125 GeV techni-dilaton at the LHC

Shinya Matsuzaki

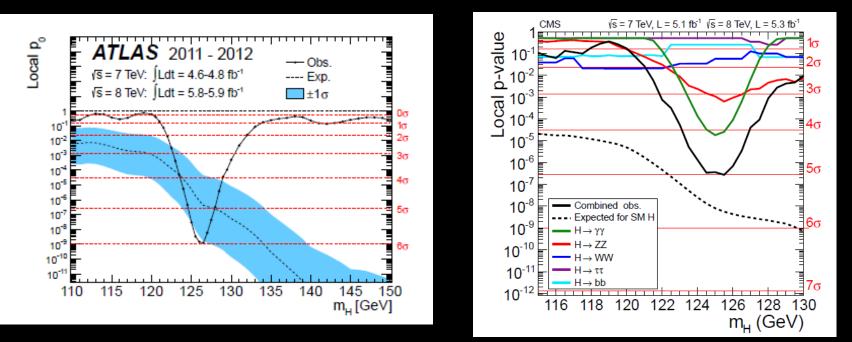
Maskawa Institute (Kyoto Sangyo U.)



Introduction

ATLAS

CMS



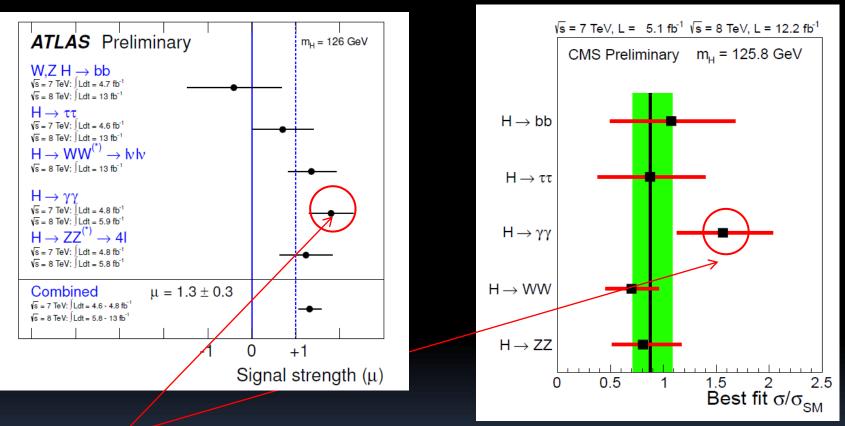
This year is exciting!!

A new boson at around 125 GeV was observed at LHC

<u>The signal strengths ($\mu = \sigma/\sigma_{SM}$)</u>

ATLAS (CONF-2012-162)

CMS (HIG-PAS-12-045)



Somewhat large diphoton event rate: μ (diphoton) ~ 2 implies a "new Higgs boson" (impostor) beyond the SM ! <u>Is it Techni-dilaton (TD) ?</u>

* TD: composite scalar; Yamawaki et al (1986); Bando et al (1986)

predicted in walking technicolor,

arising as a pNGB for (approximate) scale symmetry

spontaneously broken by techni-fermion condensate;

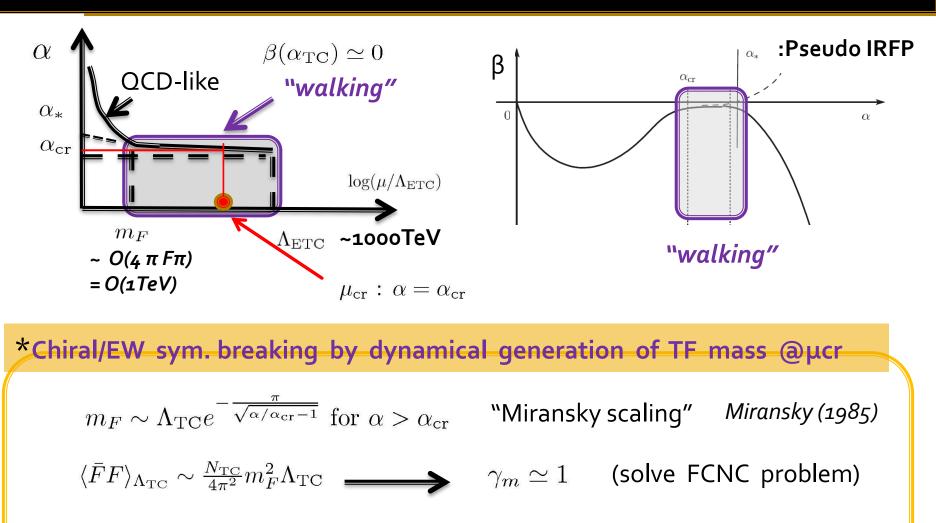
its lightness is protected by the scale symmetry,

and hence can be, say, ~ 125 GeV.

* 125 GeV TD signatures at LHC are consistent with current data!! S.M. and K. Yamawaki, PRD85 (2012); PRD86 (2012); 1207.5911; 1209.2017



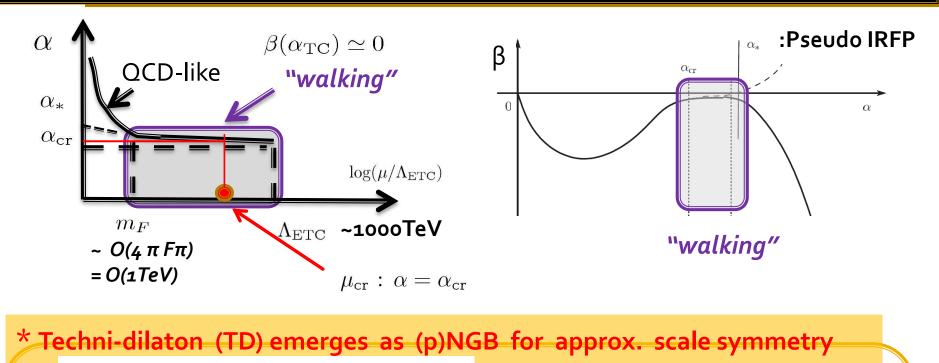
A schematic view of walking TC



wide range walking $m_F < \mu < \Lambda_{TC}$ \longrightarrow (naturalness) (approx. scale invariance)

Yamawaki et al (1986); Bando et al (1986)

★ Walking TC and techni-dilaton



$$m_{F} \sim \Lambda_{\rm TC} e^{-\frac{\pi}{\sqrt{\alpha/\alpha_{\rm cr}-1}}} \text{ for } \alpha > \alpha_{\rm cr} \qquad \text{SSB of (approximate) scale sym.}$$

$$\Rightarrow \quad \alpha \text{ starts "running"} \\ \text{(walking) up to mF} \qquad \beta(\alpha) = \Lambda_{\rm TC} \frac{\partial \alpha}{\partial \Lambda_{\rm TC}} = -\frac{2\alpha_{\rm cr}}{\pi} \left(\frac{\alpha}{\alpha_{\rm cr}} - 1\right)^{3/2}$$

$$\Rightarrow \quad \text{Nonpert. scale anomaly} \\ \text{induced by mF itself} \qquad \partial_{\mu} D^{\mu} = \frac{\beta(\alpha)}{4\alpha^{2}} \langle \alpha G_{\mu\nu}^{2} \rangle \neq 0 : \text{ TD gets massive}$$

★Ladder estimate of TD mass

* LSD + BS in large Nf QCD

Harada et al (1989); Kurachi et al (2006)

* LSD via gauged NJL

Shuto et al (1990); Bardeen et al (1992); Carena et al (1992) ; Hashimoto (1998)

