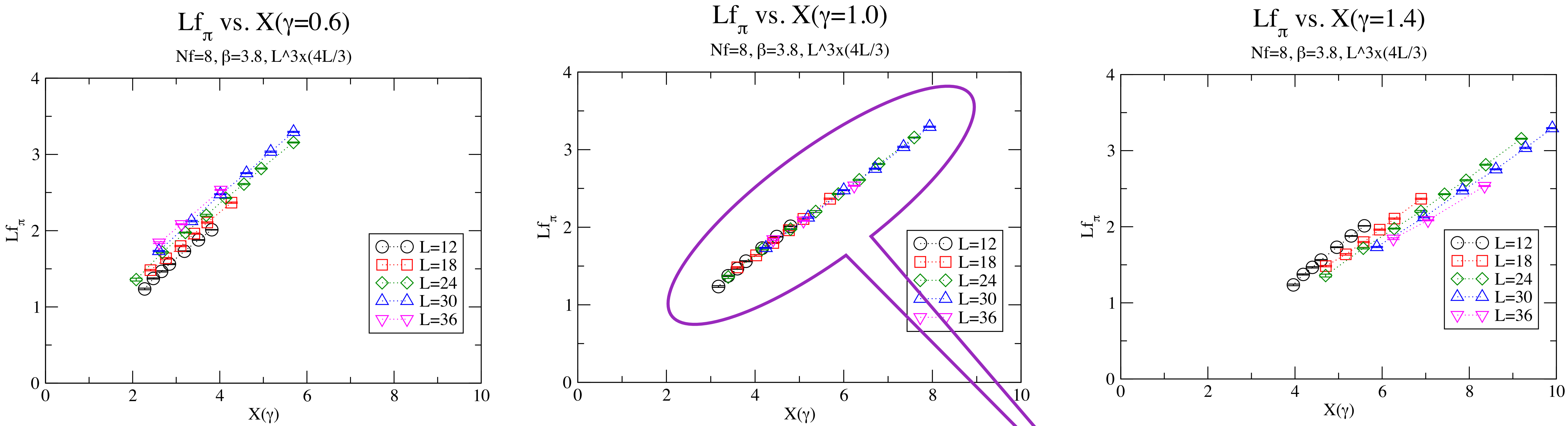
$$\xi_L(T) = L f_\xi\left(\frac{L}{\xi_\infty}\right). \qquad \xi_\infty \propto \left|\frac{T_c - T}{T_c}\right|^{-\nu}$$

Finite size hyperscaling of f_π



$$LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)$$

$$x \equiv L\, m^{1/1+\gamma}$$

How to quantify this situation?

c.f. : Finite Size Scaling (FSS) of the correlation length
in the 2nd order phase transition

$$\xi_L(T) = L f_\xi\left(\frac{L}{\xi_\infty}\right). \qquad \xi_\infty \propto \left|\frac{T_c - T}{T_c}\right|^{-\nu}$$

$P(\gamma)$ analysis

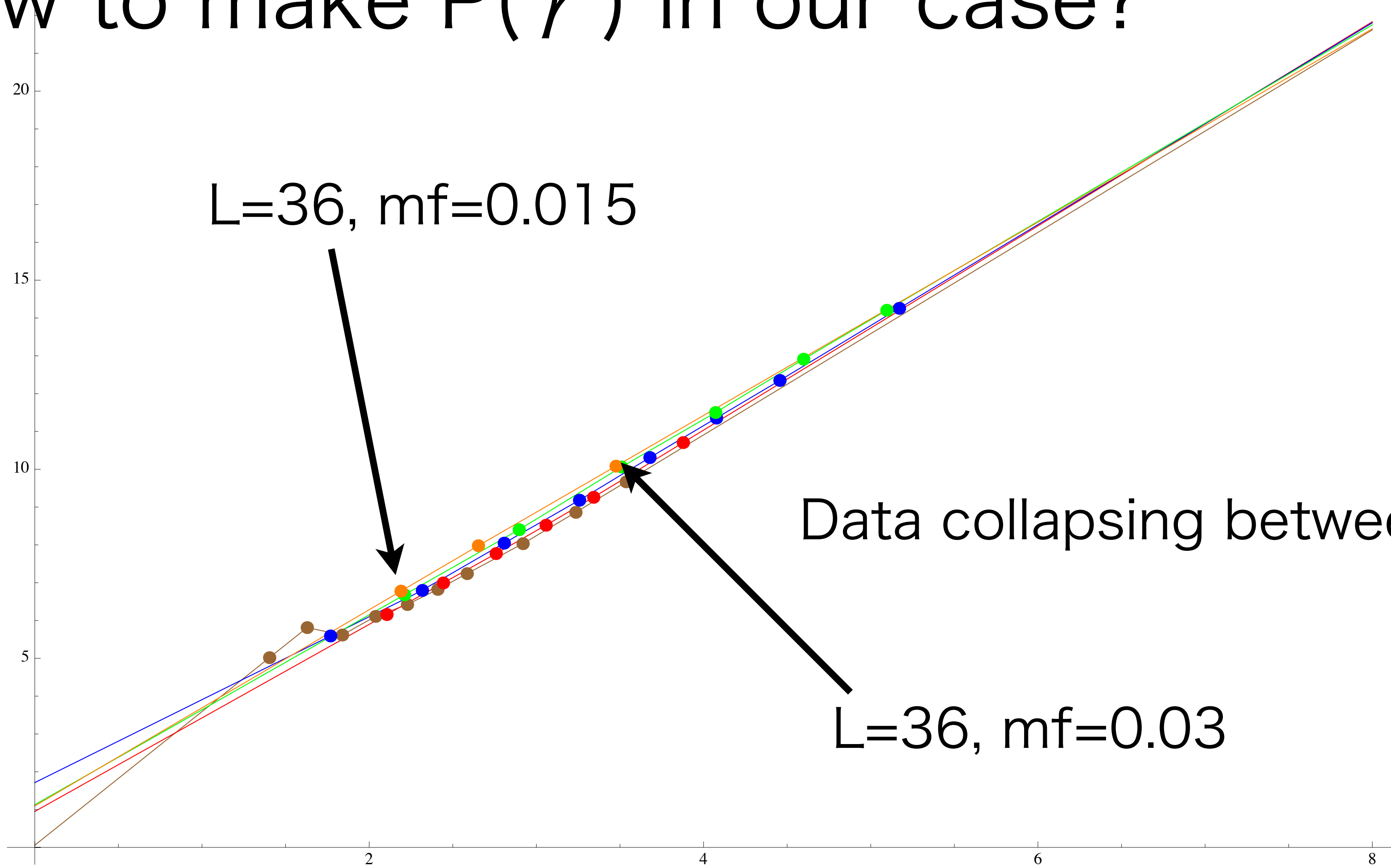
$$P(\gamma) = \frac{1}{N}\sum_{L}\sum_{j \notin K_L} \frac{\left|\xi^j - f^{(K_L)}(x_j)\right|^2}{|\delta\xi^j|^2},$$

Scaling func. f(x) is unknown.

⇒ f(x) is determined by the interpolation (or spline) of the data set K_L on the lattice L.

(Analysis of the data collapse)

