SCGT12, Nagoya, 7/12/2012

New Confinement Phases from Singular SQCD Vacua - SCGT

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Basic theme :

Conformal invariance (CFT) and confinement

UV CFT - - - - - - - - - - -> Infrared-fixed point CFT

QCD:

quarks and gluons, AF - - - - - -> collective behavior of color (confinement, XSB) ?

If confinement ~ deformation of an IR f.p. CFT

understanding of the IR degrees of freedom in CFT

is the key to see the working of confinement / XSB

Plan of the Talk

I. Confinement and XSB in QCD, Lessons from SQCD

- singular SCFT and confinement -

II. Recent developments

- Argyres-Seiberg, Gaiotto-Seiberg-Tachikawa -

III. Perturbation of singular SCFT and confinement

- New confinement picture -

I. Quark confinement vs Chiral Symmetry Breaking

- Abelian dual superconductor ? (dynamical Abelianization)

$$SU(3) \to U(1)^2 \to \mathbf{1}$$
$$\langle M \rangle \neq 0$$

't Hooft, Nambu, Mandelstam

☞ Doubling of the spectrum (*)

$$\Pi_1(U(1)^2) = \mathbf{Z} \times \mathbf{Z}$$

If confinement and XSB both induced by $\langle M^a_b \rangle = \delta^a_b \, \Lambda$

$$SU_L(N_F) \times SU_R(N_F) \to SU_V(N_F)$$

☞ Accidental SU($N_F{}^2$) : too many NG bosons (**)

- Non-Abelian monopole condensation

$$SU(3) \to SU(2) \times U(1) \to \mathbf{1}$$

$$\langle M^a \tilde{M}_b \rangle \sim \delta^a_b \, \Lambda^2$$
$$\Pi_1(SU(2) \times U(1)) = \mathbf{Z}$$

☞ Problems (*), (**) avoided but

Non-Abelian monopole are probably strongly coupled (sign flip of b_0 unlikely)

What $\mathcal{N}$=2 SQCD (softly broken) teaches us

- Abelian dual superconductivity ✔

Seiberg, Witten

SU(2) with N_F = 0, 1,2,3

monopole condensation ⇒ confinement & dyn symm. breaking

Beautiful, but don't look like QCD

$$SU(N) \ \mathcal{N} = 2 \ SYM : \ SU(N) \Rightarrow U(1)^{N-1}$$

- Non-Abelian monopole condensation for SQCD ✔

Argyres, Plesser, Seiberg, '96
Hanany-Oz, '96
Carlino-Konishi-Murayama '00

SU(N), N_F quarks

$SU(N) \Rightarrow SU(r) \times U(1) \times U(1) \times \ldots$ $\quad r \leq N_F/2$