A composite Higgs mass

$$M_{\phi} \sim 4F_{\pi}$$

~500 GeV for one-family model (1FM) still larger than ~125 GeV

* This is reflected in PCDC (partially conserved dilatation current)

$$F_{\phi}^{2}M_{\phi}^{2} = -4\langle \theta_{\mu}^{\mu} \rangle = \frac{\beta(\alpha)}{\alpha} \langle G_{\mu\nu}^{2} \rangle \simeq 3\eta m_{F}^{4}$$

$$\text{Miransky et al (1989):}$$

$$\text{Hashimoto et al (2011):}$$

$$\text{Where } \eta \simeq \frac{N_{\text{TC}}N_{\text{TF}}}{2\pi^{2}} = \mathcal{O}(1)$$

$$M_{\phi}/m_{F} \rightarrow 0.$$

$$\text{only when } F_{\phi}/m_{F} \rightarrow \infty, \text{ i.e., a decoupled limit.}$$

$$\text{No massless NGB limit:}$$

Holographic estiamte w/ techni-gluonic effects

K. Haba et al PRD82 (2010); S.M. and K.Yamawaki, 1209.2017

- * Ladder approximation : gluonic dynamics is neglected
- * Deformation of successful AdS/QCD model (Bottom-up approach)

Da Rold and Pomarol (2005); Erlich, Katz, Son and Stephanov (2005)

incorporates nonperturbative gluonic effects

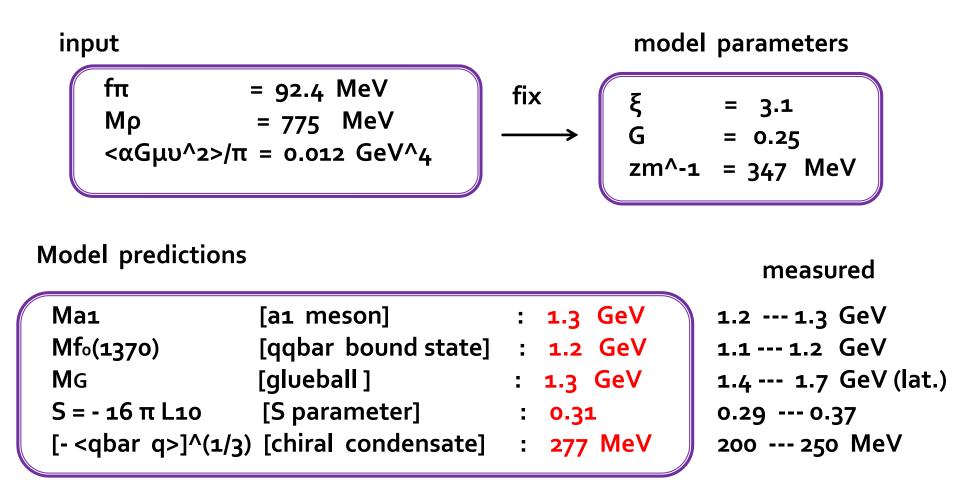
$$0 \leftarrow z = \epsilon$$

$$\int d^{4}x \int_{\epsilon}^{z_{m}} dz \sqrt{-g} \frac{1}{g_{5}^{2}} \frac{e^{cg_{5}^{2}\Phi_{X}(z)}}{q_{\mu\nu}dx^{\mu}dx^{\nu}-dz^{2}} \prod_{\mathbf{R}} z$$

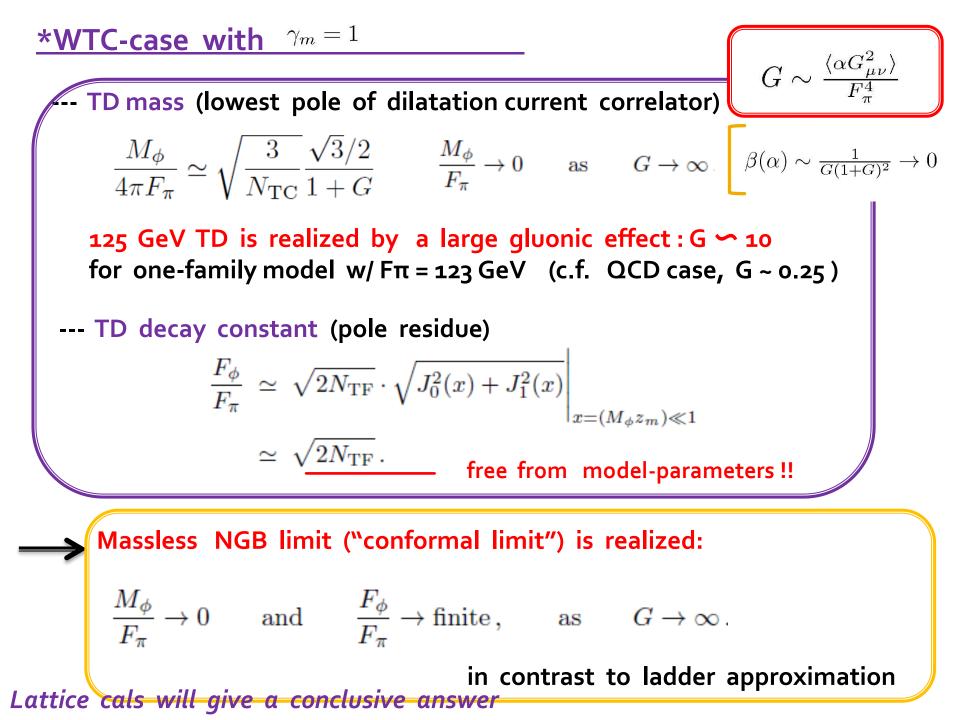
$$F_{5} = \int d^{4}x \int_{\epsilon}^{z_{m}} dz \sqrt{-g} \frac{1}{g_{5}^{2}} \frac{e^{cg_{5}^{2}\Phi_{X}(z)}}{q_{5}^{2}} \left(-\frac{1}{4} \operatorname{Tr} \left[L_{MN}L^{MN} + R_{MN}R^{MN}\right] + \operatorname{Tr} \left[D_{M}\Phi^{\dagger}D^{M}\Phi - m_{\Phi}^{2}\Phi^{\dagger}\Phi\right] + \frac{1}{2}\partial_{M}\Phi_{X}\partial^{M}\Phi_{X}\right)$$

$$m_{\Phi}^{2} = -(3 - \gamma_{m})(1 + \gamma_{m})/\tilde{L}^{2} \quad \left\{\begin{array}{c} \mathsf{QCD} & \gamma_{m} = 0 \\ \mathsf{WTC} & \gamma_{m} = 1 \end{array}\right.$$

* QCD-fit w/ $\gamma_m = 0$



Monitoring QCD works well!

