Generalized Skyrmions
and
Mass of the Lightest Electroweak Baryon


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SC theories generically exhibit SSB

$\to$ Soliton solutions in low-E L_eff

prototype: QCD & Skyrme model

but:

- Skyrme non-unique &
  many generalizations possible
- relevant strong dynamics might
  be very different from QCD

With discovery of Higgs candidate @LHC, models of strongly-interacting EW symm. breaking especially relevant, to distinguish between possible scenarios

For example, simplest possibility that it is a pseudo-dilaton of some nearly conformal strongly interacting EW sector

Existence or non-existence of soliton solutions may be a valuable diagnostic tool for discriminating between EW symmetry breaking scenarios

Low-E chiral Lagrangians:
soliton masses & other properties depend on higher order terms in derivative expansion, in particular 4-th order terms

Quadratic terms: universal

@ 4-th order:
Minimal L_eff with SU(2) x SU(2) →SU(2)
(both QCD and EW SSB):

2 possible terms: [,]^2 & { , }^2

[,]^2 ≡ Skyrme term

$$L = \frac{1}{16} F_\pi^2 \ \mathrm{Tr}\,(\partial_\mu U \partial_\mu U^\dagger) + \frac{1}{32 e^2} \mathrm{Tr}\,[(\partial_\mu U) U^\dagger, (\partial_\nu U) U^\dagger]^2$$

→ skyrmion phenomenology

What about beyond 4-th order?

contribution of higher order terms
to mass not parametrically suppressed

but no chance for exp info
in foreseeable future $\rightarrow$ do what you can

$\sim$20-30% phenomenology in QCD
with Skyrme term only

with both [,] and {,} soliton mass > m_N
so truncation hopefully OK for upper limits

next step: semiclassical quantization

- in QCD contrib. to mass 1/N_c supressed (~8% of nucleon mass)

- applicable to many models of EWSB, but need to explore case-by-case

→ study the mass in classical approximation only; interested in approximate bounds

Existence/absence of stable solitons depends on ratio of the two 4-th order coefficients:

- generic range w/o stable solution

- generic range with stable solution (within spherically stable config.) Original Skyrmion belongs here.
- in QCD stable solution range favored by large N_c & phenomenology

EWSB: very little known about possible 4-th order coeffs.

ratio very could be quite unlike Skyrme, possibly with no stable solitons

However, if EWSB due to underlying constituents that form “EW baryons”, expect masses and other properties approx. described by solitons, even if very different from baryons & skyrmions in QCD

Classical Mass and Stability of an SU(2) Soliton

$$\mathcal{L}_{\rm eff} = \mathcal{L}_2 + \mathcal{L}_4$$

$$U(x) = \exp\left(i \frac{\vec{\tau}\cdot\vec{\pi}(x)}{v} \right)$$

$$\mathcal{L}_2 = \frac{v^2}{4} {\rm Tr}\left(\partial_\mu U \partial^\mu U^\dagger \right)$$

$v = F_\pi = 93$ MeV in QCD, $v \sim 246$ GeV in EW

$$\mathcal{L}_4 = 2s\,{\rm Tr}\left[(R_\mu R_\nu)(R^\mu R^\nu) - (R_\mu R^\mu)^2 \right] + 2t\,{\rm Tr}\left[(R_\mu R_\nu)(R^\mu R^\nu) + (R_\mu R^\mu)^2 \right] ,$$

$$R_\mu = \partial_\mu U\, U^\dagger$$

truncation at 4-th order could be reliable at energies below characteristic strong interaction scale,
~ 1 GeV in QCD, ~ TeV in EW

parameters s and t:

- in principle calculable from underlying theory

  and/or

- phenomenologically extracted from data on scattering of Nambu-Goldstone bosons or massive gauge bosons in large N_c QCD: |t| << |s|

$\rightarrow$ Skyrme-like.

s $\equiv$ "Skyrme term"

t $\equiv$ "non-Skyrme term"

Skyrme: $B = \frac{1}{24\pi^2}\int d^3x \epsilon^{ijk} \mathrm{Tr}\left[(U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U)\right]$

B = baryon number

spherically-symmetric 'hedgehog' Ansatz for a static field configuration:

$$U(\vec{r}) = \exp\left(i\frac{\vec{\tau}\cdot\vec{r}}{r}P(r)\right), \qquad P(0) = \pi,\ P(\infty) = 0\,.$$

Contributions to mass

2-derivative term to always positive:

$M2 > 0$

virial theorem:

