Baryonic Matter in the Hidden Local Symmetry Induced from Holographic QCD Models

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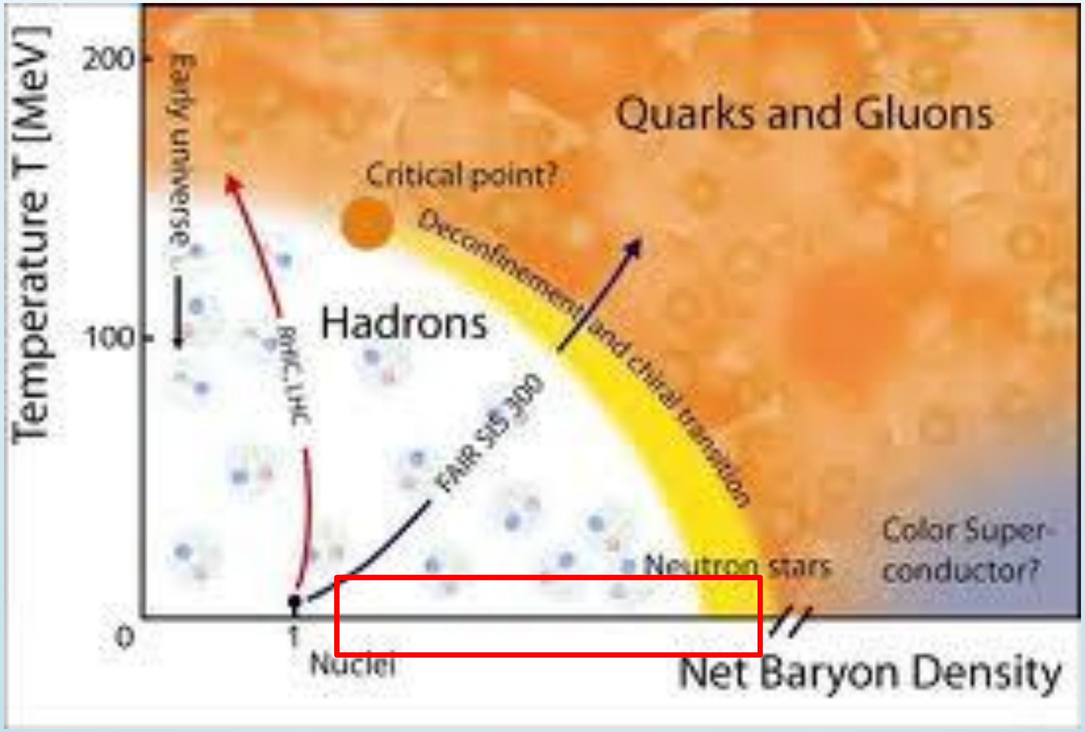
Outline

I. Background
II. Infinity tower of vector mesons for baryons
III. Baryon Properties in the Hidden Local Symmetry Induced from Holographic Models
IV. Dense baryonic mater
V. Lessons and perspectives

I. Background



Complete description:

- Must have complete understanding of dense, strongly interacting hadronic matter and strongly interacting quark matter from underlying theory (QCD).
- Requires the proper understanding of QCD in its non-perturbative regime, far from being completely understood, any theoretical advance remains very challenging.

Baryonic matter using Skyrme model

- In accesses to dense baryonic matter, the Skyrme model is a good approach since it can unify both the elementary baryons and multi-baryons system.

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Nuclear Physics **31** (1962) 556—569; © *North-Holland Publishing Co., Amsterdam*

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A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME †

A.E.R.E., Harwell, England

Received 29 September 1961

Abstract: Some aspects of a field theory, similar to but more realistic than, that examined in the preceding paper are discussed. The way in which a non-linear meson field theory of this type may contain its own sources, and how these may be idealised to point singularities, as in the conventional field theories of interacting linear systems, is formulated. The structure of the particle source in the classical theory is calculated, and some qualitative features of the interactions between these particles and mesons are described.

Skyrme model

$$\mathcal{L}_{\mathrm{Skyr}} = \frac{F_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{\epsilon^2}{4}\mathrm{Tr}\left\{\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]\left[U^\dagger\partial^\mu U, U^\dagger\partial^\nu U\right]\right\}$$

Nonlinear Sigma model

Skyrme term

Parameters: F_π , ▪ .

Many physical quantities of baryons can be calculated, agree with data quite well !

Table 2a

The low-energy parameters and masses as predicted in the Skyrme model, together with the MIT bag model results are compared to experiment. (A.N.W.) and (J.R.) are the predictions of ref. [21] and ref. [20] respectively, for $m_\pi = 0$. (A.N.) and (M.) are the results of ref. [22a] and ref. [22b] respectively, for $m_\pi \neq 0$

	A.N.W.	A.N.	J.R.	M.	MIT	Expt
$10^3\varepsilon^2$	4.21	5.34	5.52	5.13	–	–
f_π (MeV)	64.5	54.	93*	94.3*	149	93
g_A^N	1.02	1.08	1.33*	1.23*	1.09	1.23
M_N (MeV)	939*	939*	1425	1385	939*	939
$M_\Delta - M_N$ (MeV)	293*	293*	283	310	293*	293

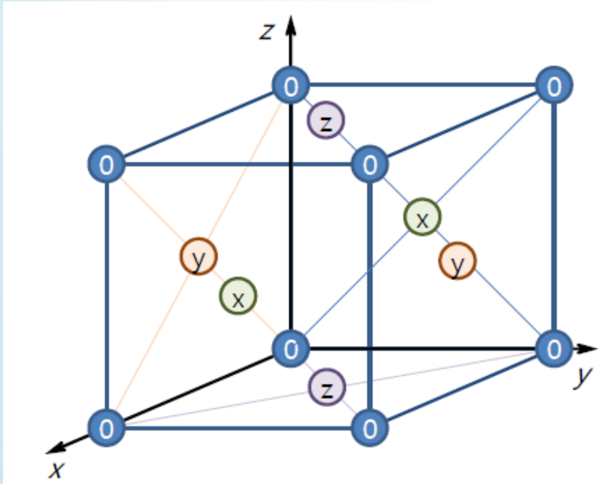
Table 2b

Same comparative table as in (a) for the electric and magnetic mean square radii together with the proton and neutron magnetic moments

	A.N.W.	A.W.	J.R.	M.	MIT	Expt
$\langle r_E^2\rangle_0^{1/2}$ (fm)	0.59	0.68	0.47	0.43	0.76	0.72
$\langle r_E^2\rangle_1^{1/2}$ (fm)	∞	1.04	∞		0.76	0.88
$\langle r_M^2\rangle_0^{1/2}$ (fm)	0.92	0.95			0.62	0.81
$\langle r_M^2\rangle_1^{1/2}$ (fm)	∞	1.04	∞		0	0.80
μ_p (n.m.)	1.87	1.97	2.74	2.43	1.90	2.79
μ_n (n.m.)	−1.31	−1.24	−2.24	−1.98	−1.27	−1.91

■ I. ZAHED and G.E. BROWN, Phys. Rept. 142, (1986) 1—102.