r vacua are local, IR free theories

Beautiful, but don't look like QCD

- Non Abelian monopoles interacting very strongly

SCFT of higher criticalities, EHIY points

Beautiful, interesting but difficult

Argyres, Plesser, Seiberg, Witten,
Eguchi-Hori-Ito-Yang, '96

Effective degrees of freedom in the quantum r vacuum of softly broken N=2 SQCD

$(r \leq N_f\ / 2\ )$

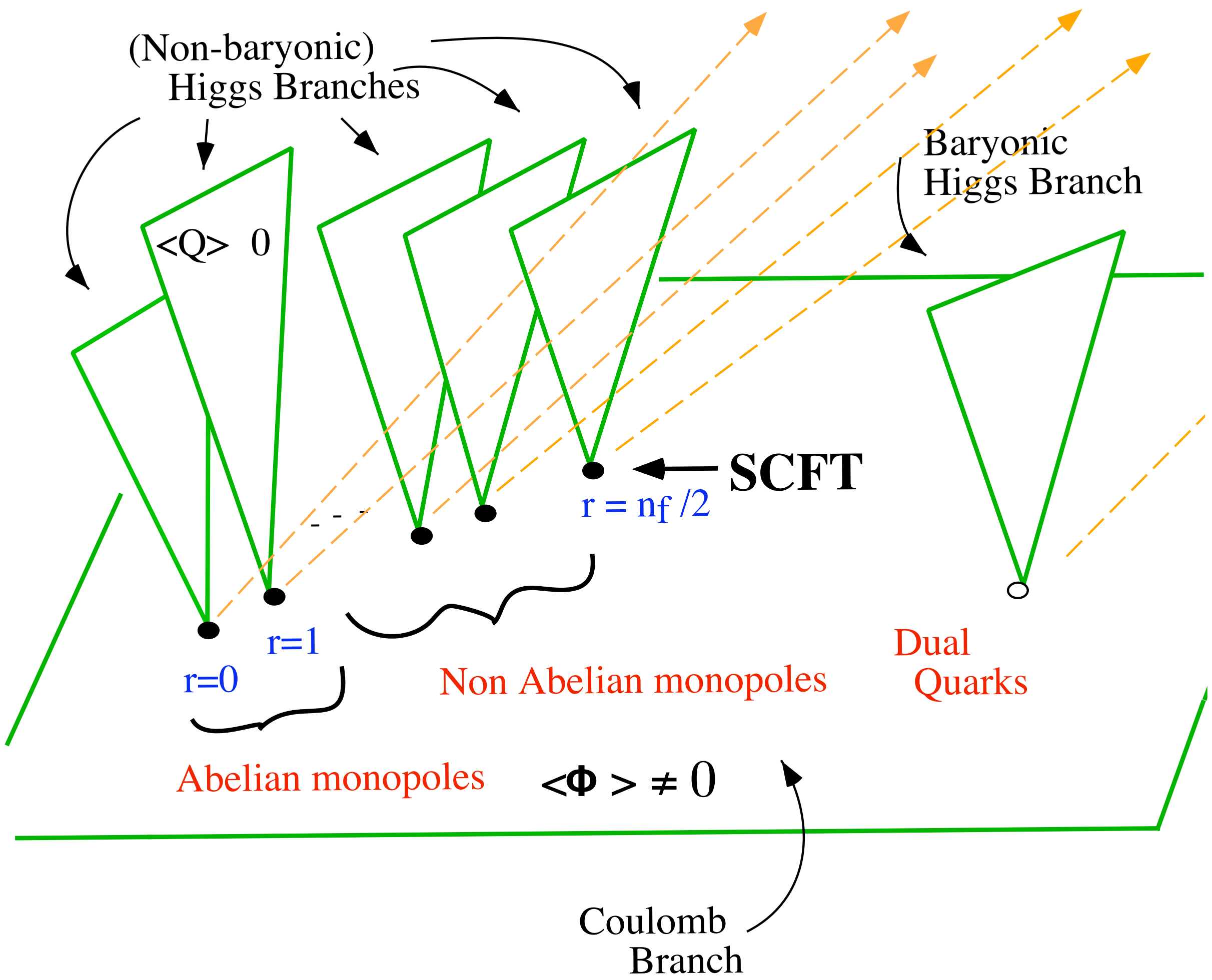
Seiberg-Witten '94
Argyres,Plesser,Seiberg,'96
Hanany-Oz, '96
Carlino-Konishi-Murayama '00

	$SU(r)$	$U(1)_0$	$U(1)_1$	$\dots$	$U(1)_{N-r-1}$	$U(1)_B$
$N_F \times \mathcal{M}$	$\underline{\mathbf{r}}$	1	0	$\dots$	0	0
M_1	$\underline{\mathbf{1}}$	0	1	$\dots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
M_{N-r-1}	$\underline{\mathbf{1}}$	0	0	$\dots$	1	0

The massless non-Abelian and Abelian monopoles and their charges at the r vacua

- "Colored dyons" do exist !!!
- they carry flavor q.n.
- $\langle q^i{}_\alpha \rangle = v\ \delta^i{}_\alpha \Rightarrow \quad U(N_f) \rightarrow U(r) \times U(N_f - r)$

QMS of N=2 SQCD (SU(n) with n_f quarks)



- N=1 Confining vacua (with Φ^2 perturbation)