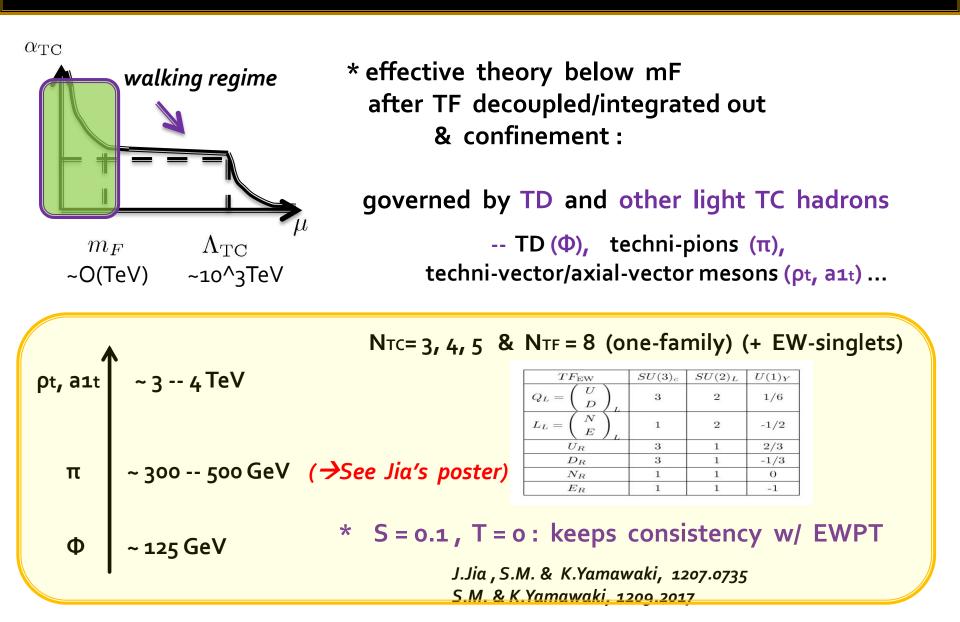


* More on the "conformal limit" G → ∞

$$G \sim \frac{\langle \alpha G_{\mu\nu}^2 \rangle}{F_{\pi}^4} \qquad \qquad \beta(\alpha) \sim \frac{1}{G(1+G)^2} \to 0$$

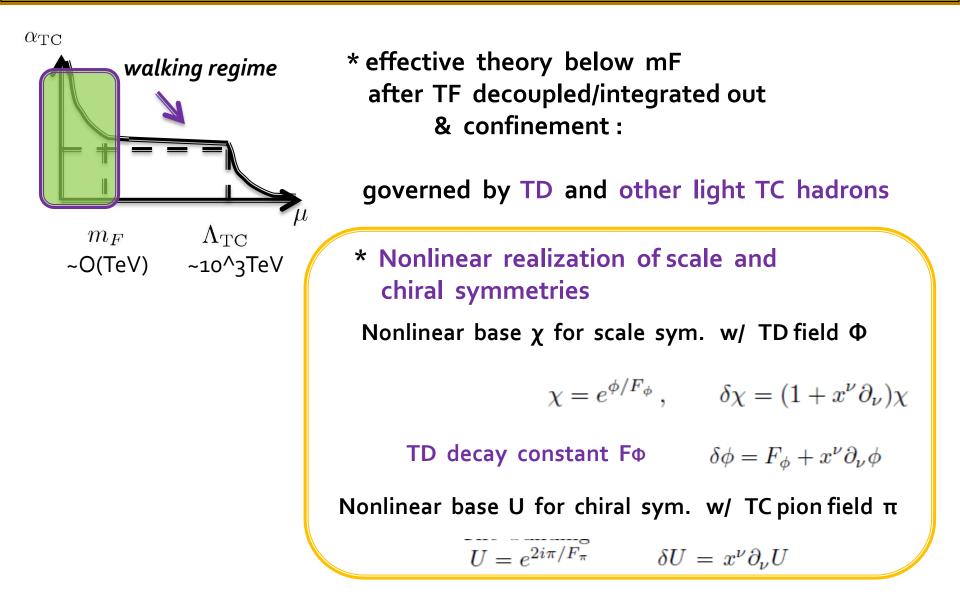
Ratios of masses of FFbar boundstates to Techni-glueball mass MG (lowest pole of $G\mu\nu^2$ correlator) TD mass/TG mass : $\frac{M_{\phi}}{M_C} \sim \frac{1}{1+G} \rightarrow 0$ Note: $\frac{M_{\phi}}{M_{c}} \rightarrow 0$ rho mass/TG mass : $rac{M_{
ho}}{M_{C}}
ightarrow 0$ Hence $\frac{M_{\phi}}{M_{C}} \ll \frac{M_{\rho}}{M_{C}} \to 0$ Interesting to check in lattice simulations !!

A low-energy theory below *m*_F



TD Lagrangian below *m*_F S.M. and

S.M. and K. Yamawaki, PRD86 (2012)



eff. TD Lagrangian
$$\mathcal{L} = \mathcal{L}_{ ext{inv}} + \mathcal{L}_S - V_{\chi}$$

i) The scale anomaly-free part:

$$\mathcal{L}_{\rm inv} = \frac{F_{\pi}^2}{4} \chi^2 \text{Tr}[\mathcal{D}_{\mu} U^{\dagger} \mathcal{D}^{\mu} U] + \frac{F_{\phi}^2}{2} \partial_{\mu} \chi \partial^{\mu} \chi$$

ii) The anomalous part (made invariant by including spurion field "S"):

$$\mathcal{L}_{S} = -m_{f} \left(\left(\frac{\chi}{S} \right)^{2-\gamma_{m}} \cdot \chi \right) \overline{f} f \qquad \qquad \text{Fellecting ETC-induced} \\ + \log \left(\frac{\chi}{S} \right) \left\{ \frac{\beta_{F}(g_{s})}{2g_{s}} G_{\mu\nu}^{2} + \frac{\beta_{F}(e)}{2e} F_{\mu\nu}^{2} \right\} + \cdots$$

iii) The scale anomaly part:

β_F: TF-loop contribution
 to beta function

$$V_{\chi} = \frac{F_{\phi}^2 M_{\phi}^2}{4} \chi^4 \left(\log \chi - \frac{1}{4} \right)$$

which correctly reproduces the PCDC relation:

$$\left. \left\langle \theta^{\mu}_{\mu} \right\rangle = -\delta_D V_{\chi} \right|_{\text{vacuum}} = -\frac{F_{\phi}^2 M_{\phi}^2}{4} \left\langle \chi^4 \right\rangle \right|_{\text{vacuum}} = -\frac{F_{\phi}^2 M_{\phi}^2}{4}$$

* TD couplings to W/Z boson (from L_inv)

$$g_{\phi WW/ZZ} = \frac{2m_{W/Z}}{F_{\phi}}$$

* TD couplings to $\gamma\gamma$ and gg (from L_S)

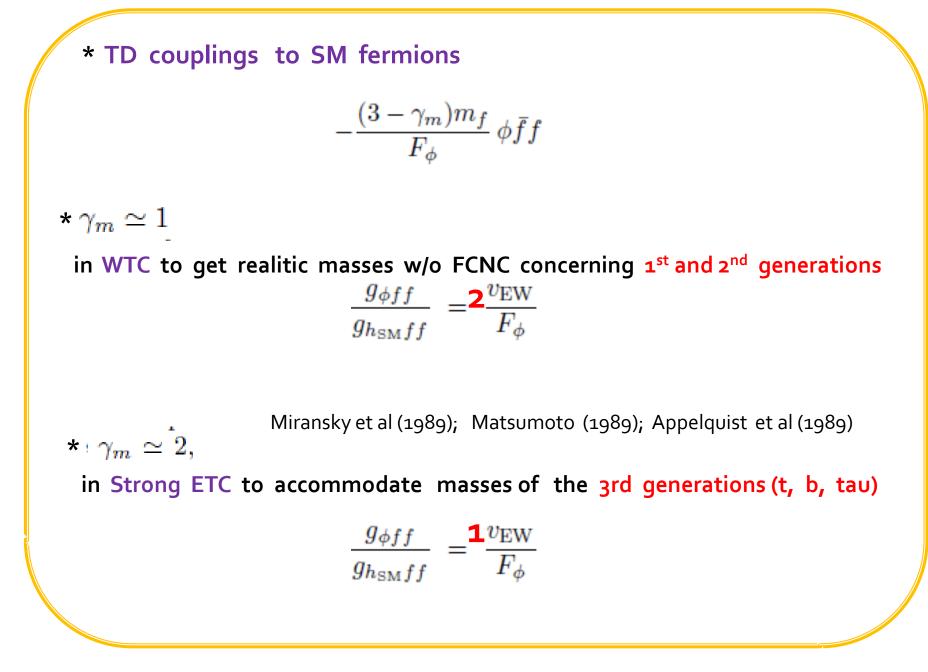
$$g_{\phi\gamma\gamma} = \frac{\beta_F(e)}{e} \frac{1}{F_{\phi}}$$