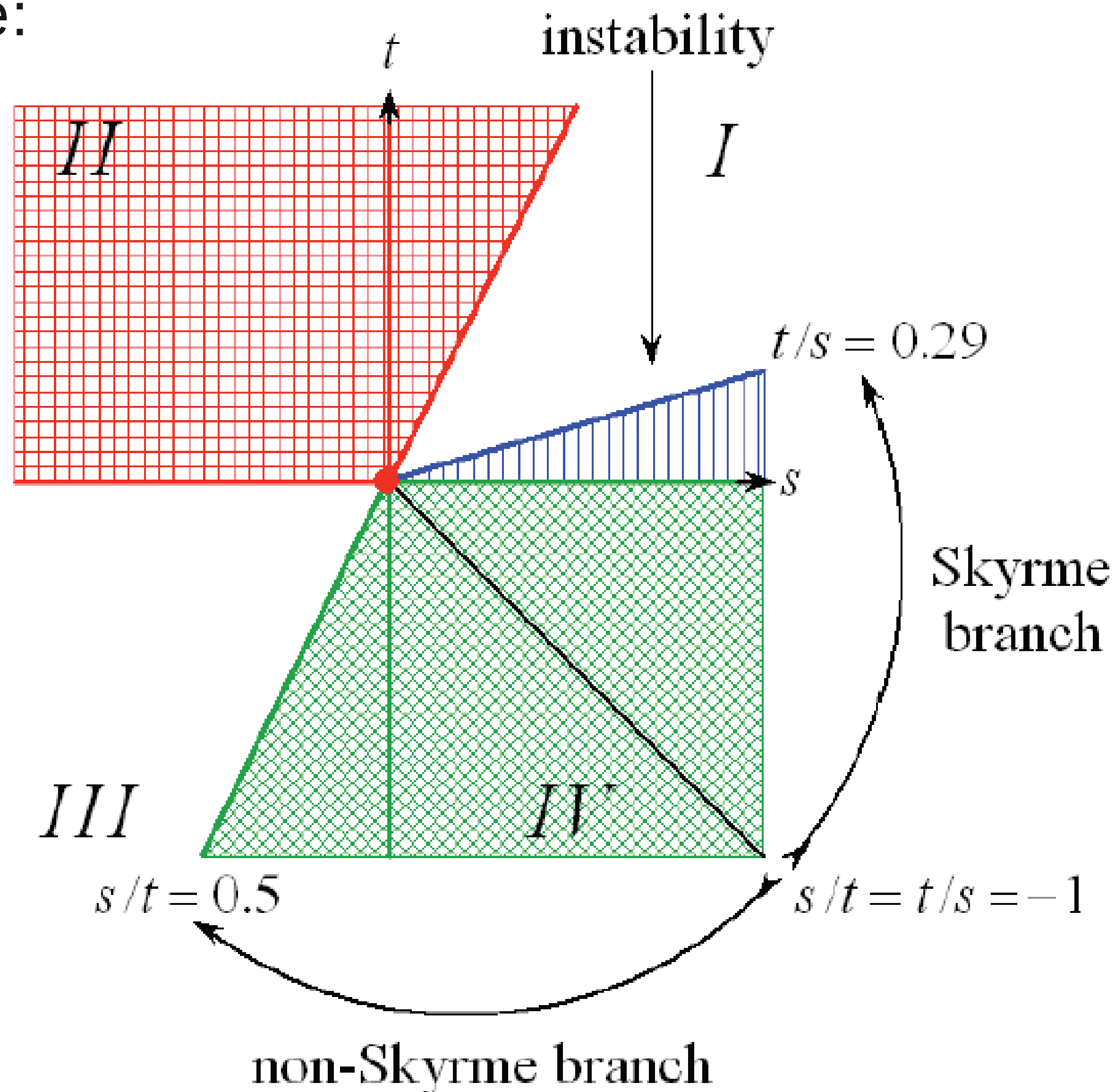
at the solution 4-derivative contribution equals

2-derivative contribution,

$M4 = M2$

$\rightarrow$ $M4 > 0$ is a condition for existence of stable solution regions in (s,t) plane:

M4 > 0
M4 < 0
metastable
(∃ M4>0 solutions but no positivity bound, so likely non-spherical unstable modes)