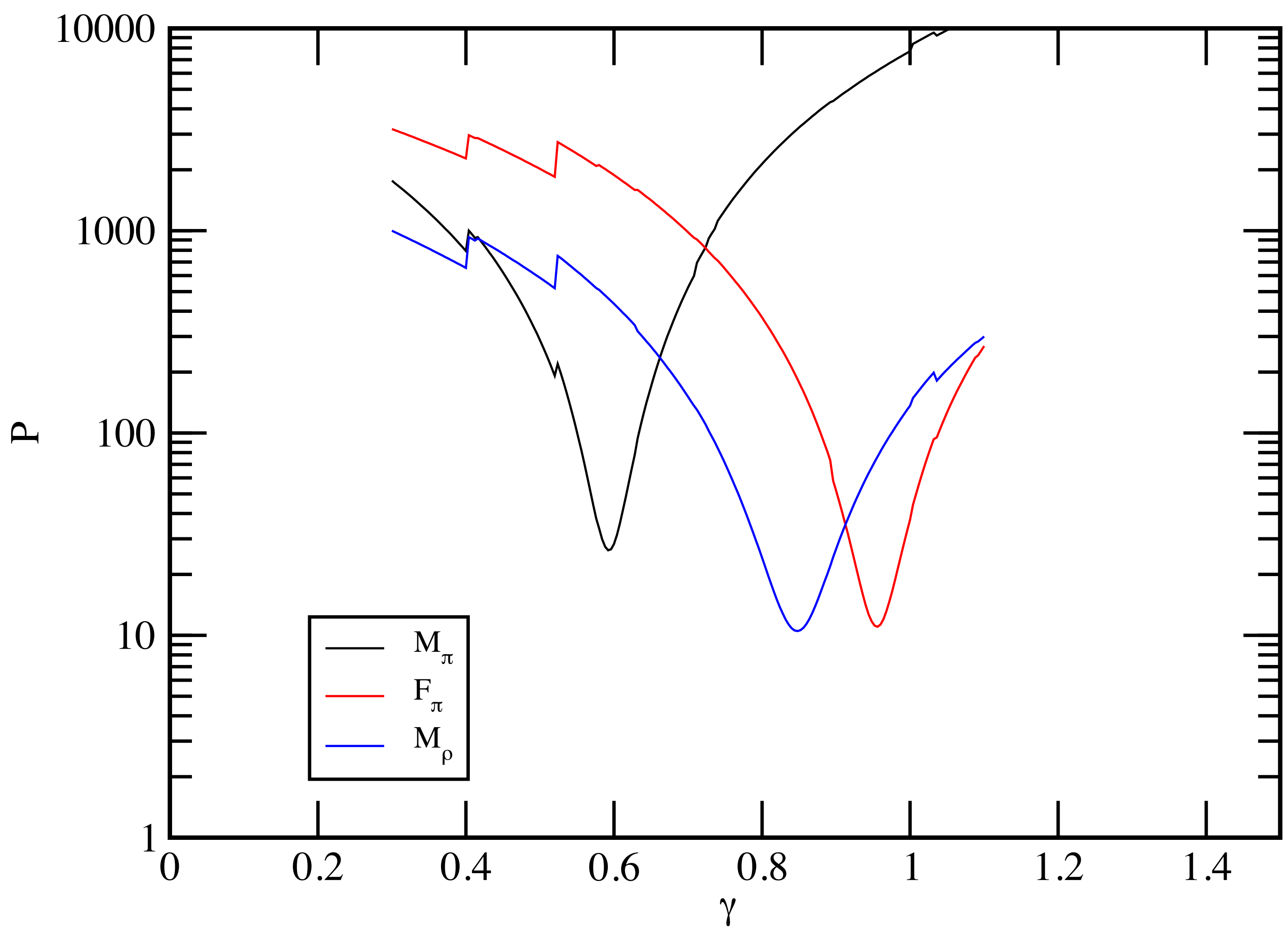
How to make P(γ) in our case?



Data collapsing between these values ⇒ P(γ)

$P(\gamma)$ analysis

$$P(\gamma) = \frac{1}{\mathcal{N}} \sum_{L} \sum_{j \notin K_L} \frac{\left|\xi^j - f^{(K_L)}(x_j)\right|^2}{|\delta\xi^j|^2},$$



quantity	γ
M_π	0.593(2)
f_π	0.955(4)
M_ρ	0.844(10)

If the theory is in conformal phase,

$$LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)$$

$$x \equiv L\,m^{1/1+\gamma}$$

mass anomalous dimension (universal value)

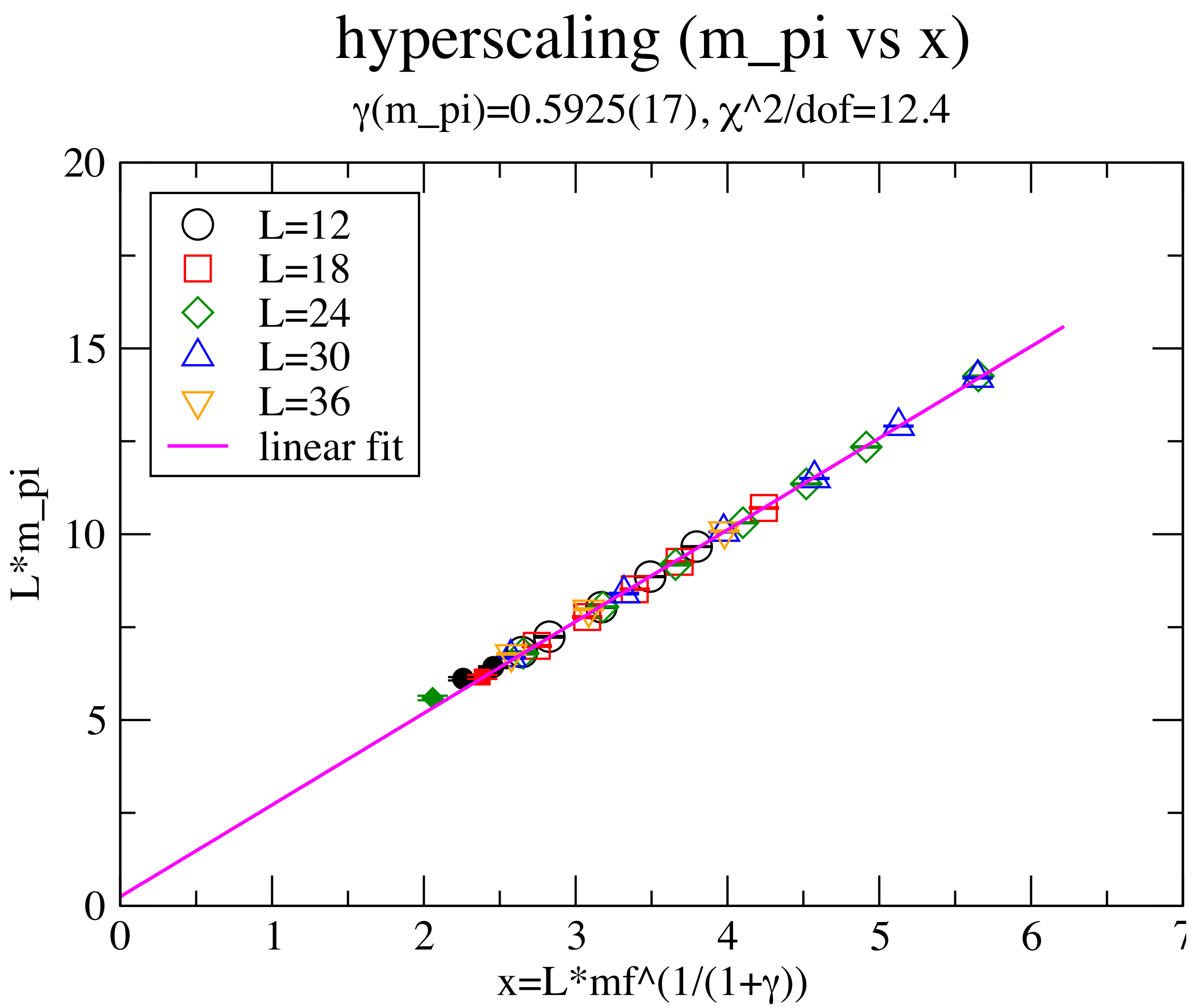
Then,

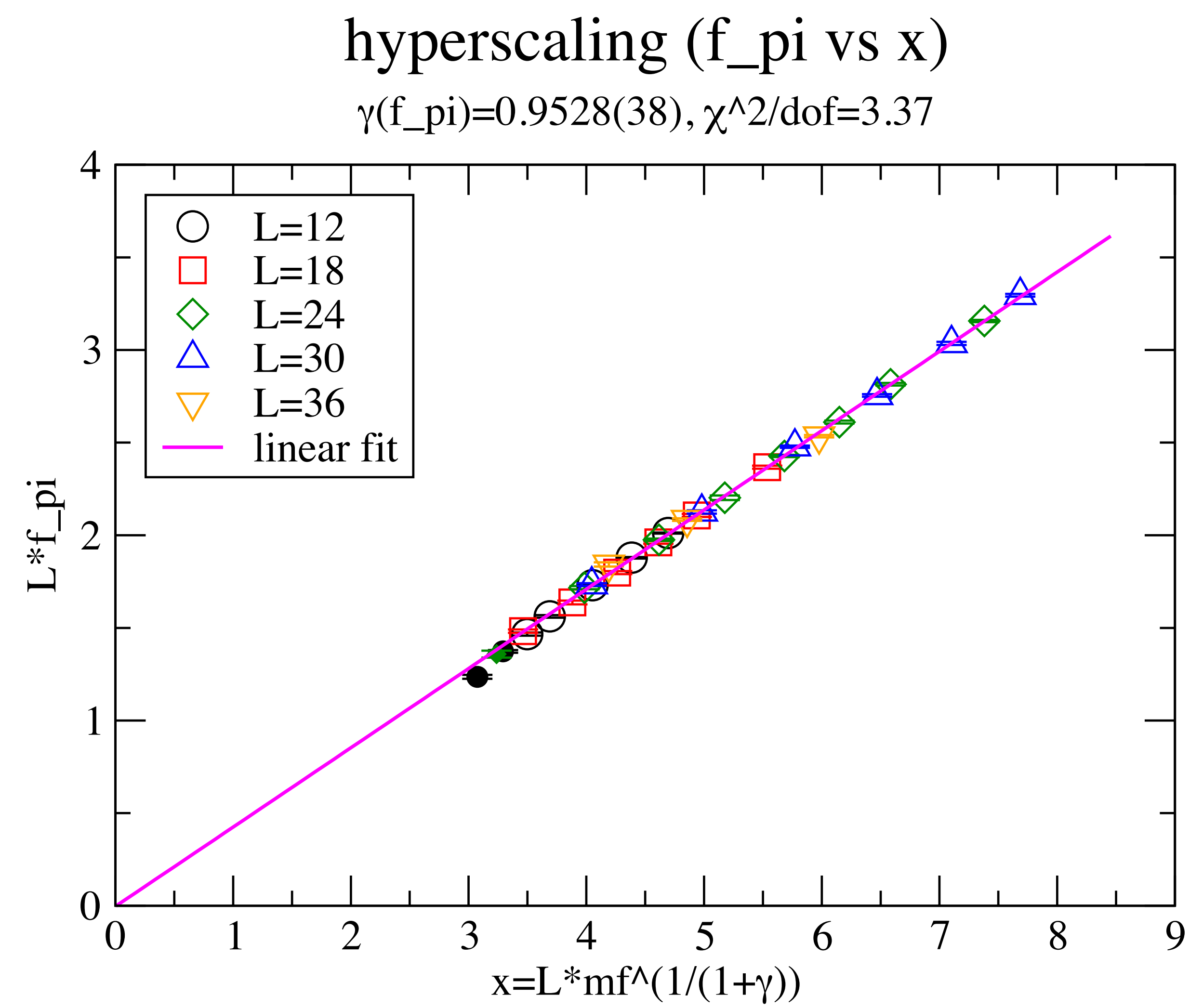
Linear approximation of $F_H(x)$ and $G_H(x)$.