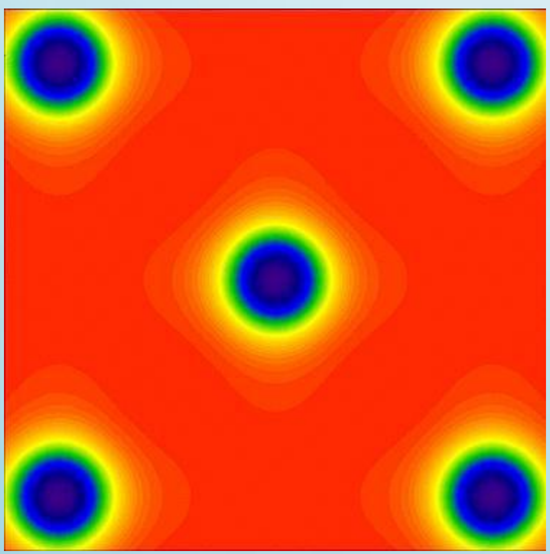
Skyrmion Crystal



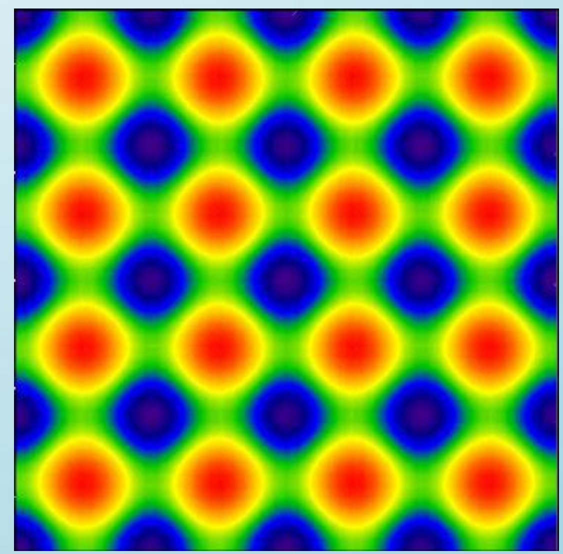
Soliton periodic in space

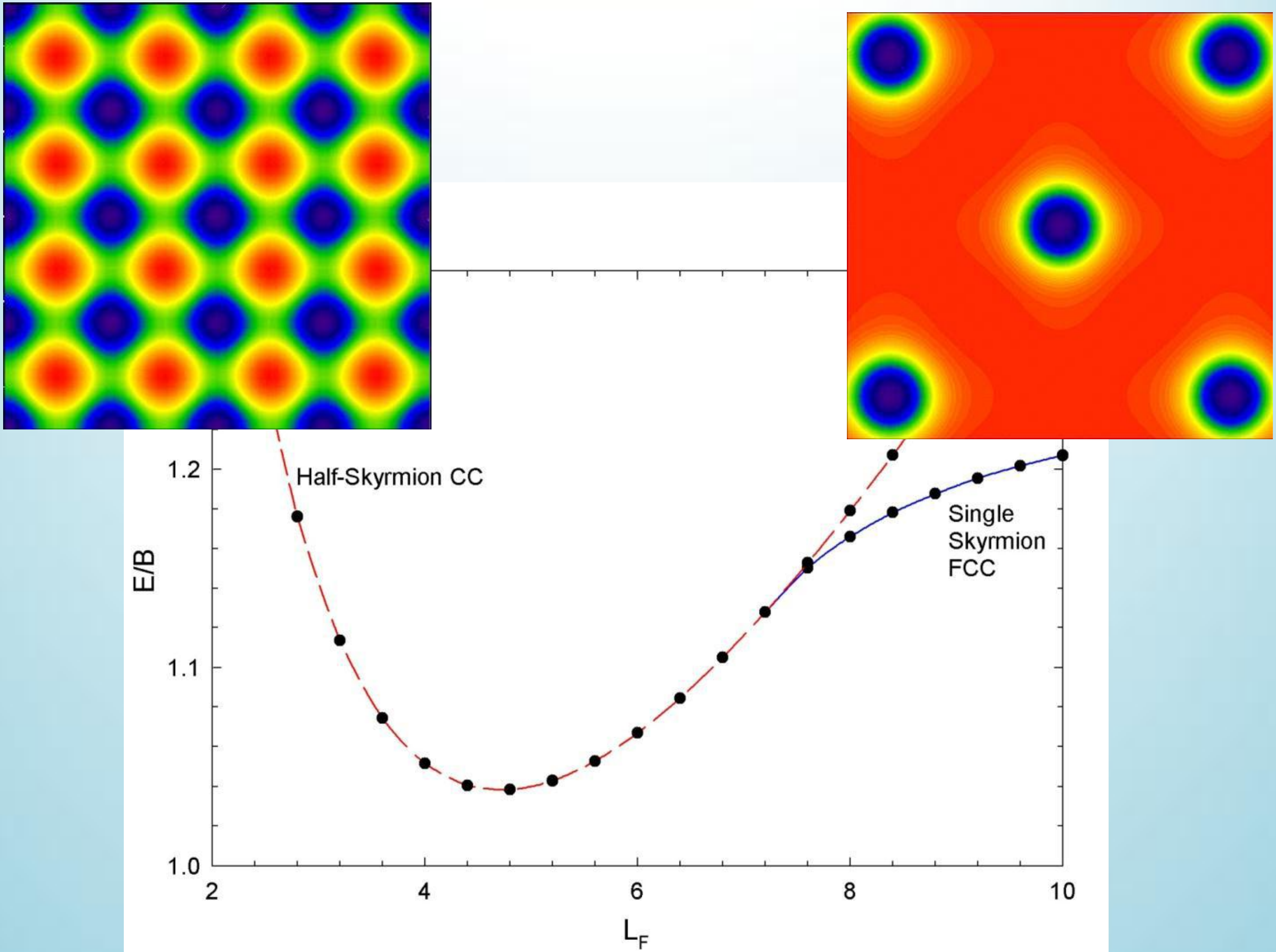
- Klebanov, Nucl. Phys. B262(1985)133.
- Goldhaber & Manton, Phys. Lett. B198(1987)231.

Low density, chiral symmetry broken by Skyrme crystal as in vacuum



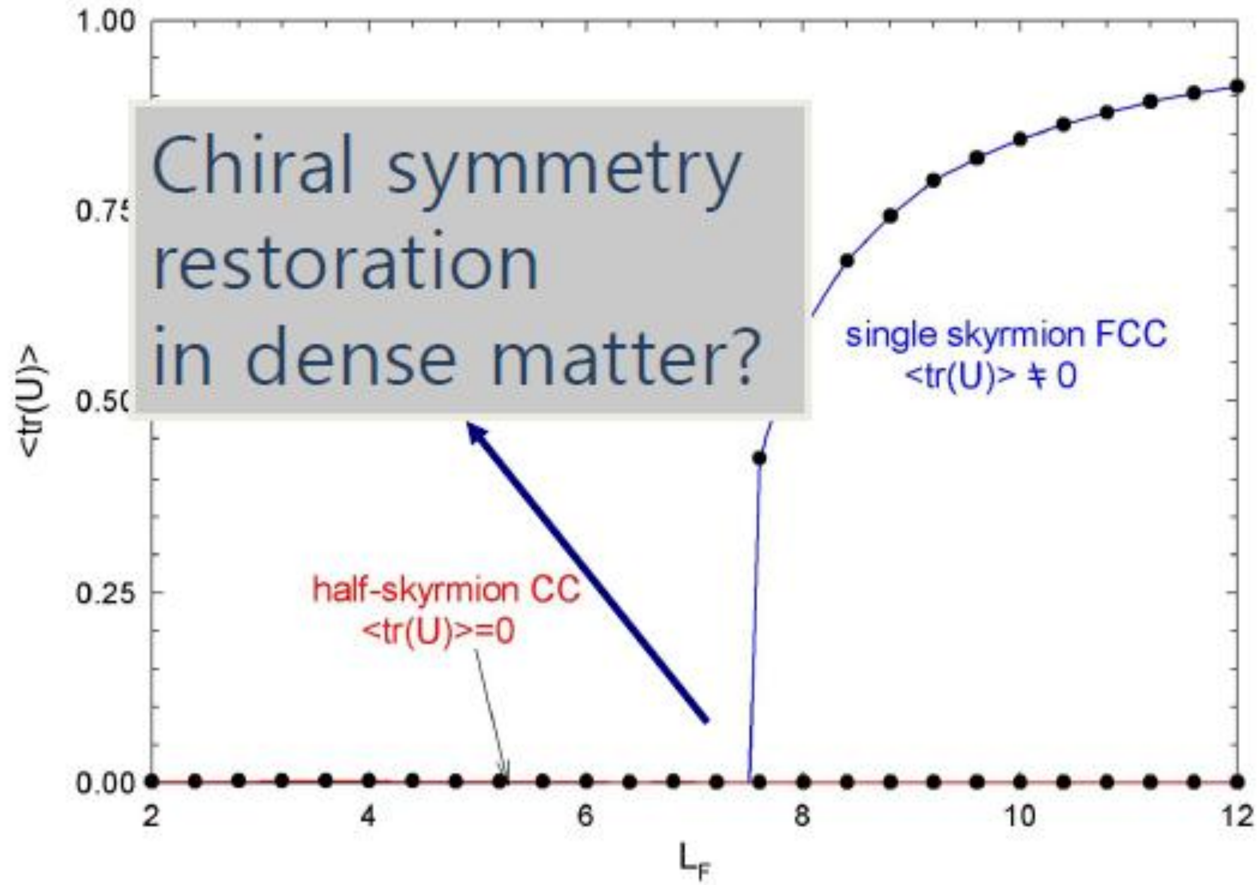
High density, chiral symm. restored: ▪U▪ = 0 in each cell (half skyrmion symm.)





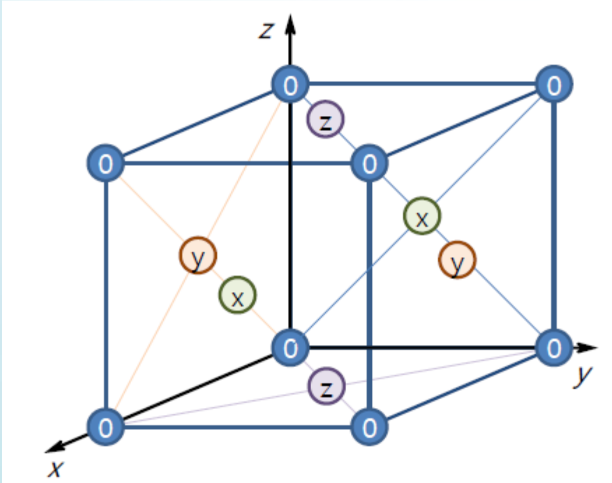
- Hee-Jung Lee, Byung-Yoon Park, Dong-Pil Min, Mannque Rho, Vicente Vento, Nucl. Phys. A723 (2003) 427–446

$\frac{1}{2}\,\mathrm{tr}(U) = \sigma$



- Hee-Jung Lee, Byung-Yoon Park, Dong-Pil Min, Mannque Rho, Vicente Vento, Nucl. Phys. A723 (2003) 427–446

Skyrmion Crystal

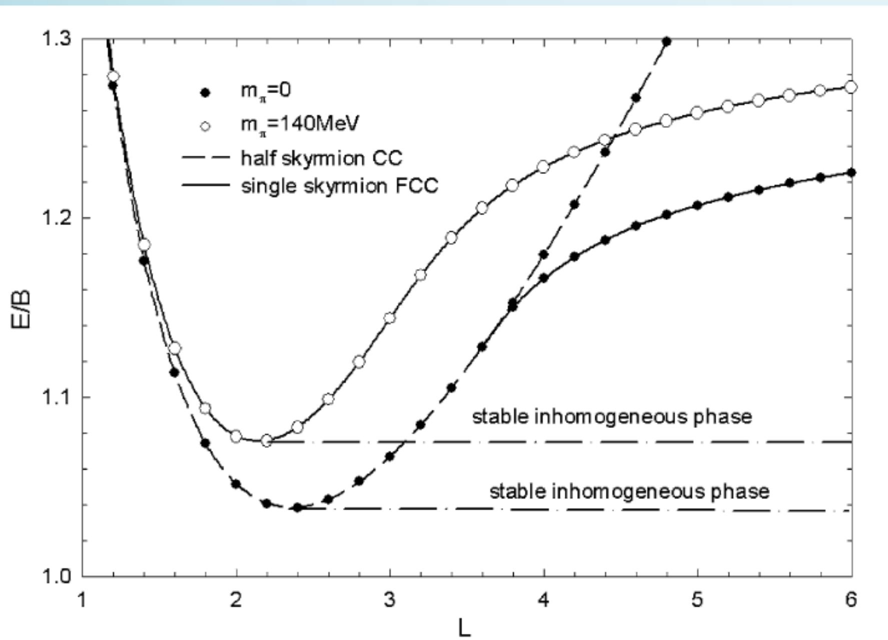


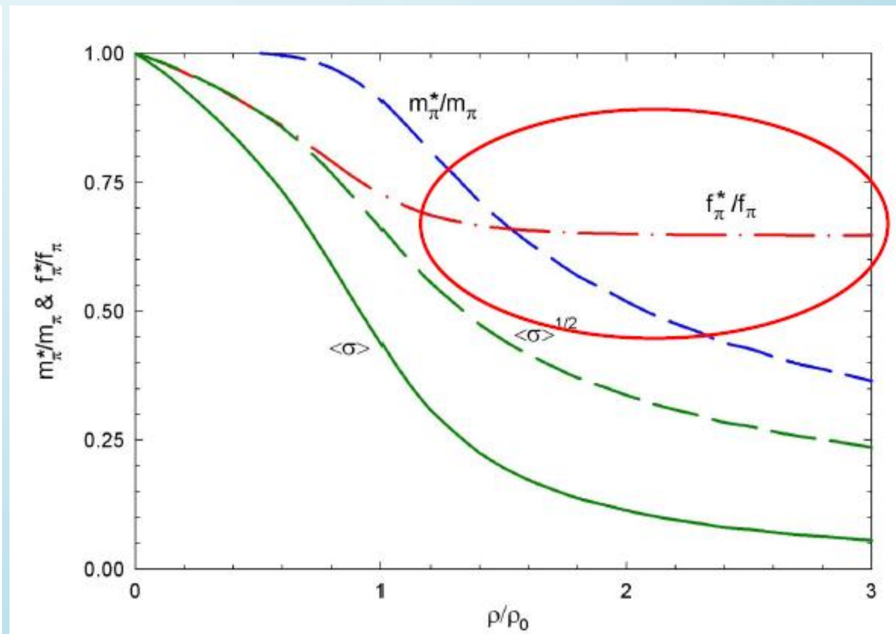
Dense Skyrmion matter

$$U(\vec{x},t) = \sqrt{U_\pi}\,U_0\,\sqrt{U_\pi}$$

fluctuating pions

classical dense matter config.