$$g_{\phi gg} = \frac{\beta_F(g_s)}{g_s} \frac{1}{F_{\phi}}$$

β_F: TF-loop contribution to beta function

* TD couplings to W/Z boson (from L_inv) $g_{\phi WW/ZZ} = \frac{2m_{W/Z}}{F_{\phi}}$ The same form as SM Higgs couplings * TD couplings to γγ and gg (from L_S) except $F\Phi$ and betas $g_{\phi\gamma\gamma} = \underbrace{\mathfrak{g}_F}_{e} \underbrace{\mathfrak{g}_F}_{e}$ $g_{\phi gg} = \underbrace{\beta_F(g_s)}_{g_s}$

β_F: TF-loop contribution to beta function



Thus, the TD couplings to SM particles essentially take the same form as those of the SM Higgs! : Just a simple scaling from the SM Higgs:

$\frac{g_{\phi WW/ZZ}}{g_{h_{\rm SM}WW/ZZ}}$	=	$\frac{v_{\rm EW}}{F_{\phi}},$		
$rac{g_{\phi ff}}{g_{h_{ m SM}ff}}$	=	$\frac{v_{\rm EW}}{F_{\phi}},$	for	$f = t, b, \tau$.

But, note ϕ -gg, ϕ - $\gamma\gamma$ depending highly on particle contents of WTC models. β_{F}

β_F: TF-loop contribution to beta function

$$\mathcal{L}_{\phi\gamma\gamma,gg} = \frac{\phi}{F_{\phi}} \left[\frac{\beta_F(e)}{2e^3} F_{\mu\nu}^2 + \frac{\beta_F(g_s)}{2g_s^3} G_{\mu\nu}^2 \right]$$

To be concerete, we consider the **one-family model (1FM**)

★Estimate of $\frac{v_{\rm EW}}{F_{\phi}}$: #1–Ladder approximation

* PCDC (partially conserved dilatation current)

$$F_{\phi}^2 M_{\phi}^2 = -4\langle \theta_{\mu}^{\mu} \rangle \qquad \langle \theta_{\mu}^{\mu} \rangle = 4\mathcal{E}_{\text{vac}} = -\kappa_V \left(\frac{N_{\text{TC}} N_{\text{TF}}}{2\pi^2}\right) m_F^4$$

* criticality condition

Appelequist et al (1996)

 $N_{\rm TF}\simeq 4N_{\rm TC}$

* Pagels-Stokar formula

$$F_{\pi} = v_{\rm EW} / \sqrt{N_D}.$$

of EW doublets

$$F_\pi^2 = \kappa_F^2 \frac{N_{\rm TC}}{4\pi^2} m_F^2$$

$$\frac{v_{\rm EW}}{F_{\phi}} \simeq \frac{1}{8\sqrt{2}\pi} \sqrt{\frac{\kappa_F^4}{\kappa_V}} N_D \frac{M_{\phi}}{v_{\rm EW}}$$

* Recent ladder SD analysis (large Nf QCD) $\kappa_V \simeq 0.7$, $\kappa_F \simeq 1.4$ Hashimoto et al (2011) * Inclusion of theoretical uncertainties

Estimate

Ladder approximation is subject to about 30% uncertainty for estimate of critical coupling and QCD hadron spectrum

critical coupling : T. Appelquist et al (1988); Hadron spectrum : K. -I. Aoki et al (1991); M. Harada et al (2004).

$$\frac{N_{\rm TF}}{4N_{\rm TC}} \simeq 1 \pm 0.3 \qquad \langle \theta^{\mu}_{\mu} \rangle = 4\mathcal{E}_{\rm vac} = -\frac{\kappa_V}{30\%} \left(\frac{N_{\rm TC}N_{\rm TF}}{2\pi^2} \right) m_F^4$$

$$F_{\pi}^2 = \frac{\kappa_F^2}{4\pi^2} \frac{N_{\rm TC}}{4\pi^2} m_F^2$$
Estimate
30%
$$\frac{v_{\rm EW}}{F_{\phi}} \simeq (0.1 - 0.3) \times \left(\frac{N_D}{4} \right) \left(\frac{M_{\phi}}{125 \,{\rm GeV}} \right)$$

$$\star Estimate of \frac{v_{\rm EW}}{F_{\phi}} : #2 - Holographic approach$$

* TD decay constant for the light TD case w/ G ~ 10:

$$\begin{array}{l} \displaystyle \left. \frac{F_{\phi}}{F_{\pi}} \ \simeq \ \sqrt{2N_{\mathrm{TF}}} \cdot \sqrt{J_0^2(x) + J_1^2(x)} \right|_{x = (M_{\phi} z_m) \ll 1} \\ \\ \displaystyle \simeq \ \sqrt{2N_{\mathrm{TF}}} \, . \end{array} \begin{array}{l} \text{free from model-parameters !!} \end{array}$$

Inclusion of typical size of 1/NTC (20% ~ 30%) corrections:

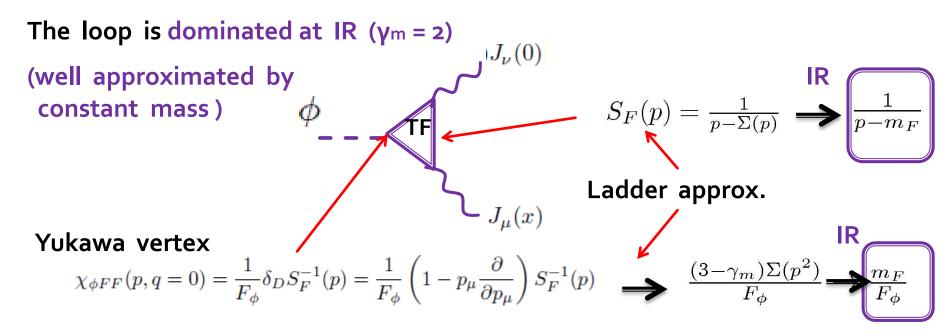
$$\left. \frac{v_{\rm EW}}{F_{\phi}} \right|_{\rm holo}^{+1/N_{\rm TC}} \sim 0.2 - 0.4$$

This is consistent with ladder estimate:

$$\frac{v_{\rm EW}}{F_{\phi}} \simeq (0.1 - 0.3) \times \left(\frac{N_D}{4}\right) \left(\frac{M_{\phi}}{125 \, {\rm GeV}}\right)$$

***** Calculation of beta functions

$$\mathcal{L}_{\phi\gamma\gamma,gg} = \frac{\phi}{F_{\phi}} \left[\frac{\beta_F(e)}{2e^3} F_{\mu\nu}^2 + \frac{\beta_F(g_s)}{2g_s^3} G_{\mu\nu}^2 \right]$$