$$LM_H(LF_H) = C_0 + C_1 x.$$

⇒ Trial of the linear fit

Hyperscaling fit (Linear of X)

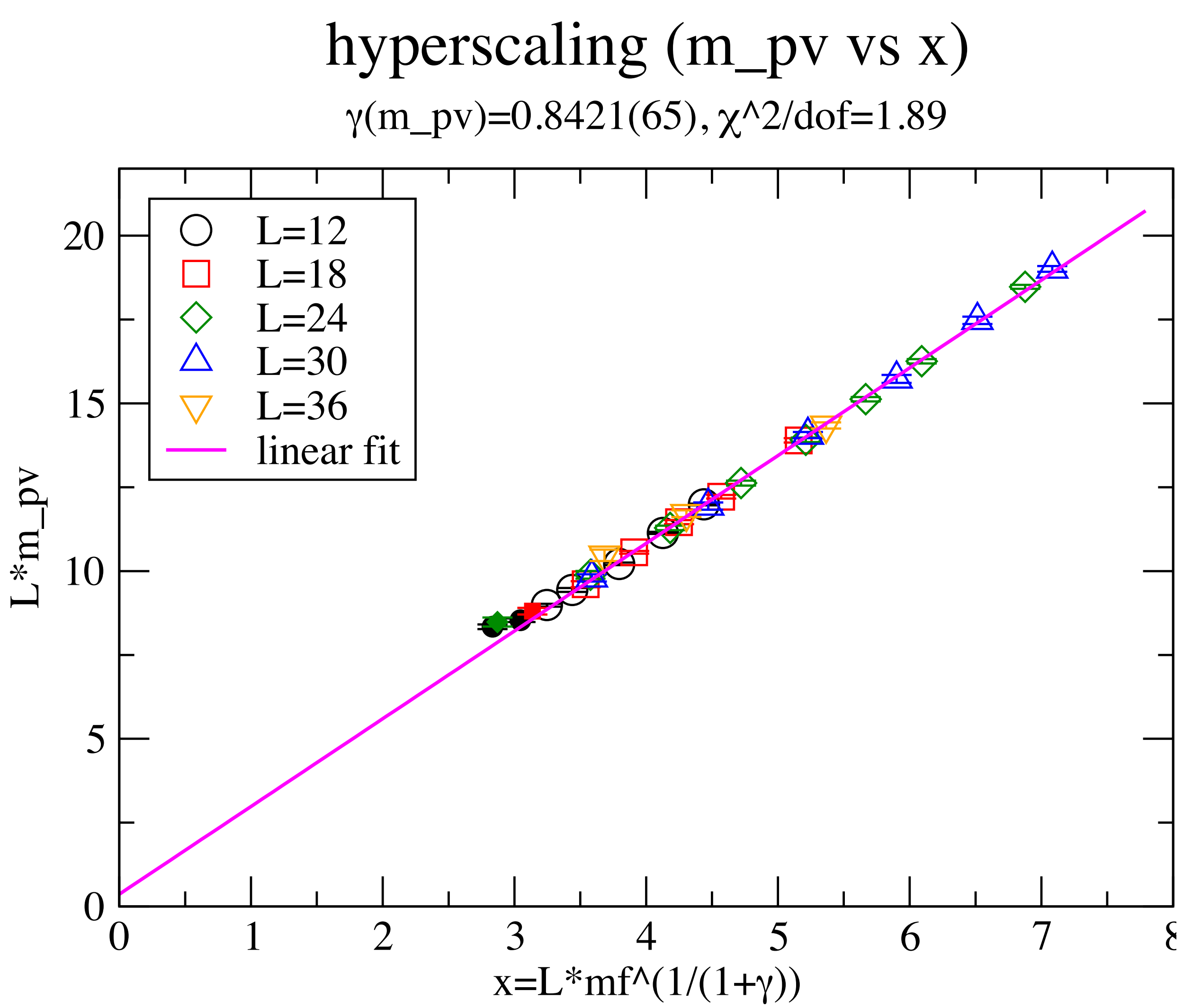


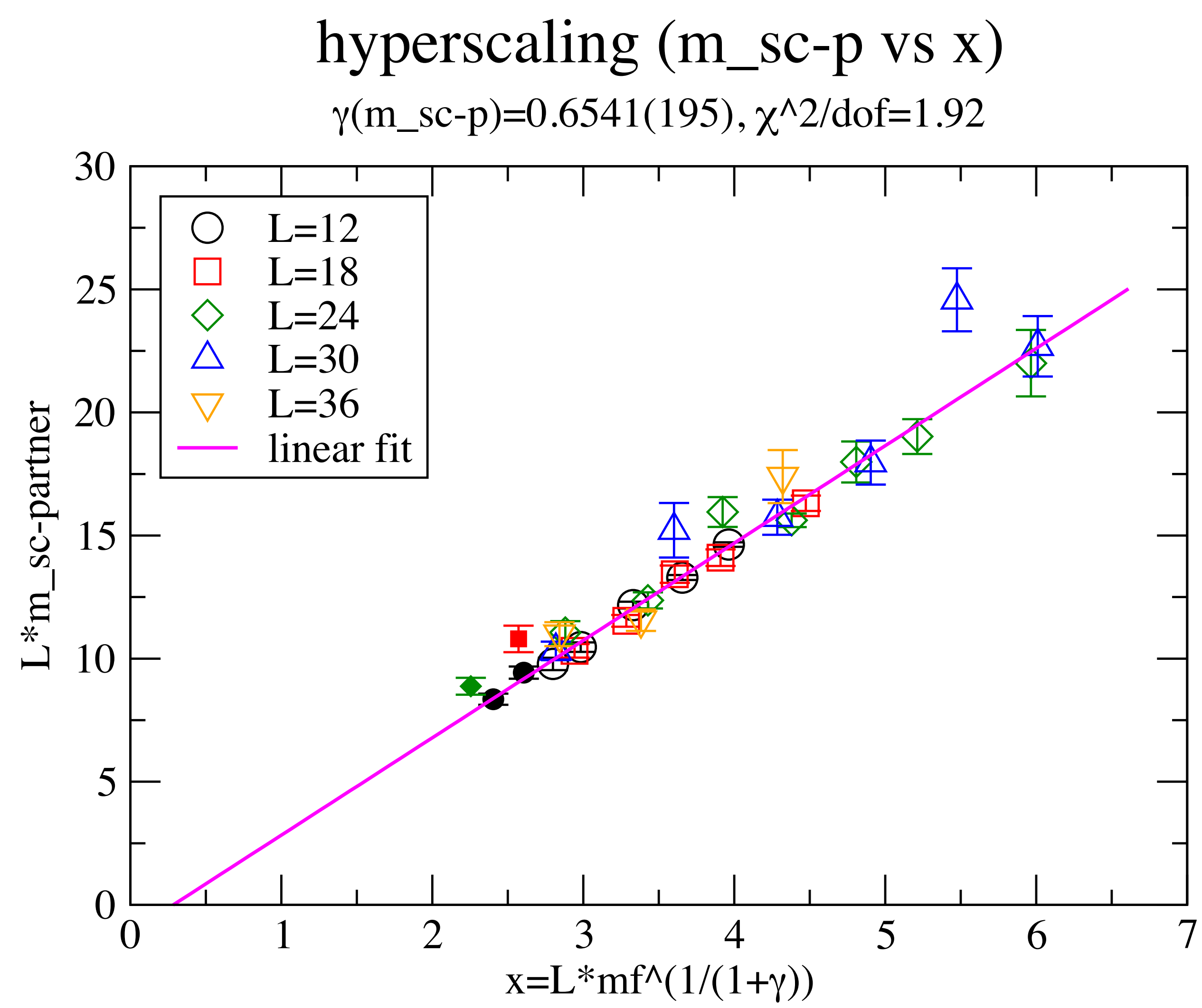


good linearity

Mρ and Msc-partner(flavor non-singlet scalar) in staggered fermions

(flavor non-singlet scalar meson)





good linearity

Finite Size Hyperscaling result

FSHS, linear fit

quantity	γ
M_π	0.5925(17)
f_π	0.9528(38)
M_ρ	0.8421(65)

FSHS, P(γ)

quantity	γ
M_π	0.593(2)
f_π	0.955(4)
M_ρ	0.844(10)

$$\gamma(M_\pi) \neq \gamma(f_\pi)$$

γ (f)~1.0

In contrast to Nf=12 case.