soliton mass as function of t/s: Skyrme and non-Skyrme branches

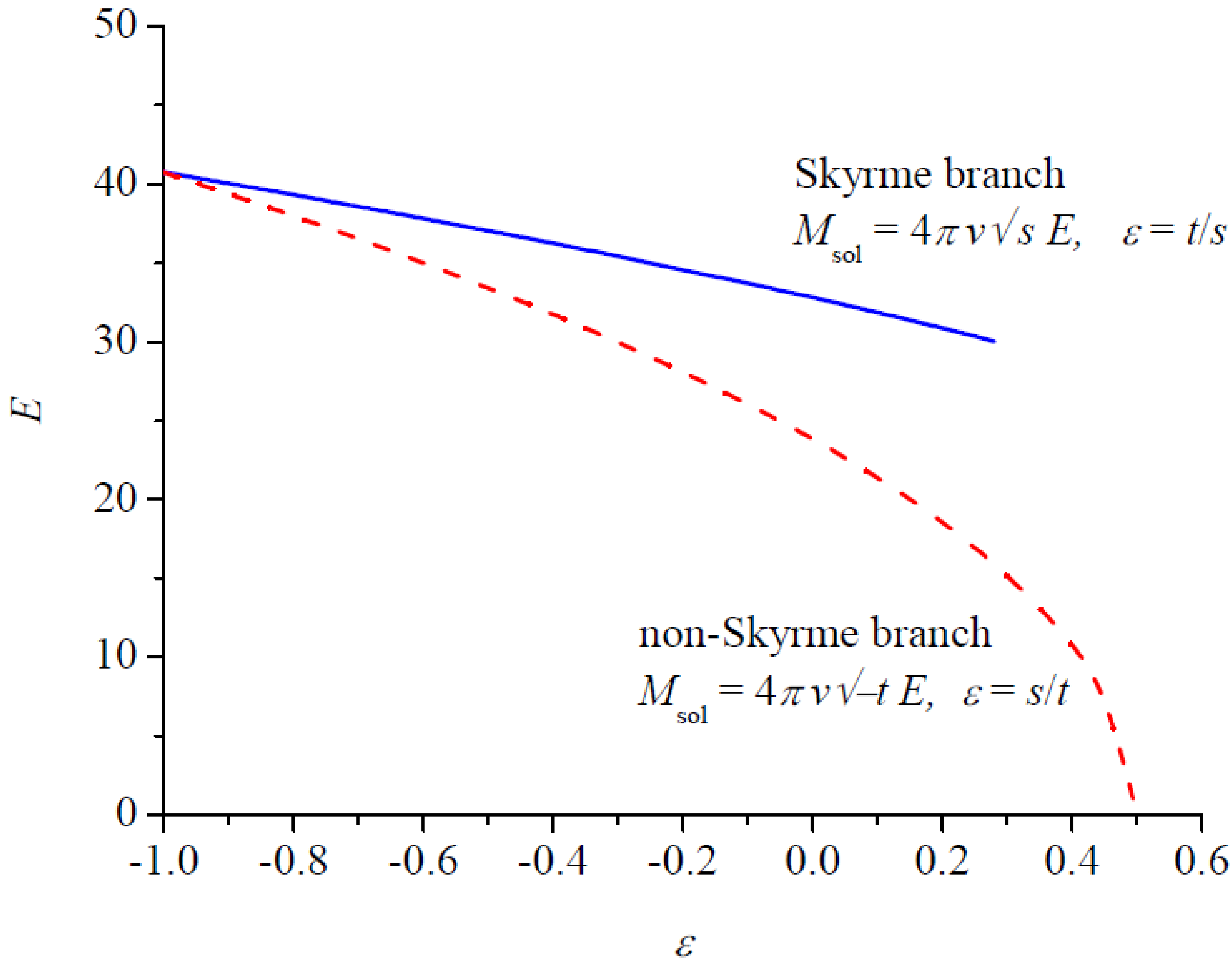
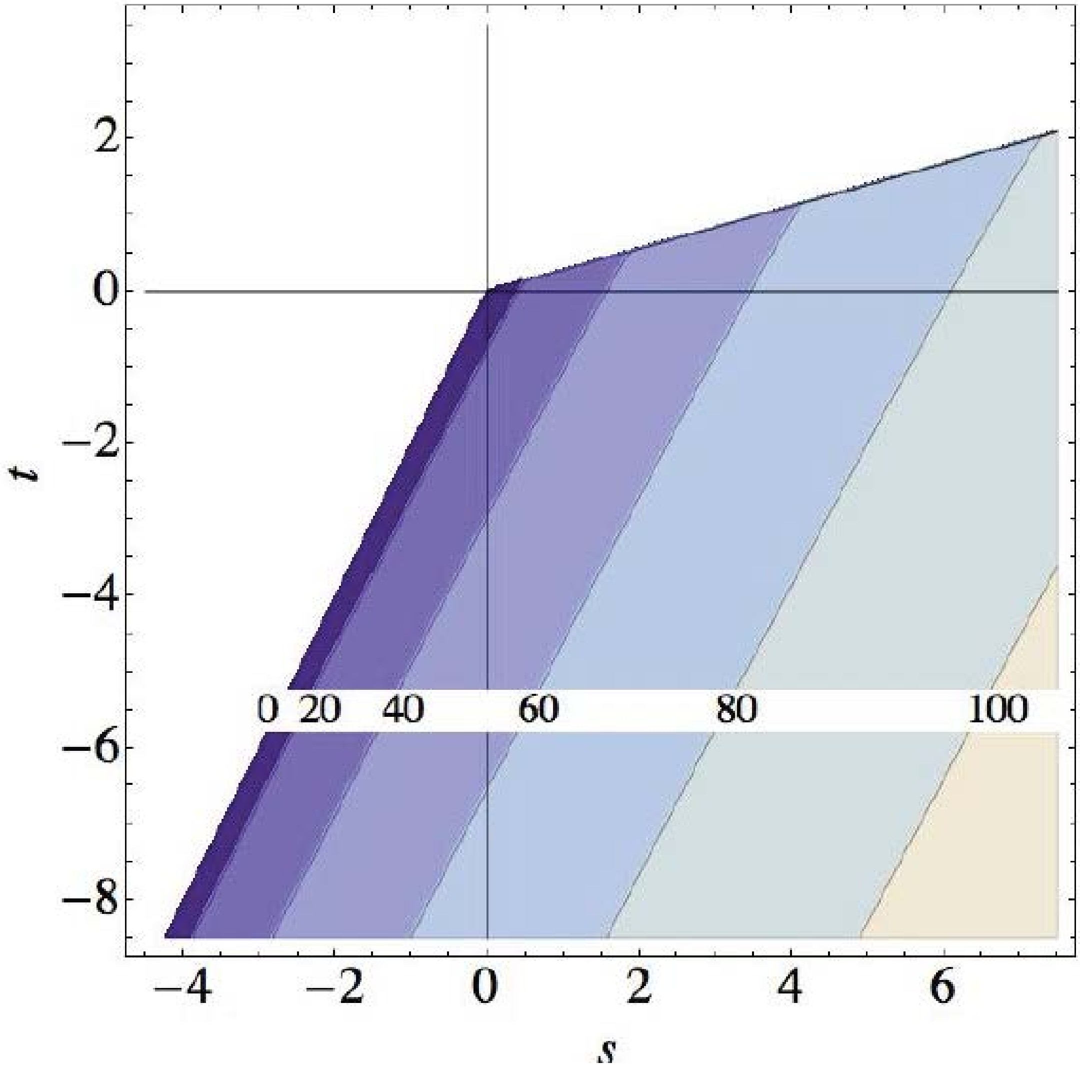


Figure 2: *The soliton mass in units of $4\pi v\sqrt{s}$ for the Skyrme branch where $\epsilon = t/s$ (solid blue line), and for the the non-Skyrme branch in units of $4\pi v\sqrt{-t}$ where $\epsilon = s/t$ (dashed red line). The Skyrme branch ends at $t/s \sim 0.29$ (see Fig. 1).*

Contour plot of soliton mass M(s,t)



Contour plot of the mass of the generalized Skyrmion in units of $4\pi v$.