○ N=1 vacua (with Φ^2 perturbation) in free magnetic pha

$m = m^{cr}$

next
slide

Di Pietro, Giacomelli '11

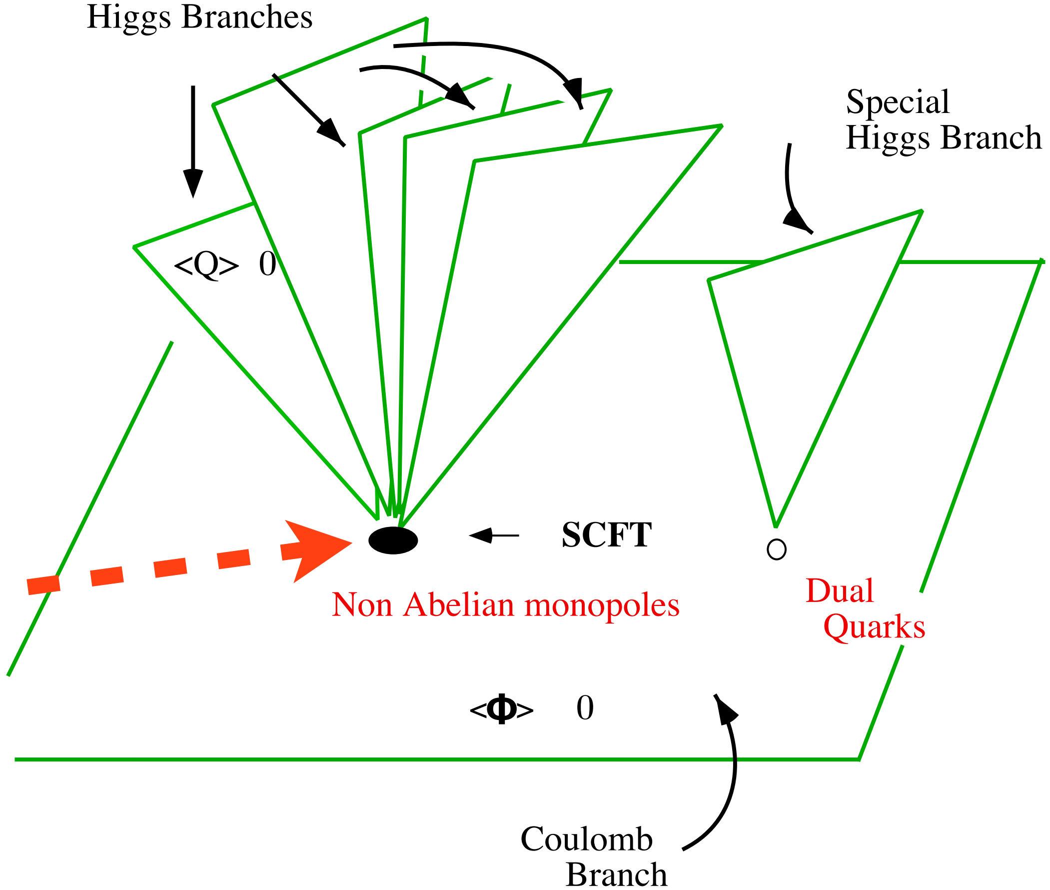
previous slide (Universality)

$m \neq 0$

QMS of N=2 USp(2n) Theory with n_f Quarks

(m = 0)

SCFT of
highest criticality
EHIY point
non-Lagrangian



Carlino-Konishi-Murayama '00

• N=1 Confining vacua (with Φ^2 perturbation)

○ N=1 vacua (with Φ^2 perturbation) in free magnetic pha

II. Recent key developments

N=2 SCFT's

Witten,Gaiotto, Seiberg, Argyres, Tachikawa, Moore, Maruyoshi, ... '09 - '12

- S-duality in SCFT at $g = \infty$ e.g. SU(N) w/ $N_F = 2N$

Argyres-Seiberg '07

- Argyres-Seiberg S-duality applied to SCFT (IR f.p.) of highest criticality (EHIY points)

Gaiotto-Seiberg-Tachikawa '11

- GST duality generalized to USp(2N), SO(N)