4. Discussion in HS

- Hyperscaling $\gamma(M_\pi) \neq \gamma(f_\pi)$

  → non-universal means this is hadronic.

  (finite mass and finite size correction?)

- Hyperscaling for each M_H seems to be good.

  → Remnant of the conformal (?)

Comparison with $N_f = 4,\ 12$

Comparison of γ with $N_f = 12$

$N_f = 12 \ (\beta = 3.7)$

quantity	γ
M_π	0.434(4)
f_π	0.516(12)
M_ρ	0.459(8)

Nf=12 is consistent with the conformal.

LatKMI,

Phys.Rev.D86 (2012) 054506

(H.Ohki's talk)

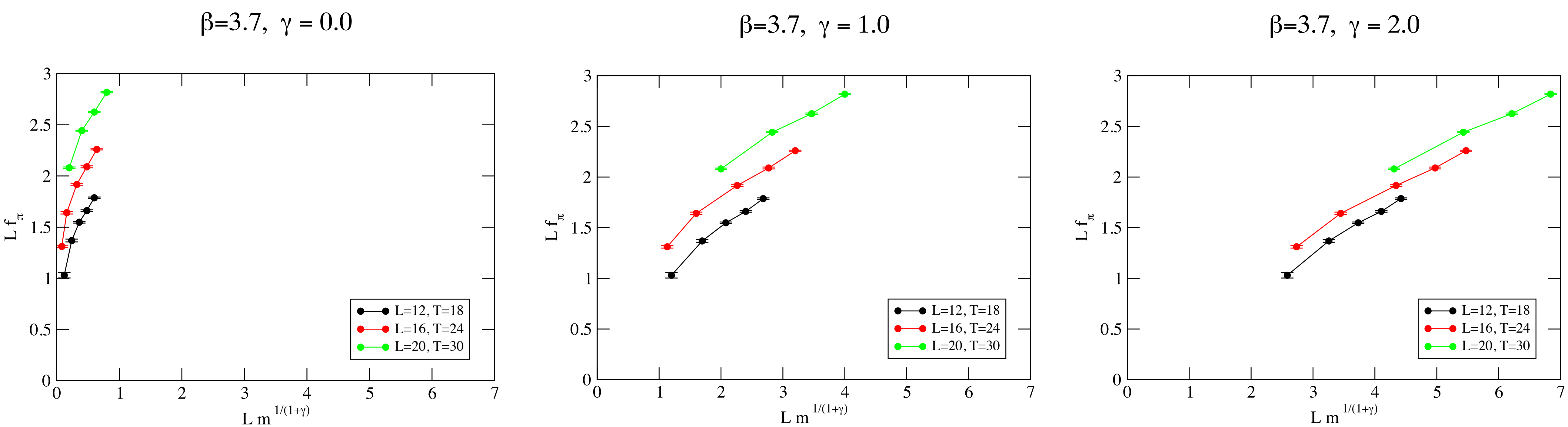
$N_f = 8$ (β=3.8)

quantity	γ
M_π	0.593(2)
f_π	0.955(4)
M_ρ	0.844(10)

γ (f) is large.

$$\gamma(M_\pi) \neq \gamma(f_\pi)$$

Comparison with $N_f = 4$
: hyperscaling of f_π (in χSB)



f_π : no scaling for $0 < \gamma < 2$

⇒ In this sense,

it seems that Nf=8 is not χSB of ordinary QCD.

Comparison with Schwinger-Dyson (SD) Eq.

LatKMI collaboration, Phys.Rev. D85 (2012) 074502

Input the 2-loop running coupling (α^*=IRFP)

→SD-eq. gives critical flavor $N_f^{\rm cr} \simeq 11.9$.

SD-eq.: $\alpha^* > \alpha_{cr}$ → SχSB phase

$\alpha^* < \alpha_{cr}$ → conformal window

$\alpha^* \sim \alpha_{cr}$ → near conformal

$$\alpha_{\rm cr} = \frac{\pi}{3C_2} = \frac{\pi}{4}(N_c = 3).$$

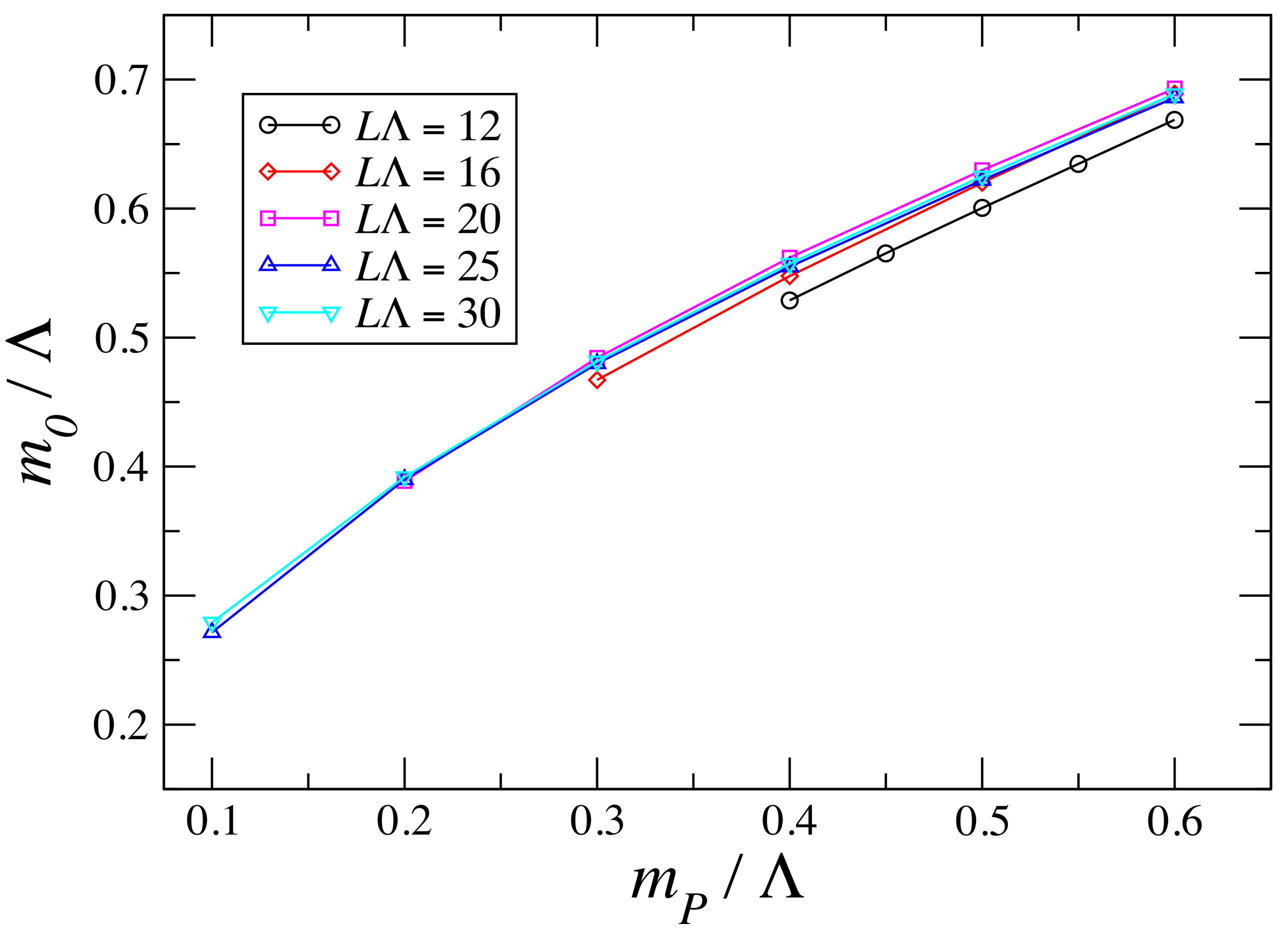
Input $\alpha^* \sim \alpha_{cr}$ for Nf=11(near conformal in SDeq analysis) on finite V and finite mf.

⇒ Hyperscaling test (γ=?)