Phenomenological Estimates of Soliton Masses

Phenomenological constraints on higher-order Lagrangian parameters are often given in terms of the coefficients $\alpha_{4,5}$ that are related to the parameters s and t

$$s = \frac{\alpha_4 - \alpha_5}{4}, \; t = \frac{\alpha_4 + \alpha_5}{4} .$$

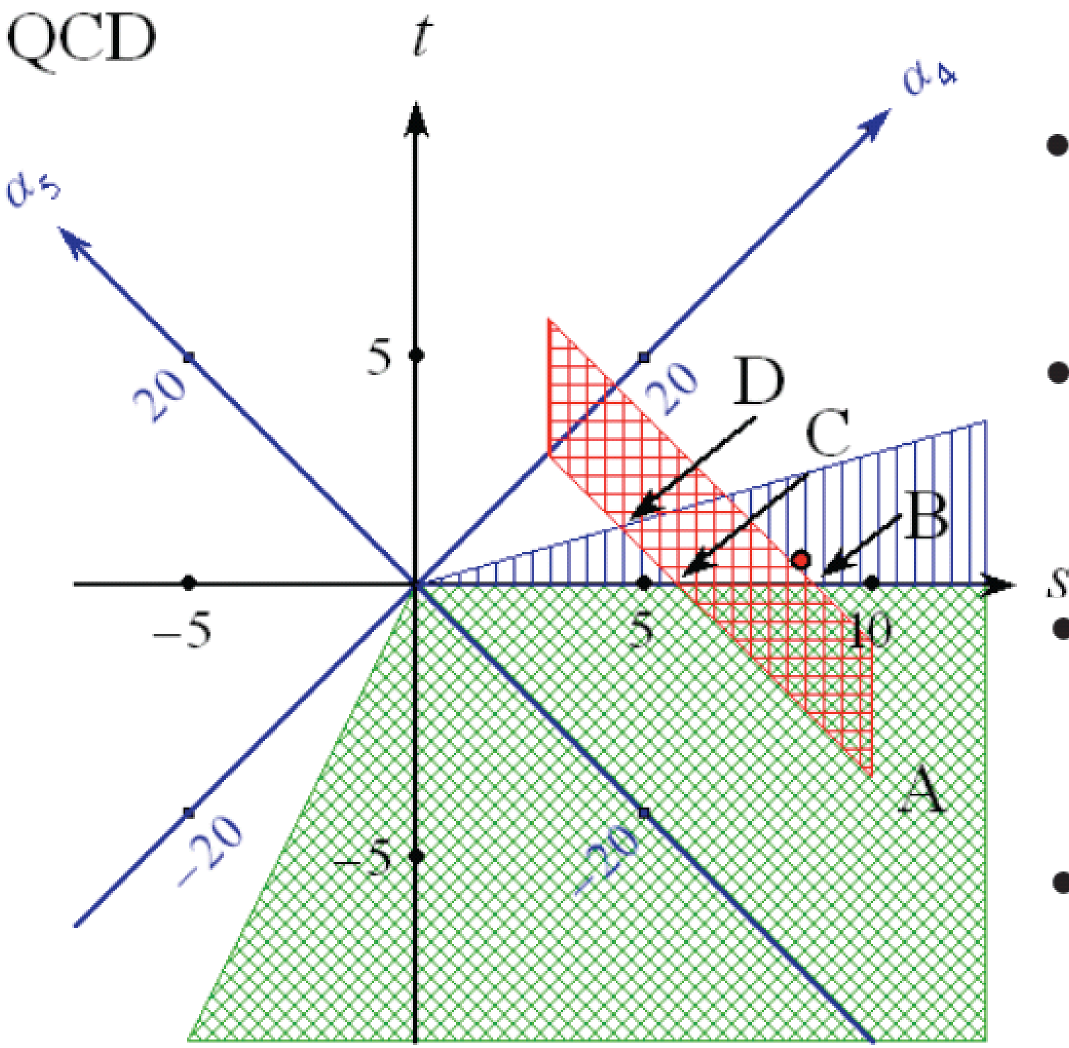
warm up exercise: QCD, comparing non-Skyrmion $t \neq 0$ with conventional Skyrmion $t=0$

from low E scattering data [red hashed area in the plot]

$$11 \times 10^{-4} < \quad \alpha_4 \quad < 17 \times 10^{-4} ,$$
$$14 \times 10^{-4} < \alpha_4 - \alpha_5 < 40 \times 10^{-4} .$$

To be compared with large N_c predictions for chiral $\mathrm{SU}(3) \times \mathrm{SU}(3) \to \mathrm{SU}(3)$ [red dot]:

$$\alpha_4 = 18 \times 10^{-4}\,,$$
$$\alpha_5 = -16 \times 10^{-4}\,.$$



- A : $(s,t) = (10, -4.5) \times 10^{-4}$ (maximal mass point overall):
- B : $(s,t) = (8.5, 0.0) \times 10^{-4}$ (maximal mass Skyrme point): $M_{\mathrm{B}} \simeq 1118$ MeV;
- C : $(s,t) = (5.5, 0.0) \times 10^{-4}$ (minimal mass Skyrme point): $M_{\mathrm{C}} \simeq 900$ MeV.
- D : $(s,t) = (4.3, 1.2) \times 10^{-4}$ (minimal mass point overall): $M_{\mathrm{D}} \simeq 728$ MeV.