Giacomelli '12

- Colliding r-vacua and EHIY in SU(N)

Giacomelli, Di Pietro '11

- GST duality in USp(2N) and SU(3), N_F =4 and confinement

Giacomelli, Konishi '12 and in preparation

Argyres-Seiberg's S duality

- SU(3) with $N_F = 6$ hypermultiplets ($Q_i, \tilde{Q}_i$ ' s) at infinite coupling

Minahan-Nemeschansky '96

$$SU(3)\ w/\ (6 \cdot \mathbf{3} \oplus \bar{\mathbf{3}}) \quad = \quad SU(2)\ w/\ (2 \cdot \mathbf{2} \oplus \mathrm{SCFT}_{E_6})$$

g = ∞ g = 0

$$SU(2) \times SU(6) \subset E_6$$

Flavor symmetry ~ SU(6) x U(1)

- USp(4) with $N_F = 12$ Q' s at infinite coupling

$$USp(4)\ w/\ 12 \cdot \mathbf{4} = SU(2)\ w/\ SCFT_{E_7}$$

$$SU(2) \times SO(12) \subset E_7$$

Gaiotto-Seiberg-Tachikawa (GST)

- Apply the basic idea of Argyres-Seiberg duality to the IR f.p. SCFT
- SU(N) with $N_F = 2n$:

$$y^2 = (x^N + u_1 x^{N-1} + u_2 x^{N-2} + \cdots + u_N)^2 - \Lambda^{2N-2n} \prod_{i=1}^{2n} (x + m_i)$$

At $u = m = 0$, $y^2 \sim x^{N+n}$ (EHIY point)

relatively non-local
massless monopoles and dyons

Note

Eguchi-Hori-Ito-Yang

- Straightforward treatment of fluctuations around u=m=0, gives an incorrect scaling laws for the masses

❀ To get the correct scale-invariant fluctuations, introduce two different scalings:

$$u_{N-n+2} \sim O(\epsilon_A^2), \quad u_{N-n+3} \sim O(\epsilon_A^3), \quad \ldots, \quad u_N \sim O(\epsilon_A^n).$$

$$u_1 \sim O(\epsilon_B), \quad u_2 \sim O(\epsilon_B^2), \quad \ldots, \quad u_{N-n+2} \sim O(\epsilon_B^{N-n+2}).$$

$$\epsilon_A^2 = \epsilon_B^{N-n+2} \qquad \square \Rrightarrow$$

$$a_i = \oint_{\alpha_i} \lambda, \quad a_{D\,i} = \oint_{\beta_i} \lambda$$

$$\lambda \sim dx\, y / x^n$$

$$m_{(n_m, n_e, n_i)} = \sqrt{2}\,|n_m a_D + n_e a + n_i m_i|$$

☞

Gaiotto-Seiberg-Tachikawa '11

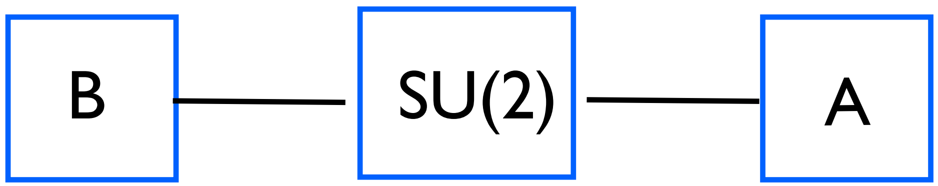
- $U(1)^{N-n-1}$ gauge multiplets

- SU(2) gauge multiplet (infrared free) coupled to the SU(2) flavor symmetry of the two SCFT's A & B

$$b_0 = \frac{N-n}{N-n+2}$$

- The A sector: the SCFT entering in the Argyres-Seiberg dual of SU(n), N_F =2 n, having SU(2)xSU(2n) flavor symmetry

- The B sector: the maximally singular SCFT of the SU(N-n+1) theory with two flavors



where

A: 3 free <u>2</u>'s (n=2); E_6 of Minahan-Nemescahnsky (n=3), etc.