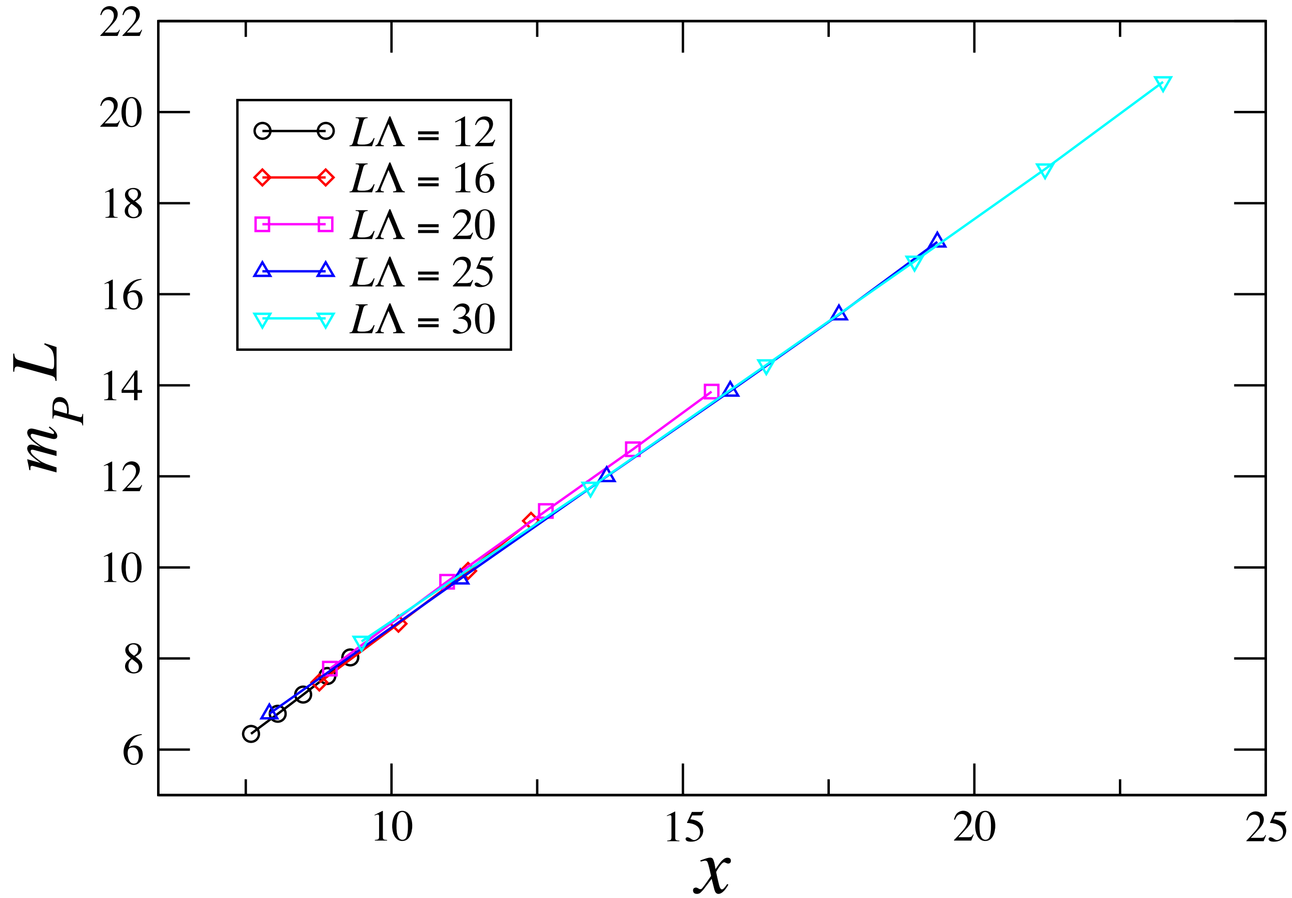
c.f. Nf=12 is conformal in SDeq analysis: γ=0.6--0.8 .