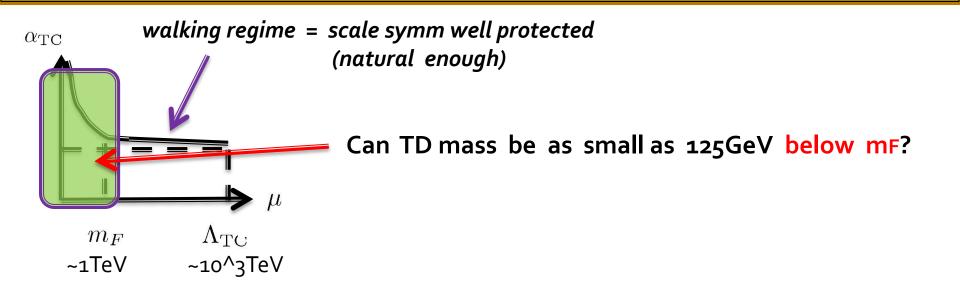


The resultant betas coincide just one-loop perturbative expressions:

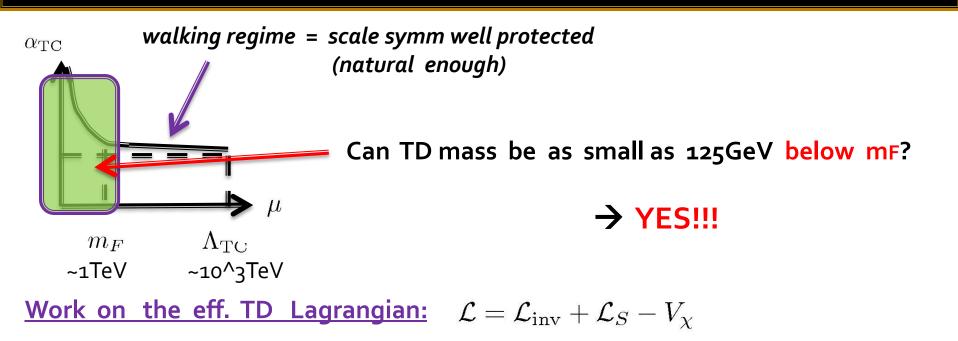
$$\beta_F(g_s) = \frac{g_s^3}{(4\pi)^2} \frac{4}{3} N_{\rm TC}$$
$$\beta_F(e) = \frac{e^3}{(4\pi)^2} \frac{16}{9} N_{\rm TC}$$

constant

TD mass stability below mF_{S.M. and K. Yamawaki, PRD86 (2012)}



TD mass stability below mF S.M. and K. Yamawaki, PRD86 (2012)



Dominant corrections come from top-loop (quadratic div.)

cutoff by mF ~4 π F π ~ 1TeV (~ F ϕ): $\delta M_{\phi}^2 \approx -\frac{3}{4\pi^2} \frac{m_t^2}{F_{\phi}^2} \cdot m_F^2$

w/
$$m_t^2 \simeq 2M_{\phi}^2$$
 $\frac{\delta M_{\phi}}{M_{\phi}(125 \text{GeV})} \approx -\frac{3}{4\pi^2} \frac{m_F^2}{F_{\phi}^2} \approx \mathcal{O}(10^{-2} - 10^{-1})$

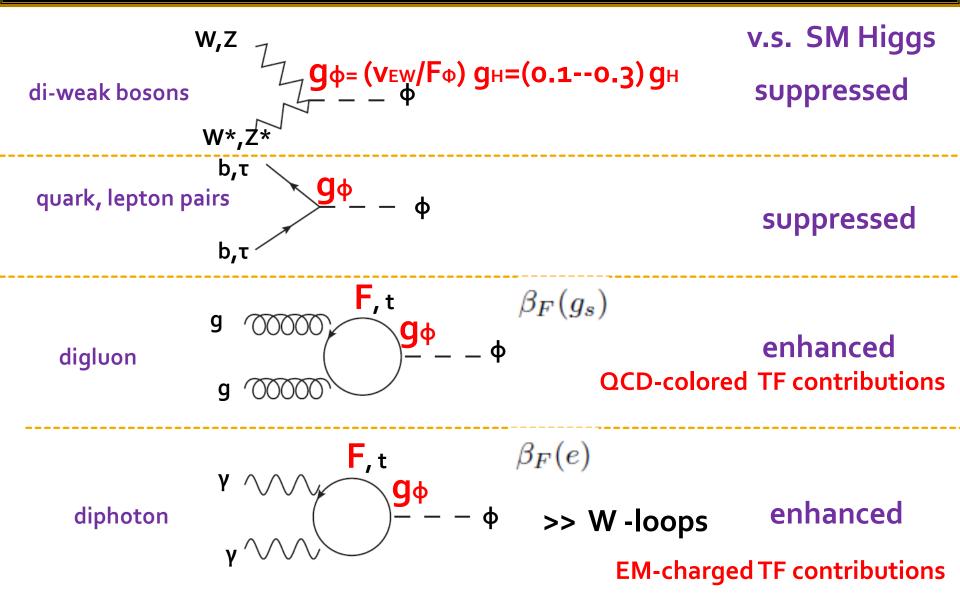
naturally light thanks to large Fo



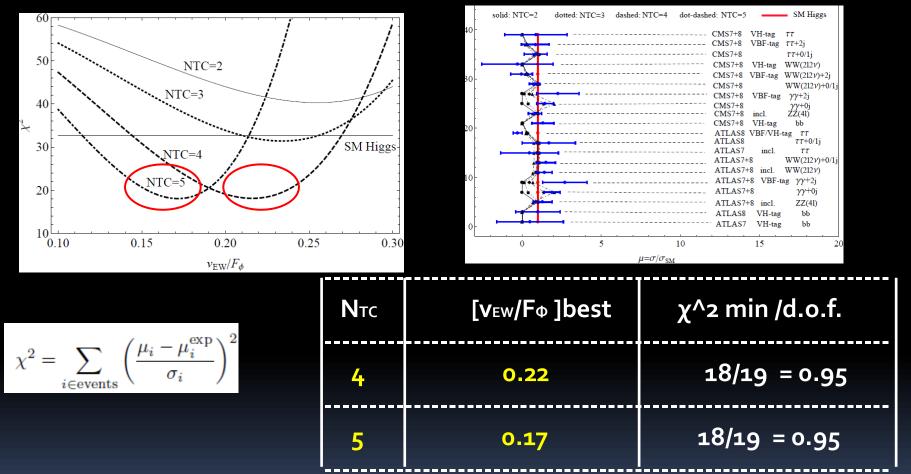
S.M. and K. Yamawaki

PRD85 (2012); PRD86 (2012); arXiv:1207.5911; arXiv:1209.2017

Characteristic features of ★ 125 GeV TD in 1FM (w/ NTC=4,5) at LHC



*updated from *updated from *1207.5911" after HCP2012 to the current Higgs search data



* TD can be better than the SM Higgs (chi^2/d.o.f= 33/20=1.6), due to the enhanced diphoton rate, by extra BSM (TF) contributions!

* The TD charasteristic signal strengths for each category

μzz	=	0.7		1.0	(inclusive)
μ bb	=	0.006		0.01	(VH-tag)
μww₀j	=	0.8		1.1	(ggF-tag)
L WW2j		0.2		0.3	(VBF-tag)
μww	=	0.006	-	0.01	(VH-tag)
μ ττο j	=	0.8		1.1	(ggF-tag)
μ ττ2j		0.2		0.3	(VBF-tag)
μττ	=	0.006		0.01	(VH-tag)
μγγοj	=	1.4		2.0	(ggF-tag)
μγγ2j	=	0.5		0.7	(VBF-tag)

VH & VBF-tags : suppressed

γγοj : enhanced



- * TD is the characteristic light scalar in WTC: the mass can be 125 GeV; protected by approximate scale invariance.
- * The couplings to the SM particles take essentially the same forms as those for the SM Higgs, except couplings to diphoton and digluon.
- * The 125 GeV TD in 1FM gives the LHC signal favored by current LHC data, notably somewhat large diphoton event rate thanks to extra TF contributions.
- * More precise measurements on exclusive categories (e.g., Vbb, ττ+dijet) will draw a definite conclusion that the TD is favored, or not.