- Hee-Jung Lee, Byung-Yoon Park, Dong-Pil Min, Mannque Rho, Vicente Vento, Nucl. Phys. A723 (2003) 427–446

- QCD at very low energy is well modeled by nonlinear sigma model .
- Nonlinear sigma model, stabilized by Skyrme term, is an effective model for baryon.
- Single and multibaryon system can be unifiedly described in Skyrme model.
- It’s possible to study dense effect on meson properties.

Questions may be asked:

- Does Skyrmion matter really describe baryonic matter?

  Deviation!

- What will happen when energy scale increases?

More resonances appear. ➡ More complicated model

E ▪ massive vector mesons appear

Find redundancies in the chiral field and introduce associated gauge symmetry to capture the physics of vector mesons

The NL▪ model is gauge-equivalent to HLS and the vector mesons so generated can be identified with the hidden local gauge fields.

To fix all the LECs, we resort to holographic models!

The effect of the heavier resonances on Skyrmion properties can be self-consistently calculated.

II. Infinity Tower of Vector Mesons for Baryons

The holographic model

$$S = S_{\text{DBI}} + S_{\text{CS}}$$ CS term

where

$$S_{\text{DBI}} \approx S_{\text{YM}} = -\kappa \int d^4x dz \frac{1}{2e(z)^2} \operatorname{tr} \mathcal{F}_{mn}^2$$

$\cdot N_c/(216\pi^3)$

Effective YM coupling depending on the holographic direction z.

Consider $N_f = 2$

$$\mathcal{A} = A_{\text{SU(2)}} + \frac{1}{2}\tilde{A}_{\text{U(1)}}.$$

$$S_{\text{YM}} = -\kappa \int d^4x dz \frac{1}{2e^2(z)} \left(\operatorname{tr} F_{mn}^2 + \frac{1}{2}\tilde{F}_{mn}^2 \right)$$

$$S_{\text{CS}} = \frac{N_c}{16\pi^2} \int \tilde{A} \wedge \operatorname{tr} F^2 + \frac{N_c}{96\pi^2} \int \tilde{A} \wedge \tilde{F}^2.$$

The role of the infinite tower of vector mesons in the baryon structure can be studied in the approximation that the space is flat and the CS term is ignored.

The BPS Skyrmion

The leading O(▪) .

- Hata *et al*, PTP117(2007)1157.
- Kaplunovsky *et al*, JHEP11(2012)047

Consider the 5D Euclidean YM action

$$S = -\frac{1}{2}\int \mathrm{tr} F_{mn}^2 d^4x\, dz,$$

where

$$A_m = T^a A_m^a$$

$$F_{mn} = \partial_m A_n - \partial_n A_m + [A_m, A_n]$$

$$E \geq 8\pi^2 B$$

$$B = \frac{1}{16\pi^2}\int \mathrm{tr}(F_{MN}{}^*F_{MN}) d^3x\, dz$$

$F_{MN} = {}^*F_{MN}$

The more vector mesons are included, the closer the static energy goes down and approaches the BPS bound. In other words, the higher tower of vector mesons drive the theory to a conformal theory.

▪ $$E^{(0)} = 1.235\,(8\pi^2 B).$$

▪, ▪ $$E^{(1)} = 1.071(8\pi^2 B)$$

▪, ▪, a_1 $$E^{(2)} = 1.048(8\pi^2 B).$$

- Paul Sutcliffe ,
  JHEP 1104 (2011) 045

The full tower will bring this to the bound $E^{(\cdot)}$ = 8 ▪^{2} B

The high-lying vector mesons make the theory flow to a conformal theory.

Comments:

- The BPS Skyrmion, no repulsive interaction. Should consider the deviation of BPS.
- To next order in ▪, the metric is curved in the holographic direction. To that order, the Chern-Simons term enters bringing in a U(1) degree of freedom, i.e., the ▪ meson and its tower.
- We know from nuclear physics that the ▪ meson brings in repulsion, without which nuclei will collapse. In the Skyrmion description, what it does is to make the soliton mass appreciably increased compared with the one without it.
- It is clear from the above consideration that both the warping of the background deviating from the BPS structure and the Chern-Simons term needs to be confronted.
- We have to address the infinite tower structure in the presence of warping and the Chern- Simons term including all the terms to O(▪^{0}) .

III. Baryon Properties in the Hidden Local Symmetry Induced from Holographic Models

- Skyrmion has been studied based on the $O(p^2)$ HLS (f. , g, a).

$$M_{\rm sol} = (667 \sim 1575)\ \mathrm{MeV} \qquad 1 \leq a \leq 4$$

- Igarashi et al, Nucl. Phys. B259(1985)721

- The ambiguity in the value of "a" results in a large uncertainty on the soliton mass, hinders systematic investigation on the properties of a single Skyrmion and baryonic matter.

- The description of baryons as Skyrmions is supported by the large N_c limit. In the HLS, the higher order terms such as the $O(p^4)$ terms are at $O(N_c)$ as well as the $O(p^2)$ terms. As a result, in the N_c counting, these higher order terms should be taken into account.

- However, including the higher order terms inevitably calls forth more complicated form of the Lagrangian and uncontrollably large number of low energy constants.

- By making use of the hQCD, we fix them in a controllable way. Furthermore, the · meson is included through the hWZ terms whose parameters are also fixed by the hQCD.

Skyrmions from the Hidden Local Symmetry

Consider two flavor, to O(p^4)

$$\hat{\alpha}_{\parallel\mu} = \frac{1}{2i}\left(D_\mu\xi_R\xi_R^\dagger + D_\mu\xi_L\xi_L^\dagger\right)$$
$$\hat{\alpha}_{\perp\mu} = \frac{1}{2i}\left(D_\mu\xi_R\xi_R^\dagger - D_\mu\xi_L\xi_L^\dagger\right)$$
$$\xi_L^\dagger = \xi_R \equiv \xi = e^{i\pi/2f_\pi}$$
$$D_\mu\xi_{R,L} = (\partial_\mu - iV_\mu)\xi_{R,L}$$
$$V_\mu = \frac{g}{2}(\omega_\mu + \rho_\mu)$$

$$\mathcal{L}_{\mathrm{HLS}} = \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \mathcal{L}_{\mathrm{anom}}, \text{-} \mathcal{L}_{\mathrm{anom}},$$
$$\mathcal{L}_{(2)} = f_\pi^2\,\mathrm{Tr}\left(\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu\right) + a f_\pi^2\,\mathrm{Tr}\left(\hat{\alpha}_{\parallel\mu}\hat{\alpha}_\parallel^\mu\right) - \frac{1}{2g^2}\mathrm{Tr}\left(V_{\mu\nu}V^{\mu\nu}\right)$$
$$\mathcal{L}_{(4)} = \mathcal{L}_{(4)y} + \mathcal{L}_{(4)z}$$
$$\begin{aligned}\mathcal{L}_{(4)y} = {} & y_1\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu\hat{\alpha}_{\perp\nu}\hat{\alpha}_\perp^\nu\right] + y_2\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_\perp^\mu\hat{\alpha}_\perp^\nu\right] + y_3\mathrm{Tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\parallel^\nu\right] + y_4\mathrm{Tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_\parallel^\nu\right] \\ & + y_5\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\parallel^\nu\right] + y_6\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_\parallel^\nu\right] + y_7\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_\parallel^\nu\hat{\alpha}_\parallel^\mu\right] \\ & + y_8\left\{\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_{\perp\nu}\hat{\alpha}_\parallel^\nu\right] + \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\perp^\nu\hat{\alpha}_\parallel^\mu\right]\right\} + y_9\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\perp^\mu\hat{\alpha}_\parallel^\nu\right],\end{aligned}$$
$$\mathcal{L}_{(4)z} = iz_4\mathrm{Tr}\left[V_{\mu\nu}\hat{\alpha}_\perp^\mu\hat{\alpha}_\perp^\nu\right] + iz_5\mathrm{Tr}\left[V_{\mu\nu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_\parallel^\nu\right].$$

$$\mathcal{L}_{\mathrm{anom}} = \frac{N_c}{16\pi^2}\sum_{i=1}^{3} c_i\mathcal{L}_i,$$
$$\mathcal{L}_1 = i\,\mathrm{Tr}\left[\hat{\alpha}_{\mathrm{L}}^3\hat{\alpha}_{\mathrm{R}} - \hat{\alpha}_{\mathrm{R}}^3\hat{\alpha}_{\mathrm{L}}\right],$$
$$\mathcal{L}_2 = i\,\mathrm{Tr}\left[\hat{\alpha}_{\mathrm{L}}\hat{\alpha}_{\mathrm{R}}\hat{\alpha}_{\mathrm{L}}\hat{\alpha}_{\mathrm{R}}\right],$$
$$\mathcal{L}_3 = \mathrm{Tr}\left[F_{\mathrm{V}}\left(\hat{\alpha}_{\mathrm{L}}\hat{\alpha}_{\mathrm{R}} - \hat{\alpha}_{\mathrm{R}}\hat{\alpha}_{\mathrm{L}}\right)\right],$$

Parameters (17):

f. , g, a, y_1 ･ y_9, z_4, z_5, c_1 ･ c_3

- M. Harada and K. Yamawaki, Phys. Rep. 381, 1 (2003).