(Nf=11), α*~α_{cr}, just below conformal window

Raw data (Mock data) → Finite size Hyperscaling test



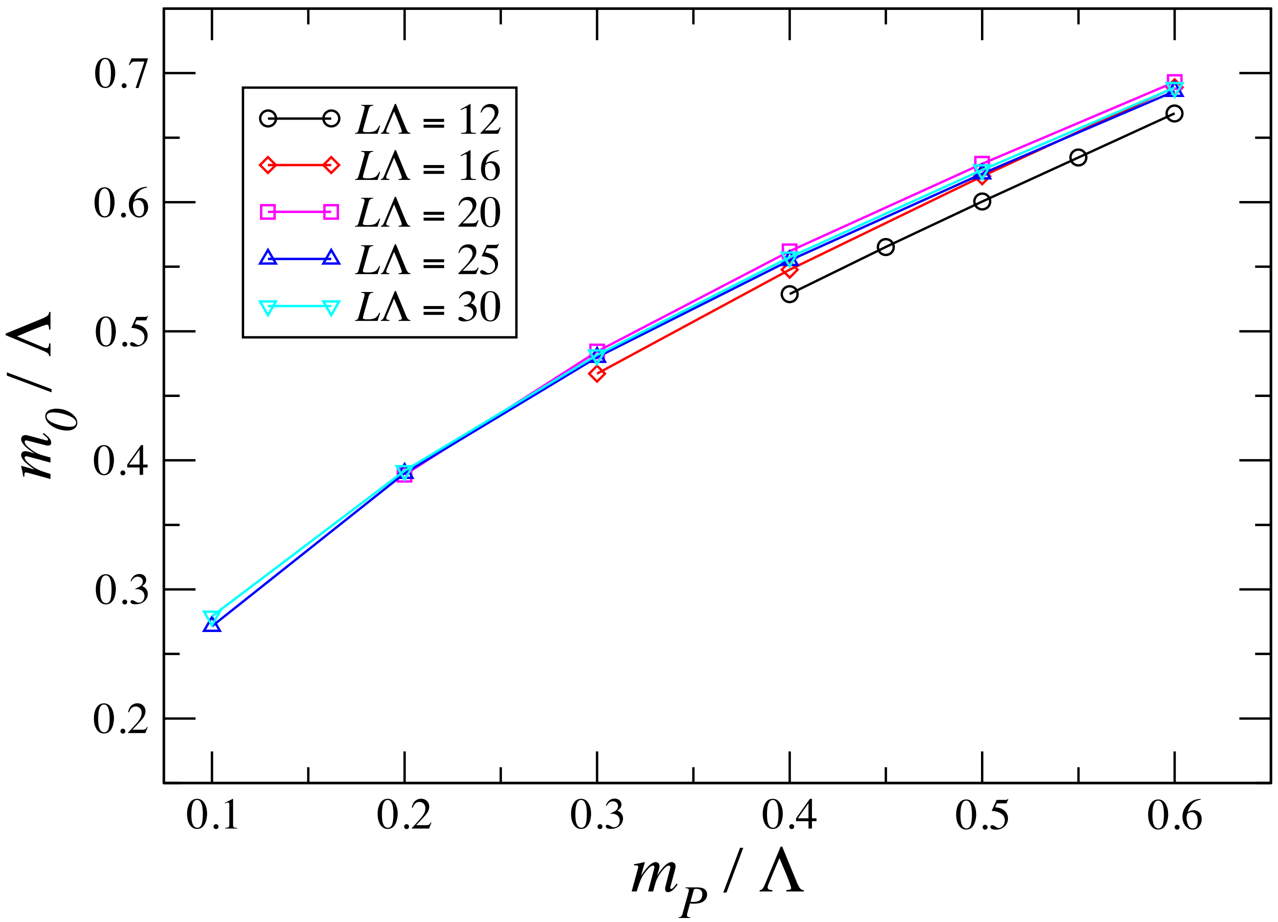
$\gamma = 1.0$

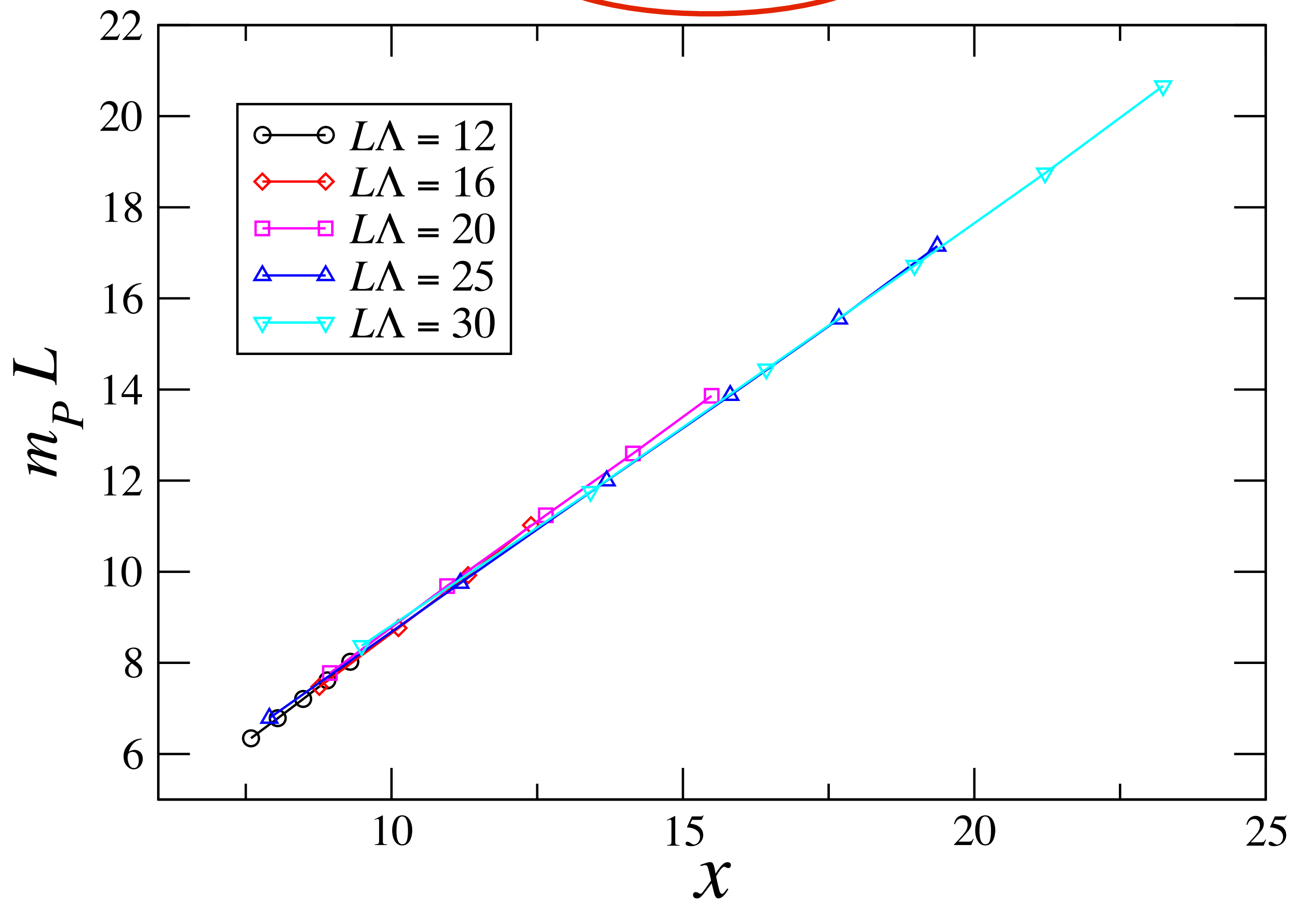


(Nf=11), $\alpha^* \sim \alpha_{cr}$, just below conformal window

Raw data
(Mock data) → Finite size Hyperscaling test

$\gamma = 1.0$





Summary-2, Hyperscaling test (preliminary)

- Hyperscaling for each observable is seen.
- $\gamma(M\pi) \neq \gamma(f\pi) \sim 1.0$
- Nf=8 is different from Nf=4 and 12.
- In HS of SDeq, $\gamma \sim 1.0$ is near conformal.
- Remnant of conformal (?)

Simulation	Nf=4	Nf=8	Nf=12
$\gamma(f_\pi)$	>2	~0.95	~0.45
SDeq	$\alpha^* > \alpha_{cr}$ (Nf=9)	$\alpha^* \sim \alpha_{cr}$ (Nf=11)	$\alpha^* < \alpha_{cr}$ (Nf=12)
$\gamma(m_{dyn})$	no align	~1.0	0.6--0.8

Summary (Preliminary)

- SU(3) gauge theory with 8 HISQ quarks.
- Preliminary result of spectrum

SχSB from ChPT analysis ($F_\pi \neq 0$, & similar F_π/M_π to Nf=4)

Remnant of conformal (not ordinary QCD) , $\gamma(F_\pi)$~0.95

from hyperscaling test and from comparison with Nf=4 and 12.

From SD-eq analysis, $\gamma \simeq 1$ indicates near conformal.