Backup Slides

★ More on holographic estiamtes

S.M. and K.Yamawaki, 1209.2017

- * Ladder approximation : gluonic dynamics is neglected
- * Deformation of successful AdS/QCD model (Bottom-up approach)

Da Rold and Pomarol (2005); Erlich, Katz, Son and Stephanov (2005)

incorporates nonperturbative gluonic effects

$$0 \leftarrow z = \epsilon$$

$$\int d^{4}x \int_{\epsilon}^{z_{m}} dz \sqrt{-g} \frac{1}{g_{5}^{2}} \frac{e^{cg_{5}^{2}\Phi_{X}(z)}}{(1 + \gamma_{m})/\tilde{L}^{2}} \left(-\frac{1}{4} \operatorname{Tr} \left[L_{MN}L^{MN} + R_{MN}R^{MN} \right] + \operatorname{Tr} \left[D_{M}\Phi^{\dagger}D^{M}\Phi - m_{\Phi}^{2}\Phi^{\dagger}\Phi \right] + \frac{1}{2}\partial_{M}\Phi_{X}\partial^{M}\Phi_{X} \right)$$

$$m_{\Phi}^{2} = -(3 - \gamma_{m})(1 + \gamma_{m})/\tilde{L}^{2} \left(\begin{array}{c} \operatorname{OCD} & \gamma_{m} = 0 \\ \operatorname{WTC} & \gamma_{m} = 1 \end{array} \right)$$

$$S_{5} = \int d^{4}x \int_{\epsilon}^{z_{m}} dz \sqrt{-g} \frac{1}{g_{5}^{2}} e^{cg_{5}^{2}\Phi_{X}(z)} \left(-\frac{1}{4} \operatorname{Tr} \left[L_{MN}L^{MN} + R_{MN}R^{MN} \right] \right.$$
$$\left. + \operatorname{Tr} \left[D_{M}\Phi^{\dagger}D^{M}\Phi - m_{\Phi}^{2}\Phi^{\dagger}\Phi \right] + \frac{1}{2}\partial_{M}\Phi_{X}\partial^{M}\Phi_{X} \right)$$
$$\Phi(x,z) = \frac{1}{\sqrt{2}} (v(z) + \sigma(x,z)) \exp[i\pi(x,z)/v(z)]$$

AdS/CFT dictionary:

$$\Phi(x,z) = \frac{1}{\sqrt{2}} (v(z) + \sigma(x,z)) \exp[i\pi(x,z)/\Phi_X(z)] = v_X(z),$$

* UV boundary values = sources

$$\alpha M = \lim_{\epsilon \to 0} Z_m \left(\frac{L}{z} v(z) \right) \Big|_{z=\epsilon}, \qquad Z_m = Z_m \left(L/z \right) = \left(\frac{L}{z} \right)^{\gamma_m}$$
$$M' = \lim_{\epsilon \to 0} Lv_X(z) \Big|_{z=\epsilon}$$

* IR boundary values:

$$\xi = Lv(z) \Big|_{z=z_m} \quad \longleftrightarrow \quad \text{chiral condensate } \langle \bar{T}T \rangle$$

$$\mathcal{G} = Lv_X(z) \Big|_{z=z_m} \quad \longleftrightarrow \quad \text{gluon condensate } \langle \alpha G_{\mu\nu}^2 \rangle$$

* AdS/CFT recipe:

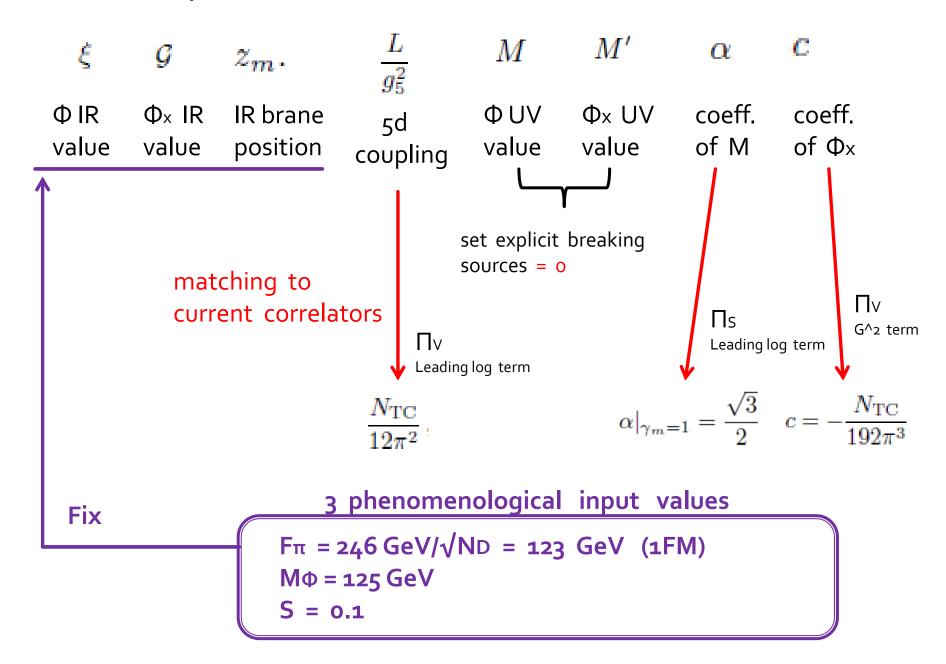
 $S_{5} \xrightarrow{} S_{5}[s, g, v, a]|_{\text{UV-boundary}} = W_{4\text{D}} \text{ generating functional}$ classical solutions $S_{5}[s, g, v, a]|_{\text{UV-boundary}} = W_{4\text{D}} \text{ generating functional}$ $S_{5}[s, g, v, a]|_{\text{UV-boundary}} = W_{4\text{D}} \text{ generating functional}$ $S_{5}[s, g, v, a]|_{\text{UV-boundary}} = W_{4\text{D}} \text{ generating functional}$ $S_{5}[s, g, v, a]|_{\text{UV-boundary}} = W_{4\text{D}} \text{ generating functional}$

$$W_{4D} \longrightarrow \langle TJ(x)J(0) \rangle \quad J = \bar{F}F, G^2_{\mu\nu}, \bar{F}\gamma_{\mu}T^aF, \bar{F}\gamma_{\mu}\gamma_5T^aF$$

Current collerators $\Pi_S, \Pi_G, \Pi_V, \Pi_A$ are calculated as a function of three IR –boundary values and γ_m :

 $\begin{cases} \xi &: \text{IR value of bulk scalar} \ \Phi_S &\longleftrightarrow \ \bar{F}F \\ G &: \text{IR value of bulk scalar} \ \Phi_G &\longleftrightarrow \ G_{\mu\nu}^2 \\ z_m &: \text{IR-brane position} \end{cases}$

The model parameters:



Other holographic predictions (1FM w/S=0.1)