To be compared with large N_c predictions for chiral $\mathrm{SU}(3) \times \mathrm{SU}(3) \rightarrow \mathrm{SU}(3)$ [red dot]:

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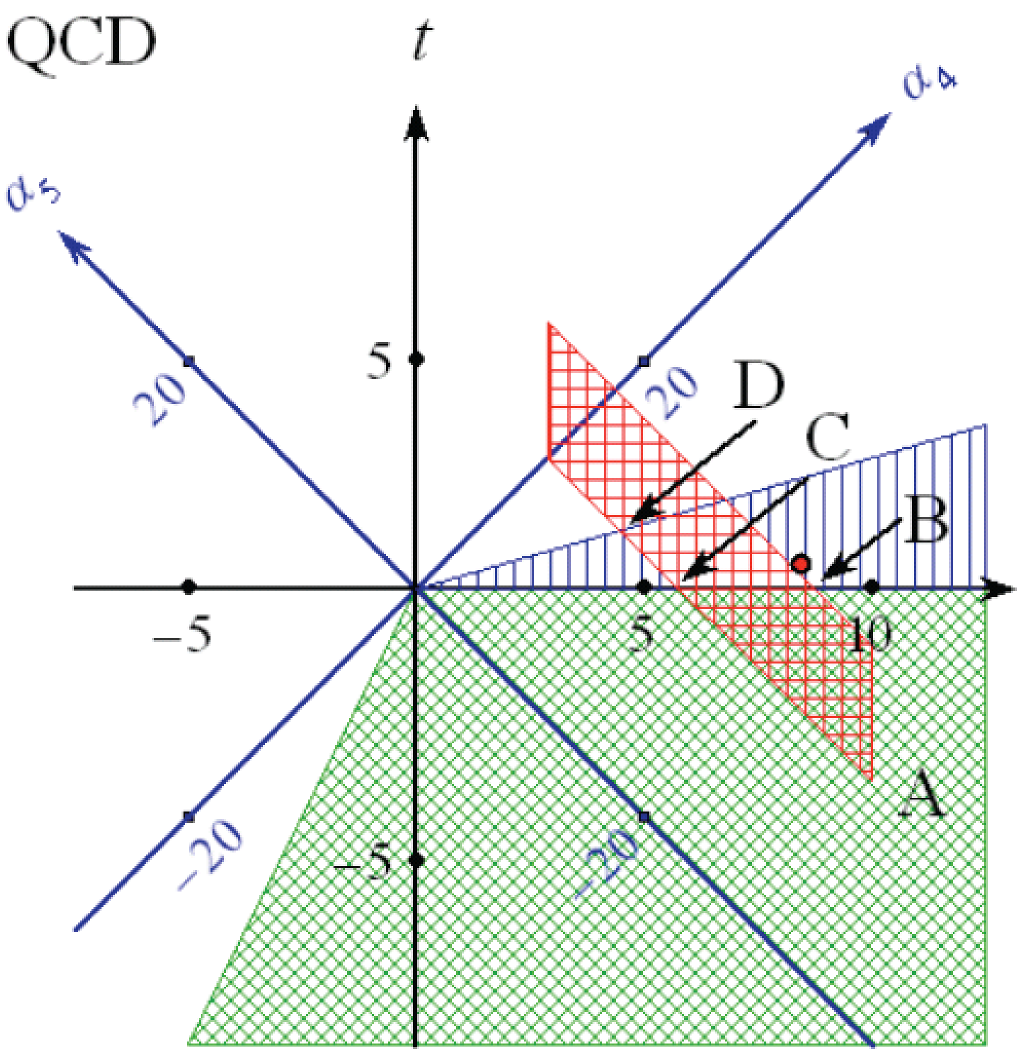


- $\mathrm{A} : (s, t) = (10, -4.5) \times 10^{-4}$ (maximal mass point overall): $M_{\mathrm{A}} \simeq 1354$ MeV;
- $\mathrm{B} : (s, t) = (8.5, 0.0) \times 10^{-4}$ (maximal mass Skyrme point): $M_{\mathrm{B}} \simeq 1118$ MeV;
- $\mathrm{C} : (s, t) = (5.5, 0.0) \times 10^{-4}$ (minimal mass Skyrme point): $M_{\mathrm{C}} \simeq$ [illegible]00 MeV.
- $\mathrm{D} : (s, t) = (4.3, 1.2) \times 10^{-4}$ (minimal mass point overall): $M_{\mathrm{D}} \simeq 728$ MeV.