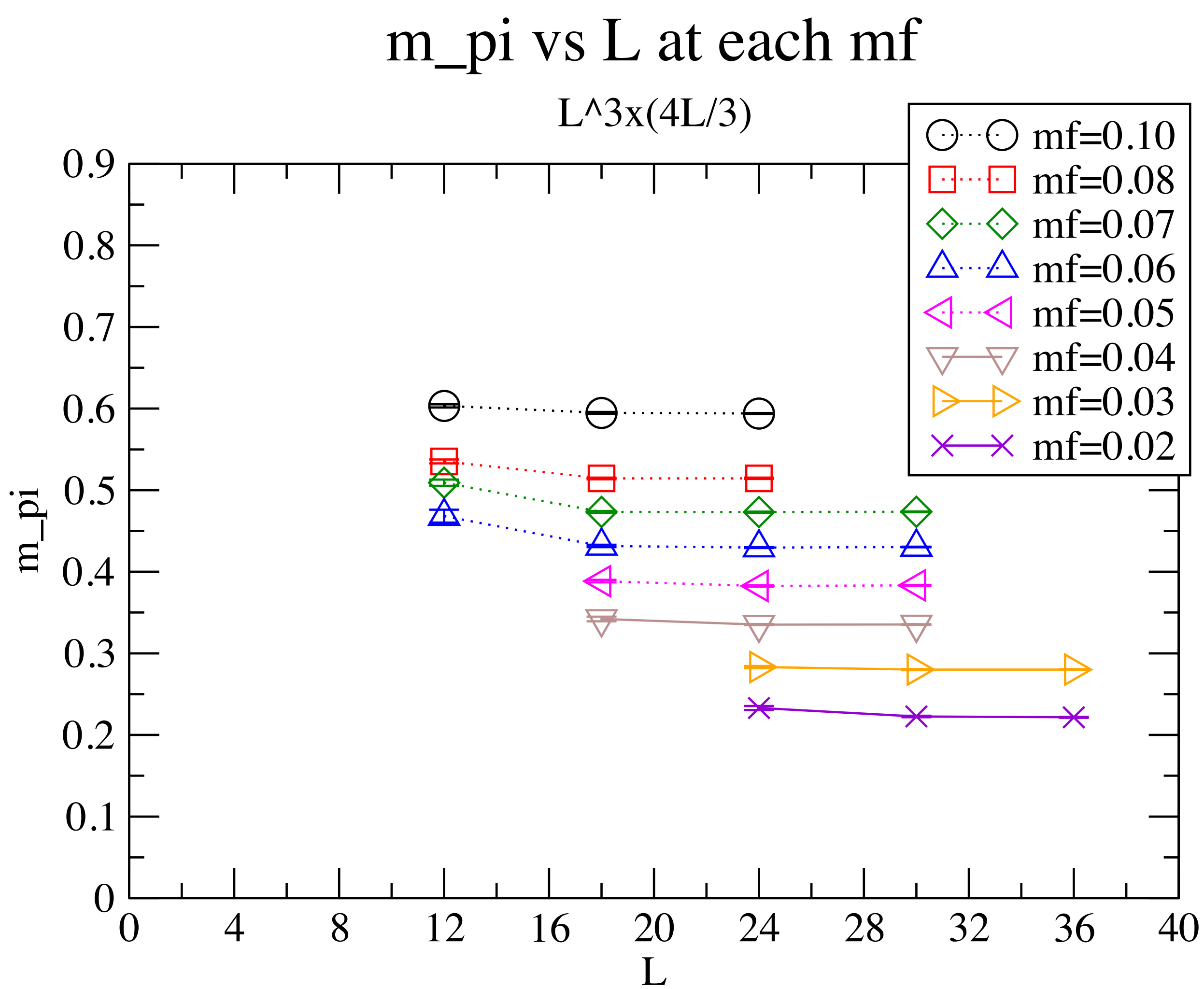
→ Candidate of Walking dynamics

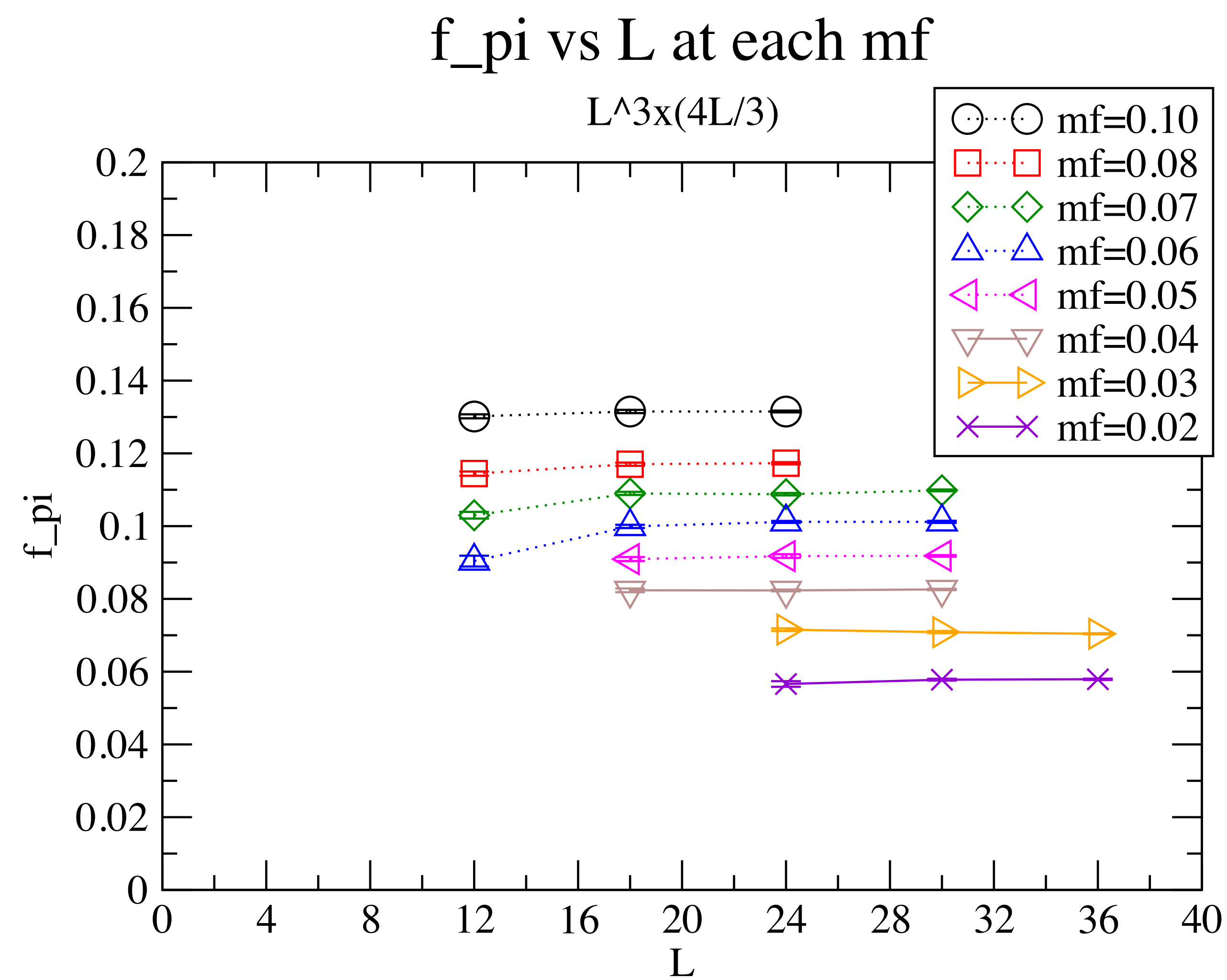
Future plan

- Simulation on larger volumes at lighter mass
- Finite Size Scaling (due to the difficulty to take V=∞ and mf→0)
- Lattice spacing dependence (UV cutoff dep.) ←many βs
- Spectroscopy ($M_{glueball}$, $M_{\text{"dilaton"}}$, M_{baryon}, M_{meson}, F_π, S-para. etc.)
- $M_{\text{"dilaton"}}$, $M_{glueball}$, M_H ⇒ 126GeV?
- $M_{glueball}$ and $M_{scalar(singlet)}$ → E. Rinaldi's talk (Nf=12)