Parameterizations and B.C.s:

$$\xi(\boldsymbol{r}) = \exp\left[i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\frac{F(r)}{2}\right]$$

$$\omega_\mu = W(r)\delta_{0\mu}, \quad \rho_0 = 0, \quad \boldsymbol{\rho} = \frac{G(r)}{gr}(\hat{\boldsymbol{r}}\times\boldsymbol{\tau})$$

$$\begin{aligned} F(0) &= \pi, & F(\infty) &= 0,\\ G(0) &= -2, & G(\infty) &= 0,\\ W'(0) &= 0, & W(\infty) &= 0. \end{aligned}$$

Collective rotation, field excitations and B.C.s:

$$\begin{aligned} \xi(\boldsymbol{r}) &\to \xi(\boldsymbol{r},t) = A(t)\,\xi(\boldsymbol{r})A^\dagger(t),\\ V_\mu(\boldsymbol{r}) &\to V_\mu(\boldsymbol{r},t) = A(t)\,V_\mu(\boldsymbol{r})A^\dagger(t), \end{aligned}$$

$$i\boldsymbol{\tau}\cdot\boldsymbol{\Omega} \equiv A^\dagger(t)\partial_0 A(t).$$

$$\begin{aligned} \rho^0(\boldsymbol{r},t) &= A(t)\frac{2}{g}\left[\boldsymbol{\tau}\cdot\boldsymbol{\Omega}\,\xi_1(r) + \hat{\boldsymbol{\tau}}\cdot\hat{\boldsymbol{r}}\,\boldsymbol{\Omega}\cdot\hat{\boldsymbol{r}}\,\xi_2(r)\right]A^\dagger(t),\\ \omega^i(\boldsymbol{r},t) &= \frac{\varphi(r)}{r}\left(\boldsymbol{\Omega}\times\hat{\boldsymbol{r}}\right)^i, \end{aligned} \qquad (21$$

$$\begin{aligned} \xi_1'(0) &= \xi_1(\infty) = 0,\\ \xi_2'(0) &= \xi_2(\infty) = 0,\\ \varphi(0) &= \varphi(\infty) = 0, \end{aligned}$$

$$M = M_{\rm sol} + \frac{i(i+1)}{2\mathcal{I}} = M_{\rm sol} + \frac{j(j+1)}{2\mathcal{I}}$$

In order to intuitively see the effects of the vector mesons on the Skyrmion size in a simple way, here we consider the winding number and energy root mean square radii.

Winding number radius

$$\langle r^2 \rangle_W^{1/2} = \left[\int_0^\infty d^3 r r^2 B^0(r) \right]^{1/2}$$

$$B^0 = -\frac{1}{2\pi^2} \sin^2 F \frac{F'}{r^2}.$$

Energy root mean square radius

$$\langle r^2 \rangle_E^{1/2} = \left[\frac{1}{M_{\rm sol}} \int_0^\infty d^3 r r^2 M_{\rm sol}(r) \right]^{1/2},$$

Soliton mass (energy) density

HLS from hQCD models

- M. Harada, S. Matsuzaki, and K. Yamawaki, Phys. Rev. D 82, 076010 (2010).

$$S_5 = S_5^{\mathrm{DBI}} + S_5^{\mathrm{CS}}$$

$$S_5^{\mathrm{DBI}} = N_c G_{\mathrm{YM}} \int d^4x dz \Big\{ -\frac{1}{2} K_1(z) \mathrm{Tr}\left[\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right] + K_2(z) M_{KK}^2 \mathrm{Tr}\left[\mathcal{F}_{\mu z}\mathcal{F}^{\mu z}\right] \Big\},$$

$$S_5^{\mathrm{CS}} = \frac{N_c}{24\pi^2} \int_{M^4\times R} w_5(A)$$

$$w_5(A) = \mathrm{Tr}\left[\mathcal{A}\mathcal{F}^2 + \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right]$$

$A_z = 0$ gauge
Mode expansion

$$A_\mu^{\mathrm{integ}}(x,z) = \hat{\alpha}_{\mu\perp}(x)\psi_0(z) + \left[\hat{\alpha}_{\mu\|}(x) + V_\mu(x)\right] + \hat{\alpha}_{\mu\|}(x)\psi_1(z),$$

$$-K_1(z)\partial_z\left[K_2(z)\partial_z\psi_n(z)\right] = \lambda_n\psi_n(z),$$

16 parameters in terms of
2 parameters: G_{YM}, M_{KK}
Inputs: $f_{\cdot}$, $m_{\cdot\cdot}$

$$f_\pi^2 = N_c G_{\mathrm{YM}} M_{KK}^2 \int dz K_2(z)\left[\dot{\psi}_0(z)\right]^2,$$

$$a f_\pi^2 = N_c G_{\mathrm{YM}} M_{KK}^2 \lambda_1 \langle\psi_1^2\rangle,$$

$$\frac{1}{g^2} = N_c G_{\mathrm{YM}} \langle\psi_1^2\rangle,$$

$$y_1 = -y_2 = -N_c G_{\mathrm{YM}} \left\langle \left(1+\psi_1-\psi_0^2\right)^2 \right\rangle,$$

$$y_3 = -y_4 = -N_c G_{\mathrm{YM}} \left\langle \psi_1^2 \left(1+\psi_1\right)^2 \right\rangle,$$

$$y_5 = 2y_8 = -y_9 = -2N_c G_{\mathrm{YM}} \left\langle \psi_1^2\psi_0^2 \right\rangle,$$

$$y_6 = -\left(y_5 + y_7\right),$$

$$y_7 = 2N_c G_{\mathrm{YM}} \left\langle \psi_1\left(1+\psi_1\right)\left(1+\psi_1-\psi_0^2\right) \right\rangle,$$

$$z_4 = 2N_c G_{\mathrm{YM}} \left\langle \psi_1\left(1+\psi_1-\psi_0^2\right) \right\rangle,$$

$$z_5 = -2N_c G_{\mathrm{YM}} \left\langle \psi_1^2\left(1+\psi_1\right) \right\rangle,$$

$$c_1 = \left\langle\!\left\langle \dot{\psi}_0\psi_1\left(\frac{1}{2}\psi_0^2 + \frac{1}{6}\psi_1^2 - \frac{1}{2}\right) \right\rangle\!\right\rangle,$$

$$c_2 = \left\langle\!\left\langle \dot{\psi}_0\psi_1\left(-\frac{1}{2}\psi_0^2 + \frac{1}{6}\psi_1^2 + \frac{1}{2}\psi_1 + \frac{1}{2}\right) \right\rangle\!\right\rangle,$$

$$c_3 = \left\langle\!\left\langle \frac{1}{2}\dot{\psi}_0\psi_1^2 \right\rangle\!\right\rangle,$$

- Physical quantities are parameter “a” independent.
- CS terms are responsible for the omega meson repulsive interaction.

- YM, G. –S. Yang, Y. Oh, M. Harada, e-Print: **arXiv:1209.3554 [hep-ph] .**
- YM, Y. Oh, G.-S. Yang, M. Harada, H. K. Lee, B. –Y. Park, M. Rho, **Phys.Rev. D86 (2012) 074025**.

Skyrme parameter from hQCD models

Original Skyrme model

$$\mathcal{L}_{\rm Sk} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{Tr}\left[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger\right]^2$$

Integrate out all vector modes

$$\begin{aligned}\mathcal{L}_{\rm ChPT} = {} & f_\pi^2 \mathrm{Tr}\left[\alpha_{\perp\mu}\alpha_\perp^\mu\right] \\ & + \left(\frac{1}{2g^2} - \frac{z_4}{2} - \frac{y_1 - y_2}{4}\right)\mathrm{Tr}\left[\alpha_{\perp\mu}, \alpha_{\perp\nu}\right]^2 \\ & + \frac{y_1 + y_2}{4}\mathrm{Tr}\left\{\alpha_{\perp\mu}, \alpha_{\perp\nu}\right\}^2,\end{aligned}$$

$$\frac{1}{2e^2} = \frac{1}{2g^2} - \frac{z_4}{2} - \frac{y_1 - y_2}{4}$$

$$\frac{1}{2e^2} = \frac{N_c G_{\rm YM}}{2}\left\langle(1-\psi_0^2)^2\right\rangle$$

$e \simeq 7.31$ SS model

$e \simeq 10.02$ Flat space

- Both values are larger than that used in the original Skyrme model because of the contributions from y1, y2, and z4 terms at O(p4).
- Skyrme parameter value depends on geometry .
- Moment of inertia ・ $1/e^3$ in the Skyrme model, larger value of e, smaller moment of inertia ・ larger ・-N mass splitting.

Numerical results:

In hQCD models, the mass scale M_{KK} and the 't Hooft coupling G_{YM} are free parameters.

$$\begin{aligned} m_\rho &= 775.49 \text{ MeV}, \\ f_\pi &= 92.4 \text{ MeV}. \end{aligned}$$

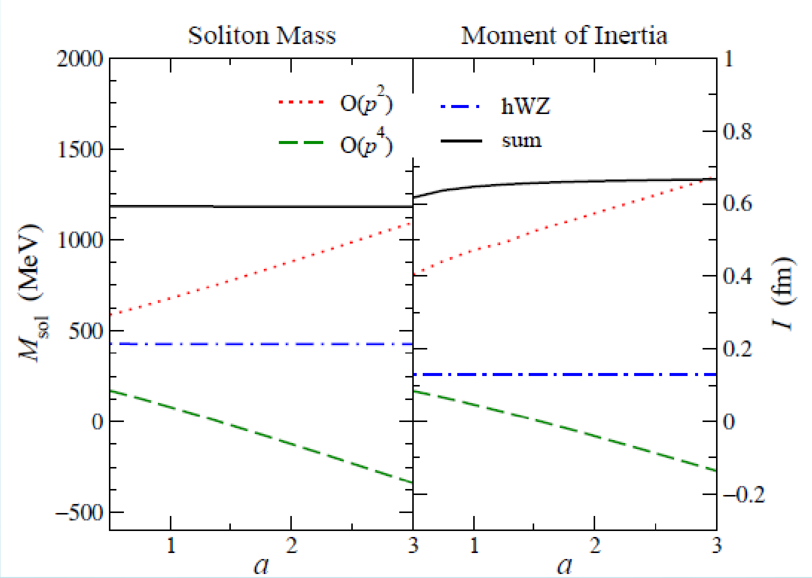
Three versions of HLS:

1. HLS including ▪, ▪, ▪.
2. HLS without hWZ.
3. Skyrme model from HLS

Two hQCD models:

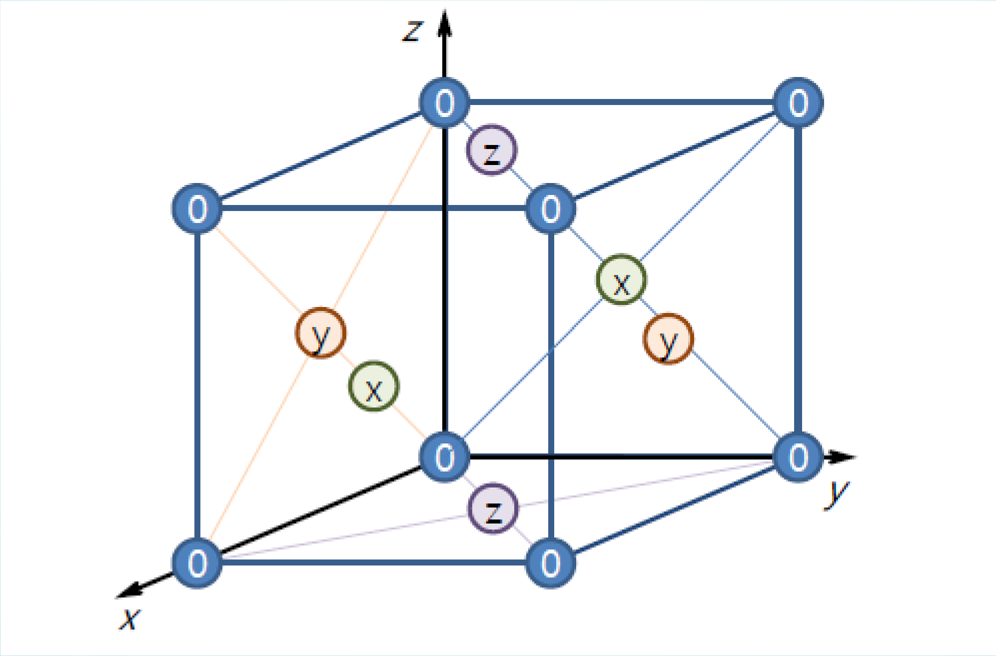
1. SS model (HLS_1): $K_1(z) = (1 + z^2)^{-1/3}$, $K_2(z) = 1 + z^2$;
   - Sakai & Sugimoto, PTP113(2005)843,114(2005)1083.
2. Flat space (BPS): $K_1(z) = K_2(z) = 1$.