1 GeV +-30%

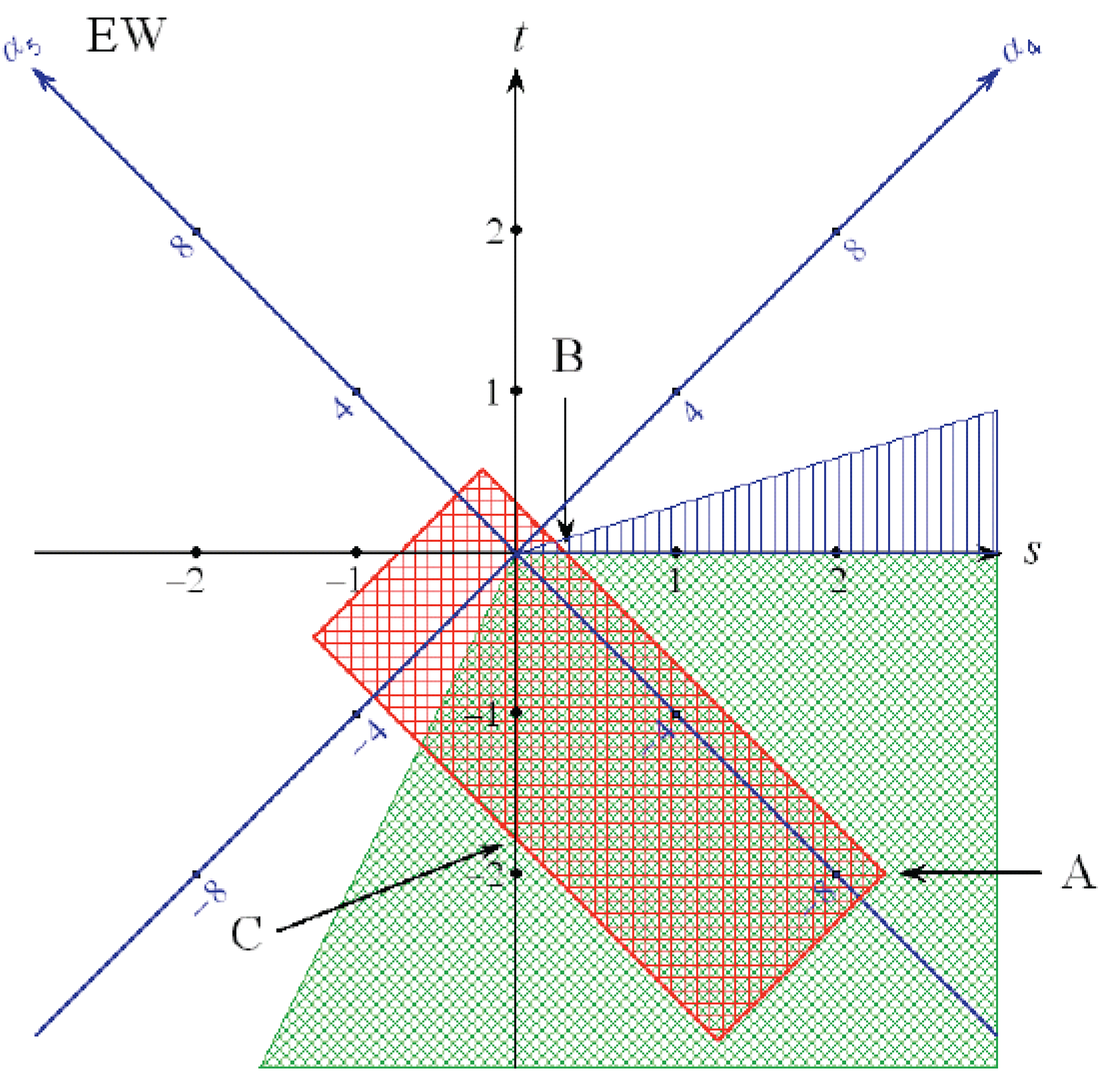
After the QCD warm-up, can get down to business

Current bounds on electroweak baryon masses

existing constraints on higher-order coeffs in L_eff of EW

$$-3.5 \times 10^{-1} < \alpha_4 < 0.6 \times 10^{-1}\,,$$
$$-8.7 \times 10^{-1} < \alpha_5 < 1.5 \times 10^{-1}\,.$$

Current bounds on EW baryon mass



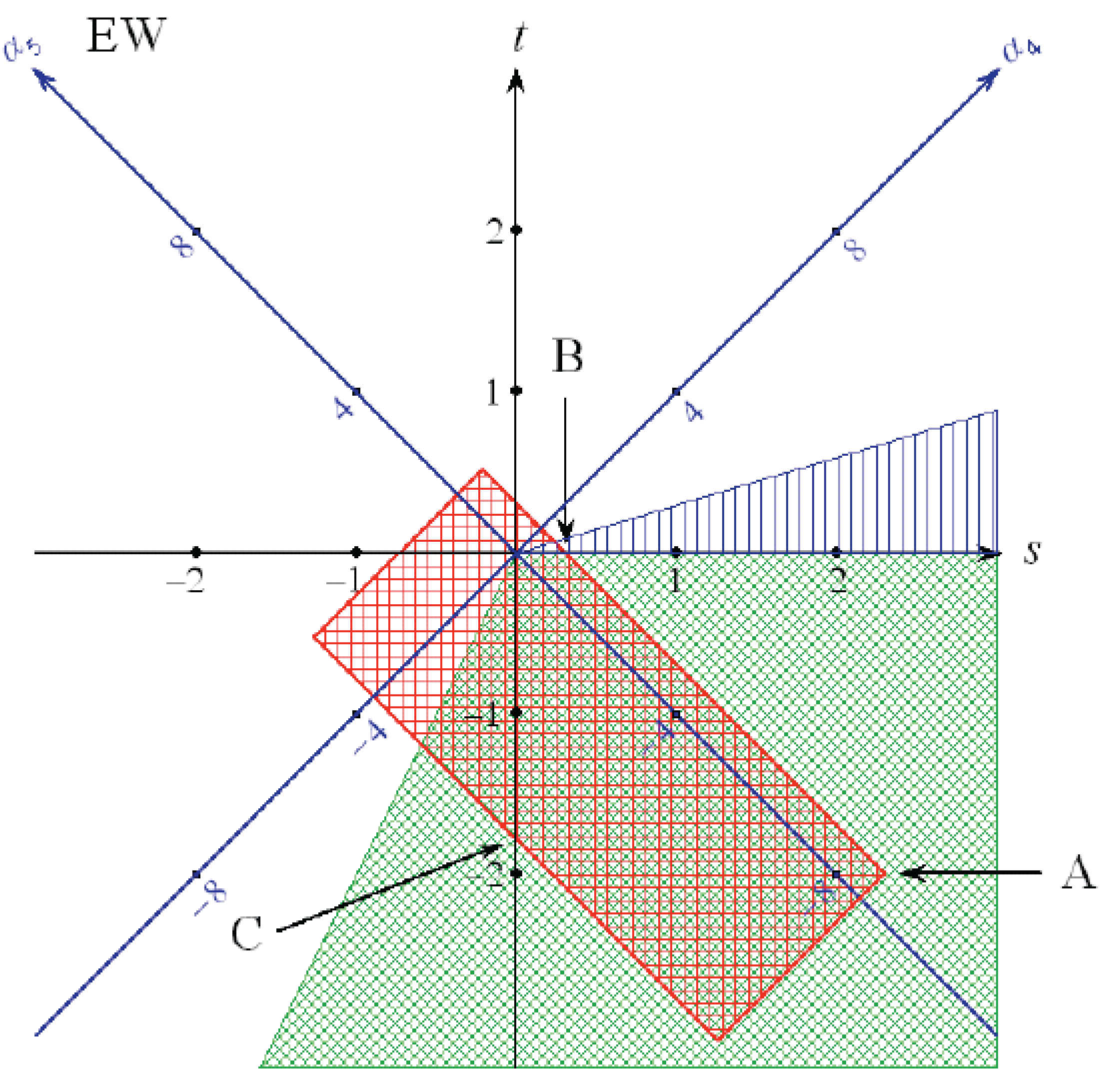
no lower bound, as constraints include t=2s <0 where M=0