NTC = 3

Techni-p, a1 masses	: Mp = Ma1 = 3.5 TeV
Techni-glueball (TG) mass	: MG = 19 TeV
TG decay constant	: FG = 135 TeV
dynamical TF mass mF	: mF = 1.0 TeV

NTC = 4

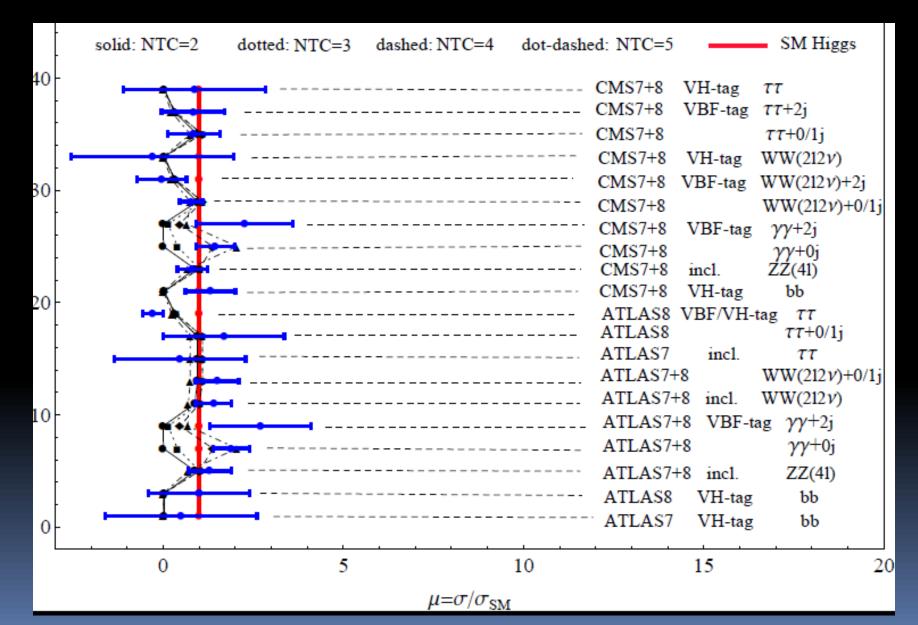
Techni-p, a1 masses	: Mp = Ma1 = 3.6 TeV
Techni-glueball (TG) mass	: MG = 18 TeV
TG decay constant	: FG = 156 TeV
dynamical TF mass mF	: mF = 0.95 TeV

NTC = 5

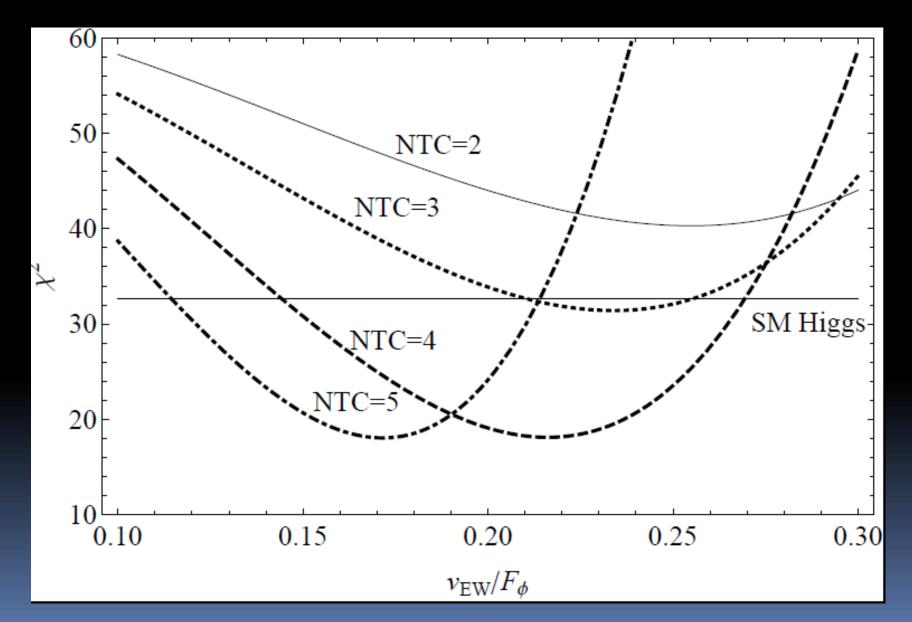
Techni-p, a1 masses Techni-glueball (TG) mass : MG = 18 TeV TG decay constant dynamical TF mass mF : mF = 0.85 TeV

: Mp = Ma1 = 3.9 TeV : FG = 174 TeV

After HCP update (I):

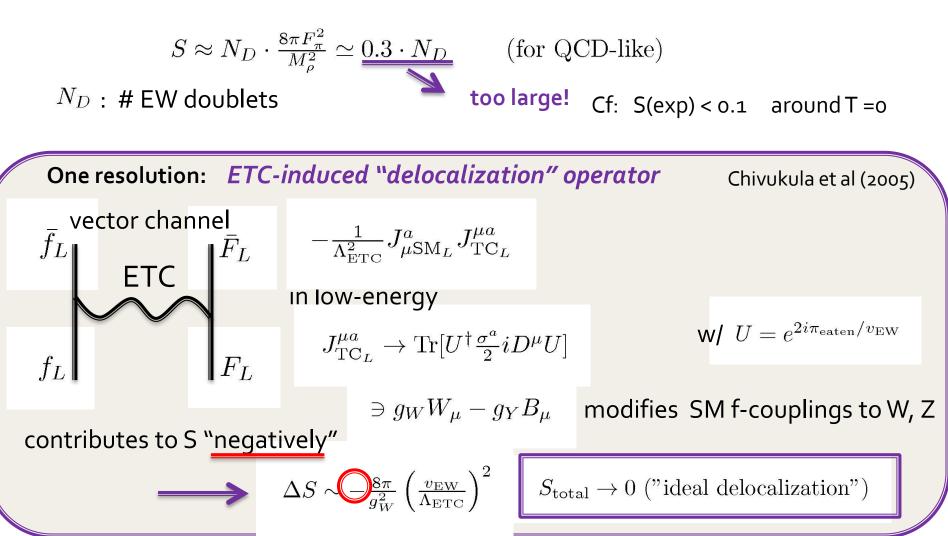


After HCP update (II):

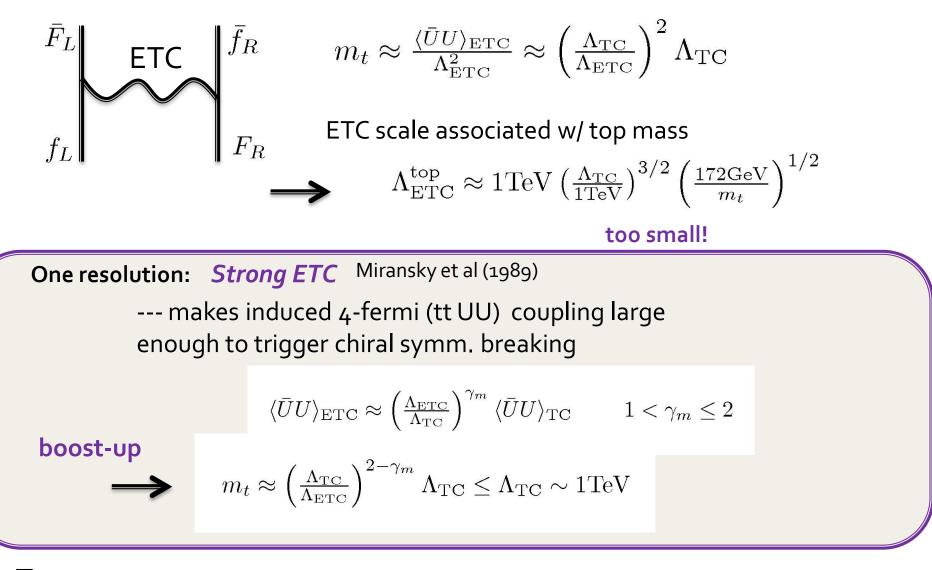


Other pheno. issues in TC scenarios

<u>S parameter</u>



Top quark mass generation



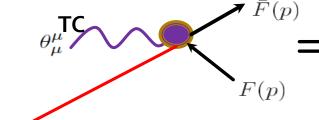
<u>T parameter</u> (Strong) ETC generates large isospin breaking → highly model-dependent issue