	$HLS_1(\pi,\rho,\omega)$	$HLS_1(\pi,\rho)$	$HLS_1(\pi)$
M_{sol}	1184	834	922
Δ_M	448	1707	1014
$\sqrt{\langle r^2\rangle_W}$	0.433	0.247	0.309
$\sqrt{\langle r^2\rangle_E}$	0.608	0.371	0.417



- Lessons from nuclear physics indicates that, the omega meson brings repulsive interaction which prevents the nuclei from collapsing and the sigma meson brings attractive interaction and the near cancellation of the these to interactions gives the small binding energy of nuclear matter. Sigma meson is important!

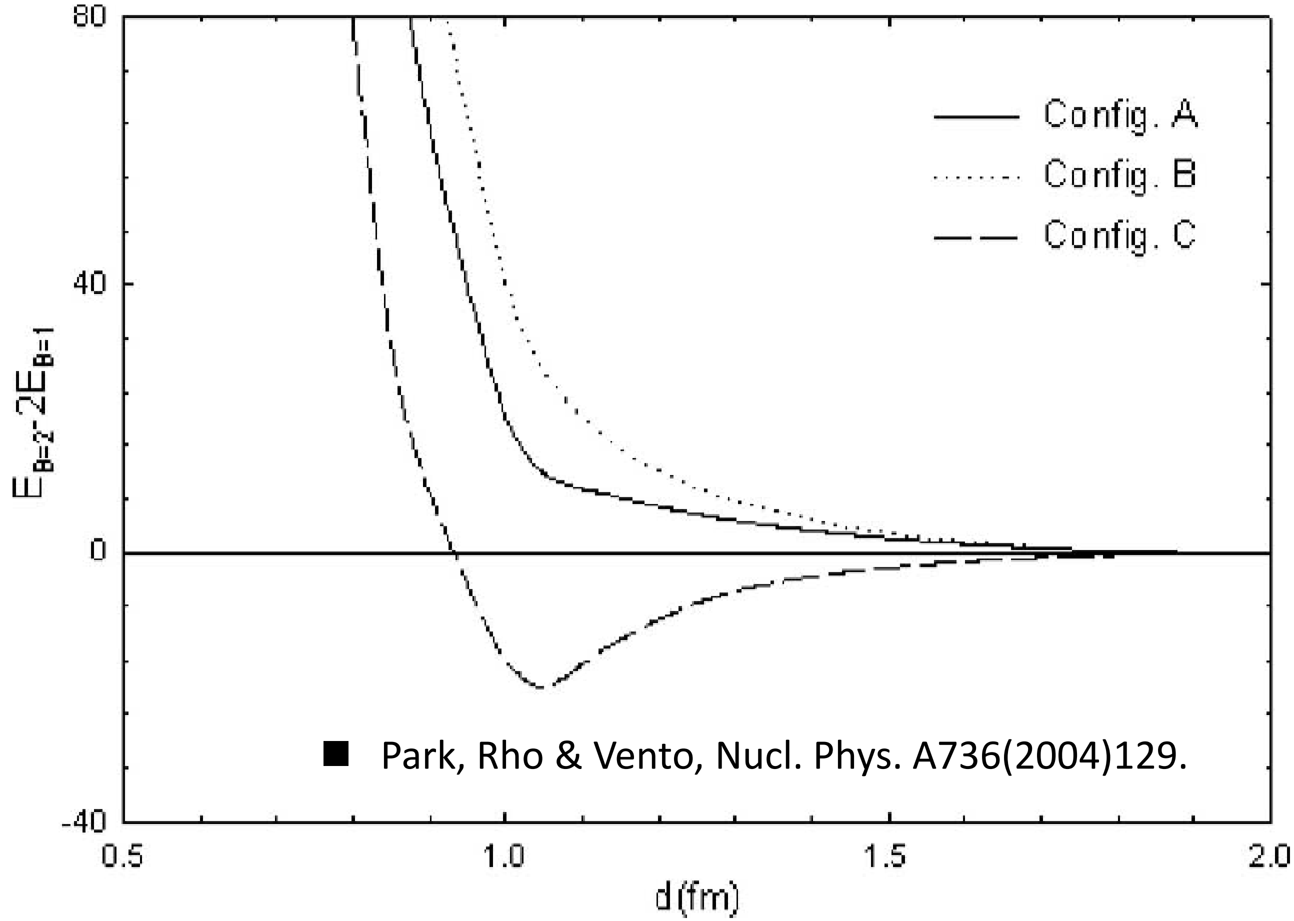
IV. Dense baryonic Matter



- YM, Y. Oh, G.-S. Yang, M. Harada, H. K. Lee, B. –Y. Park, M. Rho, **in preparation**.

Product Ansatz

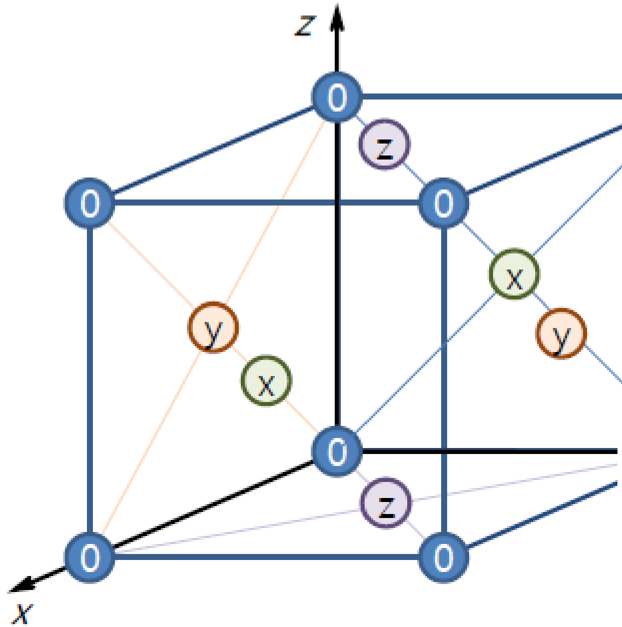
$$U(\vec{x}) = AU(\vec{x}-\vec{x}_1)A^{\dagger}BU(\vec{x}\,\cdot$$



A) no relative rotation.
B) rotated by an angle $\pi/2$ wrt the axis parallel to the line joining the two skyrmions.
C) the angle is π and the rotation axis is perpendicular to the line joining the two skyrmions.

FCC Crystal

- H.-J. Lee, B.-Y. Park, D.-P. Min, M. Rho, V. Vento, Nucl. Phys. A 723 (2003) 427.



$$U(x+L/2,y+L/2,z) = \tau_z\, U(x,y,z)\, \tau_z$$

$$U(x+L/2,y,z+L/2) = \tau_y\, U(x,y,z)\, \tau_y$$

$$U(x,y+L/2,z+L/2) = \tau_x\, U(x,y,z)\, \tau_x$$

$$\bar{\phi}_0^\pi = \sum_{abc} \beta_{abc}^\pi \cos(\pi a x/L)\cos(\pi b y/L)\cos(\pi c z/L),$$

$$\bar{\phi}_1^\pi = \sum_{hkl} \alpha_{hkl}^\pi \sin(\pi h x/L)\cos(\pi k y/L)\cos(\pi l z/L),$$

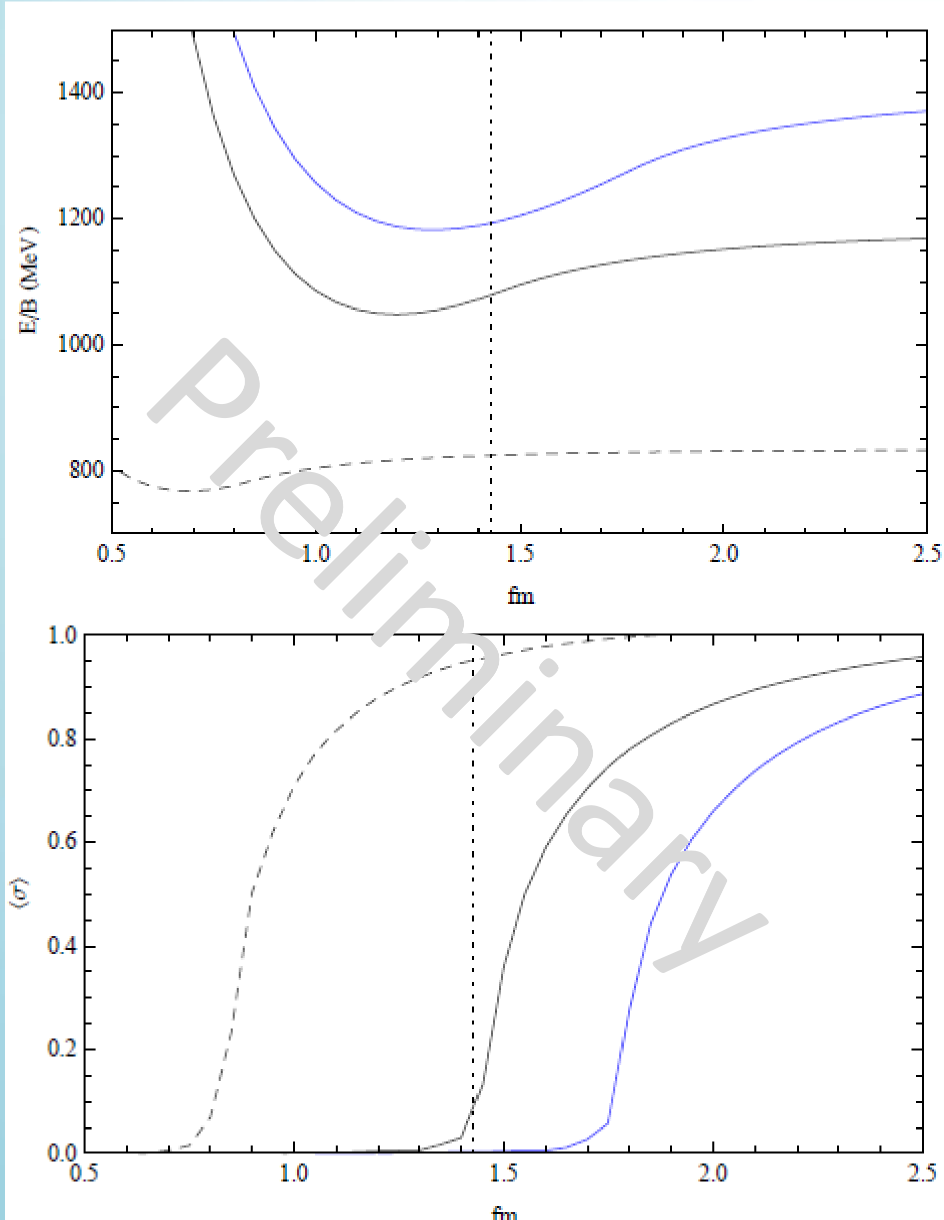
$$\bar{\phi}_2^\pi = \sum_{hkl} \alpha_{hkl}^\pi \cos(\pi l x/L)\sin(\pi h y/L)\cos(\pi k z/L),$$