B: the maximally singular SCFT of SU(2), N_F =2 (Seinberg-Witten) for N=3, n=2 , etc.

- Analogous results for USp(2N), SO(N)

Giacomelli '12

III. GST duals and confinement

Note

USp(2N) theory w/ $N_F = 2n$

- Two Tchebyshev* vacua ($\phi_1 = \phi_2 = \ldots = 0$; $\phi_n^{\,2} = \pm \Lambda^2$; ϕ_m det'd by Tch. polynom.)

$$xy^2 \sim \left[x^n(x-\phi_n^2)\right]^2 - 4\Lambda^4 x^{2n} = x^{2n}(x-\phi_n^2-2\Lambda^2)(x-\phi_n^2+2\Lambda^2).$$

$y^2 \sim x^{2n}$ singular SCFT (EHIY point);
strongly interacting, relatively non-local monopoles and dyons

- A strategy: resolve the vacuum by adding small $m_i \neq 0$ and determining the vacuum moduli (u_i's or ϕ_i's) requiring the SW curve to factorize in maximally Abelian factors (double factors) (i.e., Vacua in confinement phase surviving N=1, $\mu\,\Phi^2$ perturbation)

Carlino-Konishi-Murayama '00

□ ➔

$$\binom{N_f}{0} + \binom{N_f}{2} + \ldots \binom{N_f}{N_f} = 2^{N_f-1}$$

even r vacua, from one of the Tcheb. vacua

$$\binom{N_f}{1} + \binom{N_f}{3} + \ldots \binom{N_f}{N_f-1} = 2^{N_f-1}$$

odd r vacua, from one of the Tcheb. vacua

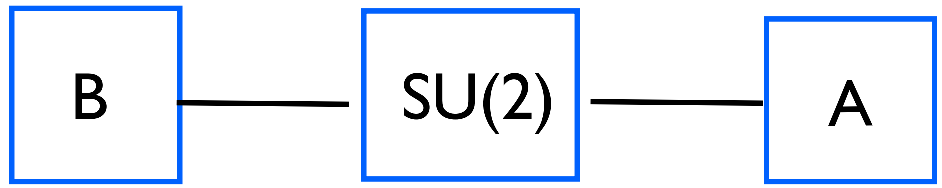
$$xy^2 = \left[x\prod_{a=1}^{n_c-1}(x-\phi_a^2)(x-2\Lambda^2-\beta) + 2\Lambda^2 m_1\cdots m_{n_f}\right]^2 - 4\Lambda^4\prod_{i=1}^{n_f}(x+m_i^2)$$

$$\xrightarrow{\phi_i\text{'s}} \quad xy^2 = x(x-4\Lambda^2-\gamma)\prod_{a=1}^{n_c}(x-\alpha_a)^2,$$

GST dual for the Tchebyshev point of USp(2N) (also SO(N))

Giacomelli '12

$$y^2 \sim x^{2n} \qquad \approx$$



- $U(1)^{N-n}$ gauge multiplets

- The A sector: a (in general) non-Lagrangian SCFT having SU(2)xSO(4n) flavor symmetry

- The B sector: a free doublet (coupled to U(1) gauge boson)

For N_F = 2n= 4, A sector ~ 4 free doublets

But this allows a direct description of IR physics !!

For $USp(2N), \; N_f = 4$

the GST dual is (both the A and B sectors are free doublets) :

Giacomelli,Konishi '12

$$\boxed{1} - SU(2) - \boxed{4}\,.$$

U(1)

the effects of m_i and $\mu\,\Phi^2$ perturbation can be studied from the superpotential:

$$\sqrt{2}\,Q_0(A_D + m_0)\tilde{Q}^0 + \sqrt{2}\,Q_0\phi\tilde{Q}^0 + \sum_{i=1}^{4}\sqrt{2}\,Q_i\phi\tilde{Q}^i + \mu A_D\Lambda + \mu\,\mathrm{Tr}\phi^2 + \sum_{i=1}^{4} m_i\,Q_i\tilde{Q}^i\,.$$