A(0.23,-0.20): max overall, M≈59 TeV

B(0.03,-0.0): max Skyrme, M≈18 TeV

C(0.0,-0.175): max with s=0, M≈31 TeV

Current bounds on EW baryon mass



no lower bound, as constraints include t=2s <0 where M=0

A(0.23,-0.20): max overall, M≈59 TeV

B(0.03,-0.0): max Skyrme, M≈18 TeV if QCD-like

C(0.0,-0.175): max with s=0, M≈31 TeV

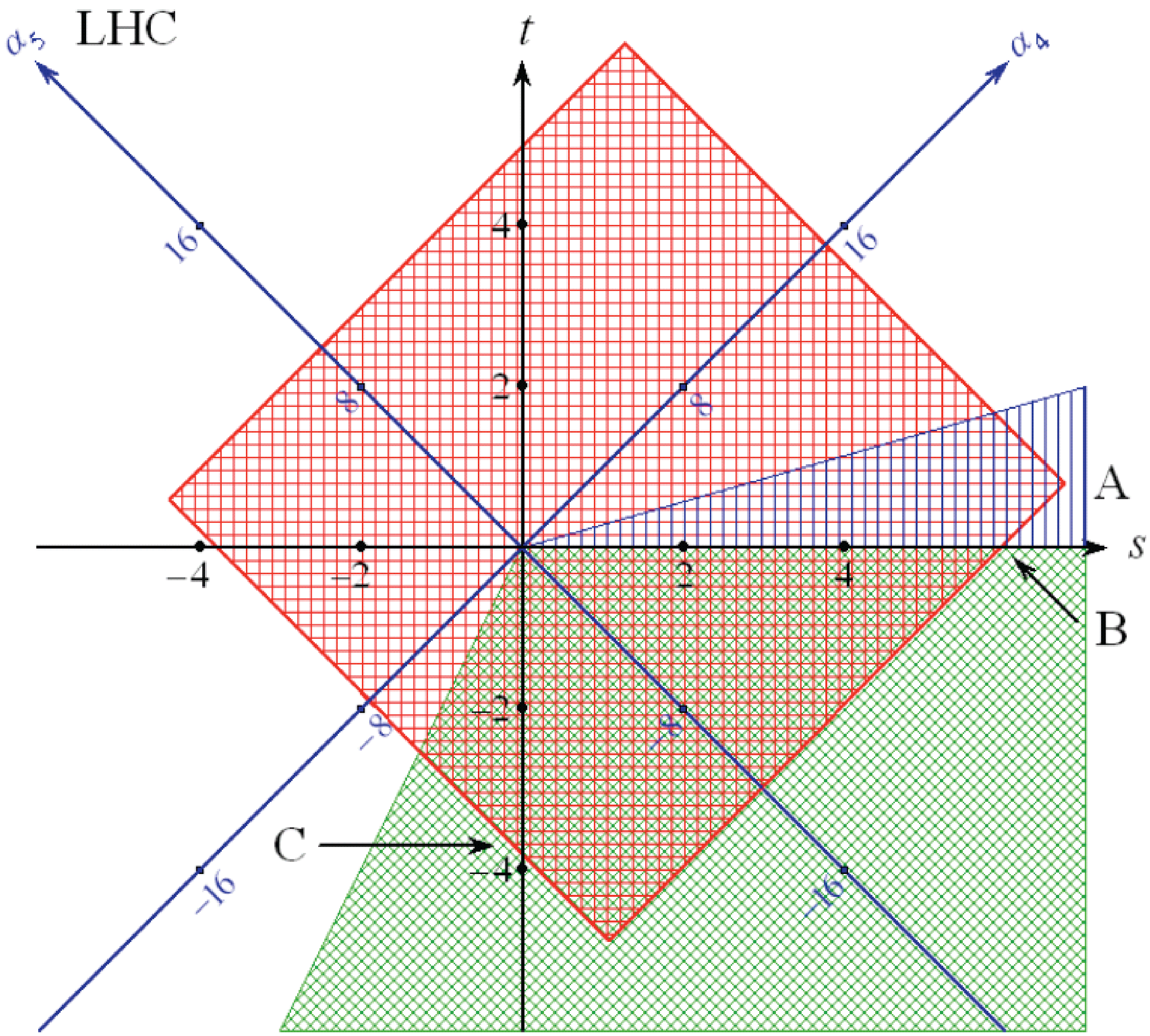
Prospective LHC bounds on EW baryon masses

estimate of LHC sensitivity

$$-7.7 \times 10^{-3} < \alpha_4 < 15 \times 10^{-3} \,,$$
$$-12 \times 10^{-3} < \alpha_5 < 10 \times 10^{-3} \,.$$

(Eboli, Gonzales-Garcia, Mizukoshi 2006)