$$\bar{\phi}_3^\pi = \sum_{hkl} \alpha_{hkl}^\pi \cos(\pi k x/L)\cos(\pi l y/L)\sin(\pi h z/L),$$

- M. Kugler, S. Shtrikman, Phys. Lett. B 208 (1988) 491; Phys. Rev. D 40 (1989) 3421.

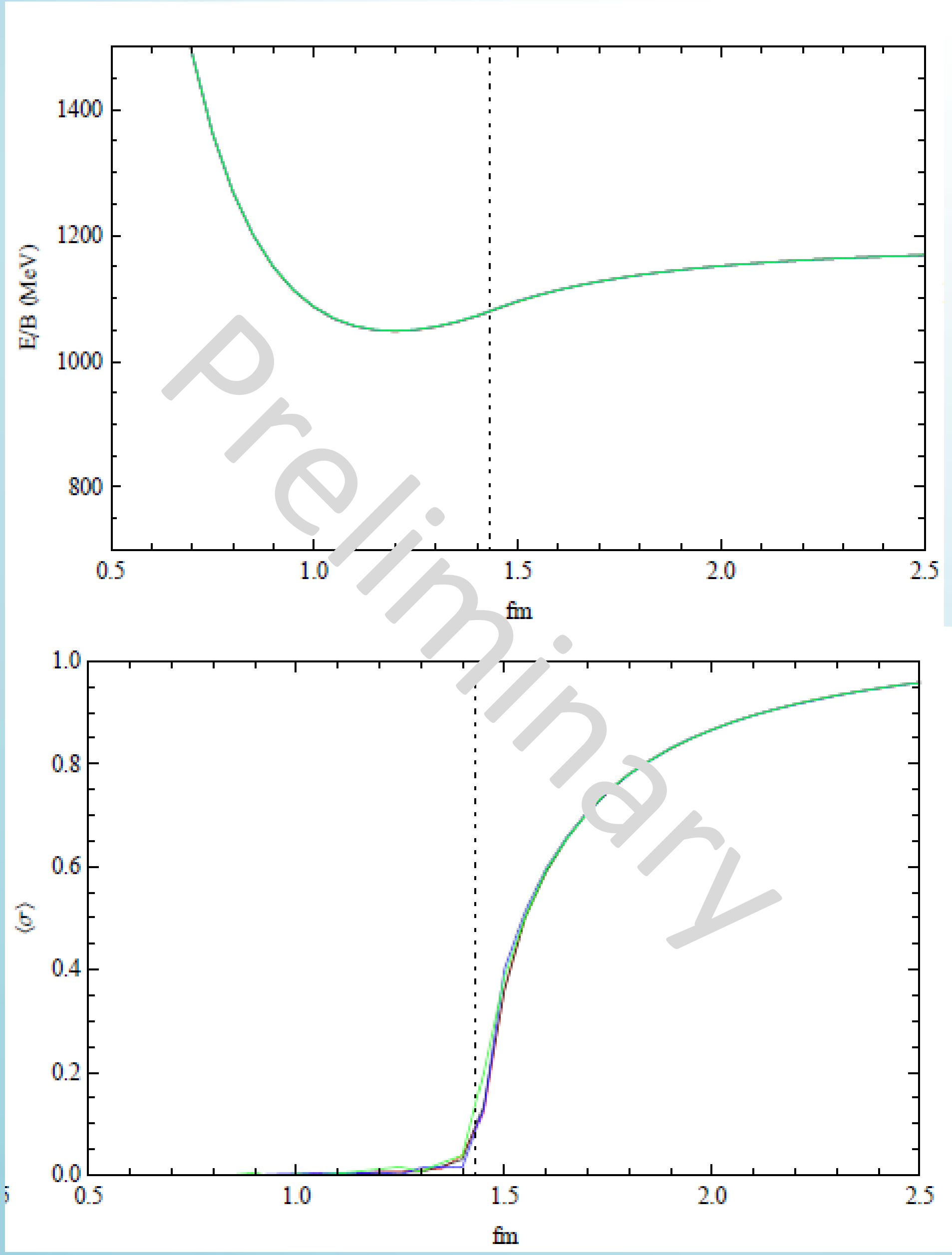
Symmetries of the FCC skyrmion crystal

		Reflection (yz-plane)	3-fold axis rotation	4-fold axis (z-axis) rot.	Translation
(x,y,z)	$\to$	$(-x,y,z)$	(y,z,x)	$(x,z,-y)$	$(x+L,y+L,z)$
$U=\sigma+i\vec{\tau}\cdot\vec{\pi}$	$\to$	$(\sigma,-\pi_1,\pi_2,\pi_3)$	$(\sigma,\pi_2,\pi_3,\pi_1)$	$(\sigma,\pi_1,\pi_3,-\pi_2)$	$(\sigma,-\pi_1,-\pi_2,\pi_3)$
$\rho_i^a \equiv \varepsilon_{aip}\tilde{\rho}_p$	$\to$	$(-\tilde{\rho}_1,\tilde{\rho}_2,\tilde{\rho}_3)$	$(\tilde{\rho}_2,\tilde{\rho}_3,\tilde{\rho}_1)$	$(\tilde{\rho}_1,\tilde{\rho}_3,-\tilde{\rho}_2)$	$(-\tilde{\rho}_1,-\tilde{\rho}_2,\tilde{\rho}_3)$
ω_0,χ	$\to$	ω_0,χ	ω_0,χ	ω_0,χ	ω_0,χ

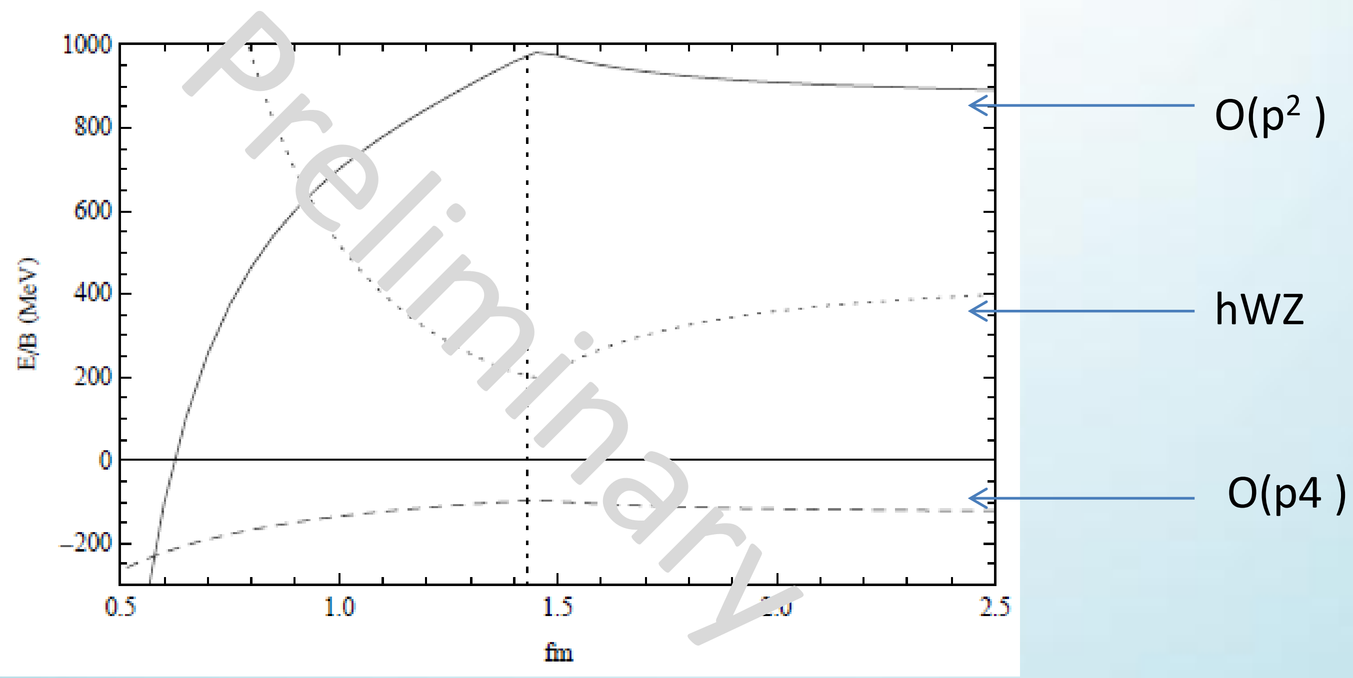


- For HLS(·,·,·) and HLS(·,·) $n_{1/2}$ appears almost simultaneously as the n_{min} of the Skyrmion energy while in the minimal model $n_{1/2}$ appears before n_{min} of the Skyrmion energy.

- For HLS(·,·,·) and HLS(·,·) both $n_{1/2}$ and n_{min} appear after the normal nuclear matter density n_0 while $n_{1/2}$ for the minimal model appear before n_0.



Independent on parameter “a”!



- At high density, the repulsive interaction from the hWZ terms is very strong.
- The $O(p^4)$ contributes only to a attractive interaction which is because, in the HLS reduced from hQCD, there is no omega meson effects in these $O(p^4)$ terms.

V. Lessons and Perspectives

- Baryon and baryonic matter properties is studied using the HLS to O(p^4) (▪,▪,▪). We use a general master formula to determine all the LECs self-consistently from a class of holographic QCD (hQCD) models.