cfr. UV Lagrangian:

$$W = \mu\mathrm{Tr}\Phi^2 + \frac{1}{\sqrt{2}}Q_a^i\Phi_b^a Q_c^i\;J^{bc} + \frac{m_{ij}}{2}Q_a^i Q_b^j\;J^{ab}$$

$$m = -i\sigma_2 \otimes \mathrm{diag}\,(m_1, m_2, \ldots, m_{n_f})\,.$$

Correct flavor symmetry for all {m}

- $m_i = m$: SU(4) x U(1) ;
- $m_i = 0$: SO(8) ; etc.,

vacuum equations

$$\sqrt{2}\, Q_0 \tilde{Q}_0 + \mu\Lambda = 0 \;;$$

$$(\sqrt{2}\,\phi + A_D + m_0)\tilde{Q}_0 = Q_0\,(\sqrt{2}\,\phi + A_D + m_0) = 0 \;;$$

$$\sqrt{2}\;\left[\frac{1}{2}\sum_{i=1}^{4} Q_i^a \tilde{Q}_b^i - \frac{1}{4}\delta_b^a Q_i \tilde{Q}^i + \frac{1}{2} Q_0^a \tilde{Q}_b^0 - \frac{1}{4}\delta_b^a Q_0 \tilde{Q}^0\right] + \mu\,\phi_b^a = 0 \;;$$

$$(\sqrt{2}\,\phi + m_i)\,\tilde{Q}^i = Q_i\,(\sqrt{2}\,\phi + m_i) = 0, \quad \forall i \;.$$

Solutions

$$Q_0 = \tilde{Q}_0 = \begin{pmatrix} 2^{-1/4}\sqrt{-\mu\Lambda} \\ 0 \end{pmatrix}$$

four solutions

$$a = -\frac{m_i}{\sqrt{2}}, \qquad Q_i = \tilde{Q}_i = \begin{pmatrix} f_i \\ 0 \end{pmatrix}; \qquad Q_j = \tilde{Q}_j = 0, \quad j \neq i.$$

four more solutions

$$a = +\frac{m_i}{\sqrt{2}}, \qquad Q_i = \tilde{Q}_i = \begin{pmatrix} 0 \\ g_i \end{pmatrix}; \qquad Q_j = \tilde{Q}_j = 0, \quad j \neq i.$$

$$f_i^2 = \frac{\mu\Lambda - 4\,a}{\sqrt{2}} = \mu(\frac{\Lambda}{\sqrt{2}} + 2m_i).$$

They are 4 + 4 , r=1 vacua ! (p. -2)

$$g_i^2 = \frac{-\mu\Lambda + 4\,a}{\sqrt{2}} = -\mu(\frac{\Lambda}{\sqrt{2}} - 2m_i).$$

But where are the even r-vacua (r=0,2) ???

Answer: in the second Tchebyshev vacuum:

$$\sqrt{2}\,Q_0(A_D+m_0)\tilde{Q}^0+\sqrt{2}\,Q_0\phi\tilde{Q}^0+\sum_{i=1}^{4}\sqrt{2}\,Q_i\phi\tilde{Q}^i+\mu A_D\Lambda+\mu\,\mathrm{Tr}\phi^2+\sum_{i=1}^{4}\tilde{m}_i\,Q_i\tilde{Q}^i$$

with

$$\tilde{m}_1=\frac{1}{4}(m_1+m_2-m_3-m_4)\;;$$
$$\tilde{m}_2=\frac{1}{4}(m_1-m_2+m_3-m_4)\;;$$
$$\tilde{m}_3=\frac{1}{4}(m_1-m_2-m_3+m_4)\;;$$
$$\tilde{m}_4=m_0=\frac{1}{4}(m_1+m_2+m_3+m_4)\;;$$

Spinor representation of $SO(2N_F)$ ...

Magnetic Monopoles!