Prospective LHC bounds on EW baryon mass

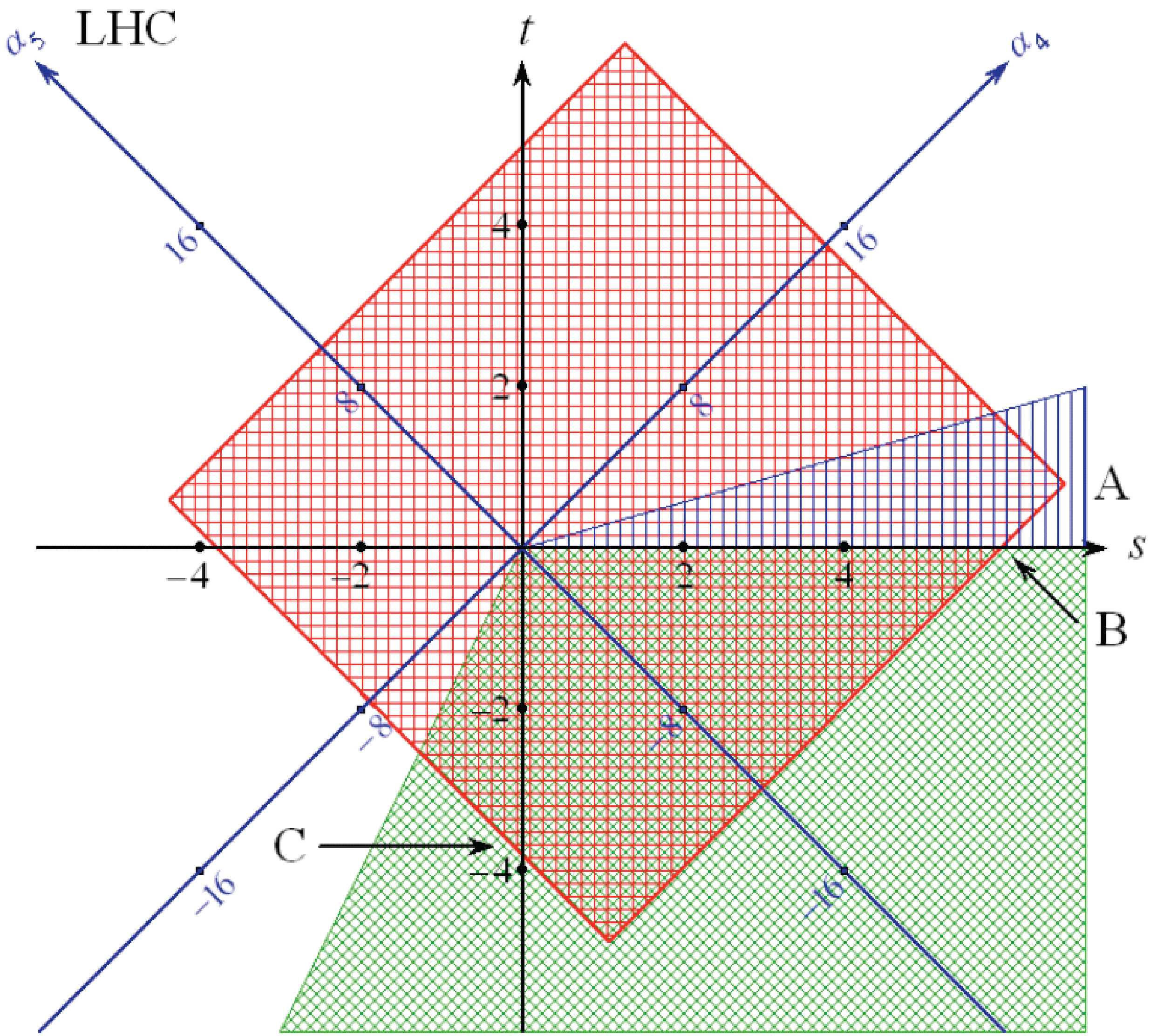


A: (6.75,0.75)x10^{-3}
max overall,
M≈8.1 TeV

B: (6.0,0.0)x10^{-3}
max Skyrme,
M≈7.9 TeV

C: (0.0,-3.85)x10^{-3}
max with s=0,
M≈4.6 TeV

Prospective LHC bounds on EW baryon mass



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max with s=0,
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LHC will *either* measure nonzero L4 *or* put these bounds on EW baryon mass

Electroweak Baryons as Dark Matter?

If EW baryons exist, should be present in the Universe today $\rightarrow$ possible cold dark matter (CDM) candidates (Nussinov)

Relic density depends on primordial EW baryon asymmetry. If small, would be wiped out, so would need other CDM.

If EW baryons do make bulk of CDM, must be electrically neutral.

Cannot be fermions, as would have too large x-section through magnetic moment couplings (Bagnasco, Dine & Thomas, 1993)

Requirement that EW baryon be a non-charged boson:

$\rightarrow$ apparent problem for models where topological analysis (WWZ) of L_eff yields solitons which are charged and/or fermions (Gillioz 2012)

e.g. SU(3) x SU(3) $\rightarrow$ SU(3): neutral fermion for N_c=3

and SU(N) $\rightarrow$ SO(N): boson with charge N_c; etc.

even if EW baryon is a neutral boson, additional problems from its DM scattering (tension with XENON100 for M > 1 TeV)

(Campbell, Ellis & Olive 2012)

Our analysis suggest:

such models should not necessarily be abandoned

This is because the topological analysis yields only the quantum numbers of the soliton and says nothing about its dynamical stability

the parameters of L_eff might be in the range where no stable baryonic soliton exists, i.e.

$t > 0$ or $t/s > 2$

Conclusions

- Analysis of existence, stability and masses of classical soliton solutions of L_eff of QCD and possible strongly interacting EW sector
- in particular, consequences of non-Skyrme quartic term
- stability and masses in (s,t) plane
- current bounds: M ~< 18÷59 TeV
- prospective LHC bounds: M ~< 5-8 TeV
- much higher precision on L4 from LHC extremely useful
- interesting interplay between dark matter and strongly interacting EW with soliton solutions