- The hWZ terms in the HLS induced from the CS term in hQCD models are crucial for the inclusion of the omega meson effect in the baryon and baryonic matter properties.

- Analytically shown and numerically checked that baronic matter properties are independent of the parameter "a" in the HLS.

- The results clearly show that both the ▪ meson attractive interaction and the ▪ meson repulsive interaction affects on the Skyrmion properties.

- Since the hQCD model used here is justified in the chiral limit, our study is performed in this limit. Therefore, we can not expect our results to give reliable prediction for all the physical quantities, especially those depend on the quark mass.

- Our results of energy are too large to reproduce the nucleon and mass. Other meson effects?
- Lessons from nuclear physics indicates that, the omega meson brings repulsive interaction which prevents the nuclei from collapsing and the sigma meson brings attractive interaction and the near cancellation of the these to interactions gives the small binding energy of nuclear matter. How to include the sigma meson?

In progress!

- This approach can be straight forwardly applied to TC Skyrmion case.

Holographic + phenomenology

Skyemion parameter e calculable.

In progress!

Thanks !

$$M_{\rm sol} = 4\pi \int dr\left[M_{(2)}(r) + M_{(4)}(r) + M_{\rm anom}(r)\right],$$

$$M_{(2)}(r) = \frac{f_\pi^2}{2}\left(F'^2 r^2 + 2\sin^2 F\right) - \frac{a g^2 f_\pi^2}{2} W^2 r^2 + a f_\pi^2 \left(G + 2\sin^2\frac{F}{2}\right)^2 - \frac{W'^2 r^2}{2} + \frac{G'^2}{g^2} + \frac{G^2}{2g^2 r^2}(G+2)^2, \qquad \text{(A2)}$$

$$\begin{aligned} M_{(4)}(r) = & -y_1 \frac{r^2}{8}\left(F'^2 + \frac{2}{r^2}\sin^2 F\right)^2 - y_2 \frac{r^2}{8} F'^2 \left(F'^2 - \frac{4}{r^2}\sin^2 F\right) - y_3 \frac{r^2}{2}\left[\frac{g^2 W^2}{2} - \frac{1}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right]^2 \\ & - y_4 \frac{g^2 W^2 r^2}{2}\left\{\frac{g^2 W^2}{4} - \frac{1}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right\} + \frac{y_5}{4}\left(r^2 F'^2 + 2\sin^2 F\right)\left[\frac{g^2 W^2}{2} - \frac{1}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right] \\ & + \left(y_8 - \frac{y_7}{2}\right)\frac{\sin^2 F}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2 + y_9 \left\{\frac{g^2 W^2 r^2}{8}\left(F'^2 + \frac{2}{r^2}\sin^2 F\right) + \frac{F'^2}{4}\left(G + 2\sin^2\frac{F}{2}\right)^2\right\} \\ & + z_4 \left\{G' F' \sin F + \frac{\sin^2 F}{2r^2} G(G+2)\right\} + \frac{z_5}{2r^2} G(G+2)\left(G + 2\sin^2\frac{F}{2}\right)^2, \end{aligned} \qquad \text{(A3)}$$

$$\begin{aligned} M_{\rm anom}(r) = & \ \alpha_1 F' W \sin^2 F + \alpha_2 W F' \left(G + 2\sin^2\frac{F}{2}\right)^2 \\ & - \alpha_3 \left\{G(G+2) W F' + 2\sin F\left[W G' - W'\left(G + 2\sin^2\frac{F}{2}\right)\right]\right\}, \end{aligned} \qquad \text{(A4)}$$

$$\alpha_1 = \frac{3gN_c}{16\pi^2}(c_1 - c_2), \qquad \alpha_2 = \frac{gN_c}{16\pi^2}(c_1 + c_2), \qquad \alpha_3 = \frac{gN_c}{16\pi^2}c_3.$$

The Euler-Lagrange equations for $F(r)$ and $G(r)$

$$\mathcal{A}_1 F'' + \mathcal{A}_2 G'' = \mathcal{B},$$
$$\mathcal{A}_3 G'' + \mathcal{A}_4 F'' = \mathcal{D},$$
$$\mathcal{A}_1 = f_\pi^2 r^2 - \frac{3}{2}(y_1 + y_2) r^2 F'^2 - (y_1 - y_2)\sin^2 F + (y_5 + y_9)\frac{g^2 W^2 r^2}{4} - (y_5 - y_9)\frac{1}{2}\left(G + 2\sin^2\frac{F}{2}\right)^2,$$
$$\mathcal{A}_2 = z_4 \sin F,$$
$$\mathcal{A}_3 = 1,$$
$$\mathcal{A}_4 = \frac{g^2}{2} z_4 \sin F,$$

$$
\begin{aligned}
\mathcal{B} = &-2f_\pi^2 rF' + f_\pi^2 \sin 2F + 2af_\pi^2 \sin F\left(G + 2\sin^2\frac{F}{2}\right) + f_\pi^2 m_\pi^2 r^2 \sin F \\
&+ (y_1 + y_2)\, rF'^3 + (y_1 - y_2)\frac{\sin 2F}{2}F'^2 - y_1\frac{\sin 2F}{r^2}\sin^2 F + (y_3 + y_4)\, g^2 W^2\left(G + 2\sin^2\frac{F}{2}\right)\sin F \\
&- y_3\frac{2}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^3 \sin F - (y_5 + y_9)\frac{g^2 rW}{2}(W + rW')\, F' + (y_5 + y_9)\frac{g^2W^2}{4}\sin 2F \\
&+ (y_5 - y_9)\left(G + 2\sin^2\frac{F}{2}\right)\left(G' + \frac{1}{2}\sin F F'\right)F' - y_5\frac{\sin 2F}{2r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2 - y_5\frac{\sin^3 F}{r^2}\left(G + 2\sin^2\frac{F}{2}\right) \\
&+ (2y_8 - y_7)\frac{\sin 2F}{2r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2 + (2y_8 - y_7)\frac{\sin^3 F}{r^2}\left(G + 2\sin^2\frac{F}{2}\right) \\
&+ z_4\frac{\sin 2F}{2r^2}G(G+2) + z_5\frac{G(G+2)}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)\sin F \\
&\quad - \alpha_1 \sin^2 F W' - \alpha_2 W'\left(G + 2\sin^2\frac{F}{2}\right)^2 - 2\alpha_2 W G'\left(G + 2\sin^2\frac{F}{2}\right) \\
&\quad + \alpha_3\left[G(G+2)W' + 2\left(G + 2\sin^2\frac{F}{2}\right)G'W + 2\cos F\left(G + 2\sin^2\frac{F}{2}\right)W' + 2\sin^2 F W'\right],
\end{aligned}
\tag{A11}
$$

$$
\begin{aligned}
\mathcal{D} = &\, ag^2 f_\pi^2\left(G + 2\sin^2\frac{F}{2}\right) + \frac{1}{r^2}G(G+1)(G+2) + y_3 g^2\left[\frac{g^2W^2}{2} - \frac{1}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right]\left(G + 2\sin^2\frac{F}{2}\right) \\
&+ y_4\frac{g^4W^2}{2}\left(G + 2\sin^2\frac{F}{2}\right) - y_5\frac{g^2}{4}\left(F'^2 + \frac{2}{r^2}\sin^2 F\right)\left(G + 2\sin^2\frac{F}{2}\right) + (2y_8 - y_7)\frac{g^2}{2r^2}\sin^2 F\left(G + 2\sin^2\frac{F}{2}\right) \\
&+ y_9\frac{g^2}{4}F'^2\left(G + 2\sin^2\frac{F}{2}\right) - z_4\frac{g^2}{2}\cos F F'^2 + z_4\frac{g^2}{2r^2}\sin^2 F(G+1) \\
&+ z_5\frac{g^2}{2r^2}\left[(G+1)\left(G + 2\sin^2\frac{F}{2}\right) + G(G+2)\right]\left(G + 2\sin^2\frac{F}{2}\right) \\
&+ \alpha_2 g^2 W F'\left(G + 2\sin^2\frac{F}{2}\right) + \alpha_3 g^2\left[2W'\sin F - WF'\left(G + 2\sin^2\frac{F}{2}\right)\right].
\end{aligned}
\tag{A12}
$$

The equation of motion of W reads

$$\begin{aligned}W'' = &-\frac{2}{r}W' + ag^2 f_\pi^2 W + (y_3 + y_4)\, g^2 W \left[\frac{g^2W^2}{2} - \frac{1}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right] \\ &- (y_5 + y_9)\,\frac{g^2W}{4}\left(F'^2 + \frac{2}{r^2}\sin^2 F\right) - \alpha_1 \frac{\sin^2 F}{r^2}F' - \alpha_2 \frac{F'}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2 \\ &+ \frac{\alpha_3}{r^2}\left[\left(G^2 + 2G + 2\sin^2 F\right)F' + 2\cos F\left(G + 2\sin^2\frac{F}{2}\right)F' + 4\sin F G'\right].\end{aligned}$$

$$\mathcal{I} = 4\pi \int dr \left[\mathcal{I}_{(2)}(r) + \mathcal{I}_{(4)}(r) + \mathcal{I}_{\rm anom}(r)\right]$$

$$\begin{aligned}\mathcal{I}_2(r) = {} & \frac{2}{3} f_\pi^2 r^2 \sin^2 F + \frac{1}{3} a f_\pi^2 r^2 \left[(\xi_1 + \xi_2)^2 + 2\left(\xi_1 - 2\sin^2\frac{F}{2}\right)^2\right] \\ & - \frac{1}{6} a g^2 f_\pi^2 \varphi^2 - \frac{1}{6}\left(\varphi'^2 + \frac{2\varphi^2}{r^2}\right) + \frac{r^2}{3g^2}\left(3\xi_1'^2 + 2\xi_1'\xi_2' + \xi_2'^2\right) \\ & + \frac{4}{3g^2} G^2 (\xi_1 - 1)(\xi_1 + \xi_2 - 1) + \frac{2}{3g^2}\left(G^2 + 2G + 2\right)\xi_2^2.\end{aligned}$$

$$\mathcal{I}_{(4)} = \sum_i y_i \mathcal{I}_{y_i} + \sum_i z_i \mathcal{I}_{z_i}.$$

$$\mathcal{I}_{y_1}(r) = -\frac{1}{3} r^2 \sin^2 F \left(F'^2 + \frac{2}{r^2} \sin^2 F \right),$$

$$\mathcal{I}_{y_2}(r) = \frac{1}{3} r^2 \sin^2 F F'^2,$$

$$\mathcal{I}_{y_3}(r) = -\frac{1}{12} g^2 \varphi^2 \left[g^2 W^2 - \frac{4}{r^2} \left(G + 2\sin^2 \frac{F}{2} \right)^2 \right] + \frac{2}{3} g^2 W \varphi \left(G + 2\sin^2 \frac{F}{2} \right) \left(\xi_1 - 2\sin^2 \frac{F}{2} \right)$$
$$+ \left[\frac{1}{2} r^2 g^2 W^2 - \frac{1}{3} \left(G + 2\sin^2 \frac{F}{2} \right)^2 \right] \left[(\xi_1 + \xi_2)^2 + 2 \left(\xi_1 - 2\sin^2 \frac{F}{2} \right)^2 \right],$$

$$\mathcal{I}_{y_4}(r) = \frac{r^2}{2} g^2 W^2 \left[(\xi_1 + \xi_2)^2 + 2 \left(\xi_1 - 2\sin^2 \frac{F}{2} \right)^2 \right] - \frac{1}{12} g^2 W \varphi \left[g^2 W \varphi - 8 \left(G + 2\sin^2 \frac{F}{2} \right) \left(\xi_1 - 2\sin^2 \frac{F}{2} \right.\right.$$
$$+ \frac{1}{3} \left(G + 2\sin^2 \frac{F}{2} \right)^2 \left[\frac{g^2 \varphi^2}{r^2} + (\xi_1 + \xi_2)^2 \right],$$

$$\mathcal{I}_{y_5}(r) = \frac{1}{6} \sin^2 F \left[r^2 g^2 W^2 - 2 \left(G + 2\sin^2 \frac{F}{2} \right)^2 \right]$$
$$- \frac{r^2}{12} \left(F'^2 + \frac{2}{r^2} \sin^2 F \right) \left[2 \left(\xi_1 - 2\sin^2 \frac{F}{2} \right)^2 + (\xi_1 + \xi_2)^2 - \frac{g^2 \varphi^2}{2r^2} \right],$$

$$\mathcal{I}_{y_6}(r) = \frac{1}{6} \sin^2 F \left(r g W - \frac{g\varphi}{2r} \right)^2,$$

$$\mathcal{I}_{y_7}(r) = \frac{1}{6} \sin^2 F \left[\left(r g W - \frac{g\varphi}{2r} \right)^2 + 4 \left(G + 2\sin^2 \frac{F}{2} \right) \left(\xi_1 - 2\sin^2 \frac{F}{2} \right) \right],$$

$$\mathcal{I}_{y_8}(r) = \frac{1}{3} \sin^2 F \left[\left(r g W - \frac{g\varphi}{2r} \right)^2 - 4 \left(G + 2\sin^2 \frac{F}{2} \right) \left(\xi_1 - 2\sin^2 \frac{F}{2} \right) \right],$$

$$\mathcal{I}_{y_9}(r) = \frac{r^2}{6} g^2 W^2 \sin^2 F + \frac{r^2}{6} F'^2 \left(\xi_1 - 2\sin^2 \frac{F}{2} \right)^2$$
$$- \frac{r^2}{12} \left(F'^2 - \frac{2}{r^2} \sin^2 F \right) (\xi_1 + \xi_2)^2 + \frac{1}{24} g^2 \varphi^2 \left(F'^2 + \frac{2}{r^2} \sin^2 F \right),$$

$$\mathcal{I}_{z_4}(r) = \frac{2}{3} \sin^2 F \left[G (1 - \xi_1) + \xi_2 \right] - \frac{2}{3} r^2 \sin F F' \xi_1',$$

$$\mathcal{I}_{z_5}(r) = \frac{2}{3} \left(G + 2\sin^2 \frac{F}{2} \right) \left\{ \left[G (1 - \xi_1 - \xi_2) - \xi_2 \right] (\xi_1 + \xi_2) + \left[G (1 - \xi_1) + \xi_2 \right] \left(\xi_1 - 2\sin^2 \frac{F}{2} \right) \right\},$$

$$
\begin{aligned}
\mathcal{I}_{\rm anom}(r) = &\frac{gN_c}{8\pi^2}\,(c_1 - c_2)\,\varphi F' \sin^2 F \\
&- \frac{gN_c}{24\pi^2}\,(c_1 + c_2)\,\varphi F' \left(G + 2\sin^2\frac{F}{2}\right)\left(\xi_1 - 2\sin^2\frac{F}{2}\right) \\
&+ \frac{gN_c}{24\pi^2} c_3 \left\{ \varphi F' \,(G\xi_1 - G - \xi_1 - 2\xi_2) + \varphi \sin F\,(\xi_1' - G') + \varphi' \sin F \left(G - \xi_1 + 4\sin^2\frac{F}{2}\right) \right\}.
\end{aligned}
$$

Then the equations of motion for ξ_1, ξ_2, and φ are

$$\begin{aligned}
\xi_1'' &= -\frac{2}{r}\xi_1' + ag^2 f_\pi^2 \left(\xi_1 - 2\sin^2\frac{F}{2}\right) + \frac{G^2}{r^2}(\xi_1 - 1) - \frac{2}{r^2}(G+1)\xi_2 + \frac{3g^2}{4r^2}(\mathcal{F}_1 - \mathcal{F}_2) \\
&\quad - \frac{g^3 N_c}{32\pi^2 r^2}(c_1 + c_2)\varphi F' \left(G + 2\sin^2\frac{F}{2}\right) - \frac{g^3 N_c}{32\pi^2 r^2} c_3 \left[2\varphi' \sin F - \varphi F' \left(G + 2\sin^2\frac{F}{2}\right)\right], \\
\xi_2'' &= -\frac{2}{r}\xi_2' + ag^2 f_\pi^2 \left(\xi_2 + 2\sin^2\frac{F}{2}\right) + \frac{G^2}{r^2}(\xi_1 + 2\xi_2 - 1) + \frac{6}{r^2}(G+1)\xi_2 + \frac{3g^2}{4r^2}(3\mathcal{F}_2 - \mathcal{F}_1) \\
&\quad + \frac{g^3 N_c}{32\pi^2 r^2}(c_1 + c_2)\varphi F' \left(G + 2\sin^2\frac{F}{2}\right) + \frac{g^3 N_c}{32\pi^2 r^2} c_3 \left[2\varphi' \sin F - \varphi F' (G + 5 - \cos F)\right], \\
\varphi'' &= \frac{2}{r^2}\varphi + ag^2 f_\pi^2 \varphi - 3\mathcal{F}_3 - \frac{3gN_c}{8\pi^2}(c_1 - c_2) F' \sin^2 F + \frac{gN_c}{8\pi^2}(c_1 + c_2) F' \left(G + 2\sin^2\frac{F}{2}\right)\left(\xi_1 - 2\sin^2\frac{F}{2}\right) \\
&\quad + \frac{gN_c}{8\pi^2} c_3 \left\{2\sin F\,(G' - \xi_1') + F'\left[G(1+\cos F) - \xi_1\left(G - 2\sin^2\frac{F}{2}\right) + 2\xi_2 + 3\sin^2 F - 4\sin^4\frac{F}{2}\right]\right\},
\end{aligned}$$

$$
\begin{aligned}
\mathcal{F}_1 = y_3 & \left\{ \frac{2}{3} g^2 W \varphi \left(G + 2\sin^2 \frac{F}{2} \right) + \left[r^2 g^2 W^2 - \frac{2}{3} \left(G + 2\sin^2 \frac{F}{2} \right)^2 \right] \left(3\xi_1 + \xi_2 - 4\sin^2 \frac{F}{2} \right) \right\} \\
& + y_4 \left\{ r^2 g^2 W^2 \left(3\xi_1 + \xi_2 - 4\sin^2 \frac{F}{2} \right) + \frac{2}{3} g^2 W \varphi \left(G + 2\sin^2 \frac{F}{2} \right) + \frac{2}{3} \left(G + 2\sin^2 \frac{F}{2} \right)^2 (\xi_1 + \xi_2) \right\} \\
& - y_5 \frac{r^2}{6} \left(F'^2 + \frac{2}{r^2} \sin^2 F \right) \left(3\xi_1 + \xi_2 - 4\sin^2 \frac{F}{2} \right) + (y_7 - 2y_8) \frac{2}{3} \sin^2 F \left(G + 2\sin^2 \frac{F}{2} \right) \\
& + y_9 \frac{r^2}{3} \left\{ F'^2 \left(\xi_1 - 2\sin^2 \frac{F}{2} \right) - \frac{1}{2} \left(F'^2 - \frac{2}{r^2} \sin^2 F \right) (\xi_1 + \xi_2) \right\} \\
& + z_4 \frac{2}{3} \left\{ -G \sin^2 F + r^2 \cos F F'^2 + r^2 \sin F \left(F'' + \frac{2}{r} F' \right) \right\} \\
& - z_5 \frac{4}{3} G \left(G + 2\sin^2 \frac{F}{2} \right) \left(1 - 2\xi_1 - \xi_2 + \sin^2 \frac{F}{2} \right),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_2 = y_3 & \left[r^2 g^2 W^2 - \frac{2}{3} \left(G + 2\sin^2 \frac{F}{2} \right)^2 \right] (\xi_1 + \xi_2) + y_4 \left[r^2 g^2 W^2 + \frac{2}{3} \left(G + 2\sin^2 \frac{F}{2} \right)^2 \right] (\xi_1 + \xi_2) \\
& - y_5 \frac{r^2}{6} \left(F'^2 + \frac{2}{r^2} \sin^2 F \right) (\xi_1 + \xi_2) - y_9 \frac{r^2}{6} \left(F'^2 - \frac{2}{r^2} \sin^2 F \right) (\xi_1 + \xi_2) \\
& + z_4 \frac{2}{3} \sin^2 F - z_5 \frac{2}{3} \left(G + 2\sin^2 \frac{F}{2} \right) \left[G(1 - 2\xi_1) - 2(G+1)\xi_2 - 2\sin^2 \frac{F}{2} \right],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_3 = -y_3 \frac{1}{6} g^2 \varphi & \left[g^2 W^2 - \frac{4}{r^2} \left(G + 2\sin^2 \frac{F}{2} \right)^2 \right] + y_3 \frac{2}{3} g^2 W \left(G + 2\sin^2 \frac{F}{2} \right) \left(\xi_1 - 2\sin^2 \frac{F}{2} \right) \\
& - y_4 \frac{1}{6} g^2 W \left[g^2 W \varphi - 4 \left(G + 2\sin^2 \frac{F}{2} \right) \left(\xi_1 - 2\sin^2 \frac{F}{2} \right) \right] + y_4 \frac{2}{3} \frac{g^2 \varphi}{r^2} \left(G + 2\sin^2 \frac{F}{2} \right)^2 \\
& + (y_5 + y_9) \frac{1}{12} g^2 \varphi \left(F'^2 + \frac{2}{r^2} \sin^2 F \right) - (y_6 + y_7 + 2y_8) \frac{g}{6} \sin^2 F \left(gW - \frac{g\varphi}{2r^2} \right).
\end{aligned}
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