GST duality is not electromagnetic duality

Flavor symmetry OK in all cases:

m_i	$\tilde{m}_i$	Symmetry in UV	Symmetry in IR
$m_i=0$	$\tilde{m}_i=0$	$SO(8)$	$SO(8)$
$m_i=m\neq 0$	$\tilde{m}_4,\ \tilde{m}_1=\tilde{m}_2=\tilde{m}_3=0$	$U(1)\times SU(4)$	$U(1)\times SO(6)$
$m_1=m_2,\ m_3,\ m_4$, generic	$\tilde{m}_2=-\tilde{m}_3,\ \tilde{m}_4,\tilde{m}_1$ generic	$U(1)\times U(1)\times U(2)$	$U(1)\times U(1)\times U(2)$
$m_1=m_2,\ m_3=m_4,\ m_1\neq m_3$	$\tilde{m}_2=\tilde{m}_3=0,\ \tilde{m}_4,\ \tilde{m}_1$, generic	$SU(2)\times U(1)\times SU(2)\times U(1)$	$SO(4)\times U(1)\times U(1)$
$\ldots$	$\ldots$	$\ldots$	$\ldots$

Solutions similar to the previous case but: $1+1+6$ in the $m_i \to m$

To recapitulate:

- Mass perturbation of the EHIY (SCFT) singularity :
  the resolution of the Tchebyshev vacua into the sum of the r-vacua:
  (local Lagrangian theories with $SU(r) \times U(1)^{N-r}$ gauge symmetry)

  - Correct identification of the N=1 vacua surviving $\mu \, \Phi^2$ perturbation

  - But physics was unclear (strongly-coupled monopoles and dyons) in $m \to 0$ limit

- But **we have now** checked that the singular EHIY (SCFT) theory is correctly described by GST duals after $\mu \, \Phi^2$ perturbation.

☞

- The limit $m \to 0$ can be taken smoothly in the GST description
  (cfr. the usual monopole picture)

Physics of USp(2N), $N_F = 4$ theory at m=0

$$\sqrt{2}\, Q_0 (A_D + m_0)\tilde{Q}^0 + \sqrt{2}\, Q_0 \phi \tilde{Q}^0 + \sum_{i=1}^{4} \sqrt{2}\, Q_i \phi \tilde{Q}^i + \mu A_D \Lambda + \mu\, \mathrm{Tr}\phi^2 \ .$$

➜

$$Q_0 = \tilde{Q}_0 = \begin{pmatrix} 2^{-1/4}\sqrt{-\mu\Lambda} \\ 0 \end{pmatrix}$$

$$(Q_1)^1 = (\tilde{Q}^1)_1 = 2^{-1/4}\sqrt{\mu\Lambda}\,, \qquad Q_i = \tilde{Q}_i = 0, \quad i = 2, 3, 4.$$

$$\phi = 0, \quad A_D = 0\ .$$

➜ XSB

$$SO(8) \to U(1) \times SO(6) = U(1) \times SU(4) = U(4),$$

OK with results at $\mu, m \gg \Lambda$

➜ Confinement

UV: $\Pi_1(USp(2N)) = \mathbf{1}$

IR: $\mathcal{L}_{GST} = SU(2) \times U(1), \qquad \Pi_1(SU(2) \times U(1)) = \mathbf{Z}$

Higgsed at low energies; the vortex = the unique (N=n) confining string

Cfr. Abelianization implies $U(1)^N$ low energy theory → multiplication of the meson spectrum

The confining string is Abelian cfr. non-Abelian vortex of r-vacua

Conclusion

- Analysis of the colliding r vacua of SU(N) (at m → m^{cr} ~ Λ) similar

  (SU(3) or SU(4) w/ N_F =4, worked out)

Giacomelli, Konishi
in preparation

- Cases with general N_F (the A sector is non Lagrangian SCFT) to understand

- Gaiotto-Seiberg-Tachikawa duals of singular, IRFP SCFT

allows to describe (confining) systems whose players are infinitely-strongly coupled monopoles and dyons

\- SCGT -

➔ New confinement phase in SQCD

Q C D ?

The End

Colliding r vacua of SU(3), N_F =4 theory

The GST dual is now:

$$D_3 - SU(2) - \boxed{3}$$

where D_3 is the most singular SCFT of the $\mathcal{N}=2$ SU(2), N_F =2, theory, and 3 is three free doublets of SU(2). D_3 is a nonlocal theory,* it not easy to analyze.