Direct consequences of Ward-Takahashi identities S.M. and K. Yamawaki, PRD86 (2012)

* Coupling to techni-fermions

$$\lim_{q_{\mu}\to 0} \int d^4y \, e^{iqy} \langle 0|T\partial^{\mu} D_{\mu}(y)F(x)\bar{F}(0)|0\rangle = i\delta_D \langle 0|TF(x)\bar{F}(0)|0\rangle$$
$$= i\left(2d_F + x^{\nu}\partial_{\nu}\right) \langle 0|TF(x)\bar{F}(0)|0\rangle$$

 $-S_F^{-1}(p) - ip_\mu \frac{\partial}{\partial p_\mu} S_F^{-1}(p)$

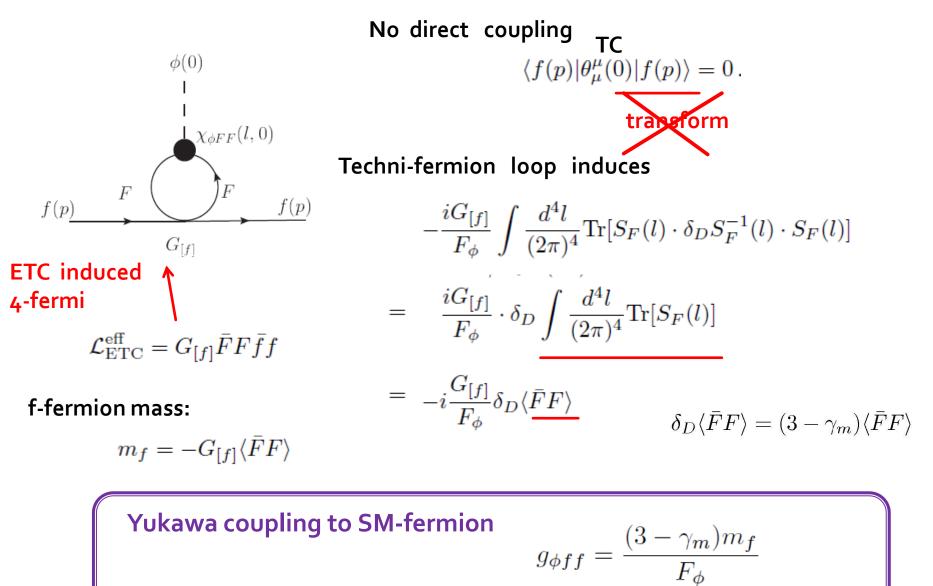


Dilaton pole dominance

 $F_{\phi} \cdot \langle \phi(q=0) | TF(x)\bar{F}(0) | 0 \rangle = \delta_D \langle 0 | TF(x)\bar{F}(0) | 0 \rangle$

w/ TD decay constant Fphi $\langle 0|D_{\mu}(x)|\phi(q)\rangle = -iF_{\phi}q_{\mu}e^{-iqx}$ $\chi_{\phi FF}(p,q=0) = \frac{1}{F_{\phi}}\delta_D S_F^{-1}(p) = \frac{1}{F_{\phi}}\left(1 - p_{\mu}\frac{\partial}{\partial p_{\mu}}\right)S_F^{-1}(p)$

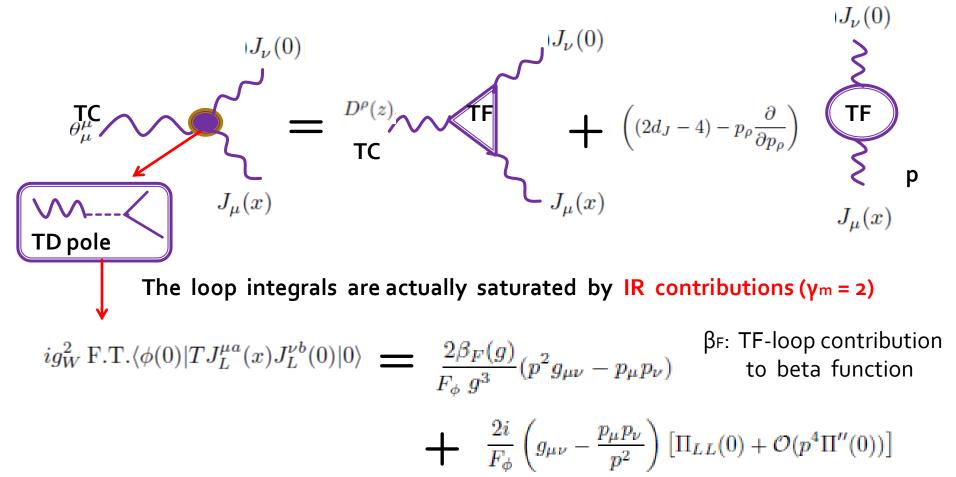
* Couplings to SM fermions



* Couplings to SM gauge bosons

WT identity \rightarrow scale anomaly term + anomaly-free term

$$\lim_{q_{\rho} \to 0} \int d^{4}z \, e^{iqz} \, \langle 0|T\partial_{\rho}D^{\rho}(z)J_{\mu}(x)J_{\nu}(0)|0\rangle \ = \ \lim_{q_{\rho} \to 0} \left(-iq_{\rho} \int d^{4}z \, e^{iqz} \, \langle 0|TD^{\rho}(z)J_{\mu}(x)J_{\nu}(0)|0\rangle \right) \\ +i\delta_{D} \langle 0|TJ_{\mu}(x)J_{\nu}(0)|0\rangle \,,$$



$$ig_W^2 \operatorname{F.T.}\langle \phi(0)|TJ_L^{\mu a}(x)J_L^{\nu b}(0)|0\rangle = \frac{2\beta_F(g)}{F_{\phi} g^3} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) \qquad \begin{array}{c} \beta_F: \operatorname{TF-loop \ contribution} \\ \text{to \ beta \ function} \\ + \frac{2i}{F_{\phi}} \left(g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2}\right) \left[\Pi_{LL}(0) + \mathcal{O}(p^4 \Pi''(0))\right] \end{array}$$

* For SU(2)W gauge bosons: W – "broken" currents

$$\Pi_{LL}(0) = N_D \frac{F_{\pi}^2}{4} = \frac{v_{\rm EW}^2}{4}$$

Coupling to W
$$\mathcal{L}_{\phi WW} = \frac{2m_W^2}{F_{\phi}} \phi W_{\mu}^a W^{\mu a}$$

* For unbroken currents coupled to photon, gluon:

$$\Pi(0) = 0.$$

$$\mathcal{L}_{\phi\gamma\gamma,gg} = \frac{\phi}{F_{\phi}} \left[\frac{\beta_F(e)}{2e^3} F_{\mu\nu}^2 + \frac{\beta_F(g_s)}{2g_s^3} G_{\mu\nu}^2 \right]$$