Backup

Size dependence of m_pi and f_pi at β=3.8

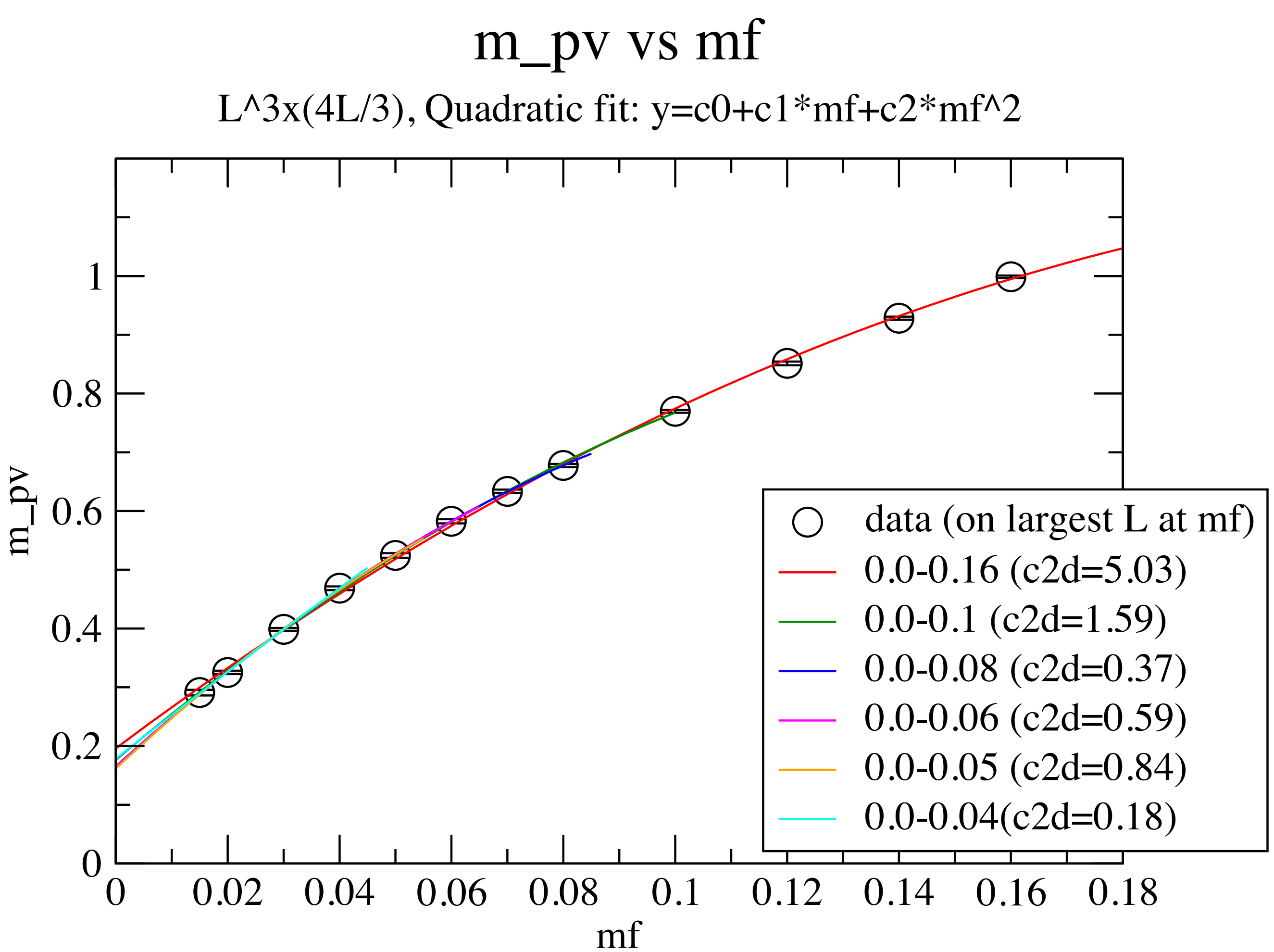


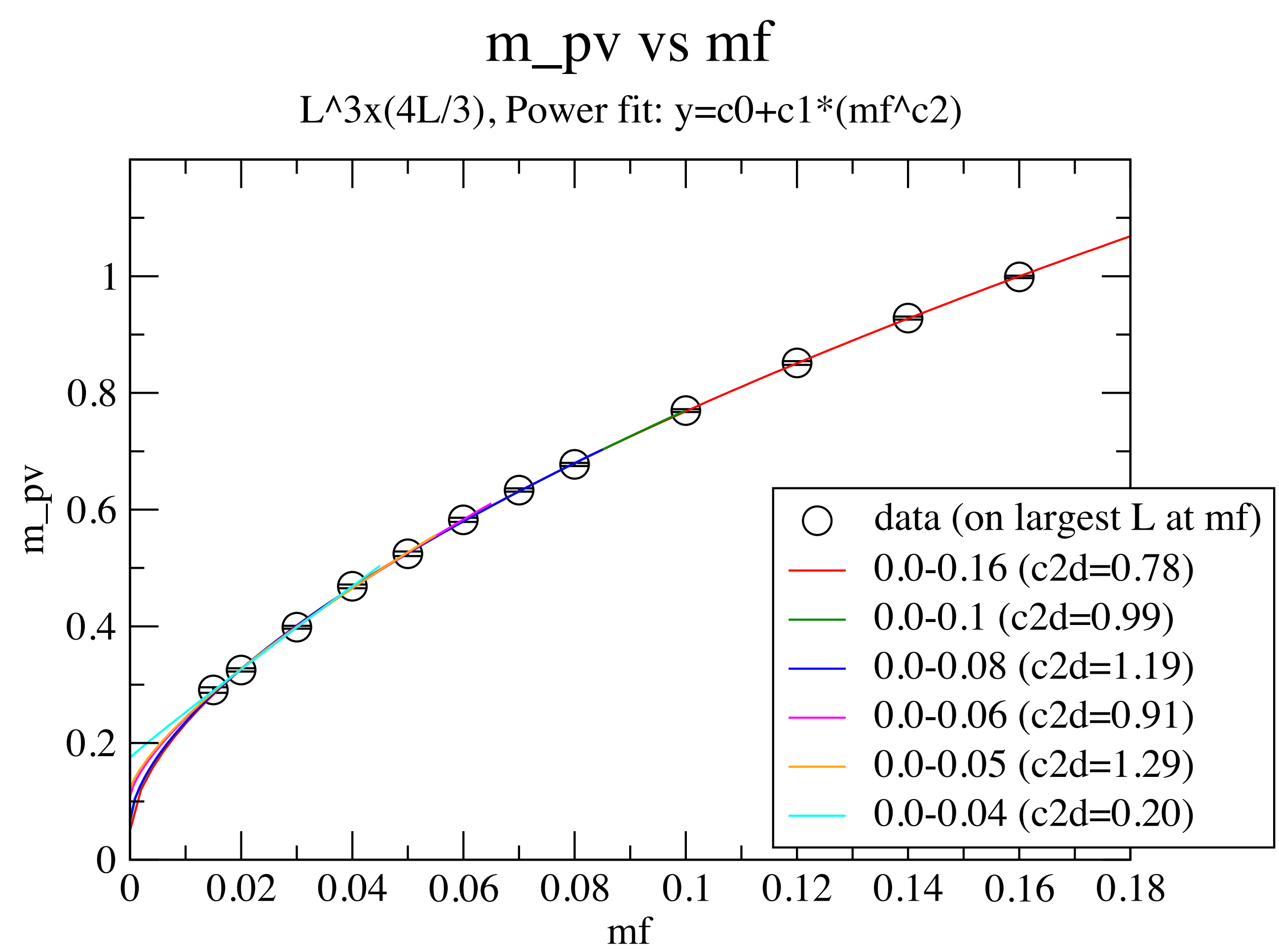


On the larger volume, there is not (or very tiny) size dependence.

We use the data on the largest volume at each mf.

$M\rho$



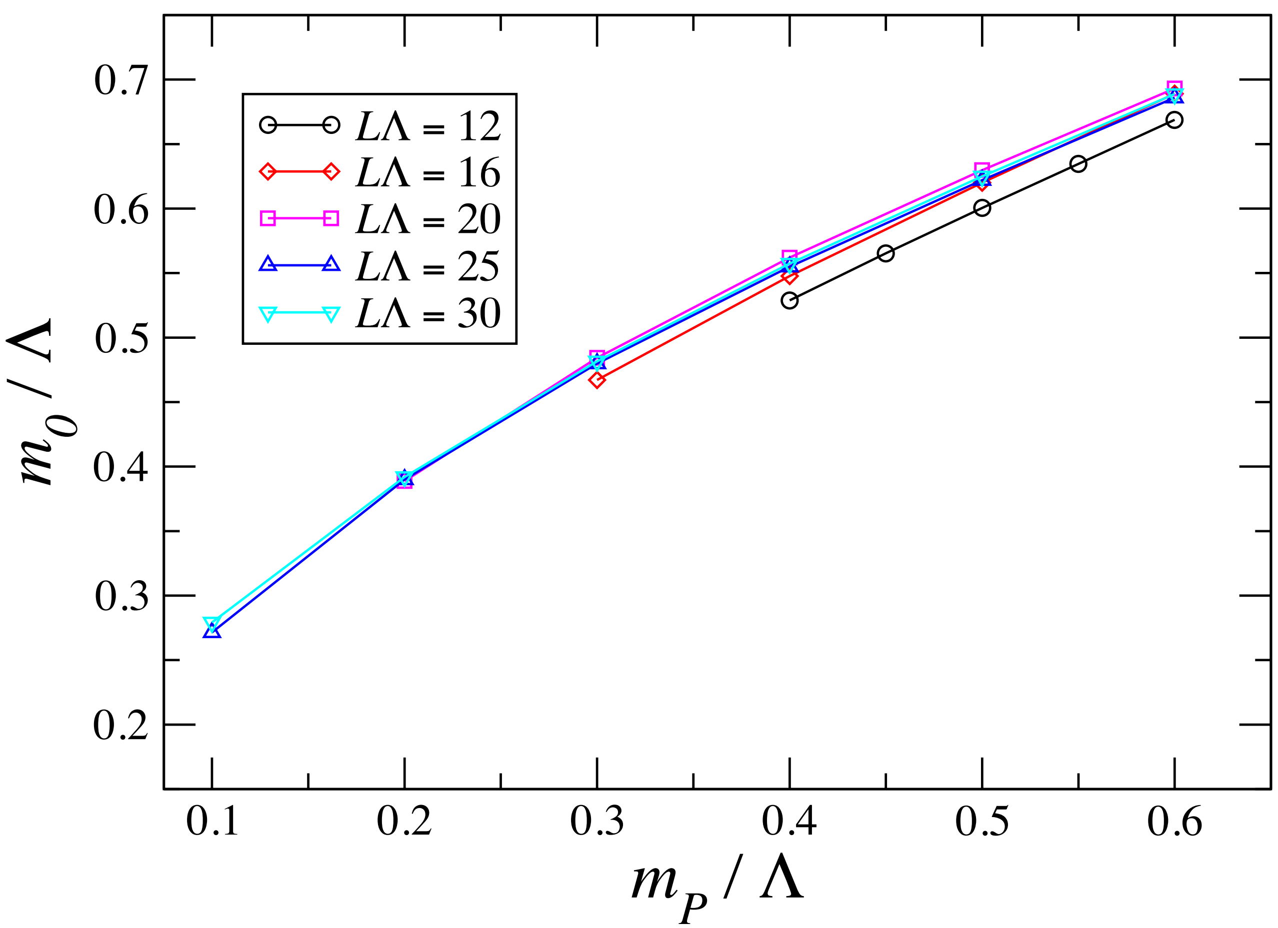


$$M_\rho \neq 0 \text{ at } m_f \to 0.$$

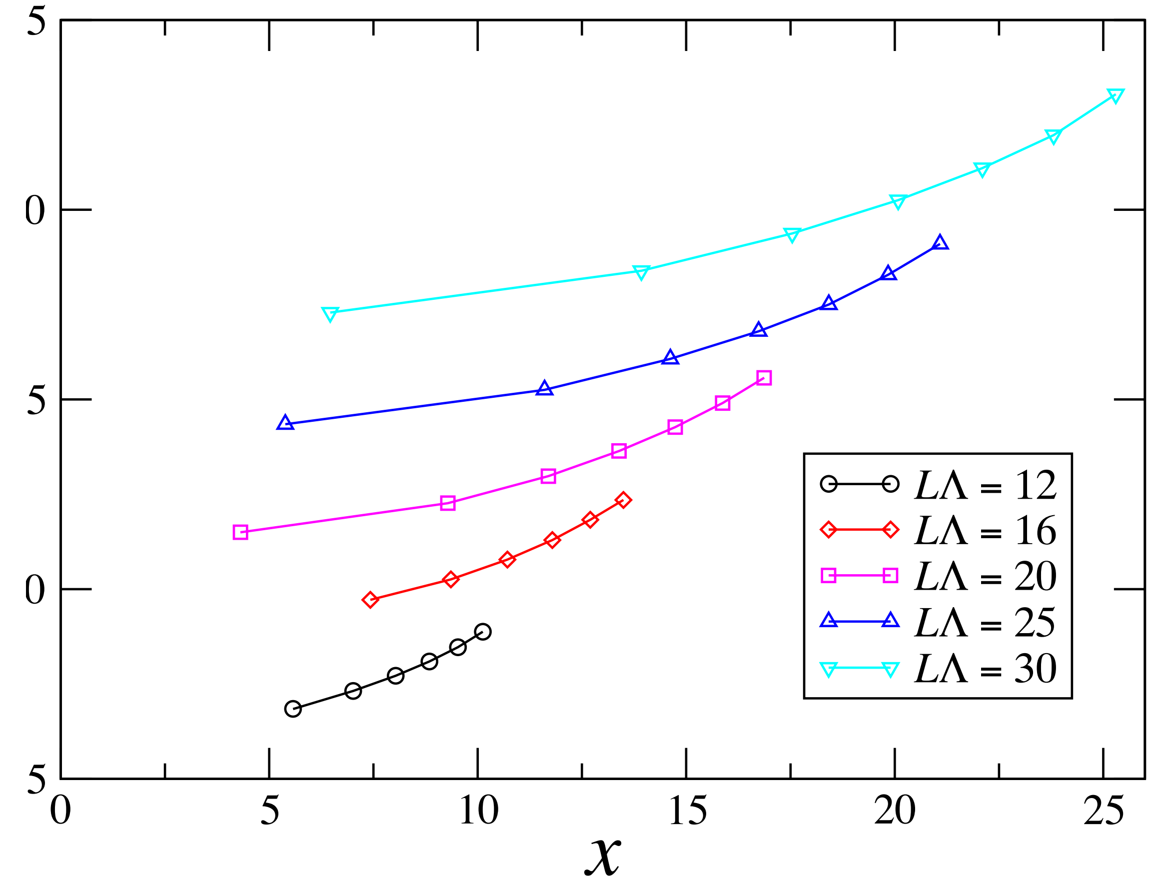
Nf=9, α*>α$_{cr}$, broken phase

Raw data
(Mock data)

→ Finite size Hyperscaling test



γ = 2.0



Nf=12, α*<αcr , conformal window

Raw data
(Mock data) → Finite size Hyperscaling test