We replace the system by

$$SU(2) \overset{P}{-} SU(2) - \boxed{3}$$

* arises from the collision of a doublet vacuum and a singlet vacuum

where P is a bifundamental field. The superpotential is:

$$\sum_{i=1}^{3} \sqrt{2} Q_i \Phi \tilde{Q}^i + \sum_{i=1}^{3} \tilde{m}_i \, Q_i \tilde{Q}^i + \mu \Phi^2 + \sqrt{2} P \Phi \tilde{P} + \sqrt{2} \tilde{P} \chi P + \mu \, \chi^2 + m^{'} \, \tilde{P} P,$$

The first SU(2) is AF: its dynamics is not affected by the second SU(2). But to exract the D_3 point, need to keep $m' \simeq \pm \Lambda^{'}$, but not exactly equal. The system Abelianizes →

Doublet vacuum (of the new, strong SU(2) N_F =2 theory)

$$\sum_{i=1}^{3} \sqrt{2} Q_i \Phi \tilde{Q}^i + \sum_{i=1}^{3} \tilde{m}_i \, Q_i \tilde{Q}^i + \mu \Phi^2 + \sqrt{2} M \Phi \tilde{M} + \sqrt{2} \tilde{M} A_\chi M + \mu \, A_\chi \Lambda^{'},$$

with m

$$\begin{aligned} \tilde{m}_1 &= \frac{1}{4}(m_1 + m_2 - m_3 - m_4) \; ; \\ \tilde{m}_2 &= \frac{1}{4}(m_1 - m_2 + m_3 - m_4) \; ; \\ \tilde{m}_3 &= \frac{1}{4}(m_1 - m_2 - m_3 + m_4) \; , \end{aligned}$$

→ correct symmetry

for all m_i :

→ six solutions (r=2 vacua)

SU(3), N_F =4 theory has r=0,1,2 vacua: where are the r=0,1 vacua? Answer:

Singlet vacuum (of the new, strong SU(2) N_F =2 theory)

$$\sum_{i=1}^{3} \sqrt{2} Q_i \Phi \tilde{Q}^i + \sum_{i=1}^{3} \tilde{m}_i \, Q_i \tilde{Q}^i + \mu \Phi^2 + \sqrt{2} \tilde{N} A N + \mu \, A \Lambda^{'} + m^{'} \, \tilde{N} N.$$

AF: becomes strongly coupled. → 4 + 1 vacua of

SW SU(2) N_F =3 theory! → r=1 (4) and r=0 (1) vacua

Vacuum structure OK

* Remarks

Tachikawa, Wecht '09

- N=2 SCFT's in UV flow into N=1 SCFT, upon N=1, $\mu\,\Phi^2$ perturbation (27/32)

- Some of them survive and brought into confinement phase; r-vacua, (r=0,1,2,...) upon N=1, $\mu\,\Phi^2$ perturbation

- Not all singular N=2 SCFT's survives N=1, $\mu\,\Phi^2$ perturbation (e.g., Aygyres-Douglas point in pure SU(3))

- N=2 IRFP SCFT's can survive and brought into confinement phase; e.g., Colliding r-vacua of SU(N) theories, m=0, USp(2N) theory (Tchebyshev vacua); m=0, SO(N) theory

- They are strongly interacting, nonlocal theory of monopoles and dyons, in confinement phase: interesting system to understand!!

r-vacua

$$y^2 = \prod_a^N (x - \phi_a)^2 - \Lambda^{2N-2n} \prod_{i=1}^{2n} (x + m_i)$$

$$\phi_a = (-m_1, -m_2, \ldots, -m_r, \phi_{r+1}, \ldots)$$

$m_i \to m,$

$$y^2 = (x+m)^{2r} \prod_{b=1}^{N-r} (x - \alpha_b)^2 \, (x - \gamma)(x - \delta)$$

This describes SU(r) x U(1)$^{N-r}